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Theory of Quasiparticle Charge Imbalance Induced
in a Superconductor by a Supercurrent in the
Presence of a Thermal Gradient

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The charge imbalance voltage induced in a clean superconductor by a supercurrent density $\vec{J}_s$ in the presence of a temperature gradient $\vec{\nabla}T$ is shown to be $V = 2\pi v_F \ell \lambda^2 \Delta (\vec{J}_s \cdot \vec{\nabla}T) / 3c^2 k_B T^2 \cosh^2(\Delta/2k_B T) g_{NS} (1 - Z)$, where v_F , ℓ , and $\lambda(T)$ are the Fermi velocity, mean free path, and penetration depth, g_{NS} is the normalized tunnel conductance of the probe junction, and $Z(\Delta/T)$ is a known function. This result agrees well with the experiments of Clarke, Fjordbøge, and Lindelof.

Pethick and Smith¹ have predicted that a quasiparticle charge imbalance Q^* and potential V will be established when the superfluid moves at a velocity $\vec{v}_s$ in a superconductor along which there exists a temperature gradient $\vec{\nabla}T$. In the clean limit and for $\Delta \ll k_B T$ (Δ is the energy gap) they find

$$\frac{eV}{E_F} = \frac{\pi}{6} \frac{\Delta}{k_B T} \frac{\tau}{g_{NS}} \vec{v}_s \cdot \frac{\vec{\nabla}T}{T}, \quad (1)$$

where τ is "a characteristic time for charge relaxation", E_F is the Fermi energy, and g_{NS} is the normalized tunneling conductance to the normal metal probe used to detect V ^{2,3}. Near T_c , v_s is proportional to $j_s (1 - t)^{-1}$, where j_s is the supercurrent density, and $t = T/T_c$. Clarke, Fjordbøge, and Lindelof⁴ observed this potential in Sn films, and found that V was proportional to $j_s \nabla T$ at a given temperature, and that $V g_{NS} T / j_s \nabla T$ diverged as $(1 - t)^{-1}$. Thus to bring the temperature dependence predicted by (1) into agreement with the experimental observations, it is necessary for τ to be proportional to Δ^{-1} . For a superconducting film in which the inelastic mean free path is much less than the elastic mean free path (not a physically realizable situation), Pethick and Smith interpret τ as the inelastic charge relaxation time, $\tau_{Q^*} = (4k_B T / \pi \Delta) \tau_E$, obtaining the correct temperature dependence but a voltage that exceeds the experimental values by 2 to 3 orders of magnitude. In one replaces τ by the temperature-independent impurity scattering time, one obtains approximately the right magnitude for the voltage at temperatures near $0.9 T_c$, but an incorrect temperature dependence near T_c , namely $(1-t)^{-1/2}$. In this Letter we give an alternative theory for this effect, applicable to

clean films where the quasiparticle momentum $\vec{p}_k$ is a good quantum number. Our result fits the observed temperature dependence over the entire experimental range ($t=0.5 - 0.99$) and it is in reasonable agreement with the experimentally determined magnitude for clean films.

We consider first the situation in which $\vec{v}_s = 0$, so that the quasiparticle excitation spectrum is symmetric around the Fermi surface, but $\nabla T \neq 0$ ⁵. Quasiparticles moving from the left in Fig. 1(a) are at an effective temperature $T - \delta T$, while those from the right are at $T + \delta T$, where T is the local temperature. Thus, there is an imbalance in the populations of the $k > k_F$ and $k < k_F$ branches on the right hand side of the Fermi surface, but an equal and opposite imbalance on the left hand side; as a result, the value of Q^* is zero. If we impose a superfluid velocity $\vec{v}_s$, the excitation energies take the form^{6,7}

$$E_k = (\epsilon_k^2 + \Delta^2)^{1/2} + \vec{p}_k \cdot \vec{v}_s, \quad (2)$$

where ϵ_k is the one-electron energy relative to the chemical potential, and $\vec{p}_k$ is the electron momentum. There is now an asymmetry in the excitation spectrum about the Fermi surface, as indicated in Fig. 1(b), so that the population imbalances on the two sides no longer cancel exactly. This net charge imbalance is the origin of the observed potential, and is the quantity that we now calculate.

