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Dynamical Emotions: Bodily Dynamics of Affect during Problem Solving

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Abstract

This paper investigates the low level bodily correlates of affective states, such as boredom, confusion, anxiety, and frustration, that spontaneously emerge during complex problem solving tasks. Participants were video recorded while they solved difficult analytical reasoning problems after which they self-reported their affective states via a retrospective affect-judgment protocol. Time series of bodily motions were automatically extracted from the videos of participants' faces and upper bodies via a motion filtering algorithm. Recurrence quantification analyses revealed that participants who reported increased levels of anxiety and frustration had less recurrent and deterministic movements compared to their counterparts. Importantly, these patterns could not be explained by the mere amount of movement or the variability in movement, but by non-obvious dynamical patterns in movement. We orient our findings towards theories that emphasize complex systems approaches to studying emotion.

Keywords: emotion; bodily movement; dynamical systems; recurrence quantification analysis.

Introduction

The significance of non-verbal channels in human-human communication is widely acknowledged, however, the impetus has been on oculesics (eye contact), peripheral physiology, facial expressions, gesture, and paralinguistic features of speech. Comparatively, little attention has been directed towards the role of gross body movement (i.e. posture, movement) as a non-verbal communicative channel. This oversight is unfortunate because it has been known for several decades that posture is an important diagnostic cue of emotions, attitudes, social relationships, personality, and a host of other affective, cognitive, and social constructs (Ekman, 1992; Oullier & Basso, 2010; Russell, Bachorowski, & Fernandez-Dols, 2003).

When it comes to affective states (general term covering moods and emotions), which are the focus of this paper, it has been claimed that the face plays a primary role in discriminating between specific emotions such as anger and fear (Ekman, 1984). Posture and body movements, on the other hand, are sometimes considered to be a mere indicator of the intensity of an emotion, thereby being downgraded in importance as an affective communicative channel (Bull, 1987). This view has been subsequently challenged by a number of studies, which showed the efficacy of the whole body in communicating specific emotions and attitudes, independent of facial expressions and paralinguistic features of speech (Castellano, Mortillaro, Camurri, Volpe, & Scherer, 2008; Coulson, 2004; Scherer & Ellgring, 2007).

There are distinctive reasons for focusing on body movements over the face and speech. First, it could be argued that body motions are ordinarily unconscious, unintentional, and thereby not susceptible to social editing, at least compared with facial expressions, speech intonation, and some gestures. Second, human bodies are relatively large and have multiple degrees of freedom, thereby making the body an effective affective communicative channel. Third, the expectation of a systematic link between bodily movements and complex mental states is supported by embodied theories of cognition and emotion (Niedenthal, 2007).

Most (but not all) of the studies that have investigated the role of body movements in the expression of affective states have focused on the degree of bodily arousal, specific postures (e.g., forward-leans, arms akimbo), and some gestures (e.g., pointing, hailing) (Bull, 1987; Coulson, 2004). Comparatively, little is known about the low-level bodily correlates of affective states like frustration, anxiety, and cognitive-affective amalgamations such as confusion and flow/engagement. This paper addresses this issue by analyzing how these affective states influence the dynamics of presumably unconscious bodily movement

In addition to simple measures of bodily motion, such as amount of movement and variability in movement, we focus on complex system measures as an index of the embodied nature of cognition and affect. Though relatively well established in the cognitive sciences, dynamical systems perspectives are only beginning to gain momentum in the affective sciences (Camras & Shutter, 2010; Coan, 2010; Lewis, 2005). Dynamical theories of emotion challenge the classical view that a central affect program coordinates the physiological, behavioral, and subjective components of an emotional episode (i.e., an anger circuit (Ekman, 1992)). Instead, they posit that these components are loosely coupled and are constantly interacting in a self-organizing fashion. When this system is perturbed, it is jolted from its state of equilibrium, until it spontaneously reorganizes and equilibrium is restored. The emotion "emerges" from the attractor (a set of states towards which the system regularly converges to) in which this dynamical trajectory converges (Camras & Shutter, 2010).

The embodied nature of cognition and affect lead us to hypothesize that variations in affective states will be associated with meaningful shifts in fluctuations of bodily movements. The present study addresses this hypothesis by showing that when individuals engage in complex problem solving tasks, patterns of bodily movement exhibit dynamic signatures that may give away whether someone is in an anxious state, a confused state, a frustrated state, etc.

