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Bit Interleaved Coded Multiple Beamforming in Millimeter-Wave Massive MIMO Systems

DISSERTATION

submitted in partial satisfaction of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

in Electrical Engineering

by

Sadjad Sedighi

Dissertation Committee:
Professor Ender Ayanoglu, Chair
Professor Lee Swindlehurst
Professor Nader Bagherzadeh

2021

DEDICATION

To the passengers of PS752.

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Thank you.

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ABSTRACT OF THE DISSERTATION

Bit Interleaved Coded Multiple Beamforming in Millimeter-Wave Massive MIMO Systems

By

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Doctor of Philosophy in Electrical Engineering

University of California, Irvine, 2021

Professor Ender Ayanoglu, Chair

An analysis of the diversity gain for bit-interleaved coded multiple beamforming (BICMB) method and BICMB with perfect coding (BICMB-PC) in millimeter-wave (mm-Wave) massive multiple-input multiple-output (MIMO) systems is carried out for both the single-user and multi-user scenarios. We show that full diversity gain and full spatial multiplexing order can be achieved in both scenarios. Also, we show that the diversity gain becomes independent of the number of transmitted data streams and the number of antennas in each remote antenna unit (RAU) at the transmitter and the receiver. Furthermore, in the multi-user scenario, the diversity gain is independent from the number of users in the system, when BICMB or BICMB-PC is utilized. Furthermore, we study BICMB with orthogonal frequency division multiplexing (OFDM) in order to achieve maximum diversity gain in mm-Wave massive MIMO distributed antenna subarray systems. The assumption here is that the channel state information (CSI) is known at both sides of the transmitter and the receiver and the number of antennas in each RAU tends to infinity. This latter assumption can be relaxed by a large number of antennas in each RAU, similar to the case for all massive MIMO research. Simulation results show that when the perfect CSI assumption is satisfied, the use of BICMB results in the diversity gain values predicted by the analysis.

Chapter 1

Introduction

Millimeter-wave (mm-Wave) massive multiple-input multiple-output (MIMO) is an important part of the Fifth Generation (5G) communication systems. It enables increasing the data rates and helps in accommodating the billions of wireless devices whose numbers increase exponentially each year [3, 4, 5]. Despite of its substantial gains, mm-Wave massive MIMO brings challenges. Severe penetration loss and path loss in the mm-Wave signals compared to signals in former and current cellular systems (e.g., 3G or 4G LTE) are two of the challenges [6].

One of the advantages of the mm-Wave frequencies is that they enable one to pack more antennas in the same area compared to a lower range of frequencies. This leads to highly directional beamforming and large-scale spatial multiplexing¹ in mm-Wave frequencies. The principles of beamforming are independent of the carrier frequency, but it is sometimes suggested not to use fully digital beamforming schemes for massive MIMO systems for practical reasons [9, 10, 11, 12]. Power consumption and cost perspectives are the main obstacles due to the high number of radio frequency (RF) chains required for the fully digital beamforming,

¹In this thesis, the terms "spatial multiplexing" or "spatial multiplexing order" are used as in [7] to describe the number of spatial subchannels. Note that this term is different from "spatial multiplexing gain" defined in [8]

i.e., one RF chain per antenna element [13]. To address this problem, hybrid analog-digital processing of the precoder and combiner for mm-Wave communications systems is being considered [14, 15, 16, 17, 18, 19, 20], where [20] proposes an algorithm to calculate the beamforming matrices in a closed form.

Recently, the diversity gain of a mm-Wave massive MIMO system with distributed antenna-subarray architecture is studied in [21]. The authors of [21] have shown that the diversity gain depends on the number of transmitted data streams. Therefore, by transmitting more data streams, we get a lower diversity gain. In [21], one can see that the diversity gain can be increased by increasing the number of remote antenna units in the distributed antenna array systems, both in the transmitter side or the receiver side. Increasing the diversity gain in mm-Wave MIMO systems by using iterative eigenvalue decomposition (EVD) is studied in [22]. In [22], the authors propose an algorithm to find the antenna weight vectors such that the full diversity gain is achieved. The system model used in [22] is co-located and the maximum achievable diversity gain is the number of multipath components (MPC), i.e., L .

The author in [23] proposes a suboptimal spatial diversity scheme by tracing the shadow process which outperforms the equal-gain spatial diversity technique in terms of link margin. The asymptotic average bit error probability is analyzed in [24] for a spatial scattering modulation in the mm-Wave MIMO system. The authors in [24] investigate the impacts of wireless channel properties and modulation order on the performance of spatial scattering modulation systems. In [25], a combination of space-time block coded spatial modulation with hybrid analog-digital beamforming is used to achieve the full diversity gain for mm-Wave MIMO systems. The proposed system in [25] benefits from the multiplexing gain of spatial multiplexing, from the transmit diversity gain of space-time block codes and from the Signal-to-Noise Ratio gain of the beamformer.

Bit-interleaved coded modulation (BICM) was introduced as a way to increase the code diversity [26, 27]. Bit-interleaved coded multiple beamforming (BICMB) has a substantial

impact on the diversity gain performance of a conventional MIMO system as stated in [28, 29, 30]. Authors in [28, 29, 30] have shown by implementing BICMB, one can achieve the maximum diversity gain in a MIMO system.

In [31, 2], we investigate the use of BICMB to increase the diversity gain for both single-user and multi-user scenario in mm-Wave massive MIMO systems with distributed antenna-subarray architecture. We show that by implementing BICMB in these systems, the maximum diversity gain can be achieved.

Perfect space-time block codes (PSTBC) were studied in [32, 33, 34] to achieve full rate and full diversity in any dimension. However, dimensions 2, 3, 4 and 6 are the only dimensions that can achieve an increase in the coding gain. In [35], perfect coding with multiple beam-forming is used to achieve full diversity and full multiplexing in a MIMO system with less decoding complexity than a system employing PSTBC and full precoded multiple beam-forming (FPMB). In [1], channel coding is added to the perfect coding and diversity gain analysis is carried out to prove that BICMB with perfect coding (BICMB-PC) achieves the maximum diversity gain.

Space-time block codes are studied in massive MIMO literature as well. In [36], space-time block codes are used to achieve full diversity gain in a flat fading non coherent wireless communication system. Also, in [37] massive space-time block code (MaSTBC) is studied. Authors in [37] proposed a novel space-time modulation scheme with PSK modulated MaSTBC for multi-user massive MIMO uplink systems.

Lately, we used BICMB with perfect coding (BICMB-PC) in the same architecture to maximize the diversity gain [38]. The comparison of BICMB-PC and BICMB can be found in Chapter 6.

1.1 Overview of Dissertation

Studying BICMB in real world scenarios shows that there are new challenges which we need to address. As an example, when the base station is serving multiple users, the singular value decomposition which is used for beamforming technique in the single user scenario in [31] is no longer valid. We need a new beamforming method not only to eliminate the inter-user-interference, but also to be a closed form beamforming method, so we can get tighter upper-bound for pairwise error probability, which leads to a closed form diversity gain formula. Furthermore, the effect of imperfect CSI is studied in Chapter 9. The importance of this Chapter is that the assumption of perfect CSI in real-life applications of mm-Wave massive MIMO systems is not valid. Chapter 9 is intended to overcome this challenge and to see if the basic result still holds when we do not have perfect CSI in our system and how much imperfection can affect the diversity gain. The rest of the thesis is organized as follows. In Chapter 4, BICMB for the single-user scenario is analyzed. We first define the system model and channel model we use in the analysis. Then, we show that by using BICMB in the system, one can achieve full spatial multiplexing without any loss in the diversity gain. That is, in Chapter 4, we show that BICMB achieves full diversity order of $M_r M_t L$ and full spatial multiplexing order of $M_r M_t L$ in a special case when the number of propagation paths is the same for all paths between remote antenna units (RAUs), i.e., $L_{ij} = L$ over the mm-Wave channel. We provide design criteria for the interleaver that guarantee full diversity and full spatial multiplexing.

In Chapter 5, the system model and channel model for the multi-user scenario are introduced. In this chapter, a hybrid beamforming method [20] is used to eliminate the inter-user interference and to maximize the achievable rate. The difference of the beamforming defined in [20] with the other beamforming methods for multi-user mm-Wave massive MIMO systems is its closed form expression where the precoder and combiner matrices can be calculated explicitly without any need for iteration. This makes it much easier to find the upper-bound

for the pairwise error probability which we use to find the diversity gain formula. Similar to Chapter 4, in Chapter 5, we show that for the multi-user scenario, BICMB achieves full diversity order of ML in a special case when the number of propagation paths is constant for all paths between RAUs, i.e., $L_{ij} = L$ over the mm-Wave channel where M is the number of RAUs at the transmitter. We will also show that the number of user does not have any impact on the diversity gain, and increasing the number of users in the system, does not change the diversity gain for other users.

In Chapter 6, we use BICMB-PC to achieve full diversity gain in a single-user mm-Wave massive MIMO systems with distributed antenna arrays. In this chapter, we use the same channel model used in previous chapter in this thesis. The system model is pretty similar to the one we used in Chapter 4, where only difference is data manipulation before precoding at the transmitter and after combining at the receiver. After setting up both the system model and channel model, the diversity analysis for this system is carried out. We show that by using perfect coding in addition to convolutional coding, i.e., BICMB-PC the diversity gain becomes independent of the number of transmitted data streams. The diversity gain is calculated as $G_d = D^2L$ where D is the perfect coding number which also is the number of RAUs at both the transmitter and the receiver, and L is the number of propagation paths between each RAU at the transmitter and the receiver. The interleaver design plays an important part in BICMB-PC as well, where we discuss the criteria for this design as well to achieve the maximum diversity gain. The comparison of BICMB-PC with BICMB where the number of RAUs at both the transmitter and the receiver in both scenarios are the same is studied in the chapter on simulation results. Furthermore, in the simulation results chapter, the comparison of BICMB-PC with conventional MIMO systems employing Rayleigh fading channel is studied.

In Chapter 7, BICMB-PC for multi-user scenario is studied. Similar to previous sections, we use perfect coding and BICMB in order to achieve the maximum diversity gain for the

multi-user scenario. We show that the diversity gain does not depend on the number of users in the system. Similar to Chapter 5, the diversity gain in a simple case scenario where the large scale fading coefficients and the number of propagation paths remain the same for all subchannels, is DL , where D is the number of RAUs at the transmitter side and L is the number of propagation paths in the system.

In Chapter 8, we study BICMB-OFDM. By employing BICMB, we prove that the maximum diversity gain can be achieved for mm-Wave OFDM massive MIMO system with distributed antenna subarray.

We would like to emphasize that the asymptotical diversity analysis obtained in the chapters studied so far is under the idealistic assumption of having perfect CSI at the transmitter and the receiver as done in similar works. The effect of imperfect CSI on the diversity gain is studied in Chapter 9. We show that the maximum diversity gain can be achieved with imperfect CSI, as long as the accuracy of the channel estimation is sufficiently large. Finally, conclusions are presented in Chapter 10.

1.2 Notation

Boldface upper and lower case letters denote matrices and column vectors, respectively. The minimum Hamming distance between any two codewords in a convolutional code is defined as the free distance d_{free} . The symbol N_s denotes the total number of symbols transmitted at a time. The minimum Euclidean distance between the two constellation points is given by d_{min} . The symbols $(\cdot)^H, (\cdot)^T, (\cdot)^*, (\bar{\cdot})$ and $\forall$ denote the Hermitian, transpose, complex conjugate, binary complement, and for all, respectively. $\mathcal{CN}(0, 1)$ denotes a circularly symmetric complex Gaussian random distribution with zero mean and unit variance. The expectation operator is denoted by $\mathbb{E}[\cdot]$. $[\mathbf{A}]_{ij}$ or $\mathbf{A}(i, j)$ gives the (i, j) th entry of matrix $\mathbf{A}$. $\mathbf{A}(i, :)$

and $\mathbf{A}(:, j)$ represent the i th row of the matrix $\mathbf{A}$ and j th column of the matrix $\mathbf{A}$, respectively. Finally, $\text{diag}\{a_1, a_2, \dots, a_N\}$ stands for a diagonal matrix with diagonal elements $\{a_1, a_2, \dots, a_N\}$.

Chapter 2

Preliminaries

In this chapter, some background knowledge related to the dissertation is presented. In Section 2.1, the mm-Wave channel models used in this dissertation for both flat fading and frequency selective fading are presented. In Section 2.2, the idea of diversity in wireless communications is introduced. In Section 2.3, two hybrid beamforming methods used in this thesis are presented. The first method is used in the single-user scenario case, while in order to eliminate the inter-user interference, the second method is more efficient.

2.1 Millimeter-Wave Massive Input Massive Output System Channel Models

2.1.1 Flat Fading Channel Model

Analytical channel models such as Rayleigh fading are not suitable for mm-Wave channel modeling. The reason for this is the fact that the scattering levels represented by these models are too rich for mm-Wave channels [15]. In this dissertation, the model is based

on the Saleh-Valenzuela model that is often used in mm-Wave channel modeling [39] and standardization [40]. For simplicity, each scattering cluster is assumed to contribute a single propagation path. The subchannel matrix $\mathbf{H}_{ij}$ is given by

$$\mathbf{H}_{ij} = \sqrt{\frac{N_t N_r}{L_{ij}}} \sum_{l=1}^{L_{ij}} \alpha_{ij}^l \mathbf{a}_r(\theta_{ij}^l) \mathbf{a}_t^H(\phi_{ij}^l), \quad (2.1)$$

where L_{ij} is the number of resolvable paths and α_{ij}^l is the complex-gain of the l th ray which follows $\mathcal{CN}(0, 1)$, the vectors $\mathbf{a}_r(\theta_{ij}^l)$ and $\mathbf{a}_t(\phi_{ij}^l)$ are the normalized receive/transmit array response and θ_{ij}^l and ϕ_{ij}^l are its random azimuth angles of arrival and departure, respectively. In particular, this dissertation adopts a uniform linear array (ULA) where both $\mathbf{a}_r(\theta_{ij}^l)$ and $\mathbf{a}_t(\phi_{ij}^l)$ are modeled as

$$\mathbf{a}_{\text{ULA}}(\varphi) = \frac{1}{\sqrt{N}} \left[1, e^{j \frac{2\pi}{\lambda} d \sin(\varphi)}, \dots, e^{j(N-1) \frac{2\pi}{\lambda} d \sin(\varphi)} \right]^T, \quad (2.2)$$

where λ is the transmission wavelength, and d is the antenna spacing.

Please note that within a narrow angular spread, there could be tens of different propagation paths. However, their net effect can be captured by a much smaller number of paths that corresponds to the effective rank of the user's channel. For this reason, we call L_{ij} as the number of resolvable propagation paths in this thesis.

2.1.2 Frequency Selective Fading Channel Model

Let K denote the number of subcarriers in the system and N_{cp} the number of cyclic prefix (CP) samples. Therefore, the OFDM symbol length and the CP length are $T_k = K T_s$ and $T_{cp} = N_{cp} T_s$, respectively, where T_s is the system sampling interval.

One can write the channel response in the frequency domain as

$$\mathbf{H}[k] = \begin{bmatrix} \mathbf{H}_{1,1}[k] & \mathbf{H}_{1,2}[k] & \dots & \mathbf{H}_{1,M_t}[k] \\ \mathbf{H}_{2,1}[k] & \mathbf{H}_{2,2}[k] & \dots & \mathbf{H}_{2,M_t}[k] \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{M_r,1}[k] & \mathbf{H}_{M_r,2}[k] & \dots & \mathbf{H}_{M_r,M_t}[k] \end{bmatrix}, \quad (2.3)$$

where $\mathbf{H}_{i,j}[k]$ is the subchannel between i th RAU at the receiver and j th RAU at the transmitter, at k th subcarrier, defined as

$$\mathbf{H}_{i,j}[k] = \sqrt{\beta \frac{N_r N_t}{L}} \sum_{l=1}^L \alpha_{i,j}^l \mathbf{a}_r(\theta_{i,j}^l) \mathbf{a}_t^H(\phi_{i,j}^l) e^{-j2\pi \frac{k}{K} \frac{\tau_{i,j}^l}{T_s}}, \quad (2.4)$$

in which L is the number of resolvable propagation paths and $\alpha_{i,j}^l$ is the complex-gain of the l th ray which follows $\mathcal{CN}(0, 1)$, $\theta_{i,j}^l \in [0, 2\pi)$, $\phi_{i,j}^l \in [0, 2\pi)$, for all i, j, l , and the vectors $\mathbf{a}_r(\theta_{i,j}^l)$ and $\mathbf{a}_t(\phi_{i,j}^l)$ are the normalized array response at the receiver and transmitter, respectively. Also, $\tau_{i,j}^l$ is the path delay between i th RAU at the transmitter and j th RAU at the receiver. Please note that in this dissertation the assumption is that the operating frequency is much larger than the total bandwidth of the OFDM system. This results in signal wavelength in all subcarriers to be considered approximately equal.

2.2 Diversity

The average probability of bit error (BER) P_E at high SNR for both coded and uncoded system can be approximated by [41, 42]

$$P_E \approx (G_c \bar{\gamma})^{-G_d}, \quad (2.5)$$

where G_c is defined as coding gain, G_d refers to diversity gain, and $\bar{\gamma}$ is the average SNR. In a log-log scale, diversity gain G_d determines the slope of the BER versus the average SNR curve in high SNR regime. Furthermore, changing G_c leads to the shift of the curve in SNR relative to a benchmark BER curve of $(\bar{\gamma}^{-G_d})$.

