Title

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Contents

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ABSTRACT

FORTRAN program KINE is a calling routine for a collection of seven subroutines. Each subroutine calculates interesting kinematical and dynamical quantities for the particle reaction $1+2 \rightarrow 3+4$ by applying special relativity to the process in the form of the Lorentz transformation. The reaction is completely specified if, in addition to the rest masses involved, two of the physical variables are specified. The subroutines perform calculations for the most useful combinations of these two variables.

I. INTRODUCTION

The application of special relativity to particle interactions is in general a straightforward but often tedious affair. This report describes a computer program KINE, that calculates relativistic kinematics and dynamics of particle interactions and allows the physicist a wide latitude in his choice of input variables and an extensive output that should cover most of his normal needs.

Symbolically the interaction for which KINE is constructed can be written as $1+2 \rightarrow 3+4$, where particle 2 is assumed to be at rest in the laboratory frame. The user specifies his problem by stating the masses of the particles in the reaction. Decays are specified by setting the mass of particle 2 equal to zero. For photoproduction the mass of particle 1 is set to zero. In three-dimensional space, each particle may be described by three variables (for example, $P_xP_yP_z$); this makes a total of nine variables for the reaction, since particle 2 is at rest. But in the reaction plane, each particle can be characterized by a pair of variables, so the number of variables is thus reduced to six. We further have three conservation equations, two in momentum and one in energy, and can arbitrarily specify the direction of particle 1; this leaves only two variables that are necessary to specify this system. KINE has seven subroutines that allow the user to specify the most useful combinations of two variables out of two of three pairs of particle variables: momentum or kinetic energy of 1 in the laboratory or in the center-of-mass frame, momentum or kinetic energy of 3 in either frame, and angle of 3 in either frame.

KINE essentially generates tables of kinematic and dynamic variables. Depending on the input, its output consists of the kinetic energies, momenta, and angles of particles 3 and 4 in both frames, as well as their η, β, and $d\Omega_{LAB}/d\Omega_{c.m.}$. In addition, the η, β, and γ of the center of mass; the relative velocity, phase space, and total opening angle of the final-state particles; and the threshold energy and momenta of the incident particle are calculated.

This program has gone through many stages, programmers, and computers over the last six years.[1] Although it reflects the needs and interests of only one high-energy physics group, it is hoped that it now can serve other groups. A program called RELKIN[2] performs some of the calculations of one of KINE's subroutines; however, RELKIN has the advantage that it can make these calculations for a moving-target particle.

II. THEORY

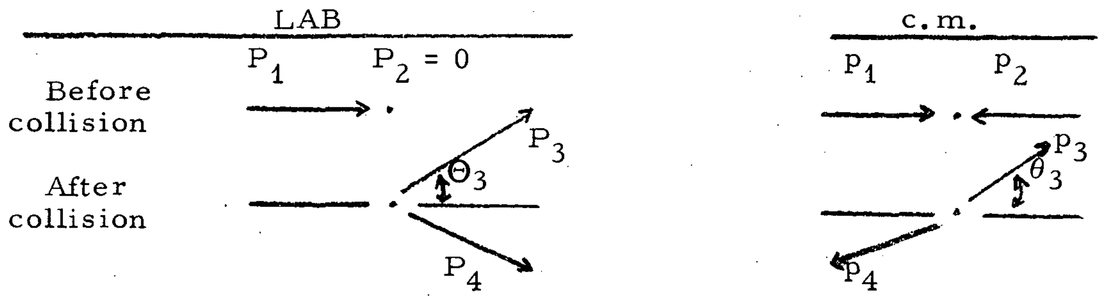
In this report we concern ourselves only with the particle reaction[3] of the form

$$1 + 2 \rightarrow 3 + 4 , \qquad (1)$$

where 1 is called the incident particle, 2 is called the target particle, and 3 and 4 are called the final particles. We look at this reaction in two frames of reference, LAB and c.m. In the LAB frame, 1 is moving and strikes 2, which is at rest. This frame is the one that a person is in when he looks at a bubble chamber or spark chamber photograph. The center-of-mass (c.m.) frame is also uniquely defined; it is the frame in which the momentum of 1 is equal in magnitude but opposite in direction to that of 2.

Now, let us adopt a convenient notation in which mass, energy, and momentum are expressed in terms of energy. Let M be an abbreviation for mass Mc^2, and P be an abbreviation for the momentum Pc. Let capital letters refer to quantities relative to the LAB frame and lower-case letters refer to quantities relative to the c.m. frame. Further, let the numerical subscripts designate the specific particle to which the quantity refers.

In summary, we construct the following diagram:



T	Kinetic energy	t
P	Momentum	p
W	Total energy	w
Θ	Angle	θ
Ω	Solid angle	ω

Since two vectors define a plane, we can think of each momentum vector as having only an x and a y component. By conservation of momentum and energy, we can write in the LAB frame the following three equations:

$$P_x = P_1 = P_3 \cos \Theta_3 + P_4 \cos \Theta_4 , \tag{2}$$

$$P_y = 0 = P_3 \sin \Theta_3 + P_4 \sin \Theta_4 , \tag{3}$$

$$W = W_1 + M_2 = W_3 + W_4 . \tag{4}$$

Two additional equations are appropriate to introduce at this time:

$$W = T + M , \tag{5}$$

$$W^2 = P^2 + M^2 . \tag{6}$$

They can be solved to give two useful relationships between T and P:

$$T = -M + (M^2 + P^2)^{1/2} , \tag{7}$$

$$P = (T^2 + 2MT)^{1/2} . \tag{8}$$

Looking at Eqs. (2) through (4), we see we have three equations and five unknowns: P_1, P_3, P_4, Θ_3, and Θ_4. The knowledge of any two will uniquely determine the other three because we have assumed a knowledge of the masses of all four particles.