Recurrence quantification analysis (RQA) (Webber & Zbilut, 2005) were used to extract dynamical measures from nonlinear time series of naturalistic bodily movements and correlate these measures to self-reported affective states. RQA is an invaluable tool to uncover patterns in nonlinear time series because it offers a window into the dynamics of complex systems, while at the same time, avoiding several of the assumptions of traditional time series analysis techniques (e.g., assumptions of stationary, homoscedasticity). It has yet to find widespread use in the cognitive sciences, hence, we begin with a brief description of analyzing dynamical systems with RQA.

Recurrence Quantification Analysis (RQA)

A dynamical system at time t consists of a *state*, which is an n-dimensional point in a n-dimensional space, called a *state space* or *phase space*. There is a fixed rule, which determines how the system transitions from one state to another and a *trajectory* represents a sequence of such state transitions. For example, the popular Lorenz attractor is a dynamical system governed by the following three equations: $\frac{dx}{dt} = \sigma(y - x);$ $\frac{dy}{dt} = x(\rho - z) - y;$ $\frac{dz}{dt} = xy - \beta z$ (σ, ρ and β are parameters) (Lorenz, 1963). Time series of length 1,000 created from the three Lorenz equations are presented in Figure 1A and the resultant phase space plot is presented in Figure 1B. It should be noted that the system is called an attractor (Lorenz attractor) because the trajectory are *attracted* to certain points in the phase space (the lines) and *repelled* from others (the white space).

Recurrence quantification analysis is a technique aimed at uncovering the intrinsic dynamics in non linear dynamical systems, such as the Lorenz system (Webber & Zbilut, 2005). It consists of computing the distance between all possible combinations of m points in the phase space, representing these distances as a $m \times m$ matrix (called a recurrence plot), and deriving measures from the matrix (i.e., quantifying the plot). A recurrence plot for the Lorenz attractor is presented in Figure 1D. Each point in the plot (matrix) consists of the distance between two points in the phase space. For example, cell (100, 130) in the recurrence plot represents the distance between the 100th and 300th point in the phase space. Matrix cells are only included in the plot if the distance between the corresponding points is within some threshold (scored as 1 or 0 otherwise). The freely available CRP toolbox (Marwan, Romano, Thiel, & Kurths, 2007) was used for all the recurrence analyses reported in this paper.

Although it is possible to derive a number of measures from the recurrence plots, the present analysis focuses on two fundamental measures: recurrence and determinism. *Recurrence* is a measure of the proportion of points in the phase space that are within some threshold distance to one another. Recurrence can be computed by simply adding up the number of points in the recurrence plot and dividing it by the total number of possible points ($m \times m$). It can range

from 0 to 1 and the recurrence rate for the Lorenz system presented in Figure 1D is .042.

Determinism is a measure of the proportion of points in the recurrence plot that form diagonal lines, which are indicative of repetitive (or deterministic) patterns in the dynamical trajectory. Determinism also ranges from 0 to 1, and the determinism of the recurrence plot of the Lorenz attractor is .99 (very deterministic).
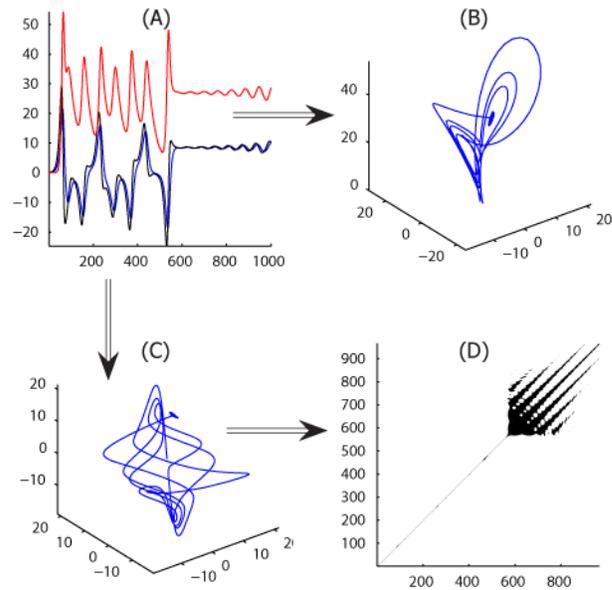

Figure 1. Recurrence quantification analysis of the Lorenz attractor.