2.3 Beamforming

2.3.1 Single-User Scenario Hybrid Beamforming

For a flat fading channel where the channel state information (CSI) is known at both the transmitter side and the receiver side, a MIMO channel can be decomposed into a number of independent parallel subchannels, i.e., beamforming [43]. By performing singular value decomposition (SVD) at both sides on the channel matrix $\mathbf{H}$, one can create multiple parallel subchannels as

$$\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V} \tag{2.6}$$

where the $N_r \times N_r$ matrix $\mathbf{U}$ and $N_t \times N_t$ matrix $\mathbf{V}$ are unitary, and the $N_r \times N_t$ matrix $\mathbf{\Lambda}$ is a rectangular diagonal matrix whose s th diagonal element, $\lambda_s \in \mathcal{R}^+$, is the singular value of $\mathbf{H}$. Please note that the singular values are sorted in decreasing order.

2.3.2 Multi-User Scenario Hybrid Beamforming

In this section, the hybrid block diagonalization beamforming for multi-user scenario is summarized based on [20]. First, by using (5.4) and SVD one can define

$$\frac{1}{\sqrt{N_t}} \mathbf{H}_k = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H \quad (2.7)$$

and

$$\frac{1}{\sqrt{N_t}} \mathbf{H}_{\text{comp}} = \mathbf{U}_{\text{comp}} \mathbf{\Sigma}_{\text{comp}} \mathbf{V}_{\text{comp}}^H \quad (2.8)$$

where

$$\mathbf{H}_{\text{comp}} = \mathbf{W}_{\text{RF}}^H \mathbf{H} = \text{diag} \left(\mathbf{W}_{\text{RF}}^{1H}, \dots, \mathbf{W}_{\text{RF}}^{KH} \right) \begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_K \end{bmatrix} \quad (2.9)$$

By using these definitions, a closed-form solution for hybrid beamforming can be derived as Algorithm 1. Here, the number of RF chains is double the least number of RF chains, i.e., $N_r^{\text{RF}} = 2N_s$ and $N_t^{\text{RF}} = 2KN_s$. After calculating the beamforming matrices by Algorithm 1, the set of equations in (2.10) is used to transform the scheme to the constrained case mentioned earlier.

Algorithm 1 Hybrid Block Diagonalization Beamforming for multi-user Scenario

Input: $\mathbf{H}$
for $k = 1$ *to* K **do**

| Calculate $\mathbf{W}_{\text{RF}}^k = \mathbf{U}_k(:, 1 : M_{\text{MS}})$ where $\mathbf{U}_k$ is defined as the previous slide

end

compute $\mathbf{W}_{\text{RF}}$ based on the previous slide

By using the previous slide, compute $\mathbf{F}_{\text{RF}} = \mathbf{V}_{\text{comp}}(:, 1 : N_t^{\text{RF}})$
for $k = 1$ *to* K **do**

| Calculate the baseband channel for the k th user

end

Compute $\mathbf{F}_{\text{BB}}$ and $\mathbf{W}_{\text{BB}}$ by using the null space of $\mathbf{H}_k$, then normalize each column of $\mathbf{F}_{\text{BB}}$

as $\mathbf{F}_{\text{BB}}(:, i) = \frac{\mathbf{F}_{\text{BB}}(:, i)}{\|\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}(:, i)\|_F}$
Output: $\mathbf{F}_{\text{RF}}, \mathbf{F}_{\text{BB}}, \mathbf{W}_{\text{RF}}^k, (\mathbf{W}_{\text{BB}}^k)_{k=1:K}$

$$\begin{aligned}
\bar{\mathbf{F}}(:, i')_{\text{RF}} &= \begin{cases} \frac{1}{\sqrt{N_t}} e^{j(\angle \mathbf{F}_{\text{RF}}(:, i) + \arccos |\mathbf{F}_{\text{RF}}(:, i)|)}, & \text{for } i' = 2i - 1 \\ \frac{1}{\sqrt{N_t}} e^{j(\angle \mathbf{F}_{\text{RF}}(:, i) - \arccos |\mathbf{F}_{\text{RF}}(:, i)|)}, & \text{for } i' = 2i \end{cases}, \\
\bar{\mathbf{W}}(:, i')_{\text{RF}}^k &= \begin{cases} \frac{1}{\sqrt{N_r}} e^{j(\angle \mathbf{W}_{\text{RF}}^k(:, i) + \arccos |\mathbf{W}_{\text{RF}}^k(:, i)|)}, & \text{for } i' = 2i - 1 \\ \frac{1}{\sqrt{N_r}} e^{j(\angle \mathbf{W}_{\text{RF}}^k(:, i) - \arccos |\mathbf{W}_{\text{RF}}^k(:, i)|)}, & \text{for } i' = 2i \end{cases}, k \in \{1, \dots, K\}, \\
\bar{\mathbf{W}}_{\text{BB}}^k &= \frac{1}{2} \begin{bmatrix} \mathbf{1}_{2 \times 1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \mathbf{1}_{2 \times 1} \end{bmatrix} \mathbf{W}_{\text{BB}}^k, k \in \{1, \dots, K\}, \\
\bar{\mathbf{F}}_{\text{BB}} &= \frac{1}{2} \begin{bmatrix} \mathbf{1}_{2 \times 1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \mathbf{1}_{2 \times 1} \end{bmatrix} \mathbf{F}_{\text{BB}}.
\end{aligned} \tag{2.10}$$

Chapter 3

An Upper Bound for Diversity Analysis

The diversity analysis in this dissertation is carried out by calculating the average Pairwise Error Probability (PEP) between the coded and decoded signals which are depicted as $\underline{c}$ and $\hat{\underline{c}}$, respectively as in, e.g., [28, 29, 35, 1]. In order to avoid repetition, finding an upper bound for the average PEP is covered in this section. We show the average PEP by $P(\underline{c} \rightarrow \hat{\underline{c}})$ and it is bounded as

$$P(\underline{c} \rightarrow \hat{\underline{c}}) \leq \mathbb{E} \left[\exp \left(-\gamma \sum_{l=1}^L \alpha_l \mu_l \right) \right] \quad (3.1)$$

where γ is the signal-to-noise ratio (SNR), α_l is a non-negative real coefficient, and μ_l is a Gamma random variable with shape of κ_l and scale of θ_l , i.e., $\mu_l \sim \mathcal{G}(\kappa_l, \theta_l)$. The inequality in (3.1) will come up multiple times in this dissertation, where the Gamma distribution parameters are decided by the channel characteristics, such as large scale fading coefficients and the path gain. One can find the upper bound for (3.1) by the following theorem.

Theorem 3.1. *Suppose that we have a mm-Wave massive MIMO system where the chan-*

nel is defined based on the Saleh-Valenzuela model. When calculating the PEP, one can see that the term $\mathbb{E} \left[\exp \left(-\gamma \sum_{l=1}^L \alpha_l \mu_l \right) \right]$ appears where this term can be upper bounded as $\mathbb{E} \left[\exp \left(-\gamma \sum_{l=1}^L \alpha_l \mu_l \right) \right]$, which can be used in the diversity analysis later in this dissertation, is

$$\mathbb{E} \left[\exp \left(-\gamma \sum_{l=1}^L \alpha_l \mu_l \right) \right] \leq \nu \gamma^{-\kappa} \quad (3.2)$$

where γ is SNR, ν is some constant, and κ is the shape of an approximated Gamma distribution

Proof. Let us denote $\alpha_{\min} = \min\{\alpha_l : l = 1, 2, \dots, L\}$. Then one can use $\alpha_{\min} \sum_{l=1}^L \mu_l \leq \sum_{l=1}^L \alpha_l \mu_l$ to find an upper bound for (3.1) as

$$P(\underline{c} \rightarrow \hat{\underline{c}}) \leq \mathbb{E} \left[\exp \left(-\gamma \sum_{l=1}^L \alpha_l \mu_l \right) \right] \leq \mathbb{E} \left[\exp \left(-\gamma \alpha_{\min} \sum_{l=1}^L \mu_l \right) \right]. \quad (3.3)$$

Since the generic summation of multiple Gamma random variables with different shapes and different scales, has no known closed form solution, one can use the Welch-Satterthwaite equation to calculate an approximation to the degrees of freedom of such summation, i.e., $X = \sum_{l=1}^L \mu_l$ [44, p.4.1-1],[45]. The new Gamma random variable will have new shapes and scales approximated as

$$\kappa = \frac{\left(\sum_{i,j} \theta_{ij} \kappa_{ij} \right)^2}{\sum_{i,j} \theta_{ij}^2 \kappa_{ij}}, \quad (3.4)$$

$$\theta = \frac{\sum_{i,j} \theta_{ij}^2 \kappa_{ij}}{\sum_{i,j} \theta_{ij} \kappa_{ij}}. \quad (3.5)$$

By using the definition of the moment generating function (MGF) [46] for the random

variable X , i.e.,

$$M_X(t) = \mathbb{E} [\exp (tX)] = (1 - \theta t)^{-\kappa} = (1 + \alpha_{\min} \theta \gamma)^{-\kappa}, \quad (3.6)$$

one can show that (3.3) is upper bounded as

$$P(\underline{c} \rightarrow \hat{\underline{c}}) \leq (1 + \theta \gamma \alpha_{\min})^{-\kappa} \quad (3.7)$$

$$\approx (\alpha_{\min} \theta \gamma)^{-\kappa} \quad (3.8)$$

for high SNR regime. Since the the power of the SNR term is $-\kappa$, this means that the diversity gain for this mm-Wave system in high SNR regimes is $G_d = \kappa$, where κ can be calculated based on (3.4). $\square$

Chapter 4

Bit-Interleaved Coded Multiple Beamforming for Single-User Scenario

4.1 System Model for Single-User Scenario

Consider a downlink single-user mm-Wave massive MIMO system with distributed antenna array as shown in Fig. 4.1. We have shown the distributed antenna array in Fig. 4.2, where it means an array consisting of several RAUs that are distributively located. In this system, the transmitter sends N_s data streams to a receiver. The transmitter is equipped with N_t^{RF} RF chains and M_t RAUs, where each RAU has N_t antennas, while at the receiver, the number of RF chains and RAUs is given by N_r^{RF} and M_r , respectively. Each RAU at the receiver has N_r antennas. When $M_t = M_r = 1$, the system reduces to a conventional co-located MIMO (C-MIMO) system.

The input to the system is N_s data streams. The vector of data symbols to be transmitted

Figure 4.1: Block diagram of a single-user mm-Wave massive MIMO system with distributed antenna subarray.

by the transmitter at each time instant, $\mathbf{x} \in \mathbb{C}^{N_s \times 1}$, can be expressed as

$$\mathbf{x} = [x_1, \dots, x_{N_s}]^T, \quad (4.1)$$

where $E[\mathbf{x}\mathbf{x}^H] = \frac{1}{N_s}\mathbf{I}_{N_s}$. The preprocessing at the baseband is applied by means of the matrix $\mathbf{F}_{BB} \in \mathbb{C}^{N_t^{RF} \times N_s}$. The last stage of data preprocessing is performed at RF, when beamforming is applied by means of phase shifters and combiners. A set of $M_t N_t$ phase shifters is applied to the output of each RF chain. As a result of this process, different beams are formed in order to transmit the RF signals. We can model this process with an $M_t N_t \times N_t^{RF}$ complex matrix $\mathbf{F}_{RF}$. Note that the baseband precoder $\mathbf{F}_{BB}$ modifies both amplitude and phases, while only phase changes can be realized by $\mathbf{F}_{RF}$ since it is implemented by using analog phase shifters. We assume a narrowband flat fading channel model and obtain

Figure 4.2: Illustration of a distributed antenna array system.

the received signal as

$$\mathbf{z} = \mathbf{H}\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\mathbf{x} + \mathbf{n}, \quad (4.2)$$

where $\mathbf{H}$ is an $M_r N_r \times M_t N_t$ channel matrix with complex-valued entries and $\mathbf{n}$ is an $M_r N_r \times 1$ vector consisting of i.i.d. $\mathcal{CN}(0, N_0)$ noise samples, where $N_0 = \frac{N_t}{\text{SNR}}$ and SNR is defined as the signal-to-noise ratio. The processed signal is given by

$$\mathbf{y} = \mathbf{W}_{\text{BB}}^H \mathbf{W}_{\text{RF}}^H \mathbf{H} \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}} \mathbf{x} + \mathbf{W}_{\text{BB}}^H \mathbf{W}_{\text{RF}}^H \mathbf{n}, \quad (4.3)$$

where $\mathbf{W}_{\text{RF}}$ is the $M_r N_r \times N_r^{\text{RF}}$ RF combining matrix, and $\mathbf{W}_{\text{BB}}$ is the $N_r^{\text{(RF)}} \times N_s$ baseband combining matrix.

The channel matrix $\mathbf{H}$ can also be written as

$$\mathbf{H} = \begin{bmatrix} \sqrt{\beta_{11}} \mathbf{H}_{11} & \sqrt{\beta_{12}} \mathbf{H}_{12} & \dots & \sqrt{\beta_{1M_t}} \mathbf{H}_{1M_t} \\ \sqrt{\beta_{21}} \mathbf{H}_{21} & \sqrt{\beta_{22}} \mathbf{H}_{22} & \dots & \sqrt{\beta_{2M_t}} \mathbf{H}_{2M_t} \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{\beta_{M_r 1}} \mathbf{H}_{M_r 1} & \sqrt{\beta_{M_r 2}} \mathbf{H}_{M_r 2} & \dots & \sqrt{\beta_{M_r M_t}} \mathbf{H}_{M_r M_t} \end{bmatrix}, \quad (4.4)$$

where β_{ij} , a real-valued nonnegative number, represents the large scale fading effect between

the i th RAU at the receiver and j th RAU at the transmitter. The normalized subchannel matrix $\mathbf{H}_{ij}$ is the MIMO channel between the i th RAU at the receiver and the j th RAU at the transmitter.

The uniform linear array (ULA) is employed by the transmitter and the receiver in our study. For an N -element ULA, the array response vector is given by

$$\mathbf{a}_{ULA}(\phi) = \frac{1}{\sqrt{N}} \left[1, e^{j\frac{2\pi}{\lambda}d\sin(\phi)}, \dots, e^{j(N-1)\frac{2\pi}{\lambda}d\sin(\phi)} \right]^T, \quad (4.5)$$

where λ is the wavelength of the carrier, and d is the distance between neighboring antenna elements.

We leverage both BICM and multiple beamforming to form BICMB [28]. An interleaver is used to interleave the output bits of a binary convolutional encoder. Then the output of the interleaver is mapped over a signal set $\chi \subset \mathbb{C}$ of size $|\chi| = 2^m$ with a binary labeling map $\mu: \{0, 1\}^m \rightarrow \chi$. The interleaver design has two criteria [28]:

1. Consecutive coded bits are mapped to different symbols.
2. Each subchannel should be utilized at least once within d_{free} distinct bits among different codewords by using proper code and interleaver.

Note that the free distance d_{free} of the convolutional encoder should satisfy $d_{\text{free}} \geq N_s$ [28].

For mapping the bits onto symbols, Gray encoding is used. Also, we are using a Viterbi decoder at the receiver. The interleaver π is used to interleave the code sequence $\underline{c}$. Then the output of the interleaver is mapped onto the signal sequence $\underline{x} \in \chi$.

The only beamforming constraint here is a total power constraint, because one can control both the amplitude and the phase of a signal. The total power constraint leads to a simple

solution based on the singular value decomposition (SVD) [15]

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H = [\mathbf{u}_1 \mathbf{u}_2 \dots \mathbf{u}_{M_r N_r}]^H \mathbf{\Sigma} [\mathbf{v}_1 \mathbf{v}_2 \dots \mathbf{v}_{M_t N_t}], \quad (4.6)$$

where $\mathbf{U}$ and $\mathbf{V}$ are $M_r N_r \times M_r N_r$ and $M_t N_t \times M_t N_t$ unitary matrices, respectively, and $\mathbf{\Sigma}$ is an $M_r N_r \times M_t N_t$ diagonal matrix with singular values of $\mathbf{H}$, i.e., σ_i , on the main diagonal with decreasing order. By exploiting the optimal precoder and combiner, the system input-output relation in (4.3) at the k th time instant can be written as

$$\mathbf{y}_k = [\mathbf{u}_1 \mathbf{u}_2 \dots \mathbf{u}_{N_s}]^H \mathbf{H} [\mathbf{v}_1 \mathbf{v}_2 \dots \mathbf{v}_{N_s}] \mathbf{x} + [\mathbf{u}_1 \mathbf{u}_2 \dots \mathbf{u}_{N_s}]^H \mathbf{n}_k, \quad (4.7)$$

$$y_{k,s} = \sigma_s x_{k,s} + n_{k,s}, \quad \text{for } s = 1, 2, \dots, N_s. \quad (4.8)$$

4.2 Diversity Gain and PEP Analysis For Single-User Scenario

In this section, we show that by using the BICMB analysis for calculating BER, the diversity gain becomes independent of the number of data streams in high SNR regimes.

Theorem 4.1. *Suppose that $N_r \rightarrow \infty$ and $N_t \rightarrow \infty$. Then the bit interleaved coded distributed massive MIMO system can achieve a diversity gain of*

$$G_d = \frac{\left(\sum_{i,j} \beta_{ij}\right)^2}{\sum_{i,j} \beta_{ij}^2 L_{ij}^{-1}} \quad (4.9)$$

for $i = 1, \dots, M_r$ and $j = 1, \dots, M_t$.

Proof. We model the BICMB bit interleaver as $\pi : k' \rightarrow (k, s, i)$, where k' represents the original ordering of the coded bits $c_{k'}$, k represents the time ordering of the signals $x_{k,s}$ and

i denotes the position of the bit $c_{k'}$ on symbol $x_{k,s}$.

We define χ_b^i as the subset of all signals $x \in \chi$. Note that the label has the value $b \in \{0, 1\}$ in position i .

