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A HEAVY UNSTABLE FOURTH FAMILY NEUTRINO AND THE τ -LIFETIME DISCREPANCY

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The existence of the fourth family heavy unstable neutrino ν_4 with $m_{\nu_4} > m_\tau$ and the dominant mixing angle $|U_{\tau 4}|^2 = 0.042 - 0.10$, is postulated from the present data on the lifetime and the leptonic branching ratios of the τ^- . The fast decay of a heavy ν_4 makes it undetectable in present experiments, while the future detection at Z^0 factories is shown to be very promising, provided $m_{\nu_4} \leq 46$ GeV. The contributions of this massive ν_4 and other fourth family fermions to the total decay widths of Z^0 and $W^\pm$ are considered. If the 90% CL of the present UA2's data on Γ_Z/Γ_W is valid, either the t-quark or ν_4 or both should be seen ($m_t \leq 76$ GeV or $m_{\nu_4} \leq 46$ GeV) in the near future. Neutrino counting via $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ is predicted to yield $N_\nu = 3$. The mass of the fourth family charged lepton is predicted to be $m_L \leq 43$ GeV if the prototype relation between the mixing angle and the mass eigenvalues is valid. The present UA1's bound on m_L , $m_L \geq 41$ GeV, is shown to be invalid, while the correct lower bound is the 25 GeV of TRISTAN, implying the likely possibility of detecting the fourth charged lepton $L^\pm$ in the present and forthcoming e^+e^- colliders. These predictions can be tested in the near future at the Z^0 factories at SLC and LEP.

1. Introduction

One of the most fundamental aspects of particle physics today is the observed replication of fermion (quark and lepton) families. Neither the mass spectra of these fundamental fermions nor their mixing angles (e.g. KM angles) are well understood, let alone the number of families itself. In particular, the physics of the fourth family of quarks and leptons, if the fourth family indeed exists, can be much different from those of the first three families, and may greatly surprise us in the forthcoming collider experiments.

In this article, we suggest that this is indeed the case at least for the lepton sector, and postulate the existence of the fourth family massive neutrino whose mass is greater than that of the τ lepton ($m_\tau = 1.78$ GeV), in order to account for the apparent discrepancy between the recently observed lifetime of the τ and the theoretical expectation for the leptonic decay rates and their measured branching ratios. This heavy (and unstable) neutrino is shown to mix among the usual three neutrinos dominantly with ν_τ , with the mixing angle of the order of the Cabibbo angle, in order to account for the observed lifetime of the τ . The observability of this neutrino in the present and the forthcoming e^+e^- colliders and the implications of this heavy neutrino for the physics of the near fu-

ture, such as the decay width of the Z^0 (and $W^\pm$), the neutrino counting experiments, and the masses of the t-quark, the heavy neutrino, and the next charged lepton L^- will be discussed.

2. Lifetime discrepancy of τ^- and the heavy neutrino ν_4

The measurements of the lifetime and the leptonic branching ratios of the τ^- have been consistently improved during the past ten years and their world average values are summarized in table 1. In particular, the measurement on the τ -lifetime has been improved to a great extent recently [6] and now allows one to compare it with the theoretically expected values, using the values of the leptonic branching ratios.

Using the most recent world average of the $BR(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)$ ($= 0.173 \pm 0.005$ [6]), e - μ - τ universality gives the theoretical value of the τ -lifetime,

$$\begin{aligned} \tau_\tau^e &\equiv \Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)^{-1} BR(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) \\ &= (G_F^2 m_\tau^5 / 192 \pi^3)^{-1} BR(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) \\ &= (1.593 \times 10^{-12} \text{ s}) BR(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) \\ &= (2.756 \pm 0.080) \times 10^{-13} \text{ s}, \end{aligned} \quad (1)$$

Table 1

World averages of lifetime and leptonic branching ratios of τ^- as a function of time (year).

