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Title
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Permalink
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Journal
Proceedings of the Annual Meeting of the Cognitive Science Society, 22(22)

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Publication Date
2000

Peer reviewed

Work at the Interface between Representing and Represented Worlds in Middle School Mathematics Design Projects

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Introduction

A central process in scientific or mathematical thinking involves being able simultaneously to look *at* and *through* the interface between representing and represented worlds (Gravemeijer, 1994; Latour, 1999). This is particularly true of thinking practices in which people construct and then explore models to gain access to situations that do not yet exist or that occur across scales of time and space that prevent direct observation. While this flexible use of modeling is central to many disciplines, pedagogy has until recently focused primarily on the notational structure of formal systems of representation. This approach can trap learners in the situation of looking *at* complex representational systems, without being able to look *through* them to construct and explore represented worlds (Greeno & Hall, 1997).

This paper adds to a line of work in cognitive studies of mathematics education that examines how learners work at the interface between representing and represented worlds to make inferences, identify and recover from conceptual errors, and manage calculation (Cobb, Yackel, & McClain, 1999; Hall, 1996; Nathan, Kintsch, & Young, 1992; Nemirovsky, in press). Empirical materials are drawn from group modeling efforts in project-based middle school mathematics classrooms (Hall, 1999). In particular, I focus on how a group of students develops an increasingly sophisticated capacity for working with the concept of predation, treated as a functional relation between animal populations (i.e., predator and prey) that can be implemented in particular computational media. In my analysis, these media and other resources available through talk and embodied action develop into systems of activity (Goodwin, 1994) that make up conceptual understanding. From this perspective, concepts and their implementation in diverse representational technologies are inseparable.

Background to the Student Design Project

Data for this paper (also for papers by Greeno, Sommerfeld, & Wiebe and Stenning & Sommerfeld, this volume) come from studies conducted in middle school mathematics classrooms where students worked on design projects. These projects were supported by curriculum units developed to embed important mathematical concepts in realistic applications (Greeno & MMAP, 1998). In the study reported in this paper, students were asked to act as biological consultants who would devise a proposal for preserving and then returning a population of guppies to a Venezuelan stream environment. As adopted for use in their classroom, the project lasted approximately four weeks and included the following: task memos directing the activities of student groups, worksheets and supporting case material for the contexts of design problems, a software tool (HabiTech™) that allowed students to model and investigate structures and processes in population biology (see Figure 2), and a set of extension scenarios asking students to model hypothetical events within the Venezuelan stream environment (e.g., harvesting by farmers or the introduction of a predatory fish).

Just before and after working on this project, student groups attempted a 20-minute design challenge in which they were asked to model the relation between mice and cat populations living in a barn over a period of several years. Both the design challenge and students' daily activities during the longer unit were filmed, and various design documents (intermediate and final) were collected. An analysis contrasting pre/post unit performance of groups on the design challenge provides evidence *that* students' understandings of population biology and associated mathematical concepts changed over time. In this paper, I focus on *how* these understandings changed, by following the work of a typical group (labeled the MLKN group) with a particular concept through the longitudinal record of their daily activities on the unit. Data materials are divided into six segments: Segments 1 and 6 come from the pre- and post test design challenge (respectively); Segments 2 through 5 come from the longitudinal record of group work during the unit.

Evidence of Conceptual Change from a Pre/Post Design Challenge

An utterance-level comparison of the MLKN group's performance on a pre and post-unit design challenge showed that they, like the majority of groups in their classroom (5 of 7 groups), were able to construct and explore a more complete functional model of population growth and predation at the end of the four week unit. At the pretest challenge, this group failed to mention deaths for either population, they did not link together overlapping timelines for otherwise correct models of mouse and cat births, and they made no mention or use of the concept of predation until questioned about it.

As evident in the following exchange[1], which was recorded at the end of the pre-test, members of this group did understand the qualitative effect of predation, in the sense that cats eat mice and so "reduce" their population. But they had no way to implement this understanding as a functional relation linking together their isolated, hand-calculated models (i.e., for mice and cat populations).

