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Heat transfer enhancement in a ribbed channel: Development of turbulence closures

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ABSTRACT

The ability to accurately predict turbulent heat transfer in massively separated flows is of immense practical importance especially in the field of heat transfer enhancement in compact heat exchangers. This paper describes new developments in the modeling of the flow and the turbulent heat fluxes in a representative benchmark flow namely that in a heated channel with periodic surface ribs. This flow, which is well-documented by experiments, poses severe challenges to conventional closures due to the significant non-equilibrium effects that are present. Several closure strategies were therefore considered ranging from the eddy-viscosity closures that are routinely used in practice, to the more sophisticated full differential transport closures that can better capture rapidly-evolving flow and thermal fields. As the heat transfer rates are largely determined by the flow conditions in the near-wall region, low Reynolds number versions of these closures were also considered. As for the turbulent heat fluxes, two alternative models were considered: the conventional Fourier's law, and a more complete, algebraic model which is explicit in these fluxes and which correctly allows for their dependence on the turbulent stresses and on the gradients of mean velocity. The models were implemented in the open source software OpenFOAM and the computations were performed with cyclic boundary conditions that are appropriate for this periodic flow. Details of models implementation are reported. Comparisons with experimental measurement indicate significant improvements over existing approaches.

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1. Introduction

There are many applications in engineering where enhancement of the heat-transfer rates by active means provides benefits that far outweigh the associated penalty of increased pressure loss. Amongst the more practical techniques used is the placement of surface ribs with axes perpendicular to the direction of flow. This is the geometry of interest here, shown schematically in Fig. 1 where the coordinates system is also defined. Typically, several identical ribs are used such that after an initial period of development, periodic flow conditions are established. Flow reversal invariably occurs in the gaps between successive ribs. Depending on the ratio of pitch to rib height, the flow within the gap may either form a single vortex that extends across the entire region in between successive ribs, or it may separate from the corner of an upstream rib, re-attach within the gap and then separates again upstream of the following rib. Our interest here is in the latter case where the conditions render the mean flow and the turbulence

structures far removed from equilibrium. The departure from equilibrium conditions poses a severe test for turbulence closures that are used in practice since non-local effects become quite large, and the assumption that the turbulent stresses and heat fluxes are determined by local conditions becomes invalid. In this paper, we report on research that was performed to improve our ability to accurately predict the extent of heat-transfer enhancement in these flows.

The geometry depicted in Fig. 1 was studied experimentally by Drain and Martin [2] who used Laser Doppler Velocimetry to obtain mean-flow and turbulence data for water flow in a ribbed channel with a relative rib height of $e/h = 0.4$, a pitch to height ratio of 7.2, and a Reynolds number of 37,000. The corresponding heat transfer measurements were conducted by Liou et al. [3] who performed real-time holographic interferometry at a Reynolds number of 12,600. A uniform heat flux boundary condition was applied at the lower wall by means of a controllable thermal foil. The upper wall was manufactured from Bakelite and was further insulated to obtain adiabatic conditions. The sidewalls of the duct were made of Plexiglas to provide optical access for the interferometry measuring devices. The measurements were performed at

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solvers suitable for a wide variety of applications. Here, interest is in solving the time-averaged equations governing conservation of mass, momentum and energy. In Cartesian-tensor notation, and for the steady flow of constant-property fluid, these equations are given as:

$$\frac{\partial U_i}{\partial x_i} = 0, \quad (1)$$

$$U_j \frac{\partial U_i}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\nu \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \overline{u_i u_j} \right) - \frac{1}{\rho} \frac{\partial p}{\partial x_i}, \quad (2)$$

$$U_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\frac{\lambda}{\rho c_p} \frac{\partial T}{\partial x_j} - \overline{u_j T} \right). \quad (3)$$

In the above, U_i , T and p are respectively the time-averaged velocity, temperature and pressure, ρ , ν and λ are the fluid density, kinematic viscosity and thermal diffusivity. Repeated indices imply summation.

2.2. Turbulence models

The turbulence correlations $\overline{u_i u_j}$ in Eq. (2) were obtained by using two distinct types of turbulence closures namely those that are algebraic in these correlations (eddy-viscosity closures), and those that obtain these correlations from the solution of differential transport equations in which they are the dependent variables (Reynolds-stress transport closures). The rationale for assessing the performance of both types is that eddy-viscosity closures are routinely used in engineering design calculations while Reynolds-stress transport closures are better able to represent the non-equilibrium effects that dominate the flows under consideration. Presenting results obtained with both offers a good opportunity to assess the merits and de-merits of each.

The eddy-viscosity closures are based on Boussinesq's assumption of a linear stress-strain relationship:

$$-\overline{u_i u_j} = 2\nu_t S_{ij} - \frac{2}{3} \delta_{ij} k, \quad (4)$$

where ν_t is the turbulent kinematic viscosity, k is the turbulence kinetic energy, and S_{ij} is the mean rate of strain:

$$S_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right). \quad (5)$$

Several alternative eddy-viscosity closures are currently in use in engineering practice and these differ mainly in the choice of variables to characterize the time and length scale of turbulence. The two most widely used models in this category are the $k-\epsilon$ and the $k-\omega$ models where ϵ is the rate of dissipation and ω is the specific dissipation.

The $k-\epsilon$ model equations are:

$$\nu_t = C_\mu \frac{k^2}{\epsilon}, \quad (6)$$

$$U_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + P_k - \epsilon, \quad (7)$$

$$U_j \frac{\partial \epsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial x_j} \right] + C_{\epsilon 1} \frac{\epsilon}{k} P_k - C_{\epsilon 2} \frac{\epsilon^2}{k}, \quad (8)$$

where P_k is the rate of production of turbulence kinetic energy:

$$P_k = 2\nu_t S_{ij} S_{ij}. \quad (9)$$

The values assigned to the various coefficients are the standard ones given in Table 1.