Year	Ref.	BR($\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$) (%)	BR($\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$) (%)	Lifetime of τ^- (s)
1978	[1]	17.9 ± 2.8	17.5 ± 1.7	undetermined
1980	[2]	17.0 ± 1.1	17.9 ± 1.5	$< 2.3 \times 10^{-12}$
1982	[3]	16.2 ± 1.0	18.5 ± 1.2	$(4.6 \pm 1.9) \times 10^{-13}$
1984	[4]	16.5 ± 0.9	18.5 ± 1.1	$(3.4 \pm 0.5) \times 10^{-13}$
1986	[5]	17.4 ± 0.5	17.6 ± 0.6	$(3.3 \pm 0.4) \times 10^{-13}$
1987	[6]	17.3 ± 0.5	17.5 ± 0.5	$(3.03 \pm 0.08) \times 10^{-13}$

while the most recent world average of BR($\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$) ($= 0.175 \pm 0.005$ [6]) gives the theoretical value

$$\begin{aligned} \tau_\tau^{\text{th}} &\equiv \Gamma(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)^{-1} \text{BR}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau) \\ &= [G_F^2 m_\tau^5 F(m_\mu^2/m_\tau^2)/192\pi^3]^{-1} \\ &\quad \times \text{BR}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau) \\ &= (1.6375 \times 10^{-12} \text{ s}) \text{BR}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau) \\ &= (2.866 \pm 0.082) \times 10^{-13} \text{ s}, \end{aligned} \quad (2)$$

where

$$\begin{aligned} F(x) &\equiv 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x, \\ F(m_\mu^2/m_\tau^2) &= 0.9728, \end{aligned} \quad (3)$$

is the phase space factor for nonvanishing m_μ/m_τ . Thus the theoretically expected τ -lifetime from e - μ - τ universality, consistent with both of the reported ^{#1} world averages of BR($\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$) and BR($\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$), is

$$\tau_\tau^{\text{th}} = (2.810 \pm 0.057) \times 10^{-13} \text{ s}, \quad (4)$$

whereas the present experimental value [6] is

$$\tau_\tau^{\text{exp}} = (3.03 \pm 0.08) \times 10^{-13} \text{ s}. \quad (5)$$

Therefore the measured τ -lifetime is clearly longer than the theoretically expected value with e - μ - τ universality.

^{#1} In ref. [6], the author advocates the use of only those data on the leptonic branching ratios that are obtained with the constraint of μ - e universality, instead of the use of the world averages. We find, however, his argument rather ad hoc and unconvincing, and use the world averages of the data whose precisions have been improved consistently over past years as one can see in table 1.

To remedy this discrepancy without spoiling the e - μ - τ universality of the gauge coupling constant, we now postulate the existence of the fourth family neutrino ν_4 , which is heavier than τ^- and mixes subdominantly with ν_τ . In other words, for the most general relation between the weak-eigenstates (ν_ℓ , $\ell = e, \mu, \tau, L$) and the mass-eigenstates (ν_j , $j = 1, 2, 3, 4$) of neutrinos (for four families)

$$\begin{aligned} \nu_\ell &\equiv \sum_{j=1}^4 U_{\ell j} \nu_j \quad (\ell = e, \mu, \tau, L), \\ L^{\text{c.c.}} &= \sum_{\ell=e,\mu,\tau,L} \bar{\ell} \gamma^\mu (1 - \gamma_5) \nu_\ell W_\mu^-, \end{aligned} \quad (6)$$

where U is a 4×4 unitary mixing matrix (i.e. the analogue of the KM matrix of the quark sector), we postulate that

$$\begin{aligned} m_{\nu_4} &> m_\tau (= 1.78 \text{ GeV}), \\ |U_{\tau 4}|^2 &\gg |U_{\mu 4}|^2, |U_{e 4}|^2. \end{aligned} \quad (7)$$

In such a case, τ^- can not decay into the heaviest component ν_4 in ν_τ since $m_\tau < m_{\nu_4}$, and the decay rate of τ^- will be reduced by a factor of $(1 - |U_{\tau 4}|^2)$ ^{#2}. This implies

$$\begin{aligned} 1 - |U_{\tau 4}|^2 &= \tau_\tau^{\text{th}} / \tau_\tau^{\text{exp}} = 0.9274 \pm 0.0310, \\ |U_{\tau 4}|^2 &= 0.0726 \pm 0.0310, \\ |U_{\tau 4}| &= 0.2694^{+0.0525}_{-0.0654}. \end{aligned} \quad (8)$$

Thus we see that $|U_{\tau 4}| = 0.20$ – 0.32 , which is of the order of the typical Cabibbo angle ($\sin \theta_C = 0.22$).

^{#2} The present experimental limits on the mass of the lighter components are: $m_{\nu_1} < 18 \text{ eV}$ [7], $m_{\nu_2} < 250 \text{ keV}$ [8], and $m_{\nu_3} < 50 \text{ MeV}$ [9]. Since these light neutrinos are much lighter than τ^- , the decay rates of τ^- into these light neutrinos are essentially the same as into the massless neutrinos since the phase space factors are essentially the same.

