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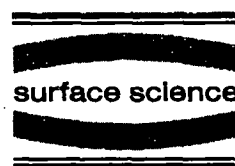
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Activated conductivity in the quantum Hall effect

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Abstract

Activated dissipative conductivity $\sigma_{xx} = \sigma_{xx}^* \exp(-A/T)$ and the activated deviation of the Hall conductivity from the precise quantization $\delta\sigma_{xy} = \sigma_{xy} - ie^2/h = \sigma_{xy}^* \exp(-A/T)$ are studied in a plateau range of the quantum Hall effect. The prefactors σ_{xx}^* and σ_{xy}^* are calculated for the case of a long-range random potential in the framework of a classical theory. There is a range of temperatures $T_1 \ll T \ll T_2$ where $\sigma_{xx}^* = e^2/h$. In this range $\sigma_{xy}^* \approx (e^2/h)(T/T_2)^{80/21} \ll \sigma_{xx}^*$. At large $T \gg T_2$, on the other hand, $\sigma_{xy}^* = e^2/h$ and $\sigma_{xx}^* = (e^2/h)(T_2/T)^{10/13} \ll \sigma_{xy}^*$. Similar results are valid for a fractional plateau near the filling factor p/q if charge e is replaced by e/q .

Keywords: Conductivity; Hall effect

The temperature dependence of the activated dissipative conductivity is widely used to study energy gaps in the quantum Hall effect. When the Fermi level lies in the middle between two Landau levels σ_{xx} has the form [1–4]

$$\sigma_{xx} = \sigma_{xx}^* \exp(-A/T), \quad (1)$$

provided the temperature T is not too low. The prefactor σ_{xx}^* has attracted a great deal of interest since σ_{xx}^* was claimed [2] to be equal to e^2/h independently of the Landau level number. This puzzling universality was observed in the fractional quantum Hall effect as well: at filling $\nu = p/q$, the prefactor $\sigma_{xx}^* = (e/q)^2/h^2$. On the other hand, both the numerical value of the prefactor [4] and the

very independence of σ_{xx}^* from T have been questioned [3].

There have been several interesting attempts to calculate the prefactor universally for any type of disorder, using the idea that at $T = 0$ an extended state exists at only one energy, and phase-breaking processes are responsible for the delocalization of electrons within a narrow band of states near the Landau-level center [2,3,5–8]. Nevertheless, it has not been shown in these works that a range of temperatures exists where $\sigma_{xx}^* = e^2/h$.

Polyakov and Shklovskii [9,10] suggested a more specific approach and considered separately the cases of a short-range and a long-range random potential. In Ref. [9] they evaluated explicitly two major contributions to the conductivity for the case of a white noise potential: (i) from a narrow band of delocalized states which appear near the Landau-level center at finite temperatures, and (ii) from the variable range hopping between localized states in the tail of the density of states. It

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was shown that, in agreement with the conjecture made in Ref. [8], the interplay of these contributions is responsible for the inflection point in the dependence of $\ln \sigma_{xx}$ on $1/T$. Polyakov and Shklovskii demonstrated that in the vicinity of the inflection point, σ_{xx} is a very slow function of the rate of phase-breaking processes, and when fitted to the activation form (Eq. (1)) gives σ_{xx}^* somewhat smaller than e^2/h . Polyakov and Shklovskii therefore concluded that, strictly speaking, no universality can be obtained for the case of a short-range potential. In Ref. [10], Polyakov and Shklovskii calculated the prefactor σ_{xx}^* in the case of a long-range random potential $\epsilon(r)$, which exists in high-mobility heterojunctions. In this case tunneling is suppressed, and at values of T which are not too low, the conductivity may be described by a completely classical theory. It was shown that there is a range of temperatures where the prefactor is constant and universal. Polyakov and Shklovskii obtained $\sigma_{xx}^* = e^2/h$ for the contribution of a single Landau level to the conductivity in this range. The Planck constant appears in this expression only through the density of states at the Landau level.