Table 1

The $k-\epsilon$ model coefficients.

C_μ	$C_{\epsilon 1}$	$C_{\epsilon 2}$	σ_k	σ_ϵ
0.09	1.44	1.92	1.0	1.3

For reasons related to the boundary conditions that are applied to the ϵ equation at the walls, it was found in several previous studies of separated turbulent flows that better predictions can be obtained by restricting the application of the $k-\epsilon$ to the outer regions of the flow where the boundary conditions for ϵ can be assigned with no ambiguity while using the $k-\omega$ model in the near-wall region. A blending function is then used to transition the solution from one model to the other. This approach, commonly referred to as SST [17], is adopted here. The relevant governing equations are:

$$\nu_t = \frac{a_1 k}{\max(a_1 \omega, \Omega F_2)}, \quad (10)$$

$$U_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left((v + \sigma_k \nu_t) \frac{\partial k}{\partial x_j} \right) + P_k - \beta^* k \omega, \quad (11)$$

$$U_j \frac{\partial \omega}{\partial x_j} = \frac{\partial}{\partial x_j} \left((v + \sigma_\omega \nu_t) \frac{\partial \omega}{\partial x_j} \right) + 2\gamma S_{ij} S_{ij} - \beta \omega^2 + 2(1 - F_1) \sigma_{\omega 2} \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}, \quad (12)$$

where Ω is the absolute value of the vorticity and F_2 is a limiter given as

$$F_2 = \tanh \left(\left[\max \left(\frac{2\sqrt{k}}{\beta^* \omega y}, \frac{500\nu}{y^2 \omega} \right) \right]^2 \right). \quad (13)$$

The model constants (ϕ) are determined by blending together the constants of the $k-\omega$ model (ϕ_1) and the $k-\epsilon$ model (ϕ_2) (Table 2) using the blending function F_1 i.e.

$$\phi = F_1 \phi_1 + (1 - F_1) \phi_2. \quad (14)$$

The blending function F_1 is given by:

$$F_1 = \tanh \left(\min \left[\max \left(\frac{\sqrt{k}}{\beta^* \omega y}, \frac{500\nu}{y^2 \omega} \right), \frac{\sigma_{\omega 2} k}{CD_{k\omega} y^2} \right]^4 \right), \quad (15)$$

where y is the distance from the wall and $CD_{k\omega}$ denotes the positive part of the cross-diffusion term:

$$CD_{k\omega} = \max \left(2\sigma_{\omega 2} \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}, 10^{-20} \right). \quad (16)$$

Turning now to Reynolds-stress transport closures, the unknown Reynolds-stresses are obtained from the solution of differential transport equations for these quantities which are of the form:

$$U_k \frac{\partial \overline{u_i u_j}}{\partial x_k} = D_{ij} + P_{ij} + \Phi_{ij} - \epsilon_{ij}. \quad (17)$$

In the above, D_{ij} is the rate of transport of $\overline{u_i u_j}$ by combined molecular, turbulent and fluctuating pressure processes (diffusion). The

Table 2

Coefficients of the SST model.

Model	σ_k	σ_ω	β	β^*	γ
$k-\omega$, ϕ_1	0.85	0.5	0.0750	0.09	0.5532
$k-\epsilon$, ϕ_2	1.0	0.856	0.856	0.09	0.4403

production of the Reynolds-stresses is represented by P_{ij} , its dissipation by viscous action is ϵ_{ij} and the fluctuating pressure–strain correlations are characterized by Φ_{ij} .

The production term is exact and in no need of modeling. The processes of diffusion and dissipation were modeled in the conventional way (e.g. [12,15]), the former via a gradient-transport model and the latter by the assumption of isotropic dissipation at high turbulence Reynolds numbers. The focus here is on the modeling of the fluctuating pressure–strain correlations Φ_{ij} . Most practical models for Φ_{ij} can be expressed in the unified form:

$$\begin{aligned} \Phi_{ij} = & -(C_1\epsilon + C_1^*P_k)b_{ij} + C_2\epsilon\left(b_{ik}b_{kj} - \frac{1}{3}b_{kl}b_{kl}\delta_{ij}\right) \\ & + \left(C_3 - C_3^*I_b^{\frac{1}{2}}\right)kS_{ij} + C_4k\left(b_{ik}S_{jk} + b_{jk}S_{ik} - \frac{2}{3}b_{kl}S_{kl}\delta_{ij}\right) \\ & + C_5k(b_{ik}W_{jk} + b_{jk}W_{ik}). \end{aligned} \quad (18)$$

In the above, $b_{ij} (= \overline{u_i u_j} / \overline{u_q u_q} - \frac{1}{3}\delta_{ij})$ is the turbulence anisotropy, $I_b (= b_{ij}b_{ij})$ is the second invariant of anisotropy, and $W_{ij} (= \frac{1}{2}(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}))$ is the mean vorticity tensor. Different models can be obtained from Eq. (18) by assigning values to the coefficients that can be deduced by reference to data from homogeneous and inhomogeneous shear flows. Table 3 lists these coefficients for some of the more commonly used models of this term. In this table, LRR refers to the widely-used model of Launder et al. [18], GL refers to the model of Gibson and Launder [19] which requires a wall-damping term to account for wall effects, SSG refers to the model of Speziale et al. [20] in which this term is absent, and DY refers to the vorticity-free model of Dafalias and Younis [21,22] which also does not require a wall-damping term. Details of all these models and of the relevant equations can be found in the original references.

