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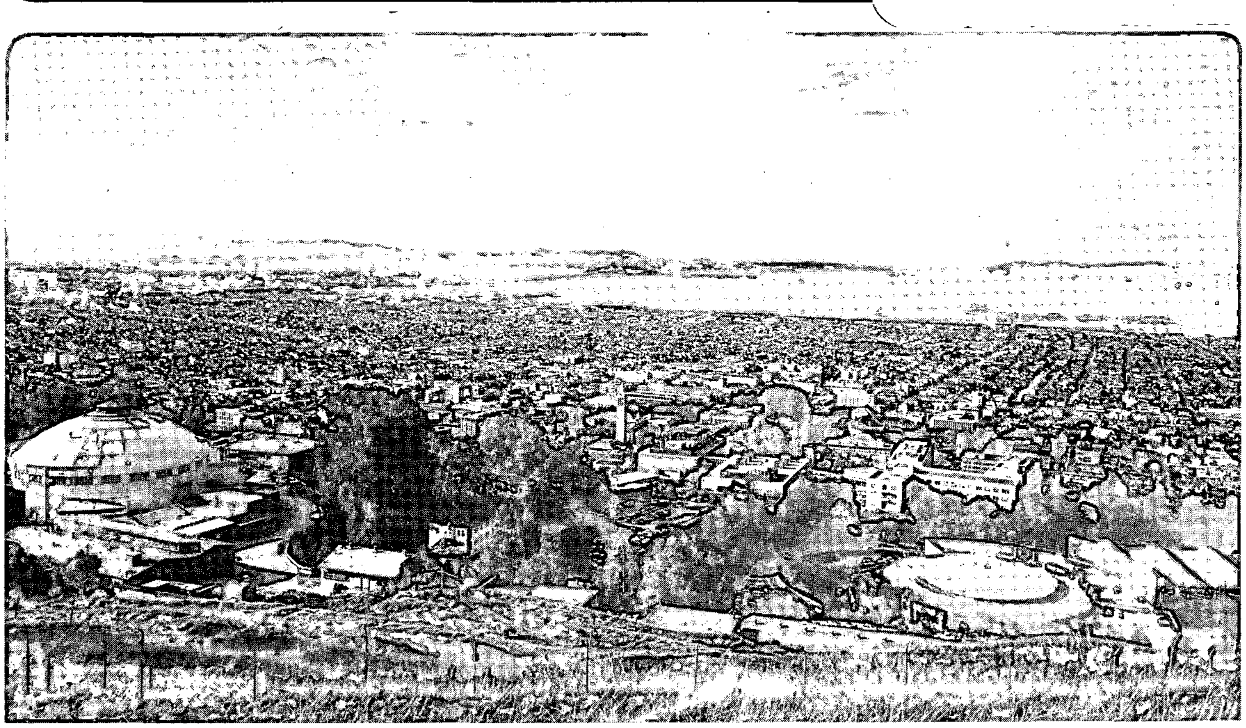

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DYNAMIC STABILITY OF A FOAM LAMELLA FLOWING THROUGH A PERIODICALLY CONSTRICTED TUBE

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Introduction

Experimental studies of foam flow in two dimensional, water-wet, porous medium micromodels reveal the importance of the capillary pressure of the medium upon the existence of foam in a given porous medium (1). Preformed foam injected into a dry medium simply cannot enter until it breaks and the released liquid imbibes into the small pores. This coalescence process increases the medium liquid saturation and reduces the capillary pressure according to a typical Leverett function (2). Upon reaching high enough liquid saturation foam then flows into the micromodel. On the other hand, if foam is injected into a previously surfactant solution saturated medium, it immediately flows to a steady state.

It has also been observed in studies of foam flow in partially liquid saturated bead packs and sand packs that breakage occurs if the gas superficial velocity is increased sufficiently (3). For a porous medium with a given permeability and saturation (and corresponding capillary pressure) there is a critical gas velocity above which lamellae rupture. Similarly, if the capillary pressure is increased at a given velocity of flow, lamellae will rupture. Khatib, Hirasaki, and Falls (3) measured this critical capillary pressure of foam coalescence for bead packs of permeabilities ranging from 72 to 8970 μm^2 and various surfactant formulations.

