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THEORY AND EMPIRICAL EVIDENCE

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Pricing Interest Rate Swaps: Theory and Empirical Evidence

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1. Introduction

An interest rate swap is a contract between two parties in which interest payments are based on a notional principal amount, which itself is never paid or received. Instead the parties agree to pay each other the interest which would be due on the notional principal if the underlying securities were bought and sold. One interest payment stream, the floating payment, is tied to a short-term money market rate, such as the Treasury bill rate or LIBOR, and adjusted periodically. The other payment stream is fixed for the life of the swap. Both fixed and floating interest payments start accruing on the swap's effective date and cease on the swap's maturity date. The effective date is generally five business days after the trade date.

A simple example illustrates how a swap works. Suppose that firm A is the floating payer and firm B is the fixed payer. The floating rate is set at three-month LIBOR on the effective date and reset every three months, while the fixed rate is set at 12% for the life of the swap. The notional principal is $10 million and the maturity is 10 years. Assuming that no further trades occur in the secondary market, this swap requires firm A to make the floating payment to firm B for the next 10 years, while firm B makes the fixed payment to firm A. For the purposes of valuation it is important to note that the cash payments are identical to the payments which would result if A issued a $10 million floating rate bond at par that was purchased by B, and B issued a $10 million fixed rate bond at par that was purchased by A.

The "price" of a swap is the spread between the fixed and floating rates. As a matter of convention the Treasury yield curve is typically used for the fixed side of the market and spreads are stated on a semiannual bond equivalent basis. For example, a 10 year swap might be quoted to the fixed-rate payer as the Treasury yield curve plus 60 basis points versus three-month LIBOR. This means that the fixed-rate payer could enter into a swap in which he receives three-month LIBOR and makes fixed payments, the internal rate of return which equates to 60 basis points over the semiannual bond equivalent yield on a 10 year Treasury bond. All the spread data used in this study conform to this convention.

The key question for the pricing of swaps is, "What causes the spreads (prices) of new swaps to vary over time?" That question is the focus of this study. The paper is organized as follows. In the next section some basic theoretical propositions regarding the pricing of swaps are developed. In section three preliminary tests of the propositions are conducted using data provided by Salomon Brothers. The final section presents a summary of the conclusions.

2. The Pricing of Swaps: Basic Theory

The theoretical propositions developed here are based on idealized swaps that trade in a competitive capital market which permits no arbitrage opportunities.[1] It worth noting at the outset that many practitioners feel that the swap market grew so

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[1] An arbitrage opportunity is defined to be a riskless investment which earns a rate of return in excess of the risk-free rate.

rapidly precisely because there were arbitrage opportunities. If such is the case during the sample period, the pricing propositions will fail to hold.

The first proposition is based on the assumption that both parties to the swap can borrow at the risk-free rate. The floating rate side of the swap is tied to the one-period, risk-free interest rate. That is, at the beginning of each period, t, the floating rate payment to be made at t+1 is $R_{1,t}$ per dollar of notional principal, where $R_{1,t}$ is the one-period, risk-free interest rate at the beginning of period t. Assuming that the swap has a maturity of n periods, the fixed-rate payment is set at R_n for the life of the swap, where R_n is the risk-free rate on n-period debt.

Proposition 1: If the two parties to a swap can borrow at the riskless rate, and if the riskless rates R_1 and R_n are used in setting the swap payment streams, then the fixed-floating spread, will be zero independent of the maturity of the swap, n, and the term structure of interest rates given by $R_n - R_1$.

The key to proving this proposition is to recognize that, net of the return of principal, the payments exchanged in a swap are identical to the payouts from investment of the notional principal in a sequence of one-period bills and an n-period bond.[2] These two investments will be of equal value if the bills and the bond

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[2] Stated another way, entering into a swap is equivalent to financing the purchase of n-period debt by issuing one-period securities and rolling them over each period.

are risk-free and if the yields on the bills are $R^1{}_{,t}$ and the yield on the bond is R_n. Thus the spread on a risk-free swap will be zero when R_1 and R_n are used to set the floating and fixed payments.[3]

The importance of proposition 1 is that it demonstrates that under most circumstances the spread on swaps will be independent of complicated factors like the term structure of interest rates and the volatility of interest rates. However, there are a number of circumstances where proposition 1 will fail to hold. First, the arbitrage argument used to prove the proposition relies on the assumption that the floating rate payment is equivalent to the interest received from rolling over investments in the short-term security to which the floating interest rate is tied. If the interval over which the interest payment is set does not match the maturity of the instrument used to determine the floating rate, the arbitrage argument does not work. For example, the floating interest rate on swaps tied to three-month Treasury bills is typically reset on a weekly basis.