Consider a superconductor with transverse dimensions small compared with the penetration depth, so that any supercurrent flows uniformly. We assume that $p_F v_s \ll \Delta$, and $\ell \nabla T / T \ll 1$, where p_F is the Fermi momentum and ℓ is the electronic mean free path. Our derivation is based on the physical

assumption that the population of the state $\vec{k}$ at point $\vec{r}$ is characterized by the temperature at the point one mean free path back along its trajectory, whether the scattering there is elastic or inelastic, since that scattering erases all memory of the direction of the previous random walk trajectory. Expressing this assumption mathematically, we have

$$\delta f_{\vec{k}} = f_o \left(\frac{E_{\vec{k}}^o + \vec{v}_s \cdot \vec{p}_{\vec{k}}}{T - \vec{\ell}_{\vec{k}} \cdot \vec{v}_T} \right) - f_o \left(\frac{E_{\vec{k}}^o}{T} \right) \quad (3)$$

Here f_o is the Fermi function, and $\delta f_{\vec{k}}$ is the departure from the occupation number when both $\vec{v}_s$ and $\vec{v}_T$ are zero. As argued above, one gets a net Q^* only if both $\vec{v}_s$ and $\vec{v}_T$ are non-zero. The relevant non-cancelling term is

$$\delta f_{\vec{k}} = - \frac{\partial^2 f_o}{\partial E \partial T} (\vec{v}_s \cdot \vec{p}_{\vec{k}}) (\vec{\ell}_{\vec{k}} \cdot \vec{v}_T) \quad (4)$$

Note that $\vec{\ell}_{\vec{k}} \equiv \ell \hat{v}_{\vec{k}} = \ell \text{sgn } \epsilon \vec{p}_{\vec{k}}/p_F$, since the group velocity along $\vec{p}_{\vec{k}}$ is $dE/d\epsilon = (\epsilon/E)v_F$, and the factor (ϵ/E) is cancelled by a factor $|E/\epsilon|$ in the scattering time⁸. Then an elementary geometrical argument shows that, averaged over a sphere, the factors in parenthesis in (4) reduce to

$\frac{1}{3} p_F \ell \text{sgn } \epsilon \vec{v}_s \cdot \vec{v}_T$. Thus, the normal charge⁹ δQ_n is given by

$$\begin{aligned} \delta Q_n &= 2N(0) \int_{-\infty}^{\infty} \frac{\epsilon}{E} \overline{\delta f} d\epsilon = \frac{4}{3} N(0) p_F \ell \vec{v}_s \cdot \vec{v}_T \int_{\Delta}^{\infty} \left(- \frac{\partial^2 f_o}{\partial E \partial T} \right) dE \\ &= \frac{4}{3} N(0) p_F \ell \vec{v}_s \cdot \vec{v}_T \frac{\partial f_o(\Delta)}{\partial T}, \end{aligned} \quad (5)$$

where $\overline{\delta f}$ refers to the angular average of $\delta f_{\vec{k}}$, and we have used the fact that $\epsilon d\epsilon = E dE$.

Now, this δQ_n is not the same as Q^* , as was first pointed out by Pethick and Smith⁹ and further elaborated by Kadin, et al.¹⁰ The argument can be summarized as follows: Maintenance of electrical neutrality requires that the total electronic charge $\sum v_k^2 + \sum (1 - 2v_k^2) f_k$ remain equal to the total charge of the positive ion cores. Thus, first order changes in v_k and f_k about their equilibrium values v_k^0 and f_k^0 are constrained by $\sum (1 - 2f_k^0) \delta v_k^2 + \sum (1 - 2v_k^{02}) \delta f_k = 0$. But all the δv_k^2 are determined from the shift δE_F in Fermi energy of the condensate by the relation $\delta v_k^2 = (\Delta^2/E_k^3) \delta E_F$. The quasiparticle charge Q^* is also directly proportional to this shift, since it is equal and opposite to the change in condensate charge. Thus we have

$$Q^* = -2N(0) \delta E_F = 2N(0) \frac{\sum (1 - 2v_k^{02}) \delta f_k}{\sum (1 - 2f_k^0) (\Delta^2/2E_k^3)} = \frac{\sum q_k^0 \delta f_k}{1 - Z} \equiv \frac{\delta Q_n}{1 - Z} \quad (6)$$

where $Z = 2 \int_{\Delta}^{\infty} f_0(E) (\Delta^2/E^3) (E/\epsilon) dE = 2 \int_{\Delta}^{\infty} (-\partial f_0/\partial E) (\epsilon/E) dE$ is the function discussed by Clarke, et al.¹¹. Near T_c , $Z \rightarrow 1 - \pi\Delta/4k_B T$, so that Q^* is "amplified" by a factor $4k_B T/\pi\Delta$ relative to δQ_n . As $T \rightarrow 0$, however, Z becomes exponentially small, and Q^* is nearly equal to δQ_n . Equating the expression for Q^* obtained from (5) and (6) with $2N(0)g_{NS}eV$, we obtain

$$eV = \frac{2}{3} p_F \ell \frac{\partial f_0(\Delta)/\partial T}{g_{NS}(1 - Z)} \vec{v}_s \cdot \vec{\nabla} T \quad (7)$$

This is our central result.

