In Nash equilibrium, when would just one country contribute to pharmaceutical research

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Publication Date
2025-11-16

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November 17, 2025

Introduction

Mancur Olson [8] argued that in the provision of international public goods, there is a tendency for "exploitation of the great by the small", where larger countries provide a disproportionate share of the public good, while smaller countries tend to "free ride". Olson and Richard Zeckhauser [9] suggest that in Nash equilibrium,

> "the larger members of an alliance should, on the average, devote larger percentages of their national incomes to defense than do the smaller nations"

In support of this theory, they showed that in 1966, the United States, which is the largest and richest member of NATO, spent a much larger proportion of its national income on defense than any of the other members.[1]

Margaret Kyle, David Ridley, and Su Zhang [5] apply this theory to government subsidized research on pharmaceuticals. H. E. Frech, Mark Pauly, William Comanor, and Joseph Martinez [4] note that "US prices for branded pharmaceuticals are much higher on average than those paid for the same products in the rest of the world." In fact, Andrew Mulcahy et al [7] estimate that the price paid by U.S. consumers is more than four times as high as the average paid by OECD countries. Frech *et al* proceed to estimate production costs for these drugs, and thus can estimate the "subsidy" that they receive from each country by pricing above costs. They estimate that in this way, the U.S. contributes about about $1,000 capita, which is about 1.6% of GNP. Although pricing is above marginal cost in most of the world's countries, the discrepancy is far less. Frech et al estimate that pricing above marginal cost leads to contributions of about $190 per capita in the rest ofthe world.

*I am grateful to Ted Frech and George Yang for stimulating comments and pointers to relevant literature.

[1] This is less dramatically the case in recent years. In 2024, the US spent about 3.2% of national income on defense, while the average for NATO members was 2.8%. Poland, Estonia, and Lithuania each spent a higher percentage of national income on defense than the US.

They suggest that the reason for this discrepancy is that the US chooses to allow pharmaceutical companies to set high prices for their patented products in order to incentivize these companies to engage in further research. They propose that the underlying reason that the US allows these high prices, while other countries do not is the same as that identified by Olson and by Olson and Zeckhauser. In Nash equilibrium, the largest and richest country or countries will bear a disproportionate cost of provision of international public goods.

Table 1 shows that in 2024, the world's richest country in terms of gross national income is the United States, while China is second with national income about 70% as large. India and China each have populations more than 4 times as large as that of the US. But India's per capita income is only about 1/7 of that in the U.S. All other countries listed in Table 1 have lower per capita incomes and smaller populations than the United States.

Country	GNI (trillions)	Pop (millions)	GNI/ Pop (thousands)	Pop/ US Pop	GNI/ US GNI
United States	26.9	334	80.5	1	1
China	18.9	1,412	13.4	4.22	0.70
Japan	4.9	125	39.4	0.37	0.22
Germany	4.6	84	54.8	0.25	0.18
India	3.6	1,417	2.5	4.24	0.13
UK	3.3	69	47.7	0.21	0.12
France	3.1	69	45.2	0.21	0.12
Canada	2.3	43	54.0	0.13	0.09
Italy	2.2	58	37.9	0.17	0.08
Russia	2.1	147	14.3	0.44	0.08
Brazil	2.0	216	9.3	0.64	0.07
South Korea	1.8	51	35.5	0.15	0.07
Australia	1.7	27	63.2	0.08	0.06
Spain	1.6	49	32.8	0.15	0.06
Mexico	1.6	134	12.0	0.40	0.06

Table 1: Gross National Income and Population

This paper argues that if the fruits of pharmaceutical research were non-excludable, and if pharmaceutical research were financed only by voluntary contributions, then the United States would be the only country to contribute to funding this research in Nash equilibrium. Frech et al [4] estimate the U.S. spends approximately 1% of its national income on payments to drug companies above their production costs while other countries pay more than marginal cost, but far less that does the U.S. The reason that we see positive contributions to pharmaceutical profits from the other countries is that these companies hold patents and can withhold access. Thus we would expect countries other than the U.S. to pay a bargained price that allows the pharmaceutical companies some "monopoly profits."

Nash equilibrium with Global Public Goods

Wolfgang Buchholz and Todd Sandler [2] describe global public goods as follows;

> "Global public goods possess partially or fully non-rival and non-excludable benefits that affect a large swath of the planet."

