Lawrence Berkeley National Laboratory

Title

A STUDY OP THE MECHANICAL IMPEDANCE TO FLOW IN THE FEMORAL PERIPHERAL
VASCULAR BED OF THE DOG

Permalink

https://escholarship.org/uc/item/61c42680

Authors

Yanof, Howard M.
Stacy, Ralph W.

Publication Date

1960-07-07

UNIVERSITY OF CALIFORNIA

Ernest O. Lawrence

Radiation

Laboratory

A STUDY OF THE MECHANICAL IMPEDANCE TO FLOW IN FEMORAL PERIPHERAL VASCULAR BED OF THE DOG

DISCLAIMER

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

A STUDY OF THE MECHANICAL IMPEDANCE TO FLOW
IN THE FEMORAL PERIPHERAL VASCULAR BED OF THE DOG

Howard M. Yanof and Ralph W. Stacy
July 7, 1960

A STUDY OF THE MECHANICAL IMPEDANCE TO FLOW
IN THE FEMORAL PERIPHERAL VASCULAR BED OF THE DOG

Howard M. Yanof

Lawrence Radiation Laboratory
University of California
Berkeley, California

and

Ralph W. Stacy

Division of Biophysics, Department of Physiology
Ohio State University, Columbus, Ohio

July 7, 1960

ABSTRACT

The mechanical impedance to blood flow in the hind limb of the dog was studied, and a dynamic relationship among the various factors affecting blood flow was derived.

The blood flow through the femoral artery was routed through external polyethylene tubing containing an electromagnetic flowmeter and a Statham pressure transducer. The pulsatile blood pressure and the blood flow were displayed simultaneously on a dual-beam oscilloscope and recorded on 35-mm film.

The pressure and the flow curves were then analyzed for their respective Fourier component frequencies. The Fourier analysis was performed with a frequency converter and a wave analyzer. The mechanical impedance is calculated by dividing the amplitude of each pressure harmonic by the amplitude of the corresponding flow harmonic.

The average mechanical impedance-vs-frequency curve has two resonant frequencies: one between 3.6 and 4.6 cps and the other between 5.6 and 7.0 cps. The impedance has a minimum value of about 1.2 peripheral resistance units at each resonant frequency. The flow-vs-frequency and the pressure-vs-frequency curves are also described.

A theoretical electrical analog is hypothesized. This circuit consists of two arms in parallel, each arm containing an inductance and a resistance in series with a parallel capacitance and resistance. The impedance equations for the analog circuit are derived, and the impedance-frequency curve of the circuit is qualitatively constructed. This constructed curve agrees with experimental observation. The dynamic relationships existing among the elastance, inertial, and frictional factors affecting blood flow are discussed.

A STUDY OF THE MECHANICAL IMPEDANCE TO FLOW
IN THE FEMORAL PERIPHERAL VASCULAR BED OF THE DOG[*]

Howard M. Yanof[‡]

Lawrence Radiation Laboratory
University of California
Berkeley, California

and

Ralph W. Stacy[§]

Division of Biophysics, Department of Physiology
Ohio State University, Columbus, Ohio

July 7, 1960

INTRODUCTION

The study of the circulatory system is one of the oldest problems in physiology. Since Harvey's original discovery of blood flow, physiologists have progressed a long way in their detailed studies of the heart, arteries, veins, and capillaries. The blood pressure developed in various parts of the circulatory system has also been carefully studied. Inspite of all these studies, however, the important relationships that integrate the properties of the elements of the circulatory system to produce the various physiological effects that are observable in the animal are for the most part unknown.

The circulatory system may be divided into three separate phases, each phase represented by a "black box." The first black box represents the heart; the second, the transmission lines from the heart to the peripheral vascular bed; and the third, the peripheral vascular bed itself.

To evaluate the contents of the third black box, it is necessary to understand the factors affecting the flow of blood through it. These factors include the elastance of the walls, the mass of the blood, and the viscous friction encountered by the flowing blood. When the pulsatile flow curves and the pulsatile pressure curves are analyzed for their respective Fourier components, the mechanical impedance to flow at various frequencies is the pressure per unit of flow for each Fourier component. The mechanical impedance calculated in this fashion and the mechanical impedance which may be calculated from the mean pressure and mean flow, together, are a measure of the peripheral resistance.

It is the purpose of this thesis to study the mechanical impedance to the flow of blood in the hind limb of the dog. Using analytical methods based on concepts founded in electrical engineering, one can derive certain relationships among the components of the circulatory system, e.g., inertia, elastance, and friction. This study of the impedance--versus--frequency curve has lead to the adoption of an electrical analog for the circulatory system.

EXPERIMENTAL PROCEDURES AND INSTRUMENTATION

Male and female mongrel dogs weighing 15 to 20 kg were used. The animals were anesthesized with 30 mg of sodium pentabarbital per kilogram of body weight. The femoral artery of the left leg was exposed in the area of Scarpa's triangle and cannulated proximally and distally with external, tight-fitting polyethylene tubing. The animal was then given 250 units of sodium Heparin per kilogram of body weight to prevent the formation of blood clots in the external tubing and flowmeter. The preparation of the animal took no longer than 20 minutes after the first incision was made, and the peripheral vascular bed was anoxic no longer than a total of 5 minutes of this time in most cases.

The blood from the femoral artery was then circuited through an electro-magnetic flowmeter and a Statham pressure transducer (P23D) before being returned to the distal segment of the femoral artery. After re-entering the distal end of the femoral artery, the blood, which was theoretically unchanged, was allowed to flow through the femoral peripheral vascular bed.

When the preparation of the animal was completed, blood was allowed to flow through the external circuit and back into the dog's leg so as to prevent anoxia in the peripheral vascular bed. Following the completeion of the animal preparation, the pressure and flow circuits were checked for proper functioning. Figure 1 is a block diagram of the instruments and circuits used.