We find that most experiments are done in the LAB frame, but that calculations are much simpler and results more significant in the c.m. frame. We therefore need to have a way to convert LAB quantities into c.m. quantities. Such a way is provided in the form of a 4-by-4 tensor called the Lorentz transformation. This matrix transforms a class of four specially related physical quantities (called four vectors) from one frame to another. The particular four vector with which we are concerned is (P_x, P_y, P_z, W), and it transforms in the following manner:

$$\begin{pmatrix} P \cos \Theta \\ P \sin \Theta \\ 0 \\ W \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \eta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} p \cos \theta \\ p \sin \theta \\ 0 \\ w \end{pmatrix} = \begin{pmatrix} \gamma p \cos \theta + \eta w \\ p \sin \theta \\ 0 \\ \eta p \cos \theta + \gamma w \end{pmatrix} . \tag{9}$$

To determine the values of γ and η, consider the system of two particles in the LAB and in the c.m. before collision. Equation (9) becomes

$$\begin{pmatrix} P_1 \\ W \end{pmatrix} = \begin{pmatrix} \gamma & \eta \\ \eta & \gamma \end{pmatrix} \begin{pmatrix} 0 \\ w \end{pmatrix} . \tag{10}$$

Thus,

$$\gamma = \frac{W}{w} \quad \text{and} \quad \eta = \frac{P_1}{w} .$$

Now

$$w^2 = W^2 - P_1^2 , \tag{11}$$

because "lengths" are preserved under Lorentz transformations and explicitly in terms of P_1 and the masses,

$$W = (M_1^2 + P_1^2)^{1/2} + M_2 , \tag{12}$$

so

$$w = [M_1^2 + M_2^2 + 2M_2 (M_1^2 + P_1^2)^{1/2}]^{1/2} . \tag{13}$$

Let us look at the energy equation for a reaction of the form of Eq. (1),

$$M_1 + M_2 = M_3 + M_4 + Q . \tag{14}$$

The term Q represents the amount of energy that must be added to or liberated from the reaction when it occurs. If Q is positive it represents a release of energy and the reaction is called exoergic. If Q is negative then the energy needed to make the reaction "go" must be supplied by the motion of particle 1. The reaction is then called endoergic and has a threshold energy below which M_3 and M_4 can not be produced. Rewriting (14) we see that

$$Q = M_1 + M_2 - M_3 - M_4 \tag{15}$$

and

$$w^{\text{threshold}} = M_3 + M_4 = (M_1^2 + P_1^{\text{threshold}^2})^{1/2} + M_2 , \tag{16}$$

$$P_1^{\text{threshold}} = [(M_3 + M_4 - M_2)^2 - M_1^2]^{1/2} . \tag{17}$$

Two other quantities we will need to know are w_3 and w_4. Recall that

$$w = w_3 + w_4 = (p_3^2 + M_3^2)^{1/2} + (p_4^2 + M_4^2)^{1/2} . \tag{18}$$

But by definition of the c.m. frame,

$$|\overline{p}_3| = |\overline{p}_4| = p_{\text{out}} \quad \text{and} \quad |\overline{p}_1| = |\overline{p}_2| = p_{\text{in}} . \tag{19}$$

So

$$w_3^2 - w_4^2 = M_3^2 - M_4^2 . \tag{20}$$

Combining Eqs. (18), (19), and (20), we see that

$$w_3 = \frac{w^2 + M_3^2 - M_4^2}{2w} ; \quad w_4 = w - w_3 . \tag{21}$$

Note that for particle 2 Eq. (9) becomes

$$\begin{pmatrix} \gamma & -\eta \\ -\eta & \gamma \end{pmatrix} \begin{pmatrix} 0 \\ M_2 \end{pmatrix} = \begin{pmatrix} -p_2 \\ w \end{pmatrix} \Rightarrow p_{in} = \eta M_2 . \qquad (22)$$

We can simply write

$$p_{out} = (w_3^2 - M_3^2)^{1/2} . \qquad (23)$$

Now let's turn to the angular quantities. By applying the law of cosines to Θ_3, we get

$$\cos \Theta_3 = \frac{P_1^2 + P_3^3 - P_4^2}{2 P_1 P_3} . \qquad (24)$$

Suitable use of this with Eq. (9) for the particle of interest will generate all interesting angular quantities.

Next let us evaluate the ratio of the solid angles subtended in the two frames by particle 3:

$$\left(\frac{d\Omega}{d\omega}\right)_3 = \frac{2\pi \sin \Theta_3 \, d\Theta_3}{2\pi \sin \theta_3 \, d\theta_3} . \qquad (25)$$

Looking at the momentum equations that come when we solve Eq. (9) for particle 3, forming

$$\frac{\cos \Theta_3}{\sin \Theta_3} = \cot \Theta_3 = \frac{\gamma p_3 \cos \theta_3 + \eta \omega_3}{p_3 \sin \theta_3} \qquad (26)$$

and differentiating each side with respect to the appropriate angles, and substituting for $d\Theta_3/d\theta_3$ in Eq. (25), we get

$$\left(\frac{d\Omega}{d\omega}\right)_3 = \frac{\sin^3 \Theta_3}{\sin^3 \theta_3} \left\{\gamma + \frac{\eta w_3}{p_3} \cos \theta_3\right\} . \qquad (27)$$