To take account of this mismatch requires an equilibrium asset pricing model. However, the discrepancies from Proposition 1 are likely to be small. Previous research by Fama and Gibbons [1982], among others, indicates that short-term rates are well approximated by a random walk. Thus the expected value of an average of future short-term rates equals the current spot rate and the

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[3] This arbitrage argument is attributable to Cox, Ingersoll and Ross [1980]. A variant of it is used in a related paper by Giddy [1985].

expected deviation of the cash flows caused by the mismatch problem is zero. If the risk associated with these cash flows is nonsystematic, Proposition 1 holds in the context of an asset pricing model.

The assumption that both parties to a swap can borrow at the risk-free rate is not critical. Proposition 2 generalizes the argument to show that as long as the rates used to set the swap payments are equal to the rates at which swap participants can borrow, the swap spread will be zero.[4]

Proposition 2: If the credit ratings of swap participants permit them to borrow at R_1+x in the one-period market and R_n+y in the n period market, and if the floating rate index equals R_1+x while the fixed rate index is R_n+y, then the swap spread will be zero.

The proof follows by extending the argument used to prove proposition 1. The swap payments equal the payouts, net of the

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[4] Throughout this paper I assume that both parties to a swap have the same credit rating or that the lower rated party purchases some form of insurance. One example of such insurance is purchasing a letter of credit. Alternatively, a third party, such as a commercial bank can interpose itself. Without insurance the party with the higher credit rating subsidizes the other party. Thus it is not surprising that the Institutional Investor reports most companies refuse to do swaps with parties whose credit rating is lower than theirs. Institutional Investor, November 1984, p. 76.

return principal, from investment of the notional principal in a sequence of one-period corporate securities at the rate $R_{1,t}+x$ and an n-period corporate bond at R_n+y. By definition these two investments are equal, and thus the equilibrium swap spread is zero, as long as the borrowing rates of the parties to the swap are R_1+x and R_n+y.

Proposition 2 makes it clear that except for the mismatch problem non-zero spreads will occur only when the rates used on the fixed and floating sides of a swap reflect credit risk which differs from that of the contracting parties. Proposition 3 gives the relation between the swap spread and the differential credit risk.

Proposition 3: If the borrowing rates of the parties to a swap are R_1+x and R_n+y, but the fixed and floating payments are based on R_1 and R_n, then the swap spread will be the floating rate flat against the fixed rate plus y-x.

Reducing the floating rate index by x basis points from R_1+x to R_1 reduces the floating payout by x/100 times the notional principal each period. Similarly, reducing the fixed-rate by y basis points reduces the fixed payout by y/100 times the notional principal each period. Assuming that these reductions are certain, the fixed stream must be increased by (y-x)/100 times the notional principal if its value is to remain equal to the value of the floating stream. That is, given that the two streams are of equal value when the payments are based on R_1+x and R_n+y, they will remain of equal value only if an identical amount is

subtracted from each. Thus the swap spread must be y-x basis
points.

3. Data and Empirical Results

The data used in this study are taken from Salomon Brothers'
Bond Market Roundup which includes weekly data, as of the close
each Thursday, on swaps tied to LIBOR and three-month Treasury
bills. The sample period runs from October 12, 1984, when swap
quotes began appearing in the Bond Market Roundup, through
November 15, 1985.

The LIBOR swaps have maturities of 2, 5, 7 and 10 years and
are quoted on the basis of three-month LIBOR flat versus a spread
to the Treasury security with a maturity equal to that of the
swap.[5] The reset period for the floating rate is three months so
there is no mismatch.

The Treasury bill swaps also have maturities of 2, 5, 7 and 10
years. The swaps are quoted on the basis of the three-month bill
rate flat versus a spread to the Treasury yield curve. Although
the floating rate payments are made quarterly, in accordance with
the maturity of the floating index, the interest rate is reset
weekly so that the floating payment depends on the history of the
three-month bill rates over the last 13 weeks.

For both the LIBOR and bill swaps only "indications" of the
spreads are available. There are no bid-ask quotes or volume

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[5] If there is not an actively traded Treasury security with a
maturity equal to that of the swap, the spread is based on an
interpolation of the Treasury yield curve as reported by Salomon
Brothers.

data. Furthermore, there is no information on the relation of the indicated spread to the last transaction, nor is the time of the last transaction reported.