The electromagnetic flowmeter used here was designed by Randall. According to Randall,[1,2] the principle of the flowmeter is that the blood, which is a source of moving conductors, passes through a magnetic field and causes potential difference to develop in a loop of wire. The magnitude of the poten-tial difference is a measure of the flow of blood past the wire loop since the artery is constrained to a constant diameter. The signal developed in this fashion is then amplified and may be displayed on an oscilloscope. In addition to the amplifier circuit, there is an electrical balancing network which reduces the dc transformer component introduced by the magnetic field in the under-modulated flow signal.

The pressure circuit consists of a commercially available strain-gauge transducer (Model P23D) manufactured by the Statham Company. This model was chosen for this work because it is capable of a frequency response of at least 350 cycles per second (cps). The signal from the Statham transducer was modified by an electrical network so as to eliminate some of the dc compo-nent of the pressure. This procedure increases the relative magnitude of the envelope and thus facilitates measurements of the pulsatile pressure curve. The pulsatile pressure signal was then amplified by a Cox amplifier (Cox Instrument Company, Type 1) so that it, too, could be displayed on an oscilloscope.
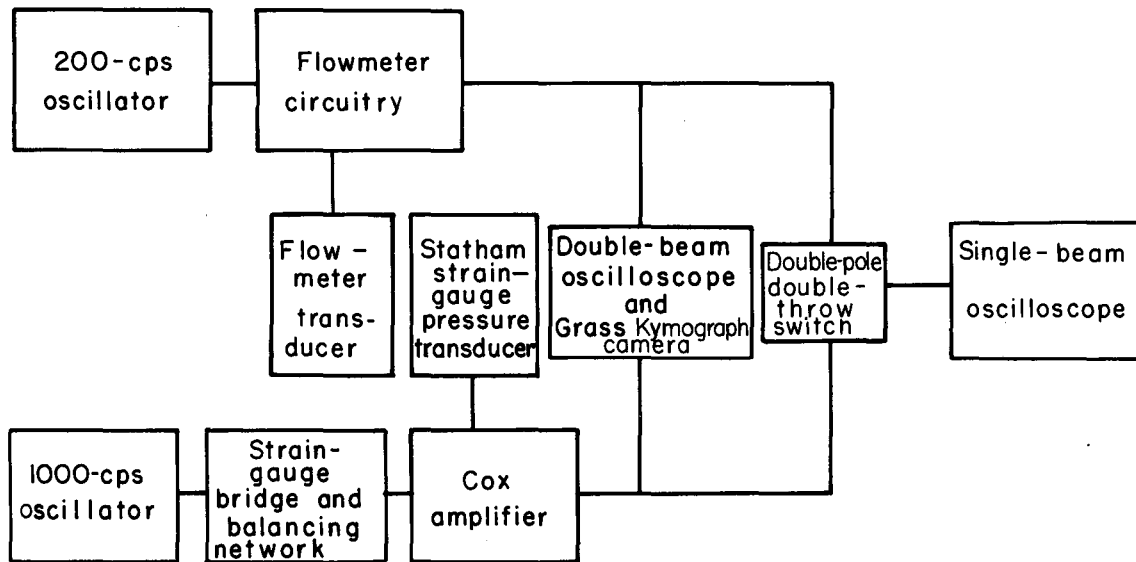
MU-21239

Fig. 1. Block diagram of experimental instrumentation.

The pressure and the flow signals were displayed simultaneously on a Dumont dual-beam oscilloscope (Model 322A). A 21-in. single-channel, Electromec oscilloscope was connected in parallel with the Dumont dual-beam oscilloscope. In this way, by using a double-pole double-throw switch, the experimenter was able to watch the pressure and the flow patterns while the dual-beam oscilloscope was being photographed, or to adjust the circuitry if needed.

The Grass kymograph camera was used to photograph the pressure and flow curves from the Dumont dual-beam oscilloscope. The Dumont oscilloscope was adjusted so that there was no horizontal sweep on either channel. The line images of pressure and flow were focused through a slit onto the film in the camera. When the film moved with a speed of 5 cm/sec, the pressure and flow curves were reproduced on the film. Kodak Tri-X film was used for this purpose.

It was then necessary to calibrate both the flow and the pressure. The flow was calibrated by disconnecting the catheters coming from and going to the animal and replacing them temporarily with a catheter from a glass-syringe constant-infusion pump. The pump delivered 20.1 cc/min to the flowmeter. A short film strip of zero flow and of a 20.1-cc/min flow was then taken.

After this procedure, the cannula from the dog were reconnected and the blood was allowed to flow freely again. The pressure circuit was then calibrated with a calibrating circuit which, by means of a calculated resistance, increased the normal pressure signal by the equivalent of 40 mm Hg. A short film strip of normal pressure was then taken. While this normal pressure was being recorded, a push-button switch was depressed several times, which introduced the additional 40-mm Hg signal into the circuit.

At this point the flow and pressure circuits were rechecked for proper balance. A normal recording of pressure and flow was then taken. In some cases it seemed advisable to take two or more such normal recordings.

ANALYTICAL PROCEDURE

The analytical procedure described below enables the experimenter to calculate the mechanical impedance to flow at various frequencies. In order to accomplish this, the pressure and flow curves must be analyzed for the various harmonics which, when added, yield the original curve. The method of analysis in which one analyzes a complex curve in this fashion is called a Fourier analysis. In this experiment both the pressure and the flow are analyzed by Fourier analysis. The mechanical impedance is then calculated by dividing the magnitude of the pressure harmonic by the magnitude of the same flow harmonic.