2.3. Heat flux modeling

In typical engineering design calculations, the turbulent heat fluxes are invariably modeled using Fourier's law where they are assumed to be linearly proportional to the temperature gradients:

$$-\overline{u_i t} = \Gamma_t \frac{\partial T}{\partial x_i}, \quad (19)$$

where Γ_t is the eddy diffusivity:

$$\Gamma_t = \frac{\nu_t}{\sigma_t} \quad (20)$$

and σ_t , the turbulent Prandtl number, is generally assigned the value of 0.86 for air flows.

The model does not account for many of the physical processes that govern the heat fluxes. This is immediately apparent from inspection of the exact equation that describes the conservation of these quantities [23]. Principal among these processes, and one which can be influential in cases where the mean flow is in rapid evolution, is the dependence of the heat fluxes on the gradients of mean velocity. A number of alternative models have been put forward with the aim of incorporating this dependence. Among these is the one proposed by Younis et al. [11] (hereafter referred

to as YSC) which was derived by using tensor representation theory and which has been shown to yield improvements in a number of heated flows (e.g. [12–15]). This model is given by:

$$-\overline{u_i t} = C_{t1} \frac{k^2}{\epsilon} \frac{\partial T}{\partial x_i} + \left(C_{t2} \frac{k}{\epsilon} \overline{u_i u_j} + C_{t3} \frac{k^3}{\epsilon^2} \frac{\partial U_i}{\partial x_j} - C_{t4} \frac{k^2}{\epsilon^2} P_{ij} \right) \frac{\partial T}{\partial x_j}. \quad (21)$$

The first term in Eq. (21) is recognizable as Fourier's law. It accounts for the generation of heat flux components aligned with the direction of the temperature gradients. The remaining terms introduce two functional dependencies that are absent from Fourier's law but are required for the model to be consistent with the exact equations governing the evolution of $\overline{u_i t}$. Thus the model contains an explicit dependence on the details of the turbulence field via the Reynolds stresses and their rates of production (P_{ij}), and a similar dependence on the gradients of mean velocity. In a non-equilibrium flow, both these dependencies become important as the velocity and temperature fields adjust to new equilibrium following flow reversal. The presence of these additional terms also allows for temperature gradients in one spatial direction to produce finite heat fluxes in the other two directions – a result which is consistent with experiments but which can not be obtained with Fourier's law. The model coefficients are assigned their original values, as listed in Table 4.

The coefficient C_{t1} is modified according to [12] to account for the presence of a solid wall:

$$C_{t1}^* = C_{t1} \left(1 - \exp(-1.5A\text{Pe}^{0.1}) \right), \quad (22)$$

where A is the stress flatness parameter and Pe is the Peclet number.

2.4. Models development

In light of results that were obtained early in this work, it was concluded that the accuracy of the predictions may be improved in two distinct ways that are described below.

The first of these relates to the use of the YSC model in conjunction with the eddy-viscosity closures. While the use of the k – ϵ or k – ω models is wide spread in industry, it is known that Boussinesq's stress–strain relationship does not provide adequate description of the turbulence field and, in particular, fails badly in capturing the response of the Reynolds stresses to rapid changes in the mean rates of strain. Since these stresses enter directly into the YSC model for the heat fluxes, it was considered that the full benefits of this model would not be realized if it was employed in conjunction with a Boussinesq-based eddy-viscosity closure. Rather than abandon eddy-viscosity closures, it was considered worthwhile to explore whether coupling them to a more complete stress–strain relationship would yield improved predictions of the turbulence field and, consequently, of the heat-transfer rates. The alternative to Boussinesq would be a non-linear stress–strain formulation of which several have been reported in the literature. After extensive testing, the one that was finally adopted is based on the proposals of [24]. In adopting this proposal, a model for the pressure–strain correlations is required and the one adopted for this purpose was the vorticity-free closure of [21,22] which had given good predictions for a range of flows. According to [24], the anisotropy tensor b_{ij} can be expressed as a function of k , ϵ , S_{ij} , W_{ij} as:

Table 3
Coefficients of the pressure–strain models.

Model	C_1	C_1^*	C_2	C_3	C_3^*	C_4	C_5
LRR [18]	3.0	0	0	0.8	0.0	1.75	1.31
GL [19]	3.6	0	0	0.8	0.	1.20	1.20
SSG [20]	3.4	1.8	4.2	0.8	1.3	1.25	0.4
DY [21,22]	4.0	3.0	0.	0.8	2.0	0.6	0

Table 4
Coefficients of the YSC heat-flux model [11].

C_{t1}	C_{t2}	C_{t3}	C_{t4}
–0.0455	0.373	–0.00373	–0.0235

$$b_{ij} = \left(\frac{C_2 - \frac{4}{3}}{C_3 - 2} \right) \left(\frac{3(1 + \eta^2)}{3 + \eta^2 + 6\zeta^2\eta^2 + 6\zeta^2} \right) \cdot \left[S_{ij}^* + \left(S_{ik}^* W_{kj}^* + S_{jk}^* W_{ki}^* \right) - 2 \left(S_{ij}^{*2} - \frac{1}{3} S_{ij}^{*2} \delta_{ij} \right) \right], \quad (23)$$

where the dimensionless strain and vorticity tensors S_{ij}^* and W_{ij}^* are defined by:

$$S_{ij}^* = \frac{1}{2} \left(\frac{1}{2} C_1 + \frac{P_k}{\epsilon} - 1 \right)^{-1} \frac{k}{\epsilon} (2 - C_3) S_{ij}, \quad (24)$$

$$W_{ij}^* = \frac{1}{2} \left(\frac{1}{2} C_1 + \frac{P_k}{\epsilon} - 1 \right)^{-1} \frac{k}{\epsilon} (2 - C_4) W_{ij}, \quad (25)$$

and

$$\eta = \left(S_{ij}^* S_{ij}^* \right)^{\frac{1}{2}}, \quad \zeta = \left(W_{ij}^* W_{ij}^* \right)^{\frac{1}{2}}. \quad (26)$$