Segment 1: Predation at the Pre-test (4/21/98)

1 Rogers: So if the mice are eating grain...
2 Manuel: Uh, huh.
3 Rogers: What are the cats eating?
4 Lisa: [Mice.
5 Manuel: [Mice.
6 Rogers: What does that do to the mouse population?
7 Manuel: Reduce them.
8 Rogers: Ok, [so, as you were doing the mice calculations]
9 Manuel: [Ah! Oh:::]
        ((Lisa and Kera look at Manuel))
10 Rogers: Sounds like you were just kinda goin with, four per litter for the mice and letting them... go=
11 Kera: =Go, ok.
12 Rogers: So they're gonna be getting rubbed out by the cats, right?
13 Manuel: [Uh huh.
14 Kera: [Right.

The absence of predation as a functionally explicit concept strikes Manuel first (line 9), then he and Kera agree that their models allow cats to grow without bound. As they go on to acknowledge (not shown), this is something that violates the entire premise of the design challenge, and they are eager to get another chance at this kind of problem.

At the post test design challenge, the MLKN group's understanding of population concepts was still unstable and dependent on particular means of implementation (see papers by Greeno et al. and Stenning & Sommerfeld, this volume), but they were also able to implement and explain a functional model of predation. For example, as Manuel struggled to combine timelines for mice and cat populations into an integrated model, Lisa recalled their earlier use of a "Special 2 thing" (i.e., a user-defined function) to model the predation of guppies by wolf fish during the classroom design project. This recalled use of a special function provided a starting point for a fully explicit implementation of predation on the post test.

In the following exchange, recorded near the end of the MLKN post test, Lisa asks Kera for an update on what they

are doing, while Manuel and Ned (silent) work to repair an error with their combined timeline. As Kera explains, they started the combined model with too many mice, generated in an earlier model of mice living alone.

Segment 6: Predation at the Post Test (5/26/98)

1 Lisa: ((to Kera)) Could you run that by me?
2 Kera: Um, we ran the model for two years. But we forgot that one year, the cats were living with them. So then they were dying [(inaudible).
3 Manuel: [Forty eight. ((resets Moose/Mice[2] to 48))
4 Lisa: ((looking at interface)) Uh huh.
5 Kera: [(Not in this year.)
6 Manuel: [Ok, so now... bring that... to negative. ((relinks Special 2 to Moose/Mice negative pole)) And we started with, how many? ((scrolls down to check Wolves/Cats)) Six, ok. Here we go. Now build... to two thousand and four. ((resets timeline)) Two thousand and four... Now, to the end. ((runs To End))
7 HT: ((huge negative value for Moose/Mice population))
8 Manuel: Oo:::
9 Lisa: So how many... [That's only]
10 Manuel: [After], after two thousand and four there's negative [mice.
11 Lisa: [Can we bring in some dogs there! ((laughing))
12 Manuel: Ok::: ((laughing, opens graph window)) Kaboom.
13 HT: ((huge negative decline for Moose/Mice))
14 Lisa: Oh gosh!

At the end of this design challenge, I (as a research interviewer) asked the group exactly when mice die off. Their first idea was to narrow the timeline, a simplification that increased the resolution of their graph in both axes for time and population abundance. They eventually used this more fine-grained graph and a table of linked values to find that, in their implementation of predation, cats consumed all the mice after only one month.

Comparing pre and post test performances (Segments 1 and 6), it is clear that the concept of predation—along with technical means for implementing, using, and interrogating this concept—changed within the working capacity of the MLKN group. While they neither mention nor implement predation on the pre-test, at the post test this group makes several important advances: (1) They combine partial results from an investigation of mice to model the introduction of cats; (2) They define a predation function that explicitly links cat and mouse populations; (3) They display, investigate, and explain a resulting crisis in the mouse

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[1] Transcript conventions include: ((activity descriptions)) appear in double parens, (uncertain hearings) in single parens; [overlapping [onset of talk is shown with left brackets; dynamic computer responses are transcribed as turns at talk.