This paper introduces a hydrodynamic theory that explains the foam breakage phenomena described above by analyzing the stability of a single foam lamella flowing through a periodically constricted sinusoidal pore.

Stability of a Static Foam Lamella

To explain the effect of capillary pressure upon foam coalescence, consider a perfectly flat, cylindrical, foam lamella of thickness, $2h_o$, circa 1000Å, and radius, R(i.e., 50-100μm), subject to a capillary pressure, P_c, at the film meniscus or Plateau border. The liquid pressure at the film meniscus is $(P_g - P_c)$, where P_g is the gas pressure. The liquid pressure in the film is $(P_g - \Pi)$, where Π is the conjoining/disjoining pressure. For $P_c > \Pi$ the pressure difference, $P_c - \Pi$, drives liquid out of the lamella until a new state is reached. This new state, however, could be an equilibrium one or could very likely be unstable. The stability limit is set by the shape of the disjoining pressure curve (4,5,6).

The concept of an excess pressure or disjoining pressure was first introduced and tested experimentally by Derjaguin (7,8). Most recently Exerowa, Kolarov, and Khristov (9) have measured $\Pi(h)$ isotherms for foam films. The classic DLVO theory provides an analytical expression for $\Pi(h)$ for the case of perfectly flat interfaces (10). In dimensionless form $\Pi(h)$ can be expressed for the case of a constant potential at the interface as;

$$\hat{\Pi} = -\frac{1}{\theta \hat{h}^3} + \exp(1-\hat{h}) \qquad 1)$$

where, $\theta = Be^{-1}/\kappa^3 A_H$, $\hat{h} = 2\kappa h$, and $\hat{\Pi} = \Pi/Be^{-1}$.
B is a known function of the ionic strength, surfactant concentration, and temperature. A_H is the Hamaker constant and $1/\kappa$ is the Debye length. Fig.1 shows a typical $\Pi(h)$ isotherm for $\theta = 5$.

For the case of a constant charge density, q_s, at the interface $\Pi(h)$ can be expressed as;

$$\hat{\Pi} = -\frac{1}{\theta \hat{h}^3} + \operatorname{csch}^2(\hat{h}/2) \qquad 2)$$

where, $\theta = \pi q_s^2/4\kappa^3 A_H \epsilon$, $\hat{\Pi} = \epsilon\Pi/2\pi q_s^2$, and ϵ is the permittivity of the surfactant solution.

For either case the $\Pi(h)$ isotherm typically looks like that shown in Fig. 1. As found by Vrij (4) through a thermodynamic analysis, or by a simple linear stability analysis (11), the critical thickness limit for metastable films, $\hat{h}_{min}$ is given by the maximum of the $\hat{\Pi}$ curve, $\hat{\Pi}_{max}$, or equivalently when $\partial\Pi/\partial h = 0$. Unbounded films thinner than $\hat{h}_{min}$ are unstable to perturbations of long wavelength; films thicker than $\hat{h}_{min}$ are stable.

Consider now a relatively dry medium for which the corresponding capillary pressure in a dimensionless sense is greater than $\hat{\Pi}_{max}$, as shown in Fig.1. Given sufficient time any foam lamella present in such a medium will thin down to a thickness less than $\hat{h}_{min}$ and rupture. On the other hand, for a relatively wet medium with $\hat{P}_c < \hat{\Pi}_{max}$ the lamella will thin or thicken, depending on its original thickness, to a new equilibrium stable state until $\hat{\Pi} = \hat{P}_c$. This explains why foam cannot enter a completely dry porous medium. Also we anticipate that it is more difficult for a foam to exist in a lower permeability medium.

