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UNIVERSITY OF CALIFORNIA,
IRVINE

Essays on Currency Competition, Institutional Restrictions and Exchange Rates

DISSERTATION

submitted in partial satisfaction of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

in Economics

by

Arghya Bhattacharya

Dissertation Committee:
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2017

DEDICATION

To my parents and to my sister for their unwavering support and immense patience.

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ABSTRACT OF THE DISSERTATION

Essays on Currency Competition, Institutional Restrictions and Exchange Rates

By

Arghya Bhattacharya

Doctor of Philosophy in Economics

University of California, Irvine, 2017

Professor Guillaume Rocheteau, Chair

This dissertation consists of three essays on currency competition, institutional restrictions and exchange rates.

When faced with currency competition, a country's government has two tools at its disposal: reduce the level of inflation or place institutional barriers to the use of foreign currency. In the first chapter, I propose a two-country, two-currency New Monetarist model to study currency competition. I model institutional barriers as a 'tax' on the real value of foreign currency holdings which tends to lower a currency's value abroad than at home. This tax-induced asymmetric valuation of currencies leads to a set of rich and unique equilibrium currency regimes thus overcoming the multiplicity of equilibrium often noticed in models of currency competition.

In the second chapter, I examine whether capital controls as well as restrictions on financial current account in certain emerging market economies had any effect in providing buoyancy to its domestic currency. Using a country-by-country SVAR approach for five emerging market economies – Chile, Colombia, India, Malaysia and Indonesia for the time period of 1970-2013 and then using a panel VAR approach, I find that there is a role for such restrictions in affecting the nominal exchange rate. However, such effects are limited to the short-run and their effectiveness vary by country.

In the third chapter, I propose a tractable model of the black market for currencies where the black market premium on foreign currency arises endogenously and depends on the relative inflation rates of domestic and foreign currencies. Using a New Monetarist framework, I offer a plausible explanation as to why this association could be sometimes positive and sometimes negative. The black market is modeled as a market that can be used by buyers to readjust their portfolio when access to the official market is infrequent and after the realization of a shock that forces them to either consume local or foreign goods. After allowing for currency substitution by agents, in the stationary monetary equilibrium the rate of black market premium could be decreasing in the domestic inflation rate if agents are not very risk averse. Else, the black market premium is increasing in domestic inflation.

Chapter 1

Introduction

1.1 Overview

My research focuses on certain monetary phenomena observed in emerging market economies and less developed countries pertaining to the use of different currencies as means of payment, as a store of value and the role played by a country's financial openness on agents' portfolio decision and exchange rates. In particular, these include issues like dollarization, capital controls/international financial frictions and the black market for currencies.

When faced with high inflation, to preserve their wealth, residents in a country would want to switch to a low inflation foreign currency such as the US dollar. However, institutional restrictions to such a switch exist and define the kind of currency regimes that would prevail. One class of such institutional restrictions include restrictions on financial openness such as capital account controls and restrictions on financial current account. These restrictions have downstream effect, directly or indirectly, on which currencies residents can use more freely. I study whether these restrictions affect the relative value (exchange rate) of currencies. Finally, when these restrictions to free use of currencies exist, some residents would try to

“fly under the radar” and participate in an illegal market for currency exchange called the *black market*. In my last chapter, I build a link between the premium on foreign currency in the black market and the inflation rate in the domestic currency.

1.2 Currency Competition under Restrictions on Foreign Currency Use

In the first chapter, I focus on two aspects: (a) how residents would respond to inflation with regard to their portfolio choice in currencies and (b) the tools a country’s government has two tools at its disposal. When a country is faced with high inflation, its authorities have two tools at its disposal: reduce the level of inflation or place institutional barriers to the use of foreign currency. I focus on the latter as many emerging market economies and less developed countries resort to restrictions when they fail to curb inflation. On the residents’ side, they would find it optimal to switch to a low inflation foreign currency to protect their wealth from inflation tax. What proportion of a country’s residents’ portfolio would be composed of foreign currency would depend on the interplay of institutional barriers to free use of different currencies, inflation rates of those currencies as well as degree of economic integration and relative size of the country. In this chapter, I propose a two-country, two-currency New Monetarist model to study currency competition more commonly known as *dollarization*. I model institutional barriers as a ‘tax’ on the real value of foreign currency holdings which tends to lower a currency’s value abroad than at home. This tax-induced asymmetric valuation of currencies by economic agents from different countries leads to a set of rich and unique equilibrium currency regimes thus overcoming the multiplicity of equilibrium often noticed in models of currency competition.

1.3 International Financial Openness and Exchange Rate Manipulation

Institutional barriers to the choice of currency in which agents carry out trades or denominate the value of goods, services and assets are often manifestations of the restrictions that exist on the financial openness of a country. In this chapter, I examine whether capital controls as well as restrictions on financial current account in certain emerging market economies had any effect in providing buoyancy to its domestic currency vis-a-vis the US dollar, an international currency. I use a country-by-country structural vector autoregression (SVAR) approach to answer this question using data from five emerging market economies – Chile, Colombia, India, Malaysia and Indonesia for the time period of 1970-2013. Then I do a panel VAR analysis using the same five countries. Overall, my analysis suggests a role for such restrictions in affecting the nominal exchange rate especially in “currency crises” like scenarios. However, such effects are limited to the short-run and their effectiveness vary by country.

1.4 The Black Market for Currencies: Theory and Evidence

In face of high inflation, a country’s residents would want to switch to a low inflation foreign currency to avoid high inflation tax. However, the authorities, in order to protect their domestic monetary base and stem dollarization, may impose a variety of institutional barriers. Some of these barriers include restricting the exchange of one currency for the another. Whenever such restrictions exist, some residents would try to “fly under the radar” and buy/ sell currencies in an informal, illegal market called the black market. In this chapter,

I propose a tractable model of the black market for currencies where the black market premium on foreign currency arises endogenously and depends on the relative inflation rates of domestic and foreign currencies. Experience from countries like Argentina, Zimbabwe etc. suggests that higher domestic inflation is associated with higher rates of black market premium on foreign currency. However, this is not always true. I analyze data from Iran, Venezuela and four South Asian economies between 1981-1997 using a panel VAR and fixed effect panel regression and find a negative association between the two variables. The pattern is also noticed in case of India when studied separately. Using a New Monetarist framework, I offer a plausible explanation as to why this association could be negative. The black market is modeled as a market of currency exchange that can be used by buyers of one country to readjust their portfolio when access to the official market is infrequent and after the realization of a shock that forces them to either consume local or foreign goods. I show that after allowing for currency substitution in the portfolio choice problem of the agents, in the stationary monetary equilibrium the rate of black market premium could be decreasing in the domestic inflation rate if agents are not very risk averse. Else, if agents are sufficiently risk averse, then the black market premium is increasing in domestic inflation.

Chapter 2

Currency Competition under Restrictions on Foreign Currency Use

2.1 Introduction

One of the interesting features of monetary systems across different countries is the coexistence (or lack thereof) of more than one currency. What incentives do agents in an economy have to hold more of one particular currency? Why sometimes one currency is not enough and why at other times only one currency is enough? These questions have intrigued monetary economists for a while. In the last two decades researchers have pointed out certain features of the economy that incentivize agents to prefer one currency over another. It is common for economic agents to prefer a stronger currency (i.e. low inflation currency) over one which suffers from high inflation. However, this does not explain all episodes of currency competition. For example, in Russia throughout the hyperinflationary period of the 90s the ruble remained the currency of choice, while individuals held, rather secretly, the US dollar for savings purposes. Therefore, although there was some *dollarization* of individual

portfolios, US dollars (or other foreign currencies) was not used for majority of day-to-day domestic trades. This was probably due to high penalties for the use of foreign currency in day-to-day domestic transactions. The ruble's backing by the government as the only legal tender for domestic trades might have also contributed to its persistence even in a hyperinflationary period. Therefore, high costs associated with the use of a foreign currency - be it due to (i) imposition of penalties by the government or, (ii) due to everybody else adhering to the local currency (due to it being the only legal tender) could also guide which currency people use more. On the other hand, Argentina has also undergone periods of high inflation and despite restrictions, Argentineans have shown to prefer the US dollar over the peso – both in their portfolio as well as for several domestic trades, during such periods. However, the US dollar has not been able to completely replace the Argentinean peso. So, both the inflation rate of a currency and given the restrictions, the cost associated with its use play an important role in determining if that currency will be the currency of choice.

This paper considers both the inflation rate of a currency as well as the legal costs associated with its use. While relatively lower inflation rate makes a particular currency more attractive, higher cost associated with its usage makes the currency less preferable. It is the combined effect of these two opposing forces that determines which currencies will be used more. One of the most common and obvious source of cost of using a currency is government imposed penalties. Such restrictions are often common in countries from the developing world. In 2009, in response to US sanctions, Cuba completely banned the use of US dollars in the country. In more recent times, African countries like Ghana, Mozambique and Angola have imposed different degrees of restrictions on the use of foreign currency. Early in 2014, in face of a weakening cedi, Ghana limited foreign currency withdrawal by individuals to only \$10,000. Similar laws exist in other countries of Africa: in Angola oil and gas companies are required to pay tax revenues and sign local contracts in kwanza, its currency; whereas in Mozambique companies are legally bound to exchange half of their export earnings for meticaïs. In face of their weakening domestic currency Zambia and Nigeria legislated the

most restrictive laws on the use of foreign currency which among other things included jail sentences. While such laws continue to be in force in Nigeria, following the kwachas gain against the US dollar, Zambia has very recently revoked its ban on dollars.

While direct restrictions imposed by the government are one obvious source of costs associated with the use of a currency, it is not the only one. The idea of transactions cost of using a currency has been discussed in Engineer (2000) and implicitly in other papers. One conclusion from Wright et al. (2001) is that when a currency circulates abroad it has lower value than it has at home. First-generation monetary search models like Kiyotaki and Wright (1993) discuss lower acceptability, thus lower liquidity of a higher return currency as an equilibrium strategy of agents arising due to greater proportion of people possessing and using the currency with lower returns. Thus transactions costs could also arise from lower acceptability and therefore, lower liquidity of a particular currency. Given that inflation isn't too high, individuals find it easier to use the local currency because greater proportion of people use and accept the same. Under such circumstances if an individual wants to use foreign currency, then it will be accepted in only a few places. To find such a place the individual has to incur a certain cost and that effectively reduces his valuation of the foreign currency. Alternatively, he can exchange his foreign currency for local currency, however, due to bid-ask spread (also known as buying and selling rates) he will incur a cost if he goes for such currency conversion. Some of the early papers that study evidence of bid-ask spreads and determinants thereof includes Bossaerts and Hillion (1991), Black (1991), Bollerslev and Domowitz (1993). Other empirical works in this area include Bessembinder (1994) and Hartmann (1999). Bessembinder (1994) studies bid-ask spread in interbank foreign exchange markets, while Hartmann (1999) discusses transaction costs in foreign exchange markets using daily dollar-yen spot data. The persistence of greater proportion of people possessing the local currency despite its relatively higher inflation is often guided by direct or indirect government policies that affect the liquidity of a foreign currency.

Matsuyama et al. (1993) is one of the earliest studies on currency competition using a search theoretic framework. Matsuyama et al. (1993) uses a two country model in which two currencies compete and circulate as media of exchange. Wright et al. (2001) extends the framework of Matsuyama et al. (1993) by endogenizing prices and exchange rates. Wright et al. (2001) also introduce some policy considerations and one of their findings is that an international currency has higher purchasing power in its home country than abroad. Other notable papers that use search-theoretic framework to discuss issues related to dual currency or currency competition include Kiyotaki and Wright (1993), Ravikumar and Wallace (2002), Trejos (2003), and Li and Matsui (2009). Curtis and Waller (2000) study how governments attempt to reduce circulation of foreign currency by lowering its purchasing power through tax induced transactions costs. Their other paper, Waller and Curtis (2003) study how government transaction policies affect the values of fiat currencies in a two country, divisible good, search model. The topic of government transaction policies and its effect on acceptance of certain fiat currencies has also been discussed in Li and Wright (1998) albeit in an indivisible goods model.

The abovementioned papers have unit inventory restrictions thereby making it difficult to study which currency agents choose for transaction purposes. Camera et al. (2004) relaxes this restriction and studies a model where relative risk of currencies and trading difficulties associated with them affect spending patterns of agents and the transactions velocities of each currency. Craig and Waller (2004) use a setup similar to Camera et al. (2004) to study dollarization. They consider a safe and a risky currency and study the dynamics of equilibrium distribution of currency portfolios and exchange rates.

In recent years, Lagos and Wright (2005) type of models have become the primary workhorse in the field of new monetarist economics. Compared to the earlier models, these models are less restrictive in terms of divisibility and distribution of assets without compromising on tractability. Lester et al. (2012) uses a multiple-asset version of Lagos and Wright (2005)

where agents' ability to recognize an asset determines its liquidity and acceptability. They endogenize recognizability by allowing agents to invest in information acquisition. This idea of imperfect recognizability of different currencies has been further explored in Zhang (2014) where sellers invest in information acquisition by incurring a fixed cost. As in Lester et al. (2012), strategic complementarities in portfolio choices and information acquisition leads to multiple equilibria. Also, in line with Li and Matsui (2009) inflation of the international currency could benefit the country that issues it (through higher seignorage), but the threat of losing international status puts inflation discipline on the country.

In this paper I consider a two-country, two-currency version of the standard Lagos and Wright (2005) model with transactions cost. A currency abroad has lesser value than it has at home because of transactions costs associated with its use. The paper models transaction costs by assuming that one unit of a currency abroad can buy only a fraction of what it can buy at home. This leads to asymmetric valuation of currencies by agents from two different countries. I employ a model of bargaining that is similar to the one used in Geromichalos and Simonovska (2014). This results in unique portfolio choice by agents. I show that by modeling transaction costs in this way and by using the specific bargaining setup it is possible to pin down the unique currency regime that will arise. The earlier search theoretic models while being useful from the point of understanding the dynamics of currency competition, has multiplicity of equilibria. The current setup helps us avoid the multiple equilibria, and one can then use the baseline model to study certain extensions.

2.2 Environment

We consider an environment where time is discrete and continues forever. There are two countries namely, Country 1 and Country 2 which are populated by infinitely-lived agents of measure 2 and $2n$ respectively, where $n > 0$ denotes the size of Country 2 relative to Country

1. Each time period is divided into two sub-periods. In the first sub-period, local and foreign goods are traded in *decentralized markets* (DM), while in the second sub-period, economic activity takes place in a *centralized market* (CM) for settlement and currency exchange.

In line with Rocheteau and Wright (2005a), agents from each country are equally divided between *buyers* and *sellers*. These labels refer to agents' role in the DM s and such roles remain unchanged over time. In a DM , a seller can produce but does not want to consume, while a buyer wants to consume but cannot produce. Therefore, there is no double coincidence of wants. During the first sub-period a distinct DM opens up in each country. There are two distinct DM s – one for each country which I will denote by DM_i , $i = 1, 2$ where the subscripts refer to the specific country. The DM s are marked by search frictions where buyers and sellers trade in anonymous pairwise meetings.

Sellers have immobile factors of production, but buyers are mobile. During the first sub-period, a buyer from Country i ($i = 1, 2$) stays in his home DM (i.e. DM_i) with probability α or visits the foreign DM (i.e. DM_j , $j \neq i$) with probability $1 - \alpha$. Due to immobile factors of production, a seller is localized and operates only in his home DM . A seller might be matched with a domestic buyer or he might be matched with a visiting foreign buyer. Following the definition of Matsuyama et al. (1993), the ratio $(1 - \alpha)/\alpha$ represents the degree of economic integration. If the two countries are perfectly integrated then a buyer is equally likely to visit the home DM and the foreign DM , i.e. $\alpha = \frac{1}{2}$ which implies the degree of economic integration, $(1 - \alpha)/\alpha = 1$. If there is zero economic integration, then the buyer shops exclusively in his home DM , i.e. $\alpha = 1$ implying $(1 - \alpha)/\alpha = 0$. And, under imperfect economic integration a buyer is more likely to stay in home DM than visit the foreign DM implying $\alpha > \frac{1}{2}$. Thus, $\alpha \in [\frac{1}{2}, 1]$.

During the second sub-period, all agents are located in their home country and trading occurs in a frictionless Walrasian market (the CM). In this sub-period all agents consume a numéraire good which is produced through a technology that is linear in labor. The supply

of labor hours in the *CM* is h . Buyers consume in both sub-periods, but supply labor only in the *CM*. Sellers consume only in the *CM* and produce in both the *DM* and the *CM*. The instantaneous utility functions of buyers and sellers are additively separable and quasilinear. Also, the preferences of buyers and sellers are assumed to be independent of their country of residence. In the second subperiod of every period, every agent has access to a linear production technology that transforms a unit of an agents labor into a unit of the numéraire good. The preferences of the buyers and sellers are given by:

$$\mathcal{U}^b(q, x, h) = u(q) + U(x) - h \quad (2.1)$$

$$\mathcal{U}^s(q, x, h) = -q + U(x) - h \quad (2.2)$$

where q denotes the amount consumed by a buyer in the *DM* and it is equal to the amount produced by a seller the buyer meets. The variables x and h are the amount of the numéraire good consumed and the amount of labor supplied by an agent in the *CM*. For simplicity I assume that the production technology for the *DM* good is also linear in labor. However, this assumption is not crucial for our results. A seller's disutility of producing q units of *DM* consumption good is therefore $c(q) = -q$. The functions $u(q)$, $U(x)$ are twice continuously differentiable with $u(0) = U(0) = 0$, $u'(q) > 0$, $U'(x) > 0$, $u''(q) < 0$, and $U''(x) < 0$. The functions $u(q)$ and $U(x)$ satisfy the Inada conditions: $u'(0) = U'(0) = \infty$ and $\lim_{q \rightarrow +\infty} u'(q) = \lim_{x \rightarrow +\infty} U'(x) = 0$. We also assume that the relative risk aversion of $u(q)$ is sufficiently low, i.e. Arrow-Pratt measure of risk aversion $\in (0, 1]$. Also, suppose that there exists a $q^* \equiv \{q \in \mathbb{R}_+ : u'(q) = 1\}$ and a $x^* \equiv \{x \in \mathbb{R}_+ : U'(x) = 1\}$. The amount q^* maximizes the total surplus, $u(q) - q$ in a *DM* trade between a buyer and a seller. Goods

are perishable both over periods and over sub-periods. Hence, agents cannot save and must consume within the the sub-period. Let R be the real interest rate, then agents discount across periods at the rate $\beta = (1 + R)^{-1} \in (0, 1)$ but do not discount within a period.

In the first sub-period, after buyers are randomly assigned a DM , let the measure of buyers in DM_i ($i = 1, 2$) be represented by \mathcal{B}_i . Then $\mathcal{B}_1 = \alpha + (1 - \alpha)n$ which is the sum of a fraction α of buyers from Country 1 and a fraction $(1 - \alpha)$ of buyers from Country 2. Similarly, $\mathcal{B}_2 = \alpha n + (1 - \alpha)$. The measure of sellers in DM_1 is given by $\mathcal{S}_1 = 1$, while that in DM_2 is given by $\mathcal{S}_2 = n$. The measure of matches in DM_i is given by the matching function, $\mathcal{M}_i(\mathcal{B}_i, \mathcal{S}_i) = \frac{\mathcal{B}_i \mathcal{S}_i}{\mathcal{B}_i + \mathcal{S}_i}$. $\mathcal{M}_i(\mathcal{B}_i, \mathcal{S}_i)$ is a constant returns to scale matching function and it satisfies the technical properties found in Berentsen et al. (2007) which specifies a general version of this matching function. Under the specification of this model, $\mathcal{M}_1(\mathcal{B}_1, \mathcal{S}_1)$ and $\mathcal{M}_2(\mathcal{B}_2, \mathcal{S}_2)$ are given by:

$$\mathcal{M}_1(\mathcal{B}_1, \mathcal{S}_1) = \frac{\alpha + (1 - \alpha)n}{1 + \alpha + (1 - \alpha)n} \quad (2.3)$$

$$\mathcal{M}_2(\mathcal{B}_2, \mathcal{S}_2) = \frac{\alpha n^2 + (1 - \alpha)n}{n + \alpha n + (1 - \alpha)} \quad (2.4)$$

Therefore, for a buyer who is in DM_i , the arrival rate of a seller is then $\lambda_i = \mathcal{M}_i(\mathcal{B}_i, \mathcal{S}_i)/\mathcal{B}_i$. The probability with which a buyer from Country i ($i = 1, 2$) meets a local seller is $\alpha\lambda_i$ and the probability with which he meets a foreign seller $(1 - \alpha)\lambda_j$ (where $j \neq i$). Table 2.1 summarizes the probabilities with which buyers meet sellers from the two countries: Similarly, for a seller in DM_i , the arrival rate of any buyer is $\mu_i = \mathcal{M}_i(\mathcal{B}_i, \mathcal{S}_i)/\mathcal{S}_i$. The probability with which a Country i (where $i = 1, 2$) seller meets a buyer from his own country is $\alpha\lambda_i \cdot (\mathcal{B}_i/\mathcal{S}_i) = \alpha\mu_i$ and that with which he meets a buyer from Country j (where $j \neq i$) is $(1 - \alpha)\lambda_i \cdot (\mathcal{B}_i/\mathcal{S}_i) = (1 - \alpha)\mu_i$. Table 2.2 summarizes the probabilities with which

	Seller from Country 1	Seller from Country 2
Buyer from Country 1	$\alpha\lambda_1$	$(1 - \alpha)\lambda_2$
Buyer from Country 2	$(1 - \alpha)\lambda_1$	$\alpha\lambda_2$

Table 2.1: Buyers' probability of being matched with a seller.

seller meet buyers from the two countries: Each country issues its own perfectly divisible

	Buyer from Country 1	Buyer from Country 2
Seller from Country 1	$\alpha\mu_1$	$(1 - \alpha)\mu_1$
Seller from Country 2	$(1 - \alpha)\mu_2$	$\alpha\mu_2$

Table 2.2: Sellers' probability of being matched with a buyer.

fiat currency. We will call the currency of Country i as Currency i . Currency i ($i = 1, 2$) is valued at ϕ_i , where ϕ_i is the value of Currency i in terms of the CM numéraire good. In the beginning of the CM due to foreign currency controls, the authorities in Country i levy a tax at the rate of $t_j \in [0, 1]$ on the current real balance of foreign currency (Currency j) held by its residents. The tax revenue collected by the authorities are not transferred back to the residents and are there to serve as a barrier to the use of foreign currency. The total supply of Currency i , M_i ($i = 1, 2$) change each period at the rate $\gamma_i \in \mathbb{R}$. Therefore future amount of Currency i , $M'_i = (1 + \gamma_i)M_i$. Changes in the money supply are implemented through lump-sum transfers of ($\gamma_i > 0$) or taxes on ($\gamma_i < 0$) domestic currency in the CM to (from) that country's buyers.

2.3 Value Functions and Optimal Behavior

In this section I discuss the equilibrium in the two-country, two-currency model. We focus on steady-state equilibrium in which the rate of return of Currency i , $i \in \{1, 2\}$ in each country is constant and in which aggregate real balances in each country are constant over time.

2.3.1 Centralized Market Value Function

During the *CM* the representative buyer from Country i chooses the level of numéraire good he wants to consume, x , amount of labor, h and real balances to bring forward to the next period. I define: $(z_1^{(i)}, z_2^{(i)}) \equiv (\phi_1 m_1^{(i)}, \phi_2 m_2^{(i)})$ and $(z_1^{(i)'}, z_2^{(i)'}) \equiv (\phi_1' m_1^{(i)'}, \phi_2' m_2^{(i)'})$. The pair $(z_1^{(i)}, z_2^{(i)})$ represents a Country i buyer's current real balances held in Currency 1 and Currency 2 respectively, while $(z_1^{(i)'}, z_2^{(i)'})$ denote his portfolio of real balances in the two currencies for the next time period. The variables ϕ_1', ϕ_2' stand for the value of Currency 1 and Currency 2 in the next time period and $(m_1^{(i)'}, m_2^{(i)'})$ is the Country i buyer's nominal balance of the two monies for the next time period. Also let $W_i^b(z_1^{(i)}, z_2^{(i)})$ and $V_i^b(z_1^{(i)'}, z_2^{(i)'})$ denote the *CM* and *DM* value functions of a Country i buyer with portfolio $(z_1^{(i)}, z_2^{(i)})$. The *CM* value function for a buyer from Country i is then

$$\begin{aligned}
W_i^b(z_1^{(i)}, z_2^{(i)}) &= \max_{x, h, z_1^{(i)'}, z_2^{(i)'}} \{U(x) - h + \beta V_i^b(z_1^{(i)'}, z_2^{(i)'})\} \\
\text{s.t. } x + \frac{\phi_1}{\phi_1'} z_1^{(i)'} + \frac{\phi_2}{\phi_2'} z_2^{(i)'} &= h + (1 - t_1 \mathbb{1}_{\{i=2\}}) z_1^{(i)} + (1 - t_2 \mathbb{1}_{\{i=1\}}) z_2^{(i)} + \mathbb{1}_{\{i=1\}} T_1 + \mathbb{1}_{\{i=2\}} T_2 \\
&\text{where } x \geq 0, 0 \leq h \leq \bar{h}, \text{ and } z_1^{(i)'} \geq 0, z_2^{(i)'} \geq 0
\end{aligned} \tag{2.5}$$

The indicator functions in (2.5) indicates that certain parameters are country-specific. The parameter t_1 is the tax rate on current real balance of Currency 1 if the agent is from Country 2. Similarly t_2 is the tax rate on current real balance of Currency 2 if the agent is from Country 1.. T_1 and T_2 are monetary transfers (in terms of the numéraire) to the buyers from Country 1 and Country 2 respectively.

Equation (2.5) can be further simplified into

$$W_i^b(z_1^{(i)}, z_2^{(i)}) = z_i^{(i)} + (1 - t_j)z_j^{(i)} + \Omega_i \quad (2.6)$$

$$\text{where } \Omega_i = T_i + U(x^*) - x^* + \max_{z_1^{(i)'}, z_2^{(i)'}} \left\{ -\frac{\phi_1}{\phi_1'} z_1^{(i)'} - \frac{\phi_2}{\phi_2'} z_2^{(i)'} + \beta V_i^b(z_1^{(i)'}, z_2^{(i)'}) \right\}$$

where $x^* \in \mathbb{R}_+$ such that $U'(x^*) = 1$, represents the optimal consumption choice of the numéraire good in the *CM*. A similar exercise yields the Country i seller's *CM* value function:

$$W_i^s(\tilde{z}_1^{(i)}, \tilde{z}_2^{(i)}) = \tilde{z}_1^{(i)} + (1 - t_j)\tilde{z}_j^{(i)} + \tilde{\Omega}_i \quad (2.7)$$

$$\text{where } \tilde{\Omega}_i = U(x^*) - x^* + \max_{\tilde{z}_1^{(i)'}, \tilde{z}_2^{(i)'}} \left\{ -\frac{\phi_1}{\phi_1'} \tilde{z}_1^{(i)'} - \frac{\phi_2}{\phi_2'} \tilde{z}_2^{(i)'} + \beta V_i^s(\tilde{z}_1^{(i)'}, \tilde{z}_2^{(i)'}) \right\}$$

The equations (2.6) and (2.7) imply that the effective value of one unit of Currency j (where $j \neq i$) to an agent from Country i is $\phi_j(1 - t_j)$. This is because in his country, with one unit of Currency j he can only buy to buy $\phi_j(1 - t_j)$ units of the numéraire since he pays a real tax of real tax of $t_j\phi_j$ for every unit of Currency j he holds.

2.3.2 Decentralized Market Terms of Trade

In the *DM*, once a match occurs, the agents bargain over terms of trade. Terms of trade is a triplet $(q^\chi, d_1^\chi, d_2^\chi)$ where q^χ is the amount of the good to be exchanged and (d_1^χ, d_2^χ) is the real payment in Currency 1 and Currency 2 that a buyer makes to a seller in a match $\chi \in X \equiv \{(11), (12), (21), (22)\}$. X is the set of all possible matches where (ii) denotes Country i buyer matched with Country i seller, (ij) denotes Country i buyer matched with Country j seller ($i \neq j$) and (ji) denotes Country j buyer matched with Country i seller ($i \neq j$). Buyer makes a take-it-or-leave-it offer to a seller which the seller can either accept or decline. If the seller declines the offer, then no trade takes place.

Bargaining with local seller

The bargaining solution, hence the terms of trade will depend on a buyer's and a seller's nationality. This difference arises due to asymmetric valuation of currencies by agents from different countries. Below I describe the bargaining problem when a buyer from Country i meets a seller from the same country (local seller):

$$\begin{aligned}
& \max_{q^{(ii)}, d_1^{(ii)}, d_2^{(ii)}} \{u(q^{(ii)}) + W_i^b(z_1^{(i)} - d_1^{(ii)}, z_2^{(i)} - d_2^{(ii)}) - W_i^b(z_1^{(i)}, z_2^{(i)})\} \\
& \text{s.t. } q = W_i^s(\tilde{z}_1^{(i)} + d_1^{(ii)}, \tilde{z}_2^{(i)} + d_2^{(ii)}) - W_i^s(\tilde{z}_1^{(i)}, \tilde{z}_2^{(i)}) \\
& \text{and } 0 \leq d_1^{(ii)} \leq z_1^{(i)}, 0 \leq d_2^{(ii)} \leq z_2^{(i)}
\end{aligned} \tag{2.8}$$

Define $S^{(ii)}(z_1^{(i)}, z_2^{(i)})$ as the surplus from DM trade that a buyer from Country i with portfolio $(z_1^{(i)}, z_2^{(i)})$ receives when he trades with a local seller. Exploiting the linearity of the CM value functions, (2.8) can be simplified to

$$\begin{aligned}
S^{(ii)}(z_1^{(i)}, z_2^{(i)}) & \equiv \max_{q^{(ii)}, d_1^{(ii)}, d_2^{(ii)}} \{u(q^{(ii)}) - d_i^{(ii)} - (1 - t_j)d_j^{(ii)}\} \\
& \text{s.to: } q^{(ii)} = d_i^{(ii)} + (1 - t_j)d_j^{(ii)} \\
& \text{and } 0 \leq d_1^{(ii)} \leq z_1^{(i)}, 0 \leq d_2^{(ii)} \leq z_2^{(i)}
\end{aligned} \tag{2.9}$$

The following proposition describes the solution to the DM bargaining problem when a buyer meets a local seller.

Proposition 1: Define $w_i \equiv z_i^{(i)} + (1 - t_j)z_j^{(i)}$, the liquid wealth held in currencies by a buyer from Country i . Then the bargaining solution when a buyer meets a local seller is given by:

$$\begin{aligned} \text{(a) If } w_i \geq q^* \text{ then } & \begin{cases} q^{(ii)} = q^* \\ d_i^{(ii)} + (1 - t_j)d_j^{(ii)} = q^* \end{cases} \\ \text{(b) If } w_i < q^*, \text{ then } & \begin{cases} q^{(ii)} = w_i \\ d_i^{(ii)} = z_i^{(i)}, d_j^{(ii)} = z_j^{(i)} \end{cases} \end{aligned}$$

Proof: In appendix.

When the buyer possesses sufficient liquid wealth (valued at local prices) to buy q^* , i.e. when we have case (a) of Proposition 1, it is not possible to pin down an unique $(d_1^{(ii)}, d_2^{(ii)})$. When $w_i \geq q^*$, the buyer will buy the optimal q^* and since both the buyer and the seller have similar valuation of both currencies, the buyer can pay a domestic seller in any currency and in any amount of those currencies as long as $q^* = d_i^{(ii)} + (1 - t_j)d_j^{(ii)}$ is satisfied. The buyers' surplus function in a DM trade with local seller, $S^{(ii)}(z_1^{(i)}, z_2^{(i)})$ is then given by

$$S^{(ii)}(z_1^{(i)}, z_2^{(i)}) = \begin{cases} u(q^*) - q^*, & \text{when } w_i \geq q^* \\ u(w_i) - w_i, & \text{otherwise} \end{cases}$$

Lemma 1: $S^{(ii)}(z_1^{(i)}, z_2^{(i)})$, the DM surplus function when a buyer from Country i ($i = 1, 2$) meets a seller from his own country is non-decreasing in each of its arguments and jointly concave in $(z_1^{(i)}, z_2^{(i)})$.

Proof: In appendix

Bargaining with foreign seller

Define $S^{(ij)}(z_1^{(i)}, z_2^{(i)})$ as the surplus from DM trade that a buyer from Country i with portfolio $(z_1^{(i)}, z_2^{(i)})$ receives when he trades with a foreign seller. When a buyer from Country i meets a foreign seller (i.e. seller from Country j , $j \neq i$), then the bargaining problem is

$$\begin{aligned} S^{(ij)}(z_1^{(i)}, z_2^{(i)}) &\equiv \max_{q^{(ij)}, d_i^{(ij)}, d_j^{(ij)}} \left\{ u(q^{(ij)}) - d_i^{(ij)} - (1 - t_j)d_j^{(ij)} \right\} \\ \text{s.t. } q^{(ij)} &= (1 - t_i)d_i^{(ij)} + d_j^{(ij)} \\ \text{and } 0 \leq d_i^{(ij)} &\leq z_i^{(i)}, 0 \leq d_j^{(ij)} \leq z_j^{(i)} \end{aligned} \quad (2.10)$$

By substituting $q^{(ij)}$ from the constraint into the objective function, eq.(2.10), the problem can be further simplified into:

$$\begin{aligned} S^{(ij)}(z_1^{(i)}, z_2^{(i)}) &= \max_{q^{(ij)}, d_i^{(ij)}, d_j^{(ij)}} \left\{ u(q^{(ij)}) - q^{(ij)} - t_i d_i^{(ij)} + t_j d_j^{(ij)} \right\} \\ \text{and } 0 \leq d_i^{(ij)} &\leq z_i^{(i)}, 0 \leq d_j^{(ij)} \leq z_j^{(i)} \end{aligned} \quad (2.11)$$

The expression $u(q^{(ij)}) - q^{(ij)} - t_i d_i^{(ij)} + t_j d_j^{(ij)}$ is the buyer's surplus from a DM trade with a foreign seller when q amount changes hand and payments of amount $d_i^{(ij)}$ and $d_j^{(ij)}$ are made in currencies i and j respectively. The following proposition lays out the solution to the DM bargaining problem when a buyer meets a foreign seller.