[2] HabiTech™ provides named population nodes for Caribou, Wolves, Moose, and Guppies. Using Moose for Mice presented students with no particular difficulty.

population; and (4) They notice that cats will, in turn, face a related crisis brought about by a lack of food.

Particularly important for an analysis of work at the interface between representing and represented worlds, these students appear to be able to move fluidly between their roles as middle school collaborators (e.g., Lisa asks for and Kera provides an explanation), technical designers (e.g., Manual and Ned implement the network, but Kera follows and can explain their implementation), and observers/consultants for a Venezuelan stream environment (e.g., Lisa's proposal that they add dogs to the environment). How students move between these figured worlds (Holland, Lachicotte, Skinner, & Cain, 1998) in a way that helps to develop and explore functionally explicit population models is a question for longitudinal analysis.

A "Net Wall" Solution to Predation

In this and the following section, I analyze several selections from MLKN's work during the unit on population modeling. First (Segments 2 and 3), I examine their elaborated response as fictional consultants to Venezuelan farmers, in the form of a "net wall" that will serve as a mechanical barrier to predatory fish. The MLKN group sees this as a solution to the problem of losing all the guppies, which farmers need to control mosquito growth, to an exotic population of upstream predators (i.e., the wolf fish). Second (Segments 4 and 5), I examine their computational implementation of predation more closely, asking how their experiences during the unit may have contributed to a more sophisticated performance on the post test design challenge.

After successfully modeling the growth of a guppy population in captivity, the MLKN group chose an extension scenario in which predatory wolf fish were released up stream from the guppies' pond, and farmers later noticed that these guppies were disappearing. The group predicted that the guppy population would flourish in the stream environment before the arrival of wolf fish, then die out as guppies were eaten by newly arriving predators.

Engaging their fictional role as consultants to Venezuelan farmers, the MLKN group began working on solutions that would preserve the guppy population, eventually settling on Kera's proposal for a mechanical "net wall." In the following exchange, Kera reprises the idea of a net in which mesh openings capture wolf fish but allow guppies to swim through. By installing this net at the upstream boundary of the pond, she proposes they can catch and remove wolf fish before they reach the farmers' guppies (see Figure 1).

Figure 1. Kera (middle) animates the path of a guppy swimming through a hole in the net during turn 1 of Segment 2: ((R hand forms opening; L hand, fingers wiggling, traces through)). Lisa (right) forms her own version of the net with her hands as Manuel (left) looks on.

Segment 2: Blocking the Arrival of Wolf Fish (5/18/98)

1   Kera: =Ok, this is (the idea with) the guppies. ((R hand forms opening; L hand, fingers wiggling, traces through)) And it goes sh:::, straight through the net. ((R hand holds opening; L hand traces in and sticks)) And the big fishes go... and they get caught=
2   Lisa: =Caught, yeh. And then, they=
3   Kera: =((hands grab at center then rise)) You pull::: it up and then you take it out.
4   Lisa: But why should we pull it out?
5   Manuel: No::: [The stream is like fi::ve fee::t deep.
6   Lisa: [Do you know...?
7   Manuel: No not even five feet, three feet[... deep.]
8   Lisa: [Ok, ok, ok, come on.]
9   Manuel: You can just pick em out.
10 Lisa: So, yeh yeh, so, so it should be like... no no, we can't HIRE anyone to pick it out. It should just like, flow::: naturally. Stuff like that, you know? You know, cause see the [guppies
11 Kera: [You gotta pull it out!
12 Lisa: No... they won't be CAUGHT in there, cause they're like, HUGE, ok? The hole will be this big.

In this first selection of work from the classroom, several phenomena are important for understanding how students shift between representing and represented worlds. First, a world of Venezuelan streams, farmers, and interacting fish populations is densely inhabited by members of the MLKN group. They literally construct the stream, fish, and a mechanical barrier in the gestural stage between Kera and Lisa, as Manuel reaches in from "downstream." Fish, the stream, and human actors are all

animated (Goffman, 1979; Ochs, Jacoby, & Gonzales, 1994) within this shared space.