Stability of a Moving Lamella

To explain dynamic foam coalescence in porous medium consider a single lamella flowing through a periodically constricted sinusoidal pore, as shown in Fig.2. The capillary pressure of the medium imposed on the lamella at the pore wall is assumed constant and is set by the local liquid saturation. Assuming constant volumetric flow, q, the lamella transports with interstitial velocity $U(\xi)$ which varies according to the rate of change of the pore radius given by the periodic function:

$$R(\xi)/R_c = (1+a) + a\cos[\pi(1+2\xi/\lambda_o)] \qquad 3)$$

where ξ is the axial distance measured from the pore constriction, (1+2a) determines the ratio of pore body, R_b, to pore constriction, R_c; λ_o is the wavelength of the periodic pore.

Upon moving from the pore constriction ($\xi=0$) to the pore body ($\xi = \lambda_o/2$), the lamella is stretched as it is pulled by the wall. To achieve volume rearrangement a radial pressure differential is induced which thins the film but results in no net fluid efflux. The converse occurs when the film is squeezed upon moving from a pore body to a pore constriction.

In addition, as the lamella is thinned (thickened) by the stretching (squeezing) rate the disjoining pressure changes accordingly and depending on the magnitude of the capillary pressure there is a net flow of liquid out or into the lamella. We adopt the simple Reynolds film model to describe this drainage (filling) flow (12,13).

The two phenomena of film-wall conformity and capillary-pressure-driven film flow can be combined linearly in the following dimensionless evolution equation:

$$\frac{\partial \hat{h}}{\partial \hat{\xi}} = \frac{2\pi a \ \hat{h} \ \sin \pi(1+\hat{\xi})}{1+a + a \cos \pi(1+\hat{\xi})} - \frac{\theta}{Ca} \left[\hat{P}_c \ \hat{h}^3 + \frac{1}{\theta} - \hat{h}^3 \ \text{csch}^2(\hat{h}/2) \right] \qquad 4)$$

where, $\hat{\xi} = 2\xi/\lambda_o$, $\hat{h} = 2\kappa h$, and:

$$Ca = \frac{3\mu q}{4\pi \ \lambda_o \kappa \ A_H} \qquad \hat{P}_c = \frac{\epsilon \ P_c}{2\pi \ q_s^2} \qquad 5)$$

The first term in 4) corresponds to the rate of stretching/squeezing and is closely related to the shape of the pore constriction. The sharper the constriction (i.e., high R_b/R_c), the larger is the contribution of this term to the total rate of thinning of the lamella. The second term represents the rate of film drainage/filling caused by the difference between the capillary pressure, kept constant in our analysis, and the instantaneous disjoining pressure. Note that if $\hat{P}_c$ is always less than $\hat{\Pi}(\xi)$ this term causes film thickening and vice versa. The scaling factor between these two terms is a modified capillary number, Ca, which determines how fast the film moves through the constriction. In the limit of $Ca \to \infty$ the second term becomes negligible compared to the first term indicating that the time of flow through the constriction is too small to allow liquid to move in or out of the lamella under the influence of the capillary pressure. In the limit of $Ca \to 0$ the lamella is virtually at rest and regardless of the shape of the constriction the capillary pressure will drive liquid in or out of the lamella until an equilibrium or unstable state is reached. In this case the limiting capillary pressure above which the lamella cannot exist, $\hat{P}_c^*$, is identical to $\hat{\Pi}_{max}$.

For a finite velocity other than zero both the stretching and the drainage rate play important roles. For instance, in the case of a lamella of initial dimensionless thickness $\hat{h}_o = 2.0$, $Ca = 2$, $a = 0.5$ (i.e., $R_b/R_c = 2.0$), and a disjoining pressure given by that in Fig.1, $\hat{P}_c^*$ lies below $\hat{\Pi}_{max}$ and above $\hat{\Pi}_o$. Under the influence of this particular critical capillary pressure the film tends to drain to the thickness denoted by $\hat{h}_\infty$ in Fig. 1 but will oscillate in thickness from constriction to constriction with large enough amplitude as to become as thin as $\hat{h}_{min}$ at the pore body. It is here that the film becomes unstable and ruptures.

