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Essays on Asset Prices, Expectations Formation and Nonlinear Macroeconomic Effects

DISSERTATION

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in Economics

by

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ABSTRACT OF THE DISSERTATION

Essays on Asset Prices, Expectations Formation and Nonlinear Macroeconomic Effects

By

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Doctor of Philosophy in Economics

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Professor Fabio Milani, Chair

My dissertation's main objective is to estimate the effects of asset prices and expectations over the business cycle, considering potential nonlinear effects. Moreover, I am interested in quantifying the impact of the financial sector, both domestic and from abroad, on the economy.

In this tenor, in Chapter 1, I study the effects of house prices on the economy by introducing adaptive learning expectations formation in a DSGE model with housing. This framework provides flexibility in beliefs to match the non-rational behavior of house price expectations. Additionally, I can capture the evolving effects of extrapolative expectations on house prices on the economy. The results suggest that the feedback from house price beliefs into the economy was more severe around the period of the housing price bubble and continued to exist, in a lower magnitude, around the Great Recession.

Meanwhile, in Chapter 2, we further consider the effects of house prices on credit markets' conditions, for which we introduced a banking sector in the model. The results suggest that agents' expectations amplify the credit supply during episodes of asset bubbles. Additionally, we find that macro-prudential policies may lessen the response of financial intermediaries to the housing shocks, magnified by the learning dynamics.

In this regard, I found it essential to consider the effects of agents' sentiment over asset prices and the business cycle. Moreover, Adaptive Learning allows me to analyze this behavioral element jointly with other factors to separate their effects.

Finally, I have also explored the asymmetry in the business cycle, intending to create a more efficient estimator. Given the documented asymmetries in business cycles, it is vital to consider nonlinear DSGE models to better approximate the data.

In this context, the "occasionally binding constraints" is one avenue used to address the nonlinearity estimation challenge. In Chapter 3, I revisit this issue with an MCMC algorithm based on a mixture model. By carefully defining the sampling scheme, I can make most of the draws directly from their conditional distribution with a Gibbs sampler step. As a result, the algorithm features fast convergence and low inefficiency factors.

Chapter 1

Macroeconomic Effects of House Prices under Adaptive Learning Expectations

1.1 Introduction

An important part of households' wealth in the United States is invested in housing. A house is a particular type of asset that provides a utility flow from its housing services and can be used as collateral for loans. The borrowing capacity for this type of loan depends on future house prices. Therefore, it is crucial to understand how agents form house price expectations.

There is evidence that suggests that house price expectations do not adhere to the rationality assumption, such that prices are prone to bubbles[1]. Glaeser and Gyourko (2006) have

[1]Rational agents would expect lower returns after a sustained rise in prices, while irrational agents would continue to predict higher returns.

difficulties explaining the positive serial correlation in house price changes observed in the data with a dynamic model for housing under rational expectations. Piazzesi and Schneider (2009), using data from the Michigan Survey of Consumers, observed that "*starting in 2004, more and more households became optimistic after having watched house prices increase for several years*". Additionally, Gelain and Lansing (2014), using a Lucas-type asset pricing model, demonstrate that the model can approximately match the volatility of the price-rent ratio in the data using near-rational agents.

Asset price bubbles could lead to relaxed credit conditions, and households would be inclined to increase their borrowing because their beliefs indicate that prices would continue to grow. When bubbles burst, there will be a tightening of credit, and debtors will have to reduce consumption and adjust their portfolios to pay their debt. In this context, bubbles amplify the effects of credit frictions on the economy.

In my paper, I study the effects of the housing price bubble on the economy. Firstly, to tackle this, I required a model that incorporates housing in the utility of households and their budget and borrowing constraints. For this, I use a DSGE model developed by Iacoviello (2005), in which the ceiling in the households' capacity to borrow depends on house price expectations.

Secondly, I needed to model how agents formed their expectations. Here, I assume that they update their beliefs using adaptive learning. This framework provides sufficient flexibility in beliefs to match the documented behavior of house price expectations, in contrast with the rationality assumption that implies a rigid structure of expectations.

My model can capture the evolving effects of a house price bubble on the economy via the expectations effects in the credit channel, and I estimated it using Bayesian methods. I should note that adaptive learning allowed me to identify the evolving response to shocks during the different stages of the house price bubble, as opposed to Rational Expectations, which assume a constant response across the sample.

