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PARTY SIZE BASELINES IMPOSED BY
INSTITUTIONAL CONSTRAINTS

THEORY FOR SIMPLE ELECTORAL SYSTEMS

Rein Taagepera

ABSTRACT

This theory specifies the party sizes expected on the basis of constraints imposed by simple electoral rules. Duverger’s law and hypothesis states that under the single-member plurality rule third parties tend to be eliminated, while proportional representation in multi-seat districts enables more than two parties to thrive. Expanding on Duverger’ statements, the seat and vote shares of parties at all size ranks are calculated here, using nothing but two institutional inputs (district magnitude and assembly size) plus the number of voters. These are the baseline values expected if (and only if) institutional constraints were the only factor, with other factors balancing themselves out. The theory is complete in the sense of leading to complete party size structure. Among the relatively simple electoral systems the institutional baselines do reflect the long-term averages in New Zealand, while the residuals indicate the imbalance of inputs by other factors in the case of The Netherlands and Finland, and especially in the case of the UK.

KEY WORDS • Duverger’s law • institutional constraints • reasoning by boundary conditions • simple electoral systems • size of parties

Seat and vote shares of parties emerge from sociopolitical factors that play themselves out in the framework of institutional constraints. The sociopolitical factors impinge first on the votes, and the vote shares are then translated into seat shares through the sieve of electoral rules. Institutional constraints, once established, follow the opposite direction: they first impinge on the seats and only later may they affect the votes. Thus single-member districts constrain the number of seat-winning parties within the district to be no more than one. Indirectly, this may also reduce the number of parties that run and receive votes. In the case of single-member districts with plurality allocation rule the well-known Duverger law results (Duverger, 1951, 1954): only two parties tend to receive appreciable shares of the votes.

Time frames matter. Party vote and seat shares in individual elections fluctuate. For long-term averages, however, the impact of current sociopolitical

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factors is hemmed in by institutional constraints (which are themselves grounded in sociopolitical history). But what about worldwide averages? With even long-term country-specific factors evened out, could we specify the pure effect of institutional constraints? If this were possible, we would have a universal institutional baseline. We could then specify to what extent an imbalance of sociopolitical factors in a given country tilts party sizes away from expectations based on the institutions a country has adopted.

The objective of the present study is to follow the impact of certain basic institutional constraints to such an extreme; i.e. to establish a logical model that specifies the seat and vote shares for first-, second- and lower-ranking parties that would be expected when the impact of all other factors is unknown. It will be seen that this can be done when the electoral rules are simple, with all seats allocated within districts. The only input variables are district magnitude $M$ (the number of seats allocated within the district), assembly size $S$ (the total number of assembly seats) and the number of voters $V$. No empirical parameters based on electoral data are involved. What we have here is an a priori logical model, not a post hoc ‘empirical model’.

The theory is ‘complete’ in the limited sense of yielding specific seat and vote share values for the complete set of parties, ranked by size. By so doing it is more specific than Duverger’s law and the companion Duverger’s hypothesis (see Riker, 1982) that states that in the case of allocation by proportional representation (PR) more than two parties obtain appreciable shares of seats and votes. The actual outcomes of individual elections are not expected to fit the institutional model. But the average seat and vote shares should bear some resemblance to the model if the impact of institutional constraints has been adequately estimated. In particular, when considering several countries, under- and overestimates should occur to a roughly equal degree.

The importance of this approach consists in disentangling the effects of current sociopolitical factors from those of basic institutional constraints. The residuals (the differences between the actual and estimated values) reflect the imbalance in impact of some sociopolitical factors and processes in the given country (including institutional factors not included in the model). Political profiles for various countries may be constructed on the basis of these residuals.

The core of the present article consists of presenting the theory. For the author, it represents the capstone of a lifelong quest: All other factors being equal, exactly how do institutions affect the number and size of parties? As a limited reality check, the results are compared to the actual seat and vote shares in four countries with simple but very different electoral rules. Within a single article one cannot do more. A more complete test must
await further studies where the validity of each link in the chain is evaluated separately.

**Institutional Constraints and Simple Electoral Systems**

The voters’ freedom to choose among parties can be constrained by electoral institutions. This is the broad message of Duverger’s law. For example, when the fifth-ranking party is completely driven out of existence, then the voters are no longer free to vote for it. Sartori (1968) implies the same message when he distinguishes between strong and feeble electoral systems. It is the distinction between systems that strongly or feebly constrain the voters’ choice.

The processes that tend to squeeze out smaller parties can be described in terms of the so-called Duverger mechanical and psychological effects. The direct ‘mechanical effect’ of electoral rules (changing vote shares into somewhat different seat shares) exerts feedback on the party system through what Duverger termed a long-term ‘psychological effect’ on party leaders and on voters. In strong electoral systems the smaller parties, underpaid in terms of seats (through mechanical effect), tend to lose votes (through psychological effect), so that the party system may eventually approach a two-party constellation.

**Extending Duverger’s Theory of Institutional Constraints**

What Duverger presented is a theory of institutional constraints on electoral and party systems. A basic institutional constraint is district magnitude \( M \). Duverger starts with \( M \) and then makes predictions about the functioning of the electoral rules and the nature of the resulting party system. But Duverger’s statements (law and hypothesis) are incomplete in that they do not fully answer the following question:

For a given district magnitude \( M \), what share of the votes and seats would one expect to go to the largest, the second-largest, third-largest etc. parties. on average?

Duverger only states that with single-member districts and plurality allocation rule (SMP) two parties obtain large shares, while with \( M > 1 \) PR more than two parties obtain appreciable shares. But how many of them, and how large a share? Sartori (1968) specifies that, with increasing \( M \), more parties gain access, but again without specific numbers given.