The leading time can be related to momentum,

$$\left(\frac{d\Omega}{d\omega}\right)_3 = \left(\frac{p_{out}}{P_3}\right)^3 \left\{\gamma + \frac{\eta w_3}{p_{out}} \cos \theta_3\right\} . \qquad (28)$$

Finally, the phase space

$$\rho = \frac{p_{out} \, w_3 \, w_4}{w} \qquad (29)$$

and the relative velocities of the initial particles

$$v_{rel} = p_{in}c \left\{ \frac{1}{\left(p_{in}^2 + M_1^2\right)^{1/2}} + \frac{1}{\left(p_{in}^2 + M_2^2\right)^{1/2}} \right\} \qquad (30)$$

have been calculated in order to enable one to have a feeling for how the cross section would behave if the matrix elements were constant.

III. PROGRAM

The calling routine and the seven calculating subroutines that comprise KINE are described in this section.

A. Main Program

This program, whose listing appears in Appendix B, designates the input (IN) and output (IO) tapes employed by the user's particular monitor system. It reads the data cards, writes error comments, and if the input data are correct, decides which subroutine is to be called to perform the calculation. It is here that input variables' limits specified as kinetic energies are converted to momentum.

B. Subroutines

The subroutines with their input variables are listed below:

Routine	α	β	γ	δ	ϵ	ξ	η
Independent	$P_1(T_1)$	$P_1(T_1)$	θ_3	$p_{in}(t_{in})$	$P_1(T_1)$	Θ_3	$P_3(T_3)$
Dependent	θ_3	Θ_3	$P_1(T_1)$	θ_3	$P_3(T_3)$	$P_1(T_1)$	$P_1(T_1)$

Once a value for the dependent variable is established, the independent variable is allowed to be incremented over its assigned range. This is called a pass. Each set of passes over the range of the dependent variable is called a job. Since the construction of all seven subroutines is similar, a listing of ALPHA only is provided in Appendix C. However, Appendix A contains a glossary of all of the mnemonics used in KINE.

C. Input

The user supplies data on cards whose forms appear in Table I.

Table I. Format of data card.

			Word 1	Word 2	Word 3	Word 4	
Card No.	1	2—10	11 — 20	21 — 30	31 — 40	41 — 50	51 — 80
1	1	(a)	M_1	M_2	M_3	M_4	--
2	(b)	(a)	I^{min}	I^{max}	ΔI	(c)	--
3	(b)	(a)	D^{min}	D^{max}	ΔD	(c)	--

(a) User's own identification information.

(b) Assignment number.

(c) A 1.0 in this space indicates that words 1 and 2 contain kinetic-energy data; a blank indicates that they contain momentum data.

The term I represents the independent variable, D is the value of the dependent variable, and Δ is the iteration increment.

Table II shows assignment numbers for each variable.

Table II. Format of data.

Assignment number	Word 1	Word 2	Word 3	Word 4
1	M_1	M_2	M_3	M_4
2	$P_1(T_1)^{min}$	$P_1(T_1)^{max}$	ΔP_1	--(1.0)
3	θ_3^{min}	θ_3^{max}	$\Delta\theta_3$	--
4	Θ_3^{min}	Θ_3^{max}	$\Delta\Theta_3$	--
5	$P_3(T_3)^{min}$	$P_3(T_3)^{max}$	ΔP_3	0.5 to 1.0
6	$p_{in}(t_{in})^{min}$	$p_{in}(t_{in})^{max}$	Δp_{in}	--(1.0)

There are two restrictions on the data. First, the minimum value of a variable cannot be greater than the maximum. Second, a variable cannot be negative. Note that although the user can specify the range of the calculation in terms of either P or T for cards with assignment numbers 2, 5, and 6, the incrementation will always be ΔP.

All data, except the assignment number which is an integer, are decimal (floating-point) numbers.

D. Output

Each subroutine has one of two basic output formats. Their contents are summarized in Tables III and IV. An example of each is displayed in Appendix D.

Rounding errors become significant when a calculated angle is very near 0 or 180 degrees. Considerable effort has been made empirically to make the results physically correct at these extremes. In most cases this has been successful.

Table III. Explanation of variables in outputs

Subroutine	α	β	γ	δ	ϵ	ξ	η
Type	A	A	B	A	A	B	B
$*^1$	–	–	θ_3	–	$P_3^{\min}$	Θ_3	P_3
$*^2$	–	–	Θ_3	–	$P_3^{\max}$	θ_3	θ_3
$*^3$	–	–	P_3	–	–	P_3	–
$*^4$	–	–	T_3	–	–	T_3	Θ_3

Table IV. Format of outputs.

Type	Line	Word														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	Master #1	M_1	M_2	M_3	M_4	-	-	-	-	-	v_{rel}	ρ	Θ_3^{max}	Θ_4^{max}	w	-
	Master #2	P_1	T_1	Q	P_1^{Thres}	T_1^{Thres}	p_{in}	p_{out}	-	$*^1$	-	$*^2$	η	γ	β	-
	Data	Θ_3	P_3	η_3	T_3	β_3	$\left(\frac{d\Omega}{d\omega}\right)_3$	θ_3	Θ_4	P_4	η_4	T_4	β_4	$\left(\frac{d\Omega}{d\omega}\right)_4$	Θ_{34}	-
B	Master	M_1	M_2	M_3	M_4	-	-	-	-	-	$*^1$	-	-	Q	T_1^{Thres}	P_1^{Thres}
	Data	$*^2$	$*^3$	$*^4$	β_4	η_4	Θ_4	P_4	T_4	P_1	w	p_{in}	p_{out}	η	γ	β

*The asterisks with superscript numbers are explained in Table III.

