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BACKGROUND ACTIVITIES, INDUCTION, AND BEHAVIORAL ALLOCATION IN OPERANT PERFORMANCE

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In experiments on operant behavior, other activities, called “background” activities, compete with the operant activities. Herrnstein’s (1970) formulation of the matching law included background reinforcers in the form of a parameter r_O , but remained vague about the activities (B_O) that produce r_O . To gain more understanding, we analyzed data from three studies of performance with pairs of variable-interval schedules that changed frequently in the relative rate at which they produced food: Baum and Davison (2014), Belke and Heyman (1994), and Soto, McDowell, and Dallery (2005). Results sometimes deviated from the matching law, suggesting variation in r_O . When r_O was calculated from the matching equation, two results emerged: (a) r_O is directly proportional to B_O , as in a ratio schedule; and (b) r_O and B_O depend on the food rate, which is to say that B_O consists of activities induced by food, as a phylogenetically important event. Other activities unrelated to food (B_N) correspond to Herrnstein’s original conception of r_O and may be included in the matching equation. A model based on Baum’s (Baum, 2012) concepts of *allocation*, *induction*, and *contingency* explained the deviations from the matching law. In the model, operant activity B , B_O , and B_N competed unequally in the time allocation: B and B_O both replaced B_N , B_O replaced lever pressing (Soto et al.), and key pecking replaced B_O (Baum & Davison). Although the dependence of r_O and B_O on food rate changes Herrnstein’s (1970) formulation, the model preserved the generalized matching law for operant activities by incorporating power-function induction.

Key words: matching law, behavioral allocation, induction, phylogenetically important event, induced activity, background activity, extraneous reinforcers

In any study of operant behavior, other activities besides the measured operant activities occur. These other activities are often called “background” activities, but they are not neutral, because they may compete with the operant activities. Researchers have occasionally tried to identify the background activities and the way that they compete (e.g., Baum, 1981; Davison, 1993, 2004; Herrnstein, 1970; Soto, McDowell, & Dallery, 2005), but no definite answers have arrived. In this paper, we extend the inquiry into the nature of background activities and their role in determining operant performance.

Herrnstein (1961) originally expressed the matching law as a relation of proportions:

$$\frac{B_1}{B_1 + B_2} = \frac{r_1}{r_1 + r_2}, \quad (1)$$

where B_1 and B_2 are rates of responding at Alternatives 1 and 2, and r_1 and r_2 are the corresponding reinforcer rates obtained from Alternatives 1 and 2.

Later, Herrnstein (1970; see also Baum & Rachlin, 1969) generalized the matching law to n alternatives:

$$\frac{B_i}{\sum B} = \frac{r_i}{\sum r}, \quad (2)$$

where the numerators left and right indicate response rate at any one alternative and reinforcer rate obtained from that alternative, and the denominators left and right represent the sum of all response rates and the sum of all reinforcer rates.

Herrnstein used Equation 2 to derive an equation for describing performance on single schedules by assuming the sum of all behavior to be a constant k and the sum of all reinforcer rates other than that programmed by the schedule to equal r_O :

$$B = k \frac{r}{r + r_O}. \quad (3)$$

For consistency, we are using the notation r_O here, even though the same parameter has sometimes been denoted other ways, such as r_e or r_b .

Subsequent research showed that Equations 1 and 2 were oversimplified, because deviations

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from matching in the form of overmatching, undermatching, and bias occur (Baum, 1974). Thus, Equations 1 and 2 are generally presented now as including power functions that capture bias in the coefficient and sensitivity in the exponent. McDowell (2005) suggested rewriting Equation 3 in a similar way, adding exponents to r and r_O . In the present paper, we opt for the theoretical coherence and simplicity of Equation 3, rather than introduce extra, possibly unnecessary, fitting parameters that lack a basis in theory.

Following the logic of Equation 3, Herrnstein derived an equation to describe responding on any one of two concurrent schedules by assuming r_O to equal the sum of all reinforcer rates other than those programmed by the two schedules, expressing the sum of all reinforcer rates as the sum of r_1 , r_2 , and r_O :

$$B_1 = k \frac{r_1}{r_1 + r_2 + r_O}. \quad (4)$$

Belke and Heyman (1994) tested Equations 3 and 4 with an experiment that included both single Variable-Interval (VI) schedules and concurrent VI schedules. Seven single schedules were presented, within each session, separated by brief blackouts, and those same seven were also presented with a constant VI schedule as a concurrent alternative. In the concurrent phase, the seven schedules were all presented unsignaled and in random order within each session along with the constant VI, with the result that each session included seven different food ratios. Their results supported Equations 3 and 4.

We (Baum & Davison, 2014) studied five different conditions in which pairs of concurrent VI schedules were arranged within sessions similarly to Belke and Heyman's (1994) concurrent schedules. In each condition a different VI schedule was held constant on the left key while the same seven different VI schedules were presented in seven different components, separated by blackouts, within each session. Each component lasted until the 10th food delivery. Each condition lasted 100 sessions, and we analyzed the last 85 sessions. To reduce the number of figures, we pooled the data of the six pigeons in the study and showed that the pooled data were representative of the individual pigeons' data [see Baum & Davison (2014) for more details]. We turn now to those pooled data.

Herrnstein's Response-Rate Hyperbola

To analyze the response rates when they were roughly asymptotic within components, we used the results from the last three food deliveries only, and all figures in this section show only data from the last three food deliveries. To summarize, we added up all the times between the seventh and eighth food deliveries, between the eighth and ninth food deliveries, and between the ninth and tenth food deliveries, and these sums constituted the time bases. We excluded the time taken to switch keys, because, in contrast with Belke and Heyman (1994), who used a changeover delay on switching, we required two pecks on a changeover key to switch alternatives. We summed all the pecks between the seventh and eighth deliveries, between the eighth and ninth, and between the ninth and tenth. Dividing the three sums of pecks by the three time bases gave three response rates. We counted all the seventh, eighth, and ninth food deliveries and divided by the time bases to compute the three food rates.

When Equation 3 is fitted to response rates, both k and r_O are usually assumed to be constant (e.g., Belke & Heyman, 1994; Herrnstein, 1970; Soto et al., 2005). The assumption that r_O is constant, however, seems incorrect when viewed in the light of theoretical treatments of other reinforcer sources. By definition, reinforcers are contingent on behavior. Just as r depends on B , so we expect r_O to depend on B_O . The dependence of reinforcers on the activities that produce them is specified by a feedback function (Baum, 1973). Every schedule is characterized by a feedback function (Baum, 1973, 1992). For example, the feedback function for a ratio schedule is direct proportionality between reinforcer rate and response rate (Baum, 1973). The feedback function for a VI schedule is curvilinear, because it approaches the programmed reinforcer rate as an asymptote as response rate increases (Baum, 1992). By extension, if r_O is the result of activities, B_O , we would expect r_O to vary with B_O (e.g., Davison, 1993). Do we have any evidence that bears on the question of whether r_O is constant or variable?

Total Pecking on Two Alternatives

We can simplify the inquiry initially by considering total responding, including both

alternatives, in relation to total food delivered, including both alternatives. The relevant equation is Equation 3. Figure 1 shows pecks per minute (ppm) plotted against food deliveries per minute (fpm) in logarithmic coordinates. The line in each graph is the best-fitting hyperbola specified in Equation 3, fitted in arithmetic coordinates using Microsoft Excel's Solver. The least-squares parameter estimates are shown in each graph. The asymptote k showed no trend across conditions, but regression showed some tendency for r_0 to increase with food rate (Baum & Davison, 2014; Fig. 3).