For homogeneous turbulence, P_k/ϵ is obtained as

$$\frac{P_k}{\epsilon} = \frac{C_{\epsilon 2} - 1}{C_{\epsilon 1} - 1} = 2.09. \quad (27)$$

The values of the coefficients in the [24] proposal that are required to reproduce an equivalent pressure strain correlation as for the differential DY model are given in Table 5. As proposed in [24], Il_b was set equal to 0.11. Hereafter, this model will be referred to as the explicit algebraic Reynolds-stress model (EARSMS)

The second way in which improvements were sought was motivated by the finding that the use of the heat-flux model of Eq. (21) in conjunction with any of the Reynolds-stress transport closures listed in Table 3 was insufficient by itself to produce a degree of improvement that can justify the extra computational efforts involved. Specifically, this combination of closure approximations failed to yield the correct behavior of Nusselt number downstream of the separation point. In particular, the experimental data of [25] show this quantity to reach a peak just upstream of the reattachment point before falling back to a lower value as the flow relaxes into a new equilibrium but the models consistently predicted a monotonic increase in this quantity and eventual leveling off at a considerable higher value than measured. The cause of this discrepancy was traced to the use of wall functions to bridge the region adjacent to the solid walls. The practice of assuming equilibrium flows in that region, while adequate for capturing the pressure-driven development of the flow field downstream of separation, does not permit the resolution of the thermal field in the near-wall region to the extent necessary to capture the non-monotonic behavior of Nu . It became clear that in order to capture this behavior, it would be necessary to abandon the high Reynolds number formulation in favor of one which allows integration through the viscous sub-layer, directly to the wall.

The extension of the DY model for integration through the viscous sublayer was guided by the proposals of Kebede et al. [26] in this regard. They argued that since the pressure fluctuations are represented exactly by a Poisson equation which is independent of the viscosity, then models for this term that were developed for homogeneous flows can be also applied through the viscous sublayer without additional damping. In sharp contrast, the term representing viscous dissipation is strongly dependent on viscosity and the usual assumption of local isotropy becomes invalid close to the wall where the turbulence Reynolds number is low.

Table 5
Coefficients of the explicit algebraic Reynolds-stress model.

Model	C_1	C_2	C_3	C_4
Dafalias and Younis (DY)	$4.0 + 1.8P_k/\epsilon$	$0.8 - 2.0Il_b^{1/2}$	0.6	0.0

Consequently, a modified formulation for viscous dissipation is required and the one proposed by Kebede et al. [26] is adopted here. In this formulation, the dissipation tensor is obtained from:

$$\epsilon_{ij} = \epsilon \left[\frac{2}{3} \delta_{ij} (1 - f_s) + \frac{f_s F}{k} (\overline{u_i u_j} + \overline{u_i u_k} n_k n_j + \overline{u_j u_k} n_k n_i + \delta_{ij} \overline{u_k u_l} n_k n_l) \right], \quad (28)$$

where n_k denotes the unit vector normal to the wall and f_s is a function that depends on the turbulence Reynolds number:

$$f_s = \exp\{-Re_t/40\}, \quad (29)$$

$$Re_t = \frac{k^2}{\nu \epsilon}. \quad (30)$$

The function F in Eq. (28) is required the dissipation tensor to contract correctly. It is given by:

$$F = \frac{1}{1 + 5 \frac{\overline{u_k u_l} n_k n_l}{\overline{u_i u_i}}}. \quad (31)$$

Finally, to obtain the correct asymptotic near-wall behavior, c_{ϵ_1} is taken to be a function of the turbulence Reynolds number. Thus,

$$c_{\epsilon_1} = 1.45(1 - f_s) + 2.0f_s. \quad (32)$$

3. Computational details

The results presented here were obtained with the open-source software OpenFOAM which was extended here for simulating periodic flows with heat transfer. To achieve this, cyclic boundary conditions were implemented. This was done following the methodology of Patankar et al. [27]. In this approach, the pressure field can be decomposed into a periodic part ($\bar{p}$) and a part (βx) which takes into account the constant global pressure gradient:

$$\bar{p}(x, y) = \tilde{p}(x, y) - \beta x. \quad (33)$$

By substituting Eq. (33) into Eq. (2), the averaged momentum equation can be rewritten, including the new source term β which, in our flow, is finite only in the x -component momentum, as follows:

$$U_j \frac{\partial U_i}{\partial x_j} = \beta \delta_{i1} - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left[\nu \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \overline{u_i u_j} \right]. \quad (34)$$

By solving Eq. (34) in conjunction with the continuity equation, the value of the pressure gradient β is adjusted iteratively until the desired flow rate is achieved.

Similar considerations were made by Patankar et al. [27] for the fully developed thermal field where a constant wall heat flux will cause a monotonic streamwise increase in bulk temperature. Again, the temperature field can be decomposed in a periodic part ($\tilde{T}$), and another that takes into account the global temperature increase (γx) for a constant wall heat flux:

$$T(x, y) = \tilde{T}(x, y) + \gamma x. \quad (35)$$

By incorporating this temperature into the energy equation (Eq. (3)), the streamwise-periodic temperature field is now described by

$$U_j \frac{\partial \tilde{T}}{\partial x_j} = -\gamma U_i \delta_{i1} + \frac{\partial}{\partial x_j} \left(\frac{\lambda}{\rho c_p} \frac{\partial \tilde{T}}{\partial x_j} - \overline{u_j \tilde{T}} \right). \quad (36)$$

From a simple global energy balance over one periodic module, the temperature gradient γ can be written as

$$\gamma = \frac{\dot{Q}}{\dot{m} c_p L}, \quad (37)$$

where $\dot{Q}$ is the imposed total amount of heat, $\dot{m}$ is the mass flow rate, c_p the specific heat capacity and L is the length of one periodic module.