Proposition 2: Define $\omega_i = (1 - t_i)z_i^{(i)} + z_j^{(i)}$, $\underline{q}(t_i) \equiv \{q \in \mathbb{R}_+ : u'(q) = 1/(1 - t_i)\}$, and $\bar{q}(t_j) \equiv \{q \in \mathbb{R}_+ : u'(q) = 1 - t_j\}$, then the bargaining solution between a buyer from Country i and seller from Country j ($i \neq j$) is given by:

$$\begin{aligned}
\text{(a) If } z_j^{(i)} \geq \bar{q}(t_j) \text{ then } & \begin{cases} q^{(ij)} = \bar{q}(t_j) \\ d_i^{(ij)} = 0 \\ d_j^{(ij)} = \bar{q}(t_j) \end{cases} \\
\text{(b) If } z_j^{(i)} \in [\underline{q}(t_i), \bar{q}(t_j)) \text{, then } & \begin{cases} q^{(ij)} = z_j^{(i)} \\ d_i^{(ij)} = 0 \\ d_j^{(ij)} = z_j^{(i)} \end{cases} \\
\text{(c) If } z_j^{(i)} < \underline{q}(t_i) \text{ and } \omega_i \geq \underline{q}(t_i) \text{, then } & \begin{cases} q^{(ij)} = \underline{q}(t_i) \\ d_i^{(ij)} = \frac{\underline{q}(t_i) - z_j^{(i)}}{1 - t_i} \\ d_j^{(ij)} = z_j^{(i)} \end{cases} \\
\text{(d) If } \omega_i < \underline{q}(t_i) \text{, then } & \begin{cases} q^{(ij)} = \omega_i \\ d_i^{(ij)} = z_i^{(i)} \\ d_j^{(ij)} = z_j^{(i)} \end{cases}
\end{aligned}$$

Proof: In appendix

Proposition 2 states that a buyer from Country i ($i = 1, 2$) should pay a seller from Country j ($i \neq j$) in currency j whenever possible. If the buyer transfers the currency that he values less to someone who values it more than him, then it raises the buyer's surplus. We can rewrite the bargaining problem in (2.10) as maximizing $u(q^{(ij)}) - q^{(ij)} + t_j d_j^{(ij)} - t_i d_i^{(ij)}$ with respect to $q^{(ij)}, d_i^{(ij)}, d_j^{(ij)}$. The more the foreign currency is transferred from buyer to seller higher are the benefits to the buyer. Whereas using home currency has a decreasing effect on buyer's surplus. If a buyer has enough foreign currency to buy $\bar{q}(t_j)$, then he should pay

$\bar{q}(t_j)$ in foreign real balances and receive $\bar{q}(t_j)$. If the buyer doesn't have enough foreign real balances to buy $\bar{q}(t_j)$, but has at least enough of the same to buy $\underline{q}(t_i)$, then he should again use his entire holding of foreign currency and none of his home currency. A buyer should use home currency only if his real balances held in foreign currency is lower than $\underline{q}(t_i)$. When, a buyer's real balances held in foreign currency is lower than $\underline{q}(t_i)$, but his total liquid wealth (valued at foreign country's prices) exceeds $\underline{q}(t_i)$, then he exhausts his holding of foreign currency to buy $\underline{q}(t_i)$ and uses his real balances held in home currency only to pay the difference. In such a case, although the buyer can afford to buy more than $\underline{q}(t_i)$, he wouldn't do so because the more home currency he uses, the lesser is his surplus. The fourth case implies that if a buyer's real balances held in foreign currency is lower than $\underline{q}(t_i)$, and his total wealth (valued at foreign country's prices) is also less than $\underline{q}(t_i)$, then he should exhaust his entire real balance and buy as much as he can.

As in the case of a *DM* trade with a local seller, here also we can rewrite the surplus as a function of of real balances $z_i^{(i)}$ and $z_j^{(i)}$. The buyers' surplus function in a *DM* trade with local seller, $S^{(ij)}(z_1^{(i)}, z_2^{(i)})$ is then given by

$$S^{(ij)}(z_1^{(i)}, z_2^{(i)}) = \begin{cases} u(\bar{q}(t_j)) - (1 - t_j)\bar{q}(t_j), & \text{when } z_j^{(i)} \geq \bar{q}(t_j) \\ u(z_j^{(i)}) - (1 - t_j)z_j^{(i)}, & \text{when } z_j^{(i)} \in [\underline{q}(t_i), \bar{q}(t_j)] \\ u(\underline{q}(t_i)) - \frac{1}{1-t_i}\underline{q}(t_i) + \left(\frac{t_i}{1-t_i} + t_j\right)z_j^{(i)}, & \text{when } z_j^{(i)} < \underline{q}(t_i) \text{ and } \omega_i \geq \underline{q}(t_i) \\ u(\omega_i) - \frac{1}{1-t_i}\omega_i + \left(\frac{t_i}{1-t_i} + t_j\right)z_j^{(i)}, & \text{when } \omega_i < \underline{q}(t_i) \end{cases}$$

Lemma 2: $S^{(ij)}(z_1^{(i)}, z_2^{(i)})$, the DM surplus function when a buyer from Country i ($i = 1, 2$) meets a foreign seller (from Country j , $j \neq i$) is non-decreasing in each of its arguments and jointly concave in $(z_1^{(i)}, z_2^{(i)})$.

Proof: In appendix

In Figure 2.1, I depict a buyer's surplus function from DM trade with a foreign seller. We assume $t_i = t_j = 0.05$ and use $u(q) = \sqrt{q}$.

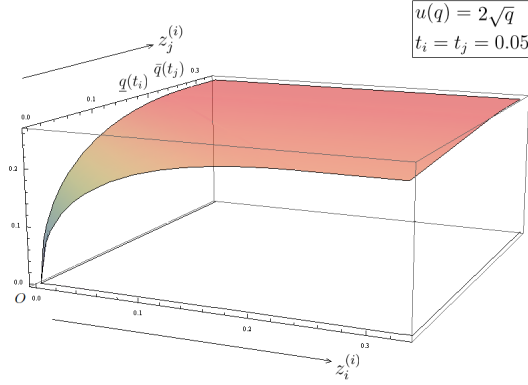


Figure 2.1: Country i buyer's surplus function when matched with a foreign (Country j) seller.

2.3.3 Decentralized Market Value Function

In the DM a buyer from Country i ($i = 1, 2$) faces three possibilities – stay in own country and meet a seller, visit foreign country and meet a seller, or not find a match at all. So, the DM value function for a buyer is the probability weighted average of these three payoffs.

$$\begin{aligned}
 V_i^b(z_1^{(i)}, z_2^{(i)}) = & \max_{q^{(ii)}, d_1^{(ii)}, d_2^{(ii)}, q^{(ij)}, d_1^{(ij)}, d_2^{(ij)}} \left\{ \alpha \lambda_i \left[u(q^{(ii)}) + W_1^b(z_1^{(i)} - d_1^{(ii)}, z_2^{(i)} - d_2^{(ii)}) \right] \right. \\
 & + (1 - \alpha) \lambda_j \left[u(q^{(ij)}) + W_i^b(z_1^{(i)} - d_1^{(ij)}, z_2^{(i)} - d_2^{(ij)}) \right] \\
 & \left. + \left(1 - \alpha \lambda_i - (1 - \alpha) \lambda_j \right) W_i^b(z_1^{(i)}, z_2^{(i)}) \right\} \quad (2.12)
 \end{aligned}$$

When a buyer meets and trades with a seller (domestic or foreign), then his payoff is the utility from DM consumption plus his continuation value. By plugging in the terms of trade, we can rewrite the DM value function for a Country i buyer as:

$$V_i^b(z_1^{(i)}, z_2^{(i)}) = \alpha \lambda_i S^{(ii)}(z_1^{(i)}, z_2^{(i)}) + (1 - \alpha) \lambda_j S^{(ij)}(z_1^{(i)}, z_2^{(i)}) + z_i^{(i)} + (1 - t_j) z_j^{(i)} + W_i^b(0, 0) \quad (2.13)$$

The DM value function for a seller is similarly the probability weighted average of the DM values under the three scenarios: (a) seller is matched with domestic buyer, (b) seller is matched with foreign buyer and (c) no match at all. The seller cannot consume in the DM , so he gets no utility from consumption. However, if a match is made, he always faces a disutility from production and this is given by the cost function $c(q) = q$. The DM value function of a seller from Country i ($i = 1, 2$) holding a portfolio of real balances $(\tilde{z}_1^{(i)}, \tilde{z}_2^{(i)})$ is given by

$$\begin{aligned} V_i^s(\tilde{z}_1^{(i)}, \tilde{z}_2^{(i)}) = & \max_{q^{(ii)}, d_1^{(ii)}, d_2^{(ii)}, q^{(ij)}, d_1^{(ij)}, d_2^{(ij)}} \left\{ \alpha \mu_i \left[-q^{(ii)} + W_i^s(\tilde{z}_1^{(i)} + d_1^{(ii)}, \tilde{z}_2^{(i)} + d_2^{(ii)}) \right] \right. \\ & + (1 - \alpha) \mu_i \left[-q^{(ij)} + W_i^s(\tilde{z}_1^{(i)} + d_1^{(ij)}, \tilde{z}_2^{(i)} + d_2^{(ij)}) \right] \\ & \left. + \left(1 - \alpha \mu_i - (1 - \alpha) \mu_i \right) W_i^s(\tilde{z}_1^{(i)}, \tilde{z}_2^{(i)}) \right\} \quad (2.14) \end{aligned}$$

As in the case of the buyer, by plugging in the terms of trade, eq.(2.14) can be rewritten as

$$V_i^s(\tilde{z}_1^{(i)}, \tilde{z}_2^{(i)}) = \tilde{z}_i^{(i)} + (1 - t_j) \tilde{z}_j^{(i)} + W_i^s(0, 0) \quad (2.15)$$

In the DM an agent, in conjunction with his trading partner has to decide on the terms of trade that will maximize their DM value functions while, in the CM , an agent decides on his portfolio of real balances to take forward to the next time period. To solve for

the agents' portfolio problem, the DM value functions are lead forward by one period and the expression is plugged back in $\left\{ -\frac{\phi_1}{\phi'_1} z_1^{(i)'} - \frac{\phi_2}{\phi'_2} z_2^{(i)'} + \beta V_i^b(z_1^{(i)'}, z_2^{(i)'}) \right\}$ (eq. (2.6)) or, $\left\{ -\frac{\phi_1}{\phi'_1} \tilde{z}_1^{(i)'} - \frac{\phi_2}{\phi'_2} \tilde{z}_2^{(i)'} + \beta V_i^s(\tilde{z}_1^{(i)'}, \tilde{z}_2^{(i)'}) \right\}$ (eq. (2.7) as the case might be and maximized with respect to $z_1^{(i)'}$ and $z_2^{(i)'}$ (or, $\tilde{z}_1^{(i)'}$ and $\tilde{z}_2^{(i)'}$).

2.3.4 Portfolio Choice

Seller's Portfolio Choice Problem

Substituting $V_i^s(\tilde{z}_1^{(i)'}, \tilde{z}_2^{(i)'})$ with its expression given by eq.(2.15) in $\left\{ -\frac{\phi_1}{\phi'_1} \tilde{z}_1^{(i)'} - \frac{\phi_2}{\phi'_2} \tilde{z}_2^{(i)'} + \beta V_i^s(\tilde{z}_1^{(i)'}, \tilde{z}_2^{(i)'}) \right\}$, the Country i seller's CM portfolio choice problem at the steady-state can be written as:

$$\max_{\tilde{z}_1^{(i)} \geq 0, \tilde{z}_2^{(i)} \geq 0} -\iota_i \tilde{z}_i^{(i)} - (\iota_j + t_j) \tilde{z}_j^{(i)} \quad (2.16)$$

where $1 + \iota_i = \frac{\phi_i}{\beta \phi'_i} = (1 + \pi_i)(1 + R)$ and $1 + \iota_j = \frac{\phi_j}{\beta \phi'_j} = (1 + \pi_j)(1 + R)$ are the marginal cost of holding currencies i and j respectively ¹ In (2.16), I use $(\tilde{z}_1^{(i)}, \tilde{z}_2^{(i)})$ and not $(\tilde{z}_1^{(i)'}, \tilde{z}_2^{(i)'})$ because at steady-state both are equal. For the problem described in (2.16), solution exists if and only if have $\iota_i \geq 0$ and $\iota_j \geq 0$. For the extreme case of $\iota_i = 0$ (or $\iota_j = 0$) following Lagos and Wright (2005) I assume that ι_i (or ι_j) approaches 0 from above. Since we never have $\iota_i < 0$ or $\iota_j < 0$, therefore as $\iota_i \rightarrow 0+$ (or $\iota_j \rightarrow 0+$). Then the solution to the above problem is $\tilde{z}_i^{(i)} = 0$, $\tilde{z}_j^{(i)} = 0$. Therefore, sellers from either country do not bring any real balance, of any currency to the DM .

¹Note, that $\phi_i/\phi'_i = (1 + \pi_i)$ where π_i is the rate of inflation. The marginal cost of holding currency i therefore comes from its inflation.

Buyer's Portfolio Choice Problem

The steady-state portfolio problem of the representative buyer from Country i can be formally written as:

$$\max_{z_1^{(i)} \geq 0, z_2^{(i)} \geq 0} \alpha \lambda_i S^{(ii)}(z_1^{(i)}, z_2^{(i)}) + (1 - \alpha) \lambda_j S^{(ij)}(z_1^{(i)}, z_2^{(i)}) - \iota_i z_i^{(i)} - (\iota_j + t_j) z_j^{(i)} \quad (2.17)$$

First order conditions for the problem described in (2.17):

$$\alpha \lambda_i \frac{\partial S^{(ii)}(z_1^{(i)}, z_2^{(i)})}{\partial z_i^{(i)}} + (1 - \alpha) \lambda_j \frac{\partial S^{(ij)}(z_1^{(i)}, z_2^{(i)})}{\partial z_i^{(i)}} - \iota_i \leq 0 \text{ and "=" } 0 \text{ if } z_i^{(i)} > 0 \quad (2.18)$$

$$\alpha \lambda_i \frac{\partial S^{(ii)}(z_1^{(i)}, z_2^{(i)})}{\partial z_j^{(i)}} + (1 - \alpha) \lambda_j \frac{\partial S^{(ij)}(z_1^{(i)}, z_2^{(i)})}{\partial z_j^{(i)}} - (\iota_j + t_j) \leq 0 \text{ and "=" } 0 \text{ if } z_j^{(i)} > 0 \quad (2.19)$$

The solution to (2.18) and (2.19) gives the optimal $z_i^{(i)}$ and $z_j^{(i)}$ ($i = 1, 2; i \neq j$) for the buyer. Since the functions $S^{(ii)}(z_1^{(i)}, z_2^{(i)})$ and $S^{(ij)}(z_1^{(i)}, z_2^{(i)})$ are jointly concave in $(z_1^{(i)}, z_2^{(i)})$ it is not necessary to check for second order conditions.

Lemma 3: *There exists an unique solution to the Country i ($i = 1, 2$) buyer's portfolio choice problem described in (2.17).*

Proof: In appendix

2.4 Equilibrium in the Two-Country Model

We begin this section with a general definition of equilibrium and focus on the symmetric, steady-state equilibria.

Definition 1: *Given (t_1, t_2) , a stationary monetary equilibrium is a list of quantities traded in the DM and their payments in the two currencies $(q^\chi, d_1^\chi, d_2^\chi) \forall \chi \in X \equiv \{(11), (12), (21), (22)\}$ and real balances $(z_1^{(i)}, z_2^{(i)}) \forall i \in \{1, 2\}$ such that:*

- *For a match of type $\chi \in X$, $(q^\chi, d_1^\chi, d_2^\chi)$ solves the bargaining problem.*
- *$(z_1^{(i)}, z_2^{(i)})$ solves the Country i ($i = 1, 2$) buyers' portfolio problem.*
- *Money markets clear.*

Since sellers will choose not to carry any money (or real balances) from *CM* to *DM* money market clearing conditions are given by

$$z_1^{(1)} + nz_1^{(2)} = \phi_1 M_1 \tag{2.20}$$

$$z_2^{(1)} + nz_2^{(2)} = \phi_2 M_2 \tag{2.21}$$

Where M_1 and M_2 are the nominal stocks of Currencies 1 and 2 respectively.

We now proceed to a more careful discussion of the optimal portfolio choice of buyers and, consequently, the equilibrium. For this purpose I use a CRRA utility function $u(q) = 2\sqrt{q}$ which has a relative risk aversion of 0.5. For starters, I assume that Country 2 is a small open economy with $n = 0.1$ and $\alpha = 0.5$. The loss in real value of currency 1 and currency 2 due to institutional barriers ('tax') in both countries is 1%, that is $t_1 = t_2 = 0.01$. In

Figure 2.2, I first show the composition of optimum portfolio for a buyer from Country 1 and Country 2 on a (ι_1, ι_2) plane and then later I show the possible currency regimes in this two-country world

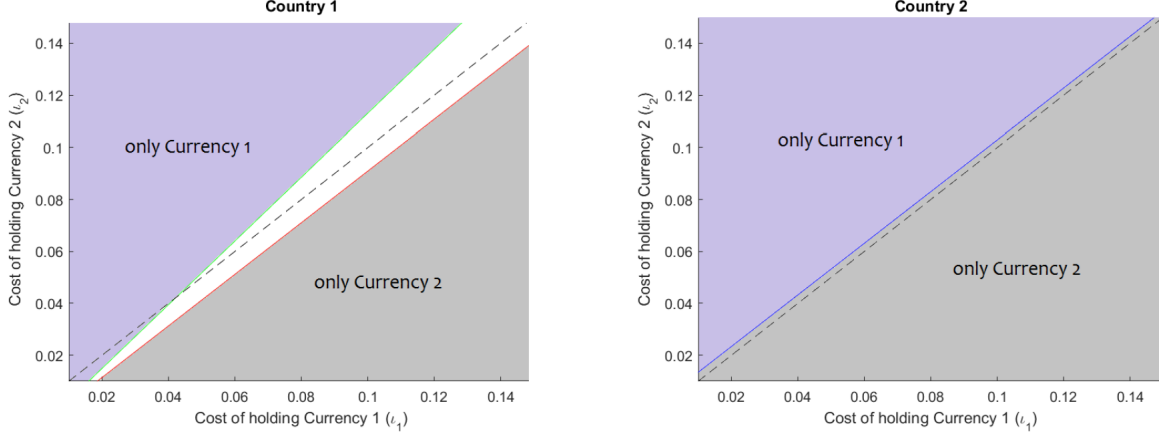


Figure 2.2: Composition of buyer's portfolio in each country when $t_1 = t_2 = 1\%$, countries are perfectly open ($\alpha = 0.5$) and Country 2 is a small economy ($n = 0.1$)

The optimum currency portfolio may be composed of either one type of currency or both currencies. The upper portion of each figure that is shaded in violet represents the (ι_1, ι_2) combinations for which it is optimal to hold a portfolio composed entirely of currency 1 under the abovementioned parameterization. The lower portion in grey, on the other hand, is the set of (ι_1, ι_2) combinations for which it is optimal to hold a portfolio composed entirely of currency 2. The portion in the middle, represents the (ι_1, ι_2) combinations for which the optimal portfolio of the buyer will comprise of both currency 1 and currency 2. For Country 1, the (ι_1, ι_2) combinations that makes dual currency portfolio optimum are represented by the white region, while for Country 2 it is a knife-edge case where the boundaries for the two single currency regions coincide. The broken 45° line represents the boundary between the two single currency regions when $t_1 = t_2 = 0$, i.e. there is no effective institutional barrier to the use of any currency in either country. In the case of $t_1 = t_2 = 0$, when $\iota_1 > \iota_2$ ($\iota_1 < \iota_2$) buyers switch to a portfolio that consists of only currency 2 (currency 1). However, when at least one of the 'tax' rates is strictly positive, it acts as a “wedge” and

the boundaries are redefined. The region permitting dual currency portfolio for the bigger country is broadened, higher *relative* marginal cost of holding currency 2 (ι_2) is necessary to allow for portfolios with only currency 1 and vice versa. For Country 2, the smaller country, a positive t_1 allows for the existence of “only currency 2” in portfolios at higher *relative* marginal cost of holding currency 2 (ι_2) than what one would have observed in a world with $t_1 = t_2 = 0$. In Figure 2.3, I describe the possible currency regimes in this two-country world.

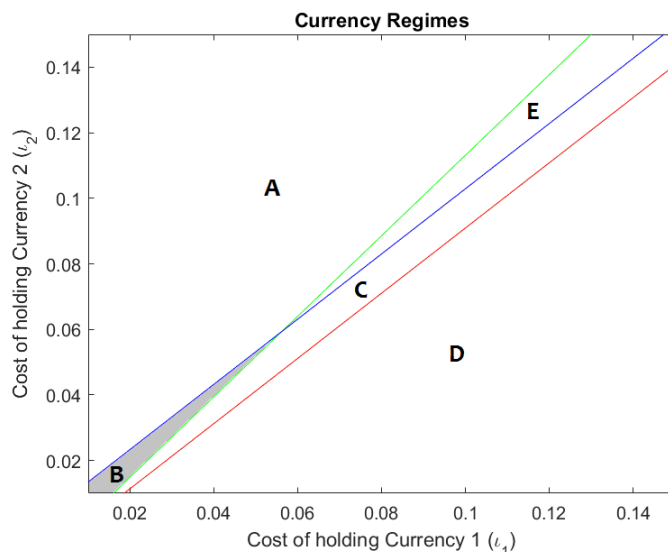


Figure 2.3: Currency regimes in the two-country world economy

The areas marked by A and D represent cases of full ‘dollarization’. In the region A, the relative marginal cost of holding currency 2 is so high, that buyers in both countries find it less expensive to hold a portfolio comprised of currency 1. This is despite the institutional barriers in Country 2 on the use of currency 1. Therefore, in this scenario, buyers use currency 1 for both domestic and international trade. The region D paints the opposite picture. Here the relative marginal cost of holding currency 1 is so high, that buyers in both countries switch to currency 2 and use it for domestic and international trade. One may think of region A as the case for Zimbabwe, a small economy whose hyperinflating currency

was completely replaced by a mix of low-inflation currencies of larger economies viz. United States, United Kingdom and European Union. To the best of my knowledge we have not witnessed a low-inflation currency of a small country replace the high-inflation currency of a larger economy. Therefore, there is no real world example for region D. However, it is a possibility that could emerge under the simple physical environment laid out in this paper. The region B represents ‘local currency regime’ which is the more common case in the real world. This is when both currencies have low levels of inflation (lower marginal costs, ι_1, ι_2). In region C represents the case where currency 2 is the internationally circulating currency. In this region buyers from Country 1 hold both currency 1 and currency 2 whereas buyers from Country 2 hold only their own currency. The line separating region E and region C represents the knife edge case where both currencies are internationally circulating currencies since buyers in both countries hold dual currency portfolios. Finally the region E represents a hypothetical case where buyers from Country 2 hold both currency 1 and currency 2 but those from Country 2 hold only currency 1 (foreign currency for them) in their portfolio. This may seem unrealistic at first glance, but it is plausible under the model specifications. Notice that in region E, inflation levels in both currencies are sufficiently high. Given that it is a small country in a well integrated world, a buyer from Country 2 is a lot more likely to meet a seller from Country 1 than a seller from her own country. Therefore, she puts a lot of weight on the surplus from the *DM* trade between her and a seller from Country 2 who prefers to be paid in currency 2. Thus, when she has to choose between two high-inflation currencies she chooses the one that will maximize her expected surplus from *DM* trade. On the other hand, for a buyer from Country 1, had there been no taxes, for these combinations of (ι_1, ι_2) , she would have held only currency 1. However, since Country 2 sellers prefer to be paid in their own currency, consume within the given period (in *CM*) and do not carry real balances to the next period, it is still a better deal for a Country 1 buyer to pay such a seller in their own currency (currency 2). Therefore, Country i buyer would have a residual demand for currency 2.

In the real world, if a scenario akin to region E arises or if Country 2 moves towards a region E like scenario, the government of Country 2 would shun their own currency and adopt the other country's currency or introduce a new currency in tandem with monetary and/or fiscal reforms. The model proposed in this paper is a simple model that chalks out all the possible scenarios with the least possible complexity. However, if policy channels like adoption of foreign currency as legal tender, introduction of new currency are added, then region E might disappear.

2.5 Increase in Tax Rates

Suppose the authorities in Country 2 in an attempt to curb currency substitution increases the 'tax'² on currency 1, t_1 . Therefore, they make it more difficult for their residents to use currency 1 by increasing institutional barriers. This tends to increase the area B, i.e. the set of (ι_1, ι_2) values that permit local currency regimes in both countries expands. This allows Country 2 to have a high-inflation currency yet maintain a local currency regime at the equilibrium. We use the same parameterization as before but change t_1 from 1% to 5% and then to 10% in Figure 2.4. In Figure 2.4, I show how the set of (ι_1, ι_2) values that permit local currency regimes in both countries expands as Country 2 increases its tax on currency 1 from 1% to 10%. The region G represents the (ι_1, ι_2) values for which Country 1 buyers hold both currencies, while Country 2 buyers hold only their own currency. The region H represents the (ι_1, ι_2) values for which Country 2 buyers hold both currencies, while Country 1 buyers hold only their own currency. Not only does the (ι_1, ι_2) set allowing for local currency regime expand, but also the (ι_1, ι_2) set that allows currency 1 to become the sole currency in the world also shrinks. A unilateral increase in taxation by Country 2, therefore limits the role

²This will of course have negative welfare consequences. However, in reality, a country facing the problem of dollarization as a consequence of persistent high inflation often use restrictions as a tool to protect their national currency. Defense of the national currency is often justified by citing issues like economic sovereignty, national pride etc. while overlooking the negative welfare consequences of high inflation tax and higher tax on real holdings of foreign currency.

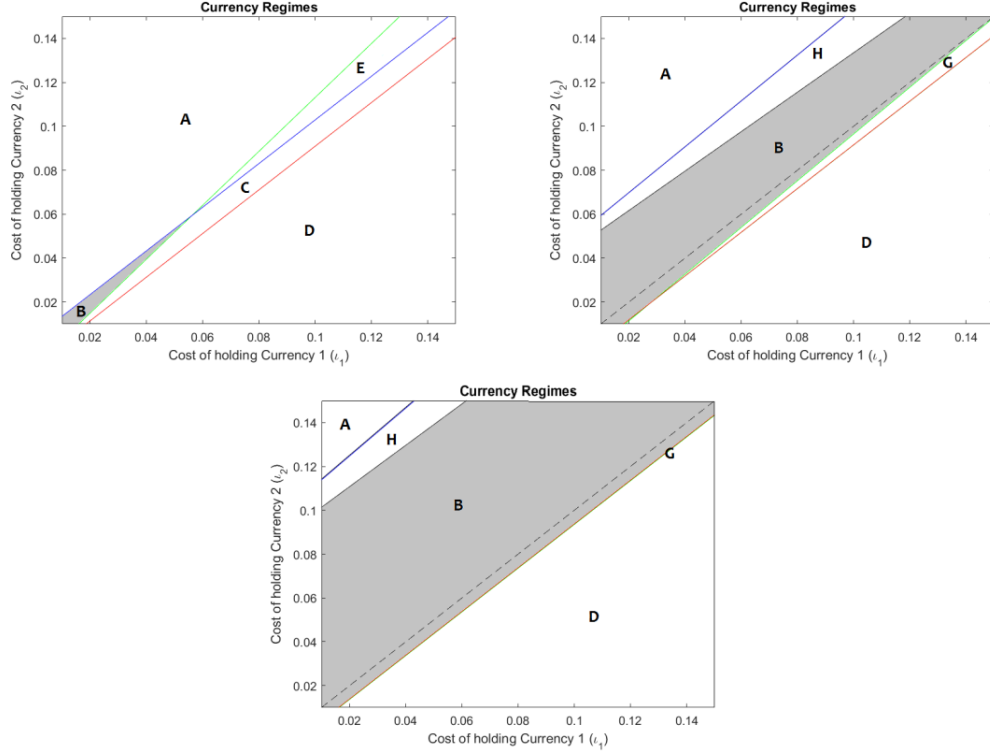


Figure 2.4: Currency regimes in the two-country world economy when $t_1 = 1\%, 5\%, 10\%$

of currency 1 as a global currency. When facing the threat of currency substitution as a consequence of high inflation, an increase in tax rate might save the currency 2 from being eroded, but it has negative welfare effects on the residents of Country 2 as well as those from Country 1. Due to high inflation in currency 2, Country 1 buyers will hold less of currency 2 and more of currency 1. Also, due to increased tax on real holdings of currency 1, in a match between buyer from Country 1 and seller from Country 2, payment in currency 1 becomes even less attractive for the seller. Therefore, quantity bought by the buyer will decrease. This reduces the surplus from the match. Buyers in both countries face similar negative welfare consequences, when Country 1 increases its institutional barriers to the use and real holdings of currency 2 by its residents. If protecting the national currency from dollarization is the objective of the game, then the equilibrium solution for the game would be to reduce inflation and reduce the tax to a very small positive number such that welfare level in Country i ($i = 1, 2$) is at least equal to a level chosen by policy-makers.

2.6 Conclusion

This paper, by including transactions cost and using asymmetric valuation of currencies by economic agents from different countries, provides a methodological framework to determine the unique currency regime that will arise. While the earlier models in monetary search literature helps us understand unique features pertaining to currency competition, this paper provides a way to pin down the equilibrium outcome and overcomes problem multiplicity of equilibrium. Although, in this simple model, we might be losing some richness by assuming that transactions cost lowers purchasing power of a currency abroad, it is possible to endogenize transactions cost to factors like extent of use and recognizability.

Chapter 3

International Financial Openness and Exchange Rate Manipulation

3.1 Introduction

In an economy that is perfectly financially open one should expect no direct or indirect institutional barriers to the choice of currency in which agents carry out trades or denominate the value of goods, services and assets. However, such barriers do exist - sometimes more in certain countries. The degree to which such institutional hurdles exist reflect the financial openness of the country. This paper tries to examine whether restrictions on financial openness in certain countries had any effect in providing buoyancy to its domestic currency vis-a-vis the US dollar (USD), an international currency. We look at empirical evidence and try to deduce if such restrictions have in effect acted as tools for manipulating relative value of currencies.

It is important to note that these controls or restrictions on financial openness do not have a uniform structure across countries, but comes in a variety of forms ranging from outright ban

on the holding and usage of foreign currency and assets denominated in foreign currency to the imposition of institutional hurdles that makes foreign currency and assets denominated in it less acceptable. For example, in India, the Foreign Exchange Management Act (FEMA) forbids the use of foreign exchange for daily transactions. Under the precursor of FEMA, the Foreign Exchange Regulation Act which was in effect till 1999, holding or usage of foreign currency was deemed a criminal offence. However, there was and is little monitoring on what currency agents use in cash transactions - yet residents in India rarely use any other currency than the rupee, the domestic currency. This due to two reasons – firstly, it is difficult to obtain foreign currency in India as there are only a limited number (relative to the country’s size) financial intermediaries authorized to buy/ sell foreign currency. Even when one finds an that are authorized dealer, the buyer of foreign currency is required by law to fill out certain forms and furnish valid reasons (e.g. proof of foreign travel, import license etc.) thereby making it difficult to obtain foreign currencies. The second reason pertains to the savings decision of agents. In India, only a few branches of banks (mostly in big cities) allow customers to hold a foreign currency account subject to the customer’s meeting of certain criteria established by the Reserve Bank of India. Thus, if one holds dollars or any other foreign currency, it is difficult for him to keep it in bank thereby foregoing the interest that could have been earned. Banks are, however, more likely to convert dollars into rupees and open usual rupee denominated-bank accounts for their customers at a little cost. This example provides a glimpse of the complex nature of these restrictions and how it affects economic agents’ choice of currency. On the other hand, if we look at other countries, we would notice a different setup. In the African country of Nigeria, in October 2013 the Central Bank of Nigeria withdrew operating licenses of twenty Bureaux de Change and banned the import of USDs and other foreign currencies without its prior approval. Mozambique, back in 2009 passed a law that mandates exporters repatriate their export earnings and convert at least 50% of their overseas earnings into meticaais (the local currency). Then there are countries where transactions in foreign currencies are allowed but agents are required to

pay a tax if they transact in foreign currencies (e.g. Kazakhstan), while in other countries daily withdrawal limit of foreign currency from bank accounts is tightened (as it happened in Ghana early 2014). Therefore, there is no one-size-fits-all structure for these restrictions. The key thing to note here is that authorities impose restrictions on certain pivotal elements in the nation's financial and payment systems and these barriers decrease the desirability of foreign currency as a medium of exchange down the stream.

As the examples mentioned above might suggest, these restrictions on financial openness, which eventually restricts domestic trades in foreign currency, are more often noticed in emerging market economies (EMEs henceforth) and less developed countries (LDCs henceforth). Some of these restrictions may also be found in advanced economies, but advanced economies in general are financially more open. The Chinn-Ito Index of Financial Openness, which I will discuss later in this paper, gives the extent of restrictions on financial openness for various countries and it usually EMEs and LDCs that are more restrictive than advanced economies. Also, such measures are not imposed exogenously, but are often spurred by events associated with poor macroeconomic health or instability in the economy. Sometimes, these restrictions could imposed with the objective to boost foreign exchange reserve position of the country, sometimes to defend a falling currency or to maintain a peg. Yet at other times, it could be the result of nationalistic pride and the political-economic philosophy of the ruling regime. The reasons for imposing these restrictions are many but usually they are employed to provide macroeconomic stability, correct trade imbalances and to defend the local currency against a rapid fall or speculative attacks. In this paper I will not go into the details of factors that make a country more likely to employ such restrictions¹, I simply recognize that these restrictions exist and focus on its effect on the value of the local currency vis-a-vis the foreign currency (I use USD as 'foreign currency'). Given the economic events

¹Grilli and Milesi-Ferretti (1995) does a 61 country panel data study to understand the economic effects and structural determinants of capital controls. Capital controls are more likely in countries with lower income, large governments and limited central bank independence. Although such measures are associated with high inflation and low interest rates.

that provide the impetus for such restrictions, it is not surprising that these controls on financial openness are often bundled with capital control measures. Although this paper is not about capital controls, given that capital control measures contribute significantly to an economy's financial openness (or lack thereof), any study of the economic effects of financial restrictions demands a discussion of the related literature on capital controls.

Capital controls have gained significant attention since the Latin American crises of 1980s and the Asian crises of the 1990s. To cushion a nation against the adverse economic effects of rapid inflows/ outflows of financial capital often triggered by domestic or international shocks, nations have time and again imposed capital controls that affected the financial openness of countries. Mathieson and Rojas-Suarez (1993) discusses the different rationales for the imposition of capital controls. They often include retention of domestic savings, sustainability of stabilization and structural reform programs and the maintenance of domestic tax base. Certain aspects of capital controls were advocated by economists like James Tobin in the 1970s. Shortly after the end of the Bretton Woods system, as a protective measure against rapid capital flows, Tobin (1978) proposed “throwing sand in the wheels” by imposing a uniform tax on foreign exchange transactions. Tobin's contention was that such a tax would temper surges in short-term capital flows, but have minimal effect in the long run. Today capital controls take a variety of forms and the IMF has provided detailed country-by-country reports in the *Annual Report on Exchange Agreements and Exchange Restrictions* which it has published since 1950. As Eichengreen and Rose (2014) notes, once imposed, such controls stay in place for high durations, often for decades. They argue that governments rarely impose or remove capital controls in response to short run fluctuations in output, terms of trade or financial stability. Capital controls have a great degree of inertia and according to Eichengreen and Rose (2014), these measures are implemented to address domestic economic issues by discriminating against foreigners. A related study, Fernández et al. (2013) do not find any empirical evidence either, to support the claim that governments use controls in a macro-prudential or counter-cyclical manner.

One argument is along the lines of trade openness, that financial openness, i.e., a higher degree of financial integration with rest of the world is a beneficial for nations – that it will pave the way for financial development, risk diversification and eventually contribute to economic growth and welfare. With this understanding the IMF has, over the years, played an important role in encouraging countries to liberalize their capital account. Therefore, a great deal of research since the 1980s have focused on the real effects of capital account liberalization. Dooley (1996) and Eichengreen (2001) are two papers that survey the literature of capital control. The literature in this area is vast and we are yet to reach a clear consensus on the effectiveness of capital account liberalization. Rodrik (1998) finds no evidence of positive correlation between capital account convertibility and investment-output ratio and the paper concludes that such controls serve only as a proxy for the reputation of the government. On the other hand Quinn (1997) and Edwards (2001) reach the opposite conclusion. Using different measures of international financial openness Edison et al. (2004) finds no significant evidence that indicates openness promotes economic growth. Some authors have also investigated the effect of controls in areas other than output or economic growth. Chinn and Ito (2002) finds significant negative correlation between capital account control and financial development. Such correlation is stronger in the case of developed countries with good institutional framework. Miniane and Rogers (2007) use a vector-autoregression approach to study if stringent capital controls better insulate countries against foreign (US in their case) monetary shocks and they do not find any robust evidence. Capital controls also influence U.S. multinational corporations' financial decisions as discussed in Desai et al. (2006). Some country-specific case studies in the capital control literature include Kaplan and Rodrik (2001), Edison and Reinhart (2001), Laurens and Cardoso (1998), De Gregorio et al. (2000) and Ariyoshi et al. (2000). Ariyoshi et al. (2000) employ a descriptive approach to study several countries and focus on the effectiveness of capital controls and their associated costs. The study concludes that capital controls cannot be a substitute for sound macroeconomic policy.