The heavy neutrino ν_4 is unstable and decays through the modes $\nu_4 \rightarrow \tau^- W^+$, $W^+ \rightarrow e^+ \nu_e$, $\mu^+ \nu_\mu$, $u\bar{d}$, ..., where W^+ is virtual (or real if $m_{\nu_4} > M_W + m_\tau$). This heavy and unstable neutrino ν_4 will lead to interesting physics, which will be the subject of the following sections.

3. Observability of ν_4 in the present and future e^+e^- colliders

The decay rate of ν_4 is proportional to $|U_{\tau 4}|^2$, and is given, to a sufficient approximation for $m_{\nu_4} < 50$ GeV by

$$\begin{aligned} \Gamma_{\nu_4} &= G_F^2 m_{\nu_4}^5 (192\pi^3)^{-1} P(y) |U_{\tau 4}|^2 \\ &\times \text{BR}(\nu_4 \rightarrow \tau^- e^+ \nu_e)^{-1} \\ &= \tau_\mu^{-1} (m_{\nu_4}/m_\mu)^5 P(y) |U_{\tau 4}|^2 \\ &\times \text{BR}(\nu_4 \rightarrow \tau^- e^+ \nu_e)^{-1}, \end{aligned} \quad (9)$$

where

$$\begin{aligned} P(y) &\equiv 1 - 8y + 8y^3 - y^4 - 12y^2 \ln y, \\ y &\equiv m_\tau^2/m_{\nu_4}^2 \end{aligned} \quad (10)$$

is a phase space factor, as in the τ^- decay rate [10]. The lifetime of ν_4 is then

$$\begin{aligned} \tau_4 &\equiv 1/\Gamma_{\nu_4} \\ &= (3.2 \times 10^{-16} \text{ s}) (10 \text{ GeV}/m_{\nu_4})^5 P(m_\tau^2/m_{\nu_4}^2)^{-1} \\ &\times (0.1/|U_{\tau 4}|^2) [\text{BR}(\nu_4 \rightarrow \tau^- e^+ \nu_e)/\frac{1}{9}], \end{aligned} \quad (11)$$

where $\text{BR}(\nu_4 \rightarrow \tau^- e^+ \nu_e)$ is a function of m_{ν_4} , and is to be calculated in a manner shown in ref. [10]. For $5 \text{ GeV} < m_{\nu_4} \leq m_t$, an excellent approximation to $\text{BR}(\nu_4 \rightarrow \tau^- e^+ \nu_e)$ is

$$\text{BR}(\nu_4 \rightarrow \tau^- e^+ \nu_e) \simeq \frac{1}{9}, \quad (11a)$$

which comes from the number of available decay channels ($e^+ \nu_e$, $\mu^+ \nu_\mu$, $\tau^+ \nu_\tau$, $3 \times u\bar{d}$, $3 \times c\bar{s}$) including color factors for the quarks. Following ref. [10], we have calculated $\text{BR}(\nu_4 \rightarrow \tau^- e^+ \nu_e)$ and therefore the lifetime τ_4 , and have presented the result ^{#3} in fig. 1, as a function of m_{ν_4} .

^{#3} The lifetime in fig. 1 and $\text{BR}(\nu_4 \rightarrow \tau^- e^+ \nu_e)$ of eq. (11a) is valid for a Dirac neutrino ν_4 . If ν_4 is a Majorana neutrino, the lifetime via eq. (11) and eq. (11a) should be further reduced by a factor of two, since the decay channels to antiparticles are also allowed in such a case.

Fig. 1. Lifetime of $\nu_4 \times (|U_{\tau 4}|^2/0.1)$ as a function of m_{ν_4} .

As one can see in fig. 1, the lifetime of a heavy and unstable ν_4 is rather short (e.g. $\tau_4 \leq 7.1 \times 10^{-14}$ for $|U_{\tau 4}|^2 \geq 0.042$ and $m_{\nu_4} \geq 5$ GeV). This implies that ν_4 will decay inside of the detector with a very short decay length. For e^+e^- collisions at the center-of-mass energy of $\sqrt{s}$, a pair produced $\nu_4 \bar{\nu}_4$ via $e^+e^- \rightarrow Z^0(\text{virtual}) \rightarrow \nu_4 \bar{\nu}_4$ will have a decay length

$$\begin{aligned} l &= c\beta\gamma\tau_4 \\ &= (\sqrt{s - 4m_{\nu_4}^2}/2m_{\nu_4}) (3 \times 10^{10} \text{ cm}) \tau_4/\text{s}. \end{aligned} \quad (12)$$