Following from these considerations, the periodic momentum and energy equations (Eqs. (34) and (36)) are solved, with cyclic boundary conditions applied for at the inlet and outlet planes. Once convergence has been attained, the actual pressure and temperature are recovered by taking into account the global pressure drop and the bulk temperature rise.

Concerning the wall boundary conditions for the mean velocities, these were usually obtained by using the log-law to provide the momentum fluxes. This is the conventional ‘wall function’ approach. It was later supplemented in this study by the low turbulence Reynolds number approach which did not invoke the log-law. Regarding the thermal boundary conditions, these were specified as constant heat flux on the ribbed wall. The upper wall was assumed to be adiabatic.

The equations were discretized using the second-order accurate MUSCL scheme and the SIMPLE algorithm. The computations were performed on block-structured non-uniform meshes. A great deal of effort was expended on minimizing the numerical discretization errors in the predictions. This was achieved by successive grid refinement and by using the Grid Convergence Index (GCI) methodology to quantify the errors [28]. As proposed in [28], three meshes with refinement ratios of about 1.3 were created, a coarse grid with 24,517 cells, a medium dense mesh with 33,898 cells and fine grid featuring 46,280 cells. In order to visualize the influence of the grid on the solution, skin friction C_f shall be plotted vs. the surface coordinate ζ in Fig. 2 for the three differently refined grids. As can be seen from the trends, the medium dense mesh is in between the coarse and the fine mesh, being however hardly discernible from the fine mesh. To determine a quantitative value for the discretization error the procedure of [28] requires the identification of a characteristic variable (ϕ) to serve as a marker for testing for discretization errors. Two such variables were chosen: the dimensionless reattachment length (X_r/e) and the section-averaged streamwise non-dimensional pressure gradient ($\partial p/\partial x$). The corresponding values for the different refinement levels are summarized in Table 6. The quantity ϕ_{ext}^{21} denotes the extrapolated value of ϕ and e_{ext}^{21} , associated with the extrapolation error. Whereas, reattachment length shows monotonic convergence, the pressure gradient indicates oscillatory convergence. Overall, grid dependence is fairly small since the GCI lies between 0.06% obtained from the reattachment length and 1.07% when considering the pressure gradient.

It is to be noted that for the considered low-Reynolds number models a comparable study has been conducted.

Table 6

Quantification of discretization errors using the Grid Convergence Index method [28]. Results are for the DY/WSC model.

N_1	46,280	
N_2	33,898	
N_3	24,517	
ϕ	X_r/e	$\partial p/\partial x$
ϕ_1	4.0303	50.6141
ϕ_2	4.0735	50.7621
ϕ_3	4.0769	50.5621
ϕ_{ext}^{21}	4.528	50.182
e_{ext}^{21}	0.95%	0.29%
GCI_{fine}^{21}	0.06%	1.07%

4. Results and discussion

The flow geometry and coordinates are shown in Fig. 1. The experimental data for the velocity field are those of Drain and Martin [2] obtained with LDV measurements of the fully developed flow in a ribbed channel with water flow. The Reynolds number (based on bulk velocity and hydraulic diameter) was 37,200. The heat transfer data are those of Liou et al. [3] and Lockett and Collins [4] who both used real-time holographic interferometry to measure the temperature distribution of fully developed air flow in the ribbed channel and reported on Nusselt number distribution at all surfaces including the vertical walls. These measurements were obtained for $Re = 12,600$ and $Re = 13,100$, respectively. This choice of experimental data is based on the recommendations of ERCOFTAC [1] who put forward these data sets as the benchmarks for the assessment of computational methods for heated flows in ribbed channels.

4.1. Flow field

An overview of the flow field as predicted with alternative models can be seen in Fig. 3. Shown there are the flow streamlines with superimposed contours of streamwise velocity. The $k-\epsilon$ model (Fig. 3(a)) clearly predicts the ribbed passage as a k -type roughness element, showing a strong interaction of core flow and the roughness layer in the region between the ribs. In particular, significant downwash occurs at around two thirds of the area between the ribs, yielding reattachment relatively early at approximately $x/e = 4$. After reattachment, the boundary layer redevelops until re-separation occurs again near $x/e = 6$ forming a second

Fig. 2. Skin friction distribution on the ribbed wall at $Re = 37,200$: comparison of different grid densities.

Fig. 3. Flow streamlines as predicted by the different turbulence models.

separation bubble upstream of the rib. Compared to the other models, the streamwise velocity field shows a significant deceleration of the flow in the region of the down-wash which spreads into the core flow. This can be explained by the increasing effective

cross-sectional area as the separating streamline is pushed towards the wall. As a result of the strong interaction between core flow and the turbulators, enhanced turbulent mixing is produced, leading to increased heat transfer rates. The $k-\epsilon$ model result also

indicate that this is the only model of all those tested here that does not succeed in capturing the secondary vortex in the bottom corner downstream of the rib. In contrast to the $k-\epsilon$ model, the SST model results (see Fig. 3(b)) suggest that the rib as an intermediate roughness between d - and k -type, since the entire region between the ribs underlies a single recirculation system. This results in the flow to coast over the cavity without considerable downwash and consequently smaller values for the heat transfer rate can be expected.