Two main pieces of equipment are necessary for this analysis: a frequency converter and a wave analyzer. Each will be considered separately. Figure 2 is an illustration of the frequency converter. The film, which is a simultaneous recording of pressure and flow, is placed on the adjustable pully arrangement [part (a) in Fig. 2]. A light source is adjusted so that the
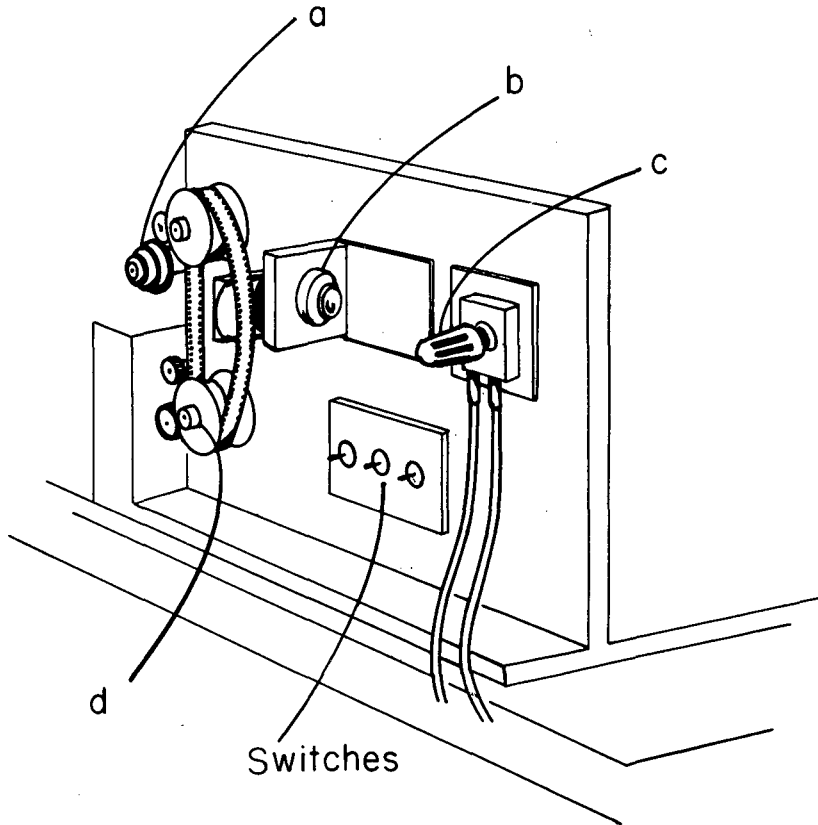
MU—21240

Fig. 2.   The frequency converter.

filament of a Mazda 4.8-volt projection bulb focuses on the film. The bulb is mounted in such a way that the filament is horizontal. The light passing through the film is incident on a lens [part (b) of Fig. 2] which focuses the flow and pressure curves on the cathode of the photo tube [part (c) of Fig. 2].

When the film is rotated past the light source, the intensity of the light incident on the phototube varies with the magnitude of the flow or pressure curve being measured. The phototube then has an output voltage that varies with the flow or pressure curve presented.

The phototube itself is an RCA 929 tube. It is connected in series with a resistance and a 150-dc-voltage supply. As light strikes the cathode of the phototube, current flows in the circuit. As this current passes through the resistance, in series with the tube, a voltage drop is observed which is proportional to the flow or pressure signal.

Part (a) of Fig. 2 is a variable-diameter pulley with which the frequency of the pressure or flow curve can be varied. It was necessary to rotate the film past the light source at a speed about 75 times as fast as the original heart beat. This was necessary because the wave analyzer was considerably more sensitive between 100 and 1000 cps than it was between 0 and 15 cps, which is the largest harmonic obtained.

A single-beam oscilloscope was placed in parallel with the phototube output. This enabled the experimenter to observe the wave form of the curve being analyzed.

The output of the phototube was amplified ten times with a wide-band decade amplifier (Technology Instrument Corp., Type 500A). The frequency response of this amplifier is such that the signal was unaltered. The amplified signal was then fed into a Model 736a wave analyzer made by the General Radio Company. The wave analyzer measures the magnitude of the individual periodic components of the complex voltage wave presented to it. The wave analyzer is sensitive to voltages between 30 μv and 300 v and having frequencies between 20 and 16,500 cps. It is, essentially, a sensitive vacuum-tube voltmeter with a four-band width.

The complete analysis is done as follows: The film strip is cut and the ends are scotch-taped together. It is important that the pressure and flow curves overlap their respective pressure and flow curves. This insures the continuity of the presentation of the curve. The light is then adjusted, and the pressure curve is brought to focus on the phototube slit. The motor rotating the film is then turned on, and the wave analyzer is adjusted for the fundamental frequency. At this time the phototube is moved back and forth till the image on the oscilloscope is undamped. Then a fine adjustment is made by moving the phototube till a peak voltage is observed on the wave analyzer. At this point the voltages associated with the fundamental and as many harmonic frequencies as possible are recorded. The same routine is then performed on the flow curve. It is important that the voltage of the fundamental be a maximum before the other readings are taken.

CALCULATIONS

The following procedures were used to arrive at the final impedance-vs-frequency curve.

(1) The voltage associated with the pressure was divided by the voltage associated with the flow for each harmonic.

(2) A correction factor was then calculated that adjusted the value of the impedance calculated in (1). This was necessary because the flow and pressure were recorded using different sensitivities. This correction factor (C. F.) in peripheral resistance units (PRU) was computed by means of the following equation:

$$(C. F.)_{PRU} = \frac{P_m F_a}{P_a F_m}$$

$$= \frac{P_m (20.1)}{F_m (40)}$$

$$= \frac{P_m}{F_m (1.99)} \quad ,$$

where P_m is the length of the 40-mm-Hg calibration measured in millimeters of displacement on the film, P_a is the value of the actual pressure calibration expressed in mm Hg F_m is the vertical distance between the peak and the trough of the flow curve measured in millimeters displacement on the film, F_a is the value of the actual flow calibration in cc/min and is equal to 20.1 cc/min. The value obtained in (1) for the impedance was divided by this correction factor.