In arithmetic coordinates, the fits to the hyperbola looked satisfactory (VAC or $r^2 > .8$), but when viewed in these logarithmic coordinates, systematic deviations from the hyperbola become evident, because they are proportionate, rather than absolute, making the deviations at lower food rates particularly evident. In every graph, the response rates for the low food rates lay above the curve, the response rates for the middle food rates fell below the curve, and the response rates for the higher food rates lay above the curve. The systematic deviations suggest some sort of systematic variation, and

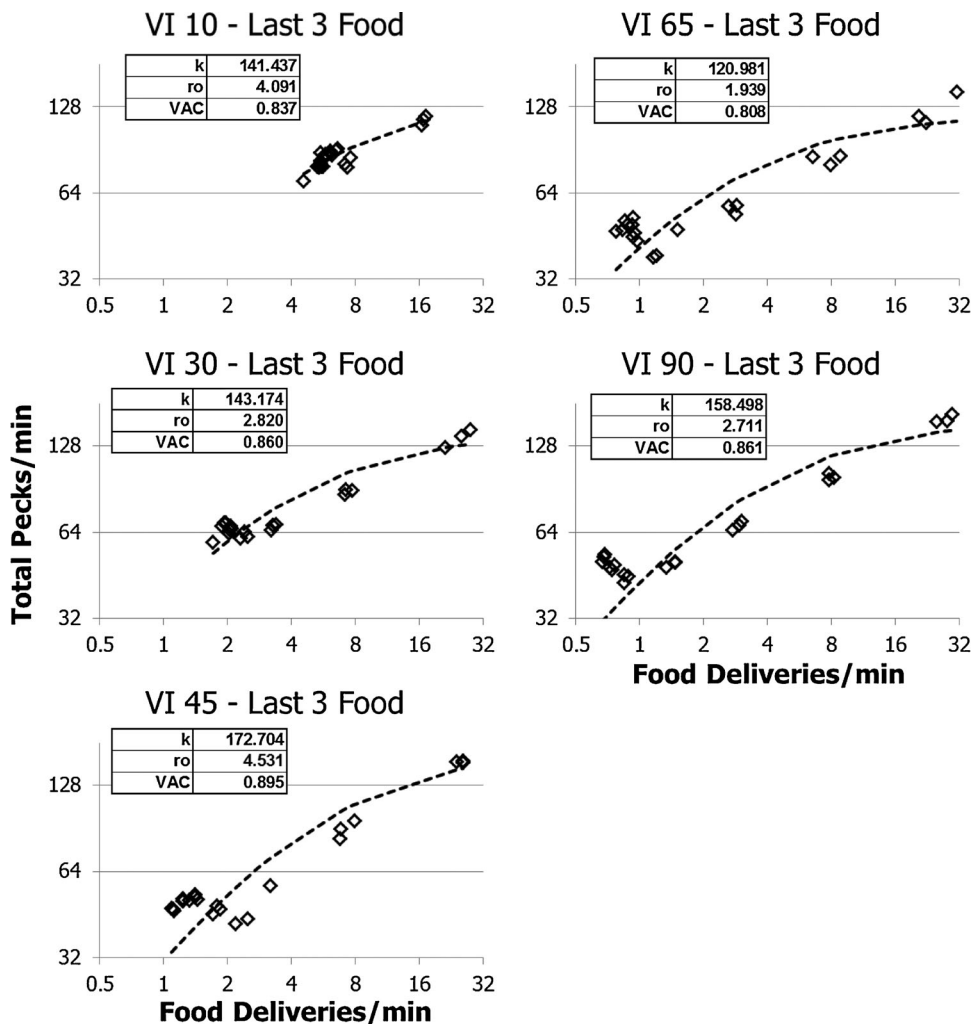


Fig. 1. Peck rate versus food rate for five conditions of Baum and Davison's (2014) study of frequently changing concurrent VI VI schedules. All points represent total pecking on two keys in the last three interfood intervals of components. Conditions differed in the VI schedule programmed on the left key, which was constant while the VI on the right key changed. Broken lines represent the best fit of Herrnstein's (1970) hyperbola in arithmetic coordinates. Note logarithmic coordinates.

since a hyperbola has only two parameters, one of them must vary systematically with food rate. Since k specifies asymptotic response rate and shouldn't change under the present conditions, the most theoretically consistent interpretation is that r_O changes. Other possibilities exist, such as McDowell's (2005) proposal to add exponents. Adding an exponent as a free parameter would help fit the data in Figure 1, but McDowell's equation still assumes constant r_O . Since the sum $B + B_O$ equals k , if B increases, B_O must decrease (and vice versa). If r_O represents reinforcers produced by B_O , then r_O must vary with B_O . Adding an exponent fails to solve this theoretical problem with Equation 3. We chose the present approach because adding free parameters with no specific meaning seems less useful than pursuing consistent theory.

Possibly r_O varies with r ; possibly r_O varies with B_O , the behavior associated with it (Staddon, 1977; e.g., magazine approach, non-key pecking, grooming, stretching, etc.). If so, the question arises, as posed by Davison (1993), "What is the feedback function for r_O ?"

To answer that question, we calculated r_O and B_O for each specific pair of response rate and food rate. Since k represents the totality of behavior measured as peck rate, we will assume it to be constant, so we may take B_O to equal $k - B$, all the behavior apart from pecking at the response keys. Then r_O may be calculated by

rearranging Equation 3, as follows:

$$r_O = \frac{r(k - B)}{B} \tag{5}$$

The left-hand graph of Figure 2 shows r_O calculated this way and plotted against r , the total food rate. Up to about 15 fpm, the points from the five conditions could all be described by a single increasing function. Above 15 fpm, r_O falls precipitously. A gap appears between 9 and 16 fpm. If we take only the estimates of r_O above 15 fpm and plot them against B_O (estimated as $k - B$), we get the result shown in the right-hand graph in Figure 2. At these high food rates, r_O was approximately directly proportional to B_O , as in a ratio schedule (Baum, 1973, 1981, 2012a; Davison, 1993). The slope of a regression line fitted to the points equals 0.16 [about equivalent to variable-ratio (VR) 6]. Equation 5 implies that across these high food rates, the ratio r/B remains approximately constant— r and B increase together in proportion.

The lines in the left graph of Figure 2 show the results of a model that accommodates the results. The model was constructed according to the principles laid out by Baum (2012a): *allocation*, *induction*, and *contingency*. The contingency between food and key pecking makes key pecking a food-related activity, with the result that food induces key pecking along with other

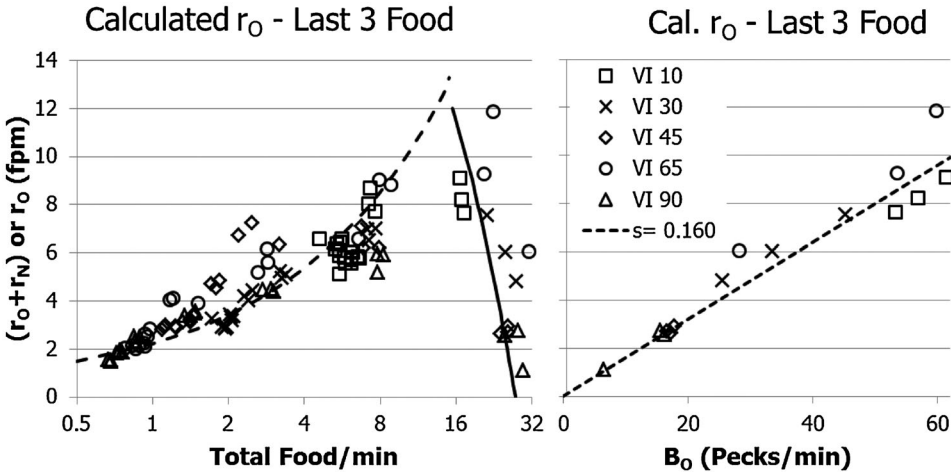


Fig. 2. Calculated r_O from Equation 5 across the five conditions of Baum and Davison's (2014) study. Left: The broken curve shows the fit of the model based on Baum's (2012a) induction theory up to the food rate at which $B + B_O$ equals k and B_N and r_N equal zero. The solid line shows the model's prediction of decline in r_O at still higher food rates. Right: Calculated r_O plotted against B_O calculated as $k - B$ including only the points in the left graph above 15 fpm. The broken line shows direct proportionality with a slope of 0.16.