Extending Duverger’s and Sartori’s work, this study offers the first complete answer to this question: it explicitly calculates the share of the seats and votes of parties at all size ranks \( s_R \) and \( v_R \), with \( R \) indicating the rank
of a party by decreasing size). The results apply only as averages over many elections – and they do not specify the identity of the $R$th ranked party in any single election. These are the same limitations that apply to Duverger’s law.\footnote{I accept Duverger’s description of the process by which the outcome is reached, but I specify this outcome more explicitly. One must distinguish between the dynamic and static aspects. In order to calculate the static equilibrium state, one does not necessarily have to solve the much more complex problem of how this equilibrium is reached. Parts of the \textit{process} by which the Duvergerian outcomes (law and hypothesis) are reached are eminently political (the psychological effect), but the eventual \textit{outcomes} themselves depend much more on institutional constraints. The present model deals with the end results. It says nothing more than Duverger does about how long a learning time it takes before a modicum of equilibrium is reached and which processes are involved.}

\textit{Sample Results}

Table 1 gives a preview of the relationship between the institutionally based expectations and the actual average seat and vote shares in a relatively simple and durable system. Finland offers an unusually long time span (from 1907 to 1995) with the same fairly simple electoral rules, assembly size ($S = 200$) and average district magnitude ($M = 15$). For these 32 elections, the actual average vote and seat shares of the largest party (at any election)

<table>
<thead>
<tr>
<th>Party rank</th>
<th>Actual</th>
<th>Instit. theory</th>
<th>Residual</th>
<th>Actual</th>
<th>Instit. theory</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31.5</td>
<td>35.1</td>
<td>−3.6</td>
<td>33.5</td>
<td>36.8</td>
<td>−3.3</td>
</tr>
<tr>
<td>2</td>
<td>22.4</td>
<td>30.1</td>
<td>−7.7</td>
<td>24.0</td>
<td>31.2</td>
<td>−7.2</td>
</tr>
<tr>
<td>3</td>
<td>17.1</td>
<td>16.2</td>
<td>+0.9</td>
<td>17.2</td>
<td>16.0</td>
<td>+1.2</td>
</tr>
<tr>
<td>4</td>
<td>13.2</td>
<td>8.7</td>
<td>+4.5</td>
<td>12.8</td>
<td>8.2</td>
<td>+4.6</td>
</tr>
<tr>
<td>5</td>
<td>7.8</td>
<td>4.6</td>
<td>+3.2</td>
<td>7.2</td>
<td>4.1</td>
<td>+3.1</td>
</tr>
<tr>
<td>6</td>
<td>4.5</td>
<td>2.5</td>
<td>+2.0</td>
<td>3.5</td>
<td>2.1</td>
<td>+1.4</td>
</tr>
<tr>
<td>7</td>
<td>(1.7)</td>
<td>1.3</td>
<td>(+0.4)</td>
<td>1.1</td>
<td>1.1</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>(1.0)</td>
<td>0.9</td>
<td>(+0.1)</td>
<td>0.4</td>
<td>0.3</td>
<td>+0.1</td>
</tr>
<tr>
<td>9</td>
<td>(0.4)</td>
<td>0.4</td>
<td>(0.0)</td>
<td>0.2</td>
<td>0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>10</td>
<td>(0.2)</td>
<td>0.2</td>
<td>(0.0)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>11 up</td>
<td>[0.3]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td></td>
<td></td>
<td></td>
<td>6.9</td>
<td>7.4</td>
<td>−0.5</td>
</tr>
<tr>
<td>$N$</td>
<td>4.9</td>
<td>4.0</td>
<td>+0.9</td>
<td>4.5</td>
<td>3.7</td>
<td>+0.8</td>
</tr>
</tbody>
</table>

\textit{Note}: Parentheses indicate inclusion of ‘Other’ parties.

$p$ = number of seat-winning parties.

$N$ = effective number of parties.
were determined, and the process was repeated for the second-largest party, etc. Table 1 shows these empirical results in the ‘Actual’ columns. This is the average seats–votes pattern for Finland over 90 years, during which much has changed in the country’s political conditions.

The ‘Institutional Theory’ columns show the model-based expectations, where no other inputs besides $S = 200$ and $M = 15$ enter. The ‘Residual’ column represents the difference between the actual values and those based on institutional theory. This residual reflects the imbalance in political impact, challenging the institutional constraints. Politics in Finland seems to depress the seat and vote shares of the largest and especially the second-largest party, compared to institutionally based expectations. As a result, the effective number of parties ($N$) is larger than expected. Yet the total number of parties represented in the assembly ($p$, the number of seat-winning parties) is close to the expected value. On this basis, all the other usual indicators can be calculated, such as the advantage ratio for each party and indices of overall deviation from PR.

One may be relieved that the residuals leave room to play beyond a forced equilibrium of political factors. Yet, the expectations based on institutional constraints do reflect the actual shares (and number of parties) to a fair extent. Other countries with relatively simple systems will be considered later on, with various combinations of $S$ and $M$, after presenting the theory. But first the notion of simple electoral systems must be clarified.

**Simple Electoral Systems**

Duverger’s law and hypothesis only apply directly to simple electoral systems where all seats are allocated in districts of roughly equal magnitude according to some usual PR rule (such as d’Hondt or LR–Hare). When such rules are applied in single-member districts, they boil down to SMP, so that the plurality rule does not have to be specified separately. Here the characteristics of an ideal simple electoral system are specified.

The basic, inescapable variable in electoral systems is what Rae (1967) first called district magnitude, $M$. This is what makes the difference between

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2. The calculations will be presented in the Theory section. In principle, the number of voters also enters, but for $M >> 1$ its impact is negligible.

3. Of course, a zero residue does not imply the absence of political factors but only that they cancel each other out.

4. The effective number of parties (Laakso and Taagepera, 1979; see also Lijphart, 1994: 67–72) can be calculated both based on votes and on seats: $N_v = 1/\sum v_R$ and $N_s = 1/\sum s_R$. All summations ($\Sigma$) in this article range over all parties.

5. For advantage ratios ($A_N = s_{R}/v_{R}$), see Taagepera and Shugart (1989: 67–76). The most usual indices of overall deviation from PR are Loosemore and Hanby’s (1971) $D = \frac{1}{2} \sum (v_{R} - s_{R})$ and Gallagher’s (1991) $Gh = \left[\frac{1}{2} \sum (v_{R} - s_{R})^2\right]^{0.5}$ see Lijphart (1994: 58–62).
strong electoral systems (where usually \( M = 1 \)) and feeble ones (where \( M \gg 1 \) and some PR rule is used). In strong systems assembly size \((S)\) may also affect the outcome (see, e.g., Lijphart, 1994), because smaller assemblies allow for more drastic elimination of third parties. \( M \) can range from 1 (as in most Anglo-Saxon countries) to \( S \) (in cases like The Netherlands, \( S = 150 \), where all the seats are allocated by countrywide PR).

Actual electoral systems most often include further features, such as legal thresholds, seat allocation at several tiers, several rounds, primaries, ability of parties to form alliances and voters to rank parties rather than cast a ‘categorical’ ballot (Rae, 1967) for only one of them.\(^6\) Such features are optional, but one cannot describe an electoral system without specifying an assembly size and some value of \( M \) (explicit or implicit). The scope of the theory to be presented here is limited to the simplest conceivable electoral system. This ideal ‘simple electoral system’ is defined to have the following characteristics.