The results suggest that the feedback from house price beliefs into the economy was more severed around the period of the housing price bubble. In this period, a monetary policy shock would have caused a severe drop in activity because the burst of the bubble would have been drastic. The expectations channel still had an effect around the Great Recession, but the effects were not as significant as during the peak of the bubble.

The remainder of my paper is organized as follows. In Section 1.2, I review the related literature on house price models. In Section 1.3, I describe the model of the economy and the expectations formation. In Section 1.4, I detail the estimation and data description, while in Section 1.5, I report my results, both under Rational Expectations and under Adaptive Learning. Finally, in Section 1.6, I give my conclusions.

1.2 Related literature

Given its importance in households' portfolios, extensive research has been created to analyze the spillover effects of house prices in the economy. Therefore, the papers that I would cite here should not be seen as an exhaustive list of all the work done in the area. I contribute to this literature by estimating a model that can seize the evolving effects of house prices on the economy via extrapolative expectations formation captured with the evolution of households' beliefs.

Iacoviello (2005) provides a framework to analyze the house prices effects in a DSGE model. In this context, Iacoviello and Neri (2010) estimate the model using US data, adding habit formation and other potential structural shocks, in a model that considers both the supply and demand dynamics of the housing sector.

Following the same framework, Guerrieri and Iacoviello (2017) estimate a nonlinear general equilibrium model where occasionally binding collateral constraints drive an asymmetry in the link between housing prices and economic activity. They found that before the financial crisis, collateral constraints became slack, so housing wealth generated a small contribution to aggregate consumption in the economy. In contrast, the collapse in the housing sector caused a tight borrowing constraint, which exacerbated the recession of 2007-2009.

In a model with frictions in credit markets used by households, where houses provide housing services and are used as collateral to lower borrowing costs, Aoki et al. (2004) showed that this amplifies and propagates the effect of monetary policy shocks on housing investment, house prices, and consumption. On the other hand, Justiniano et al. (2015) calibrated a DSGE model and found that, from the perspective of their model, the credit cycle is more likely due to factors that impact house prices more directly, thus affecting the availability of credit through a change in collateral values, while the macroeconomic consequences of leveraging and deleveraging are relatively minor in the aggregate.

Departing from the Rational Expectations assumption is the work done by Adam et al. (2012). They introduce Bayesian learning into an open economy asset pricing model and calibrate it to replicate the house price and the current account dynamics for the G7 economies over the years 2001–2008. In a closed economy framework, Branch et al. (2016) construct a theoretical search model with an adaptive learning rule and calibrate it to match the dynamics of US house prices, sectoral labor flows, and unemployment rate changes from 1996 to 2010.

Gelain et al. (2013) introduced hybrid expectations into the model's basic structure proposed in Iacoviello (2005). In their model, expectations are modeled as a weighted average of fully rational and moving average forecast rules. They calibrate the hybrid expectations model parameters to generate an empirically plausible degree of volatility in the simulated house price, household debt, and real output series.

Under a small open economy DSGE model, Milani and Park (2019) estimated macro-housing interactions under rational and non-fully rational expectations, using data from Korea. They found that spillovers from the housing market to the macroeconomy are substantially more significant under non-rational housing expectations.

1.3 Model

For this paper, I follow closely the model done by Iacoviello (2005) and introduce adaptive learning for the formation of expectations[2].

There are three types of households in the economy: entrepreneurs, patient and impatient. Patient households (identified with a prime) maximize a lifetime utility function given by:

$$\hat{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln c'_t + j_t \ln h'_t - \frac{(L'_t)^\eta}{\eta} \right)$$

subject to the following budget constraint:

$$c'_t + q_t \Delta h'_t + \tfrac{R_{t-1} b'_{t-1}}{\pi_t} = b'_t + w'_t L'_t + div_t - \xi'_{h,t}$$

where $\hat{E}_0$ is the adaptive expectations operator[3], β is the discount factor, c'_t is consumption at time t, h'_t denotes the holdings of housing, L'_t are the hours worked, and j_t represents random disturbances to the marginal utility of housing. Let $q_t \equiv Q_t/P_t$ be the real housing price, $w'_t \equiv W_t/P_t$ are real wages, $\pi_t \equiv P_t/P_{t-1}$ is gross inflation, div_t are lump-sum profits received from the retailers, Δ represents the first difference operator and $\xi'_{h,t} \equiv \phi_h \left(\frac{\Delta h'_t}{h'_{t-1}} \right)^2 \frac{q_t h'_{t-1}}{2}$ denotes the housing adjustment cost. Additionally, it is assumed that households borrow in real terms, $b'_t \equiv B'_t/P_t$ and pay back $R_{t-1} B'_{t-1}/P_t$, where R_{t-1} is the nominal interest rate