The data problems are even more serious for the bond market. The theoretical propositions imply that in equilibrium the swap spreads depend only on the relation between the credit risk of the swap participants and the credit risk of the indexes on which the fixed and floating rates are based.[6] While good data are available for the yields on government securities, only scattered data are available for the yields on A-rated corporates. Furthermore, the data for corporates are only "indications." No bid-ask quotes, transaction information, or volume data are available. What makes this problem particularly severe is that the theory predicts that the swap spread will change with y-x, which is the difference between the quality yield spread at two different maturities. With poor data on A-credit yields the variance of the measurement error in y-x is likely to be of the same order of magnitude as the variance of y-x. Finally, the data problems are compounded by the fact that active swap trading is a recent innovation, so the sample period is short. For these reasons only rough tests of the theoretical propositions are possible.

Since data on three-month A-credits are not available, I assume that LIBOR is equal to the rate on a three-month A-credit. This means that the spreads for the 7 and 10 years swaps should be approximately equal to the difference between the yield on

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[6] This assumes that both parties to the swap have an equal credit rating and that there is no mismatch problem.

intermediate term A-credits and intermediate term Treasury bonds. Evidence on this hypothesis is reported in Table 1 and Figure 1. The table shows that the average values of the 7 and 10 year swap spreads are 44.0 and 44.7 basis points, respectively, compared to an average quality yield spread of 53.5 basis points. The figure shows that the swap spreads, which are nearly identical, and the yield differential are generally within 20 basis points of each other. In addition, it is possible that the discrepancies simply reflect the fact that LIBOR is not equal to the rate on a three-month A-credit. Overall, the results are largely consistent with the predictions of Proposition 3.

Figure 2 illustrates that the 2 and 5 year swap spreads are more variable than the long-term swap spread. Unfortunately, data on the yield spread between either 2 or 5 year Treasuries and the corresponding A-credits are unavailable, so it is not possible to test whether this added variance is due to increased variation in the quality spread as the theoretical propositions predict.

However, there are indirect ways to test the propositions. Proposition 3 predicts that except for the mismatch problem, changing the floating index from LIBOR to the three-month Treasury bill rate should change all the swap spreads by the difference between LIBOR and the three-month bill rate. The data are not entirely consistent with this prediction. Figure 3 presents a plot of the difference between the LIBOR and the bill swap spreads for the 2 and 10 year maturities compared to the yield spread between LIBOR and Treasury bills. Contrary to the predictions of the model, the three series are not identical. This finding is

supported by the results reported in Table 1. The average values
of the differences between the swap spreads are 86.5, 102.6, 106.9
and 112.5 basis points at maturities of 2, 5, 7 and 10 years. In
contrast, the average LIBOR-bill yield differential is 79.3 basis
points. The differences between the means are statistically
significant for each of the four series.[7]

Another way to test the model is to check whether the swap
spreads are independent of the term structure. In the case of the
7 and 10 year swaps this is trivial. Figure 4 presents a plot of
both the yield spread between 10 year Treasury bonds and three-
month Treasury bills and the 10 year swap spread. The figure
reveals that large swings in the term structure had no impact on
the swap spread, which is virtually constant.

In the case of the 2 and 5 year swaps, however, the situation
is different. Figure 5 indicates that the 2 year swap spread
appears to be correlated with the yield spread between two-year
Treasury notes and three-month Treasury bills.[8] This is corrobo-
rated for both the 2 and 5 year maturities by the regression
results reported in Table 2. The regressions, which are estimated
using both levels of the variables and first differences, reveal
that higher term spreads are associated with larger swap spreads.

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[7] The swap spread difference minus the yield differential is
defined as XD in Table 1.

[8] The figure plots two times the swap spreads to make visual
comparison of the two series easier.

It is possible, however, that the correlations between the term structure and the swap spreads are spurious. The theory predicts that the swap spreads should widen if the spread between the yield on short-term Treasuries and short-term A-credits widens. In the regression equations the term spread may function as a proxy for the quality spread, thereby producing the spurious correlation. This problem can be solved by including the quality spread as an additional explanatory variable, but data on short-term quality spreads are not available. For this reason two other proxies, the intermediate-term quality spread and the yield spread between LIBOR and Treasury bills, are included in the regression. The hope is that these variables are more highly correlated with the 2 to 5 year quality spreads than the term structure variable, in which case the term structure variable should no longer be significant in an expanded regression.