1. The \( S \) seats are allocated in districts that all have the same \( M \). (In contrast, almost all actual countries that use several multi-seat districts feature unequal \( M \).)
2. The ballot is categorical: voters cast a ballot for one single party (or candidate within a party).
3. Seats are fully allocated in the districts, according to a simple PR rule, such as d’Hondt or LR–Hare, with no transfer to a higher tier. In the case of \( M = 1 \), such rules boil down to SMP.
4. There are no legal thresholds, multiple rounds or other such thrills.
5. There are no US-type primaries, which are akin to a first round of general elections among candidates.
6. Parties are well defined and distinct: no electoral alliances are allowed and there are no subparts of parties competing against each other.

Like an ideal gas in physics, such a simple electoral system does not exist. Real gases and electoral systems include further complexities. No engineer would base practical calculations on the Boyle–Mariotte law of ideal gases alone. But the more complex practical forms start out with this law as the baseline on which other considerations (often empirical) are grafted. The objective here is to calculate the average seat and vote shares expected to result from such a simple electoral system, if it existed.

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\(^6\) For description of the variety of rules, see e.g. Reynolds and Reilly (1997) or Lijphart (1994).
Theory of Institutional Constraints

Figure 1 supplies the general road map – the overall flowchart of calculations. (Disregard for the moment the percentage figures.) We start with district magnitude \((M)\) and assembly size \((S)\). Their product \(MS\) leads to the expected number of seat-winning parties \((p)\), which in turn leads to the expected seat share of the largest party \((s_1)\). Separately, \(S\) determines the expected seat share of the smallest seat-winning party \((s_p)\). The expected seat shares of the intermediary parties are fitted in between these two anchor points. From the seat shares \((s_R)\) the effective number of legislative parties \((N_s)\) can be calculated. Using \(M, S\) and \(V\) and reversing the seat–vote equation (Taagepera, 1986) leads us from seat shares to vote shares \((v_R)\). Then the effective numbers of electoral parties \((N_v)\) can be calculated.

The Number of Seat-winning Parties

‘Seat-winning parties’ means parties that have at least one seat in the assembly, including the independents. Taagepera and Shugart (1993) have shown that, in the absence of any other knowledge, the number \((p)\) of seat-winning parties in a single district of magnitude \(M\) should be expected to be around the square root of \(M: p = M^{0.5}\). For a multi-district country (with all districts

![Diagram](image)

\[
\begin{align*}
M & \quad 16\% \\
\text{S} & \quad p \quad 10\% \\
\text{V} & \quad s_1 \\
\text{M} & \quad s_p \\
& \quad s_{R-1} \\
& \quad N_s \quad 16\% \\
& \quad V \quad 24\% \\
& \quad N_v \\
& \quad 16\% \\
\end{align*}
\]

\(M =\) district magnitude; \(S =\) assembly size; \(V =\) number of voters; 
\(p =\) number of seat-winning parties; \(s_1 =\) seat share of the largest party; 
\(s_p =\) seat share of the smallest seat-winning party; 
\(s_R =\) seat share of the R-th largest party; \(v_R =\) vote share of the R-th largest party; 
\(N_v, N_s =\) effective numbers of electoral and legislative parties

**Figure 1.** Theoretical Calculation Flowchart (and Divergence from Actual Values at Selected Steps)
at same $M$) they expect, on average, $p = (MS)^{0.25}$ seat-winning parties. For details, see Appendix A.

As an illustrative reality check, Figure 2 shows the actual frequency distribution of $p$ over many elections in four countries with relatively simple electoral rules. These countries are selected to cover a wide range of $MS$ (from 82 for New Zealand up to 22,500 for The Netherlands) and will be discussed in more detail in the section entitled Reality Check. For the moment, note that the number of seat-winning parties varies considerably within the same country, from election to election, but the theoretically expected value is
within this range. Agreement with the empirical mean is close for New Zealand and Finland, less so for The Netherlands and poor in the case of the UK where a fair number of independents has at times boosted $p$ (see Appendix B). In other words, in the case of Finland and New Zealand one does not have to look for a specific imbalance in the sociopolitical factors to see how many seat-winning parties there are. There are as many parties as one would expect in the absence of any information beyond $M$ and $S$. In the case of The Netherlands and UK, a specific imbalance of sociopolitical factors must be considered.\footnote{Detailed checking in progress suggests that, for 30 electoral systems where all seats are allocated in the districts, the median $p/(MS)^{0.25} = 1.10$ rather than the institutionally based average expectation of 1.00. Country-specific politics can alter $p$ by a factor of two for individual countries, but $MS$ seems to determine the worldwide overall median $p$ within 10 percent.}

**The Seat Share of the Largest Party**

The fractional seat share of the largest party ($s_1$), in turn, is expected (Taagepera and Shugart, 1993) to be around $s_1 = p^{-0.5}$, in the absence of any other information (see Appendix A). This implies that $s_1^2p = 1$. While the values of $s_1^2p$ for individual elections in the four aforementioned countries range from 0.4 to 5.8, the median $s_1^2p$ values are indeed close to 1 for The Netherlands (1.05) and New Zealand (0.98), while deviating in opposite directions for Finland (0.70) and UK (2.1). Work in progress suggests that these values are quite typical.\footnote{Detailed checking in progress involves all 752 elections listed by Mackie and Rose (1991, 1997). The median $s_1^2p$ is around 1.05, instead of the expected 1.00. Country-specific politics can alter the figure by a factor of two and more for individual countries (not to mention individual elections), but the worldwide overall median seems to hold within 5 percent.}

Combining the two preceding steps, i.e. $p = (MS)^{0.25}$ and $s_1 = p^{-0.5}$, leads to $s_1 = (MS)^{-0.125}$ or $(MS)^{0.125}s_1 = 1$. The actual mean values of $(MS)^{0.125}s_1$ are as follows:

<table>
<thead>
<tr>
<th>Country</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Netherlands</td>
<td>1.03</td>
</tr>
<tr>
<td>Finland</td>
<td>0.86</td>
</tr>
<tr>
<td>UK</td>
<td>1.24</td>
</tr>
<tr>
<td>New Zealand</td>
<td>1.10</td>
</tr>
</tbody>
</table>

The deviations from 1.00 indicate the unbalanced effect of sociopolitical factors in the given country. We are out by 24 percent at most.