Then, the ML bit metrics are given by using (4.8), [28, 26, 27]

$$\gamma^i(y_{k,s}, c_{k'}) = \min_{x \in \chi_{c_{k'}}^i} |y_{k,s} - \sigma_s x|^2. \quad (4.10)$$

The receiver uses an ML decoder to make decisions based on

$$\hat{\underline{c}} = \arg \min_{\underline{c} \in \mathcal{C}} \sum_{k'} \gamma^i(y_{k,s}, c_{k'}). \quad (4.11)$$

Assume that the code sequence $\underline{c}$ is transmitted and $\hat{\underline{c}}$ is detected. Then by using (4.10) and (4.11), the pairwise error probability (PEP) of $\underline{c}$ and $\hat{\underline{c}}$ given CSI can be written as [28]

$$P(\underline{c} \rightarrow \hat{\underline{c}} | \mathbf{H}) = P \left(\sum_{k'} \min_{x \in \chi_{c_{k'}}^i} |y_{k,s} - \sigma_s x|^2 \geq \sum_{k'} \min_{x \in \chi_{\hat{c}_{k'}}^i} |y_{k,s} - \sigma_s x|^2 \right), \quad (4.12)$$

where $s \in \{1, 2, \dots, N_s\}$.

Note that in a convolutional code, the Hamming distance between $\underline{c}$ and $\hat{\underline{c}}$, $d(\underline{c} - \hat{\underline{c}})$ is at least d_{free} . In this work we assume for PEP analysis $d(\underline{c} - \hat{\underline{c}}) = d_{\text{free}}$.

For the d_{free} bits, let us denote

$$\tilde{x}_{k,s} = \arg \min_{x \in \chi_{c_{k'}}^i} |y_{k,s} - \sigma_s x|^2 \quad (4.13)$$

$$\hat{x}_{k,s} = \arg \min_{x \in \chi_{\hat{c}_{k'}}^i} |y_{k,s} - \sigma_s x|^2 \quad (4.14)$$

One can see easily that since the two sets $\chi_{c_{k'}}^i$ and $\chi_{\hat{c}_{k'}}^i$ are complementary sets of constellation

points in the signal constellation set χ and $\tilde{x}_{k,s} \in \chi_{c_{k'}}^i$ and $\hat{x}_{k,s} \in \chi_{\bar{c}_{k'}}^i$, therefore $\tilde{x}_{k,s} \neq \hat{x}_{k,s}$. Furthermore, whatever value we choose for $x_{k,s}$ from the set $\chi_{c_{k'}}^i$ the following inequality holds, since $\tilde{x}_{k,s}$ is the minimum based on (4.13)

$$|y_{k,s} - \sigma_s x_{k,s}|^2 \geq |y_{k,s} - \sigma_s \tilde{x}_{k,s}|^2. \quad (4.15)$$

By using the trellis structure of the convolutional codes, one can see that d_{free} distinct bits between any two codewords occur in consecutive trellis branches. Let us denote d such that d_{free} bits occur within d consecutive bits. The bit interleaver can be designed such that d consecutive coded bits are mapped onto distinct symbols. This guarantees that there exist d_{free} distinct pairs of $(\tilde{x}_{k,s}, \hat{x}_{k,s})$ and d_{free} distinct pairs of $(x_{k,s}, \hat{x}_{k,s})$ [28]. Then we can write the PEP as

$$\begin{aligned} P(\underline{c} \rightarrow \hat{\underline{c}} | \mathbf{H}) &= P \left(\sum_{k, d_{\text{free}}} |y_{k,s} - \sigma_s \tilde{x}_{k,s}|^2 - \sum_{k, d_{\text{free}}} |y_{k,s} - \sigma_s \hat{x}_{k,s}|^2 \geq 0 \right) \\ &= P \left(\sum_{k, d_{\text{free}}} |y_{k,s} - \sigma_s \tilde{x}_{k,s}|^2 \geq \sum_{k, d_{\text{free}}} |y_{k,s} - \sigma_s \hat{x}_{k,s}|^2 \right) \\ &\leq P \left(\sum_{k, d_{\text{free}}} |y_{k,s} - \sigma_s x_{k,s}|^2 \geq \sum_{k, d_{\text{free}}} |y_{k,s} - \sigma_s \hat{x}_{k,s}|^2 \right) \\ &= P \left(\sum_{k, d_{\text{free}}} |n_{k,s}|^2 \geq \sum_{k, d_{\text{free}}} |\sigma_s (x_{k,s} - \hat{x}_{k,s}) + n_{k,s}|^2 \right) \\ &= P \left(\zeta \geq \sum_{k, d_{\text{free}}} |\sigma_s (x_{k,s} - \hat{x}_{k,s})|^2 \right) \\ &\leq Q \left(\sqrt{\frac{d_{\text{min}}^2 \sum_{s=1}^{N_s} \alpha_s \sigma_s^2}{2N_0}} \right) \end{aligned} \quad (4.16)$$

where α_s is a parameter that indicates how many times subchannel s is used within the d_{free}

bits under consideration, and $\sum_{s=1}^{N_s} \alpha_s = d_{\text{free}}$. Please note that

$$\zeta = \sum_{k, d_{\text{free}}} \underbrace{\sigma_s(\hat{x}_{k,s} - x_{k,s})^* n_{k,s} + \sigma_s(\hat{x}_{k,s} - x_{k,s}) n_{k,s}^*}_{\zeta_{k,s}}, \quad (4.17)$$

where for the perfect channel knowledge, i.e., given $\mathbf{H}$, $\zeta_{k,s}$'s are independent zero-mean Gaussian random variables with variance $2N_0|\sigma_s(x_{k,s} - \hat{x}_{k,s})|^2$. This results in ζ as a Gaussian random variable with zero mean and variance $2N_0 \sum_{k, d_{\text{free}}} |\sigma_s(x_{k,s} - \hat{x}_{k,s})|^2$. The bound $Q(x) \leq \frac{1}{2}e^{-\frac{x}{2}}$ can be used to upper bound the PEP as

$$P(\underline{c} \rightarrow \hat{\underline{c}}) = \mathbb{E}[P(\underline{c} \rightarrow \hat{\underline{c}}|\mathbf{H})] \leq \mathbb{E}\left[\frac{1}{2}\exp\left(\frac{-d_{\min}^2 \sum_{s=1}^{N_s} \alpha_s \sigma_s^2}{4N_0}\right)\right]. \quad (4.18)$$

Let us denote $\alpha_{\min} = \min\{\alpha_s : s = 1, 2, \dots, N_s\}$. Then

$$\frac{\left(\sum_{s=1}^{N_s} \alpha_s \sigma_s^2\right)}{N_s} \geq \frac{\left(\alpha_{\min} \sum_{s=1}^{N_s} \sigma_s^2\right)}{N_s} \geq \frac{\left(\alpha_{\min} \sum_{s=1}^{L_t} \sigma_s^2\right)}{L_t}. \quad (4.19)$$

There are only $L_t = \sum_{i=1}^{M_r} \sum_{j=1}^{M_t} L_{ij}$ non-zero singular values [21].

Let us define

$$\Theta \triangleq \sum_{s=1}^{N_s} \sigma_s^2 = \|\mathbf{H}\|_F^2. \quad (4.20)$$

One can easily see from the subchannel and channel matrix definitions in (4.4) and (2.1) that there is a total of $L_s = \sum_{i=1}^{M_r} \sum_{j=1}^{M_t} L_{ij}$ resolvable propagation paths. We can write the channel matrix $\mathbf{H}$ as a summation of L_s matrices, where each matrix is rank-one. In other words

$$\mathbf{H} = \sum_{i=1}^{M_r} \sum_{j=1}^{M_t} \sum_{l=1}^{L_{ij}} \tilde{\alpha}_{ij}^l \tilde{\mathbf{a}}_r(\theta_{ij}^l) \tilde{\mathbf{a}}_t^H(\phi_{ij}^l), \quad (4.21)$$

where

$$\tilde{\alpha}_{ij}^l = \sqrt{\beta_{ij} \frac{N_r N_t}{L_{ij}}} \alpha_{ij}^l. \quad (4.22)$$

Note that in (4.21) we define $\tilde{\mathbf{a}}_r(\theta_{ij}^l)$ as a $M_r N_r \times 1$ vector whose p th entry is defined as

$$[\tilde{a}_r(\theta_{ij}^l)]_p = \begin{cases} [\tilde{a}_r(\theta_{ij}^l)]_{p-(i-1)N_r}, & (i-1)N_r < p \leq iN_r \\ 0, & \text{Otherwise.} \end{cases} \quad (4.23)$$

Similarly, we define a $M_t N_t \times 1$ vector to define $\tilde{\mathbf{a}}_t(\phi_{ij}^l)$ as

$$[\tilde{a}_t(\phi_{ij}^l)]_q = \begin{cases} [\tilde{a}_t(\phi_{ij}^l)]_{q-(i-1)N_t}, & (i-1)N_t < q \leq iN_t \\ 0, & \text{Otherwise.} \end{cases} \quad (4.24)$$

We sort $\tilde{\alpha}_{ij}^l$ s in a descending way such that $|\tilde{\alpha}_1| \geq |\tilde{\alpha}_2| \geq \dots \geq |\tilde{\alpha}_{L_s}|$. We sort the corresponding θ_{ij}^l s and ϕ_{ij}^l s based on this sort. Then, one can decompose the channel matrix defined in (4.21) as

$$\mathbf{H} = \mathbf{A}_r \mathbf{D} \mathbf{A}_t^* = [\mathbf{A}_r | \mathbf{A}_r^\perp] \tilde{\mathbf{D}} [\text{diag}([e^{j\angle \tilde{\alpha}_1}, \dots, e^{j\angle \tilde{\alpha}_{L_s}}]) \mathbf{A}_t | \mathbf{A}_t^\perp]^*, \quad (4.25)$$

where we define the matrix $\mathbf{A}_r = [\tilde{\mathbf{a}}_r(\theta_1), \dots, \tilde{\mathbf{a}}_r(\theta_{L_s})]$, $\mathbf{A}_t = [\tilde{\mathbf{a}}_t(\phi_1), \dots, \tilde{\mathbf{a}}_t(\phi_{L_s})]$, and $\mathbf{D} = \text{diag}([\tilde{\alpha}_1], \dots, [\tilde{\alpha}_{L_s}])$. Note that in (4.25), $\mathbf{A}_t^\perp$ and $\mathbf{A}_r^\perp$ are unitary matrices spanning the null space of $\mathbf{H}$ and $\mathbf{H}^T$, $\tilde{\mathbf{D}} = [\text{diag}(|\tilde{\alpha}_1|, \dots, |\tilde{\alpha}_{L_s}|) | \mathbf{0}_{(M_r N_r - L_s) \times (M_t N_t - L_s)}]$, and $\angle \tilde{\alpha}_l$ is the phase of the l th path. From Theorem 3 in [15], we can see that the singular values of $\mathbf{H}$ converge to $|\tilde{\alpha}|$ in descending order. By using the singular values of $\mathbf{H}$, (4.20) can be rewritten as

$$\Theta = \|\mathbf{H}\|_F^2 = N_r N_t \sum_{i=1}^{M_r} \sum_{j=1}^{M_t} \underbrace{\frac{\beta_{ij}}{L_{ij}} \sum_{l=1}^{L_{ij}} |\alpha_{ij}^l|^2}_{\Psi_{ij}}. \quad (4.26)$$

Note that the random variable $\sum_{l=1}^{L_{ij}} |\alpha_{ij}^l|^2$ has a χ -squared distribution with $2L_{ij}$ degrees of freedom, or equivalently a Gamma distribution with shape L_{ij} and scale 2, denoted $\mathcal{G}(L_{ij}, 2)$. Then, since $\beta_{ij} L_{ij}^{-1} > 0$, $\Psi_{ij} \sim \mathcal{G}(L_{ij}, 2\beta_{ij} L_{ij}^{-1})$ [47]. By using Theorem 3.1 in Chapter 3, one can find the shape and scale of the new estimated Gamma distribution as

$$\kappa = \frac{\left(\sum_{i,j} \theta_{ij} \kappa_{ij}\right)^2}{\sum_{i,j} \theta_{ij}^2 \kappa_{ij}} = \frac{\left(\sum_{i,j} \beta_{ij}\right)^2}{\sum_{i,j} \beta_{ij}^2 L_{ij}^{-1}}, \quad (4.27)$$

$$\theta = \frac{\sum_{i,j} \theta_{ij}^2 \kappa_{ij}}{\sum_{i,j} \theta_{ij} \kappa_{ij}} = \frac{\sum_{i,j} \beta_{ij}^2 L_{ij}^{-1}}{\sum_{i,j} \beta_{ij}}. \quad (4.28)$$

This leads to an upper bound for the PEP, which is

$$P(\underline{\mathbf{c}} \rightarrow \hat{\underline{\mathbf{c}}}) = g(d, \alpha_{\min}, \chi) \leq \frac{1}{2} \left(1 + \theta \frac{d_{\min}^2 \alpha_{\min} N_s N_t}{4L_t} \text{SNR} \right)^{-\kappa} \quad (4.29)$$

$$\approx \frac{1}{2} \left(\theta \frac{d_{\min}^2 \alpha_{\min} N_s N_t}{4L_t} \text{SNR} \right)^{-\kappa} \quad (4.30)$$

for high SNR. The function $g(d, \alpha_{\min}, \chi)$ denotes the PEP of two codewords with $d(\underline{\mathbf{c}} - \hat{\underline{\mathbf{c}}}) = d$, with $\alpha_{\min}$ corresponding to $\underline{\mathbf{c}}$ and $\hat{\underline{\mathbf{c}}}$, and with constellation χ . In (4.29) θ and κ are defined as (4.27) and (4.28).

In BICMB, P_b can be calculated as [28]

$$P_b \leq \frac{1}{k_c} \sum_{d=d_{\text{free}}}^{\infty} \sum_{i=1}^{W_I(d)} g(d, \alpha_{\min}(d, i), \chi), \quad (4.31)$$

where $W_I(d)$ denotes the total input weight of error events at Hamming distance d . For the industry standard 1/2 rate, 64-state (133, 171) convolutional code $W_I(d = d_{\text{free}} = 10) = 11$. Based on how we choose the interleaver, 11 codewords at $d_H = 10$ from the all-zeros codeword may each have a different $\alpha_{\min}$. Following (4.30) and (4.31)

$$P_b \leq \frac{1}{k_c} \sum_{d=d_{\min}}^{\infty} \sum_{i=1}^{W_I(d)} \frac{1}{2} \left(\theta \frac{d_{\text{free}}^2 \alpha_{\min}(d, i) N_s N_t}{4L_t} SNR \right)^{-\kappa}. \quad (4.32)$$

The SNR component has a power of $-\kappa$ for all summations. Hence, BICMB achieves full diversity order of

$$G_d = \kappa = \frac{\left(\sum_{i,j} \beta_{ij} \right)^2}{\sum_{i,j} \beta_{ij}^2 L_{ij}^{-1}} \quad (4.33)$$

which is independent of the number of spatial streams transmitted. $\square$

Remark 1. Under the case where N_t and N_r are large enough and assuming that $L_{ij} = L$ and $\beta_{ij} = \beta$ for any $i \in \{1, \dots, M_r\}$ and $j \in \{1, \dots, M_t\}$, it can be seen easily that the distributed massive MIMO system can achieve a diversity gain

$$G_d = L_t = M_r M_t L. \quad (4.34)$$

This result can be compared to (20) in [21] where the diversity gain with no coding is $G_d = M_t M_r L - N_s + 1$. By using BICMB, we omitted the effect of the number of transmitted data streams on the diversity gain in our work, i.e., full spatial multiplexing gain and full diversity gain is achieved. Furthermore, by comparing (4.34) with the results in [22], one can see that

using RAUs at the transmitter or the receiver side increases the diversity gain by a factor of $M_r M_t$.

Remark 2. *Theorem 4.1 implies that the diversity gain is independent of the number of data streams, i.e., the transmitter can send the maximum number of data streams $N_s \leq L_t$, and still get the same diversity gain. This will be illustrated in Section VII.*

4.3 Simulation Results

In the simulations, the industry standard 64-state 1/2-rate (133,171) $d_{\text{free}} = 10$ convolutional code is used. For BICMB, we separate the coded bits into different substreams of data and a random interleaver is used to interleave the bits in each substream. We assume that the number of RF chains in the receiver and transmitter are twice the number of data streams [14] (i.e., $N_t^{\text{RF}} = N_r^{\text{RF}} = 2N_s$) and each large scale fading coefficient β_{ij} equals $\beta = -20$ dB (except for Fig. 4.8). For the sake of simplicity, only ULA array configuration with $d = 0.5$ is considered at RAUs. For Fig. 4.4–4.7, Binary Phase Shift Keying (BPSK) modulation is employed for each data stream. For Fig. 4.8, information bits are mapped onto 16 quadrature amplitude modulation (QAM) symbols in each subchannel.

Two different cases are simulated in Fig. 4.3. In Case I, the rank of the channel is $\text{rank}(\mathbf{H}) = M_t M_r L = 30$, which shows that we can transmit up to 30 data streams through the channel. In other words, if we perform SVD on the channel, we will get 30 different singular values, creating 30 different subchannels. These subchannels enable us to transmit data streams through the channel. For the first scenario, which is shown with circle markers, $N_t = 2N_r = 100$, while in the second scenario shown with triangle markers, $N_t = 2N_r = 400$. It can be seen from Fig. 4.3 that the number of singular values of the mm-Wave channel is independent of the number of antennas at the transmitter and the receiver side. This can be seen by observing the channel defined in (4.4) and (2.1). Same result can be seen with

Figure 4.3: Singular values of the sparse mm-Wave channel with $N_t = 100$ and $N_r = 50$.

Case II, where changing the number of antennas at both transmitter or the receiver does not affect the rank of the channel. Hence, there are only limited subchannels which can be used to transmit the data. The number of available subchannels is $L_t = \sum_{i=1}^{M_r} \sum_{j=1}^{M_t} L_{ij}$ which is the rank of the channel $\mathbf{H}$ and is independent of the number of antennas in RAUs in the transmitter and the receiver side.