After some of the negative experiences of the late 1990s and the 2000s, the IMF has adopted a more nuanced approach to capital controls. One IMF staff discussion note, Ostry et al. (2010) lays out conditions under which capital controls could be useful policy tools against inflow surges or for maintaining financial stability. Controls are an important policy tool against inflow surges in case of countries having relatively strong currencies, sufficient reserves, where overheating concerns precludes easier monetary policy, and where there is fiscal discipline vis-à-vis macroeconomic and public debt considerations. Capital controls could be used as the second best to macro-prudential tools, to address financial-stability issues, when the latter tools are unavailable. Their later paper, Ostry et al. (2011) discusses factors to take into account when deciding on an optimal mix of capital control tools. Capital account controls need to be tailored to meet the specific needs of a country and to address the risks faced by that country.

In recent years a new brand of literature studies capital controls as measures for preventing crises. In this literature capital controls are second-best strategies that are desirable when economies are marked by externalities. Lorenzoni (2008), Korinek (2010), Jeanne and Korinek (2010), Bianchi (2011), Mendoza and Bianchi (2013), Fernández-Arias and Lombardo (1998), and Benigno et al. (2013) study capital controls as useful tools for maintaining financial stability. Another segment in this new brand of literature views capital controls as beneficial tools for improving macroeconomic adjustment in economies with nominal rigidities and suboptimal monetary policy, e.g. Schmitt-Grohé and Uribe (2012a,b) and Farhi and Werning (2012).

In contrast to the above mentioned papers, this paper's goal is to study the effect of not just capital account controls but the broader variable of financial openness (or lack thereof) on the value of local currency vis-a-vis the USD (foreign currency). We are specifically interested in understanding in restrictions on financial openness have been used to manipulate relative value of the local currency. Search-theoretic papers like Curtis and Waller (2000); Waller

and Curtis (2003) discuss how governments attempt to reduce circulation of foreign currency by lowering its value through taxation and how certain government transaction policies may affect the value of foreign currency. The topic of government transaction policies and its effect on acceptance of certain fiat currencies has also been discussed in Li and Wright (1998). The most common indicator of a currency's value is that measured in terms of another i.e., the nominal exchange rate. In this paper I treat USD as the foreign currency and investigate what happens to relative value of the local currency, as measured by USDs per Local Currency Unit (LCU) when the aforementioned restrictions are increased.

3.2 Restrictions on Financial Openness: Brief History

The period between 1880-1913 witnessed increased financial openness across the globe. During this period, Great Britain emerged as the central figure in the global financial network from where large amounts of savings were channelled to various parts of the world – mostly through investment in fixed-interest long-term bonds issued by governments and companies. Colonies in resource rich Australasia, North American countries of Canada and United States – even some South American countries (Argentina and Brazil), were prime beneficiaries of this free capital mobility. Other sources of capital flows included France, Germany and Netherlands. This period was also marked by the absence of government regulation of the financial and monetary systems.

Later, with the onset of the World War I in 1914, international capital flows slowed down, global growth halted and the process of financial integration was deterred. The inter-war period of 1918-1939 saw increased capital controls and a reversal of financial liberalization. The current law in India that forbids the use of any currency other than the rupee has its origin in British colonial days. At the outbreak of World War II, the then colonial government of India, under the Defence of India and Sea Customs Act, 1878 introduced new rules that

forbade residents to hold or deal in foreign currencies. Further, residents of undivided India were required by law to surrender all foreign currency and securities. Residents of India were permitted to use only the Indian rupee and in some cases the Sterling or other Sterling Area currencies. The introduction of these measures were part of a bigger plan by the British Government in order to conserve non-Sterling Area currencies, especially the US dollar. It was deemed important to have enough non-Sterling surplus so as to purchase essential war-time materials. This rules were later codified into a law in 1947. The law was later amended in 1973 and replaced by a new one in 1999. Similar laws in countries that were former British colonies, most notably the Exchange Control Act, 1953 of Malaysia have their origin in the the World War II days.

After the World War II, the broken global financial system began to be slowly mended during the Bretton-Woods era (1945-1971). This era was marked by pegged exchange rates, heavily regulated domestic financial markets and capital controls i.e., high restrictions on financial openness. With the collapse of the Bretton-Woods system, the global financial system started slowly opening up as countries gradually removed restrictions and lessened regulations. ²

However, several financial and currency crises originating in Latin America and Asia between 1980 and 2000 have time and again triggered an increased use of restrictions on financial openness. Following the crisis of 2008-2009 some emerging market economies and even Iceland, an advanced economy has made use of capital controls which further increases restrictions on financial openness. As discussed in the earlier section, some distinguished economists have suggested financial restrictions as tools for preventing crises and for countering short-term adverse fluctuations, when first best strategies are not an unimplementable.

²Using several macroeconomic and financial indicators, Rajan and Zingales (2003) provide a detailed account of the reversal in development and openness of the global financial scene during the course of 20th century.

Since this paper is about the effect of restrictions on financial openness on the relative value of the local currency, our focus will be on a rather narrow area - I will focus on the inflation differential (i.e., inflation in local currency – inflation in US dollars) and the nominal exchange rate. I have also indicated that I will use the broader variable that I call *restriction on financial openness* of which capital controls are a part. In the next section I define these variables in greater detail and run certain empirical analyses to determine if such restrictions have affected the relative value of the local currency in the post-Bretton Woods world. We use a country-by-country structural vector-autoregression approach for the few countries I consider and generate impulse response functions to understand how shocks to the variables of interest affect each other.

3.3 Empirical Analysis

To study how restrictions on financial openness affect the value of a currency, I consider five emerging market economies: Chile, Colombia, India, Malaysia and Indonesia.

3.3.1 Data

The two variables of interest in this study are: *relative value of the LCU (w.r.t USD)* and *financial openness*. For the former, we use two measures: the *nominal exchange rate* (LCU per USD) and the *inflation differential* (US dollar inflation – LCU inflation). Higher the nominal exchange rate i.e., when one US dollar buys more LCUs, weaker is the local currency. A positive inflation differential indicates the purchasing power of the LCU vis-a-vis the US dollar is decreasing, while a negative inflation differential indicates the opposite. A lower inflation differential makes it more attractive for economic agents to hold and use LCU while a higher inflation differential increases the attractiveness of US dollars. Data on

monthly averages of nominal exchange rates from 1970-2013 have been obtained from the International Financial Statistics division of the International Monetary Fund and from the Federal Reserve Economic Data (FRED). For Colombia and Indonesia, data on this variable comes from the the respective countries' central bank. Monthly CPI inflation rates, for the years 1970-2013 (1975-2013 for Chile) for the six countries: United States, India, Chile, Colombia, Malaysia and Indonesia have been obtained from the Federal Reserve Economic Data, Ministry of Labour & Employment - Government of India, the Instituto Nacional de Estadísticas - Chile, banco de la Republic - Colombia, Bank Negara Malaysia and the Bank Sentral Republik Indonesia respectively.

Measuring financial openness is slightly more complicated. It requires the transformation of qualitative information into quantitative variable. Nevertheless, economists and public policy experts have come up with such measures³. In this paper we use two such measures: the Index of Financial Openness (*KAOPEN* henceforth) by Chinn and Ito (2002, 2006) and, indicators of capital account regulations (*CAP100* henceforth) and financial current account regulations (*CURR100* henceforth) by Quinn (1992, 1997). Both these indicators are de jure indicators based on the *Annual Report on Exchange Arrangements and Exchange Restrictions* (AREAER henceforth) – a report the IMF has published since 1950. As discussed earlier, the restrictions on financial openness and on financial integration takes a variety of forms across countries and there is a lack of uniformity. However, the IMF in its AREAER provides a survey of such restrictions for each member country as they were in the December of previous year. The report also provides binary (exists/ does not exist) responses to variety of possible restrictions and regulations. Chinn and Ito's *KAOPEN* takes advantage of these binary responses provided in the IMF reports. *KAOPEN* is an “extensive” indicator of financial openness i.e., it measures the extent of these restrictions. The index is obtained through a principal component analysis on three categorical indicators of financial current account restrictions: (i) current account restrictions (ii) the requirement to surrender export proceeds

³Quinn et al. (2011) provides a survey of such measures.

surrender and (iii) the presence of multiple exchange rates plus the variable *SHARE*, which takes the rolling average of binary responses from AREAER’s categorical table over a *ve*-year window: $t - 4$ through t . Chinn and Ito’s *KAOPEN* is the first standardized principle component of four AREAER table variables. So, it doesn’t just capture capital account openness but also captures other factors that affect a country’s financial openness. The index is constructed for 182 countries for the time period of 1970-2013. The range of this index is $[-1.89, 2.39]$ with higher scores indicating greater openness. However, the *KAOPEN* is a “point in time” estimate indicating the extent of financial openness as it stands in December of every year. Therefore to get a better idea of the extent of financial openness during a given year, we use an average of the the *KAOPEN* of that year and that of the previous year. Also, since we are interested in restrictions on financial openness, we multiply the averaged value with -1 . We define modified Chinn-Ito Index (*MCI*) for year t as:

$$MCI_t = -\frac{KAOPEN_t + KAOPEN_{t-1}}{2}$$

Quinn’s *CAP100* and *CURR100* are based on the text of AREAER. These indicators are constructed using six categories: payment for imports; receipts from exports; payment for invisibles; receipts from invisibles; capital ows by residents; and by nonresidents. The first four categories are included in *CURR100* while capital flows by residents and nonresidents are included in *CAP100*. This measure also makes an assessment of the intensity of those restrictions and changes that occur over the course of the year. Both of these indicators are on a scale of 100, with higher values indicating more openness. The indicators are available for 122 countries for the period of 1950-2007. Just like we modified the Chinn-Ito Index, we modify *CURR100* and *CAP100* to *MCURR* and *MCAP* respectively using:

$$MCURR_t = 100 - CURR100_t, \quad MCAP_t = 100 - CAP100_t$$

Both *MCURR* and *MCAP* ranges between 0-100 with higher value of *MCURR* implying higher financial current account restrictions and higher *MCAP* implying higher capital account restrictions.

3.3.2 Estimating Changepoints in Nominal Exchange Rate Data

Before we go into the vector-autoregression and impulse response analysis using the variables described earlier, we test for and estimate if there are any changepoints in the nominal exchange rate data for the three countries that correspond to known changes in regulation i.e. either tightening and loosening of restrictions on financial openness and integration.

Testing and estimation of multiple changepoints is done by using the E-Divisive algorithm developed by Matteson and James (2014). This is a non-parametric approach to testing and estimation of changepoints that does not require any *a priori* knowledge of the number of changepoints.. The only distributional assumptions it places are (i) the existence of α th absolute moment, for some $\alpha \in (0, 2)$ and (ii) observations be independent over time. Given that nominal exchange rates follow a process close to random-walk as discussed in Meese and Rogoff (1983), it is reasonable to make the assumption that first difference values of nominal exchange rates are independent of each other. Methodologically, the E-Divisive algorithm is a hierarchical estimation process that combines bisection as in Vostrikova (1981) with a multivariate divergence measure from Rizzo and Székely (2010). We use first differenced values of monthly average nominal exchange rates (LCU per USD) and the estimation results (we chose $\alpha = 1$) are shown in Table 3.1.

Country	Estimates	p-value
Chile	May 1982	< 0.001
	Jan 1999	0.010
	Feb 2003	0.007
Colombia	Jun 1994	< 0.001
	Dec 1996	< 0.001
	Jun 1999	0.001
	Mar 2003	0.037
	Oct 2006	0.016
India	Mar 1988	< 0.001
	Aug 1991	0.029
	Jul 1995	0.002
	Jun 2002	0.007
	Sep 2005	0.008
	Jul 2011	0.013
Malaysia	May 1982	< 0.001
	Nov 1998	< 0.001
	Dec 2005	< 0.001
Indonesia	Dec 1997	< 0.001

Table 3.1: Changepoints estimated using E-Divisive algorithm

The changepoints January 1999 for Chile and the points November 1998 and December 2005 for Malaysia are intriguing. In Chile capital control in the form of unremunerated reserve requirement was imposed in June 1991 and was maintained through September 1998. The January 1999 changepoint is within the period shortly after September 1998 and corresponds to a switch greater financial openness in Chile. In Malaysia, following the Asian Currency Crisis, the Malaysian ringgit was pegged to the US dollar and a host of other capital control measures were imposed in September 1998 which were maintained through 2005. the changepoints of November 1998 and December 2005 correspond to these changes in Malaysia. The December 1997 changepoint in Indonesia doesn't correspond to any policy change, but it falls within the period of the Asian financial crisis and during December 1997 the Indonesian rupiah witnessed a sharp decline in its value. As for Colombia, the June 1994 changepoint corresponds to the period right after capital controls in the form of Unremunerated Reserve Requirement (URR) was introduced. During the decade of 1998-2008, several regulatory changes to capital account were made in Colombia so the changepoints in 1999, 2003 and 2006 do not seem unreasonable. In July 1991, the Indian rupee was devalued twice following the balance of payment crisis – the August 1991 changepoint for Indian rupee/US dollar exchange rate corresponds to this. However, for India we do not see any change corresponding to known changes in restrictions on financial openness (e.g. introductions of LERMS in 1992 or replacing FERA by FEMA in 2000⁴).

3.3.3 The Vector Autoregression Model

In this paper we consider a three dimensional time series y_t , $t = 1, \dots, T$, where $y_t = [FINX_t, \Delta\pi_t, XR_t]'$. $FINX_t$ is a measure of restriction of financial openness, $\Delta\pi_t$ is the

⁴In 1992, India adopted the Liberalized Exchange Rate Management System (LERMS) to smoothen the transition from fixed exchange rates to the long-term goal of floating exchange rates. LERMS was replaced in 1993 by a managed-float exchange rate system. FERA is the Foreign Exchange Management Act which classified dealings in and possession of foreign currency as a criminal offence. It was deemed draconian and out of sync with modern economic times. The FERA was replaced by the Foreign Exchange Management Act (FEMA) in 2000.

inflation differential (inflation in local currency – inflation in US dollars) and XR_t is the yearly average LCU/ USD nominal exchange rate. We postulate that y_t can be approximated by a vector autoregression of finite order p . We intend to learn about the parameters of the following structural vector autoregressive model:

$$A_0 y_t = A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + u_t \quad (3.1)$$

where u_t denotes a mean zero serially uncorrelated error term, also referred to as a structural shock. The error term is assumed to be unconditionally homoskedastic. $E(u_t u_t') = \Sigma_u$ where Σ_u is a 3×3 matrix. Since the structural shocks are mutually uncorrelated Σ_u is a diagonal matrix. We put the restriction that each of these endogenous variables depend on the lagged values of itself and on the lagged values of the two other endogenous variables. Therefore,

$$A_0 = \begin{bmatrix} a_{0,11} & 0 & 0 \\ 0 & a_{0,22} & 0 \\ 0 & 0 & a_{0,33} \end{bmatrix}$$

In the next subsections we estimate the SVAR model and generate impulse response functions for the three different shocks for the countries under consideration. We use different measures for the restriction on financial openness - the *MCI* (the modified Chinn-Ito index), the *MCURR* (modified *CURR*100) and the *MCAP* (modified *CAP*100).

Estimation Results: Chile

For Chile, using the modified Chinn-Ito index (*MCI*) and AIC criterion, we estimate that the lag, $p = 6$. The impulse response functions are shown below in Table 3.2. The top row in Table 2 shows the impulse response to a shock in nominal (Chilean peso/ US dollar) exchange rate i.e., when the peso depreciates (CLP/ USD exchange rate rises). The second

row corresponds to a positive shock in inflation differential ($\Delta\pi$) i.e., when the inflation of the local currency falls relative to US dollar. Finally the third row corresponds to sudden rise in restrictions or an increase in $FINX$. The first column of Table 3.2 shows impulse responses of nominal exchange rate, the second depicts the impulse response of inflation differential and the third column corresponds to the response of restrictions on financial openness. In these impulse response plots one notices that for a shock to the nominal exchange rate i.e., when Chilean peso depreciates relative to the US dollar, there is a positive response generated in the variable $FINX$ implying restrictions are increased. But the size of this response is very small and the confidence band is rather diffused. A shock to the variable $FINX$ (i.e., when restrictions are tightened) is associated with a depreciation of the local currency as noticed in the third row. Inflation differential responds to such a shock by falling first (Chilean peso inflation falls sharply), but then it leads to cyclical fluctuations in the inflation differential. So, tightening of restrictions on financial openness is associated with depreciation of local currency in short-run (about 5 years), long run appreciation and inflation instability.

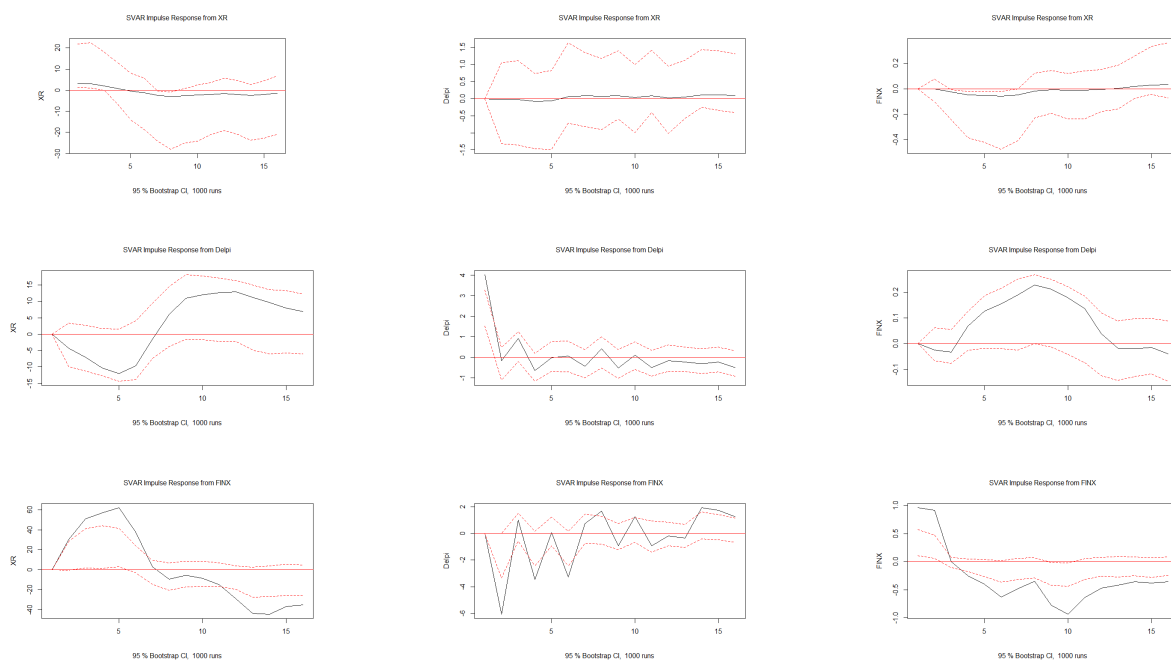


Table 3.2: Chile: impulse responses using modified Chinn-Ito index

Table 3.3 and Table 3.4 depict the impulse responses using modified Quinn's measures. In Table 3.3, first row, the impulse function of financial current account restriction in response to depreciation in Chilean peso shows no clear trend. However, tightening of restrictions (Table 3.3, third row) is associated with immediate appreciation followed by short-run depreciation (around the fifth year). The impulse function of CLP/USD nominal exchange rate in response to tightening of capital account restrictions (Table 3.4, row 3) is similar. Irrespective of the measure used for *FINX*, impulse response functions for Chile seem to suggest a role for restrictions in providing short-run boost to Chilean peso.

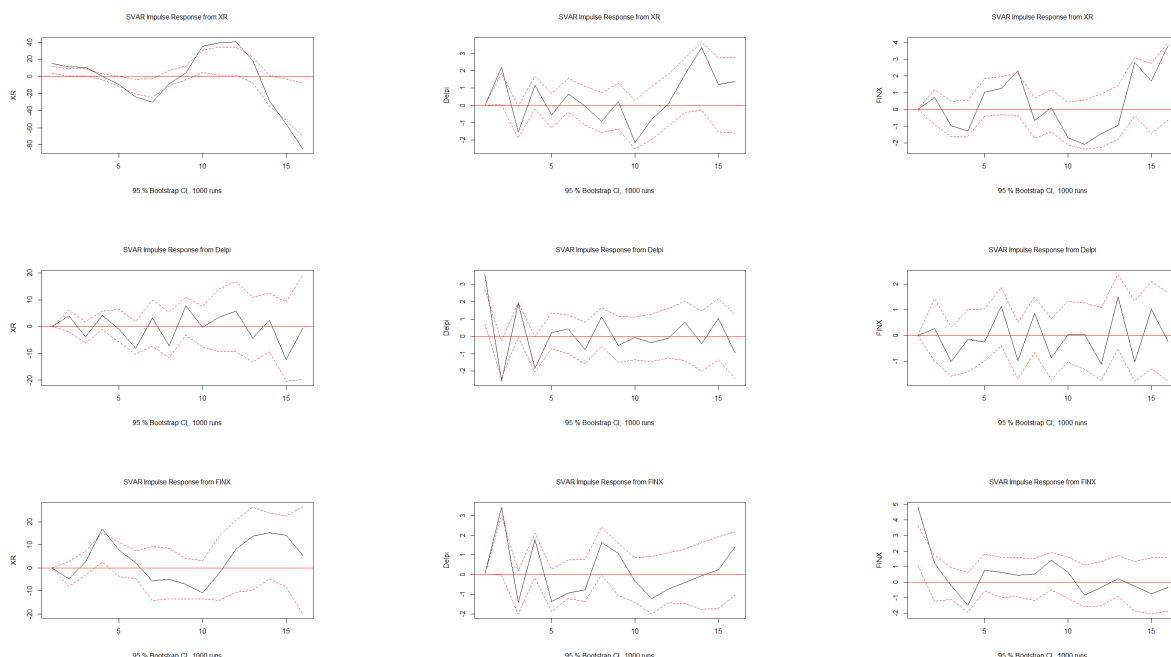


Table 3.3: Chile: impulse responses using modified Quinn's financial current account index

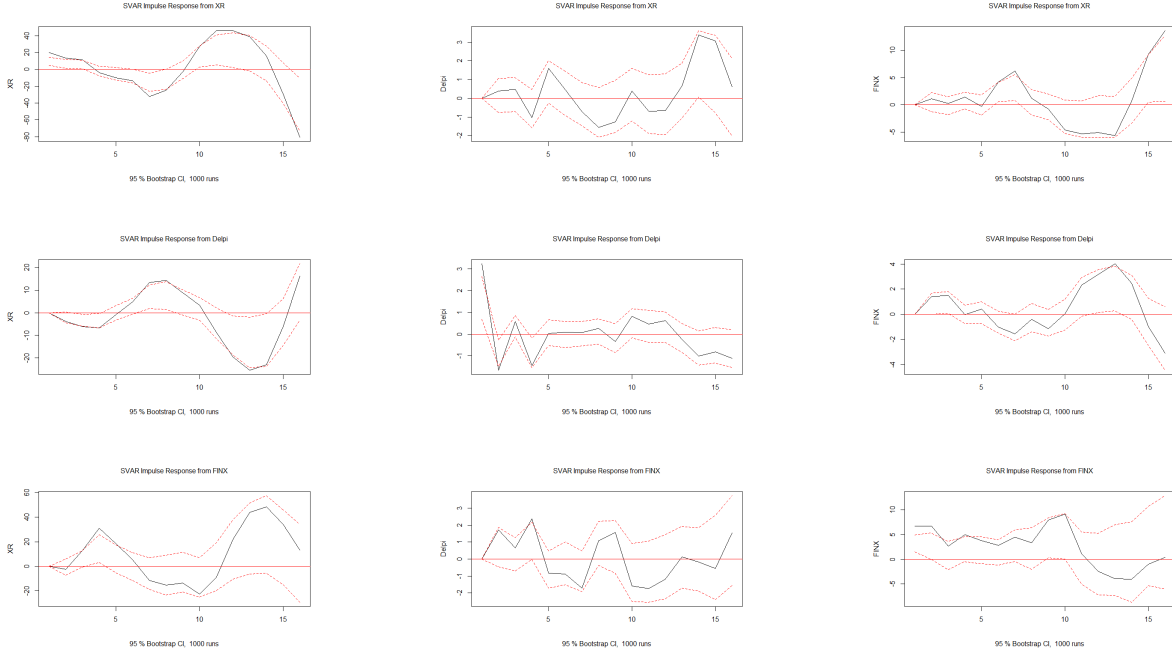


Table 3.4: Chile: impulse responses using modified Quinn's capital account index

Estimation Results: Colombia

The optimal lag length for Colombia by AIC is $p = 4$ when using modified Chinn-Ito index, $p = 6$ when using Quinn's current financial account index and Quinn's capital account index. Table 3.5 summarizes the impulse responses using modified Chinn-Ito index as a measure of restriction on financial openness. The first row plots to impulse response to a positive shock to Colombian peso/ US dollar (COP/USD) nominal exchange rate. The second row plots the impulse response to a shock to inflation differential - assuming US inflation unaffected by changes in Colombia, this can be interpreted as a sudden change in peso inflation. The third row plots the impulse response to a shock to financial openness. From the first row we observe that a sudden depreciation of the peso is associated with increase in restrictions on financial openness until the fifth year, after which it decreases and comes back to intial levels. From the third row of Table 3.5, we notice an increase in restrictions on financial openness

is associated with a decrease in COP/USD exchange rate until the seventh year thereby suggesting a role for such restrictions in improving the Colombian peso's value relative to the US dollar.

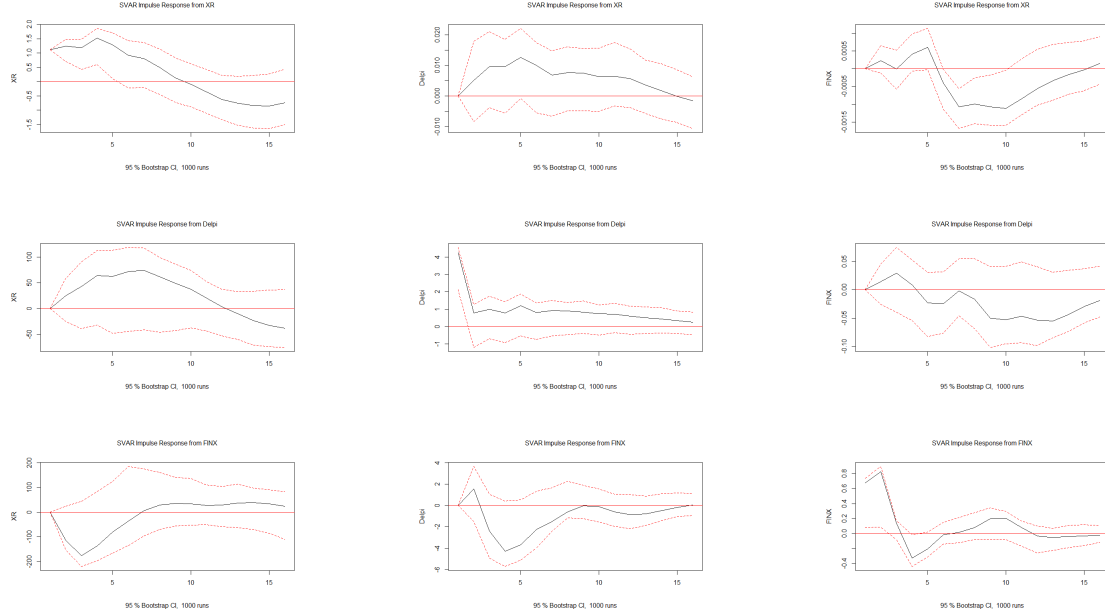


Table 3.5: Colombia: impulse responses using modified Chinn-Ito index

Tables 3.6 and 3.7 summarizes the impulse responses when using modified Quinn's index for financial current account and that for capital account. Using these measures, we notice from the first row of both tables that a shock in the COP/USD exchange rate is associated with a fall in restrictions on financial current account and on capital account. However, this estimated impulse has a very wide confidence band thereby suggesting a great deal of uncertainty in which way the restrictions would be affected. The third row is more informative as it suggests a rise in restrictions on financial current account as well as capital account is associated with a fall in COP/USD exchange rate or appreciation of the Colombian peso in the very short run (i.e. next 2-3 years).

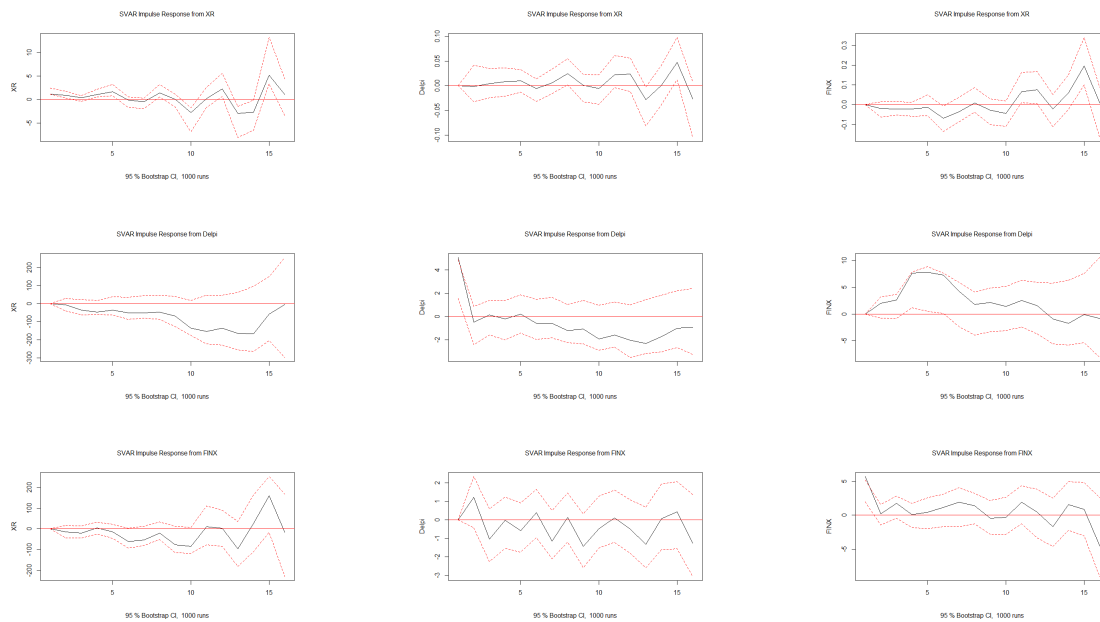


Table 3.6: Colombia: impulse responses using modified Quinn's financial current account index

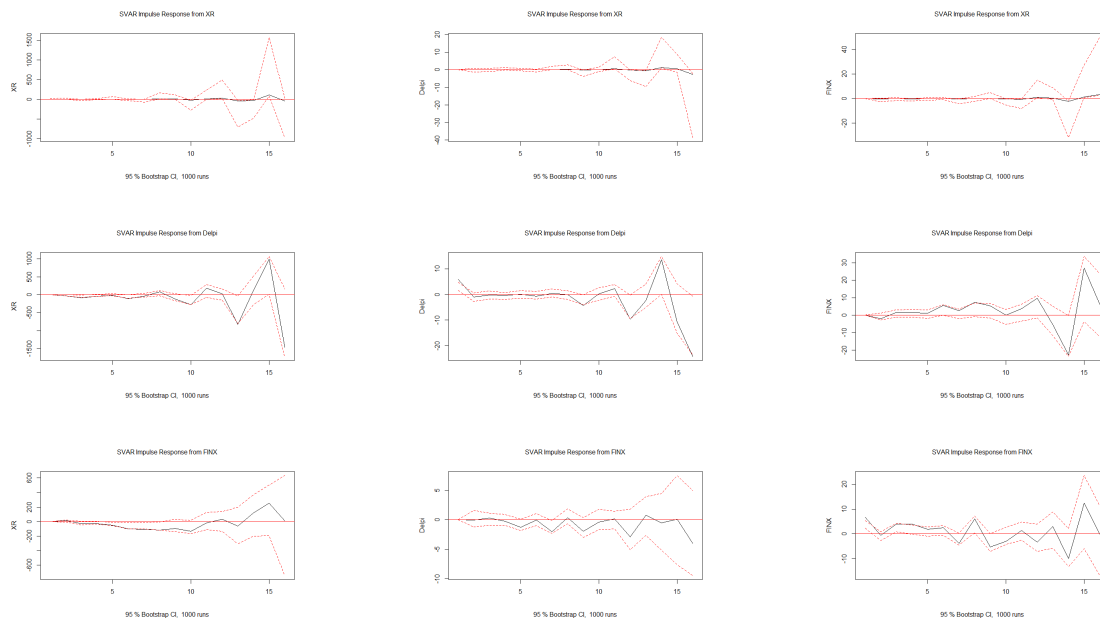


Table 3.7: Colombia: impulse responses using modified Quinn's capital account index

Estimation Results: India

In case of India, there is no variation in the Chinn-Ito index implying no change in the extent of financial openness. However, Quinn's financial current account index and capital account index changes over time. Therefore, for India we estimate Eq. (3.1) only using *MCURR* and *MCAP*. $p=6$ and $p=5$ by AIC criterion for *MCURR* and *MCAP* respectively. The impulse responses are shown in Table 3.8 and Table 3.9.

Third row of Table 3.8 indicates that tightening of of financial current restrictions are associated with short-run appreciation of the Indian rupee vis-a-vis the US dollar. The rupee starts to depreciate after the tenth year. Similar results are obtained using modified Quinn's measure for capital account restrictions as shown in third column of Table 3.9. In case of an depreciation of the Indian rupee (INR/USD exchange rate rises), the impulse response for current account restrictions (Table 3.8, row 1) show short-run perturbation (fall and then rise) but it peters out gradually.