Although $\vec{v}_s$ would be the controlled quantity in a fluxoid quantization geometry, in the usual configuration one controls $\vec{J}_s = n_s e \vec{v}_s$. Here n_s is

defined in terms of the actual penetration depth by $\lambda^2(T) = mc^2/4\pi n_s e^2 \approx \lambda^2(0)(1 - t^4)^{-1}$, using the well-known⁷ empirical formula. Inserting this, and writing $\partial f_0(\Delta)/\partial T$ explicitly, (7) becomes

$$V = \frac{2\pi}{3} \frac{v_F}{c} \ell \frac{\lambda^2(0)}{1 - t^4} \frac{\Delta}{k_B T \cosh^2(\Delta/2k_B T) g_{NS}(1 - Z)} \frac{J_s}{c} \cdot \frac{\dot{V}_T}{T} \quad (8)$$

Since the factor $\Delta/(1 - Z)$ varies only from $4k_B T_c/\pi$ at T_c to $\Delta(0) = 1.76k_B T_c$ at $T = 0$, the temperature dependence is dominated by the divergence of $\lambda^2 \sim n_s^{-1} \sim (1 - t)^{-1}$ near T_c and the exponential temperature dependences at low temperatures which reflect the freezing out of the quasiparticle population. Sufficiently near T_c , (8) simplifies to

$$V = \frac{2}{3} \frac{v_F}{c} \frac{\ell}{g_{NS}} \frac{\lambda^2(0)}{1 - t} \frac{J_s}{c} \cdot \frac{\dot{V}_T}{T} \quad (9)$$

To estimate the voltage expected near T_c for the only clean Sn sample (Nr. 4) of the experimental work⁴ we set $v_F = 0.65 \times 10^8$ cm/sec, $\ell = 4.3 \times 10^{-5}$ cm, and $\lambda(0) = 5.0 \times 10^{-6}$ cm. Then, in practical units,

(9) becomes $V_{g_{NS}} T(1 - t)/J_s \nabla T = 0.5 \times 10^{-16} \Omega \cdot \text{cm}^3$, in reasonable agreement with the measured value of $1.2 \times 10^{-16} \Omega \text{cm}^3$.

In order to test the predicted temperature dependence more fully, we return to (8). Fig. 2 shows the remarkably good fit of this function to the experimental data of ref. 4, fitted at a single point. Note that this formula reproduces even such a subtle feature as the rise of the data above the simple $1/(1 - t)$ behavior before the fall at low temperatures dictated by the $\cosh^2(\Delta/2k_B T)$ factor. This excellent fit may be fortuitous, but still it adds credibility to the analysis.

We can also extend our theory to estimate the size of the quasiparticle potential induced by a temperature gradient in the absence of an applied supercurrent. Falco¹² has performed an experiment to measure this effect. The temperature gradient induces a supercurrent $\vec{J}_s = (\eta/\rho)\vec{\nabla}T$, where η is the thermoelectric power and ρ is the normal state resistivity.¹³ Thus, from Eq. (9), $V \propto (\eta/\rho)(\nabla T)^2(1 - t)^{-1}$. If we take¹⁴ $\eta \approx 10^{-8} \text{VK}^{-1}$, $\rho \approx 2 \times 10^{-7} \Omega \text{cm}$, and $\nabla T \approx 1 \text{Kcm}^{-1}$, near T_c we find $V(1 - t) \approx 10^{-18} \text{V}$. Since the voltage observed by Falco was linear in $|\nabla T|$, showed no evidence of divergence at T_c , and was some 6 orders of magnitude larger, it appears unlikely that the voltage observed by him was generated by this mechanism.