Let us consider the case of a fully excludable and fully non-rival global public good. This assumption is not entirely realistic for drug research, since patents allow drug companies the ability to exclude access to the results of their past research. We will consider the effect of relaxing this assumption in later discussion.

Definition 1. *Let $M_i \geq 0$ be the amount contributed by Country i. Let total contributions be $M = \sum_i M_i$, and let $M_{\sim i} = M - M_i$ be the amount contributed by countries other than i.*

Definition 2. *Let n_i be Country i's the population and let y_i be its national income.*

Definition 3. *Let $x_i = \frac{y_i - M_i}{n_i}$ be per capita consumption of private goods in Country i.*

We assume that decision-makers seek to maximize the utility of a resident who has mean per capita private consumption for their country. This assumption implies that they act in the interests of a consumer with mean income for that country. Depending on the distribution of income and of political influence in a country, this assumption may be more or less accurate. Although it is possible that countries differ in their preferences over alternative combinations of the global public good and private consumption, we will here assume that these preferences are the same for all countries

Assumption 1. *Where M is the amount of the global public good and x_i is per capita private expenditures in Country i, Decision-makers for each country i seek to maximize a utility function $U(M, x_i)$.*

Assumption 1 implicitly assumes that the public good is both non-rival and non-excludable, since it assumes that all countries experience always experience the same amount of this public good.

In Nash equilibrium, each country seeks chooses a contribution to the global public good that maximizes its utility given the amounts contributed by others.

Definition 4. *A Nash equilibrium for provision of the global public good M consists of a vector of contributions $(M_1, \ldots, M_K)$ such that for all i:*
(M_i, x_i) maximizes $U_i(M_{\sim i} + M_i, x_i)$ subject to the constraints that $M_i \geq 0$ and $M_i + n_i x_i = y_i$ and $M_i \geq 0$, where n_i is Country i's population and y_i is its national income.

The case of identical Cobb-Douglas utilities

The case where consumers in all countries have identical Cobb-Douglas utility functions yields a particularly sharp characterization of Nash equilibrium.

Assumption 2. *Consumers in each country have identical utility functions $U(M, x) = M^\alpha x^{1-\alpha}$, where M is the quantity of an international public goods and where x is that consumer's consumption of private goods.*

In general, the efficient contribution to the public good by any country depends on the way in which private goods will be distributed in that country. However, when consumers have identical Cobb-Douglas utility functions, the Pareto efficient contribution to public goods will be the same regardless of the corresponding distribution of private goods.

Definition 5. *An allocation $(M, x_{i1}, \ldots, x_{in_i})$ is feasible for Country i conditional on $M_{\sim i}$ if $M \geq M_{\sim i}$ and $M + \sum_{j=i}^{n_i} \leq y_i + M_{\sim i}$.*

Definition 6. *An allocation is Pareto optimal for Country i conditional on $M_{\sim i}$ if it is feasible conditional on $M_{\sim i}$ and if no other allocation that is feasible for i conditional on $M_{\sim i}$ is Pareto superior to this allocation.*

If Assumption 2 is satisfied and if decision-makers seek an allocation that is Pareto optimal within their country, given available resources, they will seek to maximize the function $U(M, x_i) = M^\alpha x_i^{1-\alpha}$ subject to the relevant constraints.

Lemma 1. *If residents of Country i have identical Cobb-Douglas utility functions as in Assumption 2, and if the allocation that results from Country i's contribution will be Pareto optimal condtional on $M_{\sim i}$, then Country i must choose a contribution M_i that maximizes $(M_i + M_{\sim i})^\alpha x_i^{1-\alpha}$ subject to the constraints that $M_i \geq 0$ and $M_i + n_i x_i = y_i$.*

The intuition for why this lemma is true is that with Cobb-Douglas utility, the sum of marginal rates of substitution depends only on the amount of the public good and the total amount of private consumption and not on how it is distributed.