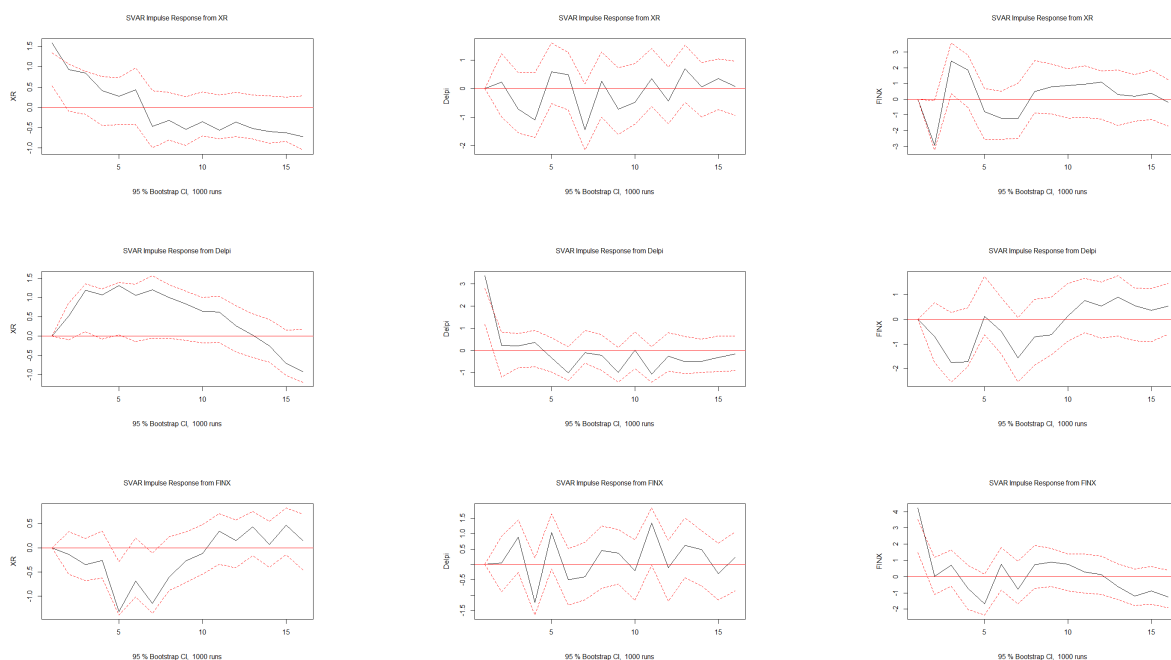


Table 3.8: India: impulse responses using modified Quinn's financial current account index

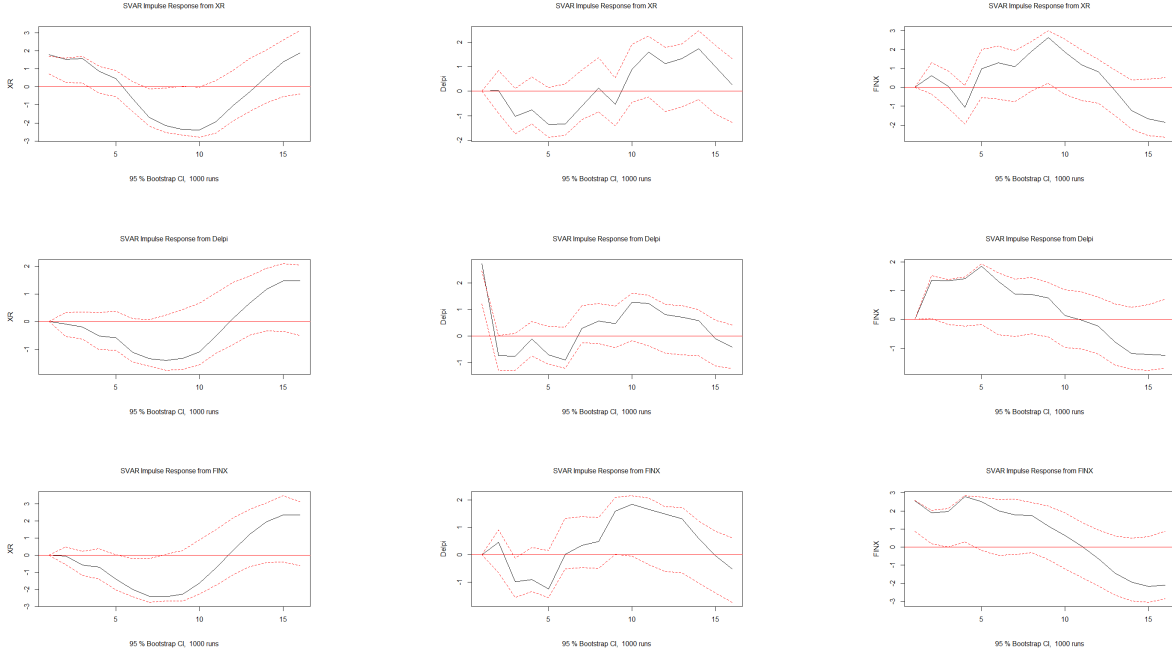


Table 3.9: India: impulse responses using modified Quinn's capital account index

Estimation Results: Malaysia

A depreciation of the Malaysian ringgit (i.e. rise in ringgit/ US dollar nominal exchange rate) is associated with a fall in $FINX$ as measured by modified Chinn-Ito's index (first row, Table 3.10). In the long-run as ringgit starts to appreciate, we notice a rise in the impulse response for $FINX$. while a the third row of Table 3.10, a positive shock to $FINX$ that is an increase in restrictions on financial openness, is associated with short-run appreciation of ringgit (or fall in ringgit/ US dollar exchange rate). But as the shock does down, exchange rate increases again before gradually coming down to its old level.

When $FINX$ is measured using the modified Quinn's financial current account restrictions (Table 3.11), the results are similar. A short-run depreciation in ringgit (followed by long-run appreciation) is associated with an impulse response of $FINX$ which is falling in the short-run and increasing in the long-run. Similarly, a sudden positive shock to $FINX$ (i.e.

decrease in openness of financial current account) is associated with a short-run increase in ringgit/ US dollar exchange rate (depreciation in ringgit). In Table 3.12 where we use a different measure for *FINX* which includes only capital account restrictions, there doesn't seem to be any significant impact on nominal exchange rate associated with a shock to *FINX* that increases *FINX*. However, we notice a positive correlation between nominal exchange rate and *FINX* in the case of an exchange rate shock.

These impulse response functions suggest that changes in the restrictions on financial openness might have an impact on the exchange rate. Also, in case of Malaysia, as the findings suggests, an increase restrictions on financial openness is associated with a depreciation of the ringgit.

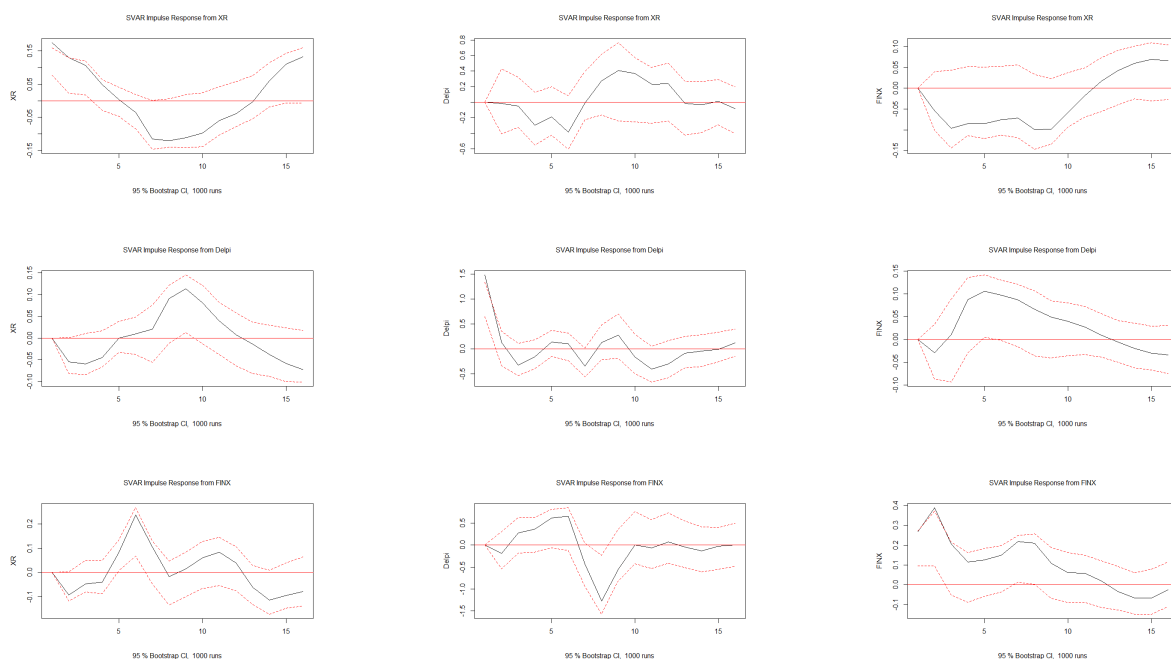


Table 3.10: Malaysia: impulse responses using modified Chinn-Ito index

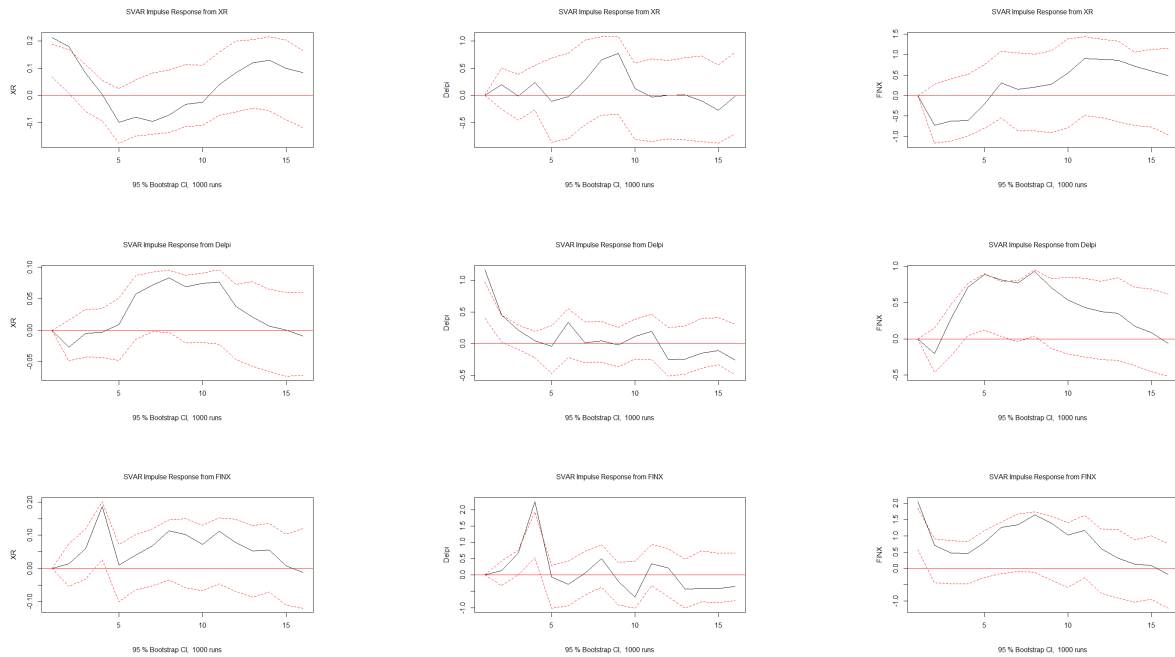


Table 3.11: Malaysia: impulse responses using modified Quinn's financial current account index

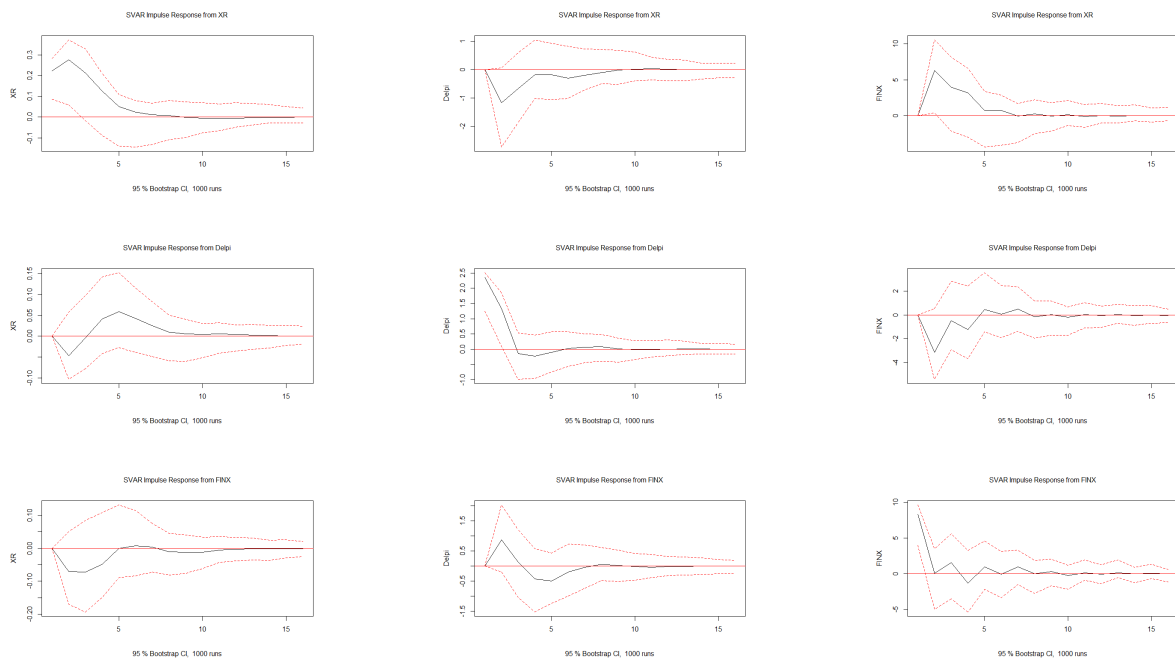


Table 3.12: Malaysia: impulse responses using modified Quinn's capital account index

Estimation Results: Indonesia

The optimal lag length using AIC, is 3 (i.e. $p = 3$) when using modified Chinn-Ito index and $p = 6$ when using the modified Quinn's capital account index. Table (3.13) shows the impulse responses when I use Chinn-Ito index as a measure for financial openness. The first row shows the effect of a shock to Indonesian rupiah (IDR)/ US dollar exchange rate. When the rupiah suddenly depreciates, i.e. IDR/USD rate goes up, it is associated with rise in inflation the rupiah (drop in $\Delta\pi$) in the next couple of years and then a drop in rupiah inflation from around the fourth year to ninth year before it comes back to the initial levels. On the other hand, a sudden depreciation of the rupiah is associated with a decrease in restrictions on financial openness (as measured by Chinn-Ito index) decreases and these restrictions remain at lower levels throughout the next fifteen years. The third row of Table (3.13) shows the impulse response for a sudden positive shock to restrictions on financial openness. Such a shock is associated, with depreciation of the Indonesian rupiah (as shown in the first column of the third row), while its effect on rupiah inflation is ambiguous.

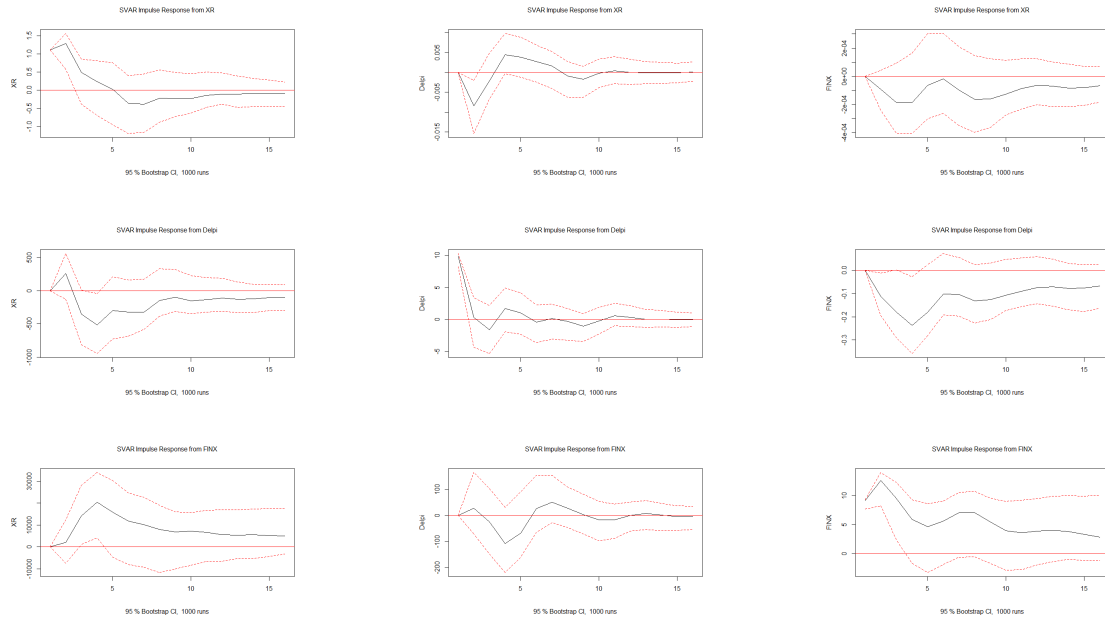


Table 3.13: Indonesia: impulse responses using modified Chinn-Ito index

Next I analyze the interrelationship between nominal exchange rates, relative inflation rates and capital account restrictions (as measured by Quinn's capital account index). Table (3.14) summarizes the impulse responses from this analysis. As before, the first row shows the impulse response from a positive shock to IDR/ USD exchange rate. If the Indonesian rupiah depreciates, it is associated with an increase in capital account restrictions that lasts for about a decade. The last row shows the impulse response associated with a sudden increase in capital account restrictions. With a shock to capital account restrictions, we notice a decrease in IDR/USD exchange rate (i.e. rupiah appreciates) after the fifth year and this pattern continues upto the tenth year after the shock after which the rupiah starts to depreciate again.

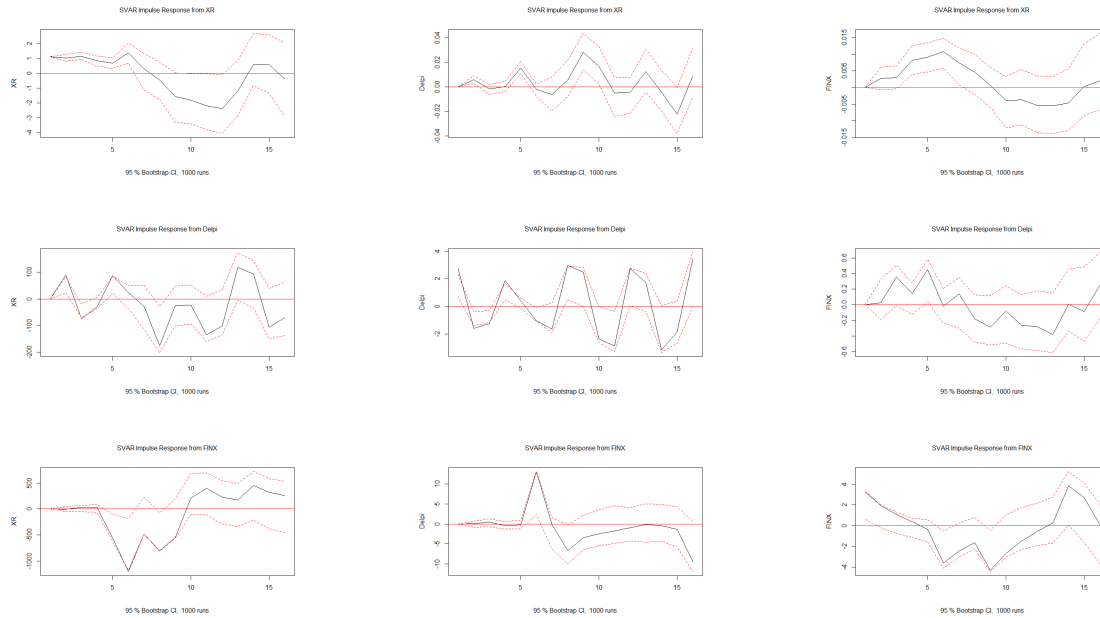


Table 3.14: Indonesia: impulse responses using modified Quinn's capital account index

From the analyses of impulse response functions for Indonesia using two different measures of financial openness we can conclude that although a sudden depreciation in the rupiah is negatively associated with a broad extent of financial restrictions and vice versa, data suggests that such an event is positively associated capital account restrictions. Therefore,

capital account restrictions have a role in stabilizing the value of the Indonesian rupiah in face of depreciation.

3.3.4 Panel VAR Model

I consider a trivariate panel VAR of order p with panel-specific fixed effects represented by the following system of linear equations:

$$\mathbf{Y}_{it} = \mathbf{Y}_{it-1}\mathbf{A}_1 + \mathbf{Y}_{it-2}\mathbf{A}_2 + \dots + \mathbf{Y}_{it-p+1}\mathbf{A}_{p-1} + \mathbf{Y}_{it-p}\mathbf{A}_p + \mathbf{u}_i + \mathbf{e}_{it} \quad (3.2)$$

where $i \in \{\text{Chile, Colombia, India, Malaysia, Indonesia}\}$ and $t \in \{1972, 1973, \dots, 2007\}$. $\mathbf{Y}_{it} = [FINX_{it}, \Delta\pi_{it}, \Delta s_{it}]$. For $FINX$ I have used the modified Quinn's index of capital account openness since this is only measure of financial restriction that shows variation for all countries. I define the variable $\Delta\pi_{it}$ = inflation in USD—inflation in local currency and Δs_{it} denotes the log difference between the current period's and the last period's local currency/US dollar nominal exchange rate⁵. In \mathbf{Y}_{it} , the variables are ordered from slow moving to fast moving. In equation Eq.(3.2), \mathbf{u}_i and \mathbf{e}_{it} are (1×3) vectors of dependent variable-specific fixed-effects and idiosyncratic errors, respectively. The errors have the following characteristics: $E(\mathbf{e}_{it}) = 0$, $E(\mathbf{e}'_{it}\mathbf{e}_{it}) = \Sigma$ and $E(\mathbf{e}'_{it}\mathbf{e}_{is}) = 0$ for $t > s$. The (3×3) matrices $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_p$ are to be estimated.

I base my model selection on the model and moment selection criterion mentioned in Andrews and Lu (2001) and select a first-order panel VAR since as this has the lowest modified AIC, modified BIC, and modified Hannan-Quinn information criterion. A first order-panel VAR model is then estimated using GMM estimation implemented by the Stata program 'pvar' developed by Abrigo and Love (2015). The estimated VAR model satisfies the stability

⁵ $s_{it} = \log_e \varepsilon_{it}$, where ε_{it} is the number of local currency units in terms for one US dollar, i.e. the nominal exchange rate

condition since the modulus of each eigenvalue of the estimated model is strictly less than one. This means the VAR is invertible and has an infinite-order vector moving-average representation, providing known interpretation to estimated impulse-response functions. The orthogonalized impulse responses based on Cholesky decomposition are summarized below in Table 3.15

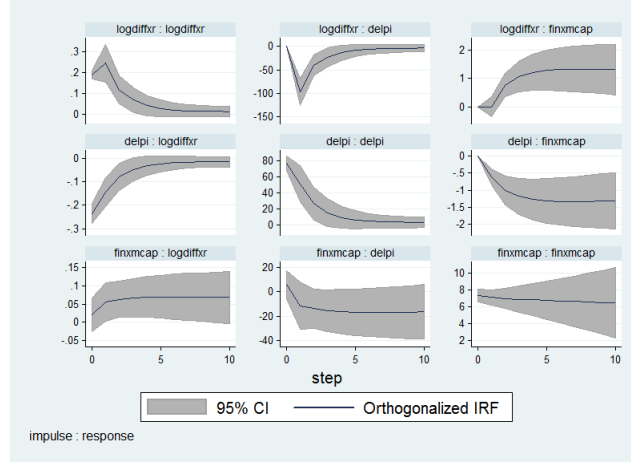


Table 3.15: Panel VAR impulse response using modified Quinn’s capital account index

The first row of Table 3.15, plots the impulse response of the three variables to a one standard deviation shock to Δs_{it} , i.e. a shock to nominal exchange rate. Such a shock is associated with an increase in capital account restriction as shown in the thrid column of the first row and an increase in local currency’s inflation vis-a-vis US dollar inflation as seen in the second column of the first row.

The second row of the table plots the impulse response to a one standard deviation positive shock to $\Delta\pi$, i.e. local currency’s inflation decreases vis-a-vis US dollar inflation (local currency’s value improves), then that is associated with an improvement in nominal exchange rate (first column, second row) and a decrease in capital account restrictions (third row, second column).

Finally, in the third row, we plot the impulse response to a positive shock to capital account index. A sudden increase in such restriction leads to higher inflation in local currency with respect to dollar inflation and deteriorating exchange rate (local currency depreciates).

3.4 Conclusion

Restrictions on financial openness vary not only in the way they are implemented across countries, but also in their effects on relative value of currencies. Here we considered five emerging market economies in this study - Chile, Colombia, India, Malaysia and Indonesia. I find no strong long term effect of such restriction on the exchange rate or inflation in local currency. Also, the effect varies depending on what measure of financial openness one uses. Restriction on financial current account to lead to different outcome than restrictions on capital account, Overall, the data seems to suggest a role for such restrictions in “currency crisis” like scenarios. In case of Chile and Colombia, we observe a sudden depreciation in the local currency is associated with a rise in such restrictions, whereas for Malaysia and Indonesia the effect seems to be reversed. Whether the opposite is true, that is does increase in restrictions lead to appreciation of currency depends on which country we are looking at. For Colombia and India we notice and appreciation in currency with rise in restrictions on financial openness. In case of Malaysia, increasing capital account controls in associated with appreciation of the ringgit at least in the short run. However, for Chile, Malaysia (using Chinn-Ito or financial current account restrictions) and Indonesia, we observe the opposite. Overall, our analysis suggests a role for restrictions on financial openness in affecting the nominal exchange rate, however, this is limited to the short-run and the effectiveness of such restrictions depend on the country’s economic structure and the nature of the crisis policy makers want to combat. Furthermore although it makes the analyses tractable use of indices obfuscates some of the micro-level country specific issues which could be important

for determining exchange rates. A country specific study that looks into the details of such restrictions, their timing and the nature of the problems facing the economy would throw more light on the dynamics of capital controls and other restrictions on financial openness. This remains subject of future studies.

Chapter 4

The Black Market for Currencies: Theory and Evidence

4.1 Introduction

This paper builds a model explaining the origins of black markets for currencies; the black market premium arises endogenously and depends on relative inflation rates of domestic and foreign currencies. I analyze the case of Iran, Venezuela and the four South Asian economies of India, Pakistan, Nepal, Sri Lanka between 1981-1997. Conventional wisdom suggests that countries with higher relative domestic inflation rates will have higher rates of black market premium. However, for these six countries using data from 1981 to 1997, I find a negative association between black market premium and domestic inflation (relative to US inflation). Using the New Monetarist framework I offer a possible explanation as to why this association could be negative and when it could be positive. Defining exchange rate as the price of foreign currency in local currency, the black market premium is the percentage deviation of the black market exchange rate from the official market's rate.

The black market for currency exchange is another market of currency exchange that exists alongside the official, legally recognized currency exchange market which includes banks, licensed financial institutions, authorized money changers, and certain websites registered for this purpose. It is a monetary phenomenon observed more often in emerging market economies (EMEs) and less developed countries (LDCs). Nevertheless, developed countries, like Iceland during the crisis of 2008-2011, have witnessed rise in black market activities as well, albeit for a brief period. A key feature of the black market for currencies, like any other black market is that it is illegal. The market for currency exchange is part of a country's financial system and dealing in foreign currencies often require sanction from the authorities who specify the channels, times, and platforms through which such trades can be carried out. This is especially true for EMEs and LDCs which have a greater degree of restrictions on the financial openness of their countries due to capital controls, stability concerns, concern about illegal economic activities, terrorism etc. This paper does not study the purpose of currency restrictions; instead it takes these as given and studies the properties of black market trade. Despite these restrictions, due to imperfect monitoring illegal trade in currencies thrive. Due to this not-so-legal status of the black market, it operates on a relatively smaller scale than its official counterpart. It is often location specific, transactions are carried out mostly in cash, leave no paper trails and carries some degree of risk. The risk comes both from the possibility of confiscation by government authorities and from the possibility of being swindled or robbed. For example, since it is unregulated, in the black market, someone could be sold fake currency notes or get robbed in a narrow alleyway. In countries where a black market for currencies exists, such trades are ubiquitous in places like convenience stores, border crossings, it involves people with stacks of US dollars on the corner of a street, or perhaps a worker at an international airport who runs a side business of buying/ selling foreign currencies etc. In Libya, a country which has recently witnessed a rise in black market trade in currencies, it has been carried out mostly in the gold souks of the major cities. Until recently, in Argentina, the participants in black market for currencies

included taxi drivers, small stores selling essential commodities, as well as the *arbolitos*, people who stand planted for hours in one spot and offer handfuls of US dollars to everyone who passes. In India, the black market has traditionally consisted of a network of unlicensed money changers, some businesses with foreign connections, foreign travelers and households who hold foreign currency¹. This pattern contrasts sharply with the officially recognized currency exchange which consists of banks and legitimate financial intermediaries (including websites), that have greater visibility, advertise the rates they offer through various channels and are required to issue receipt and register every transaction they make.

While its unregulated nature might attract individuals involved in criminal activities (like drugs, arms trafficking etc.) to exchange monies, it is not the only purpose the black market serves. In fact, currency circulating in the black market could have been acquired through official channels. Goldberg (1995) considers a model with leakage of foreign currency into the black market for the case of Russia. A more recent example will be that of Libya's, where one can get a letter of authorization from the central bank to acquire foreign currency to purchase goods abroad with the intention of selling them domestically. Sellers of foreign currency, either posing as traders or through their connections in the central bank acquire such authorization and due to lack of proper verification, sell the foreign currency in the black market instead of buying goods from abroad. This story has its parallels in other parts of the world. In India, laws specify how much foreign currency a resident can buy legally for foreign trips and how much foreign currency can they bring in and retain after a foreign trip. However, due to imperfect monitoring there is leakage from the official market to the black market.² The association between inflation rates and the black market premium rate

¹In India it is illegal to hold foreign currency without explicit authorization which can be obtained only in cases of foreign travel or by licensed exporters and importers

²Indian laws allow foreign currency worth USD 25,000 to be acquired legally from banks by furnishing proof of foreign business trips which cover a broad category of trips including attending international conference, seminar, specialized training, study tour, apprentice training etc. Indian laws also allow up to USD 100,000 worth of foreign currency to be bought by resident Indians for medical treatment abroad on self declaration basis of essential details, without insisting on any estimate from a medical service provider in India or abroad. However, how much of this is actually taken out of the country and how much left behind is not stringently monitored nor do the banks strictly verify the validity of such business/ medical trips. On

presented in this paper hints at a currency portfolio choice by economic agents. Indian rupee has been a weak currency for more than half a century. At the time of independence, in the absence of sufficient investment opportunities in interest bearing assets, it was not uncommon for the urban wealthier sections of South Asian society to hold their wealth in a combination of rupees, gold and the British pound. This sort of portfolio diversification was a response to the relatively higher cost of holding wealth only in rupees. Post-independence, following global trends the preference shifted from British pound to US dollars, but the portfolio diversification pattern has persisted among certain sections of Indian society. Although India and other South Asian countries have not witnessed dollarization in means of payment inside the country, the dollarization of portfolios is not uncommon. Although, in recent times, with increased access to capital markets and an improved economy, activity in the currency black market seems to have abated in India, if newspaper reports are to be believed it is still active in other South Asian countries like Pakistan and Nepal.

In this paper, I argue that one of the reasons behind the existence of black market is that, in presence of limited or infrequent access to official currency exchange facilities, the black market provides agents with an opportunity to sell their foreign currency to finance domestic consumption which must be paid in local currency units. It also provides the opportunity to exchange local currency for foreign currency when agents need to purchase goods from abroad, in the absence official channels. Agents tend to diversify their portfolio for two reasons: (i) to beat the inflation cost of a particular currency and (ii) in anticipation of the opportunity to consume foreign-produced goods or domestically produced goods. Once agents learn what sort of goods they are going to consume, they can access the black market

the other hand, for travelers returning to India after a foreign trip there is no upper limit to the amount of foreign currency they can bring into the country as long as they file a customs declaration form. On return from a foreign trip travelers are required to surrender unspent foreign exchange held in the form of currency notes within 90 days of return. However, they are free to retain foreign exchange up to USD 2,000 for future use. While most people spend the amount they acquire from the authorized dealers or surrender excess foreign currency they bring with them, there are instances of leakage into the black market. On a personal note, the author has spent significant time in India and has had the opportunity to come across certain agents who work as intermediaries in the black market for foreign currency. Information about the sources of foreign currency comes from informal talks with them.

in order to convert their local currency into foreign currency or vice versa. The black market for currency exchange is not “parallel” in the sense that it functions simultaneously alongside the official exchange market and agents make a choice to visit one or the other. Instead, as discussed in the examples, these two markets are sequential and agents access both. The black market provides liquidity in presence of frictions such as the timing of the official market and the time at which consumption (foreign or domestic) shocks are realized. The timing issue of the official exchange market is valid for many EMEs and LDCs where due to foreign exchange arrangements and restrictions, money changing services offered through official outlets like banks and other authorized financial intermediaries are not available at all times.

In this paper, I use a two-country, two-currency version of the Lagos and Wright (2005) model where, following the traditional international macroeconomics literature as well as Zhang (2014), I model the official currency exchange market as a frictionless spot Walrasian market in the second subperiod. The black market for currency exchange is another spot Walrasian market embedded in the first subperiod which is accessible by agents from one of the countries. The first subperiod also consists of two decentralized markets for special goods – one for each country. In the second subperiod, all agents produce and consume a consumption good and choose their portfolio of the two monies for the next time period. At the beginning of the first subperiod there are a set of agents called ‘buyers’ who only consume, but do not produce in this period, receive an idiosyncratic shock that matches them with a foreign seller or a domestic seller. ‘Sellers’ are another set of agents who produce a special good, but does not consume in the first subperiod. The labels ‘buyers’ and ‘sellers’ remain unchanged over periods. Buyers make take-it-or-leave-it (TIOLI) offers to the seller in the special goods market.

In the second subperiod buyers make an ex ante portfolio choice and choose an optimal real portfolio consisting of both monies. When the shock is realized in the next period, a

buyers who can access the black market would want to convert their portfolio into sellers currency as the seller only accepts that currency. In the stationary monetary equilibrium buyers' portfolio would consist more of the currency that has lower inflation. In the black market the relative supply of the two monies determine what the black market premium rate would be. Since the focus is on stationary monetary equilibrium, inflation rates are constant and agents would build this into their portfolio choice problem. If the cost of holding the domestic currency is higher than that of the foreign currency, buyers will dollarize their portfolios and as a result there will be more foreign currency available in the black market than domestic currency leading to a decline in the premium rate. This could be a probable explanation for the negative association between relative domestic inflation and rate of black market premium noticed in the data for certain countries.

One of the earliest works on the topic of currency black market is Dornbusch et al. (1983) for the Brazilian black market. In their model Dornbusch et al. (1983) propose a partial-equilibrium model of the black market using a stock and flow portfolio balance approach where demand for dollars depends positively on their relative yield and on wealth. Despite its simplicity and elegance, this partial equilibrium model assumes that the amount of wealth held in local currency is exogenous, which is unlikely as agents would diversify their portfolio between dollars and local currency depending upon their relative returns, wealth as well as the possibility of buying domestic or foreign goods. The black market premium depends on relative supply of both local currency and foreign currency. Therefore, the equilibrium results of Dornbusch et al. (1983) on the black market premium are not generally robust to the possibility of currency substitution. Other models of the black market include de Macedo (1982), de de Macedo (1987) which model dollar holdings by individuals as a means of diversifying a 'portfolio' of assets held to maximize expected returns on invested wealth while minimizing the variance of these returns.

