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<https://escholarship.org/uc/item/60f709tc>

ISBN

9781424451531

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Publication Date

2010-08-01

DOI

10.1109/ursi-emts.2010.5637307

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Transmission Line Model with X-Circuit for a Metamaterial Layer Made of Pairs of Dogbone-Shaped Planar Conductors

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Abstract— In this paper we analyze the propagation through a metamaterial in planar technology recently proposed. The metamaterial basic constituent cell consists of a pair of tightly coupled conductors that supports two main resonance modes corresponding to either a symmetric or an antisymmetric current distribution in the pair. We present an equivalent X-shaped lumped circuit network to be interposed in the transmission line (TL) model of propagation across a metamaterial layer to reproduce its electric and magnetic resonances. We show that reflection and transmission features of a periodic array of dogbone pairs as well as its dispersion diagram are accurately predicted by this simple but effective model.

I. INTRODUCTION

Pairs of finite-length wires have been recently suggested as constitutive particles for creating a medium with an effective negative refractive index (NRI) [1], [2], alternatively to the conventional combination of split-ring resonators and wires [3], [4]. The pairs of coupled conducting wires exhibit both a magnetic resonance (antisymmetric mode) and an electric resonance (symmetric mode) that can be properly tuned to the desired frequency by adjusting the length of the pair. Later on, the cut-wire pair arrangement has been further elaborated and the adoption of dogbone-shaped conductors in place of simple cut-wires has been proposed to achieve enhanced control on the particle resonances and to reduce the size of the metamaterial unit cell [5], [6]. As shown in [6], full transmission can be obtained through a single layer of arrayed dogbone-shaped conductor pairs, and a backward wave can be obtained when stacking such layers. The obtained interesting phenomena seem related to the occurrence of antisymmetric modes, which are not present in single layered frequency selective surfaces. For example, in [7] some of the present authors have analyzed pairs of Jerusalem crosses, for a 2D-isotropic planar metamaterial. In [8], layers of pairs of planar loaded tripoles have shown to exhibit negative refractive index properties as layers of paired Jerusalem crosses. Numerical simulations in [7], [8] show that small losses (e.g., $\tan \delta = 10^{-3}$) existing in commercial dielectric substrates do not significantly affect the total transmission property when the magnetic resonance is not very narrow. In this case, the relative bandwidth at which a metamaterial made of stacked

layers exhibits an effective negative refractive index can easily reach 10%.

An approximate method to predict the magnetic resonance f_m of metamaterials composed of pairs of dogbone conductors has been proposed in [6]. However, such formulas, based on a transmission line (TL) model of the antisymmetric mode in the pair of each dogbone, completely neglect the fringe capacitances at the TL ends and bends, and the capacitance between contiguous adjacent particles, and they work only for small thicknesses of the dielectric spacer H . However, though the TL model presented in [6] is able to estimate the magnetic resonance, it cannot predict how a plane wave interacts with a layer of dogbone pairs, which is instead one of the goals of this paper.

In this work we present a lumped element description of a metamaterial layer made of pairs of conductors. By postprocessing the reflection and transmission coefficients calculated by other means, like for instance using a full-wave code, a lumped element network is synthesized that exhibits the same frequency response when inserted in the plane-wave equivalent TL. The presence in the metamaterial response of both an electric (symmetric) and a magnetic (antisymmetric) resonance finds its correspondence in two respective resonant LC groups arranged in an equivalent balanced X-network. The equivalent network is useful for a quick numerical description of the layer but also reveals the physical operating mechanism of the metamaterial dictated by particle interactions. We show that not only the reflection and transmission features of a finite number of stacked layers of dogbone pairs are accurately predicted by stacking X-shaped networks attached to short TLs, but also the dispersion diagram of an infinite periodic stack of layers of pairs is accurately reconstructed by a Bloch analysis of a periodic arrangement of the X-shaped networks.

II. TRANSMISSION AND X-TL MODEL

The planar metamaterial is composed of a periodical arrangement of pairs of dogbone-shaped conductors, as illustrated in Fig. 1. Each constitutive element (i.e., each pair) supports two main resonances, a symmetric one (also denoted as electric resonance) and an antisymmetric one that is