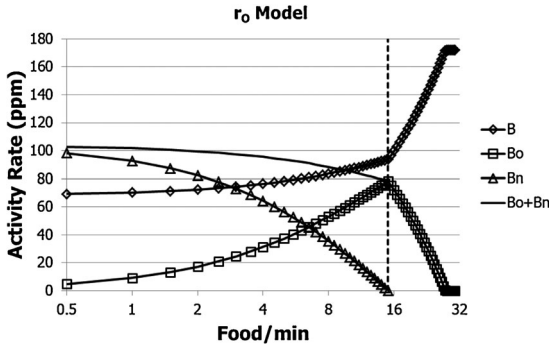


Fig. 3. Graphical representation of the induction-based model applied to Baum and Davison's (2014) results. Key pecking (B) and other food-induced activities (B_O) increase up to 16 fpm, driving out non-food-related activities (B_N). When B_N equals zero, $B + B_O$ equals k , and further increases in food rate induce more key pecking, which displaces B_O .

food-related activities. The model required five assumptions:

1. The total behavior was allocated among three activities, key pecking (B), other food-induced activity (e.g., posturing in front of the key or pecking elsewhere; B_O), and non-food related activity (e.g., running, grooming, or resting; B_N). The total equals k in units of responses per unit of time. For these data, we set k equal to 172 pecks/min (ppm), at the top of the range of estimates shown in Figure 1 and also the estimate of k when the hyperbola in Equation 4 was fitted to all the response rates on the right key.
2. These activities compete unequally in a hierarchy. As food-related activities B_O and key pecking B increase, they both replace non-food-related activities B_N , but when B and B_O sum to the maximum k , then increased key pecking B replaces other food-related activities B_O . Thus, key pecking is at the top of the hierarchy, then other food-induced activities, and then non-food-related activities.
3. Food-induced activity B_O increases according to a power function of food per minute (r), as suggested by Staddon (1977):

$$B_O = c \cdot r^s, \quad (6)$$

where c and s are free-parameter constants. For fitting the points in Figure 2, c was estimated at 12 and s at 0.7. The function

was chosen on the assumption that r_O covaries with B_O , but its importance lies in its statement that B_O depends on r .

4. r_O (fpm) is directly proportional to food-induced activity B_O , as in a ratio schedule. We set the constant of proportionality to 0.16.
5. When B_N is greater than zero, $k-B$ equals $B_O + B_N$, and B_N stands in an interval-like relation to its value r_N , on the assumption that the non-food-related activities maintain the organism and are limited in their value:

$$r_N = \frac{1}{\frac{1}{B_N} + \frac{1}{V_N}}. \quad (7)$$

The constant V_N was allowed to be a free parameter. This assumption implies that r_O in Equations 3 and 4 actually equals $r_O + r_N$. For the fit in Figure 2, V_N equaled 0.3.

Figure 3 shows a graphical representation of the model. As r increases, at first both B_O and B (diamonds and squares) increase, and r_O increases with B_O . B_N (triangles) decreases as B and B_O replace it. The sum $B_O + B_N$ (plain line), which equals $k-B$, decreases slightly. As r increases further, however, a point is reached at which B_O and B have replaced all the non-food-related activity B_N , B_N and r_N both equal zero, and the sum $B + B_O$ equals k . In Figure 2, this occurs at about 15–20 fpm; it is shown by the broken vertical line in Figure 3. As r increases further, B (key pecking) increases more rapidly and replaces B_O (other food-related activity), with the result that B_O and r_O decrease, as shown in both graphs of Figure 2. Since B_O and B sum to k when r exceeds 15 fpm, B_O equals $k-B$ in that range, as shown in the right-hand graph of Figure 2.

To fit the decrease in r_O shown in the left-hand graph of Figure 2, and to avoid intractable algebraic expressions, we substituted $0.16^*(k-B)$ for r_O in Equation 3 and solved the resulting quadratic equation, of which only one root described B less than k . As B increased and replaced B_O , r_O and B_O declined rapidly.

The idea that B_O replaces B_N was suggested also by Staddon (1977) when he argued that rats' wheel running is not an induced activity because drinking and food anticipation (activities directed at the food hopper) replace running as food rate increases.

In the light of this model, Equation 5 would be an approximation at the relatively lower food rates, because $k \cdot B$ equals $B_O + B_N$, and r_O includes a term r_N corresponding to B_N . Likely r_N is small even at low food rates and only gets smaller as B_N decreases; if so, r_N might sometimes be considered negligible, although the shape of the increase in r_O would be as shown by the squares in Figure 3. In the calculation of B from Equation 3, we avoided intractable computation by treating r_N as negligible, but in the calculation of r_N from B_N (i.e., $k \cdot B - B_O$), we used Equation 7 with V_N equal to 0.3. If r_N is not negligible, then Equation 5 would be affected as B_N diminished if B_O were taken to equal $k \cdot B$.

One implication of the model is that r_O is primarily due to induction of B_O by the food. Since no adjustment of parameters across conditions was required, food rate is the only operative variable. Food induces both key pecking and other food-related activities. The other food-related activity produces r_O according to a ratio-like relation, but as pecking increases with r , it eventually competes with the other food-related activity and replaces it, causing r_O to decrease.

The changes in r_O account for the deviations of high response rates from the hyperbola shown in Figure 1, but they do not account for the deviations of the low response rates, which must involve not only B_O but also B_N . The deviations of the low response rates indicate that response rate failed to decrease as r decreased, and they may be peculiar to this sort of rapid-change procedure, in which a food ratio is presented for such a brief time that low r would often result in all food deliveries coming from the same key (the constant VI here). In other words, these brief-exposure procedures may entail a discriminability threshold that foils discrimination among the lowest food rates.

The model offers a simple answer to the question about the feedback function for r_O : It is a ratio-like direct proportionality. The story doesn't end there, however, because r_O turns out to depend on r as a result the food's inducing key pecking and other food-related activities.

Pecking on One Alternative

To consider pecking only on the right (varied) key, we fitted the hyperbola shown in Equation 4, taking r_1 and B_1 to represent fpm and ppm on the right. Although the VI on the

left was fixed in each condition, fpm on the left varied inversely with fpm on the right. For each component, we inserted the measured fpm on the left into Equation 4. Figure 4 shows the result. As with Figure 1, the fitting was done in arithmetic coordinates, but the graphs appear in logarithmic coordinates. The fits appeared satisfactory in arithmetic coordinates (VAC or $r^2 > .97$), but in logarithmic coordinates one can see systematic deviations of the response rates from the hyperbola. The pattern differs from that in Figure 1: Response rates leveled off and lay above the curve at the lower end, as in Figure 1, but generally lay close to the curve above about 0.2 fpm.

We made the same set of five assumptions as above. We calculated r_O by first using Equation 6 to estimate B_O , with r equal to total fpm, and allowing c and s to be free parameters. Then r_O was set to 0.16 times B_O . We calculated B_N by subtracting left and right peck rates and B_O from k , set equal to 172 as before, and used Equation 7 with V_N , a free parameter. Equation 8 was used to derive a predicted peck rate on the right key (B_1):

$$B_1 = k \frac{r_1}{r_1 + r_2 + r_O + r_N}. \quad (8)$$

The graph in Figure 5 shows these predictions plotted against fpm on the right, r_1 in Equation 8. We treated the condition with VI 10 on the left separately from the other conditions, because the response rates were noticeably lower, and we treated the other conditions together, because the rates overlapped. The solid line shows the predictions for VI 10, and the broken black line shows the predictions for the rest. The vertical broken line marks fpm equal to 0.2, below which response rate apparently ceased to decrease. The model was fitted to food rates above 0.2 fpm using Excel's Solver. The fits were good (r^2 equaled .996 and .994). The constant V_N in Equation 7 for r_N was estimated at 2.5, c and s in Equation 6 were estimated at 9 and 0.162 for both VI 10 and the other left VI schedules, but k was set to 135 for VI 10 and 172 for the other left VI schedules. These different estimates of V_N , c , and s compared with those in Figure 2 probably stem from the inclusion of r_N in Equation 8, because, although r_N decreased as response rate increased, it was not negligible and thus lowered the estimates of B_O required from Equation 6. The lower estimate of k for VI 10 probably