Thermally activated electrons are also responsible for the deviation $\delta\sigma_{xy}$ from the precise quantization. At $T = 0$ the magnitude of σ_{xy} on each plateau is ie^2/h , where i is an integer. At finite temperatures, the deviation $\delta\sigma_{xy} = \sigma_{xy} - ie^2/h$ from the precise quantization appears. This deviation is the second subject of this paper. First of all, we note that directly at the center of the plateau, $\delta\sigma_{xy}$ must vanish due to the electron-hole symmetry. Therefore, we consider the case when, say, the electron contribution to σ_{xy} dominates. This occurs when the Fermi energy is in the gap between two Landau levels but is closer to the upper one by at least several T . If the temperature is not too low, so that variable-range hopping can be neglected, then $\delta\sigma_{xy}$ is determined by electrons thermally activated to the extended state at the classical percolation level $\epsilon = 0$ at the upper Landau level. One would expect

$$\delta\sigma_{xy} = \sigma_{xy}^* \exp(-A/T), \quad (2)$$

where the activation energy A is the separation between the Fermi energy and the percolation level. The prefactor σ_{xy}^* was calculated in Ref. [11].

Below we will repeat the derivation of prefactors σ_{xy}^* and σ_{xx}^* and discuss the relation between them.

Let the correlation length d of the disorder potential be much larger than the magnetic length l_B . One can think of d as the setback distance of the nearby doping layer with charged impurities in a high-mobility heterostructure. The concentration of the activated electrons

$$n = (1/2\pi l_B^2) \exp[-(A + \epsilon)/T]$$

is exponentially small, and we neglect the interaction between them. In the absence of inelastic scattering, guiding centers of electron orbits drift along the contours of constant ϵ , all of which, with the exception of the one at $\epsilon = 0$, are closed loops. In this semi-classical picture the tunneling between two loops at the same energy ϵ is neglected. The tunneling occurs in the vicinity of saddle points where the loops come close to each other. However, the probability of such tunneling falls off with increasing barrier height V as $\exp(-V/T_1)$, where $T_1 \approx W(l_B/d)^2$, with W being the standard deviation of $\epsilon(r)$ ¹². Below, we consider the wide temperature range $T_1 \ll T \ll W$ and completely neglect the tunneling. Another important element of the model are inelastic electron-phonon processes. Due to these processes, electrons can hop from one closed loop to another and thus travel across the entire sample. We assume that these inelastic processes are frequent enough, i.e. that their rate τ_{ph}^{-1} is much higher than the characteristic frequency of drift motion. This inequality also implies that quantum coherence is completely destroyed by the inelastic collisions, and together with the fact that quantum-mechanical tunneling is suppressed, it means that in our model the motion of electrons is purely classical.

Typical hops occur between two contours a distance l_B from each other, and so the characteristic phonon energy is $T_{ph} = Wl_B/d$. The scattering rate can be readily evaluated using Fermi's golden rule to be $\hbar/\tau_{ph} = \alpha T$ for $T \gg T_{ph}$. For $T \ll T_{ph}$, one has to replace α by $(\alpha T_{ph})/(\sqrt{2\pi}T)$. Here $\alpha = \Xi^2/\rho\hbar s^3 l_B^2$ is the electron-phonon coupling constant, and Ξ and ρ are the deformation potential constant and the crystal mass density, respectively. For GaAs, $\alpha \approx 0.03(100 \text{ \AA}/l_B)^2$ and this is the small parameter in this problem. Within this model we

obtained

$$\sigma_{xx}^* = \frac{e^2}{h} \begin{cases} (1 + \alpha^2)^{1/2}, & T_1 \ll T \ll T_2 \\ c_1 (T_2/T)^{10/13} + O(\alpha^2), & T \gg T_2, \end{cases} \quad (3)$$

where T_2 is defined as $\alpha^{3/10} W$. For σ_{xy}^* we obtained

$$\sigma_{xy}^* = \frac{e^2}{h} \begin{cases} c_2 (T/T_2)^{8/3} (T_{ph}/T_2)^{8/7}, & T_1 \ll T \ll T_{ph} \\ c_3 (T/T_2)^{80/21}, & T_{ph} \ll T \ll T_2 \\ 1, & T \gg T_2. \end{cases} \quad (4)$$

Here c_i s are numerical factors of order unity and we assume that $T_1 \ll T_{ph} \ll T_2$, which is equivalent to $d \gg \alpha^{-3/10} l_B$. We see that the relation between $\delta\sigma_{xy}$ and σ_{xx} is more complicated than a power law. However, the main T -dependence comes from the $\exp(-d/T)$ factors, and therefore one can say that the temperature-driven dependence $\delta\sigma_{xy}(\sigma_{xx})$ is nearly linear. We would also like to note an intriguing “complementary” behavior of the prefactors σ_{xx}^* and σ_{xy}^* . At $T \gg T_2$, $\sigma_{xy}^* = e^2/h$ and $\sigma_{xx}^* \ll \sigma_{xy}^*$, while at lower temperatures, $\sigma_{xx}^* = e^2/h$ but $\sigma_{xy}^* \ll \sigma_{xx}^*$.