Thus, solution of 4) for various values of the parameters: $\hat{h}_o$, Ca, $\hat{P}_c$, and a traces a locus of points corresponding to the critical capillary pressure of lamella stability versus the velocity of flow for a given initial foam texture and porous medium structure. Results are shown in Fig.3. by the solid line. For a given value of Ca (i.e., a given superficial velocity) this curve provides the capillary pressure above which foam cannot exist. By choosing reasonable values for the surface charge density ($1.5\mu C/cm^2$), Hamaker constant (10^{-21} J), $\hat{h}_o = 10$, and $R_b/R_c = 2$ we achieve very good agreement with experimental results by Khatib, Hirasaki, and Falls (3) for a solution of 4.2×10^{-3} $kmol/m^3$ ENORDET AOS 1618 surfactant and $0.17 kmol/m^3$ NaCl in a $81 \mu m^2$ sand pack.

Conclusions

The stability of foam lamellae has great significance in determining the fraction of gas present in the continuous phase as foam flows in porous media. By analyzing a single lamella as it percolates through a periodically constricted tube, we have developed a theory that explains the physical phenomena governing foam coalescence in porous media. The conjoining/disjoining pressure curve proves to be the crucial physical property of the surfactant/water system. It determines the maximum capillary pressure that foam can sustain at rest in a porous medium. This critical capillary pressure can be associated with a critical permeability or a critical liquid saturation for a given medium through a Leverett function. For moving foam this critical capillary pressure is reduced as lamellae are thinned by the influence of both the capillary pressure driven flow and the wall conforming flow. For a given gas superficial velocity foam cannot exist if the capillary pressure exceeds a critical value. The theory introduced here proves to be in very good agreement with available experimental data (3).

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Figure Captions

Fig. 1- Dimensionless disjoining pressure isotherm for a constant surface potential.
Fig. 2- Lamella transport through a periodically constricted sinusoidal tube.
Fig. 3- Comparison of theory with available experimental data.