[2]Here, I present a brief description of the model. For more details, please see Iacoviello (2005).

[3]I follow the convention that the rational expectations operator is defined as E_t, while the adaptive expectations operator is identified with a hat: $\hat{E}_t$.

on loans between $t-1$ and t.

Analogously, impatient households (identified with a double prime) maximize:

$$\hat{E}_0 \sum_{t=0}^{\infty} (\beta'')^t \left(\ln c''_t + j_t \ln h''_t - \frac{(L''_t)^\eta}{\eta} \right)$$

subject to the following budget constraint:

$$c''_t + q_t \Delta h''_t + \tfrac{R_{t-1} b''_{t-1}}{\pi_t} = b''_t + w''_t L''_t - \xi''_{h,t}$$

and the borrowing constraint:

$$b''_t \leq m'' \hat{E}_t \left(\tfrac{q_{t+1} h''_t \pi_{t+1}}{R_t} \right)$$

where β'' is the impatient households discount factor, m'' represents the "loan-to-value" ratio parameter, such that the maximum amount b''_t that an impatient household can borrow is bound by $m'' \hat{E}_t (q_{t+1} h''_t \pi_{t+1} / R_t)$.

In terms of entrepreneurs, it is assumed that they maximize the following lifetime utility function:

$$\hat{E}_0 \sum_{t=0}^{\infty} \gamma^t \ln c_t$$

subject to the production function:

$$Y_t = A_t K^{\mu}_{t-1} h^{\nu}_{t-1} L'^{\alpha(1-\mu-\nu)}_t L''^{(1-\alpha)(1-\mu-\nu)}_t$$

the budget constraint:

$$Y_t / X_t + b_t = c_t + q_t \Delta h_t + \tfrac{R_{t-1} b_{t-1}}{\pi_t} + w'_t L'_t + w''_t L''_t + I_t + \xi_{e,t} + \xi_{K,t}$$

and the borrowing constraint:

$$b_t \leq m\hat{E}_t\left(\frac{q_{t+1}h_t\pi_{t+1}}{R_t}\right)$$

where γ is their corresponding discount factor, Y_t is the production of intermediate good that entrepreneurs sell in a competitive market at price P_t^w, A_t is a random technology shock, K_t is capital (that depreciates at rate δ) created at the end of each period, X_t is the retailers gross markup over the intermediate goods price, $I_t = K_t - (1-\delta)K_{t-1}$ is the investment on capital, $\xi_{e,t}$ defines the entrepreneurs' housing adjustment cost, while $\xi_{K,t} \equiv \psi\left(\frac{I_t}{K_{t-1}-\delta}\right)^2 \frac{K_{t-1}}{2\delta}$ represents a capital installation cost, and the parameter m is the loan-to-value ratio for entrepreneurs.

One can notice that each household have its own discount factor. It is assumed that $\beta'' < \beta$, and $\gamma < \beta$, so that the borrowing constraint becomes binding for these types of households[4]. This assumption also implies a steady state in which the entrepreneurial return to savings is greater than the interest rate.

There is a retail sector in the economy with monopolistic competition, where it is assumed implicit costs of adjusting nominal prices. These retailers buy intermediate goods from entrepreneurs, differentiate the goods at no cost, and sell the final goods with a markup. Patient households own these firms, so the profits are rebated back to them.

Finally, there is a central bank that implements a Taylor-type interest rate rule. I assume that monetary policy responds systematically to past inflation and output (i.e., a backward-looking Taylor rule).

The market clearing conditions[5] are: for real estate ($h_t + h'_t + h''_t = 1$); for goods ($c_t + c'_t + c''_t + I_t = Y_t$); and for loans ($b_t + b'_t + b''_t = 0$). The model has a unique stationary

[4] Iacoviello (2005) assumes $\gamma = 0.98$, $\beta = 0.99$, and $\beta'' = 0.95$.