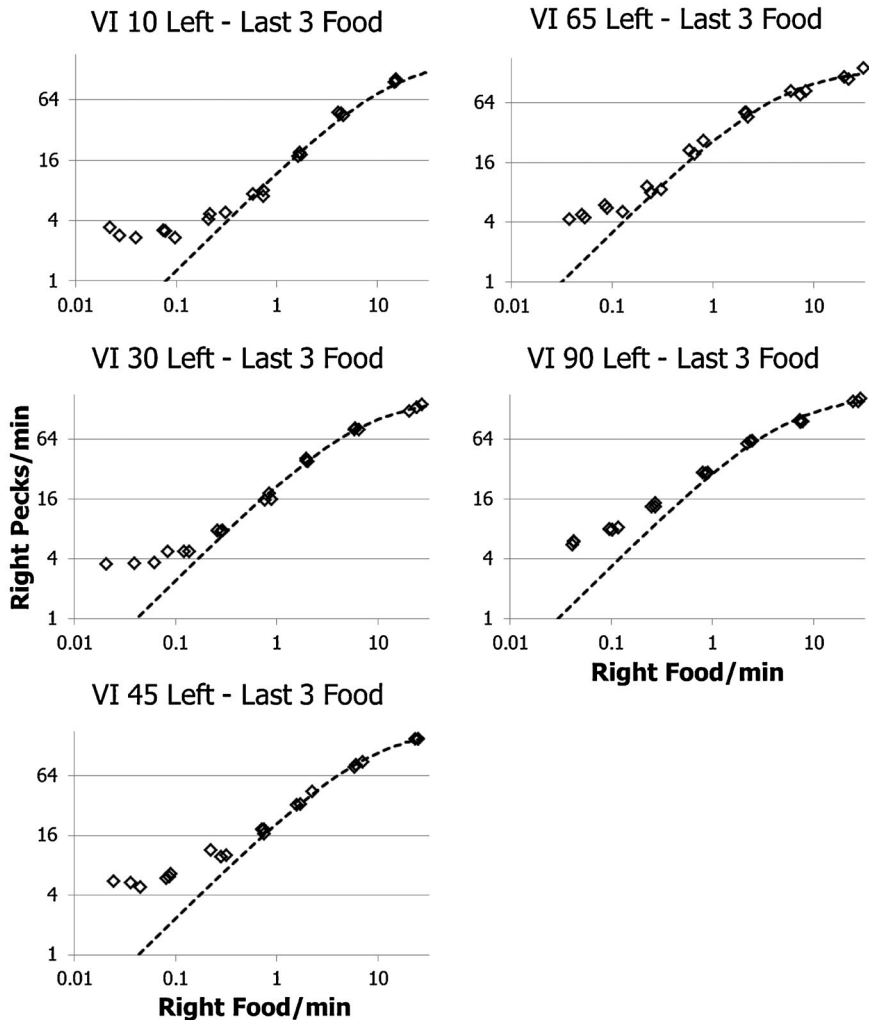


Fig. 4. Peck rate on the varied right key versus food rate on the right key across the five conditions of Baum and Davison's (2014) experiment. The broken curves show the fits of Equation 4. The fits were done in arithmetic coordinates, but are displayed here in logarithmic coordinates.

reflects a change in topography of pecking at the highest food rates—for example, pausing briefly following pecks to view the food hopper.

Belke and Heyman (1994)

Can we find any evidence of induction and dependence of r_O on B_O in the data published by Belke and Heyman (1994)? Although they studied only one choice condition, they conducted two series of single-VI conditions with the same set of varying schedules as in the choice condition. We analyzed stable data from the single-VI series and the choice condition.

Figure 6 shows the results. The top left graph shows peck rate on the seven single VI schedules, and the top right graph shows total peck rate and peck rate on the constant alternative in the choice condition (right graph) plotted against food rate. In the top left graph, the hyperbola fitted in arithmetic coordinates is shown in these logarithmic coordinates by the solid line; it remained a good fit. In the top right graph, the diamonds show total peck rate plotted against total fpm, and the solid line shows the hyperbola of Equation 3, fitted in arithmetic coordinates and displayed in these logarithmic coordinates. The squares show peck

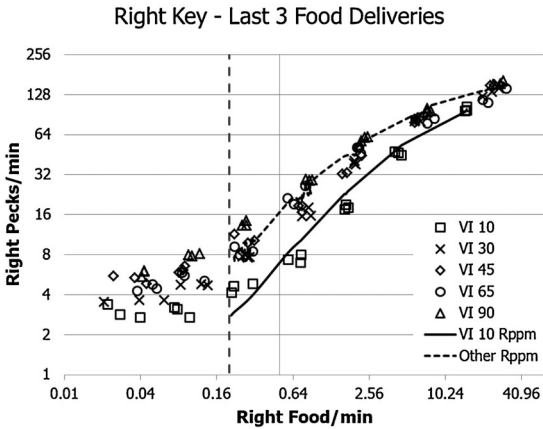


Fig. 5. The induction model (Eq. 8) fitted to peck rate on the varied right key across the five conditions of Baum and Davison's (2014) experiment. The vertical broken line marks off the low food rates that were hardly discriminated because of the brevity of components in the frequently-changing-ratios procedure. The solid black line shows the results for schedules paired with the richest schedule, VI 10, on the left key. The broken black line shows the results for all the other conditions treated together.

rate as a function of fpm on the constant VI schedule; peck rate varied enough to produce variability in the food rate. The broken curve shows a VI feedback function fitted to the points; the equation was:

$$r = \frac{1}{t + \frac{a}{B}} \tag{9}$$

where t is the average interval, the VI, and a is a free parameter that aids fitting at low response rates (Baum, 1992; Prelec & Herrnstein, 1978). Although the constant schedule was nominally VI 27 s, setting t to 27 s failed to fit the actual food rates. The curve shown, fitted by Solver in Excel with a least-squares criterion, sets the estimate of t at 17.5 s and a at 9.5. Thus, the program resulted in higher food rates for the peck rates than was intended. The parameter a applies particularly at low food rates, and its exceeding 1.0 implies that individual food deliveries produced bursts of pecking (Baum, 1992). At high food rates, when t approaches zero, a must approach 1.0 if responding is random in time, as assumed in Equation 7 (Nevin & Baum, 1980).

The middle two panels in Figure 6 show calculated r_O plotted against food rate. In the left graph, Belke and Heyman's results with single VI schedules, k was estimated according to

Equation 3, and r_O was calculated according to Equation 5. Calculated r_O increased with food rate, except for the three low filled points. Those three points correspond to the three highest peck rates. In the lower left panel, the same calculated r_O is plotted against B_O (estimated as $k \cdot B$), and the three points corresponding to the filled points in the middle panel are again shown as filled points. They stand away from the trend in the other points and represent low levels of B_O , which could indicate a hint of the dependence of r_O on B_O shown in Figure 2 (right). If the food was inducing food-related activity other than key pecking, and key pecking replaced those activities only when peck rate was high enough, then the three outstanding points might represent that replacement.

The middle right graph in Figure 6 shows calculated r_O plotted against total food rate on the left and right. We estimated k according to Equation 3, and calculated r_O according to Equation 5 with B equal to the total peck rate. The lower right graph shows calculated r_O plotted against B_O estimated as $k \cdot B$. No trend in r_O can be seen.

Soto et al. (2005): Rats' Total Pressing on Two Levers

In a related experiment with rats, Soto et al. (2005) studied seven different conditions, each with a constant VI schedule or extinction (one condition) on the right lever, ranging from VI 10 s to VI 350 s, and the 10 varied VI schedules, ranging from VI 6 s to VI 350 s, the same 10 in each condition, on the left lever. The 10 pairs of schedules were presented in two sessions per day, five in each session, and each pair for 8 minutes, separated by black-outs. Three conditions were replicated; since the replications were close to the original conditions, we averaged each replication with the corresponding original condition. The procedure differed from those of Belke and Heyman (1994) and Baum and Davison (2014) in that the different components were signaled by discriminative stimuli. Most conditions lasted for 20 pairs of sessions, but a few for only 10. Data from the last 6 days of each condition were analyzed. Paul Soto kindly sent the raw data to us.

