UC Irvine
UC Irvine Previously Published Works

Title
Estimates of dense plasma heating by stable intense electron beams

Permalink
https://escholarship.org/uc/item/620499q8

Journal
Plasma Physics, 14(12)

ISSN
0032-1028

Authors
Guillory, J
Benford, G

Publication Date
1972-12-01

DOI
10.1088/0032-1028/14/12/007

License
CC BY 4.0

Peer reviewed

ESTIMATES OF DENSE PLASMA HEATING BY STABLE INTENSE ELECTRON BEAMS

J. GUILLORY

Physics International Company, San Leandro, Calif. 94577, U.S.A.

and

G. BENFORD

Physics Department, University of California, Irvine, Calif. 92664, U.S.A.

Abstract—The plasma current induced by a high-current electron beam generates electron–ion streaming instabilities even if the electron beam itself is stable to electron–electron beam-plasma modes. We review the temperature- and current-dependence of the electron–ion mode and use a *simple model resistivity to estimate attainable electron temperatures. For beams in the 100 nsec* 10 kA/cm² range, the heating is by ion-acoustic oscillations, and if convective losses are prevented, keV temperatures can be expected in moderately dense (10^{14} cm^{-3}) plasma.

1. INTRODUCTION

ONE OF the most promising applications of high-current electron beams is heating of background plasma. The diamagnetic return current induced in the plasma is usually of the same order as the beam current, so that electron–ion streaming instabilities must obviously be generated by the return current, and very significant heating should result.* If the initial medium is weakly preionized nitrogen, for example, it has been found that beam energy loss increases from a few per cent at 1 torr to $\geqslant 50$ per cent at 0.2 torr, although it is not clear how much of the energy lost goes into plasma thermal energy. If the initial medium is not highly ionized it becomes so during the beam duration. (In a numerical example later in the paper we use fully ionized hydrogen, both for simplicity and for higher anomalous resistivity.)

The plasma return current will be approximately equal and opposite to the beam current if the magnetic diffusion time $\pi a^2/\eta c^2$ (a = beam radius, η = plasma resistivity) is long compared with the time scale of changes in beam current (YONAS *et al.*, 1969). The return current will be spatially restricted to the beam channel if $\omega_e^2 a^2/c^2 \gg 1$ (ω_e is the background electron plasma frequency) (HAMMER and ROSTOKER, 1970). Ordinarily the plasma density n_p must exceed the beam density $n_B \sim 10^{12}$–10^{13} by at least an order of magnitude. Current neutralization thus means an average drift velocity of plasma electrons $u \sim -V_B n_B/n_p$ where V_B is the beam velocity (usually $V_B \sim c$).

Currents induced by external fields will heat in the same fashion—the microscopic mechanisms are the same. Use of plasma currents induced by a penetrating beam, though, avoids the propensity of the plasma to shield out applied external fields except in a skin depth layer of width c/ω_e on the low-density plasma surface. This makes heating by beams potentially more attractive than through applied ac or dc fields. The advent of very high current beams strengthens this argument.

When large currents are induced in a plasma by an electric field, E_z, electron–ion streaming instabilities generate turbulence. Many classic papers deal with the topic (BOHM and GROSS, 1949; BUNEMAN, 1959; STRINGER, 1964); recent papers (e.g.,

---

* The possibility of such heating has been formulated independently in concurrent work by LOVELACE and SUDAN (1971).

HSIEH and LICHTENBERG, 1970) deal with the comparison of experiment and theory. For streaming plasma electrons with low thermal speed, v, the instability is that described by Buneman; when $v \geqslant u$ the instability is labeled ion-acoustic. In Section 2 we show that both collision and thermal spread have a similar and relatively weak effect upon the ion-acoustic instability, and thus do not appreciably retard heating of a collisional plasma unless the voltage rise rate is exceptionally large. The heating rate may then be estimated.