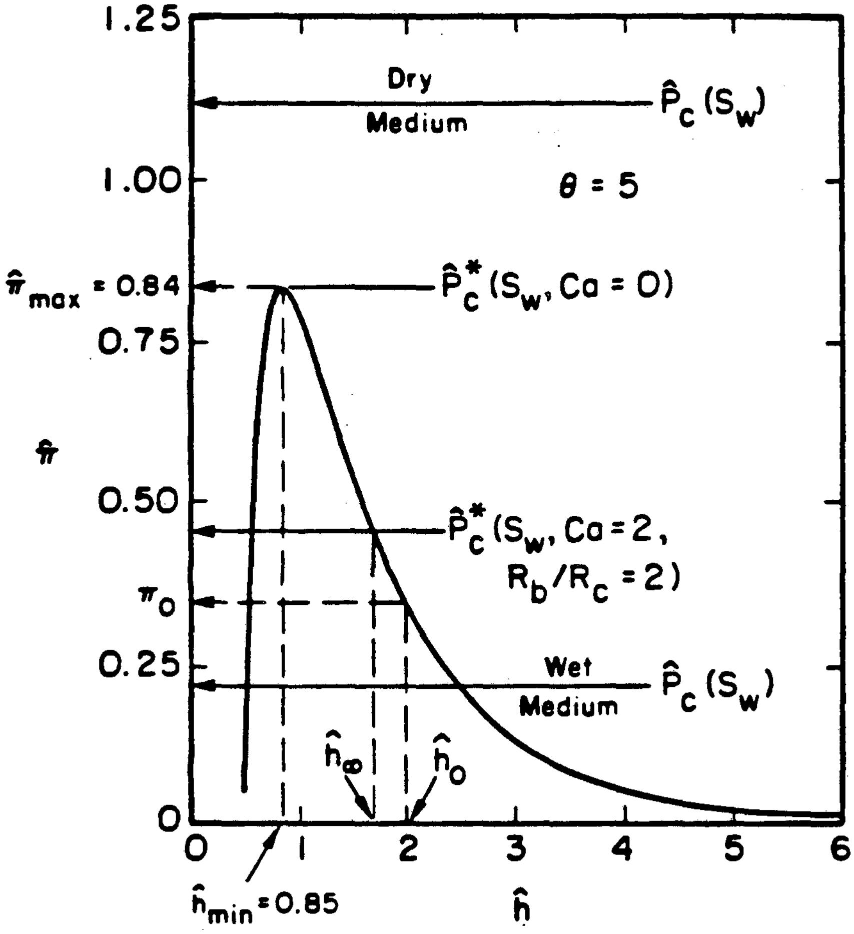

1.25
1.00
0.75
0.50
0.25
0
Dry
Medium
$\hat{P}_c(S_w)$
$\theta = 5$
$\hat{\pi}_{max} = 0.84$
$\hat{P}_c^*(S_w, Ca = 0)$
$\hat{\pi}$
$\hat{P}_c^*(S_w, Ca = 2,$
$R_b/R_c = 2)$
π_o
Wet
Medium
$\hat{P}_c(S_w)$
$\hat{h}_\infty$
$\hat{h}_o$
0
1
2
3
4
5
6
$\hat{h}_{min} = 0.85$
$\hat{h}$