Proof. The Samuelson marginal conditions for a feasible allocation to be Pareto optimal for Country i, given $M_{\sim i}$ given Assumption 2 are that

$$\sum_{j=1}^{n_i} \frac{\alpha}{1-\alpha} \left(\frac{x_{ij}}{M_i + M_{\sim i}} \right) \leq 1$$

with equality if $M_i > 0$. This condition is the same as the necessary and sufficient condition for M_i to maximize $(M_i + M_{\sim i})^\alpha x_i^{1-\alpha}$ subject to the constraints that $M_i \geq 0$ and $M_i + n_i x_i = y_i$. $\square$

Theorem 1. *Where y_i is the national income of Country i and where $y_1 > y_i$ for all countries $i \neq 1$. With Cobb-Douglas utilities as in Assumption 2, Country 1 will be the only contributor to the public good if and only if $y_i < (1-\alpha) y_1$ for all $i \neq 1$.*

Proof. Given Assumption 2, it must be that if no other country contributes, Country 1 will supply an amount $M_1 = \alpha y_1$. If Country 1 supplies M_1 and nobody else contributes, Country i will choose M_i to maximize $(M_1 + M_i)^\alpha x_2^{1-\alpha}$ subject to the constraints $M_i + x_i n_i = y_i$ and $M_i > 0$. Adding M_1 to both sides of the budget constraint, we see that this is equivalent to choosing M to maximize $M^\alpha x_2^{1-\alpha}$ subject to the constraints $M + n_i x_i = y_i + M_1$ and $M > M_1$. If $M > M_1$, then it must be that Country i will choose $M = \alpha(y_2 + M_1)$. Since $M_1 = \alpha y_1$, it must be that if $M > 0$, then $M = \alpha(y_i + \alpha y_1) > M_1 = \alpha y_1$. But $\alpha(y_i + \alpha y_1) > \alpha y_1$ if and only if $y_i > (1-\alpha) y_1$. Therefore Country i will not contribute if $y_i < (1-\alpha) y_1$. □

The estimates of Frech et al [4] and Kyle et al [5] indicate that the U.S. spends less than 2% of its GNP on research related to pharmaceutical development. Thus Theorem 1 informs us that if countries have identical Cobb-Douglas utility functions and the results of pharmaceutical research were not excludable, U.S. would be the only country to fund such research if for all $i > 1$, $y_i < .98 y_1$. According to Table 1, China has the world's second highest GNP, which is about 70% of that of the U.S. It follows that with Cobb-Douglas utilities, in Nash equilibrium, the only contributor to pharamaceutical research would be the United States.

Constant elasticity of substitution utility

With Cobb-Douglas utility, the marginal rate of substitution between two goods is inversely proportional to their quantities. Thus the elasticity of substitution is 1 and the expenditure share $\theta(n_i)$ is the same for all values of n_i. To quantify more general possibilities, we consider cases in which the elasticity of substitution takes values other than 1.

Definition 7. *The elasticity of substitution between two goods is the negative of ratio of the percentage change in the relative quantities of the two goods demanded in response to a percentage change in their relative prices.*

Two goods are said to be *substitutes* if the elasticity of substitution between them is greater than one. In this case, if the price of one of the goods decreases while the price of the other remains constant, then the amount of good whose price has fallen will rise more than proportionally to the price decrease, so the fraction of income spent on this good will rise and the fraction of income spent on the other good will fall.

If the elasticity of substitution is less than one, these goods are said to be *complements*. In this case, the consumption ratio is less responsive to the price ratio and the fraction of income spent on a good will be a decreasing function of its price. The "price" of public goods relative to that of consumption per capita differs between countries because countries with larger population, are able to share the cost of the public good among more consumers.

Let us consier a model in which preferences of decision makers are represented by a utility function that exhibits constant elasticity of substitution between the global public good and average per capita expenditures on private goods by its citizens.

Definition 8. *A quasi-concave utility function $U(x_1, x_2)$ with constant elasticity of substitution $\sigma = \frac{1}{1-\rho}$ takes the form*

$$U(x_1, x_2) = (\alpha x_1^\rho + (1-\alpha)x_2^\rho)^{\frac{1}{\rho}} \tag{1}$$

where $0 \leq \alpha \leq 1$, $\rho \leq 1$ and $\rho \neq 0$.

