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CONFIRMATION OF STRONG SECOND ORDER PROCESSES IN (p,t) REACTIONS ON DEFORMED NUCLEI[†]

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The (p,t) reaction on deformed nuclei has been computed with the inclusion of indirect transitions that go through intermediate rotational states. The indirect transitions are almost as large as the direct for the 2^+ state and their inclusion is essential to bring about agreement with the shape and magnitude of the differential cross section.

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Calculations of two-nucleon transfer reactions on spherical vibrational nuclei suggest that inelastic transitions and the multiple-step reactions that they allow produce sizable corrections to DWBA cross sections for allowed first-order transitions [1]. However, the shapes of the angular distributions are not appreciably modified by the higher order processes. No firm evidence one way or the other concerning these theoretical results for vibrational nuclei has been

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extracted from experiment. Recent (p,t) experiments performed at Minnesota [2] on rotational nuclei have stimulated our interest in investigating these questions where the inelastic cross sections are strongly enhanced, compared to the moderate collectivity characteristic of the vibrational nuclei.

We adopt the Bohr-Mottelson adiabatic hypothesis for the rotational wave functions of the ground band members [3].

$$| JM \rangle = \left(\frac{2J + 1}{8\pi^2}\right)^{1/2} D^J_{M0} \, X_o \tag{1}$$

We describe the internal wave function, X_o, as a BCS state [4] constructed from 20 levels around the Fermi surface. A pairing force of such a strength as to yield a gap parameter estimated by the Nilsson-Prior formula [5] was employed. The single-particle wave functions from which the BCS state is constructed are eigenfunctions of a deformed Woods-Saxon potential. These eigenfunctions were obtained on a harmonic oscillator basis comprising 15 oscillator shells ($0 \leqslant 2n + \ell \leqslant 15$). This large basis ensures a good representation of the single-particle radial wave functions and the use of the oscillator basis facilitates the calculation of the projected center-of-mass wave function of the transferred neutron pair [6]. The nuclear wave functions being thus defined, the (p,t) transitions between various rotational states of (A) and (A + 2) are determined by the parentage factors [6,7]

$$\begin{aligned} \beta_{(ab)LSJ}(J_p, J_t) &\equiv [(2J_p + 1)(1 + \delta_{ab})]^{-1/2} \langle J_p \, \| \, [d^+_a \, d^+_b]^{Lab}_{LSJ} \, \| \, J_t \rangle \\ &= (-)^J \, C^{J_p \; J \; J_t}_{o \;\; o \;\; o} \, \langle X_o(A + 2) \, | \, [d^+_a \, d^+_b]^{o}_{LSJ} \, | \, X_o(A) \rangle \end{aligned} \tag{2}$$

Here "a" denotes single-particle quantum numbers and d_a^+ creates a particle in such a state. In the first line the operators are expressed in the laboratory frame of axis, and in the second line, in the intrinsic frame. Note the factorization of the parentage amplitude into a Clebsch-Gordan coefficient depending on the spin of the states and a matrix element depending only on the multipolarity and the intrinsic structure of the nuclear states. This means that all (p,t) transitions are proportional to the direct transition of the corresponding multipole. (i.e., the form factor for the 4-pole transition connecting $2_p^+ \rightarrow 2_t^+$ is proportional to the $0_p^+ \rightarrow 4_t^+$.)

Inelastic transitions in deformed nuclei can be very accurately described in terms of a deformed optical potential [8]. The nucleus is treated as a rigid rotor that can be set into rotation by interaction of the free particle with the deformed field. The optical potential in this case is an effective interaction which replaces an explicit treatment of intrinsic nuclear excitations but the rotations are treated explicitly through solution of the appropriate system of coupled equations [9]. As discussed in detail elsewhere, the appropriate optical potential parameters will be those of nearby spherical nuclei [9,10]. For protons we use the average potential of Becchetti and Greenlees [11], since only spherical nuclei were used in that analysis. For tritons we extrapolated the average potential of Flynn et al. [12], with an assumed energy dependance of $dV/dE = -0.33$ and $dW/dE = 0$. These optical potentials together with the neutron potential for the bound states [13] are given in table I.