Fig. 4.4 illustrates the importance of the interleaver design. A random interleaver is used such that consecutive coded bits are transmitted over the same subchannel. Consequently, an error on the trellis occurs over paths that are spanned by the worst channel and the diversity order of coded multiple beamforming approaches to that of uncoded multiple beamforming with uniform power allocation. In other words, the BER performance decreases when the interleaving design criteria are not met. One can see this by comparing BER for different subchannels in Fig. 4.4. For example, the first subchannel enjoys the best diversity gain, while the third subchannel gets a worse subchannel, hence worse diversity gain.

On the contrary, if the interleaver criteria design is met, all different data streams can enjoy

Figure 4.4: BER with respect to SNR with $N_t = 100$, $N_r = 50$, $M_t = 2$, $M_r = 2$ and $L = 2$ for $N_s = 6$.

the maximum diversity gain. As we expect from (4.33), changing the number of streams N_s should not change the diversity gain, i.e., the slope of the BER curve in high SNR. As it can be seen from Fig. 4.5, the slope does not change by changing the number of data streams. Hence, one can get the same diversity gain by using the maximum number of data streams available (L_t). Since the rank of the channel in the system simulated in Fig. 4.5 is $2 \times 3 \times 1 = 6$, we can only transmit up to six data streams through this channel, where all different data streams can get the same diversity gain.

Fig. 4.6 illustrates BER with respect to SNR for three different cases. In each case, the number of antennas employed at each RAU at both the transmitter and the receiver changes. In Case I, the number of antennas at the receiver is twice comparing to Case II, while the number of antennas at the transmitter in both cases are the same. One can see that this difference does not change the slope of the BER curve in high SNR regime. However, the performance of the system gets better by a factor of 3 dB, which is expected, since the amount

Figure 4.5: BER with respect to SNR with $N_t = 100$, $N_r = 50$, $M_t = 3$, $M_r = 1$, and $L = 2$ for different values of N_s .

of resources in Case I is simply twice as the Case II. One can do a similar comparison for Case II and Case II, where the number of antennas at the transmitter side in Case II is twice as Case III, while the number of antennas at the receiver side remains the same for both cases.

Fig. 4.7 illustrates the results for BICMB for both co-located and mm-Wave massive MIMO systems with distributed antenna subarrays. The diversity gain for the distributed system outperforms the co-located system, even though the channel in the co-located system has richer scattering (the number of resolvable propagation paths in the co-located system is twice as the distributed system). Also, as it can be seen from Fig. 4.7, the slope of the curves for the distributed systems at high SNR are the same, which can be confirmed by (4.33). Note, for distributed systems, when $\beta_{ij} = \beta$, the diversity gain for two cases with $M_r = M_t = L = 2$ and $M_r = M_t = 2, L = [6; 2, 3; 1]$ can be calculated as $G_{d_1} = 2 \times 2 \times 2 = 8$ and $G_{d_2} = (2 \times 2)^2 / (6^{-1} + 2^{-1} + 3^{-1} + 1^{-1}) = 8$, respectively as in (4.33). This calculation

Figure 4.6: BER with respect to SNR for different number of antennas at each RAU at both the transmitter and the receiver with $M_t = 2$, $M_r = 2$, $L = 2$, and $N_s = 1$

Figure 4.7: BER with respect to SNR with $N_t = 100$, $N_r = 50$, and $N_s = 3$.

Figure 4.8: BER with respect to SNR with $N_t = 100$, $N_r = 50$, and $N_s = 1$.

shows that the diversity gain for these two scenarios must be the same, where this can be confirmed by referring to Fig. 4.7.

Fig. 4.8 illustrates the effect of the large scale fading coefficient on the diversity gain. So far, in our simulations, we assumed the large scale fading coefficients are the same for different channels. In other words, we assumed homogeneous environment for the simulations. In this figure, we consider inhomogeneous large scale fading coefficients and we assume $M_r = M_t = 2$, $L_{ij} = L = 2$, $N_t = 2N_r = 100$, and $N_s = 1$. Let $\mathbf{B} = [\beta_{ij}]$ where β_{ij} expressed in dB, as the large scale fading coefficient matrix. By these assumptions, three different cases are simulated and we use $\mathbf{B}_4$ as a reference curve to compare the diversity gain of $\mathbf{B}_3$ with it.

We used the following $\mathbf{B}$ in the simulations:

$$\mathbf{B}_1 = \begin{bmatrix} -20 & -20 \\ -20 & -20 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} -25 & -25 \\ -25 & -25 \end{bmatrix},$$

$$\mathbf{B}_3 = \begin{bmatrix} -20 & -35 \\ -35 & -20 \end{bmatrix}, \mathbf{B}_4 = -20.$$

As it can be seen from Fig. 4.8, when the system is homogeneous, the diversity gain remains the same, despite of changing the value of the large scale fading coefficient. Case I and Case II have the same slope in high SNR, which is expected because the value of β remains the same for all subchannels in both cases. In Case III, when the system is inhomogeneous, the diversity gain gets worse, although it has the same setup as the other two cases. By using (4.33), one can easily see that Case III has approximately the same diversity gain as a system with $M_r = M_t = 1$ and $L = 4$, i.e., $G_d = 4$, which is depicted in Case IV for comparison.

4.4 Discussion

The probability density function (PDF) of the azimuth angles of arrival and departure does not affect the diversity gain. One can confirm this by considering (4.26), where the singular values of the channel converge to the norm of the complex gain coefficients defined in (2.1). We can simply see that the diversity gain depends on the large scale fading coefficients, the number of resolvable propagation paths, and the number of RAUs at the transmitter and the receiver. Please note that the linear growth of diversity gain which can be seen in (4.34), is a function of the angular spread of the signals, in all of the simulations through this thesis. For narrow angular clusters, the number of resolvable propagation paths L cannot be very large, i.e., $L \leq 4$.

Figure 4.9: BER with respect to SNR with $M_r = M_t = L = N_s = 2$, $N_t = 128$, and $N_r = 64$.

Fig. 4.9 illustrates different scenarios with different PDF for the azimuth angle at the transmitter and the receiver. One can see that changing the PDF of the azimuth angles of arrival and departure does not change the diversity gain in the system.

On a side note, as it can be seen from the simulations results in the thesis, the SNR needs to be around 20 dB or higher for the distributed antenna subarray massive MIMO system operating in mm-Wave frequencies to benefit from the maximum diversity gain. This is a reasonable assumption, since we rely on the large array gain available at both the transmitter and receiver.

Chapter 5

Bit-Interleaved Coded Multiple Beamforming for Multi-User Scenario

5.1 System Model for Multi-user Scenario

Consider a downlink multi-user massive MIMO system as shown in Fig. 5.1. The antenna array at the base station (BS) consists of N_t^{RF} RF chains and M RAUs, each of which has N_t antennas. There are K different mobile stations (MS) and each one of them is equipped with N_r antennas and N_r^{RF} RF chains. The BS transmits KN_s data streams and each MS receives its N_s data streams. We constrain the number of RF chains in order to reduce the hardware complexity of the massive MIMO system. This constraint for the BS is $KN_s \leq N_t^{\text{RF}} \ll N_t$ and $N_s \leq N_r^{\text{RF}} \ll N_r$ for each MS.

We denote the RF precoder $\mathbf{F}_{\text{RF}}$ by an $MN_t \times N_t^{\text{RF}}$ matrix and the baseband precoder $\mathbf{F}_{\text{BB}}$ by an $N_t^{\text{RF}} \times KN_s$ matrix. At the BS, the transmitted symbols of K users first go through a power allocation matrix $\mathbf{P}$ which is $KN_s \times KN_s$ and $\|\mathbf{P}\|_F^2 = P_t$. Since the RF precoder matrix only changes the phase of the input signal, its magnitude is constant,

Figure 5.1: Block diagram of a multi-user mm-Wave massive MIMO system with distributed antenna arrays.

i.e., $|\mathbf{F}_{RF}(i, j)| = \frac{1}{\sqrt{N_t}}$. Also, because of the power constraint at the BS, we need to satisfy $\|\mathbf{F}_{RF}\mathbf{F}_{BB}\|_F^2 = KN_s$. We assume that the CSI is known at the transmitter and the receiver. We employ a narrowband flat fading channel model for CSI. The received signal at the k th MS after combining is given by

$$\mathbf{y}_k = \mathbf{W}_{BB}^{kH} \mathbf{W}_{RF}^{kH} \mathbf{H}_k \mathbf{F}_{RF} \mathbf{F}_{BB} \mathbf{P} \mathbf{x} + \mathbf{W}_{BB}^{kH} \mathbf{W}_{RF}^{kH} \mathbf{n}_k \quad (5.1)$$

where $k \in \{1, 2, \dots, K\}$. The channel matrix corresponding to the k th MS $\mathbf{H}_k$ is $N_r \times MN_t$ and $\mathbf{n}_k$ is a $N_r \times 1$ vector consisting of i.i.d. $\mathcal{CN}(0, N_0)$ noise samples, where $N_0 = \frac{N_t}{SNR}$. Also, $\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \dots, \mathbf{x}_K^T]^T$ is a $KN_s \times 1$ vector representing total transmitted symbols of K users, satisfying $\mathbb{E}\{\mathbf{x}\mathbf{x}^H\} = \frac{1}{KN_s} \mathbf{I}_{KN_s}$. Note that $\mathbf{x}_k$ consists of the N_s symbols transmitted to the k th user. $\mathbf{W}_{RF}^k$ is the $N_r \times N_r^{\text{RF}}$ RF combining matrix and $\mathbf{W}_{BB}^k$ is the $N_r^{\text{RF}} \times N_s$ baseband combining matrix for k th MS.

We define the baseband channel as

$$\bar{\mathbf{H}}_k = \mathbf{W}_{RF}^{kH} \mathbf{H}_k \mathbf{F}_{RF}. \quad (5.2)$$

The estimated signal can be expressed as

$$\begin{aligned} y_{k,s} = & \mathbf{W}_{BB}^{kH}(s, :) \bar{\mathbf{H}}_k \mathbf{F}_{BB}(:, k_s) \sqrt{P_{k,s}} x_{k,s} \\ & + \sum_{s'=1, s' \neq i}^{N_s} \mathbf{W}_{BB}^{kH}(s, :) \bar{\mathbf{H}}_k \mathbf{F}_{BB}(:, k_{s'}) \sqrt{P_{k,s'}} x_{k,s'} \\ & + \sum_{l=1, l \neq k}^K \sum_{s''=1}^{N_s} \mathbf{W}_{BB}^{kH}(s, :) \bar{\mathbf{H}}_k \mathbf{F}_{BB}(:, l_{s''}) \sqrt{P_{l,s''}} x_{l,s''} \\ & + \mathbf{W}_{BB}^{kH}(s, :) \mathbf{W}_{RF}^{kH} \mathbf{n}_k \end{aligned} \quad (5.3)$$

where $y_{k,s}$ is the element $(k-1)N_s + s$ of $\mathbf{y}$ in (5.1). The first term in (5.3) is our desired signal, the second term is intersymbol interference and the third term is inter-user interference. The last term is the noise.

We define $\mathbf{H}_k$ as

$$\mathbf{H}_k = \left[\sqrt{\beta_{k1}} \mathbf{H}_{k1} \dots \sqrt{\beta_{kM}} \mathbf{H}_{kM} \right]. \quad (5.4)$$

where β_{kj} is a real-valued nonnegative number which represents the large-scale fading effect between the RAU at k th MS and j th RAU at the transmitter. Note that subchannel matrix $\mathbf{H}_{kj}$ is defined as (2.1).

By modifying Algorithm 1 in [20], a two-stage hybrid beamforming is being used here to eliminate the inter-user interference. This approach maximizes the sum-rate of the communication system based on the two-stage approach in massive MIMO with double the least number of RF chains (the least number of RF chains is equal to the number of streams to

be transmitted), i.e., $N_t^{RF} = 2KN_s$ and $N_r^{RF} = 2N_s$. The details of the beamforming can be found in Section 2.3.

Based on (5.3) and by using the optimum precoders and combiners, the system input-output relation at the m th time instant for the k th user can be written as

$$y_{k,s}^m = \sqrt{P_{k,s}} \sigma_{k,s} x_{k,s}^m + \tilde{n}_{k,s}^m \quad (5.5)$$

where $\tilde{n}_{k,s}^m = \mathbf{W}_{BB}^{kH}(s, :) \mathbf{W}_{RF}^k \mathbf{n}_k^m$ and $\sigma_{k,s}$ is the s th diagonal element of $\bar{\Sigma}_k$, where $\bar{\Sigma}_k$ is calculated by using the SVD of the matrix $\mathbf{H}_{k,\text{total}}$:

$$\mathbf{H}_{k,\text{total}} = \mathbf{W}_{BB}^{kH} \mathbf{W}_{RF}^k \mathbf{H}_k \mathbf{F}_{RF} \mathbf{F}_{BB} = \bar{\mathbf{U}}_k \bar{\Sigma}_k \bar{\mathbf{V}}_k^H. \quad (5.6)$$

5.2 Diversity Gain and PEP Analysis for Multi-User Scenario

In this section, we investigate using BICMB for a multi-user scenario to increase the diversity gain while transmitting more than one data stream per user through the channel.

The inter-user interference was eliminated in Section IV by using a hybrid beamforming method for multiple users. After beamforming, the pairwise error probability (PEP) can be used in a similar way to [31] to find the upper bound for the error probability. Since this work is only concerned with high SNR regimes, we assume uniform power allocation for matrix $\mathbf{P}$. The change in achievable information rate in this case is negligible. By this assumption, without loss of generality, we assume $\mathbf{P} = \mathbf{I}_{KN_s}$. Please note that since this thesis is focused on diversity order, which is a high-SNR property, the impact of spatially colored noise should not be important.

Theorem 5.1. *When N_t and N_r are sufficiently large, the downlink transmission in a massive MIMO multi-user system can achieve a diversity gain for each user equal to*

$$G_{d,k} = \frac{\left(\sum_j \beta_{kj}\right)^2}{\sum_j \beta_{kj} L_{kj}^{-1}} \quad (5.7)$$

for $k = 1, \dots, K$.

Proof. The proof is similar to Theorem 4.1 in Chapter 4. The argument in (4.10)-(4.19) remains the same with $y_{k,s}^m$ replacing $y_{k,s}$, $\sigma_{k,s}$ replacing σ_s , $\tilde{x}_{k,s}^m$ replacing $\tilde{x}_{k,s}$, and $\hat{x}_{k,s}^m$ replacing $\hat{x}_{k,s}$.

Let us define

$$\begin{aligned} \Theta_k &\triangleq \sum_{s=1}^{L_t} \sigma_{k,s}^2 = \|\mathbf{H}_{k,\text{total}}\|_F^2 = \text{tr}(\mathbf{T}\mathbf{T}^H) = \text{tr}(\bar{\mathbf{\Sigma}}_k \bar{\mathbf{\Sigma}}_k^H) = \frac{N_t}{N_r} \sum_{j=1}^M \beta_{kj} \text{tr}(\mathbf{\Lambda}_{kj} \mathbf{\Lambda}_{kj}^H) \\ &= \frac{N_t}{N_r} \sum_{j=1}^M \sum_{s=1}^{L_t} \beta_{kj} \lambda_{kjs}^2 \end{aligned} \quad (5.8)$$

where $\mathbf{T} = \mathbf{W}_{BB}^{kH} \mathbf{W}_{RF}^{kH} \mathbf{H}_k \mathbf{F}_{RF} \mathbf{F}_{BB}$, $\bar{\mathbf{\Sigma}}_k$ is defined in (5.6), and $\mathbf{H}_{kj} = \mathbf{A}_{kj} \mathbf{\Lambda}_{kj} \mathbf{B}_{kj}^H$ is the SVD of the matrix $\mathbf{H}_{kj}$. Note that due to the similarity between the hybrid beamformer matrices at the transmitter and the receiver, same design algorithms are applicable to both sides. Therefore, as mentioned in [20, 48], by choosing the optimum precoders, $(\mathbf{V}_k^{1:N_s})^H \mathbf{F}_{RF}^{\text{opt}} \mathbf{F}_{BB}^{\text{opt}} \mathbf{F}_{BB}^{\text{opt}H} \mathbf{F}_{RF}^{\text{opt}H} \mathbf{V}_k^{1:N_s} = \mathbf{I}_{N_s}$ for $k = 1, \dots, K$, where $\mathbf{H}_k = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H$ is the SVD of the channel matrix $\mathbf{H}_k$. Same procedure can be applied to the combiner part.

Theorem 3 in [15] implies that the singular values of $\mathbf{H}_{kj}$ converge to $\sqrt{\frac{N_r N_t}{L_{kj}}} |\alpha_{kj}^l|$ in descending order when the number of antennas at the transmitter and the receiver tends to infinity. This latter assumption can be relaxed by a large number of antennas in each RAU, similar to the case for all massive MIMO research.