We averaged the data across rats and analyzed those means. Figure 7 shows mean response rate on the two levers combined across all

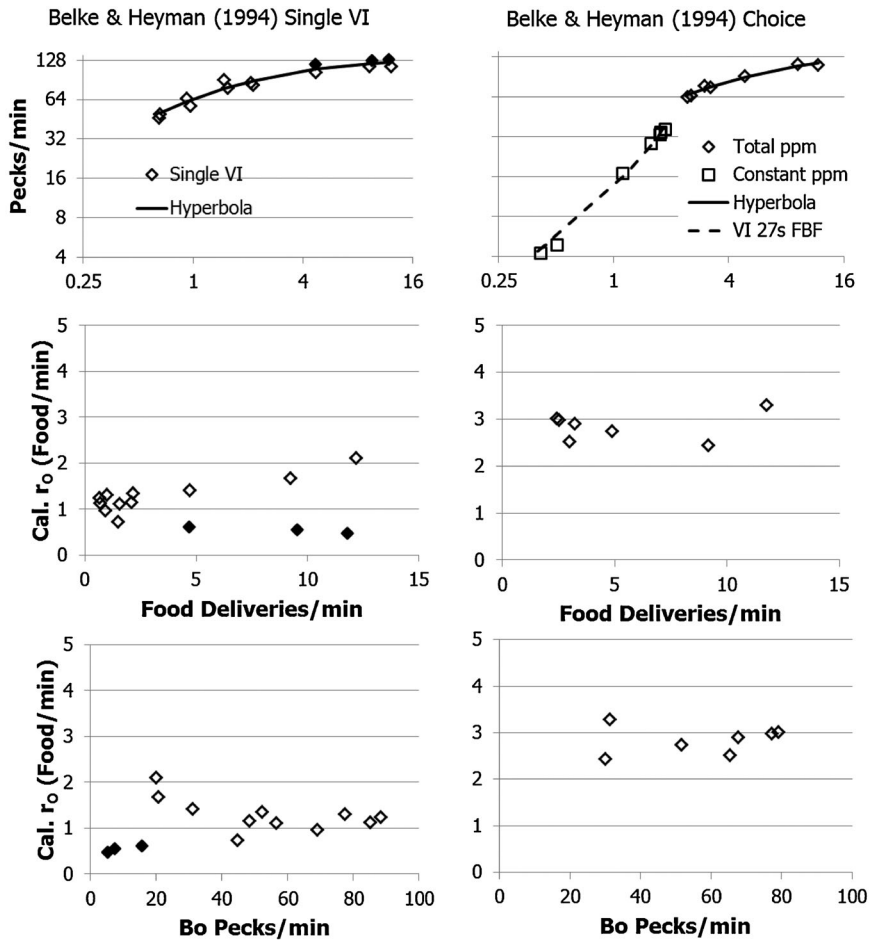


Fig. 6. Results of Belke and Heyman (1994) reanalyzed. The left column shows results for seven single VI schedules. The right column shows results for the same seven VI schedules presented with a constant concurrent VI (nominally 27 s, but actually VI 17.5 s). Top: Peck rate versus food rate. The solid black lines show the fits of the hyperbola in Equations 3 and 4. The broken curve is a VI feedback function fitted to the performances on the constant schedule. Middle: Calculated r_O from Equation 5 versus food rate. The filled symbols mark points that appear to diverge from the rest as food rate increased. Bottom: Calculated r_O versus B_O estimated as $k \cdot B$.

components and conditions (70 points) plotted against combined food rate, as in Figure 1. Conditions with extinction on the constant right lever are marked with X. The trend is for press rate to increase with combined food rate up to about 3.8 fpm and then to decrease. The lowest cluster of 10 points at the right were all from components with VI 6 s on the left (varied) lever. Thus, at the highest food rates, press rate decreased. We fitted the hyperbola of Equation 3 to the press rates for food rates below 2 fpm, keeping k and r_O constant. The solid curve in Figure 7 shows the best-fitting hyperbola, with k

equal to 80 ppm and r_O equal to 0.45 fpm. This fit gave us an estimate of k to work with.

Next, we calculated r_O for all components according to Equation 5. Figure 8 shows these calculated estimates of r_O plotted against food rate, as in the left graph in Figure 2. The points fall into four groups: Diamonds denote calculated r_O for food rates less than 2 fpm; Xs denote r_O for food rates between 2 and 3.8 fpm; squares denote r_O for food rates between 3.8 and 4.5 fpm; and triangles denote r_O for food rates greater than 4.5 fpm generated by components with VI 6 on the varied lever. The diamonds

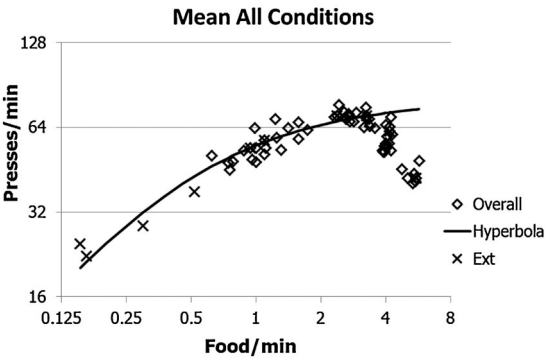


Fig. 7. Rates of pressing two concurrent levers combined from the experiment by Soto et al. (2005). The Xs show responding when the constant schedule on the right lever was extinction. The line shows the fit of Equation 3 to the conditions with food rate below 2 fpm.

show less change in calculated r_O than shown on the left in Figure 2. Instead of just one vertically varying cluster, however, three clusters can be seen: the Xs, the squares, and the triangles.

Figure 9 shows calculated r_O as a function of B_O (estimated as $k \cdot B$, with k equal to 80) for the three vertically varying clusters, in which, according to our model, B_N has been completely replaced by lever pressing (B) and other food-induced activity (B_O), and in which B_O equals $k \cdot B$. The Xs show r_O directly proportional to B_O , as in the right-hand graph in Figure 2. The constant of proportionality equals 0.0448

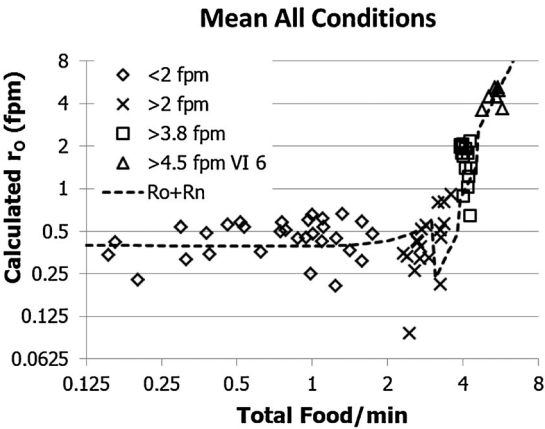


Fig. 8. Calculated r_O from Equation 5 versus food rate from the experiment by Soto et al. (2005). Different symbols indicate different ranges of food rate. The broken line shows the fit of the induction model, assuming calculated r_O to equal actual $r_O + r_N$.

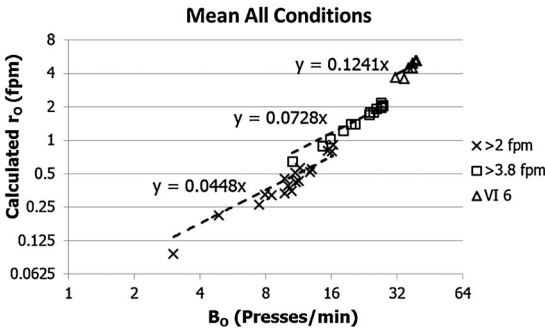


Fig. 9. Calculated r_O versus B_O estimated as $k \cdot B$ from the experiment by Soto et al. (2005). Different symbols indicate estimates from three different ranges of food rate. The lines were fitted to the estimates from the different food-rate ranges separately. Note logarithmic coordinates.

(equivalent to VR 22), less than the 0.16 of Figure 2; the difference might be attributed to any of several differences of procedure, such as rats versus pigeons, food pellets versus grain, unsignaled components versus signaled components, key pecking versus lever pressing, etc.

The squares and triangles in Figure 9 stand apart from the Xs, as they do in Figure 8. The lines drawn through the squares and triangles indicate, however, that these points also show direct proportionality between r_O and B_O . For the squares, relatively high food rates between 3.8 and 4.5 fpm, the constant of proportionality equals 0.0728 (VR 14), steeper than for food rates between 2 and 4 fpm. For food rates above 4.5 fpm, generated by VI 6s, direct proportionality appears again, but with a constant of proportionality equal to 0.1241 (VR 8), steeper still.

We may speculate that the rats' topography of lever pressing changed as food rate became extremely high. Beyond rates of 3.8 fpm, for example, the rats may have spent more time nosing in the food hopper. With this change, the activities induced by the food that comprised B_O differed too, to the point where they interfered with lever pressing at the highest food rates and caused the decrease shown in Figure 7. In all three situations, however, direct proportionality—a ratio-like relation—may be seen between r_O and B_O , supporting the conclusion that the feedback function relating r_O to B_O is a ratio-like relation. The model that applied to key pecking in pigeons also applies to lever pressing in rats.