The literature on the black market broadly consists of three different strands. The first consists of exchange rate reforms (or unification) and policy reforms in the presence of black markets, these include Goldberg (1995), Goldberg and Karimov (1997), Phylaktis and Girardin (2001) etc. Some studies like Kharas and Pinto (1989), Pinto (1991) also focus on inflationary implication of unification of black market rates and official exchange rates. A second strand of the literature which include Gupta (1981), Booth and Mustafa (1991) and Huett et al. (2014) study whether currency black markets efficiently process information about the state of the economy. Finally there are some papers that study real effect spillovers in presence of black market. For example Greenwood and Kimbrough (1987), study foreign exchange controls in an economy with black market and its impact on imports and welfare. Kamin (1995) studies how official devaluation in presence of black markets may lead to shrinking in aggregate output. In contrast to these papers, the current paper focuses on the black market premium and its association with relative inflation rates. In particular, this paper contributes to the empirical literature on black market by identifying a negative association between the black market premium rate and the relative domestic inflation. On the theoretical front it provides a model which explains this feature of the black market premium. It differs from other theoretical partial equilibrium models like Dornbusch et al. (1983), Goldberg and Karimov (1997) by providing a setup where black market exists as a result of optimal decision making by economic agents. Therefore, the model presented here justifies the existence of black market. Furthermore, the focus is on stationary monetary equilibrium wherein agents can vary the amount of both monies they hold in response to relative rates of inflation and the possibility of domestic/ foreign consumption.

The rest of this paper is organized as follows. Section 4.2 reviews the evidence between relative inflation rates and the black market premium rate. Section 4.3 describes the physical environment of the model. Section 4.4 describes value functions and the optimal behavior by economic agents in different markets. Section 4.5 defines the equilibrium and discusses equilibrium portfolio choice and welfare consequences. Finally, section 4.7 concludes.

4.2 Evidence on Black Market Premium Rate and Inflation

In this section, I analyze the relationship between black market premium rates and inflation - in domestic currency and in U.S. dollars first for a set of twenty countries and then for a subset of countries. The black market I focus on is the black market for US dollars and any exchange rate is defined as the number of local currency units that need to be paid in order to acquire one US dollar. For the purpose of this study, I consider following countries: Argentina, Brazil, Chile, Colombia, Egypt, India, Indonesia, Iran, Kenya, Malaysia, Mexico, Nepal, Nigeria, Pakistan, South Africa, Sri Lanka, Tanzania, Thailand, Uganda and Venezuela for the years 1981-1997. First I use a panel vectorautoregression (VAR) approach with these twenty countries using as variables the log values of ratio of the black market exchange rate to the official exchange rate, the log values of the ratio of domestic inflation to US inflation and the index of financial openness developed by Chinn and Ito (2006). Using orthogonalized cumulative impulse responses based on Cholesky decomposition, I find that a shock which raises domestic inflation relative to US inflation raises the log ratio of the black market to the official exchange rate only slightly. Since the black market premium varies one-to-one with ratio of black market rate to official exchange rate³, it suggests that with a sudden increase in the domestic inflation relative to the US inflation, the black market premium will rise in the short run by a small amount.

After analyzing the full sample, I choose a subset of six countries, namely, Iran, Venezuela and the four South Asian countries of India, Pakistan, Nepal and Sri Lanka. As earlier, I adopt a panel VAR approach for these six countries. Using orthogonalized cumulative impulse response, for this subset of countries, I find a positive shock to the log ratio of domestic inflation to US inflation is associated with a decline in the log ratio of the black

³black market premium = $\frac{\text{local currency/US dollar rate in black market}}{\text{local currency/US dollar rate in official market}} - 1$

market rates to the official rates. This suggests a negative relationship between the black market premium and domestic inflation rate vis-a-vis to US inflation. i.e. as domestic inflation gets bigger relative to US inflation, we would notice a decline in black market premium over time. Thereafter I run a fixed effect panel regression with data from these six countries. The results suggest a negative effect of relative domestic inflation rate on the black market premium. Finally, I focus on one country, India for which data is available at monthly frequency. For this country, I find a negative effect of relative domestic inflation rates on the black market premium. In the next paragraph I describe the sources of data and the challenges it presents.

Data and challenges: The years 1981-1997 were chosen firstly because data on black market premium for all 12 months is available only until 1997. Secondly, for the years before 1981, reliable data on inflation is not available for Brazil. Brazil is a major country that has witnessed black market trade in US dollars in the 1980s and 1990s. Therefore, it should not be excluded from the study. Monthly data on black market premium comes from the dataset constructed by Reinhart and Rogoff (2009) by culling information from annual issues of *Pick's Currency Yearbook*, *Pick's World Currency Report*, *Pick's Black Market Yearbook*. To the best of my knowledge, *Pick's* books are the only source of data on black market premia and black market exchange rates. Unlike the official forex market, where exchange rates are easily visible through different media sources, black market exchange rates are not as conspicuous. Therefore, the figures on black market premia and black market exchange rates reported in *Pick's* books are the average figures from surveys of black markets in different countries. These books were published by the International Currency Analysis, Inc. owned by Franz Pick, a New York-based currency analyst. After 1998 the publication was discontinued which explains the unavailability of data for recent years.

Data on inflation rates comes from the World Bank. It would be ideal to use monthly forecasts of annual inflation and a rational choice for that would be percentage changes in CPI from same month previous year. However, inflation forecasts or for that matter, CPI measures are not available at this level of frequency for every country for every year in my dataset. These include countries like Argentina, Brazil, Nepal. Due to these limitations in the data, for the cross-country analyses I use annual inflation rates and average annual black market premium rates.

Finally for financial openness I use the Index of Financial Openness (*KAOPEN*) by Chinn and Ito (2006). The *KAOPEN* is a de jure indicator based on the *Annual Report on Exchange Arrangements and Exchange Restrictions* (AREAER henceforth) – a report the IMF has published since 1950. The AREAER provides information in the form of binary (exists/ does not exist) responses to a variety of possible restrictions and regulations in the domain of exchange rates and international finance for every member country. *KAOPEN* takes advantage of these binary responses provided in the IMF reports. *KAOPEN* is an “extensive” indicator of financial openness i.e., it measures the extent of these restrictions. The index is obtained through a principal component analysis on three categorical indicators of financial current account restrictions: (i) current account restrictions (ii) the requirement to surrender export proceeds surrender and (iii) the presence of multiple exchange rates plus the variable *SHARE*, which takes the rolling average of binary responses from AREAER’s categorical table over a ve-year window: $t - 4$ through t . *KAOPEN* is the first standardized principle component of these four AREAER table variables. Therefore, this index does not just capture capital account openness but also other factors that affect a country’s financial openness. The index is constructed for 182 countries for the time period of 1970-2013. The range of this index is $[-1.89, 2.39]$ with higher scores indicating greater openness. I include the *KAOPEN* in my analyses because regulations and restrictions which cause international financial frictions it could potentially affect the availability and value of dollars in a country’s exchange markets.

4.2.1 Panel VAR Approach

I consider a trivariate panel VAR of order p with panel-specific fixed effects represented by the following system of linear equations:

$$\mathbf{Y}_{it} = \mathbf{Y}_{it-1}\mathbf{A}_1 + \mathbf{Y}_{it-2}\mathbf{A}_2 + \dots + \mathbf{Y}_{it-p+1}\mathbf{A}_{p-1} + \mathbf{Y}_{it-p}\mathbf{A}_p + \mathbf{u}_i + \mathbf{e}_{it} \quad (4.1)$$

where $i \in \{1, 2, \dots, 20\}$ and $t \in \{1981, 1982, \dots, 1997\}$. $\mathbf{Y}_{it} = [KAOPEN, \ln \frac{\pi_{LCU}}{\pi_{USD}}, \ln \frac{e_{black}}{e_{official}}]$. $KAOPEN$ is the Chinn and Ito (2006) index of financial openness, while π_{LCU} , π_{USD} denote the inflation rate in the local currency and US dollar respectively. The variables e_{black} , $e_{official}$ represent the local currency per US dollar exchange rate in the black market and in the official exchange market. In \mathbf{Y}_{it} , the variables are ordered from slow moving to fast moving. In equation Eq.(4.1), \mathbf{u}_i and \mathbf{e}_{it} are (1×3) vectors of dependent variable-specific fixed-effects and idiosyncratic errors, respectively. The errors have the following characteristics: $\mathbb{E}(\mathbf{e}_{it}) = 0$, $\mathbb{E}(\mathbf{e}_{it}'\mathbf{e}_{it}) = \Sigma$ and $\mathbb{E}(\mathbf{e}_{it}'\mathbf{e}_{is}) = 0$ for $t > s$. The (3×3) matrices \mathbf{A}_1 , \mathbf{A}_2 , ..., \mathbf{A}_p are to be estimated.

I base my model selection on the three model selection criteria by Andrews and Lu (2001) and select a first-order panel VAR since as this has the lowest mAIC, mBIC, and mQIC (see Table B.1). A first order-panel VAR model is then estimated using GMM estimation implemented by the Stata ‘pvar’ program developed by Abrigo and Love (2015). After estimating the model presented in Eq.(4.1) we generate orthogonalized impulse responses based on Cholesky decomposition and the impulse response of $\ln(e_{black}/e_{official})$ to one standard deviation shock to $\ln(\pi_{LCU}/\pi_{USD})$ are shown for 20 years in Figure 4.1

From Figure 4.1, it is evident that for the full sample, a shock that raises the domestic inflation-US inflation ratio is associated with a small increase in the ratio of black market rate to official market exchange rate. This means an increase in domestic inflation relative to inflation in US dollar is associated with a slight increase the black market premium. This

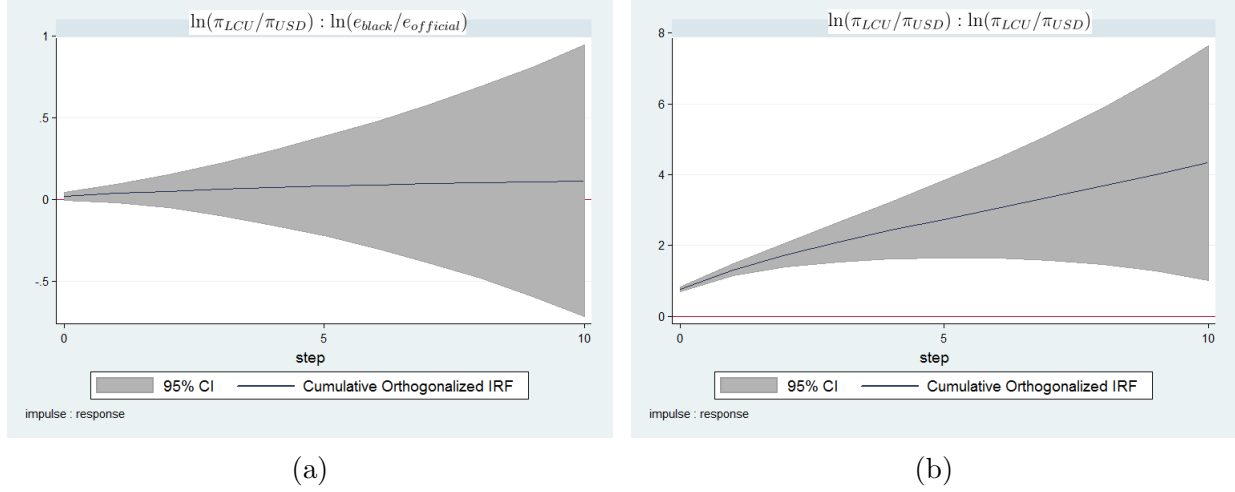


Figure 4.1: One std. deviation shock to $\ln(\pi_{LCU}/\pi_{USD})$ for full sample

corresponds to the more common belief that an increase in inflation in the local currency is associated an increase in black market premium. However, these results change when I do a panel VAR analysis for the subset of six countries.

I specify the same panel VAR model as in Eq.(4.1) and estimate it for a smaller subset of countries which include Iran, Venezuela and the four South Asian countries of India, Nepal, Pakistan and Sri Lanka. For this sample of countries, I again select a first-order panel VAR since it has the lowest mAIC, mBIC, and mQIC (see Table B.2) and after a GMM estimation of this first-order model I obtain the Cholesky decomposition-based orthogonalized impulse responses shown in Figure 4.2. For this set of countries a shock that raises the domestic inflation-US inflation ratio diminishes the ratio of black market rate to official market exchange rate from its current level. This effect persists in the for a while and the impulse response suggests a decline in black market premium with an increase in domestic inflation rate relative to US inflation. Furthermore, this panel VAR is stable as shown through eigenvalue stability conditions in Table B.3. Therefore, it is invertible and has an infinite-order vector moving-average representation, providing known interpretation to estimated impulse-response functions.

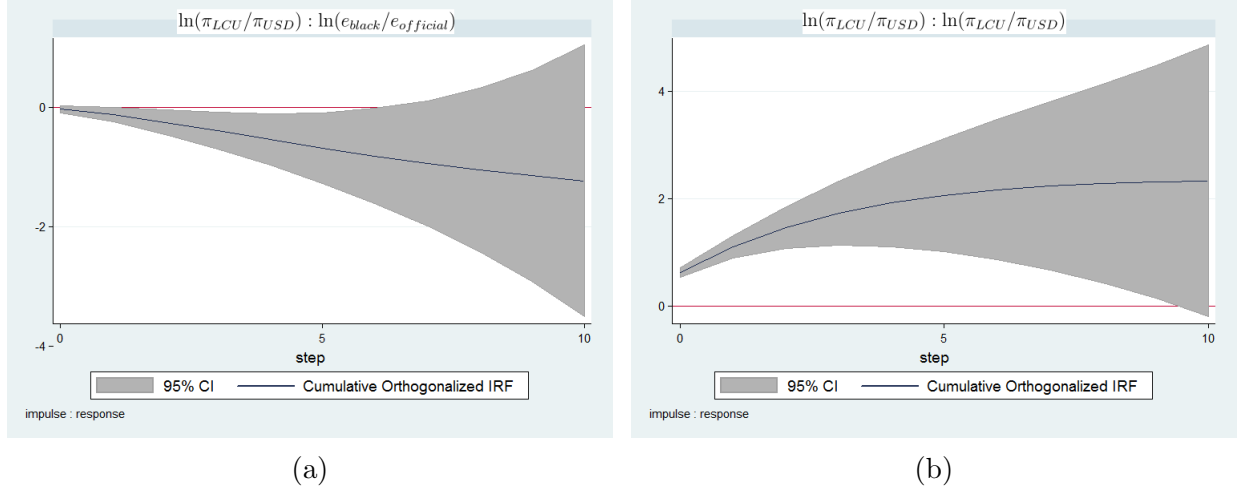


Figure 4.2: One std. deviation shock to $\ln(\pi_{LCU}/\pi_{USD})$ for the subset of countries

The results from this subset of countries is in contrast to the result from the full sample. We may predispose ourselves to misunderstand important aspects of the black market premia if we go by the results of the full sample and infer that rise in domestic inflation in relation to US inflation will always raise the black market rates vis-a-vis the official rate.

As a further confirmatory test in the next subsection I do a fixed effect panel regression with this subset of countries.

4.2.2 Fixed Effect Panel Regression Approach

In order to capture the effect of relative inflation rates on the ratio of black market rates to official market rates, for the subset of six countries discussed in the last subsection, I estimate the following fixed effect panel regression model

$$\ln\left(\frac{e_{black}}{e_{official}}\right)_{it} = \beta_0 + \beta_\pi \ln\left(\frac{\pi_{LCU}}{\pi_{USD}}\right)_{it} + \beta_f KAOPEN_{it} + \beta_d D_{it} + \theta_i + \gamma_t + \epsilon_{it} \quad (4.2)$$

where $\ln(e_{black}/e_{official})_{it}$, $\ln(\pi_{LCU}/\pi_{USD})_{it}$ and $KAOPEN_t$ have the same meaning as before. The subscript it denotes that the value is for country i in time period t , where i denotes a

country from the subset of six countries and $t \in \{1981, 1982, \dots, 1997\}$. Like before, due to the lack of data on CPI or inflation forecast at monthly frequency, I use annual averaged figures for $e_{black}/e_{official}$. The country fixed effects are denoted by θ_i , while the time fixed effects are denoted by γ_t and ϵ_{it} is an i.i.d. error term. The inclusion of country fixed effects is important to remove the bias due to the omission of country-specific time-invariant variables. Finally, D_{it} is a dummy variable that assumes the value 1 if inflation in country i in year t was high, i.e. $\pi_{LCU} > 10\%$.

The coefficient of interest is β_π . As seen in the panel VAR impulse responses, if indeed $\ln(e_{black}/e_{official})_{it}$ (or the black market premium rate) goes down with an increase in domestic inflation in relation to the US inflation, this coefficient must be negative, i.e. $\beta_\pi < 0$.

Dep. var.	(1)	(2)	(3)	(4)	(5)	(6)
Method	LSDV	LSDV	LSDV	LSDV	LSDV	LSDV
$\ln(\pi_{LCU}/\pi_{USD})_{it}$	-0.123*	-0.150*	-0.138*	-0.206**	-0.168**	-0.211**
	(0.065)	(0.082)	(0.070)	(0.094)	(0.083)	(0.093)
$KAOPEN_{it}$					-0.071	-0.063
					(0.060)	(0.063)
D_{it}			0.049	0.104		0.084
			(0.062)	(0.088)		(0.092)
Country dummies	Yes	Yes	Yes	Yes	Yes	Yes
Time dummies	No	Yes	No	Yes	Yes	Yes
Observations	102	102	102	102	102	102
Countries	6	6	6	6	6	6
Adj. R^2	0.741	0.752	0.739	0.751	0.753	0.751

Table 4.1: Fixed effect regression model: main results

Note: Heteroskedasticity-robust (Huber-White) standard errors are in brackets. Asterisks denote significance levels (** significant at 5%; * significant at 10%).

To study the relationship between relative domestic inflation rate and black market premium rate, it is worthwhile to start investigating the relationship between relative domestic inflation rate as captured through $\ln(\pi_{LCU}/\pi_{USD})_{it}$ and black market premium rate as captured through $\ln(e_{black}/e_{official})_{it}$ assuming no differences between countries in terms of financial

openness, no time fixed effects and no dummy for high inflation years. In fact, there is little variability in financial openness for these six countries. Column (1) report results obtained by Least Squares Dummy Variable (LSDV) of Eq (4.2), without the index of financial openness, dummy variable for high inflation years and time fixed effects. The estimated coefficient for $\ln(\pi_{LCU}/\pi_{USD})_{it}$ is negative and significant at 10% level (p -value = 0.061). In column (2), I add the time fixed effects to the model specified in column (1). The estimated coefficient of relative inflation, $\ln(\pi_{LCU}/\pi_{USD})_{it}$ is again negative and significant at 10% (p -value = 0.071).

Since, these countries have different political and economic regimes and in some cases different geographies, if we allow for the possibility of no single event affecting all these countries in a particular point of time, then the time fixed effects are not relevant. However, the dummy variable denoting high inflation (10%) years could be still be relevant. Therefore, in column (3) I estimate a model without index of financial openness and time fixed effects, but I include D_{it} . The estimated β_π is negative and significant at 10% level (p -value = 0.052). In column (4), I include all variables and fixed effects except for the index of financial openness. The estimated coefficient of $\ln(\pi_{LCU}/\pi_{USD})_{it}$ is negative and significant at 5% level.

In column (5), all variables except the dummy variable indicating high inflation year in country i are included and the estimated coefficient of $\ln(\pi_{LCU}/\pi_{USD})_{it}$ is negative and significant at 5% level suggesting a decline in $\ln(e_{black}/e_{official})_{it}$ with a rise in domestic inflation relative to the prevailing US inflation rate. This means black market premium rate is decreasing as domestic inflation rises with regards to US inflation. Finally in column (6), I present the estimated results of the Eq.(4.2) which includes all variables and both country and time fixed effects. In this last column, the estimated β_π is again negative and significant.

These results are supportive of the hypothesis that an increase in domestic inflation with respect to the US inflation is associated with a decline in black market premium rate, at least for the countries - India, Iran, Nepal, Pakistan, Sri Lanka and Venezuela. In the next subsection, I analyze the case of one country, India for which monthly data on inflation rates

as well as black market premium rate is available. I check whether this negative association is noticed in the case of India when using monthly data.

4.2.3 The Case of India

Monthly data on inflation rates is available both for India and the USA. Therefore, it would be prudent to check if the negative association between black market premium rate and the relative domestic inflation also holds for this country when using data at monthly frequency. For India, I specify the following time-series regression model

$$\ln\left(\frac{e_{black}}{e_{official}}\right)_t = \beta_0 + \beta_\pi \Delta \ln\left(\frac{1 + \pi_{INR}}{1 + \pi_{USD}}\right)_t + \beta_i \Delta i_t^{IN} + \beta_\ell \mathbb{I}\{t \geq 03/1993\} + \epsilon_t \quad (4.3)$$

In India's case the monthly inflation rates⁴ for some months are zero and sometimes negative. Therefore, instead of using $\ln(\pi_{INR}/\pi_{USD})_t$, here I use $\ln(1 + \pi_{INR}/1 + \pi_{USD})_t$. The variable π_{INR} denotes the inflation in Indian Rupee while i_t^{IN} is a measure of the prevailing interest rate in period t . The interest rate I use here is the bank rate offered by the Reserve Bank of India. Information about the bank rate is available on a daily basis. For those months having multiple bank rates in different portions of the month, I use a weighted average representative interest rate for the month. The rationale behind including interest rates is that if interest rates for assets denominated in rupee are sufficiently high, then the real value of the assets might be preserved despite widening difference between rupee inflation and dollar inflation. This could in turn affect relative demand for the rupee and the dollar and might have some spillover effect to the black market. $\mathbb{I}\{t \geq 03/1993\}$ is a dummy for all the months after March 1993. March 1993 is an important month in India's exchange rate and exchange arrangement history because during this month India officially switched from a basket peg to managed float exchange rate regime.

⁴These rates are calculated as percentage change in CPI from the same month last year.

The variable $\ln(e_{black}/e_{official})_t$ and the first difference variables $\Delta \ln(1 + \pi_{INR}/1 + \pi_{USD})_t$, Δi_t^{IN} are all $I(0)$. First, I estimate a time-series regression model without first difference values of interest rates and the dummy variable. These results are reported in column (1) or Table 4.2 The coefficient of $\Delta \ln(1 + \pi_{INR}/1 + \pi_{USD})_t$ is negative and significant at 1%

Dep. var.	(1)	(2)	(3)
Method	OLS	OLS	OLS
$\Delta \ln \left(\frac{1+\pi_{INR}}{1+\pi_{USD}} \right)_t$	-4.042*** (1.371)	-3.883*** (1.387)	-3.153*** (0.974)
Δi_t^{IN}		0.025* (0.013)	-0.009 (0.010)
$\mathbb{I}\{t \geq 03/1993\}$			-0.118*** (0.015)
Observations	210	210	210
Adj. R^2	0.039	0.039	0.528

Table 4.2: Regression results for India

Note: Newey-West standard errors are in brackets. Asterisks denote significance levels (***) significant at 1%; ** significant at 5%; * significant at 10%).

level suggesting that a relative increase in rupee inflation that widens the rupee inflation-dollar inflation ratio will be associated with a decline in $\ln(e_{black}/e_{official})_t$, i.e. a decline in the black maret premium rate. Next, I add Δi_t^{IN} to the model. The estimated results are shown in column (2) of Table 4.2 and the coefficient of $\Delta \ln(1 + \pi_{INR}/1 + \pi_{USD})_t$ is again negative and significant at 1% level. As a final analysis, I estimate Eq. (4.3) and the results are reported in column (3). The addition of the dummy for the months after exchange rate liberalization increases the explanatory power of the model but here also, the β_π , the coefficient of $\Delta \ln(1 + \pi_{INR}/1 + \pi_{USD})_t$ is negative and significant at 1%.

The analysis in this section lends support to the hypothesis that an increase in domestic inflation with respect to US (or foreign currency) inflation need not be associated with an increase in the black market premium rate. On the contrary, for some countries like Iran, Venezuela, the fours South Asian countries in my sample we witness a negative association. This contradicts the general idea of rising black market premium rate with increase in do-

mestic inflation. In the next section I propose a model in the New Monetarist framework that could explain this negative association between black market premium rate and the deviation between inflation in domestic currency inflation and that in foreign currency.

4.3 A Model of the Black Market

Time is discrete and continues forever. There are two countries, A and B , each populated with a continuum 2 of agents. Following Rocheteau and Wright (2005b), agents in each of the two countries are differentiated into two groups: a measure 1 of *buyers* and a measure 1 of *sellers*. Each period is divided into two stages where different activities take place. The first subperiod is for decentralized trades in local and foreign special goods and for currency exchange among country B buyers in a Walrasian *black market*. The labels ‘buyer’ and ‘seller’ refer to an agent’s role in the first subperiod and this role remains unchanged over periods. In the first subperiod, a seller can produce but does not want to consume, while a buyer wants to consume but cannot produce. Therefore, there is no double coincidence of wants. The black market is a perfectly competitive currency exchange market which country B buyers may choose to use in the first subperiod and this market is considered illegal by country B ’s authorities. If they choose to go to the black market, country B buyers can successfully transact in the black market with an exogenous probability $\alpha \in (0, 1)$ and with probability $1 - \alpha$ they lose their entire liquid wealth. This loss of can be interpreted in several ways: (i) one may interpret this as confiscation by country B ’s government as a penalty for participating in illegal currency exchange, or (ii) given the risky nature of the black market due to potential involvement of dishonest elements the agent could lose his entire wealth as a result of being robbed or swindled (due to sale of fake currency). Therefore, the parameter α captures the frictions associated in accessing the illegal currency exchange market. The black market can be thought of as a reduced form of an over-the-counter market *à la* Duffie et al.

(2005) where agents meet a dealer with probability α and a dealer's bargaining power is zero. In this special case, dealers in Duffie et al. (2005) become redundant and it is as if agents access the perfectly competitive *interdealer market* with probability α . Therefore, the black market in this paper is equivalent to the Walrasian *interdealer market* in Duffie et al. (2005). To this special case of Duffie et al. (2005), I add the additional assumption of confiscation of wealth, by country B 's authorities with probability $1 - \alpha$. In the second subperiod, there is a frictionless centralized market where agents from both countries settle debts and exchange currencies. Therefore, the second subperiod acts as a global currency exchange market. This global currency exchange market is the legally recognized, official currency exchange market where agents from both countries participate. We label the first subperiod DM (decentralized markets) and the second subperiod as the CM (centralized market). All agents discount payoffs across periods with the same factor, $\beta \in (0, 1)$. Country $i, i \in \{A, B\}$ issues its own perfectly divisible and storable fiat currency which I will call *money_i*. We use M_t^i to denote the stock of *money_i* at time t . The initial stock of *money_i* is given by $M_0^i \in \mathbb{R}_+$ which grows at a constant rate π_i over periods (therefore, $M_{t+1}^i = \pi_i M_t^i$). The growth rate of the stock of *money_i*, $\pi_i \geq \beta$ is chosen by the monetary authority in country- i .

In the CM of every period, all agents trade a consumption good produced in that stage, and the two monies, in a spot Walrasian market. The CM's spot Walrasian market serves as the official channel for the exchange of two monies. During this stage, new *money_i* ($i = A, B$) is injected ($\pi_i > 1$) or withdrawn ($\pi_i < 1$) from the economy via lump-sum transfers to buyers of country i . At the end of the second subperiod and at the start of next period's first-stage, a distinct decentralized market opens up in *each country* for the trade of special goods. We denote the two decentralized markets for special goods as $SGM_i, i \in \{A, B\}$. The $SGMs$ do not have any search frictions and the mass of bilateral matches in a SGM is given by the minimum of buyers and sellers in a market. At the beginning of first subperiod, with probability $\delta \in [0, 1)$ a country i buyer, obtains an opportunity to consume the foreign special good while with probability $(1 - \delta)$ he consumes the locally produced special good.

However, I do not assume that all buyers get to consume the local special good. The buyers who get to consume the special good produced in SGM_i are referred to as SGM_i buyers ($i \in \{A, B\}$). A simple arithmetic shows the mass of SGM_i buyers in SGM_i is 1 of which a fraction $(1 - \delta)$ are from country i , while a fraction δ are from country- j ($j \in \{A, B\}, j \neq i$). Therefore, all buyers (all sellers) in SGM_i are matched with a seller (buyer). We assume that agents cannot make binding commitments, that there is no enforcement, and that histories of actions are private in a way that precludes any borrowing and lending. Therefore, all trade must be *quid pro quo*. We assume that sellers do not recognize a foreign currency⁵. Therefore, when a seller meets a foreign buyer, the buyer must pay the seller in the seller's currency. This implies that a SGM_i country j buyer will not be able to participate in SGM_i unless he acquires $money_i$. Finally, within any given match, buyers make a take-it-or-leave-it (TIOLI) offer to the seller.

An individual buyer's preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u(q_t) + c_t - h_t)$$

where q_t is the quantity of the local or foreign special good that the buyer consumes at the end of the first subperiod of period t , c_t is his consumption of the homogeneous good that is produced, traded and consumed in the second subperiod of period t , and h_t is the utility cost from exerting h_t units of effort to produce this good. The function $u(q_t)$ is the utility a buyer derives if he consumes q_t amount of local or foreign special good in the decentralized round of trade in period t . The utility function, $u(\cdot)$ is twice continuously differentiable with $u(0) = 0$, $u'(\cdot) > 0$, $u''(\cdot) < 0$ and $u'(q_t)q_t$ is decreasing. We also assume that $u(\cdot)$ satisfies the Inada conditions: $u'(0) = \infty$ and $u'(\infty) = 0$ and that there exists a $q^* \equiv \operatorname{argmax} \{u(q) - q\}$.

The expectation operator \mathbb{E}_0 is with respect to the random matching with local or foreign

⁵This could be due to a sufficiently high cost of authentication incurred by a seller which leads to only local currencies being accepted in each DM as in Zhang (2014) or due to restriction imposed by the authorities that forbid sellers to accept foreign currency as in Curtis and Waller (2000).

seller in the decentralized trades and the random success in transacting in the black market, if a buyer chooses to access it.

An individual country i ($i \in \{A, B\}$) seller's preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (-q_t + c_t - h_t)$$

where q_t is the quantity of the local or foreign special good that the seller produces at the end of the first subperiod of period t , c_t is his consumption of the homogeneous good that is produced, traded and consumed in the second subperiod of period t , h_t is the utility cost from exerting h_t units of effort to produce this good. The expectation operator \mathbb{E}_0 is with respect to the random matching with local or foreign buyer.

4.4 Value Functions and Optimal Behavior

4.4.1 Value Functions

Let ϕ_t^A be the real price of *money_A* and ϕ_t^B be the real price of *money_B* both expressed in terms of the second-subperiod's consumption good. Then, $z_t^A = \phi_t^A m_t^A$ and $z_t^B = \phi_t^B m_t^B$ are the real balances held by an agent in *money_A* and *money_B* respectively. We use $V_i^B(\mathbf{z}_{ti})$ to denote the maximum expected discounted payoff of a country i buyer who enters the decentralized round of period t with portfolio $\mathbf{z}_{ti} \equiv (z_{ti}^A, z_{ti}^B)$. Let $W_i^B(\mathbf{z}_{ti})$ denote the maximum expected discounted payoff of a country i buyer who is holding portfolio \mathbf{z}_{ti} at the beginning

of the second subperiod of period t . Then,

$$\begin{aligned}
W_i^B(\mathbf{z}_{ti}) &= \max_{c_t, h_t, \mathbf{z}_{t+1i}} [c_t - h_t + \beta V_i^B(\mathbf{z}_{t+1i})] \\
\text{s.t. } c_t + \boldsymbol{\phi}_t \mathbf{m}_{t+1i} &= h_t + \boldsymbol{\phi}_t \mathbf{m}_{ti} + T_t^i \\
c_t, h_t &\in \mathbb{R}_+; \mathbf{m}_{t+1i}, \mathbf{z}_{t+1i} \in \mathbb{R}_+^2 \\
\mathbf{m}_{t+1i} &= (m_{t+1i}^A, m_{t+1i}^B), \mathbf{z}_{t+1i} = (z_{t+1i}^A, z_{t+1i}^B)
\end{aligned} \tag{4.4}$$

where $\boldsymbol{\phi}_t = (\phi_t^A, \phi_t^B)$, $\mathbf{m}_{ti} = (m_{ti}^A, m_{ti}^B)$ and $\boldsymbol{\phi}_t \mathbf{m}_{ti}$ (or $\boldsymbol{\phi}_t \mathbf{m}_{t+1i}$) denotes the dot product of $\boldsymbol{\phi}_t$ and \mathbf{m}_{ti} (or, \mathbf{m}_{t+1i}) which is equivalent to \mathbf{z}_{ti} (or, \mathbf{z}_{t+1i}). $T_t^i = \phi_t^i(\pi_i - 1)M_{i,t}$ is the real value of the time t lump-sum monetary transfer (or, tax, if $\pi_i < 1$). One can further simplify Eq (4.4) and it can be rewritten entirely in terms of real balances as

$$\begin{aligned}
W_i^B(\mathbf{z}_{ti}) &= \max_{c_t, h_t, \mathbf{z}_{t+1i}} [c_t - h_t + \beta V_i^B(\mathbf{z}_{t+1i})] \\
\text{s.t. } c_t + \beta(\mathbf{1} + \boldsymbol{\iota}) \cdot \mathbf{z}_{t+1i} &= h_t + \mathbf{1} \cdot \mathbf{z}_{ti} + T_t^i \\
c_t, h_t &\in \mathbb{R}_+, \mathbf{z}_{t+1i} \in \mathbb{R}_+^2 \\
\mathbf{z}_{t+1i} &= (z_{t+1i}^A, z_{t+1i}^B)
\end{aligned} \tag{4.5}$$

where $\iota_i = \frac{\phi_t^i}{\beta \phi_{t+1}^i} - 1$ and it is the cost of holding *money* _{i} , $i \in \{A, B\}$. In (4.5), eliminating h_t from the budget constraint yields

$$\begin{aligned}
W_i^B(\mathbf{z}_{ti}) &= \mathbf{1} \cdot \mathbf{z}_{ti} + W_i^B(\mathbf{0}) \\
W_i^B(\mathbf{0}) &\equiv T_t^i + \max_{\mathbf{z}_{t+1i}} \beta [-(\mathbf{1} + \boldsymbol{\iota}) \cdot \mathbf{z}_{t+1i} + V_i^B(\mathbf{z}_{t+1i})] \\
\mathbf{z}_{t+1i} &= (z_{t+1i}^A, z_{t+1i}^B) \in \mathbb{R}_+^2
\end{aligned} \tag{4.6}$$

As is standard in models that build on Lagos and Wright (2005), the buyer's value function is linear in the real balances, implying that there are no wealth effects on the choice of \mathbf{z}_{t+1} . Next, consider country i seller's value function for the second subperiod of period

t . Let $W_i^S(\mathbf{z}_{ti})$ denote the maximum expected discounted payoff of a country i seller who is holding portfolio \mathbf{z}_{ti} at the beginning of the second subperiod of period t . This agent will never want to leave the CM with any money holdings, since he does not participate in the black market and does not want to consume in the decentralized round of trade (see Rocheteau and Wright (2005b) for a rigorous proof). Then,

$$\begin{aligned}
W_i^S(\mathbf{z}_{ti}) &= \max_{c_t, h_t, \mathbf{z}_{t+1i}} [c_t - h_t + \beta V_i^S(\mathbf{0})] \\
\text{s.t. } c_t + \beta(\mathbf{1} + \boldsymbol{\iota}) \cdot \mathbf{z}_{t+1i} &= h_t + \mathbf{1} \cdot \mathbf{z}_{ti} \\
c_t, h_t &\in \mathbb{R}_+, \mathbf{z}_{t+1i} \in \mathbb{R}_+^2 \\
\mathbf{z}_{t+1i} &= (z_{t+1i}^A, z_{t+1i}^B)
\end{aligned} \tag{4.7}$$

Again, eliminating h_t from the budget constraint in (4.7), I get

$$\begin{aligned}
W_i^S(\mathbf{z}_{ti}) &= \mathbf{1} \cdot \mathbf{z}_{ti} + W_i^S(\mathbf{0}) \\
W_i^S(\mathbf{0}) &\equiv \max_{\mathbf{z}_{t+1i}} \beta [-(\mathbf{1} + \boldsymbol{\iota}) \cdot \mathbf{z}_{t+1i} + V_i^S(\mathbf{0})] \\
\mathbf{z}_{t+1i} &= (z_{t+1i}^A, z_{t+1i}^B) \in \mathbb{R}_+^2
\end{aligned} \tag{4.8}$$

In the first subperiod with a probability δ buyers of each country get the opportunity to consume a foreign special good, while with probability $(1 - \delta)$ they get to consume a local special good. However, country B buyers can successfully transact in the black market for currency exchange with probability $\alpha \in (0, 1]$ and with probability $1 - \alpha$ has their entire liquid wealth confiscated. Since sellers of country A do not accept $money_B$ and sellers of country B do not accept $money_A$, country B buyers would want to access the black market and readjust their portfolio of real balances. If a country B buyer, who gets the opportunity to consume a foreign special good, chooses to access the black market, he would want to convert his entire real holdings of $money_B$ into real holdings of $money_A$ and vice-versa. The

resulting post-trade real portfolios of country B buyer who gets to consume a special good from country A and that of a country B buyer who gets to consume a special good from country B are denoted

$$\begin{aligned} & [\bar{z}_B^A(\mathbf{z}_{tB}; \boldsymbol{\psi}_t), \bar{z}_B^B(\mathbf{z}_{tB}; \boldsymbol{\psi}_t)] \\ & [\bar{z}_{B*}^A(\mathbf{z}_{tB}; \boldsymbol{\psi}_t), \bar{z}_{B*}^B(\mathbf{z}_{tB}; \boldsymbol{\psi}_t)] \end{aligned}$$

respectively, where $\boldsymbol{\psi}_t \equiv (\phi_t^A, \phi_t^B, \varepsilon_t)$ and $\mathbf{z}_{tB} = (z_{tB}^A, z_{tB}^B)$ is the pre-trade real portfolio of country B . The asterisk (*) over B in the second post-trade real portfolio indicates that the buyer is matched with a local seller (i.e. from country B). The black market is effectively the market where country B buyers trade real balances held in the two currencies and readjust their portfolios after realizing the shock. Here real balance of $money_B$ trades at ε_t against real balance of $money_A$. In nominal terms this means, in the black market, $\varepsilon_t^{-1} \phi_t^A / \phi_t^B$ is the price of $money_A$ in terms of $money_B$. Note that ϕ_t^A / ϕ_t^B is the nominal price of $money_A$ in terms of $money_B$ (or, nominal exchange rate) in the CM which also acts as the officially recognized foreign exchange market. Therefore, $\varepsilon_t^{-1} - 1$ is the black market premium. If $\varepsilon_t^{-1} > 1$ the premium is positive. If $\varepsilon_t^{-1} < 1$, then the premium is negative and when $\varepsilon_t^{-1} = 1$, the exchange rate of the official market and the black market coincide.