By using the singular values of $\mathbf{H}_{kj}$, (5.8) can be rewritten as

$$\Theta_k = N_t^2 \sum_{j=1}^M \underbrace{\frac{\beta_{kj}}{L_{kj}} \sum_{l=1}^{L_{kj}} |\alpha_{kj}^l|^2}_{\Psi_{kj}}. \quad (5.9)$$

It can be seen easily that Ψ_{kj} in (5.9) has Gamma distribution with shape $\kappa_{kj} = L_{kj}$ and scale $\theta_{kj} = 2\beta_{kj}L_{kj}^{-1}$, i.e., $\Psi_{ij} \sim \mathcal{G}(L_{kj}, 2\beta_{kj}L_{kj}^{-1})$ [47]. One can use the Welch-Satterthwaite equation to calculate an approximation to the degrees of freedom of Θ_k (i.e., shape and scale of the Gamma distribution) which is a linear combination of the independent random variables Ψ_{kj} [44, p.4.1-1], [45]

$$\kappa_k = \frac{\left(\sum_j \theta_{kj} \kappa_{kj}\right)^2}{\sum_{i,j} \theta_{ij}^2 \kappa_{kj}} = \frac{\left(\sum_j \beta_{kj}\right)^2}{\sum_j \beta_{kj} L_{kj}^{-1}}, \quad (5.10)$$

$$\theta_k = \frac{\sum_j \theta_{kj}^2 \kappa_{kj}}{\sum_j \theta_{kj} \kappa_{kj}} = \frac{\sum_j \beta_{kj}^2 L_{kj}^{-1}}{\sum_j \beta_{kj}}. \quad (5.11)$$

By following (4.30) and (4.31) with Θ_k replacing Θ , θ_k replacing θ , and κ_k replacing k , we have

$$P_b^k \leq \frac{1}{k_c} \sum_{d=d_{\min}}^{\infty} \sum_{i=1}^{W_I(d)} \frac{1}{2} \left(\theta_k \frac{d_{\text{free}}^2 \alpha_{\min}(d, i) N_s N_t}{4L_t} \text{SNR} \right)^{-\kappa_k}. \quad (5.12)$$

The SNR component has a power of $-\kappa_k$ for all summations. Hence, BICMB achieves full diversity order of

$$G_{d,k} = \kappa_k = \frac{\left(\sum_j \beta_{kj}\right)^2}{\sum_j \beta_{kj} L_{kj}^{-1}} \quad (5.13)$$

which is independent of the number of spatial streams transmitted. $\square$

Remark 3. *Theorem 5.1 implies that each MS's diversity gain is different than that of the*

other user and depends on the number of resolvable propagation paths and the large scale fading coefficient for each user. It can be seen easily that the diversity gain is independent of the number of users.

Remark 4. Under the case where N_t and N_r are sufficiently large and assuming that $L_{kj} = L$ and $\beta_{kj} = \beta$ for any k and j , it can be seen easily that the distributed massive MIMO system can achieve a diversity gain

$$G_{d,k} = ML \quad (5.14)$$

which is independent of the number of users. Similar to the single-user scenario, the diversity gain for each user can be compared to (28) in [21] where no coding is used. Without coding, the diversity gain would be $G_d = ML - N_s + 1$. In our work, the diversity gain is independent of the number of transmitted data streams and equals to (28) in [21] when $N_s = 1$, i.e., the maximum value for the diversity gain. One can easily see that by using BICMB, the effect of the number of data streams is omitted from the diversity gain, which allows us to transmit the maximum number of data streams, while maintaining the same diversity gain.

5.3 Simulation Results

The assumptions remain the same in these simulations unless otherwise stated. We assume that each scale fading coefficient β_{kj} equals to $\beta = -20$ dB (except for Fig. 5.5). In the simulations, information bits are mapped onto 16-QAM symbols in each subchannel.

Fig. 5.2 illustrates BER with respect to SNR for three different cases. In each case, two different MSs are being served by the BS. The BS transmits three data streams to each MS. For the first MS, the number of resolvable propagation paths L in each case is $L = 3$, while for the second MS $L = 9$ for all subchannels. By comparing Case I with $N_t = 256$ and

Figure 5.2: BER with respect to SNR for two cases with different number of antennas in each RAU with $M = 2$, $K = 2$, and $N_s = 3$.

Case II with $N_t = 128$ where circle markers represent Case I and triangle markers are for Case II, one can easily see that doubling the number of antennas at the BS has no effect on the slope of the BER in high SNR, i.e., the diversity gain does not change by changing the number of antennas at the transmitter side or the receiver side. This confirms (5.13) where the diversity gain is independent of the number of antennas at the BS. The independence of (5.13) from the number of antennas at the MS side can be seen by comparing Case III with Case I, where both of them have the same number of antennas at the transmitter, but in Case III, the number of antenna elements at the MS side is twice as Case I. Note that the markers of BER of the users in Case III are a cross sign (x). Furthermore, one can see that doubling the number of antennas at the transmitter side or the receiver side approximately makes the performance 3 dB better. This happens because we are doubling the number of resources here.

Figure 5.3: BER with respect to SNR for two cases with different number of data streams sent through the channel. $M = 2$, $K = 2$, $N_t = 256$, and $N_r = 64$

Similar to Fig. 5.2, the BS serves two different MSs in two cases in Fig. 5.3. In Case I, each MS only receives one data stream, while in Case II, each MS receives three data streams. When there is no BICMB, one can get the maximum diversity gain by only sending one data stream through the channel. This can be used as a benchmark to compare the diversity gain when the number of data streams increases. It can be seen that with BICMB by sending more data streams through the channel, the slope of the BER curve does not change in high SNR. Hence, one can get the same diversity gain by transmitting maximum number of data streams through the channel. This maximum value is the rank of the channel.

Comparing Fig. 5.2 or Fig. 5.3 with Fig. 5.4 shows that by increasing the number of MSs in the system, the diversity gain does not change. Also, by using (5.13) for the second and the third user, one can calculate the diversity gain for these users as $G_{d_2} = ML = 2 \times 3 = 6$ and $G_{d_3} = \frac{M^2}{1/l_1 + 1/l_2} = \frac{2^2}{1/2 + 1/6} = 6$. We can see that they have both the same diversity gain

Figure 5.4: BER with respect to SNR when the number of user increases from two to three when each user receives three data streams. $L = [l_1 l_2]$ means that the number of resolvable ropagation paths to the user from the first RAU is l_1 and same for the l_2 and the second RAU. $M = 2$, $K = 3$, $N_t = 256$, and $N_r = 64$

at high SNR.

It is expected from (5.13) that the diversity gain is independent of the large-scale fading coefficient β_{kj} when it is constant, i.e., $\beta_{kj} = \beta$. Fig. 5.5 illustrates two different cases. In the first case with two users, $\beta_{kj} = \beta = -20$ dB. In the second case with three users, the value of β decreases to $\beta = -25$ dB for the first two users. Unlike the first two users in Case II, the values of β_{kj} for the third user are not the same for different subchannels. In both cases, we are transmitting three different data streams for each user. As we expect, the diversity gain remains the same when the large-scale fading coefficient remains constant for all subchannels. This can be concluded by comparing the first two users in each case. For the third user in Case II, based on (5.13), we expect to see different diversity gain value. By calculating the diversity gain for this user, we get $G_d = \frac{(10^{-2}+10^{-5})^2}{(10^{-4}+10^{-10})/3} \approx 3$. One can see that

Figure 5.5: BER with respect to SNR with different large-scale fading coefficient β . $M = 2$, $K = 2$, $N_t = 256$, and $N_r = 64$

when the large scale fading coefficient is small enough, the subchannel corresponding to this coefficient does not help the diversity gain.

Chapter 6

Bit-Interleaved Coded Multiple Beamforming with Perfect Coding for Single-User Scenario

6.1 System Model for Single-User Scenario

We consider a single-user mm-Wave massive MIMO scenario shown in Fig. 6.1, where the transmitter is equipped with M_t RAUs, N_t antennas at each RAU, and N_t^{RF} RF chains. The receiver has M_r RAUs, N_r antennas at each RAU, and N_r^{RF} RF chains. The transmitter sends $N_s = D$ data streams to the receiver. These data streams are generated as follows. First the bit codeword $\mathbf{c}$ is generated through a convolutional encoder with code rate R_c . Then a random bit-interleaver is used to generate an interleaved sequence. The output of the interleaver is modulated by M -quadrature amplitude modulation (M -QAM). We define a one-to-one mapping from $\mathbf{X}_k = [\mathbf{x}_{1,k}, \dots, \mathbf{x}_{D,k}]$ to $\mathbf{Z}_k$ as $\mathbf{Z}_k = \mathbb{M}\{\mathbf{X}_k\}$ where $\mathbb{M}$ denotes the PSTBC codeword generating function [1]. A PSTBC codeword, i.e., $\mathbf{Z}_k$ is generated by

using D^2 consecutive complex-valued scalar symbols [1]

$$\mathbf{Z}_k = \mathbb{M}\{\mathbf{X}_k\} = \sum_{v=1}^D \text{diag}(\mathbf{G}_D \mathbf{x}_{v,k}) \mathbf{E}^{v-1}, \quad (6.1)$$

where $\mathbf{G}_D$ is an $D \times D$ unitary matrix defined as below [32]

$$\mathbf{G}_2 = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 + j\frac{1-\sqrt{5}}{2} & \frac{1+\sqrt{5}}{2} - j \\ 1 + j\frac{1+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2} - j \end{bmatrix}, \quad (6.2)$$

$$\mathbf{G}_3 = \begin{bmatrix} 0.6603 + 0.3273i & 0.0208 + 0.3273i & -0.4921 + 0.3273i \\ -0.2939 - 0.1457i & -0.0374 - 0.5898i & -0.6136 + 0.4082i \\ 0.5295 + 0.2625i & -0.0467 - 0.7355i & 0.2731 - 0.1817i \end{bmatrix}, \quad (6.3)$$

$$\mathbf{G}_4 = \begin{bmatrix} 0.2582 - 0.3122j & 0.3455 - 0.4178j & -0.4178 + 0.5051j & -0.2136 + 0.2582j \\ 0.2582 + 0.0873j & 0.4718 + 0.1596j & 0.1596 + 0.0540j & 0.7633 + 0.2582j \\ 0.2582 + 0.2136j & -0.5051 - 0.4178j & -0.4178 - 0.3455j & 0.3122 + 0.2582j \\ 0.2582 - 0.7633j & -0.0540 + 0.1596j & 0.1596 - 0.4718j & -0.0873 + 0.2582j \end{bmatrix}. \quad (6.4)$$

Furthermore, $\mathbf{x}_{v,k}$ is a $D \times 1$ vector whose elements are the v th D input modulated scalar symbols and $D \in \{2, 3, 4, 6\}$. Matrix $\mathbf{E}$ is defined as

$$\mathbf{E} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ g & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad (6.5)$$

Figure 6.1: Block diagram of a mm-Wave massive MIMO system with distributed antenna sub-arrays employing BICMB-PC.

where

$$g = \begin{cases} j, & D = 2, 4, \\ e^{j\frac{2\pi}{3}}, & D = 3, \\ -e^{j\frac{2\pi}{3}}, & D = 6. \end{cases} \quad (6.6)$$

As it can be seen from Fig. 6.1, the complex-valued matrix $\mathbf{F}_{\text{BB}} \in \mathbb{C}^{N_t^{RF} \times N_s}$ is used for preprocessing at the baseband. A set of $M_t N_t$ phase shifters is applied to the output of each RF chain. As a result of this process, different beams are formed in order to transmit the RF signals. We can model this process with a complex-valued matrix $\mathbf{F}_{\text{RF}} \in \mathbb{C}^{M_t N_t \times N_t^{RF}}$. Note that in this chapter $M_t = M_r = N_s = D$.

By assuming a narrowband flat fading channel model, the channel matrix $\mathbf{H}$ and subchannel $\mathbf{H}_{i,j}$ are defined similar to (4.4) and (2.1), respectively.

The processed signal at the k th PSTBC codeword is

$$\mathbf{Y}_k = \mathbf{W}_{\text{BB}}^H \mathbf{W}_{\text{RF}}^H \mathbf{H} \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}} \mathbf{Z}_k + \mathbf{W}_{\text{BB}}^H \mathbf{W}_{\text{RF}}^H \mathbf{n}_k, \quad (6.7)$$

where $\mathbf{Y}_k$ is an $D \times D$ complex-valued matrix, $\mathbf{n}_k$ is an $M_r N_r \times 1$ vector consisting of i.i.d. $\mathcal{CN}(0, N_0)$ noise samples, $N_0 = \frac{N_t}{\text{SNR}}$ where SNR is defined as the signal-to-noise ratio (SNR), $\mathbf{W}_{\text{RF}}$ is the $M_r N_r \times N_r^{\text{RF}}$ RF combining matrix, and $\mathbf{W}_{\text{BB}}$ is the $N_r^{\text{(RF)}} \times N_s$ baseband combining matrix.

A solution based on singular value decomposition (SVD) of the channel matrix $\mathbf{H} = \mathbf{U} \mathbf{\Lambda} \mathbf{V}^H$ can be derived for the beamforming matrices [15, 2]. By utilizing the optimal precoder and combiner, one can write (6.7) as

$$\mathbf{Y}_k = \mathbf{\Lambda} \mathbf{Z}_k + \tilde{\mathbf{n}}_k, \quad (6.8)$$

where $\tilde{\mathbf{n}}_k = \mathbf{U}_{(1:D)}^H \mathbf{n}_k$, and $\mathbf{U}_{(1:D)}$ is the first D columns of the unitary matrix $\mathbf{U}$.

We model PSTBC codeword sequence as $k' \rightarrow (k, (m, n), j)$, where k' represents the original ordering of the coded bits $c_{k'}$, $(k, (m, n), j)$ are the index of the PSTBC codewords, the symbol position in $\mathbf{X}_k$, and the bit position on the label of the scalar symbol $x_{(m, n), k}$, respectively. We define χ_b^j as the subset of all signals $x \in \chi$. Note that the label has the value $b \in \{0, 1\}$ in position j .

The ML bit metrics for (6.8) can be written as

$$\gamma^{(m, n), j}(\mathbf{Y}_k, c_{k'}) = \min_{\mathbf{X} \in \eta_{c_{k'}}^{(m, n), j}} \|\mathbf{Y}_k - \mathbf{\Lambda} \mathbf{M}\{\mathbf{X}\}\|^2, \quad (6.9)$$

where $\eta_{c_{k'}}^{(m,n),j}$ is defined as

$$\eta_{c_{k'}}^{(m,n),j} = \{\mathbf{X} : x_{(u,v)=(m,n)} \in \chi_b^j, \text{ and } x_{(u,v) \neq (m,n)} \in \chi\}. \quad (6.10)$$

The ML decoder at the receiver makes decisions according to the rule

$$\hat{\mathbf{c}} = \underset{\mathbf{c}}{\operatorname{argmin}} \sum_{k'} \gamma^{(m,n),j}(\mathbf{Y}_k, c_{k'}). \quad (6.11)$$

6.2 Diversity Gain Analysis

In this section, the diversity gain is examined for mm-Wave massive MIMO systems employing distributed antenna-subarray architecture employing BICMB-PC. We show that the diversity gain becomes independent of the number of transmitted streams, whereas in [21] the diversity gain is dependent on the number of transmitted data streams.

Theorem 6.1. *Suppose that N_r and N_t are sufficiently large [15]. Then by utilizing BICMB-PC, mm-Wave massive MIMO systems with distributed antenna-subarray architecture can achieve a diversity gain of*

$$G_d = \frac{\left(\sum_{i,j} \beta_{ij}\right)^2}{\sum_{i,j} \beta_{ij}^2 L_{ij}^{-1}} \quad (6.12)$$

for $i = 1, \dots, M_r$ and $j = 1, \dots, M_t$.

Proof. Assume that codeword $\mathbf{c}$ is transmitted and codeword $\hat{\mathbf{c}}$ is detected. Then one can write the PEP of $\mathbf{c}$ and $\hat{\mathbf{c}}$ as

$$P(\mathbf{c} \rightarrow \hat{\mathbf{c}}|\mathbf{H}) = P\left(\sum_{k'} \|\mathbf{Y}_k - \Lambda \tilde{\mathbf{Z}}\|^2 \geq \sum_{k'} \|\mathbf{Y}_k - \Lambda \hat{\mathbf{Z}}\|^2\right), \quad (6.13)$$

where

$$\begin{aligned}\tilde{\mathbf{Z}} &= \mathbb{M}\{\tilde{\mathbf{X}}\}, \quad \text{where} \quad \tilde{\mathbf{X}} = \arg \min_{\mathbf{X} \in \eta_{c_{k'}}^{(m,n),j}} \|\mathbf{Y}_k - \Lambda \mathbb{M}\{\mathbf{X}\}\|^2 \\ \hat{\mathbf{Z}} &= \mathbb{M}\{\hat{\mathbf{X}}\}, \quad \text{where} \quad \hat{\mathbf{X}} = \arg \min_{\mathbf{X} \in \eta_{\hat{c}_{k'}}^{(m,n),j}} \|\mathbf{Y}_k - \Lambda \mathbb{M}\{\mathbf{X}\}\|^2.\end{aligned}$$

Since the bit metrics corresponding to the same coded bits between the pairwise errors are the same and $\|\mathbf{Y}_k - \Lambda \mathbf{Z}_k\|^2 \geq \|\mathbf{Y}_k - \Lambda \hat{\mathbf{Z}}_k\|^2$, (6.13) is upper bounded by

$$P(\mathbf{c} \rightarrow \hat{\mathbf{c}}|\mathbf{H}) \leq P\left(\xi \geq \sum_{k', d_H} \|\mathbf{r}\|^2\right) \quad (6.14)$$

where $\sum_{k', d_H}$ is the summation of the d_H values corresponding to the different coded bits between the bit codewords, $\mathbf{r} = \Lambda(\mathbf{Z} - \hat{\mathbf{Z}}_k)$, and $\xi = -\sum_{k', d_H} \text{tr}(\mathbf{r}^H \mathbf{n}_k + \mathbf{n}_k^H \mathbf{r})$. Since $\xi \sim \mathcal{CN}(0, 2N_0 \sum_{k', d_H} \|\mathbf{r}\|^2)$, (6.14) is replaced by the Q function as

$$P(\mathbf{c} \rightarrow \hat{\mathbf{c}}|\mathbf{H}) \leq Q\left(\sqrt{\frac{\sum_{k', d_H} \|\mathbf{r}\|^2}{2N_0}}\right). \quad (6.15)$$

By using an upper bound of the Q function $Q(x) \leq \frac{1}{2}e^{-\frac{x^2}{2}}$, the average PEP in (6.15) is upper bounded as

$$P(\mathbf{c} \rightarrow \hat{\mathbf{c}}) = \mathbb{E}[P(\mathbf{c} \rightarrow \hat{\mathbf{c}}|\mathbf{H})] \leq \frac{1}{2}\mathbb{E}\left[\exp\left(-\frac{\sum_{k', d_H} \|\mathbf{r}\|^2}{4N_0}\right)\right]. \quad (6.16)$$

By using [35], we can rewrite (6.16) as

$$P(\mathbf{c} \rightarrow \hat{\mathbf{c}}) = \frac{1}{2}\mathbb{E}\left[\exp\left(-\frac{\sum_{u=1}^D \lambda_u^2 \zeta_u}{4N_0}\right)\right], \quad (6.17)$$

where $\zeta_u = \sum_{k', d_H} \rho_{u,k}$ and $\rho_{u,k} = \sum_{v=1}^D |\mathbf{g}_u^T(\mathbf{x}_{v,k} - \hat{\mathbf{x}}_{v,k})|^2$.