Indeed, at $\sqrt{s} = 29$ GeV, a search has been made for ν_4 with a decay length greater than 0.2 cm by the MARK II group [11] with a null result. This is consistent with the calculated decay lengths for $\sqrt{s} = 29$ GeV from eq. (12),

$$\begin{aligned} l &\leq 5.8 \times 10^{-3} \text{ cm} \quad \text{for } |U_{\tau 4}|^2 = 0.042, \quad m_{\nu_4} \geq 5 \text{ GeV}, \\ l &\leq 2.4 \times 10^{-3} \text{ cm} \quad \text{for } |U_{\tau 4}|^2 = 0.10, \quad m_{\nu_4} \geq 5 \text{ GeV}. \end{aligned} \quad (13)$$

The null result of MARK II with $0.2 \text{ cm} \leq l \leq 10 \text{ cm}$ at $\sqrt{s} = 29$ GeV excludes only the very narrow mass range of ν_4 ,

$$\begin{aligned} 2.2 \text{ GeV} &< m_{\nu_4} < 3.0 \text{ GeV}, \quad \text{for } |U_{\tau 4}|^2 = 0.10, \\ 2.5 \text{ GeV} &< m_{\nu_4} < 3.5 \text{ GeV}, \quad \text{for } |U_{\tau 4}|^2 = 0.042, \end{aligned} \quad (14)$$

and the future search for the secondary vertex in ν_4 decay will not yield any positive results at all for our ν_4 with the mixing angle $|U_{\tau 4}|^2 \geq 0.042$ (see eq. (8)), since the expected decay length is much shorter than the present resolution of the vertex detectors.

On the other hand, the HRS group at PEP searched

[12] for the massive, unstable ν_4 through the event configuration $e^+e^- \rightarrow (\nu_4 \rightarrow e^\pm X^\mp(\nu))(\bar{\nu}_4 \rightarrow \text{anything})$, where $X^\mp$ is a $\mu^\mp$, $\pi^\mp$, or $K^\mp$. The reported results are: (i) At 90% CL, the present experimental limits are so poor that there is essentially no excluded region of m_{ν_4} above m_τ . (ii) At 80% CL, the range of m_{ν_4} , $2.5 \text{ GeV} \leq m_{\nu_4} \leq 5.5 \text{ GeV}$, is excluded due to the absence of an $e^\pm X^\mp$ pair in that mass range. (iii) For the effective mass of the $e^\pm X^\mp$ pair greater than 6.4 GeV, there is one candidate event. Although this search strategy with the above event configuration is in fact much more suitable for our short-lived ν_4 (with a large mixing angle $0.042 \leq |U_{\tau 4}|^2 \leq 0.10$) than the one searching for the secondary vertices, the present PEP integrated luminosity of 300 pb^{-1} gives the expected number of events to be 0.7, which is too small to yield a definite, positive result. This can be seen as follows: The cross section at $\sqrt{s}=29 \text{ GeV}$ is $\sigma(e^+e^- \rightarrow \nu_4 \bar{\nu}_4) = 0.36\beta(3+\beta^2)/4 \text{ pb}^{*4}$, while the branching ratio to the discussed event configuration is $\text{BR}(\nu_4 \rightarrow \tau W \rightarrow e^\pm X^\mp \dots) \simeq 6\%$ at $m_{\nu_4} \geq 5 \text{ GeV}^{*5}$, and the detection efficiency for $e^\pm X^\mp$ events is about 10%^{*6}. However, the situation will improve greatly at the Z^0 resonance at SLC and LEP. With $10^6 Z^0$ s per year in these forthcoming e^+e^- colliders, the expected number of events is $3.8 \times 10^2/\text{yr}$ ($=10^6 \times \text{BR}(Z^0 \rightarrow \nu_4 \bar{\nu}_4) \times \text{BR}(\nu_4 \rightarrow \tau W \rightarrow e^\pm X^\mp \dots) \times (\text{detection efficiency for } e^\pm X^\mp \text{ events}) \simeq 10^6 \times \frac{1}{16} \times 0.06 \times 0.1 \simeq 3.8 \times 10^2$), if $m_{\nu_4} \leq M_Z/2 = 46 \text{ GeV}$. Thus, at the forthcoming Z^0 factories at SLC and LEP, our postulated massive unstable ν_4 with $0.042 \leq |U_{\tau 4}|^2 \leq 0.10$ will be definitely seen if $m_{\nu_4} \leq 46 \text{ GeV}$!