Remark 1. *The limiting value as $\rho \to 0$ of the CES function in Equation 1 is the Cobb-Douglas utility function $U(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$, which has a constant elasticity of substitution of 1.*

Assumption 3. *Let M_i be the amount of public good contributed by Country i, and let $M_{\sim i}$ be the total amount contributed by all other countries. For Country i, let n_i be population, y_i be total income and let x_i be per capita expenditures on goods other than the global public good. Decision-makers for each Country i seek to maximize a constant elasticity of substitution function $U(M_i + M_{\sim i}, x_i)$ subject to the constraints that $M + n_i x_i = y_i$ and $M_i \geq 0$.*

With the budget constraint $M + n_i x_i = y_i$, the price of private consumption relative to the public good increases with population size. If the elasticity of substitution is greater than one, then larger countries will spend a larger fraction of income on the public good, and if the elasticity of substitution is less than one, they will spend a smaller fraction of income on the public good.

Recall that in the case of Cobb-Douglas utility, Country 1 will be the only contributor if $y_i < (1-\alpha)y_1$ for all $\neq 1$. Where the elasticity of substitution is σ, the condition for Country 1 to be the only contributor depends on population as well as income. Theorem 2 which is proved in the Appendix expresses this dependence.

Theorem 2. *Let Country 1 be the country with largest national income, y_1. If Assumption 3 is satisfied and if for all Countries $i > 1$,*

$$y_i n_i^{\sigma-1} < (1-\theta(n_1))y_1 n_1^{\sigma-1} \tag{2}$$

in Nash equilibrium Country 1 is the only supplier of the global public good.

Empirical estimates

To apply Theorem 2 to the case of pharmaceutical research, in addition to estimates of national incomes and populations, we need estimates of the elasticity of substitution σ and of the expenditure share $\theta(n_1)$ for pharmaceutical research in the country with largest national incoem.

We do not have data that allows us to directly estimate consumers' elasticity of substitution between pharmaceutical research and other goods. But there are studies the price-elasticity of demand for medical services and for pharmaceuticals. The RAND Health Insurance Experiment [6] conducted a randomized study of 5,809 individuals who are offered differing co-payment rates for medical services. The RAND study estimated an own price-elasticity of demand for medical services in the range from $-.1$ to $-.2$. A more recent contribution by Kai Yeung *et al* used variations in co-payment rates to estimate demand elasticities specifically for pharmaceuticals. They estimate a price elasticity of $-.16$. Yeung *et al* [10] report the results of several other studies that estimate this price elasticity. Almost all of these studies report highly inelastic demand with estimates roughly similar to those of the RAND an Yeung et al studies.

If preferences have constant elasticity of substitution, we can use the own price-elasticities to estimate the elasticity of substitution between medical services and other goods. Assuming that the elasticity of demand for expected improvements from pharmaceutical research is similar to that for existing pharmaceuticals, the following lemma allows us to estimate the elasticity of substitution between pharmaceutical research and other goods.

Definition 9. *Where the demand function for good i is $x_i(p_1, p_2)y$, the price-elasticity of demand for good i is $\xi_i = \frac{p_i}{x_i(p_1,p_2)} \frac{dx_i(p_1,p_2)}{dp_i}$*

In the Appendix we prove that with CES utility, the elasticity of substitution and the own elasticity of demand for good 1 are related as follows.

Theorem 3. *Let $U(x_1, x_2)$ be a constant elasticity of substitution utility function. Let $x_i(p_1, p_2)y$ be the demand function for good i, and let $\theta_1 = \frac{p_1 x_1(p_1,p_2)}{y}$. Where ξ_1 is the price-elasticity of good 1 and where σ is the elasticity of substitution between goods 1 and 2, it must be that*

$$1 - \sigma = \frac{1 + \xi_1}{1 - \theta_1} \tag{3}$$

Since $0 < \theta_1 < 1$, it follows from Lemma 3 that $1 - \sigma$ and $1 + \xi$ are of the same sign. This means that if the demand for good 1 is price inelastic, with $-1 < \xi < 0$, then the two goods are complements, with $0 < \sigma < 1$. If on, the other hand, demand for good 1 is price elastic, then the two goods are complements. The RAND study and the studies surveyed by Yeung *et al* [6, 10] indicate that the demand for pharmaceutical products is highly inelastic, with ξ_1 typically in the range from -0.1 to -0.2. The studies by Kyle et al [5] and Frech et al [4] suggest that the United States spends less than 2% of its GNP on research related to pharmaceuticals.