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APPENDICES

A. Glossary of Mnemonics Used in KINE

Most of the mnemonics used in KINE are constructed by appropriate combination of the following symbols:

Physical quantity	Particle	System	Magnitude
AM (mass)	1	L(LAB)	X(maximum)
P (momentum)	2	C(c.m.)	N(minimum)
T (kinetic energy)	3		D(increment)
W (total energy)	4		T(threshold)
A (angle)			I(ith iteration)

For example, the P1CT is the threshold value of the momentum of particle 1 in the c.m. system.

The following is a list of terms that are not covered by the table. If they occur for more than one particle, a representative case is given.

BETA = β
BETA3 = β_3
BRUP = $(1+\gamma^2 \tan^2 \Theta_3)$
COS 3L = $\cos \Theta_3$
DOL3C3 = $(d\Omega/d\omega)_3$
DRUP = $\left\{\left(\gamma^2+\eta^2 \frac{w_3^2}{p_{out}^2}\right)\tan^2 \Theta_3+1\right\}^{1/2}$
ETA = η
FRUP = $-\frac{\gamma\,\eta w_3}{p_{out}}\tan^2 \Theta_3$
GAMMA = γ
IA = Integral value of Q

IN = input-tape designation
IO = output-tape designation
PCIN = p_{in}
PCOUT = p_{out}
PS = phase space
Q = Q value
R = number of degrees in a radian
SAP 4 = $\left\{\frac{1}{\beta^2-\beta_4^2}\right\}^{1/2}$
SIN 3L = $\sin \Theta_3$
SIS = $w_3^2 - M_3^2$

B. LISTING OF MAIN PROGRAM

```
C      MAIN PROGRAM
       COMMON X,AM,P1L,A3C,A3L,P3L,P1C,IO,R
       DIMENSION X(4),AM(4),P1L(3),A3C(3),A3L(3),P3L(3),P1C(3)
       IN=2
       IO=3
       R=57.29578
    10 L=0
     3 READ INPUT TAPE IN,1,N,(X(I),I=1,4)
     1 FORMAT (I1,9X,4F10.0)
       L=L+1
       IF(N-1) 43,43,39
    39 DO 40 I=1,4
       IF(X(I)) 42,40,40
    40 CONTINUE
       IF(X(3)) 42,42,41
    41 IF(X(2)-X(1)) 42,42,43
    42 WRITE OUTPUT TAPE IO,6
     6 FORMAT(34H1ERROR IN DATA. EXECUTION DELETED.)
       GO TO 5
    43 GO TO (11,12,13,14,15,16,4,4,4),N
    11 DO 21 I=1,4
    21 AM(I)=ABSF(X(I))
       GO TO 3
    12 M=1
       IF (X(4)) 100, 32, 100
    32 DO 22  I=1,3
    22 P1L(I)=X(I)
       GO TO 51
    13 DO 23 I=1,3
    23 A3C(I)=X(I)
       GO TO 51
    14 DO 24 I=1,3
    24 A3L(I)=X(I)
       GO TO 51
```

```
 15 M=3
    IF (X(4)) 100,35,100
 35 DO 25  I=1,3
 25 P3L(I)=X(I)
    GO TO 51
 16 M=1
    IF (X(4)) 100,36,100
 36 DO 26  I=1,3
 26 P1C(I)=X(I)
    GO TO 51
  4 WRITE OUTPUT TAPE IO,2
  2 FORMAT(43H1 DATA CARD HAS INCORRECT ASSIGNMENT NUMBER)
  5 READ INPUT TAPE IN,1,N,(X(I),I=1,4)
    IF(N-1) 5,50,5
 50 L=1
    GO TO 11
 51 IF(L-2) 52,53,54
 52 GO TO 3
 53 K=N
    GO TO 3
 54 GO TO (4,55,58,61,62,59),K
 55 GO TO (4,4,56,57,60,4),N
 56 CALL ALPHA
    GO TO 10
 57 CALL BETA
    GO TO 10
 58 CALL GAMMA
    GO TO 10
 59 CALL DELTA
    GO TO 10
 60 CALL EPSIL
    GO TO 10
 61 CALL ZETA
    GO TO 10
 62 CALL ETA
    GO TO 10
100 DO 101 I=1,2
101 X(I) = SQRTF(X(I)**2+2.0*AM(M)*X(I))
102 GO TO (4,32,4,4,35,36),N
    END
```