[5] For the labor market, we are assuming that all the supply of patient and impatient households will be used by entrepreneurs: $L'_t + L''_t = L_t$.

equilibrium in which entrepreneurs and impatient households hit the borrowing constraint and borrow up to the limit. The model can be reduced to linearized system (see appendix), which summarized in matrix form can be express as:

$$A_0\xi_t = A_1\hat{E}_t\xi_{t+1}^f + A_2\xi_{t-1} + \Psi\epsilon_t \tag{1.1}$$

where the variables, ξ_t, and its subset of forward variables, ξ_t^f, are[6]:

$$\xi_t^f = [\hat{c}_t, \hat{c}_t', \hat{c}_t'', \hat{I}_t, \hat{X}_t, \hat{h}_t, \hat{h}_t'', \hat{q}_t, \hat{Y}_t, \hat{\pi}_t];$$
$$\xi_t = [\xi_t^f, \hat{R}_t, \hat{K}_t, \hat{b}_t, \hat{b}_t'', \hat{j}_t, \hat{u}_t, \hat{A}_t];$$

1.3.1 Expectations

In order to form their expectations, I assume that the agents have the following perceived law of motion (PLM):

$$\xi_t^f = X_{t-1}\phi_{t-1} + u_t$$

where ϕ_{t-1} is a vector containing the learning beliefs parameters, and the regressors matrix X_{t-1} is constructed in SUR form[7]. Therefore, the equation above can be described in more

[6]The hat over the variables is the notation to represent deviation from its steady-state.
[7]SUR is the acronym for seemingly unrelated regressions.

detail as[8]:

$$\begin{bmatrix} \xi^f_{1,t} \\ \xi^f_{2,t} \\ \vdots \\ \xi^f_{m,t} \end{bmatrix} = \begin{bmatrix} X_{1,t-1} & & & \\ & X_{2,t-1} & & \\ & & \ddots & \\ & & & X_{m,t-1} \end{bmatrix} \cdot \begin{bmatrix} \phi_{1,t-1} \\ \phi_{2,t-1} \\ \vdots \\ \phi_{m,t-1} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{m,t} \end{bmatrix}.$$

The errors u_t depend on a linear combination of the true model innovations, ϵ_t, with covariance matrix: $\Sigma = E[u_t \cdot u_t^T]$. With this PLM, we have that:

$$\hat{E}_t \xi^f_{t+1} = X_t \phi_t \tag{1.2}$$

By substituting these expectations in the linearized model described in equation (1), we can obtain the Actual Law of Motion (ALM).

1.3.2 Evolution of beliefs

Concerning the evolution of the beliefs, I follow Slobodyan and Wouters (2012) and assume that agents use Bayesian learning and update their beliefs with the help of a Kalman filter mechanism.

It is assumed that the beliefs follow a vector auto-regressive process around $\bar{\phi}$:

$$(\phi_t - \bar{\phi}) = F_\varrho(\phi_{t-1} - \bar{\phi}) + v_t,$$

where F_ϱ is a diagonal matrix with parameter $\rho_\phi \leq 1$ on the main diagonal, and it is assumed that v_t are i.i.d. with covariance matrix V.

[8]Each $X_{i,t-1}$ is the information set used by agents to form their PLM of variable $\xi^f_{i,t}$ and $\phi_{i,t-1}$ are the adaptive learning beliefs associated with the corresponding forward variable.

The Kalman filter updating and transition equations for the belief coefficients and the corresponding covariance matrix are given by:

$$\phi_{t|t} = \phi_{t|t-1} + R_{t|t-1}X_{t-1}^T[\Sigma + X_{t-1}R_{t|t-1}X_{t|t-1}^T]^{-1}(\xi_t^f - X_{t-1}\phi_{t|t-1}) \tag{1.3}$$

$$\text{with } (\phi_{t+1|t} - \bar{\phi}) = F_\varrho(\phi_{t|t} - \bar{\phi}).$$

$$R_{t|t} = R_{t|t-1} - R_{t|t-1}X_{t-1}^T[\Sigma + X_{t-1}R_{t|t-1}X_{t|t-1}^T]^{-1}X_{t-1}R_{t|t-1} \tag{1.4}$$

$$\text{with } R_{t+1|t} = F_\varrho R_{t|t} F_\varrho^T + V.$$

From the transition equation, we have that these best estimates for the beliefs, $\phi_{t|t-1}$, are substituted for ϕ_t in equation (2) to generate the expectations and get the ALM. Note that by using $\phi_{t|t-1}$, we can avoid the simultaneity between expectations and latent state variables[9].