We can now write the value function of a country B buyer who enters the decentralized round of period t with portfolio \mathbf{z}_{tB} and chooses to access the black market,

$$\begin{aligned} V_B^B(\mathbf{z}_{tB}) &= \alpha \delta [u(\tilde{q}_{B|bm}) + W_B^B(\bar{z}_B^A - \tilde{d}_{bm}^A, \bar{z}_B^B)] \\ &\quad + (1 - \alpha) \delta W_B^B(\mathbf{0}) \\ &\quad + \alpha (1 - \delta) [u(q_{B|bm}) + W_B^B(\bar{z}_{B*}^A, \bar{z}_{B*}^B - d_{bm}^B)] \\ &\quad + (1 - \alpha) (1 - \delta) W_B^B(\mathbf{0}) \end{aligned} \tag{4.9}$$

where $\tilde{q}_{B|bm}$ denotes the amount of special good bought by a country B buyer in SGM_A

(foreign market) given that he has adjusted his portfolio of real balances in the black market. The variable $q_{B|bm}$ denote the special good bought by a country B buyer in SGM_B (local market) conditional on him adjusting his portfolio of real balances in the black market. The real payments made with $money_A$ by a country B buyer to country A (foreign) seller after having readjusted his portfolio in the black market is \tilde{d}_{bm}^A . Similarly d_{bm}^B denotes the real payment made with $money_B$ by a country B buyer to country B (local) seller after readjusting his portfolio in the black market. the value function of a country B buyer who enters the decentralized round of period t with portfolio \mathbf{z}_{tB} and chooses not to access the black market is:

$$\begin{aligned} V_B^B(\mathbf{z}_{tB}) = & \delta[u(\tilde{q}_{B|nbm}) + W_B^B(z_{tB}^A - \tilde{d}_{nbm}^A, z_{tB}^B)] \\ & + (1 - \delta)[u(q_{B|nbm}) + W_B^B(z_{tB}^A, z_{tB}^B - d_{nbm}^B)] \end{aligned} \quad (4.10)$$

The the value function of a country A buyer who enters the decentralized round of period t with portfolio \mathbf{z}_{tA} ,

$$\begin{aligned} V_A^B(\mathbf{z}_{tA}) = & \delta[u(\tilde{q}_A) + W_A^B(z_{tA}^A, z_{tA}^B - \tilde{d}^B)] \\ & + (1 - \delta)[u(q_A) + W_A^B(z_{tA}^A - d^A, z_{tA}^B)] \end{aligned} \quad (4.11)$$

where \tilde{q}_A , denotes the amount of special good bought by a country A buyer in SGM_B (foreign market). The variable q_A denote the amount of special good bought by a country A buyer in SGM_A (local market). The real payments made with $money_B$ in SGM_B and with $money_A$ in SGM_A by country A buyer are denoted by \tilde{d}^B and d^A respectively.

4.4.2 Terms of Trade

In this section, I discuss the determination of the terms of trade in the two SGM s. Consider a meeting in SGM_i , between a country i seller and a buyer (from any country) who carries a real portfolio $\mathbf{z}_t = (z_t^A, z_t^B)$ composed of $money_A$ and $money_B$. The two parties negotiate over a quantity of special good, q_t , to be produced, and an amount of real payment in $money_A$, d_t^A and real payment in $money_B$, d_t^B to be delivered to the seller. Define $\mathbf{d}_t \equiv (d_t^A, d_t^B)$. Given that the buyer makes a TIOLI offer to the seller, the bargaining problem can be expressed as

$$\begin{aligned} & \max_{q_t, \mathbf{d}_t} \{u(q_t) + W_i^B(\mathbf{z}_t - \mathbf{d}_t) - W_i^B(\mathbf{z}_t)\} \\ & \text{s.t. } q_t = W_i^S(\tilde{\mathbf{z}}_t + \mathbf{d}_t) - W_i^S(\tilde{\mathbf{z}}_t) \\ & \text{and } q_t \in \mathbb{R}_+, \mathbf{d}_t \in [0, z_t^A] \times [0, z_t^B] \end{aligned} \tag{4.12}$$

Now, I discuss the more specific cases. Consider a meeting in SGM_i between a seller from country i and a buyer from any country who carries a real portfolio $\mathbf{z}_t = (z_t^A, z_t^B)$. Since the seller won't accept any payment in $money_j$ ($j \neq i$) it must be that $d_t^j = 0$. With $d_t^j = 0$, the bargaining problem described in (4.12) reduces to

$$\begin{aligned} & \max_{q_t, d_t^i} \{u(q_t) - d_t^i\} \\ & \text{s.t. } q_t = d_t^i \end{aligned} \tag{4.13}$$

The next lemma describes the solution to this bargaining problem.

Lemma 1. Define $q^* = \{q : u'(q) = 1\}$. Then in a SGM_i meeting between a seller from country i and a buyer from any country, who carries a real portfolio $\mathbf{z}_t = (z_t^A, z_t^B)$, the bargaining solution is given by $q_t = \min\{z_t^i, q^*\}$, $d_t^i = \min\{z_t^i, q^*\}$ and $d_t^j = 0$ where $i, j \in \{A, B\}$, $i \neq j$.

Proof. In appendix. □

The interpretation of Lemma 1 is standard. The terms of trade depend only on the buyer's real holdings of $money_i$. When z_t^i exceeds a certain level q^* , then the buyer purchases the first-best quantity, q^* , and gives up exactly q^* units of his real holdings of $money_i$. On the other hand, if z_t^i is less than q^* , then the buyer is liquidity constrained and he gives up his entire real holding of $money_i$ to receive the amount of good that the seller is willing to produce for that money, i.e., $q_t = z_t^i$.

We now proceed to the characterization of the terms of trade in the black market. Consider a country B buyer who gets the opportunity to consume foreign special goods, i.e. he is matched with a country A seller (foreign seller). This buyer would want to exchange some (or all) of his $money_B$ for $money_A$ so if he accesses the frictionless, competitive black market, he can acquire $money_A$ from other country B buyers who buys locally (and needs to exchange $money_A$ for $money_B$). The problem of the country B buyer buying foreign special goods is given by

$$\begin{aligned}
& \max_{\bar{z}_B^A, \bar{z}_B^B} [u(\tilde{q}_{B|bm}) + W_B^B(\bar{z}_B^A - \tilde{d}_{bm}^A, \bar{z}_B^B)] \\
& \text{s.t. } \bar{z}_B^A + \varepsilon_t \bar{z}_B^B = z_{tB}^A + \varepsilon_t z_{tB}^B \\
& \quad \bar{z}_B^A, \bar{z}_B^B \geq 0
\end{aligned} \tag{4.14}$$

The country B buyer matched with a foreign seller trades $money_B$ for $money_A$ to readjust his portfolio so as to maximize the sum of his utility from consumption of the foreign special good and the continuation value. However, if he successfully transacts in the black market, he cannot leave with any more than what he entered with. So, the budget constraint must be satisfied. furthermore, since the objective function is monotonic, the budget constraint must hold with equality. The following lemma solves the above problem.

Lemma 2. *Consider a country B buyer with portfolio (z_{tB}^A, z_{tB}^B) who gets the opportunity to consume foreign good in period t . If he successfully transacts in the black market, he leaves with a post-trade portfolio $(\bar{z}_B^A, \bar{z}_B^B)$ such that*

$$\begin{aligned}
(a) \text{ If } \varepsilon_t > 1, \text{ then } & \begin{cases} \bar{z}_B^A = z_{tB}^A + \varepsilon_t z_{tB}^B \\ \bar{z}_B^B = 0 \end{cases} \\
(b) \text{ If } \varepsilon_t = 1 \text{ and } z_{tB}^A + z_{tB}^B \geq q^*, \text{ then } & \begin{cases} \bar{z}_B^A \in [q^*, z_{tB}^A + z_{tB}^B] \\ \bar{z}_B^B = z_{tB}^B + z_{tB}^A - \bar{z}_B^A \end{cases} \\
(c) \text{ If } \varepsilon_t = 1 \text{ and } z_{tB}^A + z_{tB}^B < q^*, \text{ then } & \begin{cases} \bar{z}_B^A = z_{tB}^A + z_{tB}^B \\ \bar{z}_B^B = 0 \end{cases} \\
(d) \text{ If } \varepsilon_t < 1 \text{ and } z_{tB}^A + \varepsilon_t z_{tB}^B \geq \bar{\chi}, \text{ then } & \begin{cases} \bar{z}_B^A = \bar{\chi} \\ \bar{z}_B^B = z_{tB}^B + \varepsilon_t^{-1}(z_{tB}^A - \bar{\chi}) \end{cases} \\
(e) \text{ If } \varepsilon_t < 1 \text{ and } z_{tB}^A + \varepsilon_t z_{tB}^B < \bar{\chi}, \text{ then } & \begin{cases} \bar{z}_B^A = z_{tB}^A + \varepsilon_t z_{tB}^B \\ \bar{z}_B^B = 0 \end{cases}
\end{aligned}$$

where ε_t is the black market price of real balance of $money_A$ in terms real balance of $money_B$ and $\bar{\chi}$ such that $u'(\bar{\chi}) = \varepsilon_t^{-1}$.

Proof. In appendix. □

To interpret the result in Lemma 2 first observe that the objective function in (4.14) can be written as $u(\tilde{q}_{B|bm}) + \bar{z}_B^A - \tilde{q}_{B|bm} + \bar{z}_B^B$. When $\varepsilon_t > 1$, in the black market using a unit real balance of $money_B$, real balance of $money_A$ can be increased by more than one unit. Since the objective function is strictly increasing in \bar{z}_B^A the best thing to do is to convert entire $money_B$ into $money_A$. When $\varepsilon_t = 1$, real balance of $money_A$ and that of $money_B$ trades one to one. Due to the strict concavity of the utility function, an increase in real balance of $money_A$ strictly increases the objective function as long as $\bar{z}_B^A < q^*$ after that the objective function increases in $\bar{z}_B^A + \bar{z}_B^B$. Since real balance of $money_A$ and that of $money_B$ trade one to one, the buyer is indifferent between increasing or not increasing his real holding of $money_A$. When $\varepsilon_t < 1$, in the black market it gets more expensive to acquire real balance of $money_A$ as a unit real balance of $money_A$ can be only bought with more than one unit of $money_B$, while there is increase in utility from increasing real balance of $money_A$, there is a cost to it as well. The $\bar{\chi}$ represent that amount of real $money_A$ holding at which marginal utility equals marginal cost. If a buyer's real wealth is more than $\bar{\chi}$, he will increase his real balance of $money_A$ no more than $\bar{\chi}$, otherwise he will increase it as much as his wealth would permit.

Now, consider a country B buyer who is matched with a seller from his own country and gets to consume local goods. This buyer would want to exchange some (or all) of his $money_A$ for $money_B$. The problem of the country B buyer buying local special goods is given by

$$\begin{aligned}
& \max_{\bar{z}_{B*}^A, \bar{z}_{B*}^B} [u(q_{B|bm}) + W_B^B(\bar{z}_{B*}^A, \bar{z}_{B*}^B - d_{bm}^B)] \\
& \text{s.t. } \bar{z}_{B*}^A + \varepsilon_t \bar{z}_{B*}^B = z_{tB}^A + \varepsilon_t z_{tB}^B \\
& \quad \bar{z}_{B*}^A, \bar{z}_{B*}^B \geq 0
\end{aligned} \tag{4.15}$$

The following lemma provides the solution to the above problem described in 4.15

Lemma 3. *Consider a country B buyer with portfolio (z_{tB}^A, z_{tB}^B) who gets the opportunity to consume local good in period t . If he successfully transacts in the black market, he leaves with a post-trade portfolio $(\bar{z}_{B*}^A, \bar{z}_{B*}^B)$ such that*

$$(a) \text{ If } \varepsilon_t > 1 \text{ and } z_{tB}^A + \varepsilon_t z_{tB}^B \geq \varepsilon_t \bar{\psi}, \text{ then } \begin{cases} \bar{z}_{B*}^A = z_{tB}^A + \varepsilon_t(z_{tB}^B - \bar{\psi}) \\ \bar{z}_{B*}^B = \bar{\psi} \end{cases}$$

$$(b) \text{ If } \varepsilon_t > 1 \text{ and } z_{tB}^A + \varepsilon_t z_{tB}^B < \varepsilon_t \bar{\psi}, \text{ then } \begin{cases} \bar{z}_{B*}^A = 0 \\ \bar{z}_{B*}^B = \varepsilon_t^{-1} z_{tB}^A + z_{tB}^B \end{cases}$$

$$(c) \text{ If } \varepsilon_t = 1 \text{ and } z_{tB}^A + z_{tB}^B \geq q^*, \text{ then } \begin{cases} \bar{z}_{B*}^A = z_{tB}^A + z_{tB}^B - \bar{z}_{B*}^B \\ \bar{z}_{B*}^B \in [q^*, z_{tB}^A + z_{tB}^B] \end{cases}$$

$$(d) \text{ If } \varepsilon_t = 1 \text{ and } z_{tB}^A + z_{tB}^B < q^*, \text{ then } \begin{cases} \bar{z}_{B*}^A = 0 \\ \bar{z}_{B*}^B = z_{tB}^A + z_{tB}^B \end{cases}$$

$$(e) \text{ If } \varepsilon_t < 1, \text{ then } \begin{cases} \bar{z}_{B*}^A = 0 \\ \bar{z}_{B*}^B = \varepsilon_t^{-1} z_{tB}^A + z_{tB}^B \end{cases}$$

where ε_t is the black market price of real balance of money_A in terms real balance of money_B and $\bar{\psi}$ such that $u'(\bar{\psi}) = \varepsilon_t$.

Proof. In appendix. □

To interpretation of in Lemma 3 is similar to that of Lemma 2. When $\varepsilon_t > 1$, in the black market a unit real balance of money_B costs more than one unit of money_A, while utility increases with real balance of money_B, there is a cost to it as well. The $\bar{\psi}$ represent that amount of real money_B holding at which marginal utility equals marginal cost. If a buyer's real wealth is more than $\bar{\psi}$, he will increase his real balance of money_B not beyond $\bar{\psi}$,

otherwise he will increase it as much as his wealth would permit. When $\varepsilon_t = 1$, real balance of $money_A$ and that of $money_B$ trades one to one. Due to the strict concavity of the utility function, an increase in real balance of $money_B$ strictly increases the objective function as long as $\bar{z}_{B^*}^B < q^*$ after that the objective function increases in $\bar{z}_{B^*}^A + \bar{z}_{B^*}^B$. Since real balance of $money_A$ and that of $money_B$ trade one to one, the buyer is indifferent between increasing or not increasing his real holding of $money_B$. When $\varepsilon_t < 1$, it becomes cheaper to increase real balance of $money_B$ and more than one unit of real balance of $money_B$ can be obtained with a unit real balance of $money_A$. Therefore, it is optimal to convert entire $money_A$ into $money_B$.

Now, we can find the exact terms of trade in the special goods market when a country B buyer successfully transacts in the black market. First consider a country B buyer visiting SGM_A . The exact terms of trade will depend on the readjusted portfolio of the buyer. Given that payments are always made in $money_A$, from Lemma 1, whenever his real balance of $money_A$ is q^* or more, he would buy q^* amounts of the special good. Otherwise he will buy whatever amount he can afford by spending his entire holdings of $money_A$. Combining Lemmas 1 and 2 we get country B buyer's terms of trade in SGM_A after he successfully readjusts his portfolio in the black market.

Lemma 4. Consider a country B buyer who is matched with a country A seller in SGM_A . If the buyer successfully readjusts here portfolio to $(\bar{z}_B^A, \bar{z}_B^B)$ in the black market then, the terms of trade in SGM_A meeting between country B buyer and country A seller is given by

- (a) If $\varepsilon_t > 1$ and $z_{tB}^A + \varepsilon_t z_{tB}^B \geq q^*$, then $\{\tilde{q}_{B|bm} = q^*, \tilde{d}_{bm}^A = q^*, \tilde{d}_{bm}^B = 0\}$
- (b) If $\varepsilon_t > 1$ and $z_{tB}^A + \varepsilon_t z_{tB}^B < q^*$, then $\{\tilde{q}_{B|bm} = \bar{z}_B^A, \tilde{d}_{bm}^A = \bar{z}_B^A, \tilde{d}_{bm}^B = 0\}$
- (c) If $\varepsilon_t = 1$ and $z_{tB}^A + z_{tB}^B \geq q^*$, then $\{\tilde{q}_{B|bm} = q^*, \tilde{d}_{bm}^A = q^*, \tilde{d}_{bm}^B = 0\}$
- (d) If $\varepsilon_t = 1$ and $z_{tB}^A + z_{tB}^B < q^*$, then $\{\tilde{q}_{B|bm} = \bar{z}_B^A, \tilde{d}_{bm}^A = \bar{z}_B^A, \tilde{d}_{bm}^B = 0\}$
- (e) If $\varepsilon_t < 1$ and $z_{tB}^A + \varepsilon_t z_{tB}^B \geq \bar{\chi}$, then $\{\tilde{q}_{B|bm} = \bar{\chi}, \tilde{d}_{bm}^A = \bar{\chi}, \tilde{d}_{bm}^B = 0\}$
- (f) If $\varepsilon_t < 1$ and $z_{tB}^A + \varepsilon_t z_{tB}^B < \bar{\chi}$, then $\{\tilde{q}_{B|bm} = \bar{z}_B^A, \tilde{d}_{bm}^A = \bar{z}_B^A, \tilde{d}_{bm}^B = 0\}$

Proof. This proof is trivial and therefore omitted. □

On similar lines, I can specify the exact terms of trade in SGM_B for a country B buyer who has successfully traded in the black market and readjusted his portfolio. Given that payments are always made in $money_B$ in this market, from Lemma 1, whenever his real balance of $money_B$ is q^* or more, he would buy q^* amounts of the special good. Else, he will buy whatever amount he can afford by spending his entire holdings of $money_B$. Combining Lemmas 1 and 3 I can write down a country B buyer's terms of trade in SGM_A after he successfully readjusts his portfolio in the black market. This can be summarized by the following Lemma

Lemma 5. *Consider a country B buyer who is matched with a country B seller in SGM_B . If the buyer successfully readjusts here portfolio to $(\bar{z}_{B^*}^A, \bar{z}_{B^*}^B)$ in the black market then, the terms of trade in SGM_B meeting between country B buyer and country B seller is given by*

- (a) *If $\varepsilon_t > 1$ and $z_{tB}^A + \varepsilon_t z_{tB}^B \geq \varepsilon_t \bar{\psi}$, then $\{q_{B|bm} = \bar{\psi}, d_{bm}^A = 0, d_{bm}^B = \bar{\psi}\}$*
- (b) *If $\varepsilon_t > 1$ and $z_{tB}^A + \varepsilon_t z_{tB}^B < \varepsilon_t \bar{\psi}$, then $\{q_{B|bm} = \bar{z}_{B^*}^B, d_{bm}^A = 0, d_{bm}^B = \bar{z}_{B^*}^B\}$*
- (c) *If $\varepsilon_t = 1$ and $z_{tB}^A + z_{tB}^B \geq q^*$, then $\{q_{B|bm} = q^*, d_{bm}^A = 0, d_{bm}^B = q^*\}$*
- (d) *If $\varepsilon_t = 1$ and $z_{tB}^A + z_{tB}^B < q^*$, then $\{q_{B|bm} = \bar{z}_{B^*}^B, d_{bm}^A = 0, d_{bm}^B = \bar{z}_{B^*}^B\}$*
- (e) *If $\varepsilon_t < 1$ and $z_{tB}^A + \varepsilon_t z_{tB}^B \geq \varepsilon_t q^*$, then $\{q_{B|bm} = q^*, d_{bm}^A = 0, d_{bm}^B = q^*\}$*
- (f) *If $\varepsilon_t < 1$ and $z_{tB}^A + \varepsilon_t z_{tB}^B < \varepsilon_t q^*$, then $\{q_{B|bm} = \bar{z}_{B^*}^B, d_{bm}^A = 0, d_{bm}^B = \bar{z}_{B^*}^B\}$*

Proof. This proof is trivial and therefore omitted. □

4.4.3 CM Portfolio Problem

In this section, I describe the optimal portfolio choices of buyers from the countries A and B. As a first step I characterize the objective functions for these agents. First, consider a country B buyer who goes to the black market. The max operator problem in Eq. (4.6) represents the portfolio choice problem (i.e. choosing z_{t+1B}^A, z_{t+1B}^B) of a country B buyer. To derive the portfolio choice problem of a country B buyer who chooses to go to the black market I lead Eq (4.9) by one period and substitute it in the max operator of Eq. (4.6).

After replacing the $W_B^B(.)$ s with their linear expression and dropping the constant terms,

$$\begin{aligned}
\max_{z_{t+1B}^A, z_{t+1B}^B} & [\alpha\delta\{u(\tilde{q}_{B|bm}) + \bar{z}_B^A - \tilde{d}_{bm}^A + \bar{z}_B^B\} \\
& + \alpha(1 - \delta)\{u(q_{B|bm}) + \bar{z}_{B*}^A + \bar{z}_{B*}^B - d_{bm}^B\} \\
& - \{(1 + \iota_A)z_{t+1B}^A + (1 + \iota_B)z_{t+1B}^B\}]
\end{aligned} \tag{4.16}$$

where $(\bar{z}_B^A, \bar{z}_B^B)$ and $(\bar{z}_{B*}^A, \bar{z}_{B*}^B)$ are the country B buyer's post-black market trade readjusted portfolios when he is matched with a foreign (country A) seller and when he is matched with a local seller (country B) respectively. These readjusted portfolios are given by Lemmas 2 and 3. The pairs $(\tilde{q}_{B|bm}, \tilde{d}_{bm}^A)$ and $(q_{B|bm}, d_{bm}^B)$ represent the quantity bought in SGM_A (SGM_B) and real payment in $money_A$ ($money_B$) after having readjusted his portfolio in the black market. These are given by Lemmas 4 and 5.

Next, I consider the portfolio choice problem of a country B buyer who does not access the black market. Using Eq. (4.10), the max operator of Eq. (4.6) and after replacing the $W_B^B(.)$ s with their linear expression and dropping the constant terms, I get

$$\max_{z_{t+1A}^A, z_{t+1A}^B} [\delta\{u(\tilde{q}_B) - \tilde{q}_B\} + (1 - \delta)\{u(q_B) - q_B\} - \iota_A z_{t+1B}^A - \iota_B z_{t+1B}^B] \tag{4.17}$$

Finally, I consider a country A buyer's portfolio choice problem. To do this I lead Eq. (4.11) by one period and substitute it in the max operator of Eq. (4.6). After replacing the $W_B^B(.)$ s with their linear expression and dropping the constant terms, I get

$$\max_{z_{t+1A}^A, z_{t+1A}^B} [\delta\{u(\tilde{q}_A) - \tilde{q}_A\} + (1 - \delta)\{u(q_A) - q_A\} - \iota_A z_{t+1A}^A - \iota_B z_{t+1A}^B] \tag{4.18}$$

Lemma 6. *In any equilibrium, $\iota_i > 0$, $i \in \{A, B\}$.*

Proof. This is a standard result in monetary theory. If $\iota_i < 0$ (i.e. if $\phi_t^i < \beta\phi_{t+1}^i$) for any $i \in \{A, B\}$, then country A buyers will have an infinite demand for $money_i$. Therefore, the equilibrium is not well defined. Since ι_i is never negative for the extreme case of $\iota_i = 0$ following Lagos and Wright (2005) I assume that ι_i approaches 0 from above. This rules out the indeterminacy of optimal portfolio. \square

4.4.4 Entry to the Black Market

In this section I discuss a country B buyer's decision to enter the black market. After entering the first subperiod, once buyers realize the idiosyncratic shock that allows them the opportunity to consume a foreign special good or a local special good, country B buyers are separated into two groups. There is a mass $\delta \in (0, 1)$ of country B buyers who buy from country A sellers. These agents would like to sell their real holdings of $money_B$ and increase their real holding of $money_A$. This will allow them to buy a greater quantity of foreign goods. There is a mutually exclusive group of country B buyers of mass $1 - \delta$ that buy locally and pay in $money_B$ – these buyers would want to get rid of their real balance of $money_A$ and increase their real balance of $money_B$ so that they can buy more of the local goods. Therefore, these two sets of buyers create the two sides of the black market for currency exchange. For a black market to exist, both sets must want to use the black market. This would be the case when for both sets of buyers the payoff from using the black market is strictly greater than the payoff from not using it. Thus,

Lemma 7. *The black market exists if and only if the following conditions hold*

$$\alpha[u(\tilde{q}_{B|bm}) + \bar{z}_B^A - \tilde{d}_{bm}^A + \bar{z}_B^B] > u(\tilde{q}_{B|nbm}) + z_{tB}^A - \tilde{d}_{nbm}^A + z_{tB}^B$$

$$\text{and } \alpha[u(q_{B|bm}) + \bar{z}_{B*}^A + \bar{z}_{B*}^B - d_{bm}^B] > u(q_{B|nbm}) + z_{tB}^A + z_{tB}^B - d_{nbm}^B$$

Proof. In appendix. □

The first condition of this lemma suggests that the payoff from entering the black market for a country B buyer matched with a foreign (country A) seller is greater than the payoff from not entering it. The second condition suggests the same for a country B buyer matched with a local (country B) seller. Assuming that α and other macroeconomic fundamentals are such that these conditions are satisfied, in the next section I characterize the equilibrium in this two-country model with a black market in country B .

4.5 Equilibrium in the Two-Country Model with Black Market

This section describes the equilibrium of the two-country, two-monies model. The focus is on a stationary equilibrium where aggregate real balances in each country are constant over time. Therefore, the rate of return of $money_i$ in each country is constant and will equal $\pi_i^{-1} = \frac{\phi_{t+1}^i}{\phi_t^i}$. Since, the focus is on stationary monetary equilibrium, I drop the time subscripts from the variables.

Definition 1. Given α , a stationary monetary equilibrium for the two-country economy with a black market for currencies is a list of quantities traded in SGM_i , $i \in \{A, B\}$: $\{(q_A, d^A, d^B), (\tilde{q}^A, \tilde{d}^A, \tilde{d}^B)\}$ and $\{(q_{B|bm}, d_{bm}^A, d_{bm}^B), (\tilde{q}_{B|bm}, \tilde{d}_{bm}^A, \tilde{d}_{bm}^B)\}$, end of period real balances $\{z_A \equiv (z_A^A, z_A^B), z_B \equiv (z_B^A, z_B^B)\}$ of country A buyers and B respectively, post-black market trade portfolios for country B buyers, $\{(\bar{z}_B^A, \bar{z}_B^B), (\bar{z}_{B*}^A, \bar{z}_{B*}^B)\}$ and the black market terms of trade between money_A and money_B, ε , such that

1. (q_A, d^A, d^B) and $(\tilde{q}^A, \tilde{d}^A, \tilde{d}^B)$ solves country A buyer's bargaining problem when he is matched with a country A seller in SGM_A and with a country B seller in SGM_B respectively.
2. $(q_{B|bm}, d_{bm}^A, d_{bm}^B)$ and $(\tilde{q}_{B|bm}, \tilde{d}_{bm}^A, \tilde{d}_{bm}^B)$ solves country B buyer's bargaining problem when he is matched with a country B seller in SGM_B and with a country A seller in SGM_A respectively.
3. (z_A^A, z_A^B) and (z_B^A, z_B^B) solves the portfolio problem in the second subperiod for a country A buyer and country B respectively.
4. Taking ε as given $\{(\bar{z}_B^A, \bar{z}_B^B), (\bar{z}_{B*}^A, \bar{z}_{B*}^B)\}$ solves the country B buyer's black market portfolio readjustment problem.
5. ε clears the black market: $\delta \bar{z}_B^A + (1 - \delta) \bar{z}_{B*}^A = z_B^A$ and $\delta \bar{z}_B^B + (1 - \delta) \bar{z}_{B*}^B = z_B^B$
6. CM money market clears: $z_A^A + z_B^A = \phi_t^A M_t^A$ and $z_A^B + z_B^B = \phi_t^B M_t^B$

Now, I discuss some aspects of the equilibrium pertaining to the black market terms of trade and portfolio choice by agents from both countries.

Proposition 1. *The portfolio choice of a country A buyer is unaffected by the presence of black market in country B and (z_A^A, z_A^B) , its optimal end of period portfolio in the stationary monetary equilibrium solves*

$$u'(z_A^A) = 1 + \frac{\iota_A}{1 - \delta} \text{ and } u'(z_A^B) = 1 + \frac{\iota_B}{\delta}$$

This is expected since agents (i.e. buyers or sellers) from country A or sellers from country B do not participate in the black market. Also, the terms of trade in SGM_i between country A buyer and country i seller ($i \in \{A, B\}$) depends only on the amount of $money_i$ a buyer from country A carries. It is important to note the implication of the above proposition. It implies $z_A^A, z_A^B < q^*$ at the stationary monetary equilibrium. As a result a buyer from country A would always consume less than the optimal level of consumption, q^* . When $z_A^A \geq q^*$ (or $z_A^B \geq q^*$), $u(q^*) - q^*$ is flat in z_A^A (or z_A^B) and first order conditions of (4.18) would imply $\iota_A = 0$ or $\iota_B = 0$. Therefore, it is not possible to have $z_A^A \geq q^*$ or $z_A^B \geq q^*$.

Proposition 2. *In the stationary monetary equilibrium, $\varepsilon = \frac{1 + \iota_B}{1 + \iota_A}$*

Proof. In appendix. □

This result is intuitive. If $\iota_B > \iota_A$, it is costlier to hold $money_B$ and ideally agents would want to hold less of $money_B$ real balance. However, due to the presence of the black market in future there will be an additional demand of $money_B$ real balance by country B buyers matched with local sellers. Therefore, country B buyers who have been matched with local sellers, in order to acquire the extra $money_B$ real balance will have to compensate other country B buyers not matched with local sellers for holding the low return money. As a result, in the black market, a unit real balance of $money_B$ trades for more than one unit

real balance of $money_A$. In terms, of black market premium, $\varepsilon^{-1} - 1 = (\iota_A - \iota_B)/(1 + \iota_B)$, a higher ι_B implies negative black market premium, i.e. as inflation rates in a country goes up, in the stationary monetary equilibrium it will witness a decline in black market premium. If $\iota_B > \iota_A$, then in the stationary monetary equilibrium the premium would be negative. The exact opposite happens when $\iota_B < \iota_A$: agents would want to hold less of $money_A$ real balance. However, country B buyers who have been matched with foreign sellers, and who want to acquire the extra $money_A$ real balance will have to compensate other country B buyers not matched with foreign sellers for holding the low return money. As a result a unit real balance of $money_A$ trades for more than one unit real balance of $money_B$. In this case the black market is flushed with $money_B$, while $money_A$ is scarce. Therefore, in the stationary monetary equilibrium, $money_A$ despite being a low return money, will fetch a positive premium. Finally, in the case of $\iota_A = \iota_B$ agents value both currency equally and in the black market real balance of $money_A$ will trade one-to-one for real balance of $money_B$. In this case the premium would be zero.

Connection to Covered Interest Parity Condition: The result presented in Proposition 2 bears a striking resemblance to the covered interest parity condition in international finance. This is reasonable because both are arbitrage condition. In international finance the CIP condition states that one cannot buy one country's asset that pays higher rate of interest and make a profit because exchange rates will adjust and all such profit making opportunities would be eroded away. Here instead of asset markets in two countries we have two currency markets in the same country where exchange rates could be different. One could, for all practical purpose, buy a currency in one of the foreign exchange markets (e.g. official) in Country B and sell it another foreign exchange market (black) and make a profit. However, at steady-state, the exchange rates in these two markets would adjust in such a way and align with the interest rates of the two currencies such that these arbitrage opportunities would be removed.

Before, I discuss the implications of optimal portfolio choice of a country B buyer who accesses the black market, let us define the following objects

$$G(\varepsilon_t) = \alpha(1 - \delta)\varepsilon_t^{-1}u'(\varepsilon_t q^*) + \alpha\delta - 1$$

$$H(\varepsilon_t) = \alpha\delta u'(\varepsilon_t q^*) + \alpha(1 - \delta)\varepsilon_t^{-1} - 1$$

$$\bar{\alpha} = \frac{\varepsilon_t}{(1 - \delta)u'(\varepsilon_t q^*) + \delta\varepsilon_t}$$

$$\hat{\alpha} = \frac{1}{\delta\varepsilon_t u'(\varepsilon_t q^*) + (1 - \delta)}$$

Details of end of period optimal portfolio choice for a country B buyer accessing the black market is provided in the Appendix. Here I discuss the implications of this portfolio choice regarding their consumption levels in foreign and domestic special goods market. The next proposition summarizes these implications.