Fig. 1

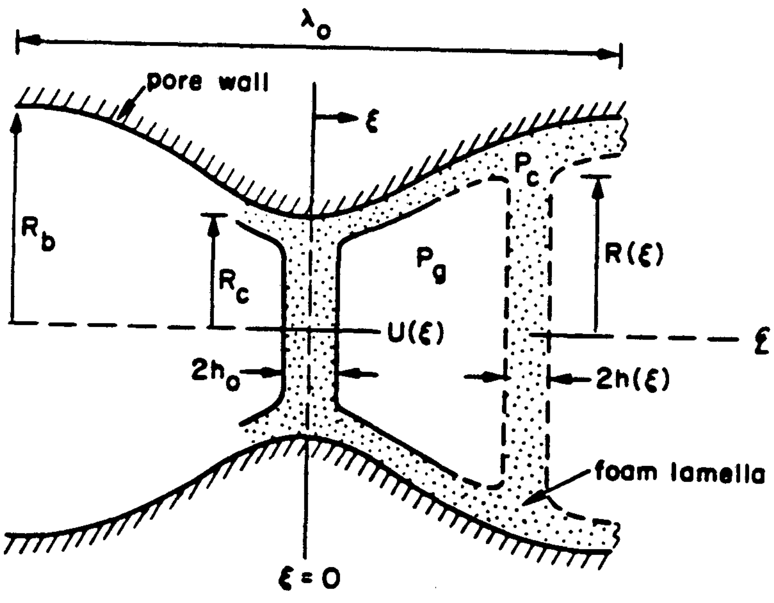
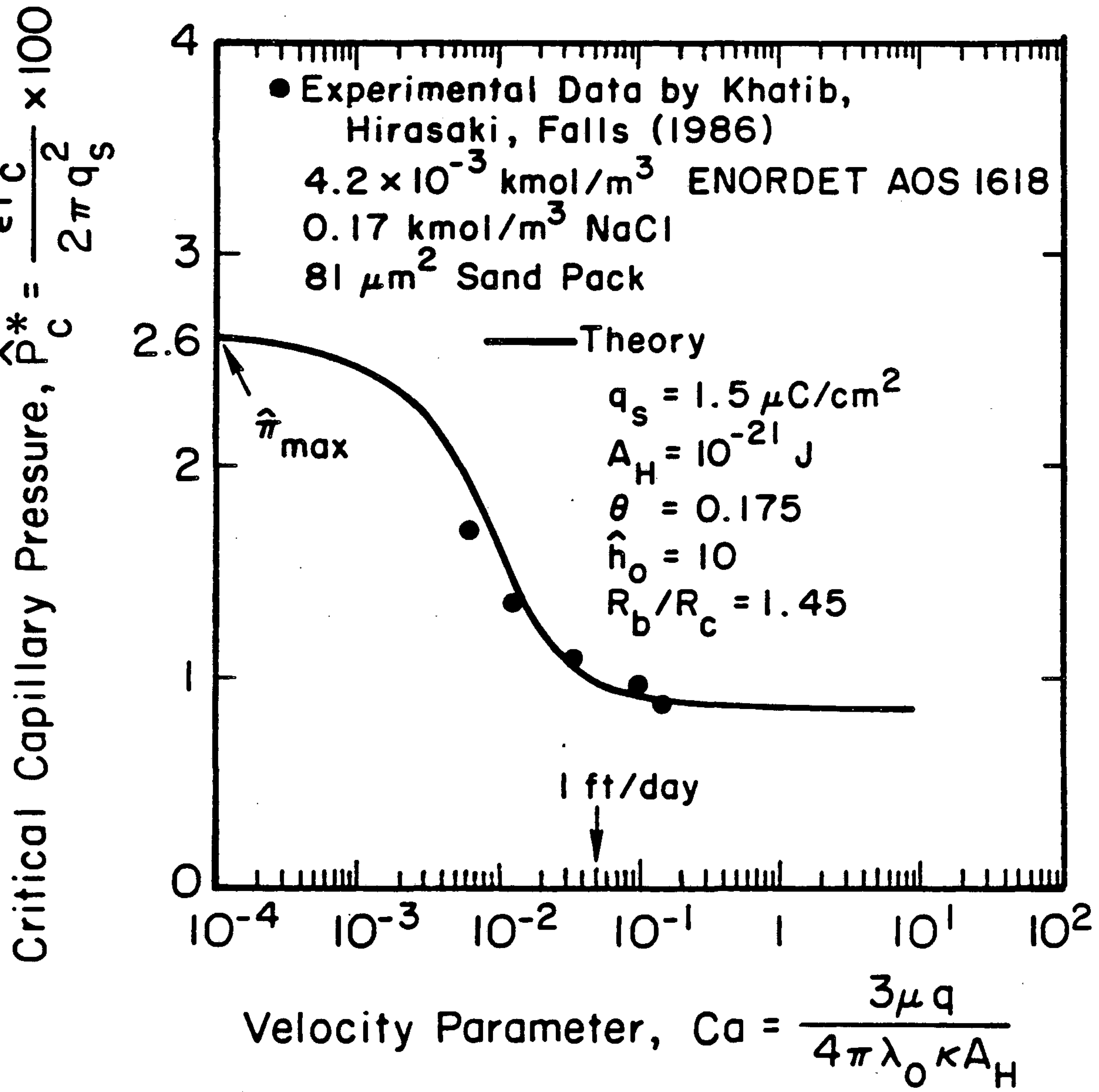

λ_o
pore wall
ξ
P_c
R_b
R_c
P_g
$R(\xi)$
$U(\xi)$
$\mathcal{L}$
$2h_o$
$2h(\xi)$
foam lamella
$\xi = 0$

Fig. 2



Critical Capillary Pressure, $\hat{P}_c^* = \frac{\epsilon P_c}{2\pi q_s^2} \times 100$
● Experimental Data by Khatib, Hirasaki, Falls (1986)
4.2×10^{-3} kmol/m^3 ENORDET AOS 1618
0.17 kmol/m^3 NaCl
81 μm^2 Sand Pack
Theory
$q_s = 1.5\ \mu C/cm^2$
$A_H = 10^{-21}$ J
$\theta = 0.175$
$\hat{h}_o = 10$
$R_b/R_c = 1.45$
$\hat{\pi}_{max}$
1 ft/day
4
3
2.6
2
1
0
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10^1
10^2
Velocity Parameter, $Ca = \frac{3\mu q}{4\pi \lambda_o \kappa A_H}$

Fig. 3