(3) The correction factor for the frequency was calculated using the following equation:

$$C. F._{frequency} = \frac{D \, S \, \pi}{C} = 73.4,$$

where s is the speed of revolution of the pulley rotating the film on the frequency converter in rps, π, a constant, D is the diameter of the pulley in cm, and C is the speed of the kymograph camera in cm/sec. The frequency of the harmonic, as noted by the wave analyzer, was divided by 73.4 to obtain the true frequency.

RESULTS

Table I presents the experimental data obtained from 12 peripheral vascular beds. These data represent eight healthy dogs having an average pulsatile pressure of 50 mm Hg and an average flow in the femoral artery of 40 cc/min. Table II is a representative sample of data taken from measurements on one day.

The Fourier analysis of the pressure and flow curves generally yielded only four discernible frequencies, and in some cases only three. This is due to the small flow and pressure components associated with frequencies higher than 9 cps, and to some extent to the unreliability of the wave analyzer. In order to analyze the pressure, flow, and mechanical impedance with respect to frequency, it was necessary to construct an average curve of all the data. The data were listed in order according to frequency and then divided into groups. Each group contained all the data from one unit of frequency which was chosen to be 1 cps. The mean frequency, flow, pressure, and mechanical impedance were then computed for each group. These data are presented in Table I.

The mean pressure was then plotted against the mean frequency. Figure 3(a) is a presentation of this curve. The pressure is measured in terms of the output of the frequency converter in millivolts. These data do not represent absolute values of pressure. The slope of the pressure-frequency curve is positive from 2.5 to 3.6 cps. Between 3.6 and 4.6 cps, however the curve changes direction and decays rapidly. The slope continues to decrease, but does so more slowly, until at a frequency of 8.3 cps the slope becomes essentially zero. The slope remains approximately zero between 8.3 and 14.0 cps.

The mean flow vs frequency is presented in Fig. 3(b). The mean flow is also measured in terms of the frequency-converter output. This curve does not represent absolute values of flow. The slope of the flow curve is five times as great as the pressure curve between 2.5 and 3.6 cps. Between 3.6 and 4.6 cps, the flow curve is parallel to the pressure curve. From 4.6 to 7.5 cps, however, the flow curve declines more slowly, but between 7.5 and 8.3 cps, the curve decreases more rapidly and is again almost parallel with the pressure curve. Between 8.3 and 14 cps, the flow and the pressure curves are roughly identical: both have zero slope.
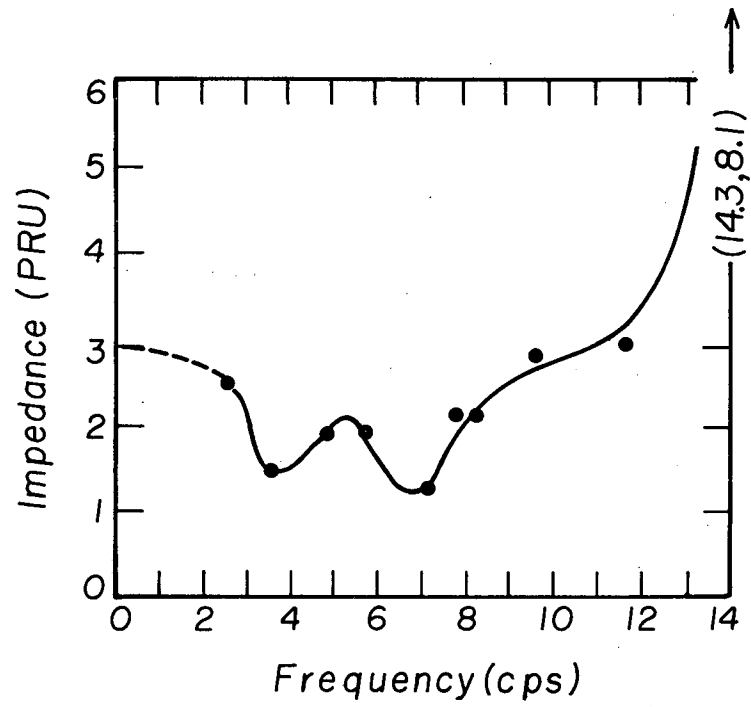
The mechanical-impedance-vs-frequency curve is presented in Fig. 4. This curve does represent absolute values of peripheral resistance. Since the flow and pressure were measured with different sensitivities because of the difference in relative magnitude, it was necessary to apply an individual correction factor to the peripheral resistance to make the peripheral resistance unit equal to 1 mm Hg per cc of blood flow per minute.