We now turn to explanations of our results. Electron–phonon scattering gives rise to diffusion with the diffusion coefficient $D \approx l_B^2/\tau_{ph}$, and therefore the local dissipative conductivity $\sigma_{xx} = \sigma_{yy} = e^2 n D/T$. We will consider explicitly only the case $T \gg T_{ph}$ and then transfer the results onto lower temperatures $T \ll T_{ph}$ replacing α with $\alpha T_{ph}/T$ where necessary. For $T \gg T_{ph}$, the diagonal component of the conductivity tensor is $\sigma_{xx} = (e^2/h)\alpha \exp[-(d + \epsilon)/T]$. As for the off-diagonal components, they are given by $\sigma_{xy} = -\sigma_{yx} = (e^2/h) \exp[-(d + \epsilon)/T]$. Suppose the system is in a uniform external electric field E . The current then satisfies the equations

$$\mathbf{j} = \frac{1}{e} \{ \sigma_{xx} \nabla \mu + \sigma_{xy} [\mathbf{z} \times \nabla \mu] \}, \quad \mathbf{j} = [\mathbf{z} \times \nabla \psi], \quad (5)$$

where $\mu(\mathbf{r})$ is the electrochemical potential and $\psi(\mathbf{r})$ is the stream function. With the appropriate boundary conditions and the condition that the spatial average of $\nabla \mu$ is equal to eE , Eq. (5) fully determines $\mathbf{j}(\mathbf{r})$. For this system there exists a

remarkable exact relation [13]

$$(\sigma_{xx}^*)^2 + (\sigma_{xy}^*)^2 = (e^2/h)^2 (1 + \alpha^2). \quad (6)$$

Unfortunately, it does not shed light on the individual behavior of σ_{xx}^* and σ_{xy}^* . This individual behavior is the subject of the present paper.

We begin by examining a chessboard geometry where

$$\epsilon = W \sin \left[\frac{\pi(x+y)}{\sqrt{2}d} \right] \left[\frac{\pi(x-y)}{\sqrt{2}d} \right],$$

with the average electric field in the $\hat{y}$ direction (Fig. 1). The regions $\epsilon(\mathbf{r}) \gtrless T$ (hills) have very low conductance, and hence the current avoids them. It flows mainly in the valleys where $\epsilon(\mathbf{r}) < 0$, and crosses from one valley to another via the saddle points (where $\epsilon = 0$). Once the current passes the saddle point it flows along the slope of the valley, gradually deviating from the contour $\epsilon = 0$ towards the more conductive bottom. The current distribution can be inferred from studying two basic elements of the chessboard: a saddle point and a slope. First consider an isolated saddle point

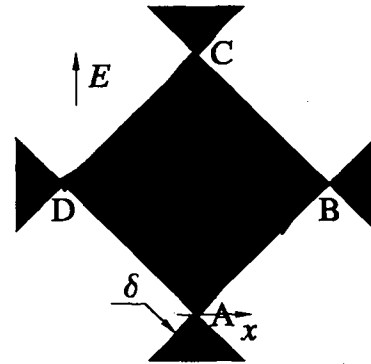


Fig. 1. Illustration of our analytical solution of Eq. (5) for a chessboard of valleys (dark) and hills (blank). The currents through saddle points A and B determine σ_{xx}^* and σ_{xy}^* , respectively. The latter current (shown by the dashed line with arrow) is exponentially small. The main current (the black arrow) bypasses B. It enters the valley through saddle point A, then deviates from the perimeter of the valley, spiraling counterclockwise towards the interior. It then spirals out clockwise to exit the valley using saddle point C. This clockwise part of the spiral can be interpreted as the stream of holes injected through saddle point C, which spiral counterclockwise to the interior, where the holes recombine with the electrons injected through saddle point A.