associated to an equivalent magnetic dipole and therefore is denoted as magnetic resonance. By properly locating these two resonances, it is possible to design materials with artificial magnetism and also materials that support backward waves, as we will see in the following. We consider here reflection and transmission through just one layer of pairs (Fig. 1(a)) and we assume that the metallic conductors are separated by a dielectric spacer with $\epsilon_r = 2.2$ (e.g., RT Duroid 5880 with $\tan \delta = 0.0009$). The various geometrical parameters characterizing the unit cell of the considered metamaterial (cf. Fig. 1(b)) are as follows (in mm): $A = 7.5$, $B = 7.5$, $A1 = 0.5$, $B1 = 4$, $A2 = 7.4$, $B2 = 0.5$, $H = 1.0$. The dimension of the unit cell along the z -direction is $C = 8.0$ mm, therefore ports are defined at $z = \pm 4$ mm. The reflection and transmission coefficients R , and T , as obtained by numerical simulations are plotted in Fig. 2. In accordance with the dogbone-pair particle phenomenology discussed in [5], [6], the *magnetic* resonance occurs at $f = 6.9$ GHz, while the *electric* resonance is at $f = 7.3$ GHz. Here we just recall that the magnetic resonance corresponds to having an antisymmetric mode on the conductor pair, with the two currents flowing in opposite directions and thus producing an equivalent magnetic dipole along the y -direction for the case under consideration. This has been recognized as a way to achieve artificial magnetism by exploiting resonances in paired structures. A detailed discussion is provided in [6]. Here we further note that total transmission occurs near f_m , whereas no total reflection occurs.

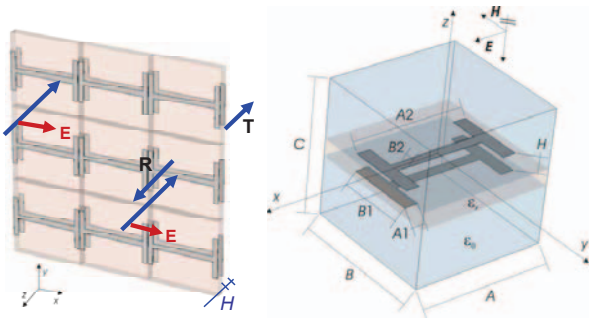


Fig. 1. (Left) Perspective view of a layer of the metamaterial formed by a periodic arrangement of pairs of dogbone-shaped conductors printed on a dielectric substrate. (Right) Metamaterial elemental particle (unit cell) with geometrical parameters quoted. The polarization of the incident electric field is along the x -direction.

A z -TL model which is able to predict the scattering parameters and the propagation characteristics through a structure layered along the z -direction is shown in Fig. 3, and consists in replacing the layer of arrayed thin pairs by the equivalent 2-ports network shown in Fig. 3, made by lumped elements. The choice of a unsuitable circuit topology would result in a non physical behavior for the lumped element impedance/admittance (e.g., a non Foster frequency dependence). A good strategy to arrange a proper circuit topology, which results in physical behaving lumped elements, is based on having a circuit whose elements directly represent the physical behavior (in terms of local magnetic and electric fields) of the various parts of the cell geometry.

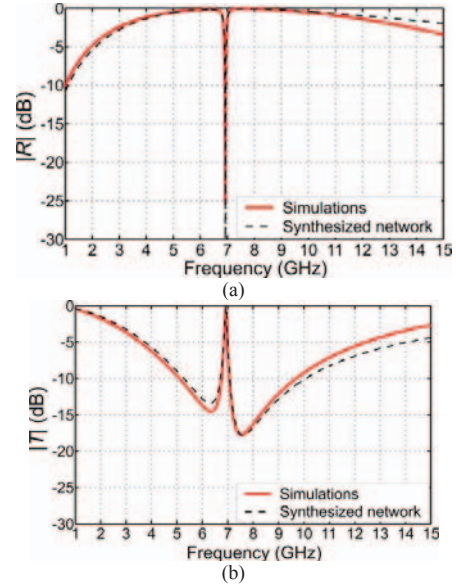


Fig. 2. Reflection and transmission coefficients vs. frequency for a layer of dogbones pairs: the *magnetic* resonance is at $f_m \approx 6.9$ GHz, whereas the *electric* resonance is at $f_e \approx 7.3$ GHz. Case with $\epsilon_r = 2.2$. Total transmission occurs near f_m . Total reflection *never* occurs. Data from a numerical analysis are compared with those from our proposed equivalent synthesized network in Sec. III.

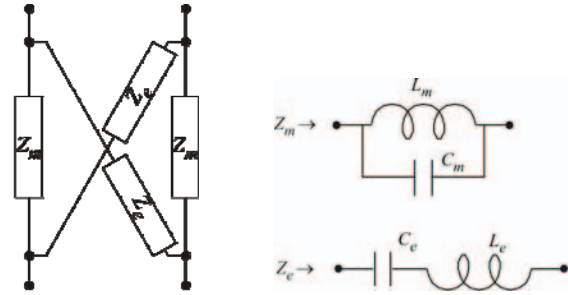


Fig. 3. (a) Plane-wave equivalent z -transmission line. (b) Synthesized symmetric X-network reproducing the metamaterial layer frequency response, which comprises two LC groups associated to the metamaterial electric and magnetic resonances.