By defining $L_t = \sum_{i,j} L_{ij}$ as the rank of the channel matrix $\mathbf{H}$, i.e., the number of singular values of the channel matrix $\mathbf{H}$, we can write

$$\frac{\left(\zeta_{\min} \sum_{u=1}^{L_t} \lambda_u^2\right)}{L_t} \leq \frac{\left(\zeta_{\min} \sum_{u=1}^D \lambda_u^2\right)}{D} \leq \frac{\left(\sum_{u=1}^D \lambda_u^2 \zeta_u\right)}{D}. \quad (6.18)$$

Similar to equations in (4.20) - (4.27), we can get the upper bound for (6.16) as

$$P(\mathbf{c} \rightarrow \hat{\mathbf{c}}) \leq \frac{1}{2} \mathbb{E} \left[\exp \left(\frac{-\zeta_{\min} D}{4N_0 L_t} \Theta \right) \right]. \quad (6.19)$$

By using the MGF of Θ , (6.19) can be written as

$$P(\mathbf{c} \rightarrow \hat{\mathbf{c}}) \leq \frac{1}{2} \left(1 + \theta \frac{\zeta_{\min} D N_t}{4L_t} SNR \right)^{-\kappa} \quad (6.20)$$

$$\approx \frac{1}{2} \left(\theta \frac{\zeta_{\min} D N_t}{4L_t} SNR \right)^{-\kappa} \quad (6.21)$$

for high SNR.

Hence, BICMB-PC achieves full diversity order of

$$G_d = \kappa = \frac{\left(\sum_{i,j} \beta_{ij}\right)^2}{\sum_{i,j} \beta_{ij}^2 L_{ij}^{-1}} \quad (6.22)$$

which is independent of the number of transmitted data streams. $\square$

Remark 5. By assuming that $L_{ij} = L$ and $\beta_{ij} = \beta$ for any $i \in \{1, \dots, M_r\}$ and $j \in \{1, \dots, M_t\}$, it can be seen easily that the mm-Wave massive MIMO system with distributed antenna-subarray architecture can achieve a diversity gain

$$G_d = L_t = M_r M_t L. \quad (6.23)$$

One can compare this result with the diversity gain calculated for the single-user scenario in [21]. As it can be seen, similar to [2], the full diversity gain is achieved in this chapter.

6.3 Decoding

By replacing (6.1) in (6.8), we can rewrite (6.8) and show that each element of $\mathbf{\Lambda}\mathbf{Z}_k$ is related to only one of the $\mathbf{x}_{v,k}$ [35, 1]. For the case $D = 3$, we can write

$$\mathbf{Y}_k = \begin{bmatrix} \lambda_1 \mathbf{g}_1^T \mathbf{x}_{1,k} & \lambda_1 \mathbf{g}_1^T \mathbf{x}_{2,k} & \lambda_1 \mathbf{g}_1^T \mathbf{x}_{3,k} \\ g\lambda_2 \mathbf{g}_2^T \mathbf{x}_{3,k} & \lambda_2 \mathbf{g}_2^T \mathbf{x}_{1,k} & \lambda_2 \mathbf{g}_2^T \mathbf{x}_{2,k} \\ g\lambda_3 \mathbf{g}_3^T \mathbf{x}_{3,k} & g\lambda_3 \mathbf{g}_3^T \mathbf{x}_{1,k} & \lambda_3 \mathbf{g}_3^T \mathbf{x}_{1,k} \end{bmatrix} + \hat{\mathbf{n}}_k. \quad (6.24)$$

The processed signal in (6.24) can be divided into D parts. Then one can write

$$\mathbf{y}_{v,k} = \mathbf{\Omega}_v \mathbf{\Lambda} \mathbf{G} \mathbf{x}_{v,k} + \hat{\mathbf{n}}_{k,v} \quad (6.25)$$

where $v = 1, \dots, D$ and $\mathbf{\Omega}_v = \text{diag}(\omega_{v,1}, \dots, \omega_{v,D})$. The elements of the matrix $\mathbf{\Omega}_v$ are defined as

$$\omega_{v,u} = \begin{cases} 1, & 1 \leq u \leq D - v + 1 \\ g, & D - v + 2 \leq u \leq D. \end{cases} \quad (6.26)$$

One can simplify (6.25) by using the QR decomposition of $\mathbf{\Lambda} \mathbf{G} = \mathbf{Q} \mathbf{R}$, where $\mathbf{Q}$ is unitary and the matrix $\mathbf{R}$ is an upper triangular matrix, one can rewrite (6.25) as [1]

$$\tilde{\mathbf{y}}_{v,k} = \mathbf{Q}^H \mathbf{\Omega}_v^H \mathbf{y}_{v,k} + \tilde{\mathbf{n}}_{k,v}, \quad (6.27)$$

where $\tilde{\mathbf{n}}_{k,v} = \mathbf{Q}^H \mathbf{\Omega}_v^H \hat{\mathbf{n}}_{k,v}$. Furthermore, one can simplify the ML bit metrics defined in (6.9)

as follows:

$$\gamma^{(m,n),j}(\mathbf{Y}_k, c_{k'}) = \min_{\mathbf{x} \in \rho_{c_{k'}}^{n,j}} \|\tilde{\mathbf{y}}_{m,k} - \mathbf{R}\mathbf{x}\|^2, \quad (6.28)$$

where $\tilde{\mathbf{y}}_{m,k} = \mathbf{Q}^H \mathbf{\Omega}_m^H \mathbf{y}_{m,k}$, and $\rho_{c_{k'}}^{n,j}$ is a subset of χ^D . This subset is defined as

$$\rho_b^{n,j} = \{\mathbf{x} = [x_1, \dots, x_D]^T : x_{d=n} \in \chi_b^i, \text{ and } x_{d \neq n} \in \chi\}. \quad (6.29)$$

As mentioned in [1], the complexity order of the simplified ML bit metrics (6.28) is proportional to M^D , i.e., $\mathcal{O}(M^D)$. By using sphere decoding (SD), the average complexity reduces, but the worst-case scenario is still in order of M^D , i.e., $\mathcal{O}(M^D)$ [49]. Furthermore, for dimensions 2 and 4, the complexity can be reduced by separating the real part and imaginary part of $\tilde{\mathbf{y}}_{m,k}$. Note that in these cases, the matrix $\mathbf{R}$ is real. By doing this separation, the complexity order for the worst-case scenario reduces to $\mathcal{O}(M^{\frac{D}{2}} - 0.5)$ [1].

6.4 Simulation Results

In the simulations, all the assumptions are similar to Section 6.1. Also, each scale fading coefficient β_{ij} equals $\beta = -20$ dB for all simulations, except for Fig. 6.5. At RAUs in both the transmitter and the receiver, ULA array configuration with $d = 0.5$ is considered. Information bits are mapped onto 16-QAM symbols in each subchannel. Fig. 6.2 illustrates the results for BICMB-PC for different values of D and L_{ij} in a mm-Wave massive MIMO system. Furthermore, we can see the comparison of the BICMB-PC with the BICMB results in [2]. Please note that for the sake of comparison, $M_r = M_t = D$ in simulations related to [2], where M_r is the number of RAUs at the receiver side and M_t is the number of RAUs at the transmitter side. The number of resolvable propagation paths are defined as $\mathbf{L} = [L_{11} \ L_{12}; L_{21} \ L_{22}]$ and $\mathbf{L} = [L_{11} \ L_{12} \ L_{12}; L_{21} \ L_{22} \ L_{23}; L_{31} \ L_{32} \ L_{33}]$. When $\mathbf{L} = L$, all

Figure 6.2: BER with respect to SNR for setups. $N_t = 128$ and $N_r = 64$.

elements in $\mathbf{L}$ are constant and equal to L . It can be seen that the diversity gain remains the same for different values for the number of resolvable propagation paths, as long as (6.22) returns the same value of G_d . For example, for the dashed-dot line curves with triangle markers and circle markers, since $\beta_{ij} = \beta$, the diversity gains can be calculated using (6.22). These calculated values are $G_d = 2 \times 2 \times 2$ and $G_d = (2 \times 2)^2 / (6^{-1} + 2^{-1} + 3^{-1} + 1^{-1})$, respectively. It can be seen that the BICMB curves in [2] which are shown in Fig. 6.2 with blue curves with no markers have the same slope in high SNR, i.e., same diversity gain as the BICMB-PC for different setups.

It can be seen from Fig. 6.3 that changing the number of antennas at the RAUs does not affect the diversity gain. This confirms (6.22) where the diversity gain is independent of the number of antennas at each RAU. Furthermore, one can see that by doubling the number of resources here, i.e., the number of antennas at the transmitter or the receiver, the performance of the system gets better by a factor of 3 dB.

Figure 6.3: BER with respect to SNR for different number of antennas at each RAU at the transmitter and receiver. $D = 2$ and $L = 2$.

In Fig. 6.4 we compare the results of BICMB-PC for single-user scenario with a conventional MIMO system utilizing BICMB-PC over the Rayleigh fading channel studied in [1]. In order to make these two comparable, we need to make the number of resolvable propagation paths in the channel equal to one, i.e., $L = 1$. Fig. 6.4 illustrates that for both cases when $D = 2$ and $D = 3$, the slope of the BER in high SNR for the conventional MIMO system is the same as the mm-Wave model in (6.22).

Fig. 6.5 illustrates the effect of large scale fading coefficients on the diversity gain. We consider three different cases for BICMB-PC with $M_r = M_t = D = 2$, $L = 2$, and $N_s = D = 2$. Furthermore, in Case IV, we use BICMB in [2] where the diversity gain is $G_d \approx 4$ with $M_r = M_t = 1$, and $L = 4$. Let $\mathbf{B} = [\beta_{ij}]$ where β_{ij} expressed in dB, as the large scale fading coefficient matrix. We use the following matrices for the large scale fading coefficients for

Figure 6.4: BER with respect to SNR, comparing Rayleigh fading channel in [1] and Saleh-Valenzuela model for mm-Wave channel in (6.22)

different cases:

$$\mathbf{B}_1 = \begin{bmatrix} -23 & -23 \\ -23 & -23 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} -30 & -30 \\ -30 & -30 \end{bmatrix},$$

$$\mathbf{B}_3 = \begin{bmatrix} -40 & -20 \\ -20 & -40 \end{bmatrix}, \mathbf{B}_4 = -20 \text{ dB}.$$

By using equation (6.22), one can see that the diversity gain for Case I and Case II is $G_d = 2 \times 2 \times 2 = 8$, whereas the diversity gain for Case III is $G_d = \frac{(10^{-4} + 10^{-2} + 10^{-2} + 10^{-4})^2}{10^{-8}/2 + 10^{-4}/2 + 10^{-4}/2 + 10^{-8}/2} \approx 4$. It can be seen from Fig. 6.5 that Case III has the same slope as Case IV, where both of them has a diversity gain of $G_d \approx 4$. One can see that in Case II, where the channel is inhomogeneous, the diversity gain decreases.

It can be seen from Fig. 6.6 that the diversity gain is independent of the PDF of the

Figure 6.5: BER with respect to SNR for different values of large scale fading coefficients. The black dashed curve is from [2] for the sake of comparison.

azimuth angle at the transmitter and the receiver. The analytical model studied in this thesis to calculate the diversity gain for the system, is independent of the PDF of the angle of arrival and angle of departure. Please note that the value of L is usually 3 or 4 in mm-Wave systems, and the linear growth of the diversity gain in (6.23) is a function of the angular spread of the signals.

Figure 6.6: BER with respect to SNR with $D = L = N_s = 2$, $N_t = 128$, and $N_r = 64$.

Chapter 7

Bit-Interleaved Coded Multiple Beamforming with Perfect Coding for Multi-User Scenario

7.1 System Model for Multi-user Scenario

In this chapter, we expand our model studied in Chapter 6 to the multi-user scenario, where multiple users are being served by the BS. Consider a downlink multi-user massive MIMO system as shown in Fig. 7.1. The antenna array at the BS consists of N_t^{RF} RF chains and D RAUs, each of which has N_t antennas. There are K different mobile stations (MS) and each one of them is equipped with D RAUs and N_r^{RF} RF chains. Each RAU at the user side contains N_r antennas. The BS transmits KD data streams through the channel and each MS receives its D data streams. In order to reduce the hardware complexity in the mm-Wave system, the number of RF chains are constrained. This constraint is $KD \leq N_t^{\text{RF}} \ll N_t$ and $D \leq N_r^{\text{RF}} \ll N_r$ for the BS and each user, respectively.

Figure 7.1: Block diagram of a multi-user mm-Wave massive MIMO system with distributed antenna arrays employing BICMB-PC.

We denote the RF precoder $\mathbf{F}_{RF}$ by an $DN_t \times N_t^{\text{RF}}$ matrix and the baseband precoder $\mathbf{F}_{BB}$ by an $N_t^{\text{RF}} \times KD$ matrix. At the BS, the transmitted symbols of K users first go through a power allocation matrix $\mathbf{P}$ which is $KD \times KD$. Since the focus is on the high SNR regimes, we assume uniform power allocation for matrix $\mathbf{P}$. The change in achievable information rate in this case is negligible. By this assumption, without loss of generality, we assume $\mathbf{P} = \mathbf{I}_{KD}$. Furthermore, since the RF precoder matrix only changes the phase of the input signal, its magnitude is constant, i.e., $|\mathbf{F}_{RF}(i, j)| = \frac{1}{\sqrt{N_t}}$. Also, because of the power constraint at the BS, we need to satisfy $\|\mathbf{F}_{RF}\mathbf{F}_{BB}\|_F^2 = KD$. We assume that the CSI is known at the transmitter and the receiver. As it can be seen from Fig. 7.1, the bit codeword $\mathbf{c}$ is first generated through a convolutional encoder with code rate $\mathbf{R}_c$. Then a random bit-interleaver is used to generate an interleaved sequence. The output of the interleaver is modulated by M -quadrature amplitude modulation (M -QAM). We define a one-to-one mapping from $\mathbf{X}_{k,j} = [\mathbf{x}_{1,(k,j)}, \dots, \mathbf{x}_{D,(k,j)}]$ to $\mathbf{Z}_{k,j}$ as $\mathbf{Z}_{k,j} = \mathbb{M}\{\mathbf{X}_{k,j}\}$ where $\mathbb{M}$ denotes the

PSTBC codeword generating function. Similar to Chapter 6, we define a PSTBC codeword, i.e., $\mathbf{Z}_{k,j}$ as a codeword generated by using D^2 consecutive complex-valued scalar symbols

$$\mathbf{Z}_{k,j} = \mathbb{M}\{\mathbf{X}_{k,j}\} = \sum_{v=1}^D \text{diag}(\mathbf{G}_D \mathbf{x}_{v,(k,j)}) \mathbf{E}^{v-1}, \quad (7.1)$$

where matrices $\mathbf{G}_D$ and $\mathbf{E}$ are defined in (6.2)-(6.5)

We employ a narrowband flat fading channel model for CSI. The received signal at the k th MS at the j th PSTBC codeword is given by

$$\begin{aligned} \mathbf{Y}_{k,j} &= (\mathbf{W}_{\text{RF}}^k \mathbf{W}_{\text{BB}}^k)^H \mathbf{H}_k \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}} \mathbf{Z}_{k,j} + (\mathbf{W}_{\text{RF}}^k \mathbf{W}_{\text{BB}}^k)^H \mathbf{n}_{k,j} \\ &= \mathbf{H}_{k,\text{total}} \mathbf{Z}_{k,j} + \tilde{\mathbf{n}}_{k,j}, \end{aligned} \quad (7.2)$$

where $\mathbf{Y}_{k,j}$ is a $D \times D$ complex-valued matrix and $k \in \{1, 2, \dots, K\}$. The channel matrix corresponding to the k th MS $\mathbf{H}_k$ is $DN_r \times DN_t$ and is defined as (5.4) in Chapter 5. $\mathbf{n}_{k,j}$ is a $DN_r \times 1$ vector consisting of i.i.d. $\mathcal{CN}(0, N_0)$ noise samples, where $N_0 = \frac{N_t}{SNR}$. The matrix $\mathbf{W}_{\text{RF}}^k$ is the $DN_r \times N_r^{\text{RF}}$ RF combining matrix and $\mathbf{W}_{\text{BB}}^k$ is the $N_r^{\text{RF}} \times D$ baseband combining matrix for k th MS. Also, $\tilde{\mathbf{n}}_{k,j} = \mathbf{W}_{\text{BB}}^{kH} \mathbf{W}_{\text{RF}}^{kH} \mathbf{n}_{k,j}$ in (7.2).