Taking the estimated range from -.1 to -.2 for price elasticity of demand, and estimating the expenditure share on pharmaceuticals as $\theta_1 = .02$, we apply Theorem 3 to estimate that $1-\sigma$ lies in the range from $\frac{1-.1}{.98} = .92$ and $\frac{1-.2}{.98} = .82$ and σ lies in the range from .08 to .18.

Theorem 2 implies that in Nash equilibrium the United States would be the only contributor to the global public good, pharmaceutical research if where the U.S. is Country 1, if for all Countries $i \neq 1$

$$y_i n_i^{\sigma-1} < (1 - \theta(n_1))\, y_1 n_1^{\sigma-1}. \tag{4}$$

Table 2 shows estimates of $y_i n_i^{\sigma-1}$ for the 15 countries with the world's highest national incomes. We see that given these estimates and an estimated value of $\theta(n_1) = .02$ and values of σ corresponding to $\xi = -.1$ and $\xi = -.2$, the inequality condition shown in 4 is satisfied for all countries other than the U.S. Thus according to Theorem 2, in Nash equilibrium, the United States would be the only provider of subsidies to pharmaceutical research.

Country	GNI (y_i) (trillions)	Pop (n_i) (millions)	$y_i n_i^{\sigma-1}$ if $\sigma = -0.072$	$y_i n_i^{\sigma-1}$ if $\sigma = -0.175$
United States	26.9	334	0.12	0.22
China	18.9	1,412	0.02	0.05
Japan	4.9	125	0.06	0.09
Germany	4.6	84	0.08	0.12
India	3.6	1,417	0.00	0.01
UK	3.3	69	0.06	0.10
France	3.1	69	0.06	0.09
Canada	2.3	43	0.07	0.10
Italy	2.2	58	0.05	0.08
Russia	2.1	147	0.02	0.03
Brazil	2.0	216	0.01	0.02
South Korea	1.8	51	0.05	0.07
Australia	1.7	27	0.08	0.11
Spain	1.6	49	0.04	0.06
Mexico	1.6	134	0.02	0.03

Table 2: Comparing $y_i n_i^{\sigma-1}$ by country

How well does Nash equilibrium explain actual behavior?

Under the assumptions of this paper, if the fruits of pharmaceutical research are not excludable and if contributions to this research are additive, then in Nash equilibrium, the U.S. would be the only provider and all other countries would "free ride".

There is compelling evidence that this extreme result does not occur. Frech et al [4] estimate that the United States contributes about 2/3 of the world's total contributions to pharmaceutical research through payments to pharmaceutical companies. Table 3 shows contributions as measured by Frech and his coauthors for the 15 OECD countries with largest GNP. The table shows that the U.S contribution both per capita and as a fraction of GNP greatly exceeds that of all other countries.

Country	GDP in billion US $	Population in millions	Contributions in billion US $	Contrib/Pop in US $	Contrib/GNP
United States	26,900	334	326.44	998	0.016
Japan	4,980	127.2	40.4	318	0.008
Germany	4.280	83.12	21.96	264	0.005
UK	3,080	67.14	11.23	167	0.004
France	2,970	67.19	14.64	218	0.005
Mexico	2,470	126.19	1.61	13	0.001
Turkey	2,210	82.34	0.00	0	0.000
South Korea	2,100	51.17	3.42	67	0.002
Spain	1,900	46.69	13.03	279	0.007
Canada	1,820	37.07	10.59	286	0.006
Australia	1,280	24.9	4.52	182	0.004
Poland	1,190	37.92	2.34	62	0.002
Netherlands	940	17.06	1.68	98	0.002
Switzerland	610	8.53	3.02	354	0.005
Belgium	580	11.48	3.18	277	0.005

Table 3: Contributions by Country, GDP, and Population

Although other countries contribute less per capita and less as a fraction of GDP than the U.S., the fact that most of them contribute significantly more than zero indicates that the observed outcome is not a Nash equilibrium for provision of a non-excludable public good.

A partial explanation for the positive contributions of these countries is that patented drugs are excludable. A company with a patent has some monopoly power and can expect to extract some rents by setting prices well above marginal cost.