It is of interest to compare the processes causing charge imbalance in this experiment with those in situations where the generation occurs at a well-defined spatial discontinuity, as in the case of tunnel injection², current flow across a normal-superconducting interface¹⁵, or phase-slip centers¹⁶. In these latter experiments, charge generation and relaxation processes are quite

distinct, being even spatially separated. Thus, for example, if one enhances the Q^* relaxation rate by the application of a magnetic field¹⁷ or addition of magnetic impurities¹⁸ while retaining the same generation rate, the steady-state value of Q^* will decrease. By contrast, in the present experiment the charge imbalance generation is a volume process, not a surface one, and generation and relaxation processes are so strongly coupled that they cannot be independently varied. As a result, the net branch imbalance charge is imposed by the presence of the temperature gradient, the magnitude of the effect depending only on the transport mean free path which limits the distance over which the gradient is effective in producing a non-equilibrium population. If this view is correct, times such as τ_E and τ_{Q^*} play no direct role in determining the magnitude of Q^* , and hence we would not expect the introduction of a small concentration of magnetic impurities (for example) to change the size of the effect, in contrast to the situation in the other experiments cited above. Our viewpoint also provides a natural explanation for the observation by Schmid and Schön¹⁹ that rather similar results are found in a diverse variety of limiting cases (clean, dirty, strong pair-breaking, weak pair-breaking) in which they make approximate solutions of the Boltzmann equation.

In closing, we also remark on the inclusion of the $(1 - Z)^{-1}$ "amplification factor" in (8). Such a factor does not enter in the "injection" experiments cited above because one is finding a steady-state equilibrium between a generation rate and a relaxation rate which are equally affected by the amplification factor. As pointed out by Kadin et al¹⁰, its effect is simply to speed up the approach to equilibrium by a factor $(1 - Z)^{-1}$, without

changing the value of Q^* at equilibrium. By contrast, in this calculation, we compute Q^* for a given δf_k , representing the non-equilibrium population imposed by the gradient, and the factor $(1 - Z)^{-1}$ enters naturally.

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References

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- (b) Senior U.S. Scientist Awardee of Alexander von Humboldt Foundation; on sabbatical leave from Department of Physics, Harvard University, Cambridge, Mass. 02138.
- 1. C.J. Pethick and H. Smith, Phys. Rev. Lett., submitted.
- 2. J. Clarke, Phys. Rev. Lett. 28, 1363 (1972).
- 3. M. Tinkham and J. Clarke, Phys. Rev. Lett. 28, 1366 (1972).
- 4. J. Clarke, B.R. Fjordbøge, and P.E. Lindelof, Phys. Rev. Lett., submitted.
- 5. If $\vec{v}_T \neq 0$, there is a small superfluid velocity even in the absence of an applied current (see later). For the moment, imagine this velocity to be cancelled by a tiny applied current.
- 6. Yu.M. Galperin, V.L. Gurevich, and V.I. Kozub, Zh. Eksp. Teor. Fiz. 66, 1387 (1974) [Sov. Phys. JETP 39, 680 (1974)]; A.G. Aronov, Zh. Eksp. Teor. Fiz. 67, 178 (1978) [Sov. Phys. JETP 40, 90 (1978)].
- 7. See, for example, M. Tinkham, Introduction to Superconductivity McGraw-Hill, New York (1975).
- 8. J. Bardeen, G. Rickayzen, and L. Tewordt, Phys. Rev. 113, 982 (1959); P. Wyder, Rev. Mod. Phys. 36, 116 (1964).
- 9. C.J. Pethick and H. Smith, Ann. Phys. (N.Y.), in press.
- 10. A.M. Kadin, L.N. Smith, and W.J. Skocpol, J. Low Temp. Phys., submitted.
- 11. J. Clarke, U. Eckern, A. Schmid, G. Schön, and M. Tinkham, Phys. Rev., submitted.
- 12. C.M. Falco, Phys. Rev. Lett. 39, 660 (1977).
- 13. V.L. Ginzburg, J. Phys. USSR 8, 148 (1944).
- 14. G.T. Pullan, Proc. Roy. Soc. A217, 280 (1953).
- 15. A.B. Pippard, J.G. Shepherd, and D.A. Tindall, Proc. Roy. Soc. A324, 17 (1971).
- 16. W.J. Skocpol, M.R. Beasley, and M. Tinkham, J. Low Temp. Phys. 16, 145 (1974).

17. A.M. Kadin, W.J. Skocpol, and M. Tinkham, J. Low Temp. Phys. 33, 481 (1978).
18. T.R. Lemberger and J. Clarke, Bull. Am. Phys. Soc. 24, 328 (1979).
19. A. Schmid and G. Schön, preprint, received while this paper was in preparation.

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