Second, while the technical details of the "net wall" barrier are still underway, the importance of isolating guppies from these predators is clearly their emergent goal. Animated from the perspective of a consultant to Venezuelan farmers, this is a response to the consequences of predation, now articulated with the developing notion of a habitat that has semi-permeable boundaries.

The importance of predation in MLKN's consulting proposal becomes clear later during this class meeting, when the group calls me over to discuss the boundaries of the stream environment. When I ask about the effect of their "net wall" on a graph of the guppy population they had drawn earlier, Kera starts a conditional response (Segment 3).

Segment 3: The Graphical Shape of Predation (5/18/98)

1	Rogers:	The graph of the guppy population. Manuel thinks its gonna continue to... [be wavy] and you all think its gonna go down and then [come back up.
2	Manuel:	[Be wavy.]
3	Lisa:	No we=
4	Kera:	=It depends. ((points to drawing of stream in notebook)) Are there still, like... wolf fish in here that are eating the guppies?
5	Rogers:	Um[::: you can
6	Kera:	[If there is, ((traces upward path)) then its gonna go a little wavy. But if NOT, then the guppies are just... gonna have their own... ((points to computer)) Like before, when... like our other, um... thingie? (You know what I'm talking about?) [Cause the guppies are living alone, and they're gonna die and (inaudible)
7	Rogers:	[Ok... I mean if you killed, if you get rid of ALL the wolf fish... then the guppies should... recover with no trouble.
8	Kera:	=Yeh.
9	Rogers:	=If there's still some wolf fish, [the wolf fish are gonna continue to grow and stuff.
10	Kera:	[Then they're gonna ((hands trace oscillation))

According to Kera (turn 6), if any wolf fish get through the net wall the graph of the guppy population will "go a little wavy." This is because "there's still wolf fish in there eating them," as she mentions several times. But if the net wall successfully closes the pond to wolf fish, then guppies will grow in isolation "like before" (i.e., referring to their earlier model of guppies alone in the pond).

Another point is important for understanding how students begin to coordinate movement between representing and represented worlds. Kera's conditional explanation crosses worlds in the sense that shapes in the representing world (i.e., graph shapes coming out of their "thingie") depend upon conditions in the represented world (i.e., the passage of fish through a net opening). As the beginning of an activity system that was intended by the curriculum, types of outcomes, as graph shapes, are being associated with types of models, as determined by their assumptions about habitat (i.e., is the pond open or closed to exotic predators) and relations between populations. And critical to a broader understanding of modeling as such an activity system, results are seen to depend upon starting assumptions.

Implementing and Exploring Predation in an Integrated Model

The "net wall" consulting proposal is an elegant solution to an emergent design problem, and it works at several levels. Guppies will be preserved for rice farmers, since the wolf fish will be blocked from moving down stream. And this can be done without killing any of these predators. As these students have elaborated the fictional world of the task, this will also keep upstream Venezuelans happy (i.e., those who, according to Lisa, must own wolf fish). Up to this point, the group's work on this proposal is closely tied to a qualitative understanding of the effects of predation. Yet they are far from a functional implementation in computational media that could produce the graphs in question. As Manuel announces at the beginning of their next class period, "Now how do we make it work?"

The three final conversational segments in this paper illustrate the kind of work these students undertook to construct a plausible (if not entirely correct) functional model of predation. In Segment 4, the group has already constructed a user-defined function that links Caribou/Wolf Fish and Guppies population nodes. With this stable network topology in view, they repeatedly adjust node parameters and run the model in an effort to produce a reasonable number of guppies. Just before this segment starts, Lisa complains that they have a "river full of not plants, not insects, but just fishes."