The shape of the nucleus enters in all three potentials, and because they all have different radii, care must be taken to ensure a consistent treatment. We solved this problem in a straightforward manner. The radius of the deformed potential for particle type k (k = p,t or n) is expressed as

$$R_k(\theta) = r_k + c\ R_\rho[1 + \sum_{\lambda=0}^{6} \beta_\lambda\ Y_{\lambda 0}(\theta)] \tag{3}$$

The constant c depends on the deformation parameters and ensures that the volume enclosed by the second part of (3) is equal to that of the sphere of radius R_ρ. For the radius R_ρ we used that of the nuclear <u>mass</u> density obtained by Myers [14] from a Thomas-Fermi treatment of the nucleus, in which agreement between the charge density and electron scattering experiments was required. The constant radius r_k may be thought of as an effective interaction radius of the particle k, and is chosen so that $r_k + R_p$ has the optical potential radius listed in table I. (This is done for both real and imaginary radii). The deformation constants β_λ in (3) were scaled from tabulated values obtained in a careful analysis of inelastic alpha scattering [8] according to

$$c\ R_\rho\ \beta_\lambda = (R\ \beta_\lambda)_{\text{alpha}}$$

which ensures that the potential with shape defined by (3) has precisely the same multipole fields as those obtained in the analysis of the alpha scattering.

Thus we have left ourselves with no adjustable parameters save a single normalization constant that scales all reaction cross sections. (No adjustments in relative cross sections are allowed).

The method used to incorporate the various indirect reactions with the direct single-step transition from the ground state, is called the source term method and is described elsewhere [1,7,15]. The results of the complete calculation is compared in fig. 1 with data of the Minnesota group [2]. The agreement is very good especially for the 0^+ and 2^+ states. This agreement would not be surprising inasmuch as the conventional DWBA treatment of the (p,t) reaction is usually successful. However, for these deformed nuclei, DWBA calculations are shown in fig. 2. The same normalization was used as for fig. 1. Now we see that while the 0^+ and 4^+ angular distributions are in fair agreement with experiment, the 2^+ is in complete disagreement. If these calculations were renormalized to the ground state, the cross sections of the 2^+ state would still be much too large. Evidently for this state, indirect processes are very large and interfere destructively with the direct amplitude so as to alter the angular distribution shown in fig. 2 and reduce the cross section to bring about the agreement shown in fig. 1. Indeed this is born out by the calculations shown in fig. 3. Cross sections corresponding to the three most important amplitudes feeding the 2^+ state are shown. It can be seen that the direct amplitude is not much stronger than the indirect amplitudes going through the target 2^+ state and the residual nucleus ground state.

We are accustomed to thinking that second-order direct reactions are considerably weaker than allowed first-order ones, if not entirely negligible. Can these unexpected results be made plausible? The answer is yes. First it is necessary to consider the meaning of the transfer of a pair of nucleons from one rotational state to the other. The cross section for the direct pickup of a pair of neutrons from the ground state to produce the rotational state J^+ measures the probability that all of the rotational motion is possessed by the pair. (This statement is more transparent for the stripping reaction). This probability clearly ought to decrease with increasing angular momentum, and indeed it does as can be seen by the decreasing cross sections for higher rotational levels in fig. 2. Now the reaction branch of the two indirect routes in fig. 3 can go by the J = 0 transition whereas the direct one has to be J = 2. Of course, the indirect routes have at least two branches, the other consisting of an inelastic transition. But these are enhanced in deformed nuclei. Moreover the crucial point is that the probability for an indirect transition is not given by the disjoint product of the probabilities for the individual branches, but by the conditional probability that one step having been taken, the second is more probable since the free particle is already in the vicinity of the nucleus.

We summarize our results: The cross section for the direct two-nucleon pickup leading to the ground state has the correct angular dependance but that for the 2^+ rotational state is much too large relative to the ground state and has a grossly incorrect angular dependance. However, when the amplitudes for indirect modes of producing the 2^+ are allowed to interfere with the direct, the resulting cross section is much reduced and strongly altered in angular dependance. This alteration comes about because each of the three most important amplitudes has a different angular shape, (see fig. 3) and it is magnified since they interfere destructively (which we know since the resultant cross section, fig. 1, is smaller than that due to any single of these three processes). That the destructive interference between three such different amplitudes results in agreement with experiment, we consider to be very hard evidence for the accurracy of each of them. To our knowledge this is the firmest confirmation of second-order direct reactions. In this instance they are almost as strong as the allowed first-order reaction.

We are very much indebted to N. M. Hintz for an early communication which stimulated our interest in the (p,t) reaction on deformed nuclei.