The "ohmic" heating rate of plasma by electron–ion streaming instability will depend strongly on the ratio of the average (streaming) velocity, u, to the electron thermal speed, v. If $u \gg v$ the turbulent resistivity will be much higher than if $u \ll v$. Actual calculation of the resistivity is quite difficult and is not attempted here. Some experimental results and some theoretical work seem to indicate $\eta \propto \gamma_{max}$ (where γ_{max} is the linear growth rate of the most unstable wavelength, for given u and v) although the constant of proportionality is poorly known.

For this reason we first review the linear theory of the electron–ion instability. Since relatively dense plasma is to be heated, and since at the onset it may also be collision-dominated, our analysis will include a phenomenological collision frequency v for electron momentum transfer by collisions with ions or neutrals. The effect of collisions was first discussed by BOHM and GROSS (1949) and subsequently by TIDMAN (1961). Our results will be stated more approximately and succinctly as a modification of the growth rate curves $\gamma_k(u, v, v)$. The results will justify neglect of collisions in the regime of interest. Without attempting to treat the nonlinear state of the instability, we will then use the dependence of γ_{max} on $u(t)$ and $v(t)$ to give an approximate resistivity for use in estimating the electron heating.

2. LINEAR THEORY OF THE ELECTRON–ION MODE WITH COLLISIONS AND THERMAL SPREAD

As a model we take the distribution of electron velocities in the axial direction to be a Lorentzian with average speed $u(t)$ with velocity spread $v(t)$,

$$f_0(V) = \frac{v/\pi}{v^2 + (V - u)^2} .$$

The perpendicular velocities may be taken Maxwellian and we will assume that heating by the instability is sufficiently isotropic that the perpendicular thermal spread is also $\approx v$. We have used a Lorentzian rather than a Maxwellian distribution for several reasons. First, it yields a tractable dispersion relation in which the relation between collisions and thermal spread is apparent. Second, if the plasma is not totally ionized as the pulse begins, further ionization during heating will preferentially contribute low-velocity electrons and thus a very thick low velocity "tail." Thus, as the rate of electron scattering by the oscillations approaches the electron–electron collision rate, $f_0(V)$ will proceed more toward equilibrium with the ions. The Lorentzian represents a more reasonable trade-off between electron–electron and electron–ion equilibration as the instability progresses well into the nonlinear stage. Computer experiments (BISKAMP and CHODURA, 1971; MORSE and NIELSON, 1971) support this assumption; they show the long-time-scale evolution of a Maxwellian to a broader, flatter (more nearly Lorentzian) form, due to ion acoustic instability. Fourth, the excess of electrons in the *high* velocity tail of the drifting Lorentzian play no important role in the instability.

It is well known that the ion-acoustic oscillations are unstable for a wide range of θ, where $\mathbf{k} \cdot \mathbf{u} = ku \cos \theta$. Indeed, in the absence of collisions the growth rate γ_k has the form $\gamma_k \sim \varepsilon^{1/2}(k_z u - |k| c_s)$ where c_s is the ion sound speed and $\varepsilon = m_e/m_i$ (SLOAN and DRUMMOND, 1970). For $u \gg c_s$ the instability is relatively isotropic. Once the wave energy saturates this means reasonably isotropic heating. To render our calculation tractable we use a one-dimensional model for the dispersion relation (integrating over velocities perpendicular to the beam) and assume that the results hold approximately for the realistic situation.

We describe electron-neutral and electron–ion collisions with an Ohm's law,

$$\frac{4\pi}{\omega_e^2} j_p(v - i\omega) = \tilde{E}_z,$$

where $\tilde{E}_z$ is the oscillating field of the instability and v is the electron collision frequency for momentum transfer. The electrostatic dispersion relation for frequency $\omega = \omega_r + i\gamma_k$ is simply

$$\frac{\omega_i^2}{\omega^2} + \frac{\omega_e^2}{(\omega - ku + ikv)(\omega - ku + ikv + iv)} = 1, \tag{1}$$

where ω_e and ω_i are the electron and ion plasma frequencies, respectively, and k is the (real) parallel wavenumber. Note that the limiting form for $v = 0$, $v = 0$ is just the usual relation for cold streaming electrons.