Figure 8 makes clear, however, a difference between rats' lever pressing and pigeons' key pecking. It shows what may be called a *role reversal*. Instead of decreasing at high food rates, as with key pecking (Fig. 2), r_O and B_O increased when the operant activity was lever pressing. This increase in B_O at the expense of lever pressing appears in Figure 7 as decreased press rate at high food rates. Instead of lever pressing replacing other food-induced behavior B_O , other food-induced activities replaced lever pressing. Thus, lever pressing affords an example of what Breland and Breland (1961) called "misbehavior of organisms." In contrast, key pecking appears to correspond to the kind of activity that is facilitated by induction, as in the Brelands' example of the "dancing chicken." Food induces pecking in pigeons even with no contingency between pecking and food (Staddon, 1977). Most likely, food with no contingency has no such effect on lever pressing, and the difference may explain when interference occurs and when facilitation occurs.

The broken line in Figure 8 shows the result of applying the same model as before, including Equations 6 and 7, but with three changes: (a) the three different constants of proportionality shown in Figure 9 were used instead of just one; (b) B_O was assumed to replace B once B_N was reduced to zero and the sum of B_O and B equaled k (equal to 80); and (c) to avoid intractable computation in calculating B from Equation 3, r_N was represented as a small constant (0.2) added to the denominator, but otherwise r_N was calculated from Equation 7 as before. A graphical depiction like that in Figure 3 would look much the same, except that B_O would increase to the right of the vertical line and B would decrease, and B_O would not increase all the way to equal k , because B could only decrease until B equaled r (Fixed Ratio 1). The estimates of c and s in Equation 6 differed from those for the pigeons' key pecking: c was 0.1, and s was 3.5, so c was much smaller, and s was much larger.

Soto et al. (2005): Rats' Pressing on One Lever

As with the pigeons' key pecking, we analyzed rats' pressing on the varied lever according to the (modified) model and Equation 8. Figure 10 shows the results. The graphs show the rates of

pressing on the varied lever with different VI schedules on the constant lever. The bottom right panel shows pressing on the varied lever with either VI 10 or VI 17 on the constant lever. The broken lines show the fits of Equation 4, with r_O assumed constant. The solid lines show the results of fitting the present model, with all parameters carried over from the model used to fit calculated r_O in Figure 8, except that k was allowed to vary across the different constant schedules to help assess the effects of lack of discrimination among components, particularly for VI 10 and VI 17.

The lack of discrimination appears in Figure 10 as the same tendency for responding to level off at low food rates as appears in Figures 4 and 5. Apparently the putative discriminative stimuli failed to distinguish among the low food rates on the varied lever. The flattening out occurred at higher and higher food rates, the richer was the schedule on the constant lever. For the single VIs, it was minimal, for VI 350 and VI 150, the lowest two food rates were indistinguishable, for VI 75 and VI 50, the lowest three food rates were indistinguishable, for VI 17, the lowest four food rates were indistinguishable, and for VI 10, the lowest six food rates were indistinguishable. Most likely, the richer the constant schedule, the more often all food during the 8-min exposure came just from that schedule, thereby preventing discrimination among the lean schedules on the varied lever. In fitting Equation 4 (broken lines) and the model (solid lines), the indistinguishable food rates were omitted. For most of the different constant schedules, the model captured the decrease in pressing at high rates (cf. Fig. 7). It was less successful with VI 17 and least successful with VI 10, presumably because of the lack of discrimination among the different schedules on the varied lever. The estimates of k from the model decreased with press rates: unsystematic variation from 75 to 90 (mean equal to 82) for the five leanest, to 70 for VI 17, and 65 for VI 10.

Figure 11 shows variation of food and press rates on the constant lever. Since the press rates on the varied and constant levers were inversely related, one would expect that as press rate on the constant lever varied, food rate might also vary, so that food rates on the two levers were inversely related [see Soto et al. (2005); Figs. 1 and 2]. The curves fitted to the points for the various constant schedules represent Equation 9, the feedback function (RFF) for a VI schedule

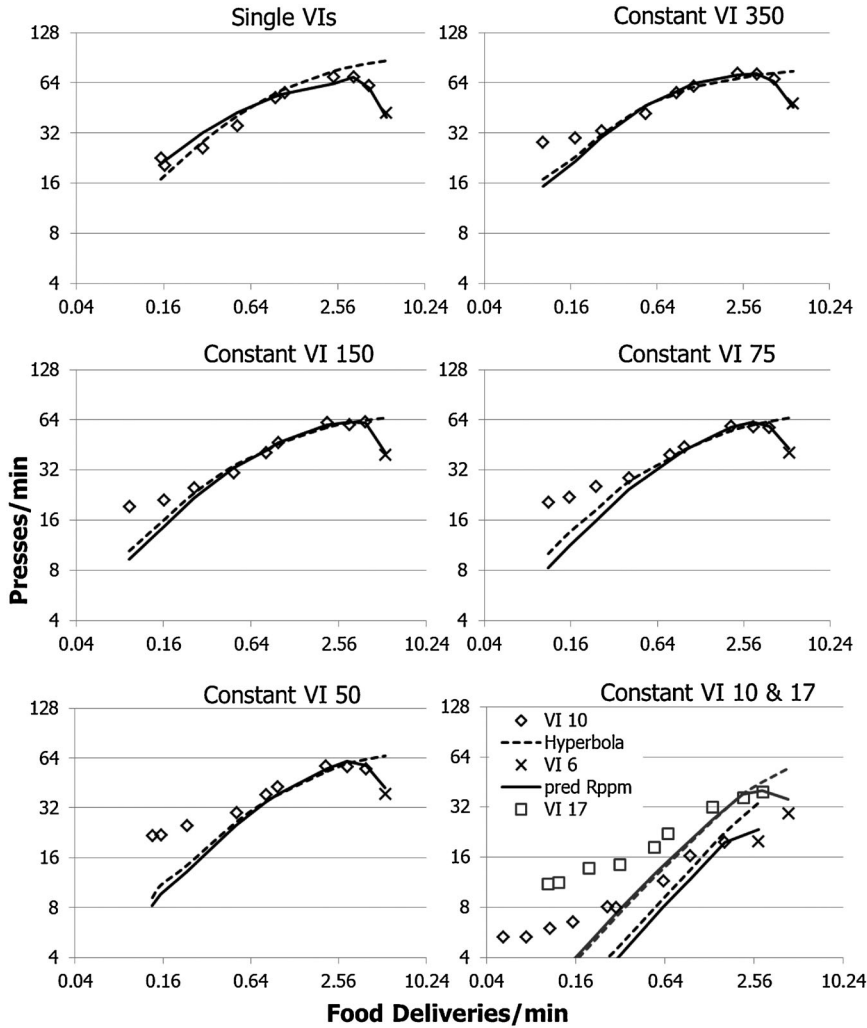


Fig. 10. Rates of pressing the varied left lever in the experiment by Soto et al. (2005). Each graph shows results from one condition with the schedule on the right held constant, except for the bottom right graph which shows results from the two conditions with the constant schedule VI 10 and VI 17. Broken lines show the fits of the hyperbola of Equation 4. Solid lines show the fits of the induction model (Eq. 8).

(Baum, 1992). Estimates of t varied with the VI, and estimates of a varied a bit with the VI, being largest (25) for VI 150 and smallest (13) for VI 10. Variation in a might reflect topographical changes in activities around the lever.

Discussion

Two results stand out. First, the results support Baum’s (1981) and Davison’s (1993) conjecture that the feedback relation between B_O and r_O is a ratio-like relation. Second, these

results show that B_O represents food-related activities other than the operant activity that are induced by the food along with the operant activity, as proposed by Baum (2012a). The success of the hierarchical model of induced activities, illustrated in Figures 2 and 8 for overall pecking and pressing, and in Figures 5 and 10 for pecking and pressing on the varied VI, supports Baum’s (2012a) proposal. Though r_O was originally thought of as independent of food rate r in Equations 3 and 4, r_O turns out to depend on r .

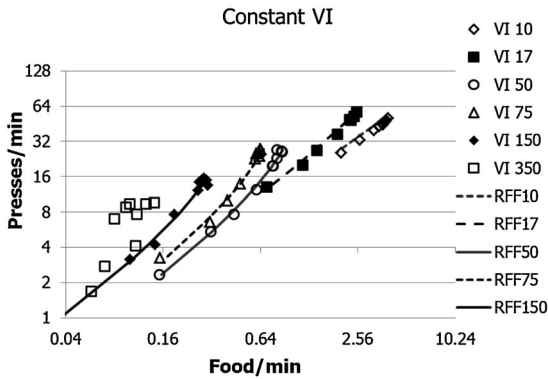


Fig. 11. VI feedback functions (Eq. 9) fitted to the press rates on different constant schedules in the experiment by Soto et al. (2005).