It may be worthwhile to attempt to explain differences among countries other than the U.S. in the amount of contributions that they make. But I will leave that for another day–or another author.

Appendix: Proofs

Proof of Theorem 2

On the way to proving Theorem 2, we introduce a definition and a series of lemmas.

Definition 10. *Let $D(I, n_i)$ be the quantity M that maximizes $U(M, x)$ subject to the constraint that $M + n_i x = I$, and let $\theta(n_i) = \frac{D(I,n_i)}{I}$.*

Lemma 2. *Where utility satisfies Assumption 3,*

$$\frac{\theta(n_i)}{1-\theta(n_i)} = \left(\frac{\alpha}{1-\alpha}\right)^{\sigma} n_i^{\sigma-1} \tag{5}$$

Proof. Setting the marginal rate of substitution equal to the price ratio, we have

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{M}{x_i}\right)^{\rho-1} = \frac{1}{n_i} \tag{6}$$

where $M = D(I, n_i)$. Equation 6 implies that

$$\frac{M}{x_i} = \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\rho-1}} \left(\frac{1}{n_i}\right)^{\frac{1}{\rho-1}} \tag{7}$$

Since $\sigma = \frac{1}{1-\rho} = -\frac{1}{\rho-1}$, Equation 6 implies that

$$\frac{M}{x_i} = \left(\frac{\alpha}{1-\alpha}\right)^{\sigma} n_i^{\sigma}. \tag{8}$$

From the definition of $\theta_i(n)$ and the fact that the budget constraint is $M + n_i x_i = y_i$, it follows that

$$\frac{\theta(n_i)}{1-\theta(n_i)} = \frac{M}{n x_i}. \tag{9}$$

Equations 8 and 9 imply that

$$\frac{\theta(n_i)}{1-\theta(n_i)} = \left(\frac{\alpha}{1-\alpha}\right)^{\sigma} n_i^{\sigma-1}. \tag{10}$$

□

Lemma 3. *In Nash equilibrium, if the total amount of public goods supplied by all other countries is $M_{\sim i}$, then if Country i has national income y_i and population n_i, it will supply $D\left(y_i + M_{\sim i}, n_i\right)) - M_{\sim i}$ if this amount is positive and will supply 0 otherwise.*

Proof. By definition, in Nash equilibrium, Country i supplies M_i^N where (M_i^N, x^N) maximizes $U(M_{\sim i}^N + M_i, x)$ subject to the two constraints that $M_i + nx \leq y_i$ and $M_i \geq 0$. Where $M = M_i + M_{\sim i}^N$, this maximization is equivalent to maximizing $U(M, x)$ subject to the constraints that $M + nx \leq y_i + M_{\sim i}$ and $M \geq M_i$. But this implies that $M^N = D(y_i + M_{\sim i}, n_i)$ if $D(y_i + M_{\sim i}, n_i) > M_{\sim i}$ and $M_i^N = 0$ if $D(y_i + M_{\sim i}, n_i) \leq M_{\sim i}$. □

Lemma 4. *Let Countries 1 and 2 have CES preferences satisfying Assumption 3 and suppose that no other country contributes to the global public good. In Nash equilibrium, Country 1 will be the only contributor if*

$$y_2 n_2^{\sigma-1} < (1 - \theta(n_1))\, y_1 n_1^{\sigma-1} \tag{11}$$

for all $i > 1$.

Proof. Let $M_1 = \theta(n_1)y_1 = D(y_1, n_1)$ be the amount of public good that that Country 1 would supply if no other country supplied any public goods. Lemma 3 implies that Country if Countries 1 and 2 are the only possible suppliers of positive amounts, then Country 2 will supply none of the public good if

$$\theta(n_2)\,(y_2 + M_1) < M_1. \tag{12}$$

Since $M_1 = \theta(n_1)y_1$, this condition is equivalent to

$$\theta(n_2)y_2 < \theta(n_1)y_1(1 - \theta(n_2)), \tag{13}$$

which in turn is equivalent to

$$\begin{aligned} \frac{y_2}{y_1} &< \frac{\theta(n_1)}{\theta(n_2)}(1 - \theta(n_2)) \\ &= (1 - \theta(n_1)) \left(\frac{\theta(n_1)}{1 - \theta(n_1)} \right) \left(\frac{1 - \theta(n_2)}{\theta(n_2)} \right). \end{aligned} \tag{14}$$