In order to initialize the Kalman filter, I assume that the initial beliefs are model consistent, and specify $\phi_{1|0} = \bar{\phi}$, $R_{1|0}$, Σ, and V using the theoretical moment matrices implied by the REE evaluated under the corresponding structural parameter vector θ. Therefore, we have that:

$$\begin{aligned}
\bar{\phi} &= E[X^T X] \cdot E[X^T \xi^f] \\
\Sigma|\bar{\phi} &= E[(\xi_t^f - X_{t-1}\bar{\phi}) \times (\xi_t^f - X_{t-1}\bar{\phi})^T] \\
R_{1|0} &= \sigma_0(X^T\Sigma^{-1}X)^{-1} \\
V &= \sigma_v(X^T\Sigma^{-1}X)^{-1}
\end{aligned}$$

Where we can see that the matrices $R_{1|0}$ and V are proportional to $(X^T\Sigma^{-1}X)^{-1}$.[10] With

[9] A similar Bayesian learning process is described in Adam et al. (2012), in the context of an asset pricing model. Adam and Marcet (2010, 2011) provide a micro-founded belief system justifying the assumption that information on prices is introduced with a delay in the formation of the beliefs.

[10] Note: the generalized least squares estimator for $\bar{\phi}$ is $\hat{\phi}_{gls} = (X^T\Sigma^{-1}X)^{-1}(X^T\Sigma^{-1}\xi^f)$. Therefore, we have that $var(\hat{\phi}_{gls}) = (X^T\Sigma^{-1}X)^{-1}$.

this initialization we can see that the scalar parameters σ_0, σ_v, and ρ_ϕ fully describe the learning dynamics.

Finally, we have that the state-space model is given by the following measurement and state transition equations:

$$Observations_t = H\xi_t \tag{1.5}$$

$$\xi_t = \gamma_t + F_t\xi_{t-1} + G_t\epsilon_t \tag{1.6}$$

where $\epsilon_t \sim N(0, \Omega)$, H is a matrix of zeros and ones just selecting observables from ξ_t, and γ_t, F_t, and G_t are matrices of coefficients, which are convolutions of structural parameters of the economy and the best estimates of agents' beliefs, $\phi_{t|t-1}$.

1.4 Estimation

Following Slobodyan and Wouters (2012), I fixed the scale parameters $\sigma_0 = 0.03$ and $\sigma_v = 0.003$, and estimate ρ_ϕ using a diffuse beta prior.

The estimation of the structural parameters of the model was done with a Metropolis-Hastings random walk algorithm:

$$\theta = \{\alpha, m'', \rho_u, \rho_j, \rho_A, \sigma_u, \sigma_j, \sigma_A, \sigma_R, \rho_R, r_\pi, r_y, \rho_\phi\}.$$

The likelihood is recursively evaluated with the Kalman filter, using equations (5) and (6). At each moment, the beliefs are updated using equations (3) and (4).

The priors used for the parameters and their description are given in Table 1. Following Iacoviello (2005) and some of the findings of Guerrieri and Iacoviello (2017), I calibrated the parameters with the values exhibit in Table 2.

	Description	Distrib.	Mean	SD
α	Patient household's wage share	B	0.65	0.05
m''	Loan-to-value for impatient households	B	0.55	0.05
ρ_u	AR coefficient - Cost-push shock	B	0.50	0.28
ρ_j	AR coefficient - Housing preference	B	0.50	0.28
ρ_A	AR coefficient - Technology	B	0.05	0.015
σ_u	SD - Cost-push shock	Γ^{-1}	1	100
σ_j	SD - Housing preference	Γ^{-1}	1	100
σ_A	SD - Technology	Γ^{-1}	1	100
σ_R	SD - Monetary Policy shock	Γ^{-1}	1	100
ρ_R	AR - interest rate	B	0.50	0.28
r_π	Monetary Policy response to inflation	N	1.5	0.25
r_y	Monetary Policy response to output gap	B	0.125	0.025
ρ_ϕ	Learning beliefs' persistence	B	0.50	0.28

Table 1.1: Prior distributions.