**Seat Shares of Other Parties**

As a novel step introduced in the present study, let us now also consider the other end of the spectrum: the smallest party that still achieves
representation. This is the one ranked \( p \)th among the \( p \) seat-winning parties. In the absence of any other information, the only assumption that can be made in the case of simple electoral systems (no legal threshold!) is that the smallest seat-winning party obtains one single seat. This may well be an underestimate, but there are no logical grounds to posit any specific value higher than one.\(^9\) With this assumption the fractional share is \( s_p = 1/S \).

Now it becomes a matter of estimating the seat shares of the parties that rank in between the largest and the smallest. By the definition of ranking, \( s_R \geq s_{R+1} \). The distribution of seat shares is subject to four conditions:

1. The total, of course, must be \( \sum s_R = 1 \);
2. The number of components is \( p = (MS)^{0.25} \);
3. The largest component is \( s_1 = (MS)^{-0.125} \) – this is an upper anchor point; and
4. The smallest one is \( s_p = 1/S \) – this is a lower anchor point.

In the absence of any other knowledge but \( M \) and \( S \) we have no grounds to posit bunching at some ranks or some numbers of seats. Bunching may well occur in individual countries, but we cannot tell where and in which direction, in the absence of country-specific information. Hence a uniform spacing between consecutively ranked share sizes is the least surprising one (i.e. the one with the least information content or the highest entropy). We should spread the seat shares of the intermediary parties uniformly between the largest and the smallest shares, according to some rule to define ‘uniformity’.

In the case of a fairly large number of parties an exponential pattern (geometric progression) determined by the last three of the previous conditions usually comes close to satisfying the first condition too (\( \sum s_R = 1 \)), but not quite. The simplest way to achieve full agreement with \( \sum s_R = 1 \) is illustrated in Figure 3 which plots seat shares (on a logarithmic scale) versus party rank by seat share. Except for the largest and smallest shares (which are theoretically anchored), the line is shifted parallel to the original line (meaning that the parties’ seat shares are multiplied by the same constant) until the sum of the shares adds up to unity. The actual seat shares for Finland are also shown in Figure 3, and there is fair agreement.

\(^9\) The actual median number of seats for the smallest seat-winning party is one in the UK and The Netherlands, two in Finland, and 2.5 in New Zealand, although in individual elections much higher numbers have occurred. During the pure two-party period in New Zealand (1946–63) it was consistently above 30 seats. Thus this assumption breaks down when \( p < 3 \), which is rarely the case in the long run.
If anything, the shares of middle-rankling parties tend to be underestimated.\footnote{10}

I realize that many a political scientist may be uneasy with my approach, feeling that there is no politics-based ‘theoretical basis’ to this estimate. In

10. The $p$ terms in the original exponential follow the formula $s_R = r^{R-1}s_1$, where the ratio $r$ between the successive terms is given by $r = (s_p/s_1)^{1/(p-1)}$. The sum of the terms can be calculated from the usual geometrical progression: $\sum s_R = s_1(r^p - 1)/(r - 1)$. This sum tends to fall somewhat short of one, while exceeding it at very large combinations of $S$ and $M$. Other simple patterns fail even worse than the exponential. In particular, the ‘rank–size’ formula $(s_R = s_1/R^k)$ cannot be fitted to satisfy conditions 1 to 4. The smoothest way to adjust the exponential pattern so as to sum up to one would be to set $s_R = r^{R-1}s_1$, where $r = (s_p/s_1)^{1/(p-1)}$ and $p = R^k$, adjusting $k$ until $\sum s_R$ becomes 1. It tends to give a better fit than the simple method adopted here. The simpler method tends to over-correct the extreme components ($s_2$ and $s_{p-1}$) while under-correcting in the middle range. Given that the smooth approach is rather complex, one might stick with the coarser but operationally unambiguous simple method.
fact, there is a very strong theoretical basis, once one admits that politics is subject to the broader laws of nature and logic. If the spread of a political variable follows the normal distribution pattern, we do not ask for a specifically political ‘theoretical basis’ to it, because the theoretical reasons for the occurrence of normal distributions are much deeper. The situation is analogous here (see Appendix A). In a situation of minimal knowledge, one can still rationally determine the most likely distribution of shares. Figure 3 (and Table 1) show to what extent nature (political nature, in this case) obliges.

We have now the expected seat shares $s_R$ for all seat-winning parties and can compare them to the actual ones (see the last column in Table 1, for Finland). The next step is to find the corresponding vote shares $v_R$.

**The Vote Shares**

Use is made of the seat–vote equation, $s_R = v_R^n / \sum v_i^n$, where $n = (\log V / \log S)^{1/M}$ and $V$ is the total number of voters (Taagepera, 1986; cf. Taagepera and Shugart, 1989: 156–98) – see Appendix C. When the seats are allocated in districts of magnitude $M$, this equation yields an estimate of the $R$th party’s seat share ($s_R$) in terms of the vote shares ($v_i$) of all the parties, for a given number of voters and seats.

The novel approach introduced in the present study is to remember that the seat–vote equation can be reversed into $v_R = s_R^m / \sum s_i^m$, where $m = 1/n = (\log S / \log V)^{1/M}$. For SMP parliamentary elections the exponent $m$ tends to be around 1/3, while for $M >> 1$ it approaches 1. Now the expected vote shares can be calculated from the expected seat shares previously determined.11 The expected effective number of electoral parties follows.

**Reality Check**

The central concern of this study is theoretical: establishing an explicit quantitative path from electoral institutions to party system in the special case of ideal simple electoral rules. A full direct empirical test of the theory is impeded by the dearth of actual electoral rules that come even close to

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11. There is a hitch. We would allocate all votes to the seat-winning parties, allowing for no votes for parties that failed to win seats. The way out is the following. For seats distribution, extend the original exponential (such as in Figure 2) to the region below the level of one seat and assume there are parties receiving fractional seats. As an average over many elections, such cases occur indeed (see Table 1 and Figure 2) – meaning that these low-ranking parties win a seat only occasionally. Inclusion of these tiny seat shares in the denominator of the seat–vote equation ($\sum s_i^m$) hardly alters it. Now we can calculate the expected vote shares for all seat-winning parties and also for the parties with appreciable votes that fail to win a seat in the average election.
ideal simplicity. Even for the simplest cases available, the number of links in the concatenation (see Figure 1) reduces the likelihood of overall agreement because each step is likely to add a random error.