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Table I. Potential Parameters. Those labelled "C.C." were used in the coupled channel calculation and are derived from Ref. 11 and 12. Those labelled "elastic" yield the same elastic cross sections obtained in the coupled channel calculation.

	V	W	W_D	r_v	r_w	r_c	a_v	a_w	V_s	r_s	a_s
					Proton						
CC	- 57.83	- 1.48	- 9.5	1.17	1.32	1.125	0.75	0.653	-6.2	1.01	0.75
elastic	- 54.65	- 1.48	-10.4	1.2	1.27	1.125	0.698	0.893	-6.2	1.01	0.75
					Triton						
CC	-169.	-12.6	0	1.16	1.498	1.125	0.752	0.817	0		
elastic	-248.	-13.6	0	0.938	1.346	1.125	0.846	1.141	0		
					Neutron						
bound	- 46.4			1.241			0.66		-8.23	1.241	0.66

FIGURE CAPTIONS

Fig. 1. Cross sections for members of the ground state rotational band in ^{174}Yb produced by 19 MeV protons in the (p,t) reaction. Calculations include the indirect modes of excitation due to inelastic excitation of rotational states in both the target and final nucleus, as well as the direct transition. The only adjusted parameter was an overall normalization which does not alter the computed relative cross sections. The experimental data is from the Minnesota Group (see Ref. 2).

Fig. 2. Two DWBA calculations without inelastic processes are compared with experiment using the same normalization as Fig. 1. Note that the relative cross sections are in disagreement and the 2^+ angular distribution does not resemble experiment. The solid curves corresponds to the same optical model parameters as were used in the coupled channel calculation of Fig. 1, while the dashed curves corresponds to adjusted parameters that reproduce the proton and triton elastic scattering obtained in the coupled channel calculation, and therefore correspond to the conventional DWBA.

Fig. 3. Cross sections for production of the 2^+ rotational state corresponding to the individual transfer processes shown. The solid curve is the direct processes while the other two proceed through an intermediate level, the 2^+ in the target, and the ground state of the final nucleus, respectively. Note that all three processes acting alone overestimate the observed cross section, relative to the ground state cross section. It is the interference among these various modes which produces the agreement with experiment shown in fig. 1.