	Description	Value	Based on
β	Patient household's discount rate	0.995	G and IA (2017)
γ	Entrepreneurs household's discount rate	0.98	IA (2005)
β''	Impatient household's discount rate	0.95	IA (2005)
δ	Variable capital depreciation rate	0.03	IA (2005)
X	Steady-state gross markup	1.2	G and IA (2017)
θ^C	Probability fixed price (Calvo parameter)	0.89	G and IA (2017)
j	Weight on housing services	0.04	IA (2005)
μ	Variable capital share	0.3	IA (2005)
ν	Housing share	0.03	IA (2005)
η	Labor supply aversion	1.01	IA (2005)
ψ	Variable capital adjustment cost	4	G and IA (2017)
ϕ^H	Housing adjustment cost	0	IA (2005)
m	Loan-to-value for entrepreneurs	0.9	G and IA (2017)
π	Steady-state Gross inflation rate	1	IA (2005)

Table 1.2: Calibrated parameters.

1.4.1 Data

As the sample for estimation, I set the period from I-1985 to I-2019. Following the work done by Iacoviello (2005), I use the following variables[11]: the GDP and GDP deflator published by the Bureau of Economic Analysis (BEA), the potential GDP published by the Congressional Budget Office (CBO), the new home sales price reported by the Census Bureau, the consumer price index from the Bureau of Labor Statistics (BLS), the Federal Reserve's Effective Federal Funds Rate and the shadow interest rate for the zero-lower-bound period estimated by Wu and Xia (2015)[12].

The output gap is simply the difference between the GDP and potential GDP series, converted to a logarithmic scale. The real house prices were constructed by deflating the new house sales median prices by the CPI, and seasonal effects adjusted it, and finally detrended using a linear trend.

The inflation was estimated from the difference, in logarithms, of the GDP deflator and demeaned. The interest rate, complemented with the shadow rate, was converted to a quarterly rate and demeaned. One can see the evolution of the variables in Figure 1.

1.5 Results

1.5.1 Rational expectations

As a first step, I estimated the model under Rational Expectations. One can see the summary of the posterior draws in Table 3. In the results, we can see that both the patient households' wage share, α, and the Loan-to-value for impatient households', m'', lie in range with the

[11] The data was downloaded from The Federal Reserve Bank of St. Louis' FRED.

[12] This series was downloaded from the Federal Reserve Bank of Atlanta website: `https://www.frbatlanta.org/cqer/research/shadow_rate.aspx`.

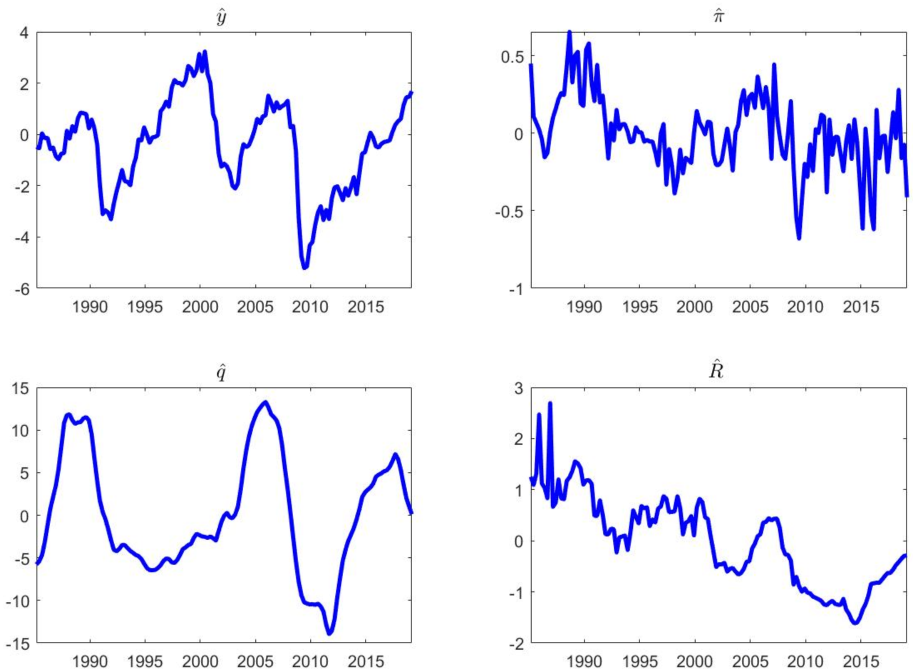

Figure 1.1: Evolution of the observed variables - Sample I-1985 to I-2019.

original estimations by Iacoviello (2005).