From Lemma 2 it follows that Inequality 14 is equivalent to

$$\frac{y_2}{y_1} < (1 - \theta(n_1))\, \frac{n_1^{\sigma-1}}{n_2^{\sigma-1}} \tag{15}$$

which in turn implies that

$$y_2 n_2^{\sigma-1} < (1 - \theta(n_1))\, y_1 n_1^{\sigma-1} \tag{16}$$

□

Proof of Theorem 2

Proof. If Country 1 is the only country to supply the global public good, and if Inequality 2 holds for all countries $i > 1$, then Lemma 4 implies that there is a Nash equilibrium in which Country 1 is the only contributor to the public good.

It has been shown elsewhere, [1, 3] that if the demand for both public and private goods are increasing in income, then Nash equilibrium is unique. Therefore given Assumption 3, the only Nash equilibrium is one in which the only contributor is Country 1. □

Proof of Theorem 3

Theorem 3. *Let $U(x_1, x_2)$ be a constant elasticity of substitution utility function. Let $x_i(p_1, p_2)y$ be the demand function for good i, and let $\theta_1 = \frac{p_1 x_1(p_1,p_2)}{y}$. Where ξ_1 is the own price-elasticity of good 1 and where σ is the elasticity of substitution between goods 1 and 2, it must be that*

$$1 - \sigma = \frac{1 + \xi_1}{1 - \theta_1} \tag{17}$$

Proof. Setting the marginal rate of substitution equal to the price ratio, we have

$$\left(\frac{x_1(p_1, p_2)}{x_2(p_1, p_2)}\right)^{\rho - 1} = \frac{p_1}{p_2} \tag{18}$$

and hence

$$\frac{p_1 x_1(p_1, p_2)}{p_2 x_2(p_1, p_2)} = \left(\frac{p_1}{p_2}\right)^{\frac{\rho}{\rho-1}} \tag{19}$$

Since $p_1 x_1(p_1, p_2) + p_2 x_2(p_1, p_2) = y$, it follows that

$$\frac{p_1 x_1(p_1, p_2)}{p_2 x_2(p_1, p_2)} = \frac{\theta_1}{1 - \theta_1}. \tag{20}$$

Since $\sigma = \frac{1}{1-\rho}$, it follows that $1 - \sigma = \frac{\rho}{\rho - 1}$. Therefore Equations 19 and 20 imply that

$$\frac{\theta_1}{1 - \theta_1} = \left(\frac{p_1}{p_2}\right)^{1-\sigma} \tag{21}$$

It follows from Equation 21 that

$$\ln \theta_1 - \ln(1 - \theta_1) = (1 - \sigma)(\ln p_1 - \ln p_2). \tag{22}$$

Therefore

$$\frac{d \ln \theta_1}{d \ln p_1} - \frac{d \ln(1 - \theta_1)}{d \ln p_1} = 1 - \sigma. \tag{23}$$

Now

$$\begin{aligned} \frac{d \ln(1 - \theta_1)}{d \ln p_1} &= - \frac{p_1 \frac{d\theta_1}{dp_1}}{1 - \theta_1} \\ &= -\frac{\theta_1}{1 - \theta_1} \frac{d \ln \theta_1}{d \ln p_1} \end{aligned} \tag{24}$$

From Equations 23 and 24 it follows that

$$\begin{aligned} 1 - \sigma &= \frac{d \ln \theta_1}{d \ln p_1} \left(1 + \frac{\theta_1}{1 - \theta_1}\right) \\ &= \frac{d \ln \theta_1}{d \ln p_1} \left(\frac{1}{1 - \theta_1}\right). \end{aligned} \tag{25}$$

Where ξ_1 is the own price elasticity of good 1, it must be that

$$\xi_1 = \frac{d \ln x_1(p_1, p_2)}{d \ln p_1} \tag{26}$$

and therefore

$$1 + \xi_1 = \frac{d \ln \theta_1}{d \ln p_1}. \tag{27}$$

From Equations 25 and 27 it follows that

$$1 - \sigma = \frac{1 + \xi_1}{1 - \theta_1}.$$

$\square$

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