The beamforming is similar to the one used in Chapter 5 where a closed form beamforming formulation is used to create multiple subchannels. Similarly, the details can be found in Section 2.3. After beamforming for each user and using the optimum precoders and combiners, the system input-output relation for the k th user at the j th PSTBC written as

$$\mathbf{Y}_{k,j} = \mathbf{\Sigma}_k \mathbf{Z}_{k,j} + \tilde{\mathbf{n}}_{k,j}, \quad (7.3)$$

where $\mathbf{\Sigma}_k = \text{diag}(\sigma_1, \dots, \sigma_D)$, if the optimum precoder and combiner are calculated based on [20].

7.2 Diversity Gain Analysis

In this section, using BICMB-PC for a multi-user scenario to increase the diversity gain while transmitting more than one data stream per user through the channel is investigated. Similar to previous chapters, the goal is to find an upper bound for the PEP.

Theorem 7.1. *When the number of antennas at both the transmitter side and the receiver side are sufficiently large, the downlink transmission in a massive MIMO multi-user system can achieve a diversity gain for each user equal to*

$$G_{d,k} = \frac{\left(\sum_j \beta_{kj}\right)^2}{\sum_j \beta_{kj} L_{kj}^{-1}} \quad (7.4)$$

for $k = 1, \dots, K$.

Proof. The proof is similar to Theorem 6.1 in Chapter 6. The argument in (6.13)-(6.19) remains the same with $\mathbf{Y}_{k,j}$ replacing $\mathbf{Y}_k$, $\lambda_{k,s}$ replacing λ_s , $\tilde{\mathbf{X}}_k$ replacing $\tilde{\mathbf{X}}$, $\hat{\mathbf{X}}_k$ replacing $\hat{\mathbf{X}}$, $\tilde{\mathbf{Z}}_k$ replacing $\tilde{\mathbf{Z}}$, and $\hat{\mathbf{Z}}_k$ replacing $\hat{\mathbf{Z}}$.

Let us define

$$\Theta_k \triangleq \sum_{s=1}^D \sigma_s^2 = \frac{N_t}{N_r} \sum_{j=1}^M \sum_{s=1}^D \beta_{kj} \lambda_{kjs}^2, \quad (7.5)$$

where λ_{kjs} s are the singular values of $\mathbf{H}_{kj}$ defined in (2.1).

Similar to the previous chapter, the PEP can be upper bounded as

$$P(\mathbf{c} \rightarrow \hat{\mathbf{c}}) \leq \frac{1}{2} \left(1 + \theta_k \frac{-\zeta_{\min} D}{4N_0 L_t} \text{SNR} \right)^{-\kappa_k} \approx \frac{1}{2} \left(\theta_k \frac{-\zeta_{\min} D}{4N_0 L_t} \text{SNR} \right)^{-\kappa_k}, \quad (7.6)$$

for high SNR regimes. Please note that κ_k and θ_k are defined in a similar way to (5.10) and (5.11).

Hence, by using BICMB with perfect coding, one can achieve the full diversity order of

$$G_{d,k} = \kappa_k = \frac{\left(\sum_j \beta_{kj}\right)^2}{\sum_j \beta_{kj} L_{kj}^{-1}} \quad (7.7)$$

for the k th user. □

The remarks for the BICMB-PC multi-user scenario are similar to Chapter 5. Furthermore, the decoding part for each user, can be implemented similar to Section 6.3.

7.3 Simulation Results

In Fig. 7.2 two different cases are simulated. In each case, three different MSs are being served by the BS, where in the first case $D = 2$, and in the second case, $D = 3$. The BS transmits D data streams to each MS. The BER for MSs in Case I is depicted with dashed line curves, whereas the solid line curves show the BER for MSs for Case II. Each MS in each case has different number of antennas. One can see that changing the number of antennas at either the BS side or the MS side does not have any effect on the diversity gain. For example, in the first MS in Case I with $N_t = 128$ and the second MS with $N_t = 256$, doubling the number of antennas at the transmitter side approximately makes the performance 3 dB better. However, this change does not affect the slope of the BER in high SNR, i.e., the diversity gain. This confirms (7.7) where the diversity gain is independent of the number of antennas at the BS.

In Fig. 7.3, we add a third MS to the system with different values of resolvable propagation paths which are shown in the legend. It can be seen that adding this new MS does not change the diversity gain of other MSs in the system, which is expected based on (7.7). Also, one can see easily that MS2 and MS3 have the same diversity gain.

Figure 7.2: BER with respect to SNR for two cases with different number of antennas in each RAU with $L = 2$.

Fig. 7.4 illustrates two different cases. In the first case, two MSs with different number of antennas and perfect coding parameter $D = 3$ are being served. However, the large-scale fading coefficient, i.e., β for the first MS and the second MS is $\beta = -20$ dB and $\beta = -25$ dB, respectively. We can see that the diversity gain for these users does not change based on the large-scale fading. In a similar way, one can see that this is true for Case II, where $D = 2$. This is expected based on (7.7) that the diversity gain is independent of the large-scale fading coefficient β_{kj} when it is constant, i.e., $\beta_{kj} = \beta$.

Figure 7.3: BER with respect to SNR when the number of user increases from two to three when each user receives three data streams. $L = [l_1 l_2]$ means that the number of resolvable propagation paths to the user from the first RAU is l_1 and same for the l_2 and the second RAU. $M = 2$, $K = 3$, $N_t = 256$, and $N_r = 64$

Figure 7.4: BER with respect to SNR with different large-scale fading coefficient β .

Chapter 8

Bit-Interleaved Coded Multiple Beamforming with Orthogonal Frequency Division Multiplexing

8.1 System Model

Consider a wideband mm-Wave distributed antenna system as in Fig. 8.1 consisting of M_t RAUs at the transmitter where each RAU consists of N_t antennas and M_r RAUs at the receiver where each RAU has N_r antennas. The BS transmits N_s data symbols per subcarrier. The number of data symbols can be different for different subcarriers. However, for the sake of simplicity, we assume that each subcarrier has an equal number of data streams. Since there are tens of subcarriers in a typical OFDM system, the analysis of the case where different subcarriers have different number of data streams would be prohibitively complex [50]. Furthermore, since implementing fully digital beamforming is expensive, we consider a hybrid beamforming architecture, where the number of RF chains is much smaller than the

Figure 8.1: Block diagram of a single-user mm-Wave massive MIMO system with distributed antenna subarray employing OFDM.

number of antennas at both the transmitter side and the receiver side, i.e., $N_t^{\text{RF}} \ll N_t$ and $N_r^{\text{RF}} \ll N_r$.

After receiving the transmitted signals from all the RAUs and all subcarriers, the received signals are processed by using the analog beamformer, W_{RF} where W_{RF} is an $M_r N_r \times N_r^{\text{RF}}$ complex matrix. Then, the receiver removes the CP and then performs a fast Fourier transform (FFT). Finally, for each subcarrier, a low-dimensional digital combiner is used to process the the frequency domain signals. The processed signal at the k th subcarrier can be written as

$$\mathbf{y}[k] = \mathbf{W}_{\text{BB}}^H[k] \mathbf{W}_{\text{RF}}^H \mathbf{H}[k] \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}}[k] \mathbf{x}[k] + \mathbf{W}_{\text{BB}}^H[k] \mathbf{W}_{\text{RF}}^H \mathbf{n}[k], \quad (8.1)$$

where $\mathbf{n}[k]$ is the additive complex white Gaussian noise with zero mean and variance N_t/SNR for subcarrier k .

Similar to [15, 2], one can find the beamforming matrices at both the transmitter and the receiver sides. We first rewrite (2.3) in a more compact form, then by assuming that the number of antennas at both the transmitter and the receiver goes to infinity, we find the optimal solutions for the beamforming matrices [50].

One can easily see from the subchannel and channel matrix definitions in (2.3) and (2.4)

that there is a total of $L_s = \sum_{i=1}^{M_r} \sum_{j=1}^{M_t} L_{ij}$ resolvable propagation paths. We can write the channel matrix $\mathbf{H}[k]$ as a summation of L_s matrices, where each matrix is rank-one. In other words

$$\mathbf{H}[k] = \sum_{i=1}^{M_r} \sum_{j=1}^{M_t} \sum_{l=1}^{L_{ij}} \tilde{\alpha}_{ij}^l \tilde{\mathbf{a}}_r(\theta_{ij}^l) \tilde{\mathbf{a}}_t^H(\phi_{ij}^l) e^{-j2\pi \frac{k}{K} \frac{\tau_{ij}^l}{T_s}}, \quad (8.2)$$

where

$$\tilde{\alpha}_{ij}^l[k] = \sqrt{\beta \frac{N_r N_t}{L}} \alpha_{ij}^l e^{-j2\pi \frac{k}{K} \frac{\tau_{ij}^l}{T_s}}. \quad (8.3)$$

Note that in (8.2) we define $\tilde{\mathbf{a}}_r(\theta_{ij}^l)$ as a $M_r N_r \times 1$ vector whose p th entry is defined as

$$[\tilde{\mathbf{a}}_r(\theta_{ij}^l)]_p = \begin{cases} [\tilde{a}_r(\theta_{ij}^l)]_{p-(i-1)N_r}, & (i-1)N_r < p \leq iN_r \\ 0, & \text{Otherwise.} \end{cases} \quad (8.4)$$

Similarly, we define a $M_t N_t \times 1$ vector to define $\tilde{\mathbf{a}}_t(\phi_{ij}^l)$ as

$$[\tilde{\mathbf{a}}_t(\phi_{ij}^l)]_q = \begin{cases} [\tilde{a}_t(\phi_{ij}^l)]_{q-(i-1)N_t}, & (i-1)N_t < q \leq iN_t \\ 0, & \text{Otherwise.} \end{cases} \quad (8.5)$$

We sort $\tilde{\alpha}_{ij}^l[k]$ s for each subcarrier in a descending way such that $|\tilde{\alpha}_1[k]| \geq |\tilde{\alpha}_2[k]| \geq \dots \geq |\tilde{\alpha}_{L_s}[k]|$. We sort the corresponding θ_{ij}^l s and ϕ_{ij}^l s based on this sort. Then, one can decompose the channel matrix defined in (8.2) as

$$\mathbf{H}[k] = \mathbf{A}_r \mathbf{D}[k] \mathbf{A}_t^* = [\mathbf{A}_r | \mathbf{A}_r^\perp] \tilde{\mathbf{D}} [\text{diag}(e^{j\angle \tilde{\alpha}_1}, \dots, e^{j\angle \tilde{\alpha}_{L_s}}) \mathbf{A}_t | \mathbf{A}_t^\perp]^*, \quad (8.6)$$

where we define the matrix $\mathbf{A}_r = [\tilde{\mathbf{a}}_r(\theta_1), \dots, \tilde{\mathbf{a}}_r(\theta_{L_s})]$, $\mathbf{A}_t = [\tilde{\mathbf{a}}_t(\phi_1), \dots, \tilde{\mathbf{a}}_t(\phi_{L_s})]$, and $\mathbf{D}[k] = \text{diag}([\tilde{\alpha}_1], \dots, [\tilde{\alpha}_{L_s}])$. Note that in (8.6), $\mathbf{A}_t^\perp$ and $\mathbf{A}_r^\perp$ are unitary matrices spanning the null space of $\mathbf{H}$ and $\mathbf{H}^T$, $\tilde{\mathbf{D}} = [\text{diag}(|\tilde{\alpha}_1|, \dots, |\tilde{\alpha}_{L_s}|) | \mathbf{0}_{(M_r N_r - L_s) \times (M_t N_t - L_s)}]$, and $\angle \tilde{\alpha}_l$ is the phase

of the l th path.

Similar to [15, 50], one can see that when the number of antennas at both the transmitter and the receiver goes to infinity, the optimal fully-digital precoders can be calculated by the hybrid precoding design in which $\mathbf{F}_{\text{RF}} = \mathbf{A}_t$ and $\mathbf{F}_{\text{BB}}[k] = \mathbf{\Gamma}[k]$, where $\mathbf{\Gamma}[k]$ is the diagonal matrix of the allocated power for each data stream. One can use the water-filling approach to calculate elements of $\mathbf{\Gamma}[k]$. Similarly, for the combiner matrices at the receiver side, the asymptotic optimal analog combiner is given by the set of columns of $\mathbf{A}_r$ corresponding to N_s largest complex gains, $\tilde{\alpha}_l[k]$.

After employing the optimal beam at both the transmitter and the receiver side, the system input-output relation at the k th subcarrier can be written as

$$y_s[k] = \sigma_s[k]x_s[k] + n_s[k] \quad (8.7)$$

for $s = 1, \dots, N_s$ and $k = 1, \dots, K$, where σ_s is the s th largest singular value of $\mathbf{H}[k]$.

As mentioned in [2], there are some criteria which we need to consider when designing the interleaver. Similarly, for the BICMB-OFDM system, the interleaver is designed such that

1. Consecutive coded bits are interleaved within one OFDM symbol.
2. Each subchannel should be used at least once within d_{free} distinct bits among different codewords.
3. Consecutive coded bits need to be transmitted over different subcarriers of an OFDM symbol.

8.2 Diversity Gain and PEP Analysis

In this section we carry out the analysis of the diversity gain for BICMB-OFDM and we find the upper bound for the pairwise error probability (PEP).

Theorem 8.1. *Suppose that $N_r \rightarrow \infty$ and $N_t \rightarrow \infty$. Then the bit interleaved coded distributed massive MIMO system employing OFDM can achieve a diversity gain of*

$$G_d = \min(d_{\text{free}}, M_r M_t L) \quad (8.8)$$

Proof. We model the BICMB bit interleaver as $\pi : k' \rightarrow (k, s, i)$, where k' represents the original ordering of the coded bits $c_{k'}$, k represents the time ordering of the signals $x_{k,s}$, and i denotes the position of the bit $c_{k'}$ on symbol $x_{k,s}$.

We define χ_b^i as the subset of all signals $x \in \chi$. Note that the label has the value $b \in \{0, 1\}$ in position i .

Then, the ML bit metrics are given by, [28, 26, 27]

$$\gamma^i(y_{k,s}, c_{k'}) = \min_{x \in \chi_{c_{k'}}^i} |y_s[k] - \sigma_s[k]x|^2. \quad (8.9)$$

The receiver uses an ML decoder to make decisions based on

$$\hat{\underline{c}} = \arg \min_{\underline{c} \in \mathcal{C}} \sum_{k'} \gamma^i(y_s[k], c_{k'}). \quad (8.10)$$

Assume we transmit the code sequence $\underline{c}$ through the channel we detect $\hat{\underline{c}}$. Then one can use

(8.7) to write the PEP of $\underline{c}$ and $\hat{\underline{c}}$ as

$$P(\underline{c} \rightarrow \hat{\underline{c}} | \mathbf{H}[k], \forall k) = P \left(\sum_{k'} \min_{x \in \chi_{c_{k'}}^i} |y_s[k] - \sigma_s[k]x|^2 \geq \sum_{k'} \min_{x \in \chi_{\hat{c}_{k'}}^i} |y_s[k] - \sigma_s[k]x|^2 \right), \quad (8.11)$$

where $s \in \{1, 2, \dots, N_s\}$.

Let's define

$$\tilde{x}_s[k] = \arg \min_{x \in \chi_{c_{k'}}^i} |y_s[k] - \sigma_s[k]x|^2 \quad (8.12)$$

$$\hat{x}_s[k] = \arg \min_{x \in \chi_{\hat{c}_{k'}}^i} |y_s[k] - \sigma_s[k]x|^2. \quad (8.13)$$

One can see that the two sets $\chi_{c_{k'}}^i$ and $\chi_{\hat{c}_{k'}}^i$ are complementary sets of constellation points in the signal constellation set χ . Also, since $\tilde{x}_{k,s} \in \chi_{c_{k'}}^i$ and $\hat{x}_{k,s} \in \chi_{\hat{c}_{k'}}^i$, $\tilde{x}_{k,s} \neq \hat{x}_{k,s}$. Furthermore, whatever value we choose for $x_{k,s}$ from the set $\chi_{c_{k'}}^i$ the following inequality holds, since $\tilde{x}_{k,s}$ is the minimum based on (8.12)

$$|y_{k,s} - \sigma_s x_{k,s}|^2 \geq |y_{k,s} - \sigma_s \tilde{x}_{k,s}|^2. \quad (8.14)$$

We want to guarantee there exist d_{free} distinct pairs of $(\tilde{x}_s[k], \hat{x}_s[k])$ and d_{free} distinct pairs of $(x_s[k], \hat{x}_s[k])$ with d_{free} distinct values of k . The interleaver design criteria 2 and 3 can make

this happen. One can rewrite the PEP as

$$P(\underline{c} \rightarrow \hat{\underline{c}} | \mathbf{H}[k], \forall k) = P \left(\sum_{k, d_{\text{free}}} |y_s[k] - \sigma_s[k]x[k]|^2 \geq \sum_{k, d_{\text{free}}} |y_s[k] - \sigma_s[k]x[k]|^2 \right) \quad (8.15)$$

$$\leq P \left(\zeta \geq \sum_{k, d_{\text{free}}} |\sigma_s[k] (x_s[k] - \hat{x}_s[k])|^2 \right) \quad (8.16)$$

$$\leq Q \left(\sqrt{\frac{\sum_{k, d_{\text{free}}} d_{\text{min}}^2 \sigma_s^2[k]}{2N_0}} \right), \quad (8.17)$$

where $\zeta = \sum_{k, d_{\text{free}}} \sigma_s[k](\hat{x}_s[k] - x_s[k])^* n_s[k] + \sigma_s[k](\hat{x}_s[k] - x_s[k]) n_s^*[k]$ for given $\mathbf{H}[k], \forall k$, and ζ is a Gaussian random variable with zero mean and variance $2N_0 \sum_{k, d_{\text{free}}} |\sigma_s[k](x_s[k] - \hat{x}_s[k])|^2$.