C. LISTING OF SUBROUTINE ALPHA

```
      SUBROUTINE ALPHA
      COMMON X,AM,P1L,A3C,A3L,P3L,P1C,IO,R
      DIMENSION X(4),AM(4),P1L(3),A3C(3),A3L(3),P3L(3),P1C(3)
      P1LI = P1L(1)
      Q = AM(1)+AM(2)-AM(3)-AM(4)+0.001
      IA=Q
      IF(IA) 100,103,103
  100 W1LT=AM(1)+(((-Q)*(AM(1)+AM(2)+AM(3)+AM(4)))/(2.0*AM(2)))
      P1LT=SQRTF((W1LT+AM(1))*(W1LT-AM(1)))
      T1LT=-AM(1)+SQRTF(AM(1)**2+P1LT**2)
  101 IF(P1LI-P1LT) 102, 104, 104
  102 P1LI=P1L(3)+P1LI
      GO TO 101
  103 P1LT=0.0
      T1LT=0.0
  104 P1L(1)=P1LI
      IF(P1L(2)-P1LI) 10,200,200
   10 WRITE OUTPUT TAPE IO,6
    6 FORMAT(46H MOMENTUM CONSTRAINTS KINEMATICALLY IMPOSSIBLE)
      GO TO 500
C     NEW PASS FOR INDEPENDENT VARIABLE.
  200 A3CI=A3C(1)
      W1L=SQRTF(P1LI**2+AM(1)**2)
      T1L=ABSF(W1L-AM(1))
      W34C=SQRTF(AM(1)**2+AM(2)**2+2.0*AM(2)*W1L)
      W3C=(W34C**2+AM(3)**2-AM(4)**2)/(2.0*W34C)
      W4C=W34C-W3C
      ETA=P1LI/W34C
      GAMMA=(W1L+AM(2))/W34C
      BETA=ETA/GAMMA
      SIS=W3C**2-AM(3)**2
      IF (SIS) 201,201,202
  201 PCOUT=0.0
      BETA3=0.0
      BETA4=0.0
      GO TO 203
  202 PCOUT=SQRTF(SIS)
      BETA4=PCOUT/SQRTF(AM(4)**2+PCOUT**2)
      BETA3=PCOUT/SQRTF(AM(3)**2+PCOUT**2)
  203 PCIN=ABSF(ETA*AM(2))
      IB3=(BETA-BETA3)*10000.0
      IB4=(BETA-BETA4)*10000.0
      VREL=PCIN*(1.0/(SQRTF(AM(1)**2+PCIN**2))+1.0/(SQRTF(AM(2)**2
     1+PCIN**2))) *2.997993E10
      PS=PCOUT*W3C*W4C/W34C
      IF(IB3)204,206,205
  204 A3L(2)=180.0
      GO TO 207
  205 SAP3 =SQRTF(1.0/((BETA+BETA3)*(BETA-BETA3)))
      A3L(2)=ATANF(BETA3*SAP3/GAMMA)*R
      GO TO 207
  206 A3L(2)=90.0
  207 IF(IB4)208,210,209
  208 A4LX=180.0
      GO TO 11
```

```
  209 SAP4=SQRTF(1.0/((BETA+BETA4)*(BETA-BETA4)))
      A4LX= ATANF(BETA4*SAP4/GAMMA)*R
      GO TO 11
  210 A4LX=90.0
   11 WRITE OUTPUT TAPE IO,1
    1 FORMAT(120H1SUBROUTINE ALPHA      INDEPENDENT VARIABLE=MOMENTUM 1
     1(LAB)         DEPENDENT VARIABLE=ANGLE 3 (CM)                    )
      WRITE OUTPUT TAPE IO,2,(AM(I),I=1,4),VREL,PS,A3L(2),A4LX,W34C
    2 FORMAT(132H-  MASS       MASS       MASS       MASS
     1                     RELATIVE       PHASE     MAX ANGLE    MAX ANG
     2LE    TOTAL /
     3       132H     1          2          3          4
     4                     VELOCITY       SPACE         3            4
     5     ENERGY /
     6       132H   (MEV)      (MEV)      (MEV)      (MEV)
     7                     (CM/SEC)     (MEV-SQ.)     (LAB)        (LAB)
     8      (CM)  /
     91X,4(F7.2,4X),30X,1PE9.3,5X,1PE9.3,4X,0PF6.2,7X,F6.2,3X,F8.2)
      WRITE OUTPUT TAPE IO, 3, ETA, GAMMA, BETA, P1LI, TIL, Q, P1LT, TIL
     1T, PCIN, PCOUT
    3 FORMAT(132H- MOMENTUM   KINETIC     Q     THRESHOLD   THRESHOLD
     1MOMENTUM   MOMENTUM                                 ETA      GAMMA
     2      BETA  /
     3       132H     1       ENERGY1   VALUE   MOMENTUM1    KINETIC