A different approach is to test each link separately. Here the amount of data for some links becomes much larger. In particular, the relationship \( S_1 = p^{0.5} \) between the number of seat-winning parties and the largest party’s share can be tested for any legislative assembly, regardless of electoral rules. So can the impact of the largest seat share on the shares of all other parties. The goals of such link-by-link testing would be the following:

1. Find out to what extent the average outcome of a large number of elections fits the institutionally based expectations.
2. Investigate the overall mean residuals at various size rankings, pondering what they say about the general nature of politics.
3. Consider the residuals of individual countries, so as to characterize the specifics of the politics in these countries.
4. Locate those links in the concatenation that add most to the residuals. The reasoning regarding these links could then be rechecked.

The number of simple cases could also be multiplied by going to sub-national data. In principle, the theory applies at individual district level (with \( S = M \)), provided that a specific number of seats is assigned to the district and that all these seats are actually allocated within the district (rather than, say, quota remainders being shifted to a second tier).\(^{12}\)

Such a detailed testing amounts to numerous separate projects. Some of these are currently being carried out. However, they are warranted only if preliminary comparison with actual relatively simple systems offers encouragement, meaning that the actual outcomes straddle the institutionally expected ones. This is the purpose of this section.

**Countries with Various Magnitudes and Assembly Sizes**

The theory refers to average outcomes in ideal simple electoral systems. Consequently, one should look for countries that have used the same relatively simple electoral rules over a large number of elections, so that fairly stable average vote and seat shares can be calculated. One would also want to cover the entire usual range of \( M \) and \( S \). Four electoral systems were chosen, trying to cover the range of \( MS \) in a roughly geometric progression.

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12. A difficulty is that district-level politics are affected by the nationwide ones. Due to out-of-district financing, a 25-seat district in a large country may have more parties running (and occasionally winning) than would be the case for a tiny sovereign country with a 25-seat parliament elected nationwide (Taagepera and Shugart, 1993). The theory would directly apply only to the latter case.
They are shown in Table 2, listed in decreasing order of MS. Hence they range from ‘feeble’ to ‘strong’ systems. The time periods chosen are the longest ones with essentially constant rules for which data are given in Mackie and Rose (1991, 1997). Although they are comparatively simple, all four systems still deviate from the criteria of simple systems in the following ways.

1. In The Netherlands parties are allowed to form alliances; and there is a mild legal threshold at 0.67 percent votes. Still, it is the ‘feeblest’ of all fairly simple electoral systems. The only comparable case available is Israel, which also has a nationwide single district.

2. Finland allows local alliances and \( M \) varies across districts (from 1 to close to 30). In view of the system’s long duration this is still the most stable representative for large-district (but not nationwide) PR.

3. The UK represents the large-assembly extreme of the SMP systems. Deviations from simple system include variations in \( S \) (from 615 to 651) and some two-seat districts in the early part of the period. The pre-1922 period is omitted because Ireland’s over-representation distorted the general pattern.

4. New Zealand (pre-1996) represents the small-assembly end of durable SMP systems and hence is the ‘strongest’ of the systems considered. Deviations from simple system include two elections with majority rule (which maintained \( M = 1 \) and therefore are included), variations in \( S \) over time (70–97), and the presence of four special Maori districts. Many other SMP systems are also fairly simple and durable, but only those with the highest and lowest MS have been included.

For each country, Tables 1 and 3–5 tabulate vote and seat shares, number of seat-winning parties (see Appendix B) and the effective number of electoral and legislative parties (\( N_v \) and \( N_s \)). For each of these, the actual and institutionally calculated values are shown, plus the residual (the difference

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>Number of elections</th>
<th>Seat allocation rule</th>
<th>( S )</th>
<th>( M )</th>
<th>MS</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Netherlands</td>
<td>1956–94</td>
<td>12</td>
<td>PR</td>
<td>150</td>
<td>150</td>
<td>22,500</td>
<td>0.99</td>
</tr>
<tr>
<td>Finland</td>
<td>1907–95</td>
<td>32</td>
<td>PR</td>
<td>200</td>
<td>15</td>
<td>3,000</td>
<td>0.94</td>
</tr>
<tr>
<td>UK</td>
<td>1922–92</td>
<td>20</td>
<td>SMP</td>
<td>629</td>
<td>1</td>
<td>629</td>
<td>0.38</td>
</tr>
<tr>
<td>New Zealand</td>
<td>1890–1993</td>
<td>34</td>
<td>SMP(^a)</td>
<td>82</td>
<td>1</td>
<td>82</td>
<td>0.32</td>
</tr>
</tbody>
</table>

**Note:** PR = proportional representation; SMP = single-member plurality; 
\( S \) = mean assembly size; \( M \) = mean district magnitude;  
\( m = (\log S / \log V)^{1/M} \) where \( V \) = median number of voters.  
\(^a\) Alternative vote in 1908 and 1911.
between actual and institutional). This residual reflects unbalanced political impact. Comments on each country are given first, before proceeding to the overall picture.

The Netherlands (Table 3). Politics and other long-term factors in The Netherlands enhance the largest and especially the second-largest party’s vote and seat shares, compared to institutionally based expectations. The lower-ranked party shares are all moderately depressed. The outcome is a balance between two top parties: a votes ratio of only 1.10, compared to the institutionally expected 1.37. In turn, the votes ratio of the second- and fourth-ranking parties is as high as 3.2 instead of the institutionally expected 1.9. The number of seat-winning parties and the effective numbers are all lower than expected by more than one unit.

Finland (Table 1). Politics and other long-term factors in Finland act in an opposite direction to those in The Netherlands. The vote and seat shares of
the largest and especially the second-largest party are depressed compared to institutionally based expectations. The contrast between the two top parties is enhanced: a votes ratio of 1.41 instead of the expected 1.17. In contrast, the fourth-largest party has larger shares than expected (see Figure 3). The outcome is a bunching of the second- to fourth-largest parties. The votes ratio of the second- and fourth-ranking parties is only 1.7 instead of the institutionally expected 3.5. The number of seat-winning parties is lower but the effective numbers are higher than expected.