Proposition 3. *At the stationary monetary equilibrium $1 + \iota_B = \varepsilon(1 + \iota_A)$*

1. *If $\iota_B > \iota_A$ and*

a. *$\iota_A \leq G(\varepsilon)$ with $\alpha > \bar{\alpha}$, then $\tilde{q}_{B|bm} = q^*$, $q_{B|bm} < q^*$.*

b. *$\iota_A > G(\varepsilon)$, then $\tilde{q}_{B|bm} < q^*$, $q_{B|bm} < q^*$.*

2. *If $\iota_B = \iota_A$, then $\tilde{q}_{B|bm} < q^*$, $q_{B|bm} < q^*$.*

3. *If $\iota_B < \iota_A$ and*

a. *$\iota_A \leq H(\varepsilon)$ with $\alpha > \hat{\alpha}$, then $\tilde{q}_{B|bm} < q^*$, $q_{B|bm} = q^*$.*

b. *$\iota_A > H(\varepsilon)$, then $\tilde{q}_{B|bm} < q^*$, $q_{B|bm} < q^*$.*

Case 1(a) of the Proposition 3 implies while $money_B$ may be costlier to hold, if country B buyers have sufficient access to the black market ($\alpha > \bar{\alpha}$) and if the cost of holding $money_A$

is not too high, then since a unit real balance of $money_B$ real balance trades for more than one unit of $money_A$ real balance, it is possible for the buyer to trade his holdings of $money_B$ to sufficiently increase his real balance of $money_A$ in the black market and consume the optimal level q^* when matched with a foreign seller. On the other hand, if ι_A is higher than a certain level ($> G(\varepsilon)$), then the rate at which a unit $money_B$ real balance trades for real balance of $money_A$ goes down and despite access to black market a buyer cannot increase his consumption to q^* in either special goods markets. In Case 2, if both currencies have equal cost, real balances of monies trade one to one in the black market and it presents no advantage like before and buyers consume below q^* . Case 3(a) implies that with sufficient access to the black market ($\alpha > \hat{\alpha}$), ι_A bounded below $H(\varepsilon)$ and a unit real balance of $money_A$ trading for more than a unit real balance of $money_B$ in the black market buyers can convert their entire $money_A$ into $money_B$ and increase their consumption to q^* when matched with a local seller. These results are in sharp contrast to the case when buyers do not access the black market. When country B buyers do not access the black market, their portfolio choice problem is given by (4.17). The optimal portfolios in that case always satisfy $u'(z_{tB}^A) = 1 + \frac{\iota_A}{\delta}$, $u'(z_{tB}^B) = 1 + \frac{\iota_B}{1-\delta}$ suggesting that $\tilde{q}_{B|bm} < q^*$, $q_{B|bm} < q^*$ always.

4.5.1 Welfare in the Presence of Black Market

This section concludes with a discussion of the model's welfare properties. Welfare in country $i \in \{A, B\}$ is defined as the steady-state sum of buyers' and sellers' utilities in country i , weighted by their respective measures in the first subperiod:

$$\mathcal{W}_i = V_i^B + V_i^S$$

Now, because sellers do not bring any real balances to the first subperiod and because their real payment is exactly equal to their amount of production through TIOLI offers from buyer,

sellers' utility in the first subperiod, $V_i^S = 0$. Therefore, welfare in country i , $\mathcal{W}_i = V_i^B$. Therefore, welfare of country B when buyers use the black market

$$\begin{aligned}\mathcal{W}_{B|bm} &= \alpha\delta[u(\tilde{q}_{B|bm}) - \tilde{d}_{bm}^A + \bar{z}_B^A + \bar{z}_B^B] + \alpha(1 - \delta)[u(q_{B|bm}) - d_{bm}^B + \bar{z}_{B^*}^A + \bar{z}_{B^*}^B] \\ &\quad + W_B^B(\mathbf{0})\end{aligned}$$

Welfare of country B when buyers do not use the black market

$$\begin{aligned}\mathcal{W}_{B|nbm} &= \delta[u(\tilde{q}_{B|nbm}) - \tilde{d}_{nbm}^A + z_{tB}^A + z_{tB}^B] + (1 - \delta)[u(q_{B|nbm}) - d_{nbm}^B + z_{tB}^A + z_{tB}^B] \\ &\quad + W_B^B(\mathbf{0})\end{aligned}\tag{4.19}$$

The existence of black market requires participation from both sides of the market: country B buyers matched with country A sellers as well as country B buyers matched with country B sellers. Therefore, existence of black market implies Lemma 7 is satisfied. If Lemma 7 is satisfied then the two conditions imply $\mathcal{W}_{B|bm} > \mathcal{W}_{B|nbm}$. Therefore, in this model, if the black market exists, it strictly raises welfare for country B 's residents. Since, the presence of black market doesn't affect country A buyers' portfolio choice and consumption patterns, \mathcal{W}_A is unchanged by the presence of black market in country B .

4.6 Asymmetric Penalty

So far, we have considered a model that only explains why black market premium goes down with rise in domestic inflation in relation to foreign inflation. While this maybe true for a subset of countries, the proposed model do not explain what is observed more often, i.e. premium rises with rise in inflation rate. In this section, I present an alternative confiscation rule where residents are penalized for participating in the black market by confiscating only

their foreign currency holding ($money_A$). This rule seems more realistic and is often in place in many countries. Thus, this is an asymmetric penalty. Proposition 4 summarizes the outcome in this case

Proposition 4. *The black market premium on foreign currency is rising in domestic inflation if the buyer's DM utility function is sufficiently elastic, else it is decreasing in domestic inflation.*

Proof. In appendix. □

High elasticity of the buyer's DM utility function implies greater risk aversion. When there is an increase in domestic inflation, to avoid inflation tax agents would find it beneficial to hold more of the foreign currency. As a result, the black market would be flush with foreign currency since in every agents' portfolio the proportion of foreign currency increases. As a result, the black market premium on foreign currency decreases. However, this increase in foreign currency holding comes with a risk. Now they are susceptible to a greater amount of consumption loss if confiscation happens which wouldn't be the case had they held more of domestic currency. Therefore, agents need to be compensated for the risk they are taking in holding more foreign currency. This has an increasing effect on the premium on foreign currency. The net effect would depend on agents' risk aversion. Higher the risk aversion, higher is the premium on black market. If the utility function is sufficiently elastic, i.e. if agents are sufficiently risk averse then the increasing effect on premium due to risk aversion outweighs the decreasing effect on premium due to increased supply of foreign currency.

4.7 Conclusion

In this paper I show that it is not necessarily true that there always a positive correlation between black market premium and the domestic inflation (relative to a foreign currency). As

shown through the empirical exercise, this correlation could be negative in some cases. When a country has gone through high periods of domestic inflation and inflation has stabilized at a high value. Agents of such a country will then build this information of high but stable domestic inflation in their portfolio decision. Since real balances in the portfolio of currencies are chosen according to their rates of return, agents would hold less of their wealth in domestic currency and more of it in the low inflation/ high return currency. Thus allowing for currency substitution (commonly known as dollarization), we could get a negative correlation between black market premium on foreign currency and domestic inflation if agents are not too risk averse. Unlike other papers where the black market is a free market that is a consequence of foreign exchange controls or those models where the black market is used to channel earnings from illegal production, the black market we present here is rather benign. It is an informal, unregulated market of currency exchange that operates in the absence of a formal market of currency exchange and increases welfare. One could extend this model by adding foreign exchange controls in the *CM* by adding a constraint that sets an upper bound to the real balance of $money_A$ a buyer from country B can hold. However, the result would be the same. If the optimal choice for real $money_A$ balance is less than the upper bound, the constraint is non-binding and it would be just like the model presented here. If it is binding, then buyers will hold $money_A$ balances up to the limit, but now the optimal choice of real balance of $money_B$ will be reduced considerably as well. Since premium rates are determined by relative excess supply of real balances of the two monies, it will have a decreasing (increasing) effect on the premium as inflation (rate of return) for $money_B$ goes up (down) in the stationary monetary equilibrium. The net effect on the premium would also be determined with agents' attitude towards risk as discussed in the penultimate section of this paper. This model is not without limitations though. Firstly, it assumes away many other uses of the black market - most importantly the features such as laundering money earned through illegal production. Secondly, this model studies behavior in the stationary monetary equilibrium. It would be also interesting to extend this model in order to study

behavior of the premium to an unanticipated inflation shock in the short run. Adding some shocks to the model would be a good start.

Chapter 5

Future Work

I intend to continue researching on the issues studied in my current and ongoing work. There are several directions in which I plan to go from here. In the first chapter, I discussed that the government introduces institutional barriers which I model as a tax. However, this is something I take as exogenously given. In reality this is a choice by the government in response to certain macroeconomic and monetary circumstances. Therefore, one important step would be to endogenize this ‘tax’ and try to understand what motivates governments to impose these barriers - is it arising out of some fiscal issues, or seignorage concerns? Or, is it a side effect of other features of the economy like capital controls? Furthermore, the currency regimes indicates certain payment patterns in international trade. This is something I intend to explore further and try to link it to Local Currency Pricing/ Producer Currency Pricing literature of International Finance.

While I have used indices to measure financial openness in my second chapter and it has given us some interesting results, such indices obfuscate many micro-details. I would like to extend this project by focusing on one single country and study its government circulars and central bank minutes in relation to changes in inflation and nominal exchange rate. That

would give a richer picture. Finally, for the paper on black market there are still many areas that are yet to be explored. I have explore only one slice of the black market and it is rather benign. More often black markets for currency exchange are used to convert or launder money earned through illegal production. Then there are countries where remittances from workers abroad are channeled through the black market and not through the banks which leads to seemingly aberrant behavior of the black market exchange rates. These are issues that I intend to explore further.

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Appendix A

Appendix for Chapter 2

Proof of Proposition 1

Proof: By replacing $d_i + (1 - t_j)d_j$ with q and using the fact $d_i \leq z_i^{(i)}$ and $d_j \leq z_j^{(i)}$, the bargaining problem between local buyer and local seller can be rewritten as

$$\begin{aligned} S^{(ii)}(z_1^{(i)}, z_2^{(i)}) &\equiv \max_q \{u(q) - q\} \\ \text{s.t. } q &\leq w_i \end{aligned}$$

Case (a): If $q^* \leq w_i$, then the solution is given by first order condition: $u'(q) - 1 = 0$ which will yield $q = q^*$ and $d_i + (1 - t_j)d_j = q^* \leq w_i$.

Case (b): If $q^* > w_i$, then note that since $u''(q) < 0$, we have $u'(q) - 1 > 0$ for all $q < q^*$. Therefore the objective function will be maximized iff $q = w_i$ and $d_i = z_i^{(i)}$, $d_j = z_j^{(i)}$.

Proof of concavity of $S^{(ii)}(z_1^{(i)}, z_2^{(i)})$

Proof:

$$\frac{\partial^2 S^{(ii)}(z_1^{(i)}, z_2^{(i)})}{\partial z_i^{(i)2}} = \begin{cases} 0, & \text{when } w_i \geq q^* \\ u''(w_i) < 0, & \text{otherwise} \end{cases}$$

$$\frac{\partial^2 S^{(ii)}(z_1^{(i)}, z_2^{(i)})}{\partial z_j^{(i)2}} = \begin{cases} 0, & \text{when } w_i \geq q^* \\ (1 - t_j)^2 u''(w_i) < 0, & \text{otherwise} \end{cases}$$

$$\frac{\partial^2 S^{(ii)}(z_1^{(i)}, z_2^{(i)})}{\partial z_i^{(i)} \partial z_j^{(i)}} = \frac{\partial^2 S^{(ii)}(z_1^{(i)}, z_2^{(i)})}{\partial z_2^{(i)} \partial z_1^{(i)}} = \begin{cases} 0, & \text{when } w_i \geq q^* \\ (1 - t_j) u''(w_i) < 0, & \text{otherwise} \end{cases}$$

Therefore $\frac{\partial^2 S^{(ii)}(z_1^{(i)}, z_2^{(i)})}{\partial z_i^{(i)2}} \leq 0$, $\frac{\partial^2 S^{(ii)}(z_1^{(i)}, z_2^{(i)})}{\partial z_j^{(i)2}} \leq 0$ and $\frac{\partial^2 S^{(ii)}(z_1^{(i)}, z_2^{(i)})}{\partial z_i^{(i)2}} \cdot \frac{\partial^2 S^{(ii)}(z_1^{(i)}, z_2^{(i)})}{\partial z_j^{(i)2}} - \left(\frac{\partial^2 S^{(ii)}(z_1^{(i)}, z_2^{(i)})}{\partial z_i^{(i)} \partial z_j^{(i)}} \right)^2 = 0$. This means $D^2 S^{(ii)}(z_1^{(i)}, z_2^{(i)})$ is n.s.d. implying $S^{(ii)}(z_1^{(i)}, z_2^{(i)})$ is concave.

Proof of Proposition 2

Proof: Suppose the buyer wants to buy an amount q^0 and pays $d_i^0 > 0, d_j^0 > 0$ for it, i.e. $q^0 = (1 - t_i)d_i^0 + d_j^0$. Then his surplus is $u(q^0) - q^0 + t_j d_j^0 - t_i d_i^0$. Now consider another payment option: $d_j^1 = q^0 = (1 - t_i)d_i^0 + d_j^0$. Under this option buyer's surplus = $u(q^0) - q^0 + t_j d_j^0 + t_j(1 - t_i)d_i^0 > u(q^0) - q^0 + t_j d_j^0 - t_i d_i^0$. Therefore, if the buyer can, then he must always pay the foreign seller in the seller's currency. The buyer should use his domestic currency only when he has exhausted his real balances held in foreign currency.

Case (a): Suppose the buyer uses only currency j . Then the problem becomes maximizing $u(d_j^{(ij)}) - (1 - t_j)d_j^{(ij)}$. The solution is given by $u'(d_j^{(ij)}) - (1 - t_j) = 0$, i.e. $d_j^{(ij)} = \bar{q}(t_j)$. Any

use of currency i will only decrease the surplus from $u(q(t_j)) - (1 - t_j)q(t_j)$. But this could be if and only if $z_j^{(i)} \geq \bar{q}(t_j)$.

Case (b): Now consider the case $z_j^{(i)} < \bar{q}(t_j)$. The surplus $u((1 - t_i)d_i^{(ij)} + d_j^{(ij)}) - (1 - t_j)d_j^{(ij)} - d_i^{(ij)}$ is increasing in $d_j^{(ij)}$ and decreasing in $d_i^{(ij)}$ as long as $z_j^{(i)} \geq d_j^{(ij)} \geq \underline{q}(t_i)$. So, the solution is given by $d_i^{(ij)} = 0$, $d_j^{(ij)} = z_j^{(i)}$ and $q^{(ij)} = z_j^{(i)}$.

Case (c): If $z_j^{(i)} < \underline{q}(t_i)$, then surplus $u((1 - t_i)d_i^{(ij)} + d_j^{(ij)}) - (1 - t_j)d_j^{(ij)} - d_i^{(ij)}$ is still increasing in $d_j^{(ij)}$ and in $d_i^{(ij)}$ only if $q \leq \underline{q}(t_i)$. So, $d_j^{(ij)} = z_j^{(i)}$ and if $\omega_i \geq \underline{q}(t_i)$, then $d_i^{(ij)} = \frac{\underline{q}(t_i) - z_j^{(i)}}{1 - t_i}$ which means $q = \underline{q}(t_i)$.

Case (d): If $\omega_i \geq \underline{q}(t_i)$, then $z_j^{(i)} < \underline{q}(t_i)$. So, this case is like Case (c) – the same reasoning holds, except that $d_i^{(ij)} = z_i^{(i)}$ and $q = \omega_i$.

Proof of concavity of $S^{(ij)}(z_1^{(i)}, z_2^{(i)})$

.

Proof:

$$\frac{\partial S^{(ij)}(z_1^{(i)}, z_2^{(i)})}{\partial z_i^{(i)}} = \begin{cases} 0, & \text{when } z_j^{(i)} \geq \bar{q}(t_j) \\ 0, & \text{when } z_j^{(i)} \in [\underline{q}(t_i), \bar{q}(t_j)) \\ 0, & \text{when } z_j^{(i)} < \underline{q}(t_i) \text{ and } \omega_i \geq \underline{q}(t_i) \\ (1 - t_i)u'(\omega_i) - 1 > 0, & \text{when } \omega_i < \underline{q}(t_i) \end{cases}$$

$$\frac{\partial S^{(ij)}(z_1^{(i)}, z_2^{(i)})}{\partial z_j^{(i)}} = \begin{cases} 0, & \text{when } z_j^{(i)} \geq \bar{q}(t_j) \\ u'(z_j^{(i)}) - (1 - t_j) > 0, & \text{when } z_j^{(i)} \in [\underline{q}(t_i), \bar{q}(t_j)) \\ \left(\frac{t_i}{1 - t_i} + t_j\right) > 0, & \text{when } z_j^{(i)} < \underline{q}(t_i) \text{ and } \omega_i \geq \underline{q}(t_i) \\ u'(\omega_i) - (1 - t_j) > 0, & \text{when } \omega_i < \underline{q}(t_i) \end{cases}$$

Therefore,

$$\frac{\partial^2 S^{(ij)}(z_1^{(i)}, z_2^{(i)})}{\partial z_i^{(i)} \partial z_j^{(i)}} = \frac{\partial^2 S^{(ij)}(z_1^{(i)}, z_2^{(i)})}{\partial z_j^{(i)} \partial z_i^{(i)}} = \begin{cases} 0, & \text{when } z_j^{(i)} \geq \bar{q}(t_j) \\ 0, & \text{when } z_j^{(i)} \in [\underline{q}(t_i), \bar{q}(t_j)) \\ 0, & \text{when } z_j^{(i)} < \underline{q}(t_i) \text{ and } \omega_i \geq \underline{q}(t_i) \\ (1 - t_i)u''(\omega_i) < 0, & \text{when } \omega_i < \underline{q}(t_i) \end{cases}$$

$$\frac{\partial^2 S^{(ij)}(z_1^{(i)}, z_2^{(i)})}{\partial z_i^{(i)2}} = \begin{cases} 0, & \text{when } z_j^{(i)} \geq \bar{q}(t_j) \\ 0, & \text{when } z_j^{(i)} \in [\underline{q}(t_i), \bar{q}(t_j)) \\ 0, & \text{when } z_j^{(i)} < \underline{q}(t_i) \text{ and } \omega_i \geq \underline{q}(t_i) \\ (1 - t_i)^2 u''(\omega_i) < 0, & \text{when } \omega_i < \underline{q}(t_i) \end{cases}$$

$$\frac{\partial^2 S^{(ij)}(z_1^{(i)}, z_2^{(i)})}{\partial z_j^{(i)2}} = \begin{cases} 0, & \text{when } z_j^{(i)} \geq \bar{q}(t_j) \\ u''(z_j^{(i)}) < 0, & \text{when } z_j^{(i)} \in [\underline{q}(t_i), \bar{q}(t_j)) \\ 0, & \text{when } z_j^{(i)} < \underline{q}(t_i) \text{ and } \omega_i \geq \underline{q}(t_i) \\ u''(\omega_i) < 0, & \text{when } \omega_i < \underline{q}(t_i) \end{cases}$$

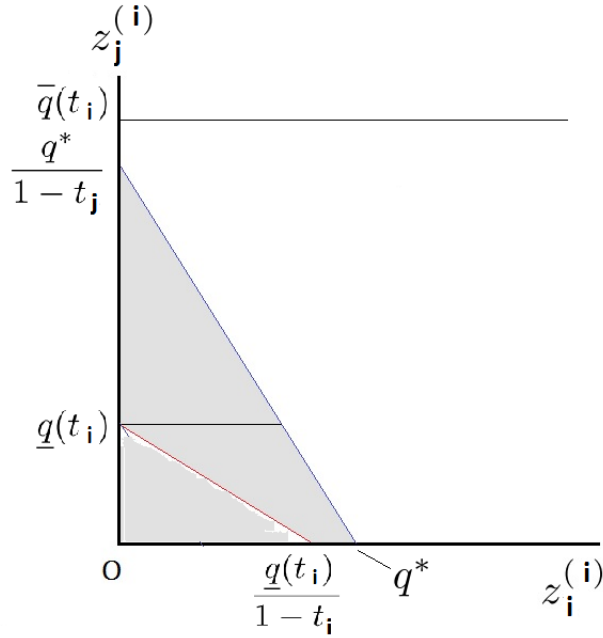
So, $\frac{\partial^2 S^{(ij)}(z_1^{(i)}, z_2^{(i)})}{\partial z_i^{(i)2}} \leq 0$, $\frac{\partial^2 S^{(ij)}(z_1^{(i)}, z_2^{(i)})}{\partial z_j^{(i)2}} \leq 0$ and

$$\frac{\partial S^{(ij)}(z_1^{(i)}, z_2^{(i)})}{\partial z_i^{(i)2}} \frac{\partial S^{(ij)}(z_1^{(i)}, z_2^{(i)})}{\partial z_j^{(i)2}} - \frac{\partial^2 S^{(ij)}(z_1^{(i)}, z_2^{(i)})}{\partial z_i^{(i)} \partial z_j^{(i)}} \frac{\partial^2 S^{(ij)}(z_1^{(i)}, z_2^{(i)})}{\partial z_j^{(i)} \partial z_i^{(i)}} = 0.$$

Therefore, $D^2 S^{(ij)}(z_1^{(i)}, z_2^{(i)})$ is n.s.d and $S^{(ij)}(z_1^{(i)}, z_2^{(i)})$ is jointly concave in $z_1^{(i)}$ and $z_2^{(i)}$.

Proof: There exists a unique solution to the Country i ($i = 1, 2$) buyer's portfolio choice problem

Since, the relative risk aversion of $u(q)$ is $< 1, -u''(q)q/u'(q) < 1 \implies u''(q)q + u'(q) > 1 \implies \frac{d}{dq}u'(q)q > 1$. This implies that for $i = 1, 2$ and $j = 1, 2$ with $i \neq j$ we have the following relationship: $\frac{\underline{q}(t_i)}{1 - t_i} < q^* < (1 - t_j)\bar{q}(t_j)$. The area $z_i^{(i)} + (1 - t_j)z_j^{(i)} \leq q^*$ and $(1 - t_i)z_i^{(i)} + z_j^{(i)} \leq \underline{q}(t_i)$ looks like the following and it is convex and compact:



Country i Buyer's Portfolio Choice Problem

$$\max_{z_1^{(i)}, z_j^{(i)} \geq 0} \alpha \lambda_1 S^{ii}(z_i^{(i)}, z_j^{(i)}) + (1 - \alpha) \lambda_j S^{ij}(z_i^{(i)}, z_j^{(i)}) - \iota_i z_i^{(i)} - (\iota_j + t_j) z_j^{(i)} \quad (\text{A.1})$$

First order conditions:

$$\alpha\lambda_i \frac{\partial S^{ii}(z_i^{(i)}, z_j^{(i)})}{\partial z_i^{(i)}} + (1-\alpha)\lambda_j \frac{\partial S^{ij}(z_i^{(i)}, z_j^{(i)})}{\partial z_i^{(i)}} - \iota_i \leq 0, \quad "=" \text{ if } z_i^{(i)} > 0 \quad (\text{A.2})$$

$$\alpha\lambda_i \frac{\partial S^{ii}(z_i^{(i)}, z_j^{(i)})}{\partial z_j^{(i)}} + (1-\alpha)\lambda_j \frac{\partial S^{ij}(z_i^{(i)}, z_j^{(i)})}{\partial z_j^{(i)}} - (\iota_j + t_j) \leq 0, \quad "=" \text{ if } z_j^{(i)} > 0 \quad (\text{A.3})$$

Equilibrium such that $w_i \geq q^*$ and $z_j^{(i)} \geq \bar{q}(t_j)$

There exists no optimum $(z_i^{(i)}, z_j^{(i)})$ such that $w_i \geq q^$ and $z_j^{(i)} \geq \bar{q}(t_j)$.*

For any $(z_i^{(i)}, z_j^{(i)})$ in this region, the expressions for first-order conditions w.r.t $z_i^{(i)}$ and $z_j^{(i)}$ become $-\iota_i$ and $-(\iota_j + t_j)$ respectively. Since, $\iota_i > 0$ (or, $\iota_i \rightarrow 0+$) and $\iota_j > 0$ (or, $\iota_j \rightarrow 0+$), we have $-\iota_i < 0$ and $-(\iota_j + t_j) < 0$. Therefore, for any $(z_i^{(i)}, z_j^{(i)})$ in this region, the buyer could improve his objective function by reducing both $(z_i^{(i)})$ and $z_j^{(i)}$. Hence no equilibrium exists in this region.

Equilibrium such that $w_i \geq q^*$ and $\underline{q}(t_i) \leq z_j^{(i)} < \bar{q}(t_j)$

There exists no optimum $(z_i^{(i)}, z_j^{(i)})$ such that $w_i \geq q^$ and $\underline{q}(t_i) \leq z_j^{(i)} < \bar{q}(t_j)$.*

The first-order condition w.r.t. $z_i^{(i)}$ yields $-\iota_i < 0$. Therefore, for any $(z_i^{(i)}, z_j^{(i)})$ in this region, a higher value of the objective function can be reached by simply decreasing $z_i^{(i)}$. This continues till $z_i^{(i)} = 0$. First-order condition w.r.t. $z_j^{(i)}$ an rearranging the terms yields

$$u'(z_j^{(i)}) \leq (1 - t_j) + \frac{1}{(1-\alpha)\lambda_j}(\iota_j + t_j) \quad (\text{A.4})$$

If the above inequality holds strictly, then there cannot be an equilibrium $(z_i^{(i)}, z_j^{(i)})$ in the

region either. For equilibrium to exist, we must have

$$u'(z_j^{(i)}) = (1 - t_j) + \frac{1}{(1 - \alpha)\lambda_j}(\iota_j + t_j)$$

Since, at equilibrium $(z_i^{(i)}, z_j^{(i)})$, real balance of Currency i , $z_i^{(i)} = 0$ and $w_i \geq q^*$, we must have $z_j^{(i)} \geq \frac{q^*}{1 - t_j} > q^* > \underline{q}(t_i)$. This implies, $u'(z_j^{(i)}) \leq u'\left(\frac{q^*}{1 - t_j}\right)$ which when combined with equation (A.4) gives us (after rearranging):

$$\iota_j \leq -(1 - \alpha)\lambda_j \left[1 - u'\left(\frac{q^*}{1 - t_j}\right) \right] - \left[1 - (1 - \alpha)\lambda_j \right] t_j < 0$$

Since $u'(q^*) = 1$ and $u''(q) < 0$, $u'\left(\frac{q^*}{1 - t_j}\right) < 1$. Also $(1 - \alpha)\lambda_j < 1$. This implies that the expression on the right hand side of the above inequality is strictly less than 0. But, ι_j cannot be < 0 . Therefore, such an equilibrium doesn't exist.

Equilibrium such that $w_i \geq q^*$ and $z_j^{(i)} < \underline{q}(t_i)$ with $\omega_i \geq \underline{q}(t_i)$

The first-order condition w.r.t $z_i^{(i)}$ is strictly negative. This implies that no interior point from this region can be optimum. The only optimum point that can exist on the line $z_i^{(i)} + (1 - t_j)z_j^{(i)} = q^*$

Given the areas where there cannot be any optimum, the only portion on the $(z_i^{(i)}, z_j^{(i)})$ plane left is the area defined by $z_i^{(i)} + (1 - t_j)z_j^{(i)} \leq q^*$ and $z_i^{(i)}, z_j^{(i)} \geq 0$. This area is convex and concave. Also, note that the both the surplus functions are strictly concave with no flat sections over this portion of the $(z_i^{(i)}, z_j^{(i)})$ plane. Therefore, there exists a unique solution to the buyer's portfolio maximization problem.

Derivation of curves that define the portfolio composition for Country i buyer using CRRA utility function

For the purpose of tractability we use a CRRA utility function: $u(q) = \frac{q^{1-\gamma}}{1-\gamma}$ such that the marginal utility is $u'(q) = q^{-\gamma}$. We derive the necessary conditions under which single currency portfolios will be optimum. Since there is an unique optimum all the cases not covered would imply dual currency equilibrium. We start with the first order condition and derive the equations of the curves that serves as boundaries for the different portfolio composition.

Optimum with only Currency j such that $w_i \leq q^*$ and $z_j^{(i)} \in [\underline{q}(t_i), q^*/(1-t_j)]$

The first order conditions are:

$$(1-t_j)^{-\gamma} z_j^{(i)-\gamma} < 1 + \frac{\iota_i}{\alpha \lambda_i}$$

$$\alpha \lambda_i (1-t_j)^{1-\gamma} z_j^{(i)-\gamma} + (1-\alpha) \lambda_j z_j^{(i)-\gamma} = \iota_j + t_j + (1-t_j)[\alpha \lambda_i + (1-\alpha) \lambda_j]$$

$$\text{which yields } (1-t_j)^{-\gamma} z_j^{(i)-\gamma} = (1-t_j)^{-\gamma} \frac{\iota_j + t_j + (1-t_j)[\alpha \lambda_i + (1-\alpha) \lambda_j]}{\alpha \lambda_i (1-t_j)^{1-\gamma} + (1-\alpha) \lambda_j} < 1 + \frac{\iota_i}{\alpha \lambda_i}.$$

Hence,

$$\iota_j < \left[1 - t_j + \frac{(1-\alpha) \lambda_j (1-t_j)^{-\gamma}}{\alpha \lambda_i} \right] \iota_i + (1-\alpha) \lambda_j [(1-t_j)^\gamma - (1-t_j)] - t_j$$

$$\text{Also, } z_j^{(i)} \geq \underline{q}(t_i) \implies z_j^{(i)-\gamma} \geq \frac{1}{1-t_i}. \text{ So, } \frac{\iota_j + t_j + (1-t_j)[\alpha \lambda_i + (1-\alpha) \lambda_j]}{\alpha \lambda_i (1-t_j)^{1-\gamma} + (1-\alpha) \lambda_j} \geq \frac{1}{1-t_i}.$$

Hence,

$$\iota_j \leq \alpha \lambda_i (1-t_j) \left[\frac{1}{(1-t_j)^\gamma (1-t_i)} - 1 \right] + (1-\alpha) \lambda_j \left[\frac{1}{1-t_i} - (1-t_j) \right] - t_j$$

$$\text{Also, } z_j^{(i)} \geq q^*/(t_j) \implies z_j^{(i)-\gamma} > \frac{q^{*- \gamma}}{(1-t_j)^{-\gamma}} = (1-t_j)^\gamma. \text{ Plugging in the expression for}$$

$z_j^{(i)-\gamma}$ and simplifying we get

$$\iota_j > (1 - \alpha)\lambda_j(1 - t_j)^\gamma[1 - (1 - t_j)^{(1-\gamma)}] - t_j$$

The above condition will always hold since $(1 - \alpha)\lambda_j(1 - t_j)^\gamma[1 - (1 - t_j)^{(1-\gamma)}] - t_j$ is decreasing in t_j and it is 0 at $t_j = 0$. Since ι_j is above Friedman rule, this will always hold.

Optimum with only Currency j such that $w_i < q^*$ and $\omega_i < \underline{q}(t_i)$

First order conditions:

$$[\alpha\lambda_i(1 - t_j)^{-\gamma} + (1 - \alpha)\lambda_j(1 - t_i)]z_j^{(i)-\gamma} = \iota_i + \alpha\lambda_i + (1 - \alpha)\lambda_j$$

$$[\alpha\lambda_i(1 - t_j)^{1-\gamma} + (1 - \alpha)\lambda_j]z_j^{(i)-\gamma} < \iota_j + t_j + (1 - t_j)[\alpha\lambda_i + (1 - \alpha)\lambda_j]$$

which gives us $z_j^{(i)-\gamma} = \frac{\iota_j + t_j + (1 - t_j)[\alpha\lambda_i + (1 - \alpha)\lambda_j]}{\alpha\lambda_i(1 - t_j)^{1-\gamma} + (1 - \alpha)\lambda_j}$. Plugging this into the first F.O.C. we get,

$$\iota_j < \frac{\alpha\lambda_i(1 - t_j)^{-\gamma} + (1 - \alpha)\lambda_j}{\alpha\lambda_i(1 - t_j)^{-\gamma} + (1 - \alpha)\lambda_j(1 - t_i)} + \left[\frac{\alpha\lambda_i(1 - t_j)^{-\gamma} + (1 - \alpha)\lambda_j}{\alpha\lambda_i(1 - t_j)^{-\gamma} + (1 - \alpha)\lambda_j(1 - t_i)} - (1 - t_j) \right] [\alpha\lambda_i + (1 - \alpha)\lambda_j] - t_j$$

Since, $\omega_i = z_j^{(i)} < \underline{q}(t_i) \implies u'(\omega_i) > \underline{q}(t_i) \implies z_j^{(i)-\gamma} > \frac{1}{1 - t_i}$. Now plugging in the expression for $z_j^{(i)-\gamma}$ and simplifying, we get

$$\iota_j > \alpha\lambda_i(1 - t_j) \left[\frac{1}{(1 - t_j)^\gamma(1 - t_i)} - 1 \right] + (1 - \alpha)\lambda_j \left[\frac{1}{1 - t_i} - (1 - t_j) \right] - t_j$$

Optimum with only Currency i such that $w_i < q^*$, $\omega_i \geq \underline{q}(t_i)$ and $z_j^{(i)} < \underline{q}(t_i)$

First order conditions:

$$u'(w_i) = 1 + \frac{\iota_i}{\alpha\lambda_i}$$

$$\alpha\lambda_i(1-t_j)u'(w_i) - \alpha\lambda_i(1-t_j) + (1-\alpha)\lambda_j\left[\frac{1}{1-t_i} - (1-t_j)\right] < \iota_j + t_j$$

The second first order condition yields:

$$\iota_j > (1-t_j)\iota_i + (1-\alpha)\lambda_j\left[\frac{1}{1-t_i} - (1-t_j)\right] - t_j$$

Also, $z_j^{(i)-\gamma} = 1 + \frac{\iota_i}{\alpha\lambda_i}$, but $(1-t_i)^{-\gamma}z_j^{(i)-\gamma} \leq \frac{1}{1-t_i} \leq z_j^{(i)-\gamma} \leq \frac{1}{(1-t_i)^{1-\gamma}}$. Therefore,
 $1 + \frac{\iota_i}{\alpha\lambda_i} \leq \frac{1}{(1-t_i)^{1-\gamma}}$ which gives us:

$$\iota_i \leq \alpha\lambda_i\left[\frac{1}{(1-t_i)^{1-\gamma}} - 1\right]$$

Optimum with only Currency i such that $w_i < q^*$, $\omega_i < \underline{q}(t_i)$

First order conditions:

$$[\alpha\lambda_i + (1-\alpha)\lambda_j(1-t_i)^{1-\gamma}]z_i^{(i)-\gamma} = \iota_i + \alpha\lambda_i + (1-\alpha)\lambda_j$$

$$[\alpha\lambda_i(1-t_j) + (1-\alpha)\lambda_j(1-t_i)^{-\gamma}]z_i^{(i)-\gamma} < \iota_j + t_j + (1-t_j)[\alpha\lambda_i + (1-\alpha)\lambda_j]$$

Solving the first order conditions we get, $z_i^{(i)-\gamma} = \frac{\iota_i + \alpha\lambda_i + (1-\alpha)\lambda_j}{\alpha\lambda_i + (1-\alpha)\lambda_j(1-t_i)^{1-\gamma}}$ Plugging this back in the second first order condition and simplifying, we get:

$$\iota_j > \frac{\alpha\lambda_i(1-t_j) + (1-\alpha)\lambda_j(1-t_i)^{-\gamma}}{\alpha\lambda_i + (1-\alpha)\lambda_j(1-t_i)^{1-\gamma}} + \left[\frac{\alpha\lambda_i(1-t_j) + (1-\alpha)\lambda_j(1-t_i)^{-\gamma}}{\alpha\lambda_i + (1-\alpha)\lambda_j(1-t_i)^{1-\gamma}} - (1-t_j)\right][\alpha\lambda_i + (1-\alpha)\lambda_j] - t_j$$

Since $\omega_i = (1-t_i)z_i^{(i)} < \underline{q}(t_i) \implies z_i^{(i)} > \frac{\underline{q}(t_i)}{1-t_i}$. Now, $z_i^{(i)-\gamma} > \frac{\underline{q}(t_i)^{-\gamma}}{(1-t_i)^{-\gamma}} = \frac{1}{(1-t_i)^{1-\gamma}}$.