     4   IN         OUT                                   (CM)      (CM)
     5      (CM)  /
     6        79H   (LAB)      (LAB)    (LAB)     (LAB)      ENERGY1
     7  (CM)        (CM)      ,24X,F9.4,2X,F8.4,2X,F8.6/
     81X,F8.2,3X,F8.2,1X,F7.2,3X,F8.2,4X,F8.2,4X,F8.2,3X,F8.2////)
      WRITE OUTPUT TAPE IO, 4
    4 FORMAT(132H  ANGLE  MOMENTUM  ETA    KINETIC   BETA   D OMEGA (LAB
     1)   ANGLE    ANGLE  MOMENTUM  ETA    KINETIC   BETA   D OMEGA (LAB
     2)     TOTAL /
     3       132H    3       3       3     ENERGY3     3    ------------
     4-     3        4       4       4     ENERGY4     4    ------------
     5-     ANGLE /
     6       132H  (LAB)   (LAB)   (LAB)    (LAB)    (LAB)  D OMEGA (CM)
     73   (CM)     (LAB)   (LAB)   (LAB)    (LAB)    (LAB)  D OMEGA (CM)
     84     (LAB) /         )
C     NEW PASS FOR DEPENDENT VARIABLE
  300 COS3C=COSF(A3CI/R)
      W3L=ETA*PCOUT*COS3C+GAMMA*W3C
      W4L=-ETA*PCOUT*COS3C+GAMMA*W4C
      IF(W3L-AM(3)) 301,301,302
  301 P3LI=0.0
      T3L=0.0
      GO TO 303
  302 P3LI=SQRTF(W3L**2-AM(3)**2)
      T3L=W3L-AM(3)
  303 IF(W4L-AM(4)) 304,304,305
  304 P4L=0.0
      T4L=0.0
      GO TO 307
  305 P4L=SQRTF(W4L**2-AM(4)**2)
      T4L=W4L-AM(4)
      IF(P1LI)500,3307,307
 3307 A3LI=A3CI
      A4L=180.0-A3CI
      GO TO 320
```

```
  307 COS3L=(P1LI**2+ P3LI**2-P4L**2) /(2.0*P1LI*P3LI)
      IF (ABSF(COS3L)-0.999995)311,308,308
  308 IF(COS3L)309, 310,310
  309 A3LI=180.0
      GO TO 313
  310 A3LI=0.0
      GO TO 313
  311 SIN3L=SQRTF(1.0-COS3L**2)
      TAN3L=SIN3L/COS3L
      A3LI=ATANF(TAN3L)*R
      IA3LI=A3LI*10.0
      IF(IA3LI)312,3312,313
 3312 A3LI=90.0
      GO TO 313
  312 A3LI=A3LI+180.0
  313 COS4L=(P1LI**2-P3LI**2+P4L**2)/(2.0*P1LI*P4L)
      IF(ABSF(COS4L)-0.999995)317,314,314
  314 IF(COS4L)315, 316,316
  315 A4L=180.0
      GO TO 320
  316 A4L=0.0
      GO TO 320
  317 SIN4L=SQRTF(1.0-COS4L**2)
      TAN4L=SIN 4L/COS 4L
      A4L=ATANF(TAN4L)*R
      IA4L=A4L*10.0
      IF(IA4L)318,319,320
  318 A4L=180.0+A4L
      GO TO 320
  319 A4L=90.0
  320 DOL3C3=(PCOUT/P3LI)**3*(GAMMA+(ETA*W3C*COS3C/PCOUT))
      IF(P3LI)321,321,322
  321 DOL3C3=0.0
  322 DOL4C4=(PCOUT/P4L)**3*(GAMMA-(ETA*W4C*COS3C/PCOUT))
      IF(P4L)323,323,324
  323 DOL4C4=0.0
  324 ETA3L=P3LI/AM(3)
      BETA3L=P3LI/SQRTF(AM(3)**2+P3LI**2)
      ETA4L=P4L/AM(4)
      BETA4L=P4L/SQRTF(AM(4)**2+P4L**2)
      A34L=A3LI+A4L
      IF(A34L-180.0)326,325,325
  325 A34L=180.0
  326 WRITE OUTPUT TAPE IO,5,A3LI,P3LI,ETA3L,T3L,BETA3L,DOL3C3,A3CI,A4L,
     1P4L,ETA4L,T4L,BETA4L,DOL4C4 ,A34L
    5 FORMAT(1X,F6.2,1X,F8.2,1X,F6.2,1X,F9.2,1X,F8.6,   F12.5,5X,F6.
     12,1X,F6.2,1X,F8.2,1X,F6.2,1X,F9.2,1X,F9.7,   F12.5,6X,F6.2)
C     INCRIMENT AND TEST VARIABLES.
      A3CI=A3CI+A3C(3)
      IF(A3C(2)-A3CI)400,300,300
  400 P1LI=P1LI+P1L(3)
      IF(P1LI-P1L(2))200,200,500
  500 RETURN
```