One can use an upper bound for the Q function $Q(x) \leq \frac{1}{2}e^{-x^2/2}$, to find an upper bound for PEP in (8.17) which can be upper bounded as

$$P(\underline{c} \rightarrow \hat{\underline{c}}) = \mathbb{E} [P(\underline{c} \rightarrow \hat{\underline{c}} | \mathbf{H}[k], \forall k)] \leq \mathbb{E} \left[\frac{1}{2} \exp \left(-\frac{d_{\text{min}}^2 \sum_{k, d_{\text{free}}} \sigma_s^2[k]}{4N_0} \right) \right]. \quad (8.18)$$

Similarly to [2], one can see that the singular values for the channel matrix $\mathbf{H}[k]$ converge to $\alpha_{i,j}^l[k]$. And since by definition $\alpha_{i,j}^l[k] \sim \mathcal{CN}(0, 1)$, therefore $\sigma_s^2[k] \sim \chi_2^2$ has a chi-square distribution with two degrees of freedom. By defining $\omega_s[k] = \sigma_s^2[k]$, one can see that the probability density function (PDF) of $\omega_s[k]$ is $f(\omega_s[k]) = \frac{1}{2\eta} \exp \left(-\frac{\omega_s[k]}{2\eta} \right)$, where $\eta = \frac{N_r N_t}{L}$. Furthermore, α_s is defined as the number of the times the s th subchannel is used within d_{free}

bits such that $\sum_{s=1}^{N_s} \alpha_s = d_{\text{free}}$, we can rewrite (8.18) as

$$P(\underline{c} \rightarrow \hat{\underline{c}}) \leq \frac{1}{2} \prod_{s=1}^{N_s} \left[\int_0^\infty \exp \left(-\frac{d_{\min}^2}{\omega_s} [k] 4N_0 \right) f(\omega_s[k]) d\omega_s[k] \right]^{\alpha_s} \quad (8.19)$$

$$= \frac{1}{2} \prod_{s=1}^{N_s} \left(\int_0^\infty \frac{1}{2\eta} \exp \left(-\frac{d_{\min}^2}{4N_0} \omega_s[k] \right) \exp \left(-\frac{\omega_s[k]}{2\eta} \right) d\omega_s[k] \right)^{\alpha_s} \quad (8.20)$$

$$= \frac{1}{2} \prod_{s=1}^{N_s} \left(\frac{1}{2\eta} \frac{1}{\frac{d_{\min}^2}{4N_0} + \frac{1}{2\eta}} \right)^{\alpha_s} \quad (8.21)$$

$$= \frac{1}{2} \prod_{s=1}^{N_s} \left(1 + \frac{d_{\min}^2 N_r}{4L} \text{SNR} \right)^{-\alpha_s}. \quad (8.22)$$

We define the function $g(d, \boldsymbol{\alpha}, \chi)$ as the PEP of two codewords with Hamming distance of d from each other, where $\boldsymbol{\alpha} = [\alpha_1, \dots, \alpha_{N_s}]$. Please note that the coefficient α_s depends on the codewords $\underline{c}$, $\hat{\underline{c}}$, $d(\underline{c} - \hat{\underline{c}})$, and the interleaver. Similar to [28], we can write down the union bound to illustrate the diversity order of the system. Therefore, one can calculate the BER P_b as

$$P_b \leq \frac{1}{k_c} \sum_{d=d_{\text{free}}}^{\infty} \sum_{i=1}^{W_I(d)} g(d, \boldsymbol{\alpha}(d, i), \chi). \quad (8.23)$$

Let us define the following

$$\Delta(\boldsymbol{\alpha}(d, i)) = \sum_{s=1}^{N_s} \alpha_s(d, i) \quad (8.24)$$

$$\boldsymbol{\alpha}(d_{\text{free}}, j) = \arg \min_{\substack{\boldsymbol{\alpha}(d_{\text{free}}, i) \\ i=1, \dots, W_I(d_{\text{free}})}} \Delta(\boldsymbol{\alpha}(d_{\text{free}}, i)). \quad (8.25)$$

Based on the definition in (8.25), we can see that the $\Delta(\boldsymbol{\alpha}(d, j))$ is the minimum value for $d \geq d_{\text{free}}$. Therefore, $\Delta(\boldsymbol{\alpha}(d, i)) \geq \Delta(\boldsymbol{\alpha}(d, j))$ for $i = 1, \dots, W_I(d)$. By using equations

(8.21)-(8.25), one can calculate P_b as

$$P_b \leq \frac{1}{k_c} \left[\sum_{d=d_{\text{free}}}^{\infty} \sum_{\substack{i=1 \\ i \neq j}}^{W_I(d)} g(d, \boldsymbol{\alpha}(d, i), \chi) + \frac{1}{2} \prod_{s=1}^{N_s} \left(1 + \frac{d_{\text{free}}^2 N_r}{4L} \text{SNR} \right)^{-\alpha_s(d_{\text{free}}, j)} \right]. \quad (8.26)$$

Since we are working in the high SNR regime, we are only interested in the smallest power of SNR in (8.26). The first term in high SNR regimes disappears. Therefore, the diversity gain, i.e., the power of SNR term would be $\Delta(\boldsymbol{\alpha}(d, j))$ when we transmit N_s different symbols through the system. Furthermore, BICMB-OFDM achieves full diversity order of $M_r M_t L$ when $M_r M_t L \leq \Delta(\boldsymbol{\alpha}(d, j))$ for spatial multiplexing order of N_s , which will be covered in the next section. $\square$

8.3 Simulation Results

The configurations for the simulation results in this chapter are similar to the previous chapters, unless it is mentioned. Each OFDM symbol has 64 subcarriers, and has 4 μs duration, of which 0.8 μs is CP.

Fig. 8.2 illustrates the results for BICMB-OFDM for different number of resolvable propagation paths. In this figure, the number of RAUs at both the transmitter and the receiver side is two, i.e., $M_t = M_r = 2$. Since the spectrum of (133, 171) is used in the simulations for this paper, one can see that in this spectrum 11 different codewords can be found with a Hamming distance of $d_{\text{free}} = 10$ from the all-zeros codeword, where the codeword [1110010100010101110000000...] has the worst performance for BICMB-OFDM [28]. On this codeword, we can find the number of subchannel use vector $\boldsymbol{\alpha}$ to be $\boldsymbol{\alpha} = [3, 7]$. This means that for the case of $L = 1$ and $L = 3$, the system achieves a diversity gain of $G_d = 4$ and $G_d = 8$, respectively. However, based on (8.23), when the number of resolvable propagation paths increases, such that $M_r M_t L \geq \Delta(\boldsymbol{\alpha}(d, i))$, the maximum achievable diversity

Figure 8.2: BER with respect to SNR for a 2×2 with different values of L and N_s .

gain which is $\sum_{l=1}^2 \alpha_l = 10$, is achieved. This result is confirmed by comparing two different cases, where $L = 3$ and $L = 6$ in the first and second case, respectively. Both of them achieve the same maximum diversity gain of 10, which can be seen from Fig. 8.2.

Similarly, it can be seen from Fig. 8.3 that the maximum diversity gain is achieved for the case when $M_r = M_t = 4$. In this case, the codeword $[001110010100010101110000 \dots]$ has the worst performance for BICMB-OFDM, when compared to the all-zeros codeword [28]. The α coefficients are given as $\alpha = [1, 3, 2, 4]$, which leads to a maximum diversity order of 10 based on (8.24).

Fig. 8.4 illustrates the effect of the number of antennas at both the receiver and the transmitter on the diversity gain. As we expect from (8.26), the diversity gain does not depend on the number of antennas at the transmitter or the receiver. Fig. 8.4 confirms this for the case where $M_r = M_t = 2$. For example, changing the number of antennas at the receiver from 64 antennas per RAU to 128 per RAU, does not affect the slope of the BER curve

Figure 8.3: BER with respect to SNR for a 4×4 with different values of L and N_s .

and only shifts the BER curve to the right, around 3 dB. This shift is expected since we are doubling the resources at the receiver. One can confirm the same for the number of antennas at the transmitter, by comparing the blue curve with circle markers and the red curve with triangle markers.

Figure 8.4: BER with respect to SNR for different number of antennas at each RAU at the transmitter and receiver. $M_r = M_t = 2$.

Chapter 9

Sensitivity to Channel Estimation Error

Obtaining perfect CSI at the transmitter and the receiver side in practice is not feasible. Therefore we consider the effect of imperfect CSI on the diversity gain for both single-user and multi-user scenarios. The channel matrix obtained in (4.4) can be used to model the imperfect CSI channel matrix as [51, 52, 53]

$$\tilde{\mathbf{H}} = \eta \mathbf{H} + \sqrt{1 - \eta^2} \mathbf{E} \quad (9.1)$$

where $0 \leq \eta \leq 1$ shows the accuracy of the channel estimation and $\mathbf{E} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ is the error matrix. The rank of $\mathbf{H}$ is $L_t \leq \min(M_t N_t, M_r N_r)$. Also, we denote $r = \text{rank}(\tilde{\mathbf{H}}) = \min(M_t N_t, M_r N_r)$. One can approximate $\tilde{\mathbf{H}}$ by a low-rank matrix $\mathbf{H}_{\tilde{r}}$ based on the theory of principal component analysis [54, 55]. By writing the vector forms of the SVD for both matrices, where $\tilde{\mathbf{H}} = \sum_{i=1}^r \lambda_i \mathbf{u}_i \mathbf{v}_i^H$ and $\mathbf{H}_{\tilde{r}} = \sum_{i=1}^{\tilde{r}} \lambda_i \mathbf{u}_i \mathbf{v}_i^H$, we can see that they have the same principal components, where λ_i is i th singular value such that $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_r$, and $\mathbf{u}_i$ and $\mathbf{v}_i$ are the left and right singular vectors corresponding to λ_i , respectively. By

comparing $\mathbf{H}$ and $\eta\mathbf{H}$, one can see they have the same principal components. Again, we can approximate $\eta\mathbf{H}$ with the low rank approximation matrix $\mathbf{H}_{L_t}$ with rank L_t of $\tilde{\mathbf{H}}$.

As it can be seen in (4.6) for the single-user scenario and (2.7)-(2.9), for multi-user scenario, SVD of the channel matrix is the core of the beamforming and by having a good approximation of the principal components of the channel matrix, beamforming error can be minimized. Therefore, since the principal components for the approximation $\mathbf{H}_{L_t}$ are the same as $\mathbf{H}$, we expect to get the same results for the diversity gain, with slightly different BER.

The approximation of $\mathbf{H}_{L_t}$ becomes more accurate when the coefficient η increases and gets closer to one. Also, when the value of η remains constant, one can increase the number of antennas or the number of RAUs at the transmitter and the receiver to increase the accuracy of approximation between $\eta\mathbf{H}$ and $\mathbf{H}_{L_t}$.

9.1 Simulation Results

To demonstrate the effect of the channel estimation error for the diversity gain, the BER for various levels of channel estimation errors are presented in Fig. 9.1 and Fig. 9.2. Fig. 9.1 shows that by increasing the value of channel accuracy, i.e., η to one, the BER performance gets better, however the diversity gain remains the same. Likewise, in the multi-user scenario in Fig. 9.2, increasing η leads to a better performance of BER, yet to the same diversity gain when all other parameters remain constant. Furthermore, in the case of the same channel accuracy, increasing the number of antennas at the transmitter and the receiver in both scenarios increases the performance of BER. This is shown in both figures with black curves. In high SNR, the slope of the BER remains the same, which means the diversity gain does not change by changing the number of antennas in the system. Also, as it can

Figure 9.1: BER with respect to SNR for different values of the channel estimation error coefficient (η) and number of antenna at the transmitter and the receiver for the single-user scenario. $N_s = 2$, $M_t = 3$, $M_r = 1$ and $L = 2$

be seen from both Fig. 9.1 and Fig. 9.2, after decreasing the value of η , the performance of the system degrades, i.e., the diversity gain does not remain the same. It can be seen from the simulation results that at $\eta = 0.85$, the diversity gain does not stay the same. The beamforming matrices which are calculated based on Section 2.3, are affected by the value of η . When the value of η is not sufficiently close to one, these matrices are not accurate anymore, therefore the diversity gain is not what we expect from Theorem 4.1 for single-user scenario and Theorem 5.1 for multi-user scenario.

The effect of the channel estimation error for the diversity gain, is illustrated in Fig. 9.3. The BER for various levels of channel estimation errors are presented in this figure. One can see that in Fig. 9.3, by increasing the value of channel accuracy, i.e., η to one, the BER performance gets better, however the diversity gain remains the same. Furthermore, when η remains the same for the channel, changing the number of antennas at the transmitter and

Figure 9.2: BER with respect to SNR for one user for different values of the channel estimation error coefficient (η) and number of antenna at the transmitter and the receiver for MS1 in the multi-user scenario. $N_s = 3$, $K = 3$, $M = 2$, and $L = [3 \ 3]$

the receiver does not have any effect on the diversity gain and only shifts the BER curve to the left or right around 3 dB. Also, as it can be seen from Fig. 9.3, after decreasing the value of η , the performance of the system degrades, i.e., the diversity gain does not remain the same. It can be seen from the simulation results that at $\eta = 0.87$, the diversity gain does not stay the same. Similar to the previous section, when the value of η is not sufficiently close to one, the beamforming matrices are not accurate anymore, therefore the diversity gain is not what we expect from the theorems we discussed in previous chapters.

Similar to previous figures in this section, the effect of the imperfect CSI on the diversity gain can be seen in Fig. 9.4. The solid line with triangle markers illustrates MS1 employing BICMB-PC with perfect CSI and $D = 3$, and $L = 2$. In other cases, where the value of η decreases, the diversity gain remains the same, as long as the accuracy of the channel is close to one. Furthermore, changing the number of antennas at both the transmitter and

Figure 9.3: BER with respect to SNR for different values of the channel estimation error coefficient (η) and number of antenna at the transmitter and the receiver for the single-user scenario. $D = 2$, and $L = 2$

the receiver does not affect the diversity gain as long as this number is large enough.

Figure 9.4: BER with respect to SNR for different values of the channel estimation error coefficient (η) and number of antenna at the transmitter and the receiver for the single-user scenario. $D = 2$, and $L = 2$

Chapter 10

Conclusion

In this work, we analyzed using the methods of bit-interleaved coded multiple beamforming (BICMB) and BICMB with perfect coding (BICMB-PC) in mm-Wave massive MIMO systems with distributed antenna-subarray architecture. We showed that by implementing BICMB, one can achieve the maximum achievable diversity gain in these systems. In the single-user scenario where we have M_t RAUs at the transmitter side and M_r RAUs at the receiver side, the maximum diversity of $\frac{(\sum_{i,j} \beta_{ij})^2}{\sum_{i,j} \beta_{ij}^2 L_{ij}^{-1}}$ can be achieved. Here β_{ij} and L_{ij} are the large scale fading coefficient and the number of resolvable propagation paths between the i th RAU at the transmitter and j th RAU at the receiver, respectively. This means, by increasing the number of RAUs in the distributed system with BICMB, one can increase the diversity gain and the multiplexing gain. Also, it can be seen that by increasing the number of data streams, the diversity gain value does not change and remains the same. Furthermore, the diversity gain formula for the single-user system is independent from the number of antennas in each RAU for the transmitter and the receiver. A special case of the diversity gain where $L_{ij} = L$ and $\beta_{ij} = \beta$ would be $M_r M_t L$ which is similar to the diversity gain of a convential MIMO system.

In a multi-user system, BICMB achieves full spatial diversity of $\frac{M^2}{\sum_j L_{kj}^{-1}}$ over M RAU transmitter for the k th user. This means one can increase the diversity gain for all users in such a system by increasing the number of RAUs at the transmitter side. Similar to the single-user scenario, the diversity gain is independent from the number of transmitted data streams, which means one can transmit more data streams while achieving the same diversity gain comparing to when only one data stream is transmitted. Another result is that the diversity gain is independent of the number antennas in the transmitter and the receiver side. Finally, the number of users does not affect the diversity gain. In other words, the diversity gain is independent from the number of users in the system which can be seen from the diversity gain formula. In a special case when $L_{kj} = L$, the diversity gain can be simplified into $G_d = ML$.

After implementing BICMB for distributed antenna array systems, we approached the same problem with a different tool. By combining BICMB and perfect coding [32] which was studied first in [1], we got the same result for the maximum achievable diversity gain. The main difference between BICMB and BICMB-PC is that in BICMB-PC the number of RAUs at both the transmitter and the receiver should be the same. This number should be chosen from the set $D = \{2, 3, 4, 6\}$. The maximum achievable diversity gain in the case of BICMB-PC for the single-user scenario is $G_d = \frac{(\sum_{i,j} \beta_{ij})^2}{\sum_{i,j} \beta_{ij}^2 L_{ij}^{-1}}$ which reduces to $D^2 L$ in a special case where $\beta_{ij} = \beta$ and $L_{ij} = L$ for $i = 1, \dots, D$ and $j = 1, \dots, D$.

In Chapter 8, unlike previous chapters, the system model is a wideband mm-Wave distributed antenna subarray system. By employing both BICMB and OFDM in this system, we could find the upper bound for the PEP which leads to finding the maximum achievable diversity gain for the BICMB-OFDM system. We showed that the diversity gain, similar to previous chapters, will become independent from the number of data streams and the number of antennas in the system. Furthermore,

In all the simulations in first chapters of the thesis, the assumption was that the perfect

channel state information is available at both the transmitter and the receiver. Since in practice, this cannot happen, we defined a parameter as sensitivity to channel estimation error to study the effect of imperfection in CSI. The parameter shows how accurately we have estimated the CSI and based on the simulations, we showed that the diversity gain remains the same for cases where η , i.e., the channel estimation accuracy, is close to one. In other words, as long as the error in the estimation is small enough, we can get the same slope in the BER curve with respect to SNR, which shows the diversity gain.

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