Consistent with more recent estimations done by Iacoviello and Neri (2010) and Guerrieri and Iacoviello (2017), we can see a reduction in the standard deviation of the housing preference shock, σ_j, in contrast to the results reported in Iacoviello (2005) of 24.89[13]. In addition, my estimation suggests an autocorrelation coefficient of the housing preference shock, ρ_j, of 0.994, which is higher than the 0.85 estimated in Iacoviello (2005), but in line with the 0.96 posterior mean of Iacoviello and Neri (2010) and 0.9835 posterior mode of Guerrieri and Iacoviello (2017).

Consistent with the phenomenon called "Great Moderation," my estimation suggests a considerable reduction in the standard deviation of the technology shock, σ_A, compared with

[13]Iacoviello and Neri (2010) estimated a mean of 4.16 percent, with data from I-1965 to IV-2006. Guerrieri and Iacoviello (2017) reported a posterior mode of 5.13 percent, and the data sample period is from I-1985 to IV-2011. Nonetheless, in both cases, the models also consider an intertemporal preference shock

Variable Name	Mode	Mean	5%	95%
α	0.5957	0.5991	0.5331	0.6629
m''	0.6094	0.6056	0.5435	0.6647
ρ_u	0.2624	0.2602	0.1685	0.3595
ρ_j	0.9941	0.9941	0.9899	0.9973
ρ_A	0.0299	0.0378	0.0096	0.0805
σ_u	0.1358	0.1378	0.1125	0.1664
σ_j	12.788	12.918	9.411	17.385
σ_A	0.8136	0.9685	0.4695	1.7989
σ_R	0.3527	0.3515	0.3179	0.3884
ρ_R	0.6767	0.6755	0.6228	0.7242
r_π	1.7310	1.7423	1.5763	1.9240
r_y	0.2740	0.2711	0.2313	0.3131
MgLikelihood		-1,098.4		

Table 1.3: REE - Posterior draws.

Note: 500,000 draws, set burn-in of 25 percent.

the 2.24 estimated in Iacoviello (2005)[14]. I obtained similar results in terms of the autocorrelation coefficient of the technology shock, ρ_A, which suggests that this shock has low persistence.

My estimation for the standard deviation for the cost-push shock, σ_u, of 0.136 percent is slightly lower than Iacoviello's 2005 estimation of 0.17 percent. Regarding this shock's autocorrelation coefficient, ρ_u, my estimation of 0.26 is considerably lower than the 0.59 estimated originally by Iacoviello.

Finally, concerning the Taylor rule estimated parameters, I found slightly more inertia than Guerrieri and Iacoviello (2017) most recent estimation of 0.5509 posterior mode[15]. My rational expectations results of the response to inflation have a slightly higher mode (1.73 vs. 1.54 in G and IA (2017)). However, the range of both estimations overlaps, while the response to the output gap is considerably higher in my analysis (0.274) compared to Guerrieri and Iacoviello's 0.0944 posterior mode.

[14]The estimation in Iacoviello (2005) was made with VAR with data from I-1974 to II-2003.

[15]I considered the most recent estimation of the Taylor rule to compare with my results since both computations consider the period of the ZLB. Iacoviello (2005) ran an OLS regression, for the period that spanned from I-1974 to II-2003, of the Fed Funds rate on its lag, past inflation, and detrended output yields and found $\rho_R = 0.73$, $r_y = 0.13$, and $r_\pi = 1.27$.