D. SAMPLE OUTPUTS

TYPE A FORMAT

```
SUBROUTINE GAMMA      INDEPENDENT VARIABLE = ANGLE 3 (CM)          DEPENDENT VARIABLE = MOMENTUM 1 (LAB)

  MASS       MASS       MASS       MASS                                   ANGLE                                                 Q     THRESHOLD   THRESHOLD
   1          2          3          4                                       3                                                 VALUE    KINETIC    MOMENTUM1
 (MEV)      (MEV)      (MEV)      (MEV)                                    (CM)                                               (MEV)    ENERGY1      (LAB)
139.59     938.21     497.80    1115.36                                  165.00                                             -535.36     767.75      896.54

 ANGLE MOMENTUM  KINETIC  BETA      ETA    ANGLE  MOMENTUM  KINETIC  MOMENTUM    TOTAL  MOMENTUM MOMENTUM    ETA      GAMMA     BETA
   3      3      ENERGY3    3        3       4        4     ENERGY4      1       ENERGY     IN      OUT      (CM)      (CM)     (CM)
 (LAB)  (LAB)     (LAB)                    (LAB)    (LAB)    (LAB)     (LAB)     (CM)     (CM)     (CM)
  2.31  237.41     53.72 0.43048   0.477   0.83    662.85   182.10    900.00  1615.15   522.80    37.02  0.55722   1.1448 0.486755
  9.15  173.08     29.23 0.32841   0.348   2.09    754.62   231.29    925.00  1629.44   532.60   106.31  0.56768   1.1499 0.493579
 15.41  142.13     19.89 0.27455   0.266   2.66    813.86   265.36    950.00  1643.62   542.28   145.93  0.57799   1.1550 0.500418
 22.19  121.33     14.57 0.23680   0.244   3.04    863.87   295.42    975.00  1657.68   551.83   177.06  0.58817   1.1601 0.506979
 29.66  106.50     11.27 0.20922   0.214   3.32    908.98   323.48   1000.00  1671.63   561.26   203.62  0.59822   1.1653 0.513371
 37.77   96.02      9.18 0.18940   0.193   3.55    950.92   350.34   1025.00  1685.47   570.56   227.22  0.60814   1.1704 0.519599
 46.31   89.01      7.90 0.17602   0.179   3.73    990.61   376.40   1050.00  1699.21   579.75   248.69  0.61793   1.1755 0.525670
 54.95   84.89      7.19 0.16811   0.171   3.87   1028.60   401.89   1075.00  1712.84   588.83   268.53  0.62761   1.1806 0.531589
 63.32   83.16      6.90 0.16477   0.167   4.00   1065.26   426.98   1100.00  1726.37   597.81   287.09  0.63718   1.1857 0.537363
 71.09   83.33      6.93 0.16510   0.167   4.11   1100.83   451.75   1125.00  1739.80   606.67   304.59  0.64663   1.1909 0.542996
 78.07   84.97      7.20 0.16826   0.171   4.20   1135.49   476.29   1150.00  1753.13   615.44   321.21  0.65597   1.1960 0.548493
 84.19   87.69      7.66 0.17348   0.176   4.28   1169.39   500.65   1175.00  1766.36   624.11   337.07  0.66521   1.2010 0.553860
 89.47   91.18      8.28 0.18016   0.183   4.35   1202.62   524.86   1200.00  1779.50   632.68   352.27  0.67435   1.2061 0.559101
 94.00   95.19      9.02 0.18782   0.191   4.41   1235.29   548.96   1225.00  1792.55   641.16   366.89  0.68338   1.2112 0.564220
 97.86   99.54      9.86 0.19608   0.200   4.46   1267.45   572.97   1250.00  1805.50   649.55   380.99  0.69233   1.2163 0.569221
101.15  104.11     10.77 0.20470   0.209   4.51   1299.16   596.90   1275.00  1813.37   657.85   394.64  0.70118   1.2213 0.574109
103.98  108.78     11.75 0.21349   0.219   4.55   1330.47   620.78   1300.00  1831.15   666.07   407.87  0.70994   1.2264 0.578888
106.40  113.51     12.78 0.22232   0.228   4.59   1361.42   644.61   1325.00  1843.84   674.21   420.72  0.71861   1.2314 0.583560
108.50  118.24     13.85 0.23109   0.238   4.62   1392.04   668.40   1350.00  1856.45   682.25   433.22  0.72719   1.2365 0.588130
110.32  122.93     14.95 0.23974   0.247   4.65   1422.37   692.17   1375.00  1868.98   690.24   445.40  0.73570   1.2415 0.592601
111.90  127.56     16.08 0.24824   0.256   4.67   1452.42   715.91   1400.00  1881.42   698.14   457.29  0.74412   1.2465 0.596975
113.29  132.13     17.24 0.25654   0.265   4.70   1482.22   739.64   1425.00  1893.79   705.97   468.91  0.75246   1.2515 0.601257
114.51  136.61     18.41 0.26465   0.274   4.72   1511.80   763.35   1450.00  1906.08   713.72   480.27  0.76073   1.2565 0.605449
115.59  141.01     19.59 0.27254   0.283   4.73   1541.16   787.06   1475.00  1918.29   721.41   491.39  0.76892   1.2614 0.609554
116.54  145.31     20.78 0.28022   0.292   4.75   1570.32   810.76   1500.00  1930.42   729.02   502.29  0.77703   1.2664 0.613574
```