Upon examination of the peripheral resistance (or mechanical impedance)-vs-frequency curve, two resonant frequencies may be seen, one between 3.6 and 4.6 cps, and the other between 5.6 and 7.0 cps. The minimum for the first resonance point is about 1.3 PRU, while the minimum value for the second resonant frequency is about 1.2 PRU. Between the two resonant
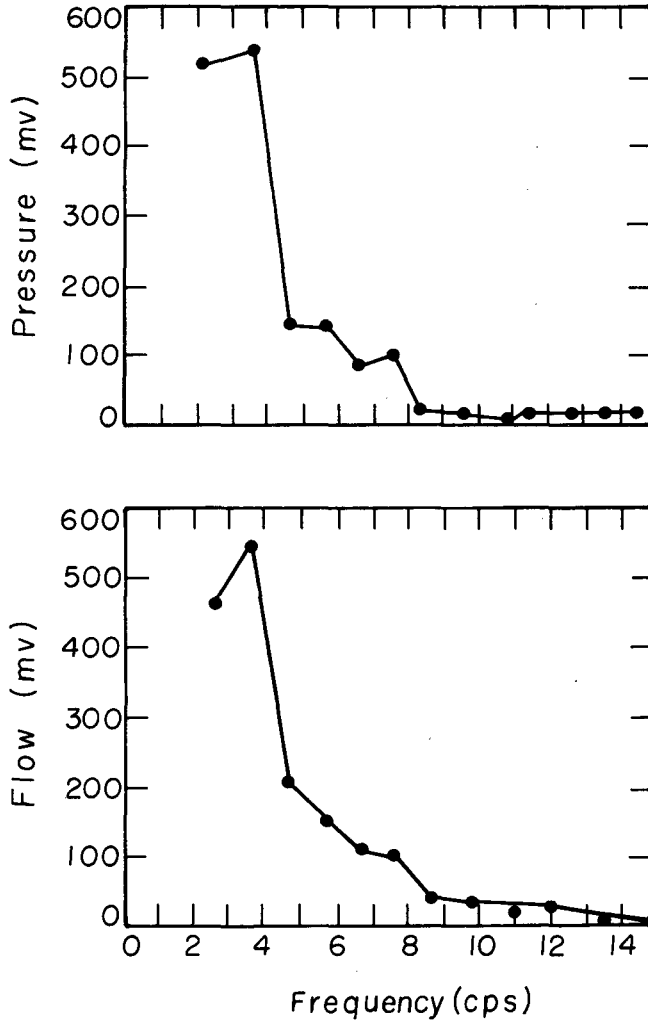
Fig. 3 (a).   Mean pressure - frequency curve;
      3 (b).   mean flow - frequency curve.

Fig. 4.   The mechanical impedance - frequency curve.

Table I

Data used to construct Figs. 4, 5, and 6.				
Number of points represented in mean	Mean frequency (cps)	Mean pressure (mv)	Mean flow (mv)	Mean peripheral resistance[a]
9.0	2.6	515.0	473.7	2.308
3.0	3.6	533.3	573.3	1.584
3.0	4.7	165.7	211.7	1.870
6.0	5.6	175.0	169.2	1.864
2.0	6.5	70.0	123.0	1.275
4.0	7.5	101.5	110.5	2.183
7.0	8.3	30.7	29.7	1.989
4.0	9.5	21.3	22.0	2.872
[b]3.0	10.9	11.6	10.8	1.365
4.0	11.4	14.0	11.9	2.929
[b]1.0	12.3	15.0	16.0	1.077
[b]1.0	13.6	12.0	9.0	1.159
3.0	14.3	14.3	4.5	8.159

[a]Mean peripheral resistance = $\dfrac{\text{mean pressure (mv)}}{\text{mean flow (mv)}}$.

[b]The data represented by these points were not used.

Table II

Sample of data from experiment of May 17, 1956.

Pressure		Flow		corrected	
frequency (cps)	amplitude (mv)	frequency (cps)	amplitude (mv)	frequency (cps)	peripheral resistance (mm Hg/cc/min)
			Part 1		
210	500	210	460	2.86	2.090
420	200	420	190	5.72	2.023
630	36	630	51	8.58	1.356
840	16	840	14	11.44	2.198
1050	14	1050	2	14.31	13.462
			Part 2		
210	465	210	213	2.86	4.198
420	210	420	85	5.72	4.750
630	60	630	17	8.58	6.787
840	20	840	7	11.44	5.494
1050	15	1050	3.5	14.31	8.242

frequencies is an antiresonant point, which is to be expected. It was impossible, however, to estimate the maximum value of the peripheral resistance at the antiresonant frequency. At frequencies higher than 7.5 cps, the mechanical impedance rises toward infinity. The impedance at zero frequency has been estimated to be about 2.5 PRU.

DISCUSSION

By analyzing the mechanical-impedance-vs-frequency curve, it is hoped that some relationship can be derived between the elastic, frictional, and inertial components of the circulatory system. It is to be expected that any relationship among these components that could be derived would be complex. Peterson has estimated that if such a relationship could be derived, it would be a nonlinear expression in which all of the variables are dependent.[3]

The mechanical--vs--frequency curve has two resonant frequencies: one between 3.5 and 4.5 cps, and the other between 6.5 and 7.5 cps. A consideration of the electrical circuit that would be analagous to the system whose behavior is described by this curve would lead one to the assumption that two separate circuits exist. Each circuit would have its own resonant frequency.

Randall, in 1955, assumed that only one circuit existed.[2] He maintained that the elastance, inertia, and friction formed a system that had only one resonant frequency. He found this frequency to be approximately 3.5 cps, which agrees with the data presented in this thesis. An explanation may be offered as to why Randall failed to observe the second resonant frequency. In Randall's method of analysis, a Fourier analysis was performed manually on the flow and pressure curves. This method of analysis was extremely tedious and time-consuming and yielded very few points on the impedance-frequency curve. For this reason Randall's data were not reliable at frequencies higher than 4 cps, whereas the analytical method employed in this thesis, when considered along with the improved frequency response of the recording equipment, has yielded data reliable to 9 cps. In essence, then, this thesis has confirmed Randall's finding of a resonant frequency at about 3.0 cps, and in effect it offers an extension of his findings.

If the circulatory system consists of two main systems, they may be defined as follows: The first would consist of the lumped characteristics of the elastic, inertial, and frictional forces of both the forward and the lateral flow in the large arteries. The second would then consist of the lumped elastic, inertial, and frictional forces associated with both the forward and lateral flow in the peripheral vascular bed.