Segment 4: Opening Boxes and Adjusting Parameters (5/19/98)

1	Lisa:	It's not enough! As long (as you go over) ten thousand ((changes Caribou/Wolf Fish births to 30% every month)) (inaudible) per cent.
2	HT:	((huge positive population value for Guppies))
3	Lisa:	It's still a lot. (inaudible) about guppies. Yeh, that's the problem.
4	Manuel:	Yeh, see, but the special two is gonna, do (3 sec)
5	Lisa:	Alright. Could you guys explain this to us? hh
6	Manuel:	Uh, explain what?
7	Lisa:	What's a... special two.
8	Manuel:	Special two is like when you die because of the caribou.

9 Lisa: OH! Really?
10 Manuel: Yes.
11 Lisa: ((mouse circles over Special 2/Predation node)) Oh this is eighteen? And um... how many guppies do they=
12 Manuel: =No, let's do three... times thirty is... thirty, ninety. So its caribou times ninety. ((Lisa changes Special 2/Predation)) Every month, and (then) go... That's it, just... Go to build, go to the thing that says build. Then go to the end.
13 HT: ((huge negative population value for Guppies))
14 Lisa: Negative? [That's a little too (much), yeh.]
15 Manuel: [Oh ok ((sighs))] Now we need to reduce the births. Go to births. No don't touch that, do the births. Reduce the births to ten percent every month.
16 Lisa: ((changes Caribou/Wolf Fish births to 10% every month))

With the work of implementing predation in these particular computational media well underway, several phenomena are worth noticing. First, Lisa has been adjusting model parameters without understanding how the predation function works. When she asks "you guys" (Manuel and Ned) for an explanation, Manuel describes what the node does from the perspective of Guppies: it is a type of death caused by Caribou/Wolf Fish.

Second, as Lisa looks inside this function and questions how many Guppies are eaten by Caribou/Wolf Fish (turn 11), Manuel proposes and Lisa executes a change in how the predation node is defined. Manuel's proposal unpacks the monthly value into a daily rate of consumption (i.e., 3 per day, times 30 days in a month, gives 90 guppies per Caribou/Wolf Fish per month).

This exchange is one of many in which students move back and forth between changing model parameters and running their updated model (these are called "Build" and "Play" modes in the interface) to produce a new set of population values. Over the entire series, each adjustment is sensible within the network topology of their model, but none of these changes produce an outcome that the group finds reasonable (e.g., negative assessments after turns 2 and 13). In the face of this stalled progress, Manuel recalls from their earlier research that overcrowding will cause the guppy birth rate to fall. He reduces this parameter and runs the updated model.

Segment 5: Arriving at a Guppy Crisis (5/19/98)

1 HT: ((running Fast))
2 Lisa: Too much.
3 Manuel: [No::: its not gone into the e's yet. And it hasn't.
4 HT: [[((Guppies value in population node rises for awhile, but becomes negative and ends with -2.71826 * 10^6 Guppies))
5 Manuel: ((opens a graph))

6 HT: ((graph shows Guppy population rising, then an extinction crisis part way into third year))
7 Manuel: Oh my [god:::
8 Rogers: [YES::::!
9 Lisa: Oh, it's so funny! [What?
10 Rogers: [Yes:::
11 Manuel: Yes what?

Lisa begins to classify this as another unsuccessful run of their model (turn 2), but Manuel, who has been monitoring the value displayed in the Guppy population node, announces that the positive growth of guppies has not yet reached scientific notation. Then as they watch the interface, the value displayed in the Guppy population node goes hugely negative (i.e., the software automatically shifts into scientific notation) and Manuel opens a graph window (see Figure 2).
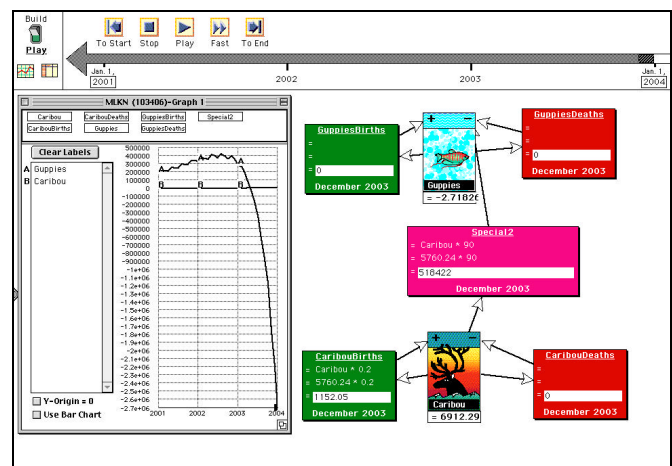