It is important to emphasize one critical point pertaining to recurrence analyses of natural time series, such as time series of bodily motion fluctuations analyzed in this paper. Unlike mathematical dynamical systems, such as the Lorenz attractor, we rarely know the precise laws (or equations) that govern the dynamics of phenomenon in the behavioral sciences. What we usually have is one or more measures of a complex system, such as a time series of bodily fluctuations. Fortunately, it is possible to reconstruct the dynamics of a complex system with a single time series using time-delayed surrogates of that time series (Takens, 1981). This is illustrated by the reconstructed phase space of the Lorenz system depicted in Figure 1C. This phase space was constructed from only one of the three time series presented in Figure 1A and the reconstructed (Figure 1C) and actual (Figure 1B) phase spaces are remarkably similar (the similarity is more obvious after rotating Figure 1C). Hence, an essential step in RQA is to reconstruct the dynamics of the system (phase-space reconstruction) prior to creating and quantifying the recurrence plots. In fact, the recurrence plot in the figure was computed from the reconstructed phase space (Figure 1C) instead of the actual phase space (Figure 1B).

Method

Participants

Participants were 41 undergraduate students who were enrolled in a preparatory course for the Law School Admissions Test (LSAT); this test is required for admission to Law School in the U.S. There were 26 females (63%) and 15 males (37%). 78% were Caucasians and the remaining 22% were African-Americans. All of the participants indicated that they were interested in attending law school and were paid $30 for their participation.

Procedure

Phase 1: Problem Solving. Participants solved difficult analytical reasoning problems taken from the LSAT over the course of the session. They interacted with a customized software program on a Tablet PC that delivered the questions, monitored their responses, and provided feedback (i.e. "Correct" or "Incorrect"). Effectively solving the analytical reasoning problems requires a considerable amount of knowledge representation, drawing diagrams, taking notes, and other related activities. Participants used a software application, Windows Journal™ (a computerized program that simulates a notepad) to take notes and draw.

The experimenter left the room after demonstrating the software interfaces to the participants. They were told that they would be paid two dollars for each correct answer. All participants were paid $30. Each problem had a scenario (e.g., a flight schedule with constraints) and approximately 5-6 sub-questions pertaining to the scenario. Participants interacted with the system for 35 minutes and videos of the participant's face and computer screen were recorded.

Phase 2. Judging Affective States. Participants provided self-judgments of their affective states immediately after the tutorial session; learning activities during the session were not interrupted. Participants were provided with a checklist of 14 states (anger, anxiety, boredom, contempt, confusion, curiosity, disgust, eureka, fear, frustration, happiness, sadness, surprise, and neutral) along with definitions.

Similar to a cued-recall procedure (Rosenberg & Ekman, 1994), the judgments for a participant's session proceeded by playing a video of the face along with the screen capture video of the computer interface. Videos of the screen were included to facilitate the affect judgment procedure by allowing participants to incorporate contextual factors of the problem solving process with their facial expressions. Participants provided affect ratings over the course of viewing these videos. Specifically, the states were tracked at points halfway between the presentation of the problem and the submission of the response. These center points were in order to capture their states while participants were in the midst of active problem solving.

Mean proportional scores for the six most frequent states were *confusion* (.135), *frustration* (.071), *curiosity* (.186), *boredom* (.115), *anxiety* (.043), and *neutral* (.363). These remaining eight states comprised a mere 8.7% of the observations; hence, the subsequent analysis focuses on this set of six frequent states.

Data Treatment

Participants' gross body movement was monitored from the videos of the face and upper body via a motion-filtering algorithm. The algorithm computes the amount of motion in a given frame F_t by measuring the proportion of pixels in F_t that have been displaced (i.e., motion is greater than a predefined threshold) from a moving background model constructed on the basis of N earlier frames (see Figure 3A-D; $N = 4$ for present analysis). The proportion of pixels with motion provides an index of the amount of movement in each frame.