4. Decay widths of Z^0 and $W^\pm$, fourth family of quarks and leptons, and neutrino counting

The decay width of the Z^0 will be measured with a good resolution in the forthcoming Z^0 factories at SLC and LEP. This will enable us to determine the number of neutrino species and the existence of the fourth family fermions lighter than $M_Z/2 = 46 \text{ GeV}$. In this section, we consider the contributions of the heavy,

^{*4} For $e^+e^- \rightarrow Z^0 \rightarrow \nu_4 \bar{\nu}_4$, the cross section at the center-of-mass energy $\sqrt{s}$ is given by $\sigma(e^+e^- \rightarrow \nu_4 \bar{\nu}_4) = (1/24\pi) \{G_F^2 s / [(1 - s/M_Z^2)^2 + (\Gamma_Z/M_Z)^2]\} (1 - 4 \sin^2\theta_W + 8 \sin^4\theta_W) (3 + \beta^2) \beta/4$, where $\beta \equiv \sqrt{1 - (4m_{\nu_4}^2/s)}$ is the velocity of ν_4 .

^{*5} See fig. 2 of ref. [12].

^{*6} See fig. 4 of ref. [12].

unstable ν_4 to the total decay widths (Γ_Z and Γ_W) of the Z^0 and $W^\pm$, as well as of other fourth family fermions, and examine the implications of the present UA2 data on Γ_Z/Γ_W . We begin by considering the contributions to Γ_Z . The contributions from the neutrinos are

$$\Gamma(Z^0 \rightarrow \nu_i \bar{\nu}_i) = G_F M_Z^3 / 12\pi\sqrt{2} \equiv \Gamma_Z^0 = 0.170 \text{ GeV}$$

$$\text{for } M_Z = 92.0 \text{ GeV, } i = 1, 2, 3,$$

and [13]

$$\Gamma(Z^0 \rightarrow \nu_4 \bar{\nu}_4) = \Gamma_Z^0$$

$$\times N_D(r_4) \equiv (1 - r_4) \sqrt{1 - 4r_4} \quad \text{for Dirac } \nu_4$$

$$\times N_M(r_4) \equiv (1 - 4r_4)^{3/2} \quad \text{for Majorana } \nu_4, \quad (15)$$

where $r_4 \equiv (m_{\nu_4}/M_Z)^2$, while the contributions from the charged leptons are given by

$$\Gamma(Z^0 \rightarrow e^+e^-) = \Gamma(Z^0 \rightarrow \mu^+\mu^-) = \Gamma(Z^0 \rightarrow \tau^+\tau^-)$$

$$= \Gamma_Z^0 (1 - 4 \sin^2\theta_W + 8 \sin^4\theta_W),$$

$$\Gamma(Z^0 \rightarrow L^+L^-) = \Gamma_Z^0 G(r_L)$$

$$\equiv \Gamma_Z^0 \sqrt{1 - 4r_L} [(1 - 4 \sin^2\theta_W)^2 (1 + 2r_L)$$

$$+ (1 - 4r_L)],$$

$$r_L \equiv (m_L/M_Z)^2. \quad (16)$$

The contributions from the quarks are

$$\Gamma(Z^0 \rightarrow u\bar{u}) = \Gamma(Z^0 \rightarrow c\bar{c})$$

$$= \Gamma_Z^0 3 [1 + \alpha_s(M_Z)/\pi] (1 - \frac{8}{3} \sin^2\theta_W + \frac{32}{9} \sin^4\theta_W),$$

$$\Gamma(Z^0 \rightarrow d\bar{d}) = \Gamma(Z^0 \rightarrow s\bar{s}) = \Gamma(Z^0 \rightarrow b\bar{b})$$

$$= \Gamma_Z^0 3 [1 + \alpha_s(M_Z)/\pi] (1 - \frac{4}{3} \sin^2\theta_W + \frac{8}{9} \sin^4\theta_W),$$

$$\Gamma(Z^0 \rightarrow t\bar{t}) = \Gamma_Z^0 3 [1 + \alpha_s(M_Z)/\pi] H_{2/3}(r_t),$$

$$\Gamma(Z^0 \rightarrow b'\bar{b}') = \Gamma_Z^0 3 [1 + \alpha_s(M_Z)/\pi] H_{1/3}(r_{b'}),$$