Figure 1 shows the computed growth rate γ_k vs. k for a hydrogen plasma. As expected, maximum growth $\gamma_{\max}$ for given u and v occurs at $k \approx \omega_e/u$ when $u > v$. The $v = 0$ curve in Fig. 1a reproduces the result of Buneman (BUNEMAN, 1959). Collisions appreciably lower γ_k for small thermal spread v/u, but are unimportant when $v/u \gtrsim \xi$, where ξ is some number small compared with unity and it appears for a limited range of ion masses that $\xi \sim \varepsilon^{1/3}$, with ε the electron–ion mass ratio, m/M. The influence of v for small v/u is somewhat greater for heavier ions. The overall effect of collisions is rather slight for $v/u \gtrsim 1$, as predicted qualitatively by Buneman; this is important for heating of collisional plasma.
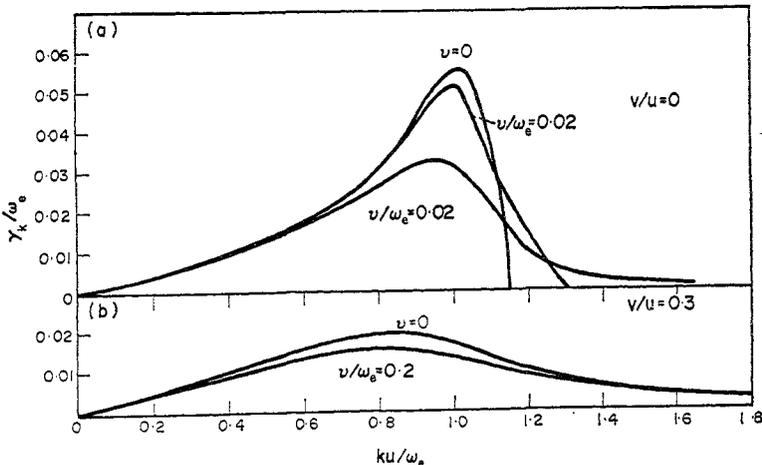

FIG. 1.—Growth rate γ_k vs. ku/ω_e. (a) zero thermal spread. (b) $v/u = 0.3$.

The effect of increasing v/u on $\gamma_{\max}$ is shown in Fig. 2 for hydrogen and argon plasma. The region $v/u \ll 1$ represents the Buneman instability for which $\gamma_{\max} \sim \varepsilon^{1/3}\omega_e$. There is a smooth but pronounced transition to the region of ion-acoustic instability $(v/u \gtrsim 1)$, for which $\gamma_{\max} \sim \varepsilon^{1/2}\omega_e u/v$. The transition occurs at smaller v/u for the Lorentzian distribution than for the Maxwellian used by Stringer (STRINGER, 1964) which is also shown. This is expected because of the broader tail of the Lorentzian; there are more electrons at lower velocities. Larger ion mass further sharpens
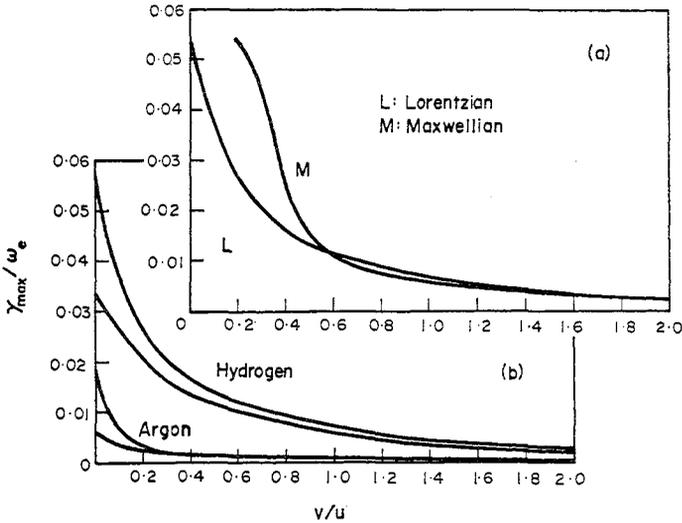