Figure 5 is a schematic diagram of the electrical analog for the proposed theoretical model of the circulatory system. In this model the left arm represents the effects of the various components on the flow through the arterial transmission lines. The right arm represents the effects of the component forces on the flow in the peripheral bed itself. The viscous friction encountered by the blood as it passes along the large arteries to the peripheral bed is represented by R_1. The inductance L_1 is representative of the mass of the
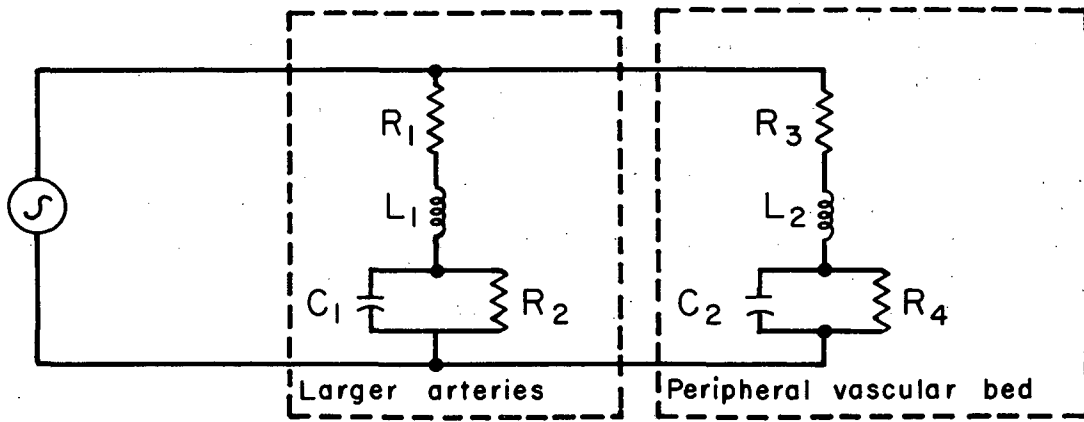
MU-21241

Fig. 5. Electrical - analog circuit of the circulatory system.

blood. Some of the energy of the blood is dissipated when the elastic wall of the vessel is expanded. This elastic component is represented by C_1. The viscous friction associated with the lateral stretching of the wall is represented by R_3.

In general, the same factors apply to the blood flow in the peripheral bed. Here R_2 is the viscous friction associated with the flow of blood in the bed, L_2 represents the inertia of the blood in the bed, C_2 represents the elasticity of the walls of the peripheral-bed capillaries, and R_4 represents the lateral viscous friction associated with the stretching of the walls.

It is possible to calculate the total impedance of the analog circuit by using standard electrical-circuit theory. The impedance Z_1 associated with the arterial transmission lines is given by

$$Z_1 = R_1(1 + j\omega \frac{L_1}{R_1}) + \frac{R_3}{1 + j\omega C_1 R_3} \quad , \tag{1}$$

which may be simplified to

$$Z_1 = R_1(1 + j\omega T_1) + \frac{R_3}{1 + j\omega T_3} \quad . \tag{2}$$

In the following equations, the time constants have been substituted for the appropriate component elements as follows:

$$T_1 = \frac{L_1}{R_1} \qquad\qquad T_3 = C_1 R_3$$

$$T_2 = \frac{L_2}{R_2} \qquad\qquad T_4 = C_2 R_4 \, .$$

Further simplification of Eq. (2) yields

$$Z_1 = \frac{R_1(1 + j\omega T_1)(1 + j\omega T_3) + R_3}{1 + J\omega T_3} \tag{3}$$

It is easier, mathematically, to work with the reciprocal of impedance, the admittance Y:

$$Y_1 = \frac{1 + j\omega T_3}{R_1(1 + j\omega T_1)(1 + j\omega T_3) + R_3} \quad , \tag{4}$$

which equals

$$Y_1 = \frac{1}{R_3 + R_1(1 + j\omega T_1)(1 + j\omega T_3)} + \frac{j\omega T_3}{R_3 + R_1(1 + j\omega T_1)(1 + j\omega T_3)} \tag{5}$$

From Eq. (5) the admittance associated with the peripheral vascular bed may be written by inspection as

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_4 + R_2(1+j\omega T_2)(1+j\omega T_4)} + \frac{j\omega T_4}{R_3 + R_1(1+j\omega T_2)(1+j\omega T_4)} . \quad (6)$$

The total admittance of the whole circuit is given by

$$Y_{total} = Y_1 + Y_2 = \frac{1}{R_3 + R_1(1+\rho T_1)(1+\rho T_3)} + \frac{1}{R_4 + R_2(1+\rho T_2)(1+\rho T_4)}$$

$$+ \frac{\rho T_3}{R_3 + R_1(1+\rho T_1)(1+\rho T_3)} + \frac{\rho T_4}{R_4 + R_2(1+\rho T_2)(1+\rho T_4)} , \quad (7)$$

where $\rho = j\omega$

From the expression for the total admittance, we can proceed to construct mathematically a qualitative picture of the impedance--vs--frequency curve for the electrical-analog theoretical model. This is done as follows:

When ρ approaches zero, Eq. (7) simplifies to

$$Y_{total}_{(\rho \to 0)} = \frac{1}{R_1 + R_3} + \frac{1}{R_4 + R_2} . \quad (8)$$

This tells us that at zero frequency the mechanical impedance has some finite value which depends on the resistive components of the circuit. This would represent the dc level of the impedance.

When ρ approaches infinity, the total admittance becomes

$$Y_{total}_{(\rho \to \infty)} = \frac{1}{R_1 T_1 T_3 \rho^2} + \frac{1}{R_2 T_2 T_4 \rho^2} + \frac{1}{T_1 \rho} + \frac{1}{T_2 \rho} , \quad (9)$$

which shows us that as the frequency approaches infinity, so does the impedance.