The UK (Table 4). Here an imbalance among political factors seems to over-rule institutional constraints appreciably. The largest party’s seat and vote shares are markedly larger than expected. The second-largest party obtains more votes but fewer seats than expected. The third-largest party is uniformly depressed compared to expectations. With a huge total number of seats, institutional constraints would allow even the fourth-largest party to get an appreciable share of votes (though not of seats), but this is not the case. The number of seat-winning parties (and independents) is much higher than expected (see Figure 2), the effective number of legislative parties is close to the expected value, and that of electoral parties is lower than expected. Before concluding that here politics completely over-rides institutional constraints, note that the constraints model still correctly generates the essentials of Duverger’s law: The two top parties are expected to dominate the assembly with a combined 87 percent of the seats (actual average: 93 percent). However, the institutionally based theory expects a

<table>
<thead>
<tr>
<th>Party rank</th>
<th>Actual</th>
<th>Instit. theory</th>
<th>Residual</th>
<th>Actual</th>
<th>Instit. theory</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>44.7</td>
<td>32.0</td>
<td>+12.7</td>
<td>55.5</td>
<td>44.7</td>
<td>+10.8</td>
</tr>
<tr>
<td>2</td>
<td>37.4</td>
<td>31.3</td>
<td>+6.1</td>
<td>37.1</td>
<td>42.4</td>
<td>−5.3</td>
</tr>
<tr>
<td>3</td>
<td>13.9</td>
<td>18.4</td>
<td>−4.5</td>
<td>4.5</td>
<td>10.2</td>
<td>−5.7</td>
</tr>
<tr>
<td>4</td>
<td>1.8</td>
<td>10.5</td>
<td>−8.7</td>
<td>1.6</td>
<td>2.5</td>
<td>−0.9</td>
</tr>
<tr>
<td>5</td>
<td>0.7</td>
<td>3.9</td>
<td>−3.2</td>
<td>0.5</td>
<td>0.2</td>
<td>+0.3</td>
</tr>
<tr>
<td>6</td>
<td>0.4</td>
<td>2.3</td>
<td>−1.9</td>
<td>0.3</td>
<td>0.04</td>
<td>+0.3</td>
</tr>
<tr>
<td>7</td>
<td>(0.3)</td>
<td>1.4</td>
<td>−1.1</td>
<td>(0.3)</td>
<td>0.01</td>
<td>−0.3</td>
</tr>
<tr>
<td>8 up</td>
<td>[0.9]</td>
<td>[0.2]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p</td>
<td>6.9</td>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
<td>+1.9</td>
</tr>
<tr>
<td>N</td>
<td>2.8</td>
<td>4.0</td>
<td>−1.2</td>
<td>2.2</td>
<td>2.6</td>
<td>−0.4</td>
</tr>
</tbody>
</table>

Note: Parentheses indicate inclusion of ‘Other’ parties. 

p = number of seat-winning parties.

N = effective number of parties.
combined vote share of only 63 percent for the two top parties, when the actual figure is 82 percent. In a way, institutional constraints account for the mechanical effect but underestimate the psychological effect of fading on the third and fourth parties.

New Zealand (Table 5). Here the institutionally based expectations materialize quite extensively. No residual is larger than 5 percentage points. The number of seat-winning parties and the largest party’s seat share fit well, and so do the effective numbers of parties. The two largest parties are expected to share over 98 percent of the seats and 81 percent of the votes (actual averages: 94 and 84 percent, respectively), thus agreeing with Duverger’s law. Long-term politics seems to be such that various trends away from institutionally determined averages cancel out.

The Balance of Residuals

Correlations between the institutionally expected and actual seat and vote shares in Tables 1 and 3–5 are visibly very high, but this is spurious, due to ranking parties by size. How large are the residuals; and do they pull in one direction? Table 6 shows the ranges and the means for the residuals for the four largest parties in each country. (Beyond four, we run out of parties in New Zealand.) These invariably range from negative to positive values, thus encompassing the zero (pure institutional theory). The means also straddle the zero point fairly evenly. Given the smallness of the sample this is no proof that the institutional model represents some worldwide mean. It merely means that this is plausible and worth detailed testing.

For the second- to fourth-ranking parties the mean residuals are within 2 percentage points of zero, but for the largest one they are more strongly

<table>
<thead>
<tr>
<th>Party rank</th>
<th>Actual</th>
<th>Instit. theory</th>
<th>Residual</th>
<th>Actual</th>
<th>Instit. theory</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>47.3</td>
<td>42.5</td>
<td>+4.8</td>
<td>58.4</td>
<td>57.7</td>
<td>+0.7</td>
</tr>
<tr>
<td>2</td>
<td>37.0</td>
<td>38.1</td>
<td>−1.1</td>
<td>35.9</td>
<td>40.9</td>
<td>−5.0</td>
</tr>
<tr>
<td>3</td>
<td>9.2</td>
<td>12.4</td>
<td>−3.3</td>
<td>3.3</td>
<td>1.2</td>
<td>+2.1</td>
</tr>
<tr>
<td>4 up</td>
<td>[6.5]</td>
<td>7.0</td>
<td>−0.5</td>
<td>[2.4]</td>
<td>0.2</td>
<td>+2.2</td>
</tr>
<tr>
<td>P</td>
<td></td>
<td>3.5</td>
<td></td>
<td>3.0</td>
<td></td>
<td>+0.5</td>
</tr>
<tr>
<td>N</td>
<td>2.7</td>
<td>2.9</td>
<td>−0.2</td>
<td>2.1</td>
<td>2.0</td>
<td>+0.1</td>
</tr>
</tbody>
</table>

Note: Parentheses indicate inclusion of 'Other' parties.

p = number of seat-winning parties.

N = effective number of parties.
Table 6. Residuals (Differences Between Actual Values and Institutional Theory) for Top-Ranking Parties

<table>
<thead>
<tr>
<th>Party rank</th>
<th>Range</th>
<th>Mean</th>
<th>Range</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3.6 to +12.7</td>
<td>4.0</td>
<td>-3.3 to +10.8</td>
<td>2.8</td>
</tr>
<tr>
<td>2</td>
<td>-7.7 to +7.1</td>
<td>1.1</td>
<td>-7.2 to +7.9</td>
<td>-2.4</td>
</tr>
<tr>
<td>3</td>
<td>-4.5 to +0.9</td>
<td>-1.7</td>
<td>-5.7 to +7.1</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>-8.7 to +4.5</td>
<td>-1.6</td>
<td>-1.7 to +4.6</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Note: Calculated from Tables 1 and 3–5.

positive, especially for votes. One may ask whether there are prevailing political factors (such as bandwagon psychology) that tend to give the largest party consistently more votes than one would expect on the basis of institutional constraints. Once more, the sample is too small to yield an answer.