FIG. 2.—Growth rate of fastest growing wave (in units of ω_e) as a function of v/u. (a) comparison of collisionless results (for small T_i) using Maxwellian (M) and Lorentzian (L) electron distributions in hydrogen. (b) effect of collisions on growth rate for hydrogen (upper 2 curves) and argon (lower 2 curves). For each species the upper curve is for $v = 0$ and the lower one for $v = 0.2\omega_e$.

the transition and shifts it to smaller thermal spread. We also note that with cold ions no cutoff of growth occurs at $v \sim u$ (BUNEMAN, 1959; JACKSON, 1960; STRINGER, 1964).

For the fastest growing mode the real part of the frequency ω_r decreases with increasing v/u, as does γ_k, but in a manner almost completely independent of v. The parallel phase velocity of this mode (also independent of v) increases from $0.4\varepsilon^{1/3}u$ to the ion sound speed $c_s = \varepsilon^{1/2}v$ as v increases from zero to $v \gg u$. (Of course, at very high $v/u \sim \varepsilon^{-1/2}$ the IA instability ceases. The actual stability boundary depends on magnetic field and ion temperature, but this limit is uninteresting for our purposes.)

Holding v constant while varying u or v is not strictly consistent. In general the u and v dependence must be folded in, but the important point is that, for $v/u \gtrsim 1$, v is unimportant even for quite large values, $v \approx 0.2\omega_e$. This contrasts with the importance of collisions in damping the electron–electron beam-plasma mode,[*]

_____

[*] Linear theory of the electron-ion mode is formally identical, in the lab frame; with the electron–electron mode viewed in the beam frame; one simply replaces m/M by n_b/n_p (beam density/plasma density). Plasma electrons in either case are nonresonant. The nonlinear theory differs in the time scale for ion or beam dynamics as compared with those for the background electron distribution.

an effect even more important when the beam velocity spread is large (BOHMER, CHANG and RAETHER, 1971).

3. HEATING BY INTENSE RELATIVISTIC BEAMS

The large deviation of electron trajectories from paraxial in intense relativistic electron beams (BENFORD and ECKER, 1969; CLARK and LINKE, 1969; YONAS and SPENCE, 1970) gives rise to a spread in beam velocity* which is usually sufficient to stabilize electron–electron two-stream interactions with the background plasma electrons. The usual Landau damping of the instability when $\mathbf{k} \cdot \Delta V_B >$ growth rate is somewhat enhanced if electron collisions with ions or neutrals are frequent (SINGHAUS, 1964; BOHMER, CHANG and RAETHER, 1970).

As a result, the relativistic electron beam itself may be stable and serve only to drive the unstable reverse current in the plasma. The rigidity of the beam electrons (negligible change in velocity for large oscillatory forces) means that the beam is unaffected by the electron–ion instability of the plasma through which it passes. We estimate that the scattering of beam electrons on ion-acoustic turbulence is insignificant for distances less than 10 m in the plasma, whereas present and projected experiments occur in less than 3 m.

The same transverse beam thermal energy just described, plus the fact that at high density j_p nearly cancels j_B resulting in negligible net magnetic field, means that some magnetic field must be applied to contain the beam. Experiments with axial applied fields B_z (ANDREWS et al., 1970; STALLINGS et al., 1971) and azimuthal fields B_θ from Z-pinches (BENFORD and ECKER, 1971) have been done, as well as some in which the confinement of the beam is incomplete and appears due to self fields (poor current neutralization) or to image currents in the walls of the tube (YONAS and SPENCE, 1969; ANDREWS et al., 1970; BRADLEY et al., 1971). Yet the applied field strengths required for radial beam containment are relatively small: the cyclotron frequency ω_c is much less than ω_e in most cases of plasma density $n > 10^{12}$. Because of this the spectrum of unstable waves in the electron–ion instability is broad in angle whenever n is high enough to give $\omega_c \ll \omega_e$ (BISKAMP and CHOUDRA, 1971).