Thus far, from Eqs. (8) and (9) we have constructed the following impedance-vs—frequency curve:
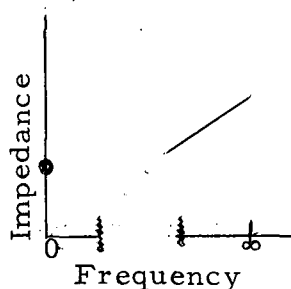
In the middle frequency range, we have chosen to assume that R_1 and R_2 are very small. This seems to be a justifiable assumption. If this assumption is allowed, Eq. (7) reduces to

$$Y_{total} = \frac{1}{R_3 + L_1 \rho} + \frac{1}{R_4 + L_2 \rho} + \frac{\rho R_3 C_1}{R_3 + L_1 \rho} + \frac{\rho R_4 C_2}{R_4 + L_2 \rho} \, , \qquad (10)$$

which equals

$$Y_{total} = \frac{1 + R_3 C_1 \rho}{R_3 + L_1 \rho} + \frac{1 + R_4 C_2 \rho}{R_4 + L_2 \rho} \, . \qquad (11)$$

Now consider

$$\theta_1 = \frac{1 + R_3 C_1 \rho}{R_3 + L_1 \rho} = \tan^{-1} \frac{R_3 C_1 \omega - \omega \frac{L_1}{R_3}}{1 + \omega^2 L_1 C_1} \qquad (12)$$

$$\theta_2 = \frac{1 + R_4 C_2 \rho}{R_4 + L_2 \rho} = \tan^{-1} \frac{R_4 C_2 \omega - \omega \frac{L_2}{R_4}}{1 + \omega^2 L_2 C_2} \qquad (13)$$

From standard electrical theory, we know that resonance occurs when the sum of θ_1 and θ_2 is zero.

If we now divide the total admittance into real and imaginary parts, we obtain

$$Y_{total} = \frac{R_3 + R_3 C_1 L_1 \omega^2}{R_3^2 + L_1^2 \omega^2} + \frac{j R_3 C_1 \omega - \omega L_1}{R_3^2 + L_1^2 \omega^2}$$

$$+ \frac{R_4 + R_4 C_2 L_2 \omega^2}{R_4^2 + L_2^2 \omega^2} + \frac{j R_4 C_2 \omega - \omega L_2}{R_4^2 + L_2^2 \omega^2} \, . \qquad (14)$$

Resonance will occur when the imaginary parts of the total-admittance equation are equal to zero:

$$\frac{R_3^2 C_1 \omega - \omega L_1}{R_3^2 + L_1^2 \omega^2} + \frac{R_4^2 C_2 \omega - \omega L_2}{R_4^2 + L_2^2 \omega^2} = 0. \qquad (15)$$

Solving Eq. (15) for ω, we obtain first

$$\left[R_3^2 C_1 \omega - \omega L_1\right]\left[R_4^2 + L_2 \omega^2\right] + \left[R_4^2 C_2 \omega - \omega L_2\right]\left[R_3^2 + L_1^2 \omega^2\right] = 0. \qquad (16)$$

Simplifying further, we have

$$R_3^2 R_4^2 C_1 \omega + R_3^2 L_2^2 C_1 \omega^3 - R_4^2 L_1 \omega - L_1 L_2^2 \omega^3 + R_3^2 R_4^2 C_2$$

$$+ R_4^2 C_2 L_1^2 \omega^3 - R_3^2 \omega L_2 - L_1^2 L_2 \omega^3 = 0. \qquad (17)$$

Gathering like terms in ω, we obtain

$$R_3^2 R_4^2 C_1 \omega + R_4^2 C_2 \omega - R_4^2 L_1 \omega - R_3 L_2 \omega$$

$$+ R_3^2 L_2^2 C_1 \omega^3 + R_4^2 L_1^2 C_2 \omega^3 - L_1 L_2^2 \omega^3 - L_1^2 L_2 \omega^3 = 0. \qquad (18)$$

Thus we have

$$\left\{\left[R_3^2 R_4^2\right]\left[C_1 + C_2\right] - \left[R_4^2 L_1 + R_3^2 L_2\right]\right\} \omega$$

$$+ \left\{\left\{\left[R_3^2 L_2^2 C_1 + R_4^2 L_1^2 C_2\right] - \left[L_1 L_2 (L_2 + L_1)\right]\right\}\right\} \omega^3 = 0, \qquad (19)$$

which is equivalent to

$$\left\{a\right\} + \left\{\left\{b\right\}\right\} \omega^2 = 0. \qquad (20)$$

Since ω, in Eq. (20) is raised to the second power, we conclude that there are two possible solutions for ω. This indicates that there are two resonant frequencies in the middle frequency range.

The completed derived impedance-frequency curve looks, at least qualitatively, like the impedance-frequency curve obtained experimentally. This confirms the assumption that the electrical analog is representative of the circulatory system. Equation (19) is an expression that relates the various parameters of the circuit. When the values of these electrical components are converted into their mechanical equivalents, it is found that the elastic, inertial, and frictional components have indeed a complex relationship.

Peterson[3] has concluded that there are three forces responsible for the genesis of the pulse wave. These three forces are : (a) the driving force of the heart, (b) the force associated with viscous friction of the flowing blood, and (c) the force associated with the distensibility of the vessel itself. Peterson, in showing that these forces are nonlinearly dependent, has to some extent confirmed the fact that any relationship between the components of the circulatory system must be complex.

The assumption that the components of the circulatory system may be lumped into two groups is consistent with the findings of Karreman, who considered the circulatory system to consist of two elastic chambers. He hypothesized that only one resonant frequency exists and that this resonant frequency is associated with the standing waves that have been observed. This thesis disagrees with Karreman in that two resonant frequencies were found.

Jochim has constructed electrical analogs of the circulatory system.[5,6] In these electrical networks, Jochim has worked out the values of the various parameters. It would be interesting to insert these parameters into Eq. (20) to see if the values for resonant frequencies agree with the experimental results. This is not possible, however, because Jochim did not publish the analog circuits that he hypothesized.

In the past, attempts have been made to arrive at a differential equation that would describe static relationships among the components of the circulatory system. This thesis has dealt with the dynamic relationships, and therefore it is meaningless to compare this work with that done by Whittaker and Winton,[7] Nutting,[8] Reiner,[9] Bayliss,[10] and others.