The streamlines obtained with the Reynolds-stress transport closures noticeably differ from each other in their behavior of modeling the flow between the ribs. While, the DY (Fig. 3(f)) and the LRR (Fig. 3(c)) models predict two separate zones of recirculating flow, the SSG model suggests the existence of a third zone in between, whereas the GL model suggest a merger between the two (Fig. 3(d)). Concerning reattachment and re-separation, the LRR model significantly underpredicts flow separation indicating reattachment shortly before $x/e = 4$ and is insensitive to the recirculation upstream of the rib. Also the DY model underestimates reattachment with $x/e = 4.5$, while the SSG lies slightly above with reattachment at $x/e = 5$. From the streamlines plot, no reattachment can be observed for the GL model. The DY_{low-Re} model results are presented in Fig. 3(g)). Compared to the high Reynolds number version, reattachment now occurs earlier, while the downstream recirculation bubble is more pointed. Finally, the streamlines obtained by the Launder and Sharma (EARS) model (Fig. 3(h)) show some distinct differences from the high Reynolds-number version including the ability to capture the secondary vortex in the downstream corner.

The models predictions of the wall skin-friction coefficient along the ribbed surface are compared in Fig. 4. The horizontal axis in that figure represents the distance along the surfaces of both the ribs and the channel, with the origin being at the mid-plane of the upstream rib (see Fig. 1). For the backward-facing side of the downstream rib, the change in the sign of C_f apparent in the Reynolds-stress closures is associated with the secondary vortex that is located in the corner. In contrast, the $k-\epsilon$ model, which does not capture this vortex, yields a negative value throughout. The $k-\omega$ -SST model predicts no reattachment and hence the skin friction remains negative over the entire enclosure. Regarding the Reynolds-stress closures, the GL model appears to be the most susceptible to separation, yielding the longest reattachment length. After reattachment, the developing boundary layer in the accelerating flow yields the steepest velocity gradients, resulting in the steady rise of C_f . The DY model shows moderate recirculation and recovery, and a small secondary separation bubble in

the corner of the downstream rib. The low-Reynolds number version of the same model (DY_{low-Re}) shows a significantly more rapid recovery of the boundary layer, especially near the upstream corner of the rib, where the skin friction coefficient appears to be overpredicted.

The predicted and measured cross-stream profiles of mean streamwise velocity are compared in Fig. 5. The velocity is non-dimensionalized using the local value of the bulk velocity. The results are presented at four streamwise locations that are located, respectively, in the middle of the rib ($x/e = 0$), between the ribs ($x/e = 3.68$ and $x/e = 4.82$) and in the vicinity of the upstream corner ($x/e = 6.8$). Overall, the asymmetric reduction in velocity due to the placement of the ribs on one side only of the channel is well reproduced by the models with the Reynolds-stress transport closures yielding essentially identical results. At section $x/e = 0$, the results from the stress transport closures overlap quite closely and are generally in good overall agreement with the experimental data except, perhaps, very close to the wall where some differences in the profile shape are obtained. The $k-\epsilon$ model obtains the most rounded shape in that region but the overall shape is too asymmetric with the velocity overpredicted near to the upper wall but underpredicted further away. Between the ribs, at $x/e = 3.68$ the shape of the velocity profile is well reproduced by the DY, GL, SSG and DY_{low-Re} models, although all appear to overestimate the depth of the recirculation zone. The differences between the DY model with wall functions and the DY_{low-Re} are visible near the wall. At the upstream corner of the rib ($x/e = 6.8$) for example, only the DY_{low-Re} model predicts reversed flow.

The predicted and measured cross-stream profiles of the normal Reynolds-stress component $\overline{u^2}$ are compared in Fig. 6. Above the rib, all models tend to underpredict the measurements, whereas the cross-plots between the ribs (at $x/e = 3.68$ and $x/e = 4.82$) are well predicted. The local peak in $\overline{u^2}$ at a distance from the wall corresponding to the rib height is accurately captured by the DY_{low-Re} and the DY models. The GL and the SSG model are slightly below the DY model, whereas the LRR model overpredicts that peak but succeeds in capturing the one above the rib at $x/e = 0$.

The distributions of the wall-normal stress component $\overline{v^2}$ are shown in Fig. 7. In general, the GL and SSG model show the best accord with the measurements, whereas both formulations of the DY model and the LRR model overpredict the maximum. The predicted and measured shear stress $-\overline{uv}$ profiles are compared in Fig. 8. All Reynolds stress closures yield good correspondence with the measurements except for the LRR model which shows significant overprediction. Overall, the best predictions are seen for the DY_{low-Re} model.

Fig. 4. Predicted skin-friction distribution on the ribbed wall at $Re = 37,200$: comparison of different turbulence closures.

Fig. 5. Predicted and measured mean velocity profiles. $Re = 37,200$; — DY-WF, -- GL-WF, --- LRR-WF, ... SSG-WF, --- $k-\epsilon$ -WF, --- DY_{low-Re} , $\circ$ Exp. Drain and Martin [2].

Fig. 6. Predicted and measured normal-stress component $\overline{u^2}/U_b^2$. $Re = 37,200$; — DY-WF, -- GL-WF, --- LRR-WF, ... SSG-WF, --- DY_{low-Re} , $\circ$ Exp. Drain and Martin [2].

Fig. 7. Predicted and measured normal-stress component $\overline{v^2}/U_b^2$. $Re = 37,200$; — DY-WF, -- GL-WF, --- LRR-WF, ... SSG-WF, --- DY_{low-Re} , $\circ$ Exp. Drain and Martin [2].

4.2. Thermal field

Turning now to the thermal field, we first explore the extent to which the choice of turbulence closure influences the predicted

distribution of Nusselt number on the ribbed side of the channel. Computations were therefore made with alternative stress-transport closures and with the heat fluxes evaluated by using the YSC model. The Nusselt number is defined as:

Fig. 8. Predicted and measured shear-stress component $\overline{u'v'}/U_b^2$. $Re = 37,200$: — DY-WF, -- GL-WF, -.- LRR-WF, ... SSG-WF, --- DY_{low-Re}, LRR, ... SSG, ○ Exp. Drain and Martin [2].

$$Nu = \frac{-\frac{\partial T}{\partial n}|_w H_d}{T_w - T_b}, \quad (38)$$

where H_d is the hydraulic diameter and T_b is the mass-flow averaged (bulk) temperature $(= \int_0^{2h} T U dy / \int_0^{2h} U dy)$. It should be noted that, as in the experiments, the value of T_b used for calculating Nu over the vertical faces is that of T_b in the adjacent slice between the ribs. In the experiments, estimation of the gradients of mean temperature at the wall is subject to a great deal of uncertainty due to the rapid variation of temperature and because of the difficulty of obtaining accurate measurement close to the wall where interference effects can be quite significant.