$$\Gamma(Z^0 \rightarrow b'\bar{b}') = \Gamma_Z^0 3 [1 + \alpha_s(M_Z)/\pi] H_{1/3}(r_{b'}),$$

$$\Gamma(Z^0 \rightarrow t'\bar{t}') = \Gamma_Z^0 3 [1 + \alpha_s(M_Z)/\pi] H_{2/3}(r_{t'}), \quad (17)$$

where

$$H_{2/3}(r) \equiv \frac{1}{2} \sqrt{1 - 4r} [(1 - \frac{8}{3} \sin^2\theta_W)^2 (1 + 2r)$$

$$+ (1 - 4r)],$$

$$H_{1/3}(r) \equiv \frac{1}{2} \sqrt{1 - 4r} [(1 - \frac{4}{3} \sin^2\theta_W)^2 (1 + 2r)$$

$$+ (1 - 4r)],$$

$$r_f \equiv (m_f/M_Z)^2 \quad (f = t, t', b'). \quad (18)$$

Using $\alpha_s(M_Z)=0.15$ and $\sin^2\theta_W=0.230$ [14], the sum of all contributions of eqs. (15)–(17) gives the total width ^{#7}

$$\Gamma_Z = \Gamma_Z^0 \{ 15.09 + [N_D(r_4) \text{ or } N_M(r_4)] + G(r_L) + 3.143[H_{1/3}(r_{b'}) + H_{2/3}(r_t) + H_{2/3}(r_{t'})] \}. \quad (19)$$

To compare with the present UA2 data [15,16]

$$\begin{aligned} \Gamma_Z/\Gamma_W &\leq 1.16 \quad (90\% \text{ CL}), \\ &\leq 1.27 \quad (95\% \text{ CL}), \end{aligned} \quad (20)$$

we also consider the contributions to Γ_W . The contributions are [14]

$$\begin{aligned} \Gamma(W^- \rightarrow e^- \bar{\nu}_e) &= G_F M_W^3 / 6\pi \sqrt{2} \\ &\equiv \Gamma_W^0 = 0.230 \text{ GeV} \quad \text{for } M_W = 80.7 \text{ GeV} \end{aligned}$$

and

$$\begin{aligned} \Gamma(W^- \rightarrow \mu^- \bar{\nu}_\mu) &= \Gamma(W^- \rightarrow \tau^- \bar{\nu}_\tau) = \Gamma_W^0, \\ \Gamma(W^- \rightarrow L^- \bar{\nu}_4) &= \Gamma_W^0 F(\tilde{r}_L, \tilde{r}_4), \\ \Gamma(W^- \rightarrow \bar{u}d) &= \Gamma(W^- \rightarrow \bar{c}s) = \Gamma_W^0 3[1 + \alpha_s(M_W)/\pi], \\ \Gamma(W^- \rightarrow \bar{t}b) &= \Gamma_W^0 3[1 + \alpha_s(M_W)/\pi] F(\tilde{r}_t, 0), \\ \Gamma(W^- \rightarrow \bar{t}'b') &= \Gamma_W^0 3[1 + \alpha_s(M_W)/\pi] F(\tilde{r}_{t'}, \tilde{r}_{b'}), \end{aligned} \quad (21)$$

where

$$\begin{aligned} \tilde{r}_f &\equiv (m_f/M_W)^2 \quad (f=L^-, \nu_4, t, t', b'), \\ F(x, y) &\equiv F(y, x) \\ &\equiv [(1-x-y)^2 - 4xy]^{1/2} [1 - \frac{1}{2}(x+y) - \frac{1}{2}(x-y)^2] \\ &\quad \text{for } \sqrt{x} + \sqrt{y} < 1, \\ F(x, y) &= 0 \quad \text{for } \sqrt{x} + \sqrt{y} \geq 1. \end{aligned} \quad (22)$$

Summing all contributions and using $\alpha_s(M_W)=0.15$, one finds

$$\begin{aligned} \Gamma_W &= \Gamma_W^0 \{ 9.29 + F(\tilde{r}_L, \tilde{r}_4) \\ &\quad + 3.143[F(\tilde{r}_t, 0) + F(\tilde{r}_{t'}, \tilde{r}_{b'})] \}. \end{aligned} \quad (23)$$