The results in Table 2 show that this procedure is only partially successful. In the level regressions the term structure variable remains significant after the quality spread variables are added. In addition, the Durbin-Watson statistics show that the autocorrelation of residuals is still very high. It is likely, therefore, that an autocorrelated variable, perhaps the true quality spread, has been omitted. This interpretation is supported by the regressions using first differences of the data. Differencing the variables reduces the autocorrelation and thereby reduces the chance of spurious correlation. In the first difference regressions the term structure coefficient is only significant in the case of the 5 year swap. When the quality

spread variables are added to the equation, the t-statistic on the term structure coefficient falls from 2.5 to 1.8.

In summary, the results are mixed. While the prices for the 7 and 10 year swaps are basically consistent with the theory, there are discrepancies between the theoretical predictions and the actual prices, particularly at the short maturities. Unfortunately, the data are not precise enough to determine whether these discrepancies represent arbitrage opportunities or measurement error. In this context it is worth reiterating that the pricing propositions were derived under the assumption of a capital market in which there are no arbitrage opportunities. However, the explosive growth of the swap market indicates that profit opportunities were available, at least initially. If regulation, information costs, underwriting costs and trading costs make it expensive for corporations to alter the maturity structure of their debt in the cash market or lead to the mispricing of different quality bonds, then there will be an incentive to do swaps. Furthermore, the incentive will remain as long as the pricing discrepancy is less than the added cost of dealing in the cash market.

5. Summary and Conclusions

The theoretical propositions on the pricing of swaps derived in this paper predict that the swap spread should depend only on the relation between the credit risk of the parties to the swap and the implicit credit risk of the indexes on which the swap is based. If both parties to a swap have A-ratings and if the swap payments are based on the interest rates on A-rated securities,

then the swap spread will be zero. The spread is independent of the term structure because the impact of the yield curve is impounded in the interest rates on which the swap is based.

While the rough nature of the data make detailed empirical tests impossible, the initial results are largely consistent with the theoretical propositions. The 7 and 10 year swap spreads are almost constant, independent of the term structure, at a level close to the average value of the yield differential between A-credits and Treasury securities. The 2 and 5 year swap spreads are more variable, but it could not be determined whether this variation presented profit opportunities because of the lack of data on 2 to 5 year A-credits. Finally, the difference between the bill swap spreads and the LIBOR swap spreads is not equal to the yeild spread between LIBOR and bill yields, but the discrepancies are relatively small.

Whether the divergences of the swap prices from the predictions of the theory present profit opportunities is a question that must await better data. In particular, transaction data on the yield spreads between Treasuries and A-credits at all swap maturities are required. In addition, information on the costs of dealing in the A-credit cash market would be helpful because these costs determine the maximum amount by which observed prices can differ from model prices without presenting arbitrage opportunities.

References

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TABLE 1

Summary Statistics

Variable	Mean	Standard Deviation	Observations
L2	58.5	7.3	58
L5	56.8	5.9	58
L7	44.0	3.0	58
L10	44.7	2.1	58
D2	86.5	12.0	58
D5	102.6	8.9	58
D7	106.9	5.2	58
D10	112.5	2.8	58
TS2	165.7	22.0	58
TS5	253.4	28.9	58
TS7	288.5	33.9	58
TS10	293.2	32.6	58
L-B	79.3	14.6	58
A-T	53.5	8.9	58
XD2	7.2	16.7	58
XD5	23.3	9.9	58
XD7	27.7	12.1	58
XD10	33.2	16.6	58

L2, L5, L7 and L10 = the LIBOR swap spreads for maturities of 2, 5, 7, and 10 years.

D2, D5, D7, and D10 = the difference between the LIBOR and bill swap spreads at each of the four maturities.

TS2, TS5, TS7, and TS10 = the yield spread between three month Treasury bills and Treasury bonds with each of the four maturities.

L-B = three-month LIBOR minus the three-month Treasury bill rate.

A-T = the yield on an intermediate term A-credit minus the yield on an intermediate term Treasury.

XD2, XD5, XD7, XD10 = the difference between the LIBOR and bill swap spreads - (L-B).