The predicted and measured distributions of Nu are compared in Fig. 9. To quantify the extent of heat-transfer enhancement relative to a smooth channel, the computed values were non-dimensionalized by $Nu_s = 39.4$ which was the measured value in the experiments of Lockett and Collins [4]. Along the backward-facing side of the upstream rib, Nu decreases since the flow is almost stagnant with much reduced convection towards the bottom corner B. Interestingly, the heat transfer rate there drops to a value lower than in a smooth channel. Along the bottom surface, between the ribs, Nu gradually increases reaching a local maximum slightly downstream of reattachment. As the flow enters the second separation area upstream of the rib, Nusselt number decreases again

towards corner D, implying a value that is only slightly higher than for the smooth channel. Thereafter, the Nusselt number increases rapidly along the forward facing wall owing to the steep gradients there. The global maximum occurs close to corner E, where the flow deflects and the gradients of velocity and temperature are at their maximum. Downstream of this point, the measurements suggest a steady decrease as the newly established thermal boundary layer increases in thickness. The trends in Nusselt number are generally well produced by the stress-transport closures although all seem to predict the location of maximum heat transfer to occur further downstream of the measurements of Liou et al. [3]. Closer agreement is obtained with the measurements of Lockett and Collins [4] which are more asymmetric. Regarding the maximum of the Nusselt number between the ribs, this is seen to occur 7–10% forward of the maximum of the skin friction. This behavior seems physically correct, since the location of the maximum skin friction is associated with the steepest velocity gradients which are responsible to enhanced heat transfer. It should be noted, that LES predictions of Ciofalo and Collins [5] and DNS data of Nagano [9] showed a similar asymmetric trend as that obtained here.

In detail, the distribution between the ribs is best predicted by the DY model in both its high and low Reynolds-number forms. The SSG and GL models underpredict Nusselt number, whereas the LRR model overpredicts the measurements by quite a significant margin. Along the rib itself, on the forward face between the corners

Fig. 9. Nusselt number distribution on the ribbed wall ($Re = 12,600$): predictions with the YSC model in conjunction with various Reynolds-stress transport closures.

D and E the DY_{low-Re} model provides the most accurate results. Compared to those, the results of the DY model incorporating wall functions are slightly lower. Reasonable results are also obtained for the LRR model, whereas GL and SSG underpredict the heat transfer rates there. On the top surface of the rib, the decrease of Nusselt number is not well captured by any of the models, except for the DY_{low-Re} which shows a decreasing trend downstream of corner E . Again, on the backward face between the corners B and C the DY_{low-Re} model shows better results compared to the high Reynolds-number version.

In order to assess the sensitivity of the heat-transfer predictions to the choice of heat-flux closure, predictions were obtained with Fourier's law (FL) and the YSC model, both being used in conjunction with the DY model. The results are shown in Fig. 10 where the predictions of Nusselt number are compared. On the backward-facing wall, a virtually indistinguishable the Nusselt number distribution is obtained but significant differences appear further downstream where Fourier's Law returns lower values. When the same model is used with the standard $k-\epsilon$ closure, very different results are obtained emphasizing the importance of accurate prediction of the turbulence field. The YSC model results are generally in good accord with the measurements of Liou et al. [3].

In Fig. 11, the Nusselt number is compared for different implementations of the SST-based EARSM closures in conjunction with the YSC, as well as the standard SST model with FL. In what follows, EARSM1 refers to the case where the Reynolds stress tensor was

obtained *a posteriori* from a frozen velocity field using Eqs. (24)–(27) whereas in EARSM2, the output of these equations was used in calculating the velocity field. From the outset it is noted that the SST model underpredicts turbulence motion and therefore flow separation is present throughout the cavity, with consequently lower heat transfer rates. Indeed, particularly low values are present for SST/FL. Applying the YSC in conjunction with the EARSM1 approach led to a slight increase of the Nusselt number for the main part of the ribbed surface. However, as the main flow field is not altered by this approach the positive effect is limited. As also the EARSM2 entailed relatively small influence on mean velocity and Reynolds-stresses, the improvements compared to EARSM1 are fairly small and limited to the downstream part of the bottom surface. Regarding the distribution along the faces of the rib, the Nusselt number along the backward face reveals a bulge of smaller values which spreads out to the beginning of the bottom surface, due to the large secondary vortex located in corner C . Reasonably accurate results are obtained with the models along the forward- and top-face of the rib. The EARSM2 model captures especially well the heat transfer peak around corner E , and the decreasing trend resulting from the thickening temperature boundary layer.

The mean temperature profiles are compared at different streamwise locations in Fig. 12. The results presented there were obtained with the models DY/YSC , DY/FL , $k-\epsilon/YSC$, SST (EARSM2)/YSC and DY_{low-Re}/YSC . The $k-\epsilon/FL$ model shows the most rapid reduction of the wall-normal temperature gradients, and also

Fig. 10. Nusselt number distribution on the ribbed wall ($Re = 12,600$): comparison of the YSC and FL heat-flux models.

Fig. 11. Nusselt number distribution on the ribbed wall predictions ($Re = 12,600$): comparison of different SST (EARSM) variants.