Note that validity range of the equivalent network concept is usually limited just to the first resonances. Once the equivalent network topology is given, the steps that lead to the calibration of the involved circuitual elements are given in the following. The effectiveness of the choice of the X-shaped network relies on the fact that its elements can be represented by simple physical L - C circuits; indeed Z_m and Z_e satisfy the Foster's theorem.

For simplicity, in our simulations we neglect the losses in the metals and in the ambient. Indeed, the dielectric substrate separating the pair elements has been chosen to be lossless, since as we have shown [7], [8] a small amount of losses do not affect the performance of the metamaterial. Therefore, the impedances of the X-shaped network are made only by L - C

elements because we aim at highlighting the main features of our proposed network. We assume that the elements of our proposed equivalent X-network are as shown in Fig. 3, where the resonant frequencies are

$$\omega_m = 2\pi f_m = \frac{1}{\sqrt{C_m L_m}}, \quad \omega_e = 2\pi f_e = \frac{1}{\sqrt{C_e L_e}}, \quad (1)$$

which are denoted as *magnetic* and *electric* resonances, respectively. We now use the L, C representations in Fig. 3, for Z_e and Z_m . By data fitting the curves in Fig. 2, the L, C parameters are evaluated as $C_m = 7.62$ pF, $L_m = 0.07$ nH, and $C_e = 0.125$ pF, $L_e = 3.8$ nH. Note that $C_m \gg C_e$ and that $L_m \ll L_e$, therefore the resonance ω_m can be of the same order of ω_e . The scattering parameters in Fig. 2 obtained with the synthesized TL and X-network are in good agreement with those obtained numerically. The electric and magnetic frequencies (1) are now determined as: $f_m = 6.89$ GHz, and $f_e = 7.30$ GHz. Again, the electric frequency f_e corresponds to the wide stop band, whereas the magnetic frequency f_m corresponds to the narrow band phenomena with the local maximum of transmission.

III. DISPERSION DIAGRAMS AND THEIR PREDICTION BY THE TL MODEL

In the following we show that our proposed X-shaped circuit in the z -TL model in Fig. 3 is able to predict correctly also the dispersion diagram when layers are stacked periodically along the z -direction. Here we assume that the period along z is $C = 2.6$ mm. The dimensions relative to each layer of arrayed pairs are as follows (in mm): $A = 7.5, B = 7.5, A1 = 1.0, B1 = 4, A2 = 7.4, B2 = 0.8$. In each layer, the dogbone conductors are separated by a spacer with $\epsilon_r = 1$ and various thicknesses $H = 0.2, 0.4, 0.6$ mm are considered. The dispersion diagram in Fig. 4 obtained by CST Microwave Studio shows the frequency versus normalized propagation constant $k_M C$ (therefore k_M is the Bloch wavenumber along the z -direction). Fig. 4(a) shows a low frequency pass band, followed by a bandgap and another passband around 10 GHz. As can be inferred from the negative slope branch (zoom in Fig. 4(b)), the structure clearly supports backward propagation along the z -direction (for each choice of H) as already observed in [6], [7], [9].

IV. CONCLUSION

A metamaterial in planar technology has been proposed recently. In the context of a plane-wave transmission line model, we have synthesized a circuit network reproducing the frequency response of a metamaterial made of a periodic arrangement of conductor pairs. The transmission line model is an effective tool to predict propagation through a number of layers. This network description offers the possibility to evaluate Bloch wavenumbers and characteristic impedances, and to match metamaterials formed by stacked layers to the free space impedance. Furthermore, it provides a neat physical

description in terms of transmission lines and lumped elements.

The proposed TL network is also used to explain maxima and minima of the transmission and reflection coefficient, providing necessary and sufficient conditions for their occurrence.

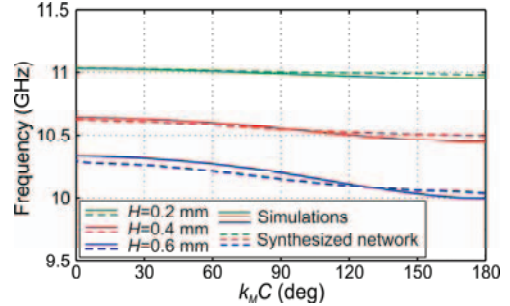


Fig. 4. Dispersion diagrams for layers of arrayed dogbone pairs periodically stacked along the z -direction with period C . We show only the backward wave curve for three different thicknesses of the pairs spacer.

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