In the literature may be found several statements to the effect that the pressure and flow curves have the same shape. Such an observation was made by Machella.[11] The curves obtained in the work represented by this thesis failed to bear out this relationship. Flow and pressure curves were not similar in shape when compared on a time base.

The expression derived by Womersley[12] for flow through an elastic tube is not considered to apply to studies made here. At the beginning of this thesis it was stated that the circulatory system may be divided into three "black boxes." In this analogy, Womersley's equation is concerned with the second black box, while the relationships derived in this thesis are applicable only to the third black box.

To study the respiratory tract Brody used an approach similar to the one used in this thesis.[13] By using the impedance-vs-frequency curve he was able to derive relationships between the elasticity and the inertia of the respiratory tract.

There are several problems which may be interpreted as sources of error in this experiment. The first of these is the fact that definite alterations may take place in the peripheral vascular bed as a result of hypoxia. Other chemical changes may be taking place in the external tubing, but at present this is difficult to evaluate.

It would be interesting to obtain the impedance-vs-frequency curve for peripheral vascular beds other than the one associated with the femoral artery. This is indeed something that must be done in order to fully evaluate the experiments presented here.

Another serious criticism of the present experiment is the method of measuring flow. In 1956 a careful study was made to determine the precise characteristics of the electromagnetic flowmeter used in this work. The study showed that the flowmeter is accurate within 5% from 0 to 9 cps.

It has been found that each heart beat produces pressure and flow curves that are slightly different in shape. In this experiment film loops were used that represented twenty to thirty heart beats. If the change in shape of these curves is enough to alter the impedance-frequency curve, then it is obvious that some method of analysis is necessary that would consider the time changes of the impedance-vs-frequency curve.

Knowing the general shape of the impedance--vs-frequency curve, location of the resonant frequencies, location of the antiresonant frequency, points of singularity, and Eqs. (11) and (20), it may be possible to evaluate the components of the circulatory system that affect flow and pressure. When this is accomplished, we will have taken a big step forward toward the total understanding of the circulatory system.

SUMMARY AND CONCLUSIONS

This thesis has presented an approach to the understanding of the rela-
tionships existing among the physical factors of the circulatory system. These
factors are, in general, the inertia of the blood, the elastance of the vessel
walls, and the viscous friction associated with the flow of the blood. The
analytical procedure provides a method for doing a Fourier analysis electrically
on any periodic curve.

The mechanical impedance--vs--frequency curve derived in this thesis
shows two resonant frequencies-- one between 3.5 and 4.5 cps, and the other
at 5.5 to 6.5 cps. The average flow--vs--frequency and the average pressure-
vs--frequency curves have also been presented.

The impedance--vs--frequency curve has suggested the theoretical elec-
trical analog presented in Fig. 5. Using standard electrical-circuit theory,
the total impedance of the circuit in Fig. 5 was computed [Eq. (13)]. From
the equation for the total impedance, the impedance--vs frequency curve was
qualitatively constructed. The constructed curve has the same shape as the
experimental curve obtained in this thesis.

The author concludes from the experimental data that the relationship
among the elastance, inertial, and frictional factors cannot be simply stated.
The relationships arrived at in this thesis represent lumped parameters. The
solution of the total-impedance equation will most probably not involve lumped
parameters. If an exact solution were possible, it would add significantly to
our basic understanding of the circulatory system.

ACKNOWLEDGMENTS

REFERENCES

1. J. E. Randall and J. W. Clark, An Electromagnetic Blood Flowmeter, Rev. Sci. Instr. 20, 951-954 (1949).

2. J. E. Randall, Pulsatile Relationship Between Blood Pressure and Blood Flow in the Hind Limb of the Dog. Ph. D. Dissertation, Dept. of Physiology, The Ohio State University, 1955.

3. L. Peterson, The Dynamics of Pulsatile Blood Flow, Circulation Research 2, 127-139 (1954).

4. G. Karreman, The Resonance of the Arterial System, Bull. Math. Biophysics 16, 159-170 (1954).

5. K. E. Jochim, Further Studies on Electrical Analogs of the Circulatory System, Fed. Proc. 8, 82 (1949).

6. K. E. Jochim, Pressure and Flow Pulses in Carotid Artery of the Dog Compared with Voltage and Current Pulses in Electrical Anologs of the Circulatory System, Fed. Proc. 10, 70 (1951).

7. S. R. F. Whittaker and F. R. Winton, The Apparent Viscosity of Blood Flowing in the Isolated Hind-Limb of the Dog, and its Variation with Corpuscular Concentration, J. Physiol. 78, 339 (1933).

8. P. G. Nutting, J. Franklin Inst. 191, 679 (1921).

9. M. Reiner, Deformation and Flow, An Elementary Introduction to Theoretical Rheology (H. K. Lewis and Co. Ltd., London, 1949).

10. L. E. Bayliss, Rheology of Blood and Lymph , from Deformation and Flow in Biological Systems, Frey-Wyssling, Ed., (Interscience Publishers, Inc., New York, 1952).

11. T. E. Machalla, The Velocity of Blood Flow in Arteries of Small Animals, Am. J. Physiol. 115, 632-644 (1936).

12. J. R. Womersley, Oscillatory Motion of a Viscous Liquid in a Thin-Walled Elastic Tube - I: The Linear Approximation for Long Waves, Phil. Mag. Ser. 7, 46, 199 (1955).

13. A. W. Brody, A. B. DuBois, O. Nisell, and J. Engerber, Inertance and the Response of the Chest Wall and Abdomen to Forced Oscillations of Air Volume, Proc. Am. Physiol. Soc., Am. J. Physiol 179, 622 (1954).

14. M. P. Spencer and A. B. Dennison, An Explanation of the Major Features of the Arterial Pulse by Means of a Simple Electrical Analogy, Fed. Proc. Abstracts, 19, No. 1, 87 (1960).