Fig. 12. Nondimensional temperature profiles ($Re = 12,600$): — DY-WF/YSC, -- DY-WF/FL, -.- $k-\epsilon$ -WF/FL, ... SST (EARS2)/YSC, ---- DY_{low-Re} /YSC.

predicts the lowest temperatures adjacent to the wall. By far the highest values of temperature are predicted by the SST (EARS2)/YSC model. For the latter, the excessive increase of the wall-temperature directly downstream of the rib is responsible for the low Nusselt numbers in that region. However, in the cross-plots above the rib the predictions are similar as obtained for the DY/YSC which consistently provides slightly higher values for the wall temperature than $k-\epsilon$ /FL. The DY_{low-Re} /YSC is in between the DY/YSC and $k-\epsilon$ /FL. The temperature field predicted by DY/FL differs from that of the DY/YSC such that the profiles are shifted to larger values implying a thicker thermal boundary layer and higher near-wall temperatures.

The profiles of $-\overline{u}t$, the streamwise component of the turbulent heat fluxes, obtained with the various models are compared in Fig. 13. Consistent with the formulation of Fourier's law, the small temperature gradients predicted in the streamwise direction lead to the prediction of negligible turbulent heat fluxes in that direction. In contrast, the YSC model predicts quite substantial levels for these fluxes consistent with the presence of large gradients of temperature and velocity in the separated flow, and in the recovering boundary layer. The high levels of the turbulent shear-stresses $\overline{u}v$ which occur there, as well as the significant rates of production of the Reynolds-stresses, both of which enter the formulation of this heat-flux component in the YSC model, also contribute to this

result. The same model also obtains counter-gradient diffusion, since for the most part of the domain, the turbulent heat fluxes are pointed into the opposite direction of mean temperature gradient $\partial T / \partial x$.

The predicted cross-stream profiles of the heat-flux component $\overline{v}t$ are compared in Fig. 14. Overall, Fourier's law and the YSC provide similar trends. However, due to the direct dependence of Fourier's law on the eddy viscosity, there is a strong dependence of the turbulent heat fluxes on the turbulence model results. When employing the $k-\epsilon$ model, the high levels of turbulence kinetic energy obtained with that model produces relatively large values of the turbulent heat fluxes, while the lower level of turbulence kinetic energy predicted by the stress-transport closures yields smaller values for $\overline{v}t$. This is clearly evident from the DY model results. Regarding the YSC closure, the main influencing parameters can be identified from Eq. (21). As the streamwise temperature gradient is small, the correlations connected with $\partial T / \partial x$ are negligible. The dominant terms refer to the scalar diffusivity and the wall-normal Reynolds-stresses $\overline{v}^2$. Also the production term of the Reynolds-stresses is of considerable importance since the gradients of the wall-normal mean velocity in both directions are quite large. In view of these considerations, the smaller values for the turbulent heat fluxes $\overline{v}t$, and therefore the smaller heat transfer rate, predicted by the SST EARS2 model, can be

Fig. 13. Turbulent streamwise heat flux profiles $-\overline{u}t$ ($Re = 12,600$): — DY/YSC, -- DY/FL, -.- $k-\epsilon$ /FL, ... SST (EARS2)/YSC, ---- DY_{low-Re} /YSC.

Fig. 14. Turbulent wall-normal heat flux profiles $\bar{v}t$ ($Re = 12,600$): — DY/YSC, -- DY/FL, --- $k-\epsilon$ /FL, ... SST (EARS2M2)/YSC, ---- DY_{low-Re} /YSC.

explained. It should be noted that the mean temperature gradients themselves are smaller. Moreover, studies on the gradients $\partial V/\partial x$ and $\partial V/\partial y$ showed that especially the component in x -direction is smaller compared to the DY. Since this term is further multiplied with the shear-stress $\bar{u}v$, which is considerably underpredicted by the SST EARS2M2, the C_{t4} -term in Eq. (21) is poorly represented. In addition, investigations on the eddy viscosity showed that the scalar diffusivity is one order of magnitude smaller than for the DY, since dissipation of turbulence kinetic energy due to the dissipation rate ω is fairly higher compared to ϵ .

5. Conclusions

Predictions of the flow and thermal fields in a heated channel with periodic ribs that are obtained using industry-standard turbulence closures fail to reproduce the observed effects of the large-scale separation that occurs between successive ribs. These effects are manifest in significant departures from equilibrium conditions, and in enhancement in the heat-transfer rates relative to smooth channels. The work reported in this paper was motivated by the need to improve the predictive capabilities of RANS-based approaches in this important class of flows. It was found that this can be achieved via two separate mechanisms. The first relates to the modeling of the effects on the Reynolds stresses of rapid evolution in the mean strain field. It was found that this can adequately be accounted for by using a Reynolds-stress transport closure in which the difficult fluctuating pressure-strain correlations term is modeled using a vorticity-free formulation that does not require the use of wall-damping functions. In order to correctly capture the wall heat-transfer rates, it was found necessary to enhance this model by extending its applicability via the viscous sub-layer directly to the wall. Both, results obtained for turbulent stresses and for Nusselt number showed superior results compared to the model in combination with wall functions. The second relates to modeling the turbulent heat fluxes. Results obtained with Fourier's law confirmed previous findings that showed this model to yield incorrect distribution of Nusselt number in the region between successive ribs. In particular, both the magnitude and location of the peak value of this parameter are badly predicted by this model. The alternative approach used here was to replace Fourier's law with one which, consistent with the exact equations governing the evolution of these fluxes, introduces direct dependence on both the Reynolds stresses and the mean rates of strain. The computations, which were checked for numerical accuracy using the Grid Convergence Index method, show that these developments make it possible to obtain, using RANS,

improved predictions of the enhanced heat-transfer rates in a ribbed channel.

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