$\epsilon(r) = W(\pi^2/2d^2)(x^2 - y^2)$. We choose the boundary conditions $\mu(x, -\infty) = 0$, $\mu(x, \infty) = eU$ and look for the solution that has the full symmetry of the problem: $\mu(x, y) + \mu(-x, -y) = eU$. The desired solution is [14]

$$\psi = (GU/2) \operatorname{erfc}[(x + \beta y)/\sqrt{2}\delta],$$

$$\mu = (U/2) \operatorname{erfc}[(x - \beta y)/\sqrt{2}\delta],$$

where $\beta = \alpha + \sqrt{1 + \alpha^2}$, $\delta^2 = 2(d/\pi)^2 \alpha \beta T/W$. The quantity $G = (e^2/h)\sqrt{1 + \alpha^2} \exp(-\Delta/T)$ can be identified as the conductance of the saddle point. One observes from this solution that the current flows through the saddle point in a thin stream of width δ (Fig. 1). The stream deviates by a small angle $\alpha/2$ from the contour $\epsilon = 0$ towards the bottom of the valley. With E in the y direction, the current through the saddle point A (Fig. 1) determines σ_{xx}^* , and the current through saddle point B gives σ_{xy}^* . Our solution for an isolated saddle point is of use if the deviation of the current stream upon traveling the distance $AB = d$ exceeds the width of the stream δ . In this case, only an exponentially small fraction of the current which enters through saddle point A, leaves the valley through saddle point B.

The equivalent circuit of the chessboard is a square network of identical conductances $(e^2/h)\sqrt{1 + \alpha^2} \exp(-\Delta/T)$ (saddle points) connecting the reservoirs (valleys). The current through saddle point A is GU , where $U = eEd\sqrt{2}$ is the voltage drop across this saddle point. Similarly, the current through B is almost zero since there is no voltage drop across this saddle point. Therefore

$$\sigma_{xy}^* = 0, \quad \sigma_{xx}^* = \frac{e^2}{h} \sqrt{1 + \alpha^2}. \quad (7)$$

To examine the range of validity of Eq. (7), we need to know how a thin stream of current propagates along the slope of the chessboard.

For simplicity, consider a uniform slope $\epsilon(X, Y) = -WX/d$. Eq. (5) may be written as $-D\nabla^2\psi + v\nabla\psi = 0$, where $v_y = (I_B^2/\hbar)(W/d)$ and $v_x = \alpha v_y$. This equation is identical to the one describing the diffusion of a tracer in a hydrodynamic flow of constant velocity v [15]. This analogy suggests that the current flows in a thin stream;

the stream is directed along v , thus making only a small angle with respect to the contour of constant ϵ . As the current flows along, the stream spreads according to the diffusion law $\delta \approx \sqrt{2Dt}$, where $t = p/v_y$ is the "time" to travel the distance p from the origin of the stream to a given point. Correspondingly, if the stream originates at $(0, 0)$ then the fraction of current that reaches the point $(0, p)$ is of the order of $\exp[-(v_x t)^2/2\delta^2] = \exp(-p/L)$, where $L = 4dT/\alpha W$. We see that this fraction is exponentially small if p exceeds the "relaxation length" L . Applying this consideration to the slope between saddle points A and B (Fig. 1), we find that Eq. (7) is valid if the temperature is low enough: $T \ll T_2^{\text{ch}}$, where $T_2^{\text{ch}} = \alpha W$. In fact, we understand now that in this regime a non-zero σ_{xy}^* only appears due to the fraction of current propagating along the path AB.

Our calculation gives

$$\sigma_{xy}^* = (e^2/h)\sqrt{8T/\pi T_2^{\text{ch}}} \exp(-T_2^{\text{ch}}/2T).$$

Similarly, σ_{xx}^* differs from the value given by Eq. (7) due to the current propagating along the ABC, which is twice as long (Fig. 1). Correspondingly, the correction to σ_{xx}^* is of the order of $(\sigma_{xy}^*)^2$ (both in units of e^2/h), in agreement with Eq. (6).

Let us return to the random system. In this case, reservoirs are not single valleys but collections of many of them with the typical diameter $\xi = d(W/T)^v$ and perimeter (or hull) $p = d(\xi/d)^{d_H}$, where $v = 4/3$ and $d_H = 7/4$ is the fractal dimension of the hull [15]. In the spirit of our analysis of the chessboard, we can say that the fraction $\exp(-p/L)$ of the current entered through a saddle point of type A can exit through the neighboring saddle point of type B. This fraction is exponentially small if $T \ll T_2$, where $T_2 = \alpha^{3/10}W$. It means that, as in the case of a chessboard, the equivalent circuit is the network of independent conductances. It has been proven [10] that for this network σ_{xx}^* is still given by Eq. (7). But the conclusion that σ_{xy}^* is also exponentially small would be incorrect. The reason is that in a random system, the distance p along the perimeter between two critical saddle points might be as small as L , i.e., much less than a typical distance. Such an event gives rise to a "Hall generator" contributing $\sim e^2/h$ to σ_{xy}^* . The