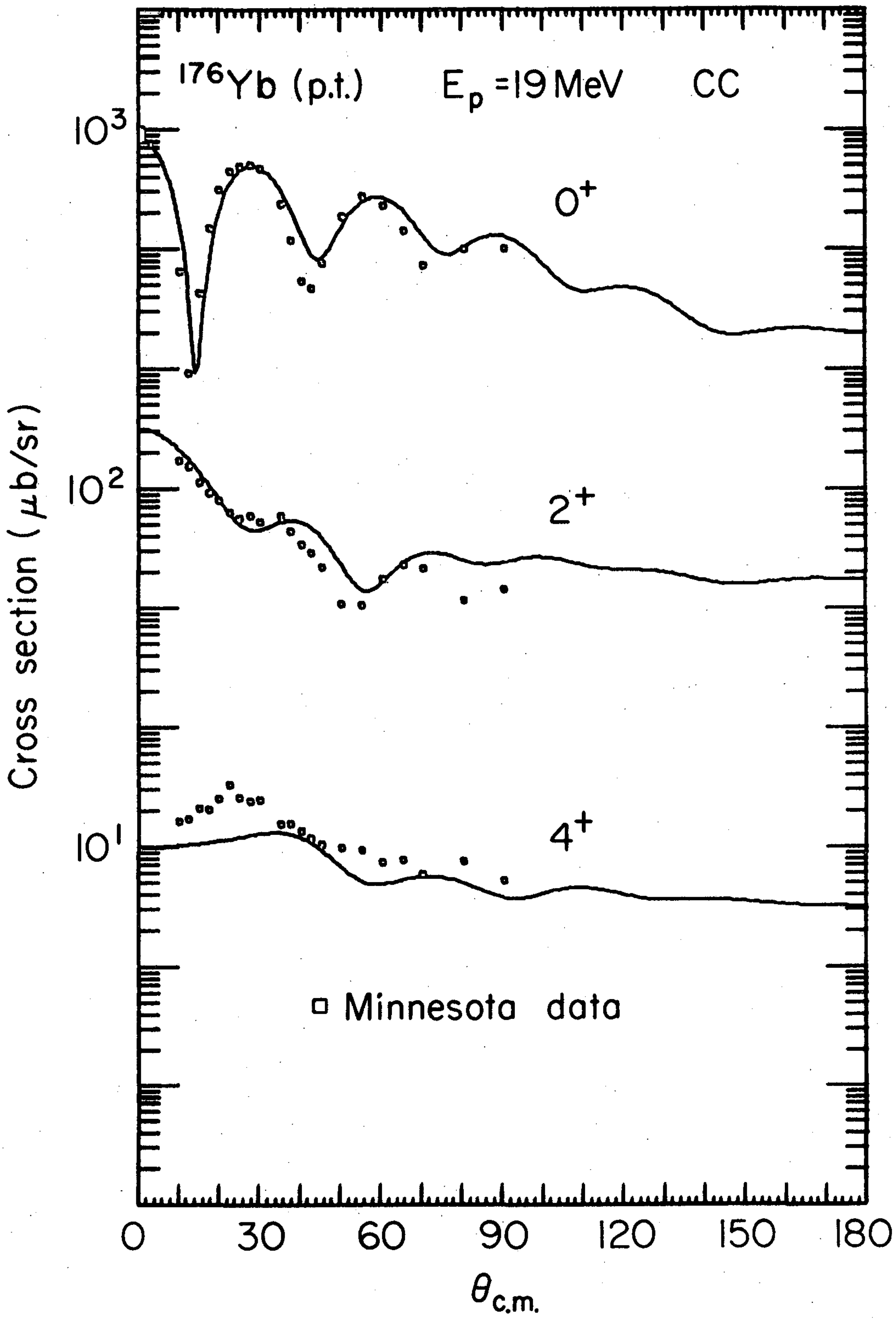

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Fig. 1

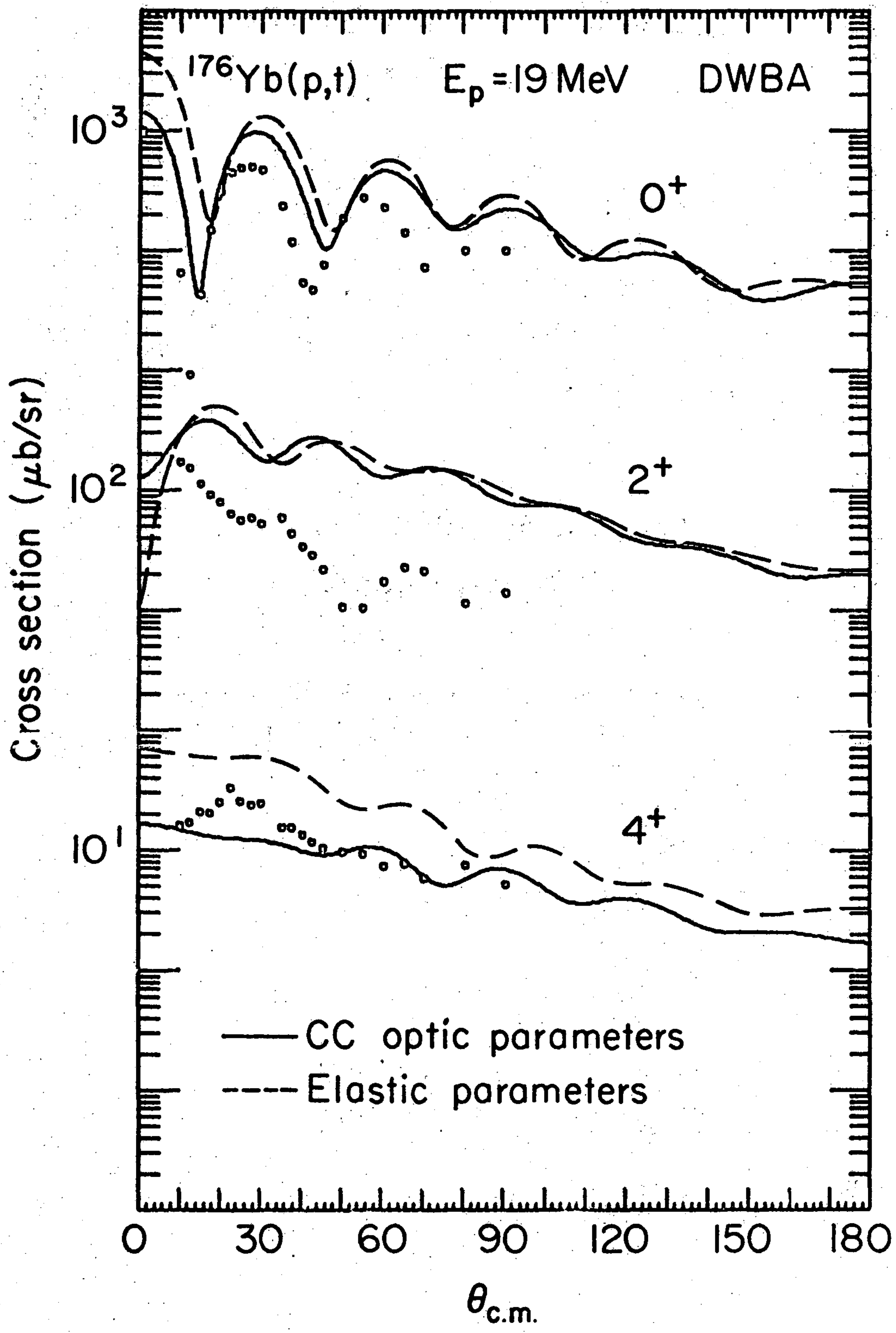

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Fig. 2