Substituting the expression for $z_i^{(i)-\gamma}$ and simplifying we get:

$$\iota_i > \alpha \lambda_i \left[\frac{1}{(1 - t_i)^{1-\gamma}} - 1 \right]$$

Appendix B

Appendix for Chapter 4

Selection criteria for full sample

Lag	mBIC	mAIC	mQIC
1	-111.842	-17.865	-55.731
2	-75.445	-12.793	-38.037
3	-45.655	-12.329	-24.952

Table B.1: Selection criteria for full sample

Selection criteria for subset

Lag	mBIC	mAIC	mQIC
1	-88.292	-26.822	-51.294
2	-59.401	-18.421	-34.736
3	-31.183	-10.693	-18.850

Table B.2: Selection criteria for Iran, India, Nepal, Pakistan, Sri Lanka and Venezuela

Eigenvalue stability condition in panel VAR for subset

Real	Imaginary	Modulus
0.801	-0.069	0.804
0.801	0.069	0.804
0.548	0.000	0.548

Table B.3: Eigenvalue stability condition in panel VAR with Iran, India, Nepal, Pakistan, Sri Lanka and Venezuela

Proof of Lemma 1

Lemma 1.

Proof.

$$\begin{aligned} & \max_{q_t, d_t^i} \{u(q_t) - d_t^i\} \\ & \text{s.t. } q_t = d_t^i \end{aligned}$$

The above problem can be rewritten as: $\max_{q_t} \{u(q_t) - q_t\}$ which yields $q_t = q^*$ and therefore $d_t^i = q^*$. Note that for $q_t < q^*$, $u(q_t) - q_t$ is strictly increasing in q_t . Therefore, if $z_t^i < q^*$, the optimal solution would entail $q_t = z_t^i$ and $d_t^i = z_t^i$. \square

Proof of Lemma 2

Lemma 2.

Proof. Using linearity of $W_B^B(\cdot)$, the objective function can be rewritten as $u(\tilde{q}_{B|bm}) + \bar{z}_B^A + \bar{z}_B^B - \tilde{d}^A + W_B^B(\mathbf{0})$. Dropping the term $W_B^B(\mathbf{0})$ and using $\tilde{d}^A = \tilde{q}_{B|bm}$ (from Lemma 1), the

problem simplifies to maximization of $u(\tilde{q}_{B|bm}) - \tilde{q}_{B|bm} + \bar{z}_B^A + \bar{z}_B^B$, with respect to \bar{z}_B^A and \bar{z}_B^B subject to the budget constraint. From the budget constraint: $\bar{z}_B^B = \varepsilon_t^{-1} z_{tB}^A + z_{tB}^B - \varepsilon_t^{-1} \bar{z}_B^A$. We plug this back in the objective function and a country B buyer's portfolio readjustment, when he is matched with a foreign seller, becomes:

$$\max_{\bar{z}_B^A} u(\tilde{q}_{B|bm}) - \tilde{q}_{B|bm} + \varepsilon_t^{-1} z_{tB}^A + z_{tB}^B + (1 - \varepsilon_t^{-1}) \bar{z}_B^A$$

When $1 - \varepsilon_t^{-1} > 0$, i.e. $\varepsilon_t > 1$: there would be two cases

Case 1: If $z_{tB}^A + \varepsilon_t z_{tB}^B < q^*$, then it must be $\bar{z}_B^A < q^*$. When $\bar{z}_B^A < q^*$, then the objective function is strictly increasing in \bar{z}_B^A . Therefore, the solution is $\bar{z}_B^A = z_{tB}^A + \varepsilon_t z_{tB}^B$ and $\bar{z}_B^B = 0$.

Case 2: If $z_{tB}^A + \varepsilon_t z_{tB}^B \geq q^*$. From Lemma 1, $\tilde{q}_{B|bm}$ is increasing in \bar{z}_B^A as long as $\bar{z}_B^A < q^*$. And, for $\bar{z}_B^A \geq q^*$, $\tilde{q}_{B|bm} = q^*$. So $u(\tilde{q}_{B|bm}) - \tilde{q}_{B|bm}$ is weakly increasing in \bar{z}_B^A , while $(1 - \varepsilon_t^{-1}) \bar{z}_B^A$ is strictly increasing in \bar{z}_B^A . Therefore, it is optimal to convert entire holdings of $money_B$ to $money_A$. Therefore, the solution is $\bar{z}_B^A = z_{tB}^A + \varepsilon_t z_{tB}^B$ and $\bar{z}_B^B = 0$.

When $1 - \varepsilon_t^{-1} = 0$, i.e. $\varepsilon_t = 1$: the objective function becomes $u(\tilde{q}_{B|bm}) - \tilde{q}_{B|bm} + z_{tB}^A + z_{tB}^B$.

There would be two cases:

Case 1: If $z_{tB}^A + z_{tB}^B < q^*$, it must be always $\bar{z}_B^A < q^*$. When $\bar{z}_B^A < q^*$, then using Lemma 1, $u(\tilde{q}_{B|bm}) - \tilde{q}_{B|bm}$ is strictly increasing in \bar{z}_B^A . Therefore, the optimal solution will entail $\bar{z}_B^A = z_{tB}^A + z_{tB}^B$ and $\bar{z}_B^B = 0$.

Case 2: If $z_{tB}^A + z_{tB}^B \geq q^*$, then the objective function, $u(\tilde{q}_{B|bm}) - \tilde{q}_{B|bm} + z_{tB}^A + z_{tB}^B$ is increasing in \bar{z}_B^A only until $\bar{z}_B^A \rightarrow q^* -$. For $\bar{z}_B^A \geq q^*$ it becomes flat – so the buyer is indifferent between increasing or not increasing his real balance of $money_A$ beyond q^* . Therefore, the solution is $\bar{z}_B^A \in [q^*, z_{tB}^A + z_{tB}^B]$ and $\bar{z}_B^B = z_{tB}^B + z_{tB}^A - \bar{z}_B^A$.

When $1 - \varepsilon_t^{-1} < 0$, i.e. $\varepsilon_t < 1$: The $(1 - \varepsilon_t^{-1}) \bar{z}_B^A$ part of the objective function is always decreasing in \bar{z}_B^A while $u(\tilde{q}_{B|bm}) - \tilde{q}_{B|bm}$ is increasing in \bar{z}_B^A as long as $\bar{z}_B^A < q^*$. In the case

of $\varepsilon_t < 1$ we can never have $\bar{z}_B^A = q^*$ since that would mean $\tilde{q}_{B|bm} = q^*$ and derivative of the objective function with respect to \bar{z}_B^A would be $1 - \varepsilon_t^{-1} < 0$. So, the optimal solution entails $\bar{z}_B^A < q^*$. In the set $0 \leq \bar{z}_B^A < q^*$, after plugging in values given by Lemma 4 the derivative of the objective function w.r.t \bar{z}_B^A is

$$u'(\bar{z}_B^A) - \varepsilon_t^{-1}$$

Now, at $\bar{z}_B^A = 0$, we have $u'(\bar{z}_B^A) \rightarrow \infty$. Therefore, we will have an interior solution for \bar{z}_B^A . First order conditions for interior solution imply $u'(\bar{z}_B^A) - \varepsilon_t^{-1} = 0$ or $u'(\bar{z}_B^A) = \varepsilon_t^{-1}$. Define $\bar{\chi}$ such that $u'(\bar{\chi}) = \varepsilon_t^{-1}$. Then, there are two possibilities:

Case 1: If $z_{tB}^A + \varepsilon_t z_{tB}^B < \bar{\chi}$, then $u'(\bar{z}_B^A) - \varepsilon_t^{-1}$ the derivative of the objective function w.r.t. \bar{z}_B^A is strictly positive. So, $\bar{z}_B^A = z_{tB}^A + \varepsilon_t z_{tB}^B$ and $\bar{z}_B^B = 0$.

Case 2: If $z_{tB}^A + \varepsilon_t z_{tB}^B \geq \bar{\chi}$, then as discussed above $\bar{z}_B^A = \bar{\chi}$ and $\bar{z}_B^B = z_{tB}^B + \varepsilon_t^{-1}(z_{tB}^A - \bar{\chi})$. \square

Proof of Lemma 3

Lemma 3.

Proof. Using linearity of $W_B^B(\cdot)$, the objective function can be rewritten as $u(q_{B|bm}) + \bar{z}_{B^*}^A + \bar{z}_{B^*}^B - d^B + W_B^B(\mathbf{0})$. Dropping the term $W_B^B(\mathbf{0})$ and using $d^B = q_{B|bm}$ (from Lemma 1), the problem simplifies to maximization of $u(q_{B|bm}) - q_{B|bm} + \bar{z}_{B^*}^A + \bar{z}_{B^*}^B$, with respect to $\bar{z}_{B^*}^A$ and $\bar{z}_{B^*}^B$ subject to the budget constraint. From the budget constraint: $\bar{z}_{B^*}^A = z_{tB}^A + \varepsilon_t z_{tB}^B - \varepsilon_t \bar{z}_{B^*}^B$. We plug this back in the objective function and a country B buyer's portfolio readjustment, when he is matched with a local seller, becomes:

$$\max_{\bar{z}_{B^*}^B} u(q_{B|bm}) - q_{B|bm} + z_{tB}^A + \varepsilon_t z_{tB}^B + (1 - \varepsilon_t) \bar{z}_{B^*}^B$$

When $1 - \varepsilon_t < 0$, i.e. $\varepsilon_t > 1$ We claim that $\bar{z}_{B^*}^B$ cannot be $\geq q^*$ because for $\bar{z}_{B^*}^B \geq q^*$, using Lemma 1, the derivative of the objective function with respect to $\bar{z}_{B^*}^B$ is strictly negative. So, optimal $\bar{z}_{B^*}^B < q^*$. In this range, the derivative of the objective function is:

$$u'(\bar{z}_{B^*}^B) - \varepsilon_t$$

At $\bar{z}_{B^*}^B = 0$ we have $u'(\bar{z}_{B^*}^B) \rightarrow \infty$. Therefore, there exists an interior solution. First order condition for interior solution gives us $u'(\bar{z}_{B^*}^B) = \varepsilon_t$. Define $\bar{\psi}$ such that $u'(\bar{\psi}) = \varepsilon_t$. Then,

Case 1: If $z_{tB}^A + \varepsilon_t z_{tB}^B \geq \varepsilon_t \bar{\psi}$, then $\bar{z}_{B^*}^A = z_{tB}^A + \varepsilon_t(z_{tB}^B - \bar{\psi})$ and $\bar{z}_{B^*}^B = \bar{\psi}$.

Case 2: If $z_{tB}^A + \varepsilon_t z_{tB}^B < \varepsilon_t \bar{\psi}$, then it must be $\bar{z}_{B^*}^B < \bar{\psi}$. In this range the objective function is strictly increasing in $\bar{z}_{B^*}^B$. therefore, optimal portfolio: $\bar{z}_{B^*}^A = 0$ and $\bar{z}_{B^*}^B = \varepsilon_t^{-1} z_{tB}^A + z_{tB}^B$.

When $1 - \varepsilon_t = 0$, i.e. $\varepsilon_t = 1$: the objective function becomes $u(q_{B|bm}) - q_{B|bm} + z_{tB}^A + z_{tB}^B$.

There would be two cases:

Case 1: If $z_{tB}^A + z_{tB}^B < q^*$, it must be always $\bar{z}_{B^*}^B < q^*$. When $\bar{z}_{B^*}^B < q^*$, then using Lemma 1, $u(q_{B|bm}) - q_{B|bm}$ is strictly increasing in $\bar{z}_{B^*}^B$. Therefore, the optimal solution will entail $\bar{z}_{B^*}^A = 0$ and $\bar{z}_{B^*}^B = z_{tB}^A + z_{tB}^B$.

Case 2: If $z_{tB}^A + z_{tB}^B \geq q^*$, then the objective function, $u(q_{B|bm}) - q_{B|bm} + z_{tB}^A + z_{tB}^B$ is increasing in $\bar{z}_{B^*}^B$ as long as $\bar{z}_{B^*}^B < q^*$. For $\bar{z}_{B^*}^B \geq q^*$ it becomes flat – so the buyer is indifferent between increasing or not increasing his real balance of $money_B$ beyond q^* . Therefore, the solution is $\bar{z}_{B^*}^A = z_{tB}^B + z_{tB}^A - \bar{z}_{B^*}^B$ and $\bar{z}_{B^*}^B \in [q^*, z_{tB}^A + z_{tB}^B]$.

When $1 - \varepsilon_t > 0$, i.e. $\varepsilon_t < 1$ there would be two cases

Case 1: If $z_{tB}^A + \varepsilon_t z_{tB}^B < q^*$, then it must be $\bar{z}_{B^*}^B < q^*$. When $\bar{z}_{B^*}^B < q^*$, then using Lemma 1, the objective function is $u(\bar{z}_{B^*}^B) - \bar{z}_{B^*}^B + z_{tB}^A + \varepsilon_t z_{tB}^B + (1 - \varepsilon_t) \bar{z}_{B^*}^B$ which strictly increasing in $\bar{z}_{B^*}^B$. Therefore, the solution is $\bar{z}_{B^*}^A = 0$ and $\bar{z}_{B^*}^B = \varepsilon_t^{-1} z_{tB}^A + z_{tB}^B$.

Case 2: If $z_{tB}^A + \varepsilon_t z_{tB}^B \geq \varepsilon_t q^*$. From Lemma 1, $q_{B|bm}$ is increasing in $\bar{z}_{B^*}^B$ as long as $\bar{z}_{B^*}^B < q^*$.

And, for $\bar{z}_{B^*}^B \geq q^*$, $q_{B|bm} = q^*$. So $u(q_{B|bm}) - q_{B|bm}$ is weakly increasing in $\bar{z}_{B^*}^B$, while $(1 - \varepsilon_t)\bar{z}_{B^*}^B$ is strictly increasing in $\bar{z}_{B^*}^B$. Therefore, it is optimal to convert entire holdings of $money_A$ to $money_B$. Therefore, the solution is $\bar{z}_{B^*}^A = 0$ and $\bar{z}_{B^*}^B = \varepsilon_t^{-1}z_{tB}^A + z_{tB}^B$. \square

Proof of Lemma 7

Lemma 7.

Proof. The payoff from going to the black market for a country B buyer matched with foreign (country A seller) = $\alpha[u(\tilde{q}_{B|bm}) + W_B^B(\bar{z}_B^A - \tilde{d}_{bm}^A, \bar{z}_B^B)] + (1 - \alpha)W_B^B(\mathbf{0}) = \alpha[u(\tilde{q}_{B|bm}) - \tilde{d}_{bm}^A + \bar{z}_B^A + \bar{z}_B^B] + W_B^B(\mathbf{0})$

The payoff from not going to the black market for a country B buyer matched with foreign (country A seller) = $[u(\tilde{q}_{B|nbm}) + W_B^B(z_{tB}^A - \tilde{d}_{nbm}^A, z_{tB}^B)] = [u(\tilde{q}_{B|nbm}) - \tilde{d}_{nbm}^A + z_{tB}^A + z_{tB}^B] + W_B^B(\mathbf{0})$

Therefore, a country B buyer matched with a foreign seller will access the black market if and only if: $\alpha[u(\tilde{q}_{B|bm}) - \tilde{d}_{bm}^A + \bar{z}_B^A + \bar{z}_B^B] > [u(\tilde{q}_{B|nbm}) - \tilde{d}_{nbm}^A + z_{tB}^A + z_{tB}^B]$

The payoff from going to the black market for a country B buyer matched with local (country B seller) = $\alpha[u(q_{B|bm}) + W_B^B(\bar{z}_{B^*}^A - d_{bm}^A, \bar{z}_{B^*}^B)] + (1 - \alpha)W_B^B(\mathbf{0}) = \alpha[u(q_{B|bm}) - d_{bm}^A + \bar{z}_{B^*}^A + \bar{z}_{B^*}^B] + W_B^B(\mathbf{0})$

The payoff from not going to the black market for a country B buyer matched with local (country B seller) = $[u(q_{B|nbm}) + W_B^B(z_{tB}^A - d_{nbm}^A, z_{tB}^B)] = [u(q_{B|nbm}) - d_{nbm}^A + z_{tB}^A + z_{tB}^B] + W_B^B(\mathbf{0})$

Therefore, a country B buyer matched with a foreign seller will access the black market if and only if: $\alpha[u(q_{B|bm}) - d_{bm}^A + \bar{z}_{B^*}^A + \bar{z}_{B^*}^B] > [u(q_{B|nbm}) - d_{nbm}^A + z_{tB}^A + z_{tB}^B]$ \square

Proof of Proposition 2 and 3

Proposition 2 and 3.

Proof. Using Lemma 1, country B buyer's end of period (CM) portfolio choice problem (described in 4.16) can be rewritten as:

$$\begin{aligned} \max_{z_{tB}^A, z_{tB}^B} & [\alpha\delta\{u(\tilde{q}_{B|bm}) + \bar{z}_B^A - \tilde{q}_{B|bm} + \bar{z}_B^B\} \\ & + \alpha(1-\delta)\{u(q_{B|bm}) + \bar{z}_{B^*}^A + \bar{z}_{B^*}^B - q_{B|bm}\} \\ & - \{(1+\iota_A)z_{tB}^A + (1+\iota_B)z_{tB}^B\}] \end{aligned}$$

The first order conditions with respect to z_{tB}^A :

$$\begin{aligned} & \alpha\delta \left\{ u'(\tilde{q}_{B|bm}) \frac{\partial \tilde{q}_{B|bm}}{\partial \bar{z}_B^A} \frac{\partial \bar{z}_B^A}{\partial z_{tB}^A} + \frac{\partial \bar{z}_B^A}{\partial z_{tB}^A} - \frac{\partial \tilde{q}_{B|bm}}{\partial \bar{z}_B^A} \frac{\partial \bar{z}_B^A}{\partial z_{tB}^A} + \frac{\partial \bar{z}_B^B}{\partial z_{tB}^A} \right\} \\ & + \alpha(1-\delta) \left\{ u'(q_{B|bm}) \frac{\partial q_{B|bm}}{\partial \bar{z}_{B^*}^B} \frac{\partial \bar{z}_{B^*}^B}{\partial z_{tB}^A} + \frac{\partial \bar{z}_{B^*}^A}{\partial z_{tB}^A} + \frac{\partial \bar{z}_{B^*}^B}{\partial z_{tB}^A} - \frac{\partial q_{B|bm}}{\partial \bar{z}_{B^*}^B} \frac{\partial \bar{z}_{B^*}^B}{\partial z_{tB}^A} \right\} \\ & - (1+\iota_A) = 0 \end{aligned} \tag{B.1}$$

The first order conditions with respect to z_{tB}^B :

$$\begin{aligned} & \alpha\delta \left\{ u'(\tilde{q}_{B|bm}) \frac{\partial \tilde{q}_{B|bm}}{\partial \bar{z}_B^B} \frac{\partial \bar{z}_B^B}{\partial z_{tB}^B} + \frac{\partial \bar{z}_B^B}{\partial z_{tB}^B} - \frac{\partial \tilde{q}_{B|bm}}{\partial \bar{z}_B^B} \frac{\partial \bar{z}_B^B}{\partial z_{tB}^B} + \frac{\partial \bar{z}_B^A}{\partial z_{tB}^B} \right\} \\ & + \alpha(1-\delta) \left\{ u'(q_{B|bm}) \frac{\partial q_{B|bm}}{\partial \bar{z}_{B^*}^A} \frac{\partial \bar{z}_{B^*}^A}{\partial z_{tB}^B} + \frac{\partial \bar{z}_{B^*}^A}{\partial z_{tB}^B} + \frac{\partial \bar{z}_{B^*}^B}{\partial z_{tB}^B} - \frac{\partial q_{B|bm}}{\partial \bar{z}_{B^*}^A} \frac{\partial \bar{z}_{B^*}^A}{\partial z_{tB}^B} \right\} \\ & - (1+\iota_B) = 0 \end{aligned} \tag{B.2}$$

When $u'(q)q$ is strictly decreasing, given ε_t , there are eight possible regions in the (z_{tB}^A, z_{tB}^B) space that supports CM portfolio choice by country B buyers who access the black market for currencies:

1. $\varepsilon_t > 1$ and $z_{tB}^A + \varepsilon_t z_{tB}^B \geq \varepsilon_t \bar{\psi} > q^*$
2. $\varepsilon_t > 1$ and $q^* \leq z_{tB}^A + \varepsilon_t z_{tB}^B < \varepsilon_t \bar{\psi}$
3. $\varepsilon_t > 1$ and $z_{tB}^A + \varepsilon_t z_{tB}^B < q^* < \varepsilon_t \bar{\psi}$
4. $\varepsilon_t = 1$ and $z_{tB}^A + z_{tB}^B \geq q^*$
5. $\varepsilon_t = 1$ and $z_{tB}^A + z_{tB}^B < q^*$
6. $\varepsilon_t < 1$ and $z_{tB}^A + \varepsilon_t z_{tB}^B \geq \bar{\chi} > \varepsilon_t q^*$
7. $\varepsilon_t < 1$ and $\varepsilon_t q^* \leq z_{tB}^A + \varepsilon_t z_{tB}^B < \bar{\chi}$
8. $\varepsilon_t < 1$ and $z_{tB}^A + \varepsilon_t z_{tB}^B < \varepsilon_t q^* < \bar{\chi}$

We show that cases 1 and will violate Lemma 6 and then characterize the conditions under which an optimum will exist in the rest.

Case 1: The first order conditions become:

$$\alpha\delta + \alpha(1 - \delta) - (1 + \iota_A) = 0$$

$$\alpha\delta\varepsilon_t + \alpha(1 - \delta)\varepsilon_t - (1 + \iota_B) = 0$$

These yield: $\iota_B = \alpha\varepsilon_t - 1$ and $\iota_A = \alpha - 1 < 0$ which violates Lemma 6. Therefore, there cannot be an optimal portfolio in this region when $\varepsilon_t > 1$.

Case 2: The first order conditions are:

$$\alpha\delta\varepsilon_t + \alpha(1 - \delta)u'(\varepsilon_t^{-1}z_{tB}^A + z_{tB}^B) - (1 + \iota_A)\varepsilon_t = 0$$

$$\alpha\delta\varepsilon_t + \alpha(1 - \delta)u'(\varepsilon_t^{-1}z_{tB}^A + z_{tB}^B) - (1 + \iota_B) = 0$$

These conditions yield: $(1 + \iota_A)\varepsilon_t = (1 + \iota_B)$ or, $\varepsilon_t = (1 + \iota_B)/(1 + \iota_A)$. Also, $u'(\varepsilon_t^{-1}z_{tB}^A + z_{tB}^B) = \frac{(1 + \iota_A)\varepsilon_t - \alpha\delta\varepsilon_t}{\alpha(1 - \delta)} = \frac{(1 + \iota_B) - \alpha\delta\varepsilon_t}{\alpha(1 - \delta)}$. Since, $q^* \leq z_{tB}^A + \varepsilon_t z_{tB}^B$ it must be $u'(\varepsilon_t^{-1}z_{tB}^A + z_{tB}^B) =$

$\frac{(1+\iota_A)\varepsilon_t - \alpha\delta\varepsilon_t}{\alpha(1-\delta)} \leq u'(\varepsilon_t q^*) \implies \iota_A \leq \alpha(1-\delta)\varepsilon_t^{-1}u'(\varepsilon_t^{-1}q^*) + \alpha\delta - 1$. The expression $\alpha(1-\delta)\varepsilon_t^{-1}u'(\varepsilon_t^{-1}q^*) + \alpha\delta - 1$ is increasing in α , so to make sure that ι_A is not less than zero, it must be $\alpha \geq [(1-\delta)\varepsilon_t^{-1}u'(\varepsilon_t^{-1}q^*) + \delta]^{-1}$. On the other hand, since, $z_{tB}^A + \varepsilon_t z_{tB}^B < \varepsilon_t \bar{\psi}$, so $\frac{(1+\iota_A)\varepsilon_t - \alpha\delta\varepsilon_t}{\alpha(1-\delta)} > \varepsilon_t \implies \iota_A > \alpha - 1$ which will always be satisfied since at the equilibrium $\iota_A > 0$.

Similarly, $\frac{(1+\iota_B)\varepsilon_t - \alpha\delta\varepsilon_t}{\alpha(1-\delta)} \leq u'(\varepsilon_t q^*) \implies \iota_B \leq \alpha(1-\delta)u'(\varepsilon_t^{-1}q^*) + \alpha\delta\varepsilon_t - 1$. The expression $\alpha(1-\delta)u'(\varepsilon_t^{-1}q^*) + \alpha\delta\varepsilon_t - 1$ is increasing in α , so to make sure that ι_B is not less than zero, it must be $\alpha \geq [(1-\delta)u'(\varepsilon_t^{-1}q^*) + \delta\varepsilon_t]^{-1}$. On the other hand, since, $z_{tB}^A + \varepsilon_t z_{tB}^B < \varepsilon_t \bar{\psi}$, $\frac{(1+\iota_B)\varepsilon_t - \alpha\delta\varepsilon_t}{\alpha(1-\delta)} > \varepsilon_t \implies \iota_B > \alpha\varepsilon_t - 1$, which after substituting for ε_t , boils down to $\iota_B > -1$. This is always true because $\iota_B > 0$.

If $\varepsilon_t > 1$, then $[(1-\delta)\varepsilon_t^{-1}u'(\varepsilon_t^{-1}q^*) + \delta]^{-1} > [(1-\delta)u'(\varepsilon_t^{-1}q^*) + \delta\varepsilon_t]^{-1}$. SO a necessary condition for optimal portfolio to exist in this region is $\alpha \geq [(1-\delta)u'(\varepsilon_t^{-1}q^*) + \delta\varepsilon_t]^{-1}$. Also, $0 < \iota_A \leq \alpha(1-\delta)\varepsilon_t^{-1}u'(\varepsilon_t^{-1}q^*) + \alpha\delta - 1$. Since $(1+\iota_A)\varepsilon_t = (1+\iota_B)$, the upper bound for ι_B : $\iota_B \leq \alpha(1-\delta)u'(\varepsilon_t^{-1}q^*) + \alpha\delta\varepsilon_t - 1$ will be satisfied if the upper bound for ι_A is satisfied.

For this optimal end of period portfolio $\bar{z}_{B^*}^A = 0$, $\bar{z}_{B^*}^B = \varepsilon_t^{-1}z_{tB}^A + z_{tB}^B$, $\bar{z}_B^A = z_{tB}^A + \varepsilon_t z_{tB}^B$, $\bar{z}_B^B = 0$. From the market clearing conditions of black market it can be show that $z_{tB}^B = \frac{1-\delta}{\delta\varepsilon_t}z_{tB}^A$. Plugging this back in $\varepsilon_t^{-1}z_{tB}^A + z_{tB}^B$, we get $u'\left(\frac{z_{tB}^A}{\delta\varepsilon_t}\right) = \frac{1+\iota_B - \alpha\delta\varepsilon_t}{\alpha(1-\delta)} = \frac{(1+\iota_A)\varepsilon_t - \alpha\delta\varepsilon_t}{\alpha(1-\delta)}$

Case 3: The first order conditions are:

$$\begin{aligned} \alpha\delta u'(z_{tB}^A + \varepsilon_t z_{tB}^B) + \alpha(1-\delta)\varepsilon_t^{-1}u'(\varepsilon_t^{-1}z_{tB}^A + z_{tB}^B) - (1+\iota_A) &= 0 \\ \alpha\delta\varepsilon_t u'(z_{tB}^A + \varepsilon_t z_{tB}^B) + (1-\alpha)\delta u'(\varepsilon_t^{-1}z_{tB}^A + z_{tB}^B) - (1+\iota_B) &= 0 \end{aligned}$$

These conditions yield: $(1+\iota_A)\varepsilon_t = (1+\iota_B)$ or, $\varepsilon_t = (1+\iota_B)/(1+\iota_A)$. Also, $u'(\varepsilon_t^{-1}z_{tB}^A + z_{tB}^B) > u'(\varepsilon_t^{-1}q^*)$ and $u'(z_{tB}^A + \varepsilon_t z_{tB}^B) > 1$. Therefore, $(1+\iota_A) = \alpha\delta u'(z_{tB}^A + \varepsilon_t z_{tB}^B) + \alpha(1-\delta)\varepsilon_t^{-1}u'(\varepsilon_t^{-1}z_{tB}^A + z_{tB}^B) > \alpha(1-\delta)\varepsilon_t^{-1}u'(\varepsilon_t^{-1}q^*) + \alpha\delta \implies \iota_A > \alpha(1-\delta)\varepsilon_t^{-1}u'(\varepsilon_t^{-1}q^*) + \alpha\delta - 1$.

Therefore, $\iota_B > \alpha(1 - \delta)u'(\varepsilon_t^{-1}q^*) + \alpha\delta\varepsilon_t - 1$

For this optimal end of period portfolio $\bar{z}_{B^*}^A = 0, \bar{z}_{B^*}^B = \varepsilon_t^{-1}z_{tB}^A + z_{tB}^B, \bar{z}_B^A = z_{tB}^A + \varepsilon_t z_{tB}^B, \bar{z}_B^B = 0$.

From the market clearing conditions of black market it can be show that $z_{tB}^B = \frac{1-\delta}{\delta\varepsilon_t}z_{tB}^A$.

Plugging this back in the expressions for $\varepsilon_t^{-1}z_{tB}^A + z_{tB}^B$ and $z_{tB}^A + \varepsilon_t z_{tB}^B$, we get the following expression that solves the optimal z_{tB}^A :

$$\alpha\delta u'\left(\frac{z_{tB}^A}{\delta}\right) + (1 - \alpha)\delta\varepsilon_t^{-1}u'\left(\frac{z_{tB}^A}{\delta\varepsilon_t}\right) = 1 + \iota_A$$

Case 4: The first order conditions are:

$$\alpha\delta + \alpha(1 - \delta) - (1 + \iota_A) = 0$$

$$\alpha\delta + \alpha(1 - \delta) - (1 + \iota_B) = 0$$

These conditions yield: $\iota_a = \iota_b = \alpha - 1 < 0$ which is not possible. Therefore, there cannot be an optimal portfolio in this region.

Case 5: The first order conditions

$$\alpha u'(z_{tB}^A + z_{tB}^A) - (1 + \iota_A) = 0$$

$$\alpha u'(z_{tB}^A + z_{tB}^A) - (1 + \iota_B) = 0$$

These yield: $\iota_A = \iota_B$. Note here also $\varepsilon_t = \frac{1+\iota_B}{1+\iota_A}$ is satisfied (trivially). Also, it must be $u'(z_{tB}^A + z_{tB}^A) = \frac{1+\iota_A}{\alpha} = \frac{1+\iota_B}{\alpha} > 1$ which will be satisfied as long as $\iota_A, \iota_B \geq 0$. For this optimal end of period portfolio $\bar{z}_{B^*}^A = 0, \bar{z}_{B^*}^B = z_{tB}^A + z_{tB}^B, \bar{z}_B^A = z_{tB}^A + z_{tB}^B, \bar{z}_B^B = 0$. From the market clearing conditions of black market it can be show that $z_{tB}^B = \frac{1-\delta}{\delta}z_{tB}^A$. Therefore, the optimal z_{tB}^A solves $u'\left(\frac{z_{tB}^A}{\delta}\right) = \frac{1+\iota_A}{\alpha} = \frac{1+\iota_B}{\alpha}$.

Case 6: first order conditions are:

$$\begin{aligned}\alpha\delta\varepsilon_t^{-1} + \alpha(1-\delta)\varepsilon_t^{-1} - (1+\iota_A) &= 0 \\ \alpha\delta + \alpha(1-\delta) - (1+\iota_B) &= 0\end{aligned}$$

These yield: $\iota_A = \varepsilon_t^{-1} - 1$ and $\iota_B = \alpha - 1 < 0$. Now, ι_B cannot be negative. Therefore, an optimal end of period (CM) portfolio in this region is inadmissible.

Case 7: first order conditions are:

$$\begin{aligned}\alpha\delta u'(z_{tB}^A + \varepsilon_t z_{tB}^B) + \alpha(1-\delta)\varepsilon_t^{-1} - (1+\iota_A) &= 0 \\ \alpha\delta u'(z_{tB}^A + \varepsilon_t z_{tB}^B)\varepsilon_t + \alpha(1-\delta) - (1+\iota_B) &= 0\end{aligned}$$

Again, these yield: $\varepsilon_t = \frac{1+\iota_B}{1+\iota_A}$ and $u'(z_{tB}^A + \varepsilon_t z_{tB}^B) = \frac{1+\iota_A - \alpha(1-\delta)\varepsilon_t^{-1}}{\alpha\delta} = \frac{1+\iota_B - \alpha(1-\delta)}{\alpha\delta\varepsilon_t}$. Now $u'(z_{tB}^A + \varepsilon_t z_{tB}^B) \leq u'(\varepsilon_t q^*) \implies \iota_A \leq \alpha\delta u'(\varepsilon_t q^*) + \alpha(1-\delta)\varepsilon_t^{-1} - 1$. Now, $\alpha\delta u'(\varepsilon_t q^*) + \alpha(1-\delta)\varepsilon_t^{-1} - 1$ is increasing in α and we cannot have $\iota_A < 0$. Therefore, one necessary condition is $\alpha \geq [\delta u'(\varepsilon_t q^*) + (1-\delta)\varepsilon_t^{-1}]^{-1}$. Also, $u'(z_{tB}^A + \varepsilon_t z_{tB}^B) > \varepsilon_t^{-1} \implies \iota_A > \alpha\varepsilon_t^{-1} - 1 \implies (1+\iota_B - \alpha)(1+\iota_A) > 0$ which is always true because $\iota_A, \iota_B > 0$. \square

Proof of Proposition 4

Proof. As in the proof for Proposition 2 and 3, on similar lines it can be shown that when only $money_A$ is confiscated with probability $(1-\alpha)$, the following condition holds true:

$$\frac{e_{formal}}{e_{black}} = \varepsilon_t = \frac{1+\iota_B - (1-\alpha)[(1-\delta)u'(z_{tB}^B) + \delta]}{1+\iota_A}$$

Since they are price takers in the competitive black market, buyers take ε as given and

choose the optimum z_{tB}^B . The optimal real balance of $money_B$ for Country B buyers, z_{tB}^B is necessarily decreasing in ι_B and ε adjusts accordingly so as to satisfy one of the eight cases mentioned in the proof of Proposition 2 and 3.

So, an increase in domestic inflation would increase ι_B leading to an increase in ε or fall in the premium on $money_A$. But at the same time $u'(z_{tB}^B)$ would increase since z_{tB}^B decreases. This leads to a fall in ε or a rise in the premium on $money_A$. The reverse happens when domestic inflation decreases or ι_B falls. Now which effect will be stronger depends on the elasticity of $u(\cdot)$. If $u(\cdot)$ is more elastic, then the drop in ι_B will be stronger leading to a larger increase in $u'(z_{tB}^B)$. If $u(\cdot)$ is sufficiently elastic, it would lead to a fall in ε with an increase in ι_B and a rise in premium on $money_A$, else it would lead to a drop in premium on $money_A$ with an increase in ι_b . □