Hall conductivity of the sample will then be proportional to the probability of finding such a rare generator in a square with the sides of length ξ [16]. The estimate of this probability [11] leads to the result $\sigma_{xy}^* = (e^2/h)(T/T_2)^{80/21}$ (Eq. (4)). As explained above, the results for the case $T_1 \ll T \ll T_{ph}$ are obtained by replacing α with $\alpha T_{ph}/T$, which changes the exponent from 80/21 to 8/3 (Eq. (4)).

Until now, we have considered a situation when the Fermi level is far enough from the percolation level so that $\exp(-\Delta/T) \ll (T/T_2)^{80/21}$. Consider now the opposite case, where almost all of the area of each valley is covered by the “lake” of totally occupied states, and the spiral in Fig. 1 reaches the line DB, already inside the lake. It means that almost all the current I flows through the lake and therefore induces a Hall voltage Ih/e^2 across the lake in the direction DB. Due to the periodicity of the chessboard, a voltage of the same magnitude is applied to the saddle point B, producing the current GIh/e^2 in the y direction. Using $G = \sigma_{xx}$ and $I = \sigma_{xx}U$, we obtain [17] $\delta\sigma_{xy} = (h/e^2)\sigma_{xx}^2$, in agreement with Refs. [14,18]. Thus, with decreasing Δ , crossover from the linear dependence $\delta\sigma_{xy} = f(T)\sigma_{xx}$ to $\delta\sigma_{xy} \propto \sigma_{xx}^2$ is predicted. Note that in the latter regime, the activation energy for $\delta\sigma_{xy}$ is twice as large as in Eq. (2).

Finally, we briefly discuss transport at temperatures away from the range $T_1 \ll T \ll T_2$. If $T \lesssim T_1$, tunneling and variable-range hopping become important. The calculation of the Hall conductivity in this regime remains an unsolved problem. In the opposite limit $T \gg T_2$, σ_{xy} approaches e^2/h . Indeed, at these temperatures the deviation of the current stream from the contour $\epsilon = 0$ is much smaller than the width of the stream, i.e. $\alpha p \ll \delta$. The current flows mainly within the strip of width δ centered around the contour $\epsilon = 0$. It may be verified that within the strip, $|\epsilon| \ll T$ and, therefore the Hall conductivity $\sigma_{xy} = (e^2/h) \exp[-(\Delta + \epsilon)/T]$ is approximately constant. In this case, the sample averaging of σ_{xy} necessary for calculating σ_{xy}^* trivially gives $\sigma_{xy}^* = e^2/h$. As for σ_{xx}^* , it can be found from the prefactor ρ_{xx}^* of the diagonal resistivity $\rho_{xx} = \rho_{xx}^* \exp(-\Delta/T)$ with the help of the relation $\sigma_{xx}^* = \rho_{xx}^*(\sigma_{xy}^*)^2$. Now, ρ_{xx}^* can be estimated as the resistance of a wire of width δ , length p , resistiv-

ity $\alpha h/e^2$, which yields $\sigma_{xx}^* = (e^2/h)(T_2/T)^{10/13}$, in agreement with Refs. [10,15].

In conclusion we would like to say a few words about related experimental data. Unfortunately, in high-mobility samples the temperature dependence of only σ_{xx} , and only at its minima, was studied. We propose to investigate the temperature dependence of both $\delta\sigma_{xy}$ and σ_{xx} away from the very minima of σ_{xx} to verify the predictions of Eqs. (3) and (4), and the relationship between them given by Eq. (6).

When the Fermi level is exactly in the middle between two Landau levels in the minima of σ_{xx} , the absolute values of contributions of electrons and holes to σ_{xx} and σ_{xy} should be equal, and we arrive at $\sigma_{xx}^* = 2e^2/h$ and $\sigma_{xy}^* = 0$. Our result for σ_{xx}^* differs by a factor of two from the values claimed in Ref. [2]. Note, however, that larger values of σ_{xx}^* were reported by another group [4]. Moreover, recently σ_{xx}^* has been found [3] to be proportional to $1/T$. We do not know how to resolve these contradictions. It seems that more experimental evidence is needed.

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