For simplicity we will assume an axial magnetic field in order to avoid considerations of cyclotron-type instabilities.

Thus, it is reasonable to assume that the electron–ion instability produces roughly isotropic heating of electrons by "effective collisions" with unstable waves. Ions are also heated, to an extent dependent on v/u (STRINGER, 1964; SAGDEEV and GALEEV, 1969), but if T_e/T_i is large initially, then it remains large (SAGDEEV and GALEEV, 1969; TSYTOVICH, 1971). For simplicity it will be assumed that T_e/T_i is large. Treating these "effective collisions" with waves as an isotropic joule-heating mechanism, one has roughly (ignoring convective and radiative energy losses)

$$\tfrac{3}{2} n \frac{\mathrm{d}T_e}{\mathrm{d}t} \approx \eta j_p^2 \tag{2}$$

where $\eta = 4\pi\nu_s/\omega_e^2$, with ν_s the effective collision rate. This joule heating model also assumes that heating by the waves is limited spatially to the region where j_p is flowing.

---

* In the absence of applied magnetic fields the transverse energy $\varepsilon_\perp$ is related to the directed energy $\varepsilon_\parallel$ through the beam density n_B: $\varepsilon_\perp/\varepsilon_\parallel = e^2 n_B/\gamma m c^2$, with γ the usual relativistic factor.

When $\omega_p a/c \gg 1$ (a = beam radius), this region is just the relativistic beam region*
(HAMMER and ROSTOKER, 1970).

The dependence of the anomalous resistivity η (or equivalently, of v_s) on u and
v is in considerable dispute (for a review, see SELF, 1970). Here both u and v are
time-varying quantities. The dependence of v_s on "d.c." electric field strength E_z is
through $u(t)$, which is assumed here to be given by approximate current neutralization
of the known beam current density $j_B(t)$. Since E_z and v_s vary throughout the duration
of the beam current, there is no terminal velocity as in fixed-field heating problems
(FIELD and FRIED, 1964; SIZONENKO and STEPANOV, 1970; HIROSE et al., 1970).
The quantity v_s has been estimated at widely different values by different authors,
with some experimental justification for the diversity (BUNEMAN, 1959; SAGDEEV
and GALEEV, 1969; DUPREE, 1970; SIZONENKO and STEPANOV, 1970; RUDAKOV
and TSYTOVICH, 1971; TSYTOVICH, 1971). As a somewhat optimistic (high) value
for v_s we use $v_s \approx \gamma_{\max}(u, v)$ as first predicted by BUNEMAN (1959) for $v/u < 1$.
The extension to $1 < v/u < \varepsilon^{-1/2}$ appears approximately justified by the more con-
vincing recent work (e.g., TSYTOVICH, 1971).

A good fit to the electron-ion mode growth rate (Fig. 2) for hydrogen is

$$\frac{\gamma_{\max}(u, v)}{\omega_e} \approx \frac{0{\cdot}055}{1 + 5{\cdot}9v/u}$$

and if $v/u \gtrsim 1$ this can be simplified to

$$\frac{\gamma_{\max}(u, v)}{\omega_e} \approx 0{\cdot}9 \times 10^{-2}u/v \tag{3}$$

for the ion acoustic regime. Using $\eta = 4\pi\gamma_{\max}/\omega_e{}^2$ and $T_e \approx \frac{1}{2}m_s v^2$† in (2) then gives
an approximate differential equation for the heating of a constant-density plasma, in
which u and v change with time and in which loss processes are neglected as discussed
above.