Defining ΔW and ΔZ by

$$\begin{aligned} \Gamma_W &\equiv \Gamma_W^0 (9.29 + \Delta W), \\ \Gamma_Z &\equiv \Gamma_Z^0 (15.09 + \Delta Z), \end{aligned} \quad (24)$$

where

$$\begin{aligned} \Delta W &\equiv F(\tilde{r}_L, \tilde{r}_4) + 3.143[F(\tilde{r}_t, 0) + F(\tilde{r}_{t'}, \tilde{r}_{b'})], \\ \Delta Z &\equiv G(r_L) + [N_D(r_4) \text{ or } N_M(r_4)] \\ &\quad + 3.143[H_{1/3}(r_{b'}) + H_{2/3}(r_t) + H_{2/3}(r_{t'})]. \end{aligned} \quad (25)$$

Eq. (20) and

$$\begin{aligned} \frac{\Gamma_Z}{\Gamma_W} &= \frac{1}{2} \cos^{-3}\theta_W \frac{(15.09 + \Delta Z)}{(9.29 + \Delta W)} \\ &= 0.740 \times \frac{(15.09 + \Delta Z)}{(9.29 + \Delta W)} \end{aligned} \quad (26)$$

implies

$$\begin{aligned} \Delta Z &\leq 1.57\Delta W - 0.53 \quad (90\% \text{ CL}), \\ \Delta Z &\leq 1.72\Delta W + 0.85 \quad (95\% \text{ CL}). \end{aligned} \quad (27)$$

In figs. 2a and 2b, we have plotted the numerical values of the contributions to ΔZ and ΔW from each fermion as function of masses of L^- , ν_4 , b' , and t . For

Fig. 2. (a) Contribution of each fourth family fermion to ΔZ as a function of its mass. (b) Contributions of $(L^- \bar{\nu}_4)$ and $(t \bar{b})$ to ΔW as a function of the masses m_{ν_4} and m_t .

^{#7} For $r \equiv m^2/M_Z^2 \geq \frac{1}{4}$, $N_D(r) = N_M(r) \equiv G(r) = H_{2/3}(r) = H_{1/3}(r) = 0$.

the most updated experimental bounds on the masses, $m_t \geq 56$ GeV [17], $m_b \geq 41$ GeV [17], $m_L \geq 25$ GeV^{#8}, and $m_{\nu} > m_\tau$, figs. 2a and 2b give $F(\tilde{r}_L, \tilde{r}_4) \leq 0.86$, $3.143F(\tilde{r}_L, 0) \leq 1.02$, $F(\tilde{r}_L, \tilde{r}_{b'}) = 0$, $G(r_L) \leq 0.30$, $3.143H_{1/3}(r_{b'}) \leq 0.63$, $H_{2/3}(r_L) = 0 = H_{2/3}(r_{L'})$, $N_D(r_4) \leq 1.0$, and $N_M(r_4) \leq 1.0$. Thus we have

$$0.0 \leq \Delta W \leq 1.88, \quad 0.0 \leq \Delta Z \leq 1.93, \quad (28)$$

or, equivalently,

$$\begin{aligned} 2.14 \text{ GeV} &\leq F_W \leq 2.57 \text{ GeV}, \\ 2.57 \text{ GeV} &\leq F_Z \leq 2.90 \text{ GeV}. \end{aligned} \quad (29)$$

The allowed region of $(\Delta W, \Delta Z)$, consistent with (28) and (27) of UA2, are shown in fig. 3. In fig. 3 we have also considered various possibilities for the fourth family fermion masses and m_t , and the largest possible values of ΔZ and ΔW are shown for each case. Figure 3 shows that those cases with both $m_t \geq 76$ GeV and $m_{\nu_4} \geq 46$ GeV are excluded by 90% CL of UA2, while 95% CL allows them. Thus, if the ΔW and ΔZ fall within the 90% CL allowed region, then

$$m_t \leq 76 \text{ GeV} \quad \text{or} \quad m_{\nu_4} \leq 46 \text{ GeV}. \quad (30)$$

^{#8} UA1's bound $m_t \geq 41$ GeV [18] is not valid [19] for a massive unstable ν_4 . Hence, 25 GeV of the present TRISTAN collider energy is the correct bound.

Fig. 3. Allowed 90% CL and 95% CL regions of $(\Delta W, \Delta Z)$ by UA2's present data on F_Z/F_W are to the right of the correspondingly labeled lines. The boxes containing possible $(\Delta W, \Delta Z)$ for all combinations of fourth family particles are shown and labeled at the largest possible $(\Delta W, \Delta Z)$ by those particles whose masses are below $M_Z/2$ for b' , L , and ν_4 (labeled by 4), and for the t -quark if $m_t < M_W - m_b$.