TABLE 2

Regression Results [a]

Level Regressions

L2 = 23.8 + 0.209*TS2 R^2=0.40
     (4.2)   (6.1)   DW=0.61

L2 = 21.4 + 0.213*TS2 + 0.070*L-B - 0.071*A-T R^2=0.43
     (2.5)   (6.2)   (1.3)   (0.8)   DW=0.69

L5 = 42.2 + 0.058*TS5 R^2=0.08
     (6.4)   (2.2)   DW=0.11

L5 = 31.3 + 0.063*TS5 + 0.154*L-B - 0.052*A-T R^2=0.24
     (3.7)   (2.6)   (3.2)   (0.7)   DW=0.32
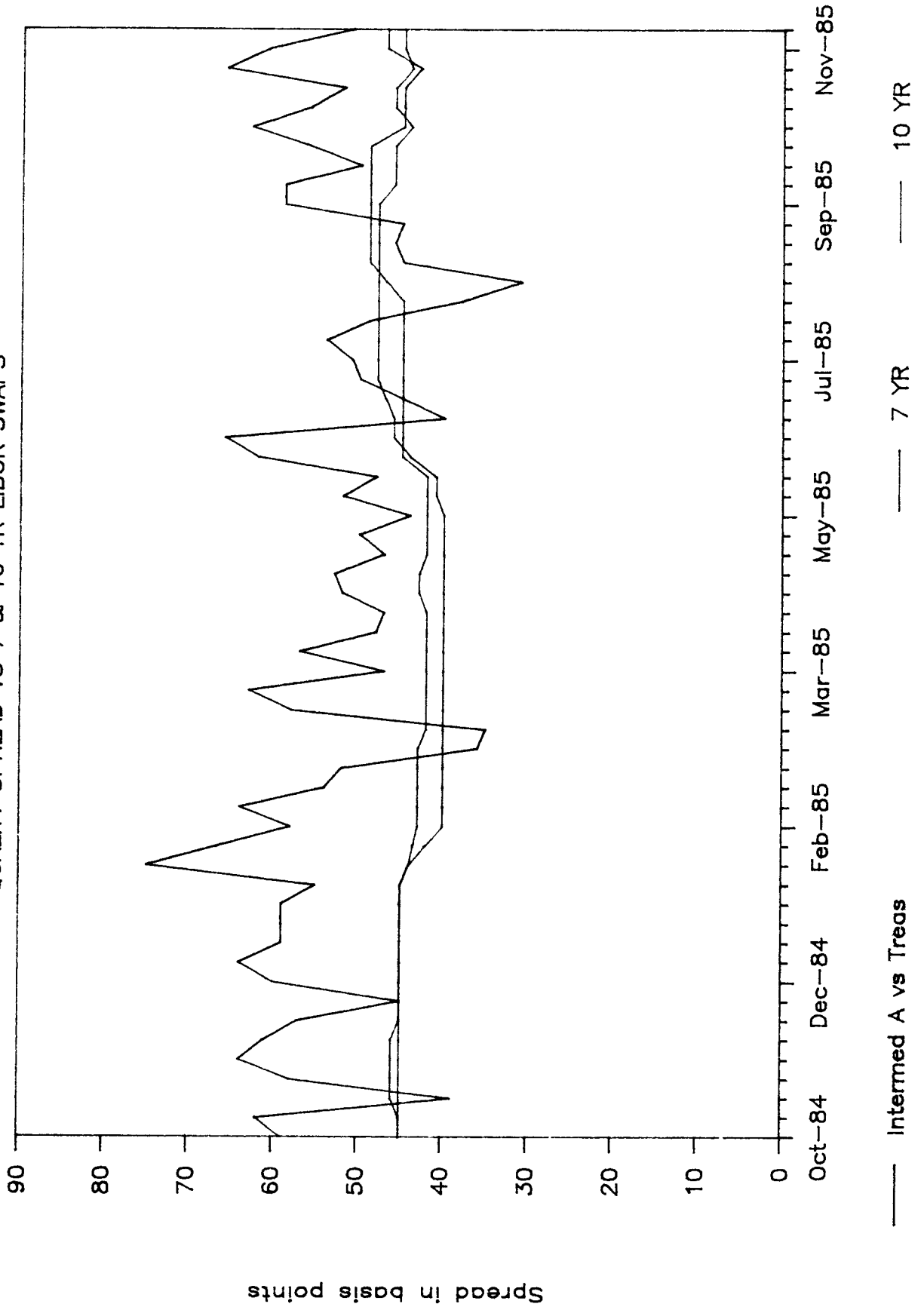