Sample output of the motion tracking algorithm is presented in Figure 2. Panels A shows a single frame extracted from a video sequence, while the output of the motion filtering algorithm is shown in Panel B. It is important to note that background noise (i.e. the patterns on the walls and ceilings) have been correctly filtered out. A sample time series of bodily movements is presented in Figure 3.

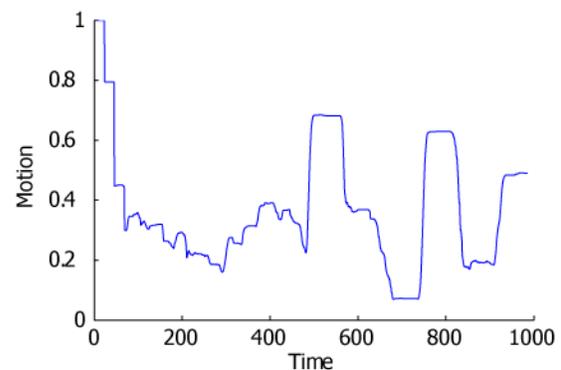Figure 2. Sample output from motion filtering algorithm

Figure 3. Sample time series of bodily movements

Results and Discussion

Time series were extracted from each of the videos with the motion filtering algorithm. Four measures were computed for each participant's time series. The first two measures consisted of the mean and standard deviation of the time series. These basic measures reflect simple movement (mean) and variations in movement (standard deviation). The two dynamical measures consisted of recurrence and determinism as described above. RQA parameters were

embedding dimension = 4; delay = 50, range (1024, see below), norm = maximum norm, radius = 0.3, and line = 2; see Webber & Zbilut (2005) for details.

It is important to emphasize one critical point pertaining to the computation of these measures. The face videos were recorded at 15 frames per second and each video was approximately 35 minutes long. These large time series of 31,500 frames (15 × 60 × 35) introduced some computational problems because it was difficult to store the 31,500 × 31,500 recurrence matrix in memory. These computational problems were circumvented by dividing the time series into 1024-frame windows and computing the four dependent measures for each window. An aggregate score for each participant was then computed by averaging across windows. A similar procedure was adopted for the computation of the basic descriptive measures (mean and standard deviation).

We computed correlations between the four dependent measures to test whether the basic and dynamical measures were capturing unique aspects of the time series. Values outside of a -2SD to 2SD range were identified as outliers and removed prior to computing the correlations.

Mean motion did not significantly ($p < .05$ unless specified otherwise) correlate with either recurrence ($r = -.165$) or determinism ($r = -.149$). It did, however, correlate with standard deviation of movement ($r = .490$), thereby indicating that amplified movement was associated with heightened variability of movement. Standard deviation did not significantly correlate with determinism ($r = -.244$), but it did correlate with recurrence ($r = -.613$). Hence, it appears that increased variability of movement was related to less recurrence, which is what would be expected. Recurrence and determinism were also strongly correlated ($r = .766$), so participants with more recurrent movements also yielded more deterministic patterns. Since only one of the four correlations between the basic and dynamical measures was significant (i.e., the inverse correlation between standard deviation and recurrence), we concluded that, to some extent, these simple and dynamical measures are capturing different aspects of bodily motion.

Next, we computed a 6 × 4 across-subjects correlation matrix between the proportional occurrence of six frequent affective states and the four measures of motion (see Table 1). Standard deviation did not correlate with any of the affective states. This indicates that simple variability in movement is not very diagnostic of affect. Recurrence and determinism demonstrated similar correlational patterns; hence, the subsequent analyses simply refer to these as the dynamical measures.

Simple motion (mean) positively correlated with boredom and confusion, and negatively correlated with the neutral state. This suggests that the experience of boredom and confusion during the problem solving sessions was associated with more bodily movement. In contrast, the neutral state was associated with less movement.

A rather different pattern emerged from the dynamical measures. Both recurrence and determinism were negatively correlated with anxiety and frustration, but were positively correlated with neutral. Hence, it appears that participants who reported increased levels of cognitive distress (anxiety and frustration) had less fluid and less deterministic movements than their counterparts.