Herrnstein's (1961, 1970) original formulation of the matching law, which Belke and Heyman (1994) relied on, implicitly assumed that alternatives were independent sources of reinforcers; each r depended only on its B and not on any other activity. The unlooked-for result, not included in Herrnstein's conception of matching, is that r_O depends on r in Equations 3 and 4. The reason r_O depends on r is that the food induces both operant activity and other food-related activities B_O that depend on r —are induced at different rates by different food rates produced by the operant activity—and, as Figures 2 and 9 show, r_O in Equation 8 is directly proportional to B_O . Equations 3 and 4, to be accurate, ought to include a term r_N in the denominator, as in Equation 8. The term r_N , however, conforms to the original conception of r_O , because r_N arises from activities (B_N ; e.g., grooming and stretching) that occur independently of food.

The model for right-key pecking incorporates an effect of food from the left key inducing activities that affect pecking on the right key (Eq. 7), because food induces the same food-related activities (B_O) regardless of the food source. Thus, the r_O term that appears in Equation 8 is equal to r_O in Equations 3 and 4 minus r_N ; the total of induced activities' value r_O appears in the denominator regardless of whether we are considering total operant responding (Eq. 3) or responding only on the varied VI (Eqs. 4 and 8). In contrast with concurrent schedules, successive components in multiple schedules might differ in both r_O and r_N . Some researchers have proposed to explain

behavioral contrast by supposing that B_O might shift from a richer component to a leaner component, decreasing operant activity in the leaner component and increasing operant activity in the richer component (McLean, 1992, 1995; McLean & White, 1983; Staddon, 1982). According to the present account, likely B_N would be fungible this way, but B_O might be less so, because B_O might tend to remain in the component with the food that induces it. The fungibility of B_N might explain why essential bodily maintenance activities like grooming and stretching can decrease to zero, as shown in Figure 3, even in concurrent schedules; the activities are simply postponed until after the session.

A striking difference appeared between pigeons' key pecking and rats' lever pressing: Key pecking replaced other food induced activities B_O , whereas those activities replaced lever pressing—a role reversal. The difference corresponds to the difference in induced activities that Breland and Breland (1961) described. When induced activities interfered with operant activities, as they did for lever pressing, the Brelands called the interference "misbehavior," but they also noted that induced activities could facilitate operant training, as occurred for key pecking.

Even though key pecking replaced other food-induced activity in the present results, that replacement should not be confused with the reduction in key pecking that results from addition of a second concurrent source of food, as represented by the difference between Equations 3 and 4. Equation 3 was meant to apply to a single source of food, and we applied it here to the combined food from two alternatives as if they were one source (cf. Baum, 2002, Figure 3), whereas Equation 4 was meant to apply to one alternative out of two. The inclusion of a nonzero term for r_2 decreases key pecking B_I below what would occur if B_I were alone. Reduction in B_O and reduction in B_I both result from competition between activities for the time available, but the reduction in B_O is the result of dominance of key pecking, whereas the reduction in B_I is the result of adding a second food source. That second food source need not depend on key pecking, as Rachlin and Baum (1972) found. In their experiment, a constant VI schedule was maintained on a response key while additional food was presented, varying in rate and amount. When they added response-

independent food, it affected pecking on the constant VI just like an independent second source of food. Consistent with the idea that more food usually induces more food-related activities, Rachlin and Baum reported that, when they applied a version of Equation 4 that incorporated the product of amount times rate (v instead of r), the estimate of v_O analogous to r_O was much higher with the presence of the added food.

Baum (2005, 2012a) argued that phylogenetically important events (PIEs) such as food induce activities related to them both by phylogeny and ontogeny. Food delivered independently of behavior induces several different activities in hungry pigeons, including moving about, wing flapping, and pecking at screws, floor, or in the air (Staddon, 1977). When made contingent on key pecking, food comes quickly to induce key pecking too, because the contingency makes key pecking a food-related activity also. Similarly, food induces various activities in rats, such as drinking and nosing in the food hopper (Staddon, 1977). When food is made contingent on lever pressing, lever pressing, like key pecking in pigeons, becomes a food-induced activity as a result of its contingency with food. Thus, Baum (2012a) argued, reinforcement may be recast as induction—operant activities are induced by the PIEs to which they are related by contingencies.

That r_O depends on r doesn't invalidate Equation 3 or Equation 4. Rather, the dependency makes the equations more plausible, because it brings our understanding of r_O into line with our understanding of other PIE-related phenomena. That every schedule is characterized by a feedback function allows us to write a matching equation

$$\frac{B_O}{B} = \frac{r_O}{r}, \quad (10)$$

confident that r_O depends on B_O according to a feedback function—a ratio-like function—just as r depends on B according to a feedback function—a VI function in all the studies we have considered in the present paper, and which appears in Figures 6 and 11 (Eq. 9; Baum, 1992).

If the relation between r_O and B_O is a ratio relation, as we have concluded, then Equation 10 appears to be incorrect. For example, it implies that B/r ought to match B_O/r_O . If the ratio B_O/r_O

is constant, it should fix the point where response rate should fall on a VI feedback function (Baum, 1973, 1992). If B_O/r_O is constant, then B/r should be constant, which would mean that B is directly proportional to r , contrary to Figures 1, 6, and 7 and all previous research. Looked at the other way around, we would have to conclude that the ratio B_O/r_O should change across VI schedules, because the ratio B/r changes, but Figure 2 appears to show that, at least for pigeons' key pecking, B_O/r_O is constant. Possibly the third category B_N plays a role yet to be determined at low or moderate food rates, when $k \cdot B$ equals $B_O + B_N$. Possibly the correct equation is the one corresponding to Equation 5:

$$\frac{B_O + B_N}{B} = \frac{r_O + r_N}{r}. \quad (11)$$

Equation 11 allows the possibility that B_N might change as the VI schedule changes and might preserve the relation that B/r matches $(B_O + B_N)/(r_O + r_N)$ even though B_O/r_O is constant.

A key to solving this conundrum, at least for pigeons' key pecking, may lie in Baum's (1993) study of variable-ratio (VR) and VI schedules. We speculated that topography of lever pressing changes as food rate gets high, but key pecking at high food rates may change also. Baum (1993) studied multiple schedules in which one component was a VR schedule and the second component was a VI schedule programmed by playing back the intervals generated in the VR to match the food rate and distribution of intervals across the two schedules. VR and VI components alternated, with blackouts separating the components. Varying the VR over a wide range, all the way to requiring only a single peck, resulted in a wide range of food rates, from moderate to extremely high. The top panel in Figure 12 shows the results averaged across pigeons. As food rate increased, peck rate on the VI increased, started to level off over a middle range, greatly increased again for higher food rates and then decreased at the highest food rates. The high peck rates could not be accounted for by decreases in r_O , because r_O was too small, perhaps because B_O and B_N were reallocated to the blackouts between components (McLean, 1992, 1995; McLean & White, 1983; Staddon, 1982). The lower panel shows that the decrease at the highest food rates is accommodated if one assumes that B , B_O , and B_N

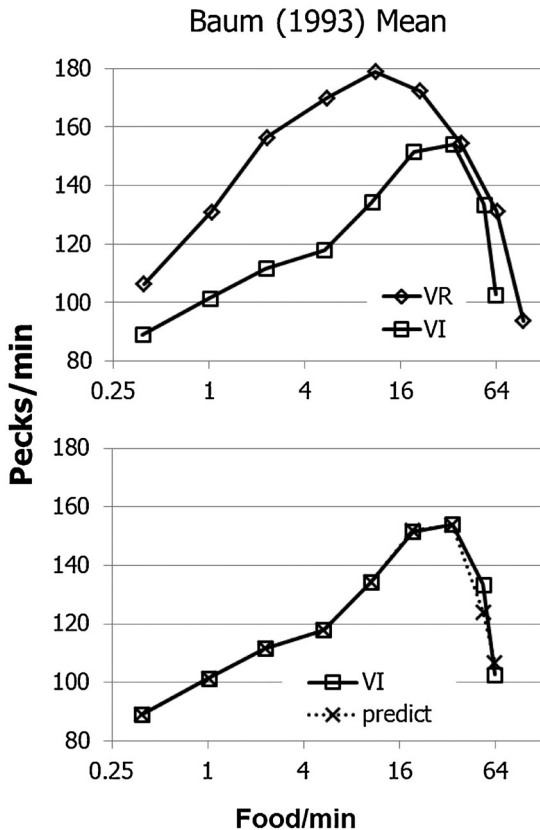


Fig. 12. Peck rates and food rates averaged across four pigeons in the experiment by Baum (1993). Top: Rates from the VR components and the yoked VI components. Bottom: Rates from the VI components predicted by assuming power-function induction of B_O .

sum to a constant k equal to 180 ppm and that Equation 6 governs B_O , with a power s of 1.679 and a coefficient c of 0.07. Peck rates for lesser food rates are predicted perfectly, but that is to be expected, because B_N was calculated as $k \cdot B_O$.