Note also that, from this experiment alone, the point $\Delta W=0$, $\Delta Z=0$ with no fourth generation particle lighter than $M_Z/2$ (or $m_t < M_W - m_b$) falls outside the 90% CL but within the 95% CL bound.

5. Neutrino counting, helium abundance, and bound on the fourth family charged lepton mass

Since ν_4 is heavy ($m_{\nu_4} > m_\tau$) and unstable, several remarks are in order.

(i) The rate of observing only a single photon in the neutrino counting experiment $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ [20] will receive no contribution from ν_4 since it will decay into charged particles, and the result will yield $N_\nu=3$.

(ii) The primordial helium abundance, which is a sensitive function of the number of light neutrino species of mass less than a MeV, will receive no contribution from ν_4 . In fact, the standard big bang cosmology and nucleosynthesis yields the best fit to the observed He abundance at $N_\nu=3$ [21]. Thus our heavy ν_4 is "better" than the conventional sequential massless (or very light) ν_4 in this respect.

(iii) The present UA1 bound on the mass of the fourth charged lepton, $m_L \geq 41$ GeV [18] is not valid [19] since our ν_4 will decay inside the detector, carrying no missing P_T . Therefore the best bound on m_L if $m_{\nu_4} > m_\tau$ is $m_L \geq 25$ GeV of TRISTAN, and there exists a large possible range of m_L for observing L^+L^- in the present and the forthcoming e^+e^- colliders.

(iv) The unitary matrix U of eq. (6) is a product of two unitary matrices, $U = U_L^\dagger U_\nu$, where U_L (U_ν) is the diagonalizing unitary matrix of the charged-lepton mass matrix (neutrino mass matrix). If the prototype relation between the mixing angle and the mass eigenvalues $|U_{\tau 4}| = |\sqrt{m_\tau/m_L} - e^{i\phi}\sqrt{m_{\nu_3}/m_{\nu_4}}|$ ^{#9} (where ϕ is a phase) is valid, one expects

$$|U_{\tau 4}| \simeq \sqrt{m_\tau/m_L}, \quad (31)$$

since the best fit to the He abundance being at $N_\nu=3$ implies $m_{\nu_3} \leq 1$ MeV, while $m_{\nu_4} > m_\tau = 1.78$ GeV. Thus, $0.042 \leq |U_{\tau 4}|^2 \leq 0.104$ of eq. (8) implies

$$17.1 \text{ GeV} \leq m_L \leq 42.4 \text{ GeV}. \quad (32)$$

^{#9} This is the analogue of the four-family KM angle of ref. [22] or the three-family lepton mixing angle of ref. [23].

Hence, if the τ -lifetime discrepancy is due to ν_4 , then the fourth family charged lepton $L^\pm$ is very likely to be discovered at any time in the present e^+e^- colliders or in the Z^0 factories at SLC and LEP.

6. Summary and conclusion

In this article, we have postulated the existence of the fourth family heavy unstable neutrino ν_4 , with $m_{\nu_4} > m_\tau$ and the dominant mixing angle $|U_{\tau 4}|^2 = 0.042\text{--}0.10$, from the present data on the lifetime and the leptonic branching ratios of the τ^- . Our calculations show that present experiments would not have detected this ν_4 , while the future detection at Z^0 factories is shown to be very promising, provided $m_{\nu_4} \leq 46$ GeV. The contributions of this massive ν_4 and other fourth family fermions to the total decay widths of Z^0 and $W^\pm$ were considered and summarized in fig. 3. It has been shown that, if the 90% CL of the present UA2's data on Γ_Z/Γ_W is valid, either the t-quark or ν_4 or both should be seen ($m_t \leq 76$ GeV or $m_{\nu_4} \leq 46$ GeV) in the near future. Neutrino counting via $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ is predicted to still yield $N_\nu = 3$, while the He abundance of the big bang nucleosynthesis will still find the number of light (less than a MeV) neutrino species to be three. The mass of the fourth family charged lepton is predicted to be $m_L \leq 43$ GeV if the prototype relation between the mixing angle and the mass eigenvalues is to yield $|U_{\tau 4}|^2 = 0.042\text{--}0.10$. The present UA1 bound on m_L , namely $m_L \geq 41$ GeV, is shown to be invalid, while the correct lower bound is 25 GeV from TRISTAN, implying the very possibility of detecting the fourth charged lepton $L^\pm$ in the present and forthcoming e^+e^- colliders. These predictions can be tested in the near future at the Z^0 factories at SLC and LEP.

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