Table 1. Correlations between proportional affect and bodily fluctuation measures

Affect	Basic Measures		Dynamical Measures	
	Mean	*Stdev*	*Recurrence*	*Determinism*
Anxiety	-.009	-.012	*-.323	**-.385
Boredom	**.530	.237	-.147	-.223
Confusion	**.412	.236	-.081	.011
Curiosity	-.173	-.097	-.248	-.271
Frustration	-.026	.153	**-.334	**-.339
Neutral	*-.293	-.222	**.469	**.430

Note. ** $p < .05$; * $p < .10$.

The results so far indicate that both the simple and the dynamical measures are correlated with different affective states. Importantly, both measures appear to detect departures from the neutral state, albeit for different affective states. While the interpretation for the correlations between magnitude of movement (mean motion) and boredom and confusion are straightforward, the dynamical patterns are more non-obvious. Since recurrence was correlated with standard deviation, there is the question of whether these dynamical measures explain additional variance above and beyond the more simple descriptive measures (mean and standard deviation).

This question was addressed with a partial correlation between the dynamical measures and anxiety, frustration, and neutral after controlling for the magnitude (mean of each time series) and variability in movement (standard deviation of each time series). The results yielded significant relationships between recurrence and both anxiety ($r = -.426$) and neutral ($r = .495$). Similarly, the partial correlations between determinism and both anxiety ($r = -.400$) and neutral ($r = .400$) were significant. The results were mixed for frustration. There were marginally significant correlations between proportional occurrence of frustration and recurrence ($r = -.292$, $p = .094$) and determinism ($r = -.321$, $p = .069$). Taken together, these results indicate that the movements of individuals experiencing anxiety and frustration cannot be simply attributed to the magnitude or variability in movement, but rather less fluid and less predictable movements.

As an illustrative example, a 1024 frame excerpt of time series for participants reporting high and low anxiety levels are presented in Figure 4A and B, respectively. The time series shows smoother motions for the low anxiety participants compared to the jerky fluctuations for the high anxiety participant. These patterns are also evident in the reconstructed phase spaces and recurrence plots presented in Figure 4. Note that the trajectory visits a much larger

portion of the phase space of the high anxiety participant (Figure 4C), compared to the participant who reported being less anxious (Figure 4D). Consequently, there is considerably less recurrence associated with the high anxiety participant, as demonstrated by the nearly empty recurrence plot ((Figure 4E). In contrast, there are clearly visible patterns of recurrent motion for the participant reporting less anxiety (Figure 4F).
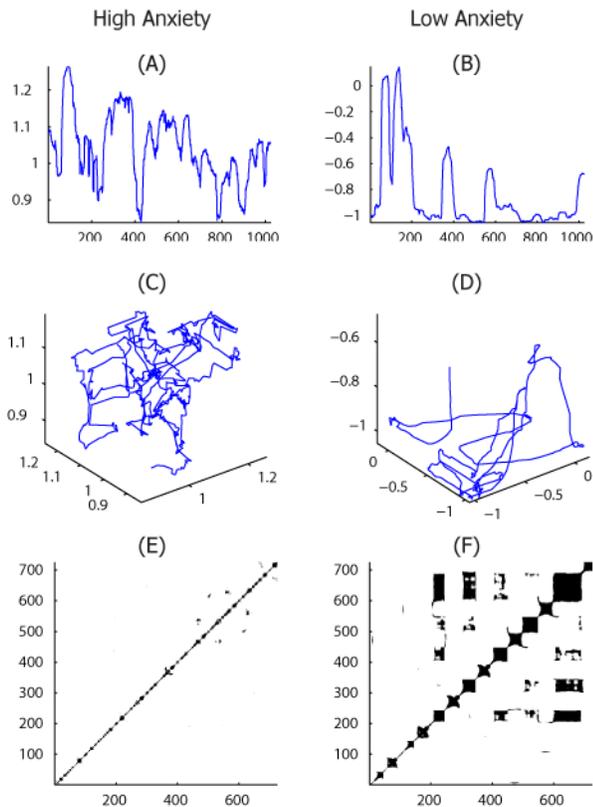

Figure 4. Bodily dynamics of high and low anxiety participants.