As a specific example, we shall estimate the electron heating of a fully ionized
plasma due to a high current pulse of duration $\sim$100 nsec, such as would be induced
by an intense electron beam with peak current density $j_B \sim 10^4 A/\mathrm{cm}^2$. The differential
equation is integrable if the Buneman $v \ll u$ regime lasts for only a negligibly short
time and we find for a beam pulse of duration t_0,

$$T_e^{3/2}(t \leq t_0) \approx 10(n_p/10^{14})^{-5/2} \int_0^t j_B{}^3(t')\,\mathrm{d}t' \tag{4}$$

with T_e in eV, n_p in cm^{-3}, t in sec, and j_B in A/cm^2.

For fully ionized hydrogen at $n_p = 10^{14}$ cm^{-3} and a linear rise in j_B to 10^4 A/cm^2
in 100 nsec, T_e reaches 3^+ keV. The plasma current falls with the beam current at
the end of the pulse because the large inductance of the plasma maintains $j_B - j_p$
small. For all but very early times in the pulse, one has

$$v/u \approx 10^{-11}n_p T_e^{1/2}/j_B(t) > 1$$

---

* This radius, for short beam propagation distances, is determined by the initial beam radius at
the generating diode; at much larger distances from the diode it is determined by magnetic pressure
balance.

† Because v in Fig. 2 is the velocity spread parameter of a Lorentzian distribution in parallel
velocities, its use as a "thermal" velocity is only approximate.

(v/u varies as $t^{1/3}$ during the triangular pulse) so that essentially all the heating is by the ion-acoustic mode and no significant runaway electrons should be generated for such a modest current rise rate. This justifies our use of $v/u > 1$ in the resistivity formula.

The ion temperature is limited to less than $T_e \varepsilon^{1/2} v/u$ (SAGDEEV and GALEEV, 1969) and does not linearly stabilize the ion-acoustic mode or affect the magnitude of the growth rate (STRINGER, 1964). Electrons and ions will equilibrate in a time of order $(\nu_s \varepsilon)^{-1}$, long compared with the effective scattering time and usually longer than one beam duration after the beam is turned off. Very little heating occurs after beam turnoff because the decaying net current neu is small and v/u is large. (The net current density decays from a peak value of only about $j_B [1 - \exp(-t_0/\tau_M)]$, with τ_M a typical magnetic diffusion time during the pulse.)

A unique feature of heating using an intense beam as voltage driving mechanism is that the voltage is applied by dj_B/dt rather uniformly throughout the beam volume, without the skin-depth phenomena that occur when voltage is applied to bounding end electrodes (WARNKE, 1970) or the impedance collapse when plasma current becomes large in the usual open-ended turbulent heating experiment (HSIEH and LICHTENBERG, 1970). In this respect it resembles Tokamak heating mechanisms but has a faster time scale.

Of course induced plasma currents are not the only possible source of ion-acoustic wave excitation in the presence of relativistic beams. If the momentum spread of the beam can be made small enough or the plasma resistivity low enough, Langmuir oscillations will grow to sufficient amplitudes that they can decay into ion-acoustic waves and thus heat the plasma (TSYTOVICH, 1970). This mechanism has recently been proposed (ALTYNTSEV et al., 1971) to explain the dramatic results of a series of heating experiments with a nearly monoenergetic beam, where direct excitation of ion-acoustic instability was ruled out as unimportant for heating at the lower densities ($n_p \sim 10^{11}$). Even at $n_p \sim 10^{14}$ such agencies may play prominent roles in ion-acoustic excitation. We have shown here that temperatures in the several keV range may optimistically be expected even without instability of the beam itself, at easily attainable current densities and interestingly high plasma densities.