To characterize politics in individual countries, with some factors chafing against the institutional constraints, the residuals of the effective number (as given in Tables 1 and 3–5) might be informative. Finland has one effective party more than expected, reflecting the depressed seat and vote shares of the top two parties. The Netherlands has one effective party less than expected, reflecting the strength of the top two parties. New Zealand has about as many effective parties as expected. The UK is a mixed case: The effective number of electoral parties is less than expected by more than one unit, indicating the voting strength of both top parties, while that of the legislative parties is rather close to expectation, because the second-largest party falls short of expectations.

The UK has often been considered the benchmark for SMP systems, but here it presents surprises from the very first step on, when it produces appreciably more seat-winning parties than expected (see Figure 2). This involves not only regional parties but also many independents at some time periods (see Appendix B). In comparison, New Zealand looks prototypical regarding the expected effects of SMP allocation rule. Lijphart’s observation (1984: 16, 1999: 25) that pre-1996 New Zealand is ‘a better example of the Westminster model than British democracy’ seems confirmed.

The Links That Enhance Residuals

In a preliminary way, I have tried to pin down the main locations where divergences from reality set in during the successive steps of the theoretical procedure. Consider relative (percentage) divergence at the two first steps
in the concatenation (the number of seat-winning parties and the seat share of the largest party) and at the two final outputs – the effective numbers of parties. The means of the absolute values of these divergences are indicated in Figure 1.

One would expect the divergence to increase at successive steps, for two distinct reasons. One is sheer accumulation of random error. The other is that the relative impact of institutional constraints fades as one proceeds from seats to votes, while the impact of politics increases. However, this is not quite the case. Starting with the product of district magnitude and assembly size, the mean divergence for the number of seat-winning parties is 16 percent. Instead of increasing, it actually falls to 10 percent for the largest party’s seat share. The divergence for \( N_v \) is no larger than for the number of seat-winning parties, suggesting that additional divergence introduced while calculating the other seat shares is small. The disparity on \( N_v \) (24 percent) is larger than on \( N_s \), reflecting appreciable additional divergence cumulated when going from seat shares to vote shares, plus strengthening of political factors.

In sum, a major source for residuals seems to occur right at the first step: the number of seat-winning parties. Either this link is strongly affected by country-specific politics or this link in the theory may need refinement. It may also be that it is the empirical method for determining the actual \( p \) that needs correction (see Appendix B). The second possibly questionable link may be the conversion from seat share to vote share.

**Discussion: How Can it Be?**

What has been achieved? Half a century ago Duverger (1951, 1954) noted that certain electoral rules (SMP) tend to impose two-party dominance, first in the representative assembly (through a mechanical effect) and then also for popular vote (through a psychological effect). For multi-seat PR, Duverger observed no such institutional restrictions on the popular vote, and he left the number of parties unspecified. Sartori (1968) replaced this SMP–PR dichotomy by a more gradual shift from strong to feeble systems: as \( M \) increases, an increasing number of parties can achieve appreciable representation and hence maintain an appreciable vote share. Again, no numbers were specified.

Within this framework various concepts were later operationalized and some quantitative interconnections were established – notably by Rae (1967), Theil (1969), Taagepera and Shugart (1993), and Lijphart (1994). The present study has joined the existing links and added new ones so as to establish an unbroken concatenation from institutional inputs to expected average distribution of votes among parties. The present theory is the first
and only one whereby one can operationally stipulate an expected average effect of basic institutional constraints on the distribution of seats and popular votes. Duverger’s qualitative concepts of ‘large’ and ‘small’ parties have been replaced by specific numerical values. This has been done on the basis of extremely parsimonious institutional inputs: district magnitude and assembly size (plus the number of voters).

At first glance, one may rebel against the idea that one can even pretend to be able to estimate the vote distribution of parties based on solely institutional inputs \((S, M \text{ and, secondarily, } V)\). Don’t the votes received by a party depend on the political decisions made by voters? They do. Voters decide \textit{which} party receives the most votes, which one comes in second, and so on. Their votes also decide whether the margin between the first- and second-ranking parties (and so on down the list) is large or small \textit{in a given election}. But for the average over many elections institutional constraints do creep in. At this level the present approach is no more preposterous than Duverger’s – it is only more systematic and explicit.

Like Duverger’s law and hypothesis, the present theory deals only with long-term averages. And these are not long-term averages for specific parties but for parties at a certain rank at a given election. One particular party may rank third in one election and first in the next. In the former case its votes are averaged in with the votes of all third-ranking parties in all elections. In the latter it will join the pool of all first-ranking parties.

The rank of a specific party at a given election certainly depends on voters’ choice. However, the votes received by the \(R\)th ranked party are not completely up to the free will of the voters. Institutional constraints do restrict the vote shares of low-ranked parties. The results of the theory presented here are estimates of what the seat and vote patterns would be, if political factors enter in an average way, balancing each other out. The extent of the imbalance of political factors is indicated by the residuals.

In the case of SMP the present theory reproduces Duverger’s law, but with extra details: estimates of the vote and seat shares of the two large parties separately as well as those of the minor parties. In addition, the theory also yields specific estimates for multi-seat PR systems, where Duverger’s hypothesis only states that more than two parties will obtain appreciable shares. This is where the present theory is superior to Duverger’s – not by contradicting Duverger, but by adding more precision.

The free will of the voters makes itself felt but within bounds – so do political factors and processes. By the same token, an institutionally based theory has its uses but also within limits. It can give a numerical estimate for the expected vote share of, say, the third-ranking party. For fairly large \(M\), this figure tends to be around 16 percent, as in Finland. For \(M = 1\) (and no primaries or second rounds), it is around 11 percent. No institutional theory will tell us whether the third-ranking party in New Zealand is called
Labour (as it was in 1914) or Social Credit (as in the 1960s). But it does tell us that whichever party falls to the third position is likely to get more votes in Finland than in pre-1996 New Zealand. If there is a large deviation from institutionally based expectation, this residual is a measure of the force and direction of special political factors.

The theory of institutional constraints also posits that in Finland the third-ranking party wins almost the same share of seats as votes, but in New Zealand it tends to be drastically under-represented (winning only about 4 percent of the seats). A purely institutionally based theory cannot do more than deal in average expectations. But that much it can do – not only approximately (in the form of Duverger’s law) but in more detail.