General Discussion

The majority of research on the embodiment of emotion has focused on facial movements, speech contours, and physiological measures, such as electromyography and electrocardiography (Calvo & D'Mello, 2010; Russell, et al., 2003). In our view, the discovery of systematic co-variation between the dynamics of bodily movement and affective experiences highlights the importance of the body as an important channel of affective expressions. Although facial expressions are considered to be strongly associated with affective expression, and entire theories of emotion are based on facial behavior alone (Ekman, 1984), meta-analyses on correlations between facial expressions and self-reported emotions have yielded small to medium effects for spontaneous expressions (Camras & Shutter, 2010; Fridlund, Ekman, & Oster, 1987; Ruch, 1995; Russell, et al., 2003). Although our results warrant replication, the reported correlations between the fractal dimension and affect were

consistent with medium to large effects. We also selected tasks that had some real-world relevance and methodologies with limited experimenter control thereby yielding naturalistic expressions of spontaneous affective states.

It is important to address a couple of potential concerns with the present methodology. This retrospective affect judgment methodology was adopted because it affords monitoring participants' affective states at multiple points, with minimal task interference, and without participants knowing that these states were being monitored. Although this affect judgment method has been previously used (Rosenberg & Ekman, 1994), producing similar distributions of states as online methods (Craig, D'Mello, Witherspoon, & Graesser, 2008), and the affective labels obtained correlate with online recordings of facial activity in expected directions (D'Mello & Graesser, 2010), there is the concern that showing participants videos of their faces might have introduced some methodological artifacts. The concern stems from the possibility that participants could have inferred their bodily motions from the videos of their faces and based their judgments on these bodily movements. This is an unfortunate complication that is difficult to mitigate because it is difficult to automatically segregate facial movement from general body movement. We would argue though that it is quite unlikely that participants could have perceived variations in the dynamical patterns, guessed our hypotheses, and selected their judgments accordingly.

We conclude by discussing some of non-obvious findings with respect to how affect is embodied in bodily movement. Boredom is typically considered to be associated with lower arousal (Pekrun, Goetz, Daniels, Stupnisky, & Raymond, 2010), while increased arousal is associated with engagement (Bianchi-Berthouze, Kim, & Patel, 2007). Hence, one counterintuitive finding is that boredom was associated with increased instead of diminished movement, a finding that has received some previous empirical support (Mota & Picard, 2003).

It is tempting to speculate on the exact nature of the bodily movements during the experience of anxiety and frustration. Are the movements associated simply more pronounced, jerky, and less fluid for these states of distress and subtle, smooth, and calm during normal cognitive functioning? This is a challenging question to answer by simply eyeballing the videos because the movements of individuals who experience heightened anxiety and frustration cannot be simply attributed to greater movement (mean of each time series) or larger variability in movement (standard deviation of each time series). Hence, it is not the magnitude or variability of bodily movements, but the *dynamics* of these movements that best explains these correlations.

It is important to align the present findings within some of the more recent complex systems approaches to emotion (Camras & Shutter, 2010; Coan, 2010; Lewis, 2005). Dynamical theories of emotion conceptualize emotions as emergent attractor states that trajectories converge upon when the cognitive-affective system is perturbed by an

internal or external event. The attractor landscape and the control parameters that modulate the dynamics of the system are ostensibly organized based on past experience, affective traits, social constraints, developmental changes, and a host of other factors. One can envision an attractor for anger, anxiety, frustration, and so on, each tightly coupled to an individuals' past experiences and evolutionary niche in the environment. The present results support the notion of cognitive-affective states causing and being caused by (circular causality) complex dynamical interactions between loosely-coupled entities by showing that there is systematic covariation between the experience of states, which are diagnostic of cognitive distress, and the body's ability to self-organize. A critical next step of this research is to test causal links between emotional states and bodily fluctuations in order to uncover the dynamical signature of specific discrete emotions, i.e. identifying an attractor for anxiety, or frustration, or confusion.

Acknowledgments

References

Bianchi-Berthouze, N., Kim, W., & Patel, D. (2007). Does body movement engage you more in digital game play? And Why? In A. Paiva, R. Prada & R. Picard (Eds.), *Proceedings of the Second International Conference on Affective Computing and Intelligent Interaction (ACII2007)* (pp. 102-113): Springer.

Bull, P. (1987). *Posture and gesture*. Oxford Pergamon Press.