REFERENCES

ALTYNTSEV A. T., BREIZMAN B. N., ESKOV A. G., ZOLOTOVSKY O. A., KOROTEEV V. I., KURTMULLAEV R. K., MASALOV V. L., RYUTOV D. D. and SEMENOV V. N. (1971) *International Atomic Energy Agency Conference on Plasma Physics and Controlled Nuclear Fusion Research*, Madison, Wisconsin (IAEA/CN-28) Paper E20.
ANDREWS M., BZURA J., FLEISCHMAN H. and ROSTOKER N. (1970) *Phys. Fluids* **13**, 1312.
BENFORD J. and ECKER B. (1971) *Phys. Rev. Lett.* **26**, 1160.
BISKAMP D. and CHODURA R. (1971) *International Atomic Energy Conference on Plasma Physics and Controlled Nuclear Fusion Research*, Madison, Wisconsin (IAEA/CN-28) Paper E17.
BOHM D. and GROSS E. P. (1949) *Phys. Rev.* **75**, 1864.
BOHMER H., CHANG J. and RAETHER M. (1971) *Phys. Fluids* **14**, 150.
BRADLEY L. P., MARTIN T. H., PRESTWICH K. R., BOERS J. E., JOHNSON D. L. and COOPERSTEIN G. (1971) to be published.
BUNEMAN O. (1959) *Phys. Rev.* **115**, 503.
CLARK J. J. and LINKE S. (1969) Cornell University Laboratory for Plasma Studies Report LPS-23.
DUPREE T. H. (1970) *Phys. Rev. Lett.* **25**, 789.
FIELD E. C. and FRIED B. D. (1964) *Phys. Fluids* **7**, 1937.
HAMMER D. A. and ROSTOKER N. (1970) *Phys. Fluids* **13**, 1831.
HIROSE A., ALEXEFF I., JONES W. D., KUSH S. T. and LONNGREN K. E. (1970) *Phys. Rev. Lett.* **25**, 1563.

HSIEH C. L. and LICHTENBERG A. J. (1970) *Phys. Fluids* **13**, 2317.
JACKSON E. A. (1960) *Phys. Fluids* **3**, 786.
JACKSON J. D. (1960) *J. nucl. Energy C* **1**, 171.
LOVELACE R. V. and SUDAN R. N. (1971) *Phys. Rev. Lett.* **27**, 1256.
MORSE R. L. and NIELSON C. W. (1971) *Phys. Rev. Lett.* **26**, 3.
RUDAKOV L. I. and TSYTOVICH V. N. (1971) *Plasma Phys.* **13**, 213.
SAGDEEV R. Z. and GALEEV A. A. (1969) *Nonlinear Plasma Theory* (Edited by T. M. O'Neil and D. L. Book) W. A. Benjamin, New York.
SELF S. A. (1970) Stanford University Institute for Plasma Research Report SU-IPR 403.
SINGHAUS H. E. (1964) *Phys. Fluids* **7**, 1534.
SIZONENKO V. L. and STEPANOV K. N. (1970) *Nucl. Fusion* **10**, 155.
SLOAN M. L. and DRUMMOND W. E. (1970) *Phys. Fluids* **13**, 2554.
STALLINGS C., SHOPE S. and GUILLORY J. (1972) *Phys. Rev. Lett.* **28**, 653.
STRINGER T. E. (1964) *J. nucl. Energy* C6, 267.
TIDMAN D. A. (1961) *Phys. Fluids* **4**, 1379
TSYTOVICH V. N. (1970) *Nonlinear Effects in Plasma*, Plenum Press, New York.
TSYTOVICH V. N. (1971) *Plasma Phys.* **13**, 741.
WARNKE D. F. (1970) Ph.D. Thesis, Ohio State University.
YONAS G., SPENCE P., ECKER B. and RANDER J. (1969) Physics International Company, San Leandro, California, Report PIFR-106-2.
YONAS G. and SPENCE P. (1970) in *Tenth Symposium on Electron, Ion and Laser Beam Technology*, (Edited by L. Marton), p. 143. San Francisco Press, California.