Residuals (divergences between calculated and actual party sizes) remain – and this is to be expected, unless one takes an utterly institutionalist view. The theory deals only with certain institutional constraints, so the residuals represent the imbalance in the impact of all other factors. This being so, it is still remarkable to what extent the party sizes in the sample countries investigated are accounted for by these institutional constraints, even though the theory applies directly only to ideal simple systems.\(^\text{13}\)

More systematic testing of various steps in the concatenation remains to be done. But a basic pattern seems to emerge, centering around a model based on institutional constraints. The present study is about establishing the broad outline of this theoretical framework. To repeat, for the first time ever, a continuous quantitative and operational concatenation now extends all the way from institutional constraints to numerical electoral outputs and to the broad outline of the party system.

\(^{13}\) How good is the fit compared to competing theories? There is no competing theory offering specific percentage shares, unless such specific percentage shares are read into Duverger’s or Sartori’s semi-quantitative statements – which could be construed as building up a straw man. One anonymous referee has voiced the concern that ‘unlike Duverger’s law, this theory cannot be disproved. The residuals, far from indicating any error in the estimates, merely show the effect of political factors!’ Here I am between Scylla and Charybdis. If I treat the residues as error, I will be branded as a blockhead institutionalist who sees anything non-institutional as merely an error term. Yet, if I allow for political factors unfettered by institutional balance, the theory is considered non-disprovable and hence non-scientific. In fact, any data that would disprove Duverger would even more glaringly disprove my elaboration on Duverger’s theory, because more specific predictions offer more potentially disprovable detail. The UK (Table 4) offers an example. My numbers reflect the Duvergerian results (two major parties and much smaller third parties), which the data confirm. But I also supply further details, and although they basically fit, an explicit maximum discordance of 12 percentage points for the two largest parties may look unacceptably large, even while Duverger’s law cannot be pinned down to that degree of precision! In general, fuzzier theories may look more impressive precisely because they are easier to satisfy with data.
APPENDIX A: IGNORANCE-BASED QUANTITATIVE MODELS

The broad approach that underlies estimating the number of seat-winning parties is an ‘ignorance-based quantitative model’, as explained in detail by Taagepera (1999). As a specific example, consider a district where 25 seats are allocated. At most 25 parties can win seats (one seat each) and at least one party must win seats (all 25 of them). These constraints are the ‘boundary conditions’. The median value of actual observations is likely to be far from these conceptually possible extremes. For reasons elaborated by Taagepera (1999), the geometric mean is here the best guess, in the absence of any other information, meaning about five parties to win seats, and five seats for the median party. This is the ‘expectation value’ in the sense of actual values being distributed around it in a balanced way. More generally, the expectation is that \( p = M^{0.5} \).

In the case of nationwide PR, the nationwide number of seat-winning parties would then be \( S^{0.5} \). When the country is divided into many districts of magnitude \( M \), the boundaries for the nationwide number of seat-winning parties are those for a single district (\( M^{0.5} \)) and for nationwide PR (\( S^{0.5} \)). Their geometric mean is \( p = (MS)^{0.25} \).

For the seat share of the largest party, the logical constraints are the following. At one extreme, all \( p \) seat-winning parties could have the same share of seats (\( 1/p \)). At the other, the largest party could have almost all the seats, which rounds off to a share of one. The geometric mean of these extremes is \( s_1 = 1/p^{0.5} \). This is an extremely general logical expectation (see Taagepera, 1999), independently of even institutional constraints. Thus it can be applied to any election, regardless of its rules, so as to pinpoint unusual patterns.

Is this approach overly mechanical, devoid of political explanation? There is one ignorance-based quantitative model that we all accept: the normal distribution curve. This is the shape mathematically produced (subject to certain boundary conditions) in the absence of any substantive information. If a distribution is normal, we do not ask for any specifically sociopolitical reasons behind it. Only deviations from the normal call for such reasons. The situation is analogous here.

APPENDIX B: THE NUMBER OF SEAT-WINNING PARTIES

The observed number of seat-winning parties is, in principle, simple to count for each election. However, a practical difficulty may arise. Data sources such as Mackie and Rose (1991, 1997) lump the seats won by small parties and independents into a single ‘Other’ category. Over the time periods considered, it involves 2.7 percent of all seats in New Zealand (a mean of 2.1 seats per election), 0.4 percent in the UK (2.8 seats per election), 0.06 percent in Finland, and none in The Netherlands. Four ‘Other’ seats could mean four seats for a single short-lived party or one seat each for four parties (or independents) or various intermediary combinations (3–1, 2–2, 2–1–1).

The operational rule followed here is to add the square root of the ‘Other’ seats to the number of seat-winning parties. In the absence of any other knowledge no other approach can be justified. In particular, merely omitting the independents
would distort the picture. The operational rule may understate the number of independents winning seats, but once elected, the independents may join a party group; in such cases the operational rule may actually overstate the functioning number of seat-winning parties. The problems with the ‘Other’ category surface again when determining the actual vote shares of low-ranking parties.

The conceptual difficulty lies in the nature of independents. Is each of them to be counted as a separate party? They often lack a platform and, once in the assembly, may attach themselves to a party. The method used here may give excessive weight to the independent representatives. This might account for part of the disparity for the UK, in particular.

**APPENDIX C: THE SEAT–VOTE EQUATION**

The background of the seat–vote equation is the following. For SMP systems the relationship with \( n = 3 \) was noticed a century ago and was termed the ‘cube law of Anglo-Saxon elections’: \( s_A/s_B = \left(\sum (v_A/v_B)\right)^3 \) for two parties, A and B, which can also be expressed as \( s_R = v_R^{1/3}\sum v_i^{1/3} \) (see March, 1957). Thell (1969) pointed out that a relationship of the form \( s_R = v_R^n/\sum v_i^n \) between \( s_R \) and \( v_R \) was the only one that did not lead to inconsistencies in the presence of more than two parties. He left the value of \( n \) open.

Taagepera (1973) showed that the form \( n = \log V/\log S \) satisfies certain logical constraints in the case of seat allocation by plurality rule. It agrees with actual results not only in representative assembly elections with SMP (where \( \log V/\log S \) is usually close to three, thus leading to the cube law) but also in US Electoral College elections (where \( \log V/\log S \) is around five) and certain labour union elections (where \( \log V/\log S \) is below two). Finally, the extension from plurality rule to PR rules in multi-seat districts was made (Taagepera, 1986) by stipulating on the basis of logical constraints that \( n = (\log V/\log S)^{1/M} \). In the case of Japan the seat–vote equation has received detailed testing by Reed (1996).

**REFERENCES**


Mackie, Thomas T. and Richard Rose (1997) *A Decade of Election Results: Updating the International Almanac*. Glasgow: Centre for the Study of Public Policy, University of Strathclyde.


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