First Difference Regressions

XL2 = 0.003 + 0.185*XTS2 R^2=0.05
      (0.4)   (1.6)   DW=1.45

XL2 = 0.004 + 0.156*XTS2 + 0.017*L-B - 0.048*A-T R^2=0.07
      (0.4)   (1.2)   (0.3)   (1.1)   DW=1.47

XL5 = 0.004 + 0.186*XTS5 R^2=0.10
      (0.8)   (2.5)   DW=1.82

XL5 = 0.003 + 0.148*XTS5 + 0.023*L-B + 0.021*A-T R^2=0.14
      (0.7)   (1.8)   (0.9)   (1.0)   DW=1.80

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[a] The t-statistics are in parentheses

FIGURE 1

QUALITY SPREAD VS 7 & 10 YR LIBOR SWAPS

Spread in basis points

Oct-84   Dec-84   Feb-85   Mar-85   May-85   Jul-85   Sep-85   Nov-85

——— Intermed A vs Treas   ——— 7 YR   ——— 10 YR

FIGURE 2

LIBOR SWAP SPREADS FOR 2, 5, & 10 YRS

Spread in basis points

2 YR ——   5 YR ——   10 YR ——
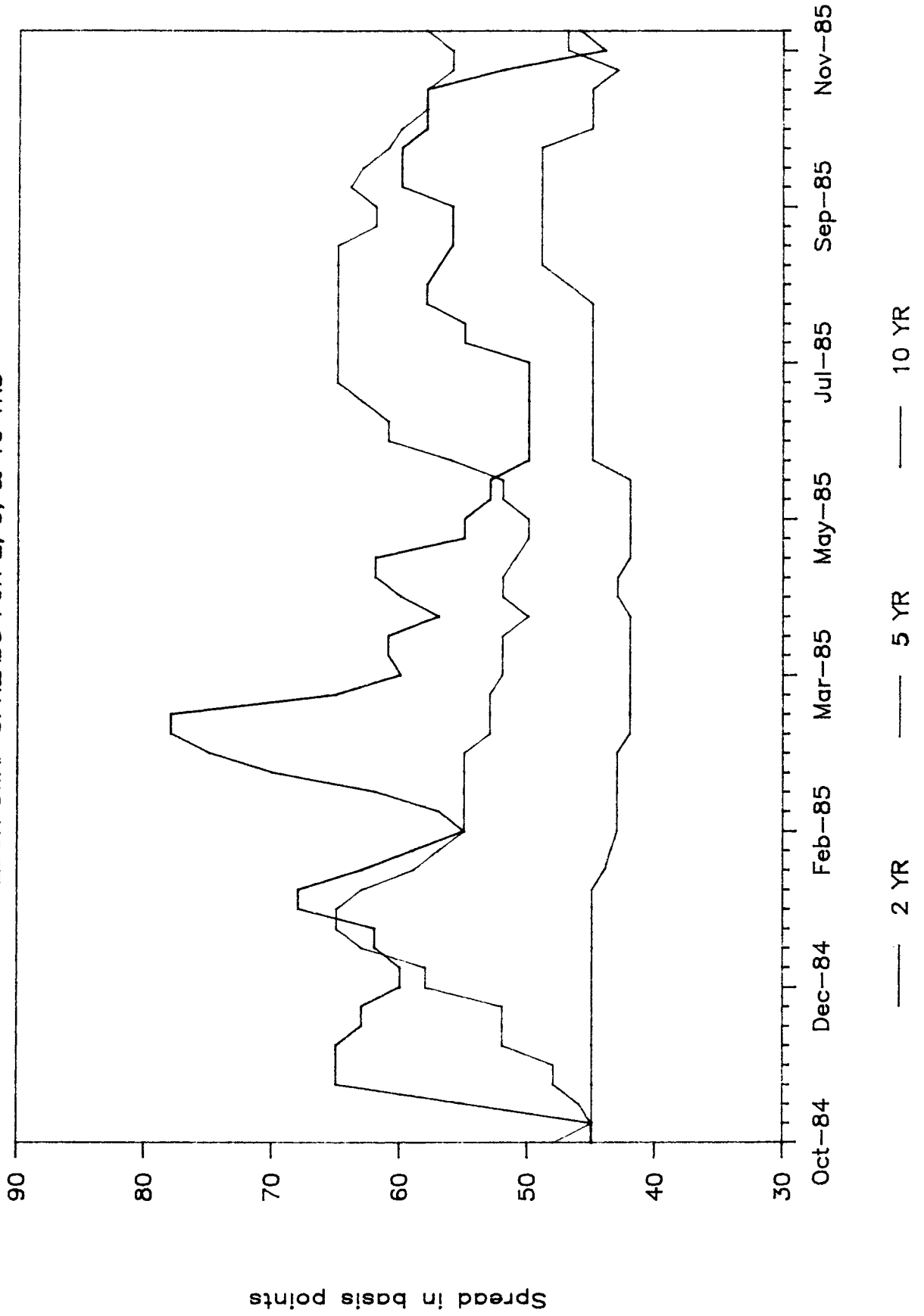
FIGURE 3

SWAP SPREAD DIFFERENTIALS: LIBOR VS BILL

Difference in basis points

Oct-84   Dec-84   Mar-85   May-85   Jul-85   Sep-85