TYPE B FORMAT

```
SUBROUTINE ALPHA      INDEPENDENT VARIABLE=MOMENTUM 1 (LAB)        DEPENDENT VARIABLE=ANGLE 3 (CM)


  MASS       MASS       MASS       MASS                                          RELATIVE      PHASE     MAX ANGLE    MAX ANGLE     TOTAL
   1          2          3          4                                            VELOCITY      SPACE         3            4         ENERGY
 (MEV)      (MEV)      (MEV)      (MEV)                                         (CM/SEC)    (MEV-SQ.)      (LAB)        (LAB)       (CM)
139.59     938.26     497.80    1115.36                                         4.670E 10   1.703E 05    180.00        31.01     1831.20


MOMENTUM  KINETIC     Q     THRESHOLD  THRESHOLD  MOMENTUM  MOMENTUM                                      ETA       GAMMA      BETA
   1      ENERGY1   VALUE   MOMENTUM1   KINETIC      IN       OUT                                         (CM)       (CM)      (CM)
 (LAB)     (LAB)    (LAB)     (LAB)     ENERGY1     (CM)      (CM)                                       0.7099     1.2264   0.578877
1300.00   1167.88 -535.31    896.45      767.66    666.08    407.92


 ANGLE  MOMENTUM   ETA     KINETIC    BETA   D OMEGA (LAB)     ANGLE   ANGLE MOMENTUM  ETA     KINETIC    BETA    D OMEGA (LAB)     TOTAL
   3       3        3      ENERGY3     3     -------------       3       4       4       4     ENERGY4      4     -------------     ANGLE
 (LAB)   (LAB)    (LAB)     (LAB)    (LAB)   D OMEGA (CM)3     (CM)    (LAB)   (LAB)   (LAB)    (LAB)     (LAB)   D OMEGA (CM)4     (LAB)
   0.      957.15   1.92      581.07 0.887187       0.18163        0.     0.     342.85   0.31       51.50 0.2938178     -1.41565          0.
   2.13    955.91   1.92      579.96 0.886941       0.18201        5.00   5.89   346.58   0.31       52.61 0.2967358     -1.35758          8.02
   4.27    952.19   1.91      576.67 0.886201       0.18315       10.00  11.43   357.53   0.32       55.90 0.3052538     -1.20162         15.69
   6.41    946.02   1.90      571.20 0.884959       0.18506       15.00  16.35   375.06   0.34       61.37 0.3187285     -0.99072         22.76
   8.56    937.43   1.88      563.60 0.883197       0.18778       20.00  20.51   398.25   0.36       68.97 0.3362687     -0.76923         29.07
  10.72    926.46   1.86      553.93 0.880894       0.19133       25.00  23.86   426.14   0.38       78.64 0.3569055     -0.56734         34.59
  12.91    913.20   1.83      542.27 0.878021       0.19576       30.00  26.46   457.81   0.41       90.30 0.3797181     -0.39867         39.36
  15.11    897.71   1.80      528.69 0.874541       0.20115       35.00  28.37   492.45   0.44      103.88 0.4039007     -0.26526         43.47
  17.33    880.08   1.77      513.31 0.870410       0.20756       40.00  29.69   529.38   0.47      119.25 0.4287835     -0.16330         47.02
  19.59    860.42   1.73      496.25 0.865574       0.21508       45.00  30.52   568.05   0.51      136.32 0.4538295     -0.08706         50.10
  21.87    838.83   1.69      477.62 0.859970       0.22383       50.00  30.93   607.99   0.55      154.95 0.4786188     -0.03086         52.80
  24.19    815.44   1.64      457.58 0.853526       0.23395       55.00  31.00   648.83   0.58      174.99 0.5028313      0.01016         55.19
  26.55    790.37   1.59      436.27 0.846155       0.24559       60.00  30.78   690.23   0.62      196.30 0.5262284      0.03983         57.33
  28.95    763.75   1.53      413.86 0.837762       0.25897       65.00  30.34   731.92   0.66      218.71 0.5486386      0.06109         59.29
  31.40    735.74   1.48      390.52 0.828234       0.27431       70.00  29.70   773.65   0.69      242.05 0.5699437      0.07615         61.10
  33.90    706.46   1.42      366.43 0.817445       0.29191       75.00  28.90   815.18   0.73      266.14 0.5900686      0.08664         62.80
  36.46    676.06   1.36      341.76 0.805255       0.31212       80.00  27.98   856.32   0.77      290.81 0.6089718      0.09377         64.43
  39.07    644.69   1.30      316.71 0.791506       0.33539       85.00  26.94   896.85   0.80      315.85 0.6266381      0.09844         66.02
  41.76    612.50   1.23      291.48 0.776023       0.36228       90.00  25.82   936.60   0.84      341.09 0.6430723      0.10132         67.58
  44.52    579.61   1.16      266.24 0.758614       0.39348       95.00  24.62   975.39   0.87      366.33 0.6582944      0.10288         69.14
  47.35    546.17   1.10      241.19 0.739075       0.42991      100.00  23.36  1013.03   0.91      391.38 0.6723350      0.10350         70.72
  50.27    512.30   1.03      216.52 0.717185       0.47277      105.00  22.05  1049.37   0.94      416.04 0.6852321      0.10346         72.33
  53.29    478.13   0.96      192.43 0.692717       0.52367      110.00  20.70  1084.23   0.97      440.14 0.6970287      0.10296         74.00
  56.42    443.78   0.89      169.09 0.665441       0.58485      115.00  19.32  1117.46   1.00      463.48 0.7077700      0.10215         75.74
  59.66    409.33   0.82      146.68 0.635130       0.65949      120.00  17.91  1148.90   1.03      485.89 0.7175019      0.10115         77.57
  63.04    374.89   0.75      125.37 0.601577       0.75230      125.00  16.47  1178.41   1.06      507.20 0.7262700      0.10004         79.51
  66.58    340.53   0.68      105.33 0.564607       0.87048      130.00  15.02  1205.86   1.08      527.24 0.7341181      0.09890         81.60
  70.32    306.33   0.62       86.70 0.524092       1.02566      135.00  13.55  1231.11   1.10      545.86 0.7410876      0.09778         83.87
  74.31    272.36   0.55       69.64 0.479982       1.23758      140.00  12.07  1254.05   1.12      562.93 0.7472168      0.09670         86.38
  78.62    238.67   0.48       54.26 0.432325       1.54216      145.00  10.58  1274.56   1.14      578.31 0.7525408      0.09571         89.20
  83.38    205.33   0.41       40.68 0.381307       2.01031      150.00   9.08  1292.54   1.16      591.89 0.7570908      0.09481         92.46
  88.84    172.43   0.35       29.02 0.327306       2.79703      155.00   7.57  1307.91   1.17      603.55 0.7608942      0.09404         96.41
  95.40    140.14   0.28       19.35 0.270986       4.28794      160.00   6.06  1320.59   1.18      613.22 0.7639742      0.09339        101.47
 104.00    108.81   0.22       11.75 0.213539       7.61267      165.00   4.55  1330.52   1.19      620.82 0.7663499      0.09287        108.55
 116.79     79.35   0.16        6.29 0.157421      16.75366      170.00   3.04  1337.65   1.20      626.28 0.7680361      0.09250        119.83
 139.39     54.62   0.11        2.99 0.109068      46.06220      175.00   1.52  1341.94   1.20      629.58 0.7690436      0.09228        140.91
 180.00     43.37   0.09        1.89 0.086792      88.47095      180.00   0.    1343.37   1.20      630.68 0.7693788      0.09221        180.00
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FOOTNOTES AND REFERENCES

*Present address: Physics Department, Virginia Polytechnic Institute, Blacksburg, Virginia.

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2. John H. Poirier (Lawrence Radiation Laboratory), RELKIN (unpublished data), 1962.
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