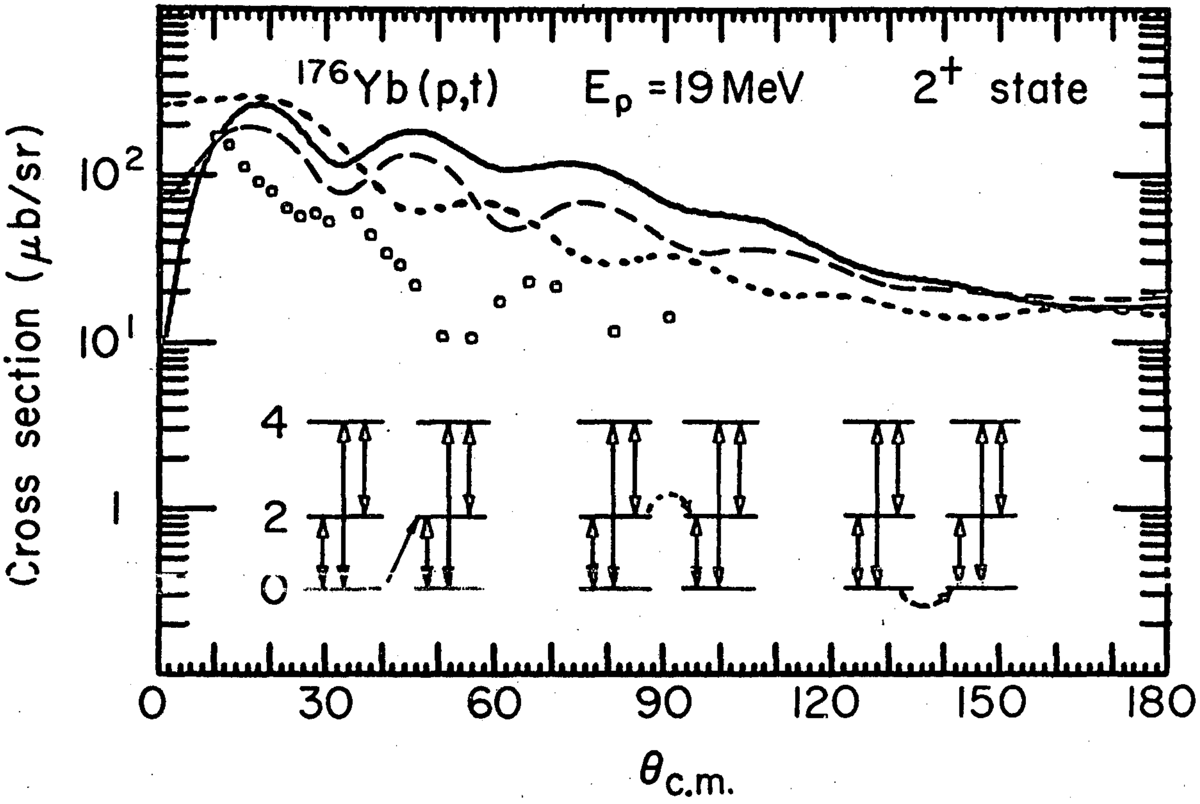

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Fig. 3

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