——— 2 YR DIFF   ——— 10 YR DIFF   ——— LIBOR – BILL

19
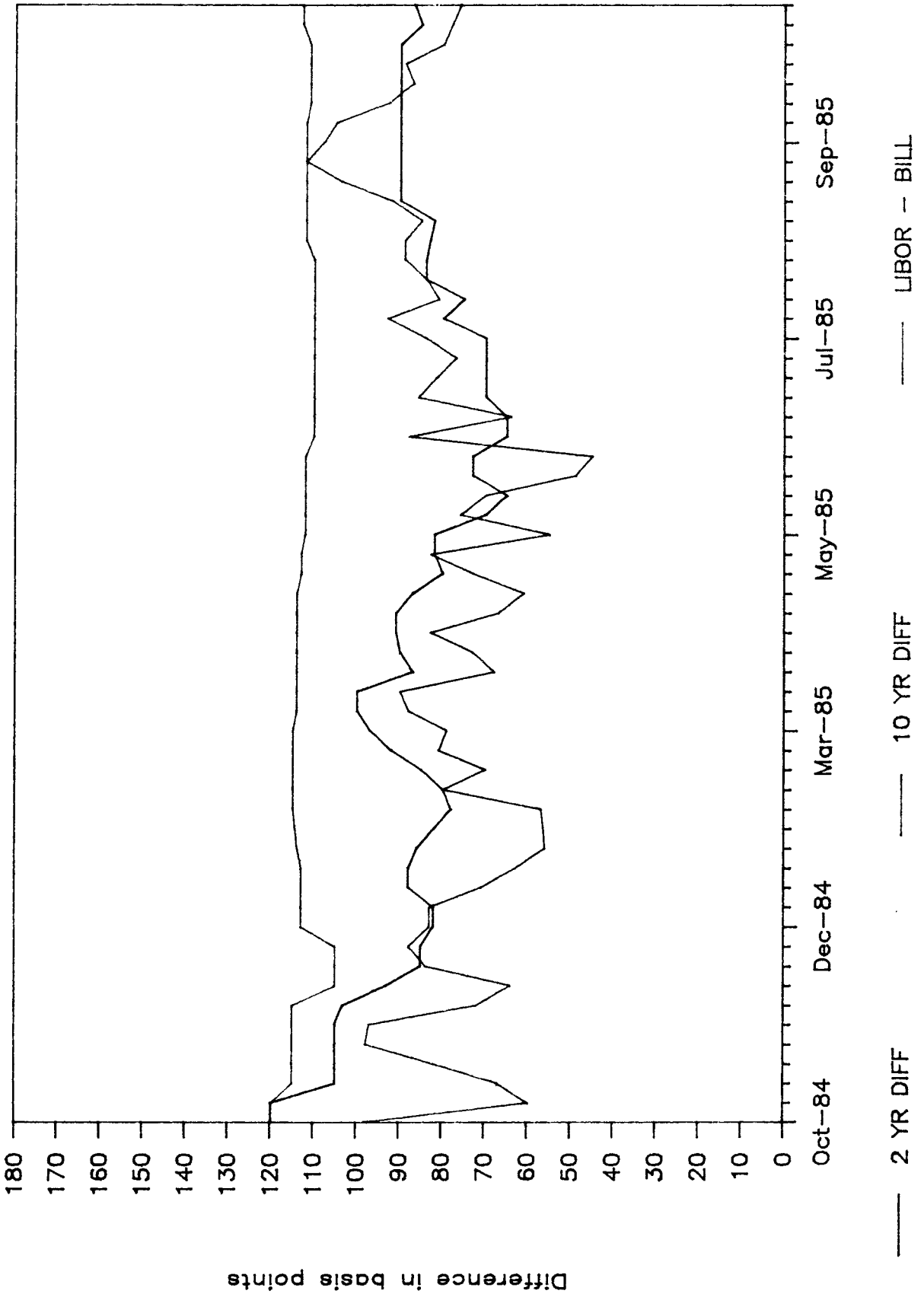
FIGURE 4

10 YR SWAP SPREAD & THE TERM STRUCTURE

Spread in basis points

400   350   300   250   200   150   100   50   0

Oct-84   Dec-84   Feb-85   Mar-85   May-85   Jul-85   Sep-85   Nov-85

———— 10 YR SPREAD   ———— 10 YR — 3 MO TREAS
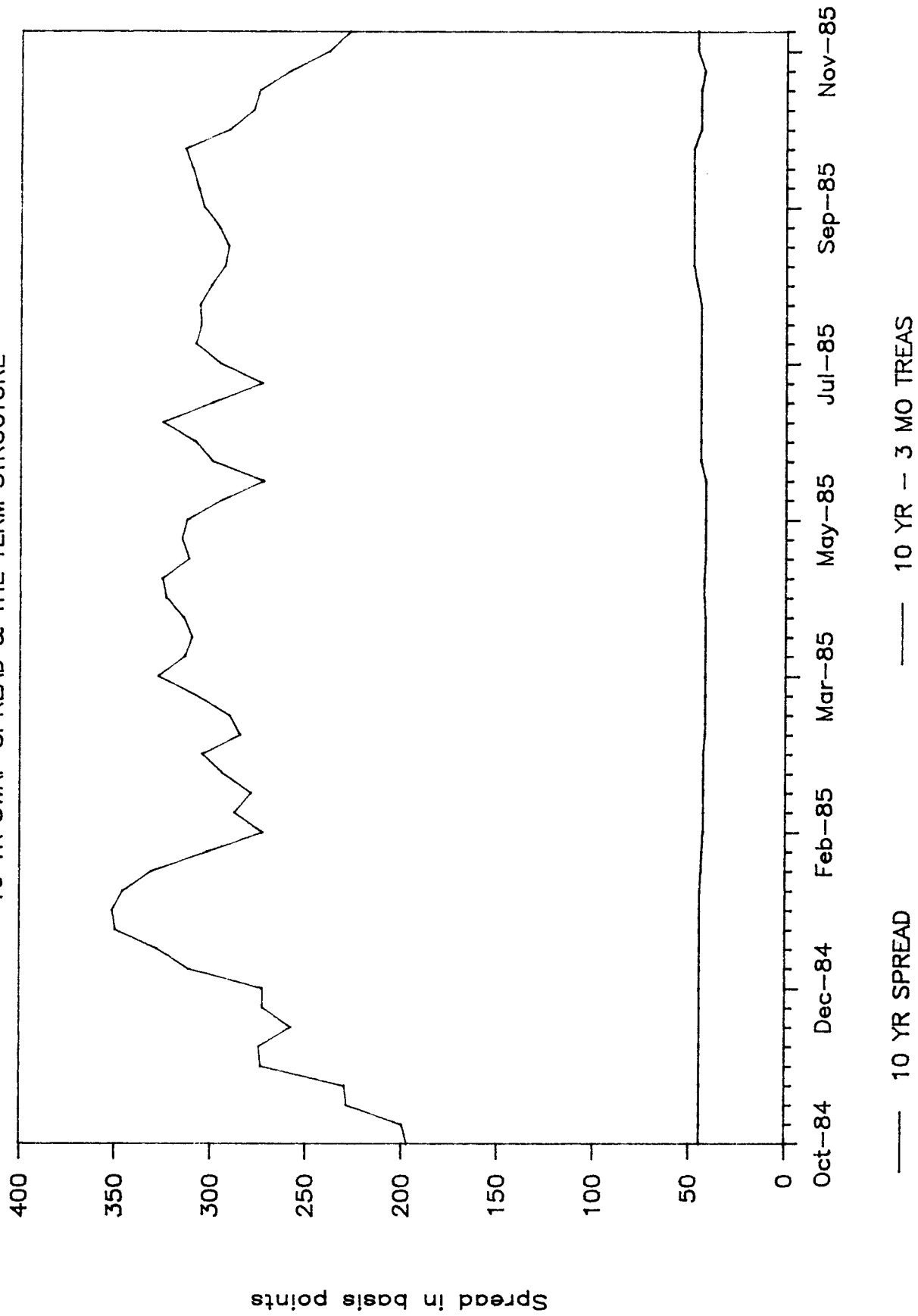
FIGURE 5

2 YR SWAP SPREAD & THE TERM STRUCTURE

Spread in basis points

——— 10 YR – 3 MO TREAS

——— 2*2 YR SPREAD