Figure 2. MLKN's network model of predation and a graph showing an extinction crisis for Guppies during their third year in a Venezuelan pond. The predation node is at the center of the network (right), linking together Guppies (above) and Caribou/Wolf Fish (below).

This graph is striking for members of the MLKN group both because it shows an extinction crisis for Guppies, but also because it catches my eye (lines 8 and 10) as I was working with a group on the other side of the classroom. In a subsequent conversation about this network model and graph, Manuel insists on the influence of overcrowding in lowering Guppies births, while both he and Lisa recount their decision to increase the level of Caribou/Wolf Fish predation. As a final part of their modeling effort, they implemented Kera's "net wall" as an emigration function, something that was suggested by their teacher as a general strategy for modeling negative influences on population growth.

Discussion

By the end of the curriculum unit from which these longitudinal selections were drawn, the MLKN group had a sensible and fully implemented model of their consulting proposal, and its behavior was consistent with what they

hoped to achieve in Segment 3 (i.e., Kera's conditional explanation, lines 4 and 6). Since the net wall was implemented as a yearly reduction in the Wolf Fish population, these predators still made it into the pond environment. As a result, some level of predation was ongoing (i.e., this appeared as a scalloped or "wavy" graph of the Guppies population over seasons). But the mechanical "net wall," which they used to remove predators at a regular interval, reversed the outcome of their earlier crisis scenario (i.e., the Guppies population grew steadily over the duration of their scenario).

Predation, as a concept that can be implemented within these particular computational media, was one among several influences in a more complex model of the Venezuelan pond habitat. These influences included (with varying levels of correctness): (a) the starting value established over an earlier period in which Guppies lived alone in the pond, (b) the production of a Guppy crisis after the unregulated arrival of predators, (c) the regulated influence of predation during smaller time cycles within the "net wall" model, and (d) the idea of birth rate suppression during conditions of overcrowding in the pond.

These explicit model components, worked out through repeated cycles of adjusting parameters and holding outcomes accountable to students' qualitative expectations, provided a rich set of resources for their activities on the post test design challenge.

Across these selections from a longitudinal record of group work, more complex forms of coordination appear in the ways that students move between representing and represented worlds. While still far from a technical implementation of their model in computational media (Segment 2), students were able to develop an elegant solution to the problem of stopping or limiting predation. Their work included conversations carried out over a stream environment that was jointly constructed in a shared gestural stage. Also central in these conversations were processes of animation in which students spoke for (or as) fish in the constructed stream environment, Venezuelan farmers who had diverging interests in these fish, biological consultants concerned with finding a solution for the loss of guppies to predation, and middle school students working on a design project (i.e., as themselves).

As these elaborations of the represented world were carried into computational media, new forms of coordination were required (Segments 3 and 4). These included forms of explanation that linked computational media to aspects of worlds being modeled (e.g., Kera's conditional explanation associates graph shapes with physical events at the net wall in Segment 3). As the structural components of their network model were settled, members of the MLKN group also managed to establish cycles of modeling activity in which they adjusted parameters and compared results with their qualitative expectations.

Through these kinds of activities, students encounter the need to simultaneously look *at* and *through* the interface between representing and represented worlds. As they work through design problems, new conceptual understanding depends upon putting existing concepts and a broader set of representational technologies into coordination. In this sense concepts—as systems of activity—develop in ways that are inseparable from the representational technologies that implement them.

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