Calvo, R. A., & D'Mello, S. K. (2010). Affect detection: An interdisciplinary review of models, methods, and their applications. *IEEE Transactions on Affective Computing, 1*(1), 18-37.

Camras, L., & Shutter, J. (2010). Emotional facial expressions in infancy. *Emotion review, 2(2)*(120-129).

Castellano, G., Mortillaro, M., Camurri, A., Volpe, G., & Scherer, K. (2008). Automated analysis of body movement in emotionally expressive piano performances. *Music Perception, 26*(2), 103-119.

Coan, J. A. (2010). Emergent ghosts of the emotion machine. *Emotion Review, 2*(3), 274-285.

Coulson, M. (2004). Attributing emotion to static body postures: Recognition accuracy, confusions, and viewpoint dependence. *Journal of Nonverbal Behavior, 28*(2), 117-139.

Craig, S., D'Mello, S., Witherspoon, A., & Graesser, A. (2008). Emote aloud during learning with AutoTutor: Applying the facial action coding system to cognitive-affective states during learning. *Cognition & Emotion, 22*(5), 777-788.

D'Mello, S., & Graesser, A. (2010). Multimodal semi-automated affect detection from conversational cues, gross body language, and facial features. *User Modeling and User-adapted Interaction, 20*(2), 147-187.

Ekman, P. (1984). Expression and the nature of emotion. In K. Scherer & P. Ekman (Eds.), *Approaches to emotion* (pp. 319-344). Hillsdale, NJ: Erlbaum.

Ekman, P. (1992). An argument for basic emotions. *Cognition & Emotion, 6*(3-4), 169-200.

Fridlund, A. J., Ekman, P., & Oster, H. (1987). Facial expressions of emotion. In A. W. Siegman & S. Feldstein (Eds.), *Nonverbal behavior and communication* (2nd ed., pp. 143–223). Hillsdale, NJ: Erlbaum.

Lewis, M. D. (2005). Bridging emotion theory and neurobiology through dynamic systems modeling. *Behavioral and Brain Sciences, 28*(2), 169-245.

Lorenz, E. N. (1963). Deterministic Nonperiodic Flow. *Journal of Atmospheric Sciences, 20*, 130-141.

Marwan, N., Romano, M., Thiel, M., & Kurths, J. (2007). Recurrence plots for the analysis of complex systems. *Physics Reports, 438*(5-6), 237-329.

Mota, S., & Picard, R. (2003). Automated Posture Analysis for Detecting Learner's Interest Level *Computer Vision and Pattern Recognition Workshop, 2003. CVPRW '03. Conference on* (Vol. 5, pp. 49-49).

Niedenthal, P. M. (2007). Embodying emotion. [Review]. *Science, 316*(5827), 1002-1005.

Oullier, O., & Basso, F. (2010). Embodied economics: how bodily information shapes the social coordination dynamics of decision-making. *Philosophical Transactions of the Royal Society B-Biological Sciences, 365*(1538), 291-301.

Pekrun, R., Goetz, T., Daniels, L., Stupnisky, R. H., & Raymond, P. (2010). Boredom in achievement settings: Exploring control–value antecedents and performance outcomes of a neglected emotion. *Journal of Educational Psychology, 102*(3), 531-549.

Rosenberg, E., & Ekman, P. (1994). Coherence between expressive and experiential systems in emotion. *Cognition & Emotion, 8*(3), 201-229.

Ruch, W. (1995). Will the real relationship between facial expression and affective experience please stand up - the case of exhilaration. *Cognition & Emotion, 9*(1), 33-58.

Russell, J. A., Bachorowski, J. A., & Fernandez-Dols, J. M. (2003). Facial and vocal expressions of emotion. *Annual Review of Psychology, 54*, 329-349.

Scherer, K., & Ellgring, H. (2007). Are facial expressions of emotion produced by categorical affect programs or dynamically driven by appraisal? *Emotion, 7*(1), 113-130.

Takens, F. (1981). Detecting strange attractors in turbulence. *Lecture Notes in Mathematics*, 366-381.

Webber, C. L., & Zbilut, J. P. (2005). Recurrence quantification analysis of nonlinear dynamical systems. In M. A. Riley & C. C. V. Orden (Eds.), *Contemporary nonlinear methods for behavioral scientists : A webbook tutorial*.