Baum (1993) speculated that pigeons' key pecking contains a mix of two topographies: one in which the beak meets the key and then pulls away, operating the key's switch once, and one in which the beak swipes the key so as to operate the key's switch multiple times. Palya (1992) made the same suggestion, calling the multiple-operation topography a "flick." As the food rate got high in Baum's experiment, key swiping may have replaced key pecking, resulting in increasing measured peck rates (switch operating rates). He reasoned that the change in units

of pecking changed the correspondence between pecking and time; the higher peck rate might affect the amount of time taken up, because the time per switch operation was smaller. The change of unit would be reflected in a change in k in Equation 3. The present account makes no assumption that k changes, but instead assumes that B_N decreased to zero at the highest peck rate and that B_O increased according to the power function. Thus, the decrease in calculated r_O shown in Figure 2 might actually be a result of a change in topography and k at the highest food rates, and possibly actual r_O did not decrease. The possibility of changes in k may need to be incorporated into future research.

The leftmost four points for the VI component in Figure 12 could be described by a hyperbola like Equation 3, suggesting an approximately constant value for background reinforcers $r_O + r_N$. Possibly, decreasing r_N tended to offset increasing r_O at low and moderate food rates, until B_N decreased to zero and B (composed of key pecking and key swiping) began replacing B_O as r increased past that point. All these speculations may be addressed by future research.

The absence of pecking below about 0.3 fpm indicates ratio strain; if pecking ceased in the VR component, the yoking of the components caused food rate to drop to zero in the VI component as well as the VR component. If the relation between r_O and B_O is a ratio relation, ratio strain might be understood as the outcome of competition between pecking and B_O . A VR schedule and the r_O schedule would constitute a pair of concurrent ratio schedules. Performance on concurrent ratio schedules moves toward exclusive preference for the richer schedule. The food produced by pecking on a VR schedule, like the food produced by pecking on a VI schedule, induces other food-related activities B_O . Whenever the relation between r_O and B_O is richer than the relation between r and B on the VR schedule, pecking on the VR schedule will cease. That cessation could occur at moderate food rates, if r_O/B_O were moderate, and it would explain ratio strain. As long as the VR schedule (r/B) was richer than the r_O/B_O schedule, pecking on the VR would be maintained and would tend to occur at the highest rate possible (exclusive preference). That doesn't explain why peck rate increased as food rate increased, unless ratio strain is a

variable effect and modulates the peck rate below the maximum. The decrease at the highest food rates, however, might be explained in the same way as the decrease for the VI peck rates—the effect of food-induced activities B_O . Future research may address these questions also.

The picture for rats' lever pressing may be more complicated. Possibly the proportionality between r_O and B_O might change with food rate, as Figure 9 suggests. Since B_O probably represents multiple activities, change in proportionality would reflect change in the activities constituting B_O . As food rate increased for the rats, new activities apparently appeared in B_O . As food rate increased for the pigeons, some activities in B_O apparently dropped out. Further research may clarify these issues.

Explaining decreased response rate at high food rates as competition with induced adjunctive activities is not new. For example, Rachlin (1978; Rachlin & Burkhard, 1978) and Staddon (1983) developed optimality models that explained the downturn at high food rates as competition. Our approach doesn't assume that individual organisms optimize, but instead seeks mechanisms that might be optimal in the narrower sense that they were selected by natural selection. In this respect only, our approach resembles Killeen's (1994), in which he developed an equation (his Eq. 12) that combined a hyperbola, in which a constant c stands in place of r_O , with a subtractive term that also would predict a downturn at high food rates. Rachlin's, Staddon's, and Killeen's explanations all entail far more assumptions and parameters than the model we propose here. In particular, because Killeen's "mathematical principles of reinforcement," rely on a molecular view of behavior that takes discrete responses and response-reinforcer contiguity as primary, his approach requires many assumptions and parameters, and appeals to hypothetical constructs such as memory (in place of extended activities) and coupling (in place of a feedback function). The present approach based on a molar view of induction may offer a simpler account of the same phenomena.

Equation 6 gives a function for the dependence of B_O on r —that is, for induction of food-related activities (Baum, 2012a). In the present context, Equation 6 served our purposes, but it may be inaccurate. Induced activity sometimes follows a bitonic function of food rate, increas-

ing with r over a range, and then decreasing at relatively high food rates (Staddon, 1977). The decrease at high food rates may be related to the decrease in r_O at high food rates seen in Figure 2. Possibly, with no operant activity present, new constituent activities of B_O are induced at high food rates that are not measured by the usual methods, which rely on specific manipulanda, like water spouts (polydipsia) or mirrors (induced aggression). For example, if a rat or pigeon began keeping its head in the food hopper, that might not register as the measured activity.

The highest food rates that Soto et al. (2005) studied apparently induced new constituent activities. Figure 9 suggests that new constituents arose either in B_O or in lever pressing at the high food rates, because the proportionality between r_O and B_O changed. Possibly, as new constituent activities begin taking up time, the estimates of B_O in units of presses/min begin systematically to fall short, because the correspondence between B_O in presses/min and time changes. If so, a correction might be derived that would move the Xs and triangles in Figure 9 sufficiently to the right to have them fall in line with the squares representing lesser food rates.

Modeling the induction of key pecking and lever pressing as we have done here has a theoretical benefit, because it preserves the generalized matching law (Baum, 1974, 2012b). If we assume that operant activities in general are induced by food or other consequences, we may write:

$$\frac{B_i}{B_j} = \frac{c_i r_i^{s_i}}{c_j r_j^{s_j}} \quad (12)$$

for all pairs of concurrent operant activities i and j ($i \neq j$). The ratio c_i/c_j corresponds to bias. Although s_i and s_j are usually assumed to be equal, they need not be equal, for example, if the activities differed in topography or with different PIEs as consequences (Baum, 2012b). Possibly, variation in B_O and B_N leave Equation 12 intact, because they only affect the total time available for the operant activities

In the final analysis, all behavior consists of time allocation (Baum, 2010, 2012a, 2013; Baum & Rachlin, 1969). Attaching a switch to a manipulandum like a key or lever is an easy way to estimate the time taken in interacting with the manipulandum. For example, rats typically lick, bite, chew, and paw a lever that

provides food, and these activities jiggle the lever and operate the switch fairly often (Baum, 1976). The method has its limitations, however, particularly when the activities with respect to the manipulandum change. As long as the activity time per switch operation remains the same, the correspondence between switch operations and time taken remains the same. When the activities with respect to the manipulandum change—say, involving the food hopper more or swiping the key instead of pecking it—then the conversion from switch operations to time may change, and one may be at a loss as to how to characterize the new conversion rate. More research on variation in the activities that compose operant activities may help to clarify the behavioral changes that correspond to changes in time taken up.

Conclusion

The present results support Baum's (2012a) formulation of operant activity based on *allocation*, *induction*, and *contingency*. The novel result is that r_O , so-called "extraneous" reinforcers, depends on the reinforcers produced by the operant activities. It does so because the activities on which r_O depends, represented by B_O , are induced by the reinforcers produced by the operant activities. The dependence of r_O on B_O appears to be simple proportionality, similar to a ratio schedule. The dependence of B_O on food rate was modeled here by a power function, but another function might prove more accurate in future research. The results raise questions about the matching law, because they appear to challenge the assumption that all terms in the matching equations are independent. The generalized matching law may remain unaffected, as shown in Equation 12, but the present results suggest some complexity may arise when non-operant-induced activities are incorporated in the matching equations.

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