1.5.2 MSV Adaptive learning expectations

To proceed with the estimation, I further assume that the agents hold near-rational expectations by updating their beliefs around the minimal state variable solution (MSV). Therefore, I posit that the information set available to the agents to form the PLM of each variable in ξ^f is the same and equal to:

$$X_{i,t} = [\hat{h}''_t, \hat{h}_t, \hat{Y}_t, \hat{\pi}_t, \hat{R}_t, \hat{K}_t, \hat{b}_t, \hat{b}''_t, \hat{j}_t, \hat{u}_t, \hat{A}_t].$$

The summary of posterior draws is in Table 4, where we can observe an improvement in the marginal likelihood compared with the estimation under rational expectations. It is also noteworthy the reduction in the cost-push shock persistence, ρ_u, which is a result consistent with previous literature. This result is in line with the idea that the propagation of these shocks under learning is captured by the expectations' mechanism and by the internal dynamics of the decision rules.

Note that the estimation for the loan-to-value ratio for impatient households, m'', and the patient households' wage share, α, are similar in both analyses. We can see that the structural parameters that govern the agents' decision rules remain relatively robust under both versions considered for the expectations. Slobodyan and Wouters (2012) mentioned that learning dynamics might explain what was previously thought to be exogenous persistence but leave the model's structural parameters unaffected.

1.5.3 Adaptive learning and house prices as a state variable

Turning our attention to the borrowing constraint of the agents, we can notice that, considering the assumption that is binding, the evolution of house prices affects how much agents

Variable Name	Mode	Mean	5%	95%
α	0.5931	0.5783	0.4899	0.6608
m''	0.6208	0.6048	0.5257	0.6680
ρ_u	0.0018	0.0150	0.0006	0.0650
ρ_j	0.9913	0.9916	0.9862	0.9967
ρ_A	0.0421	0.0451	0.0252	0.0720
σ_u	0.0877	0.0903	0.0640	0.1207
σ_j	13.640	13.324	8.885	18.032
σ_A	0.3549	0.4724	0.1895	0.9195
σ_R	0.3431	0.3453	0.3111	0.3835
ρ_R	0.6623	0.6607	0.6033	0.7128
r_π	1.5306	1.5107	1.3300	1.7004
r_y	0.2763	0.2736	0.2333	0.3136
ρ_ϕ	0.9467	0.9471	0.9346	0.9600
MgLikelihood		-1,033.9		

Table 1.4: AL-MSV Posterior draws.

Note: 500,000 draws, set burn-in of 25 percent.

will be able to borrow each period. Therefore, we could assume that agents will use house prices as a state variable instead of the current debt amount. This assumption is plausible since agents determine how much to borrow given the value of their housing holdings, and they take house prices as given.

With this in consideration, I change the information set used by the agents in their PLM and include house prices as a state variable:

$$X_{i,t} = [\hat{h}''_t, \hat{h}_t, \hat{q}_t, \hat{Y}_t, \hat{\pi}_t, \hat{R}_t, \hat{K}_t, \hat{j}_t, \hat{u}_t, \hat{A}_t].$$

I present the results obtained under this assumption in Table 5, where we can observe that this specification delivers a higher marginal likelihood than in my previous estimations. Once again, the structural parameters remain relatively robust. More importantly, by having house prices as a state variable, we can now study the feedback effects of house prices through the agents' beliefs evolution.

In Figure 2, I present the evolution of the real house prices cycle and the beliefs persistence.

Variable Name	Mode	Mean	5%	95%
α	0.4785	0.4921	0.4273	0.5672
m''	0.5783	0.5811	0.5356	0.6252
ρ_u	0.0119	0.0271	0.0061	0.0526
ρ_j	0.9906	0.9900	0.9846	0.9936
ρ_A	0.0567	0.0497	0.0236	0.0769
σ_u	0.0979	0.1026	0.0702	0.1413
σ_j	11.701	12.198	9.954	15.417
σ_A	0.3545	0.5433	0.1937	1.1694
σ_R	0.3222	0.3351	0.3035	0.3706
ρ_R	0.6589	0.6497	0.5915	0.6974
r_π	1.4492	1.4608	1.3395	1.5867
r_y	0.3026	0.3024	0.2662	0.3392
ρ_ϕ	0.9615	0.9609	0.9553	0.9653
MgLikelihood		-1,016.1		

Table 1.5: AL Posterior draws.

Note: 500,000 draws, set burn-in of 25 percent.

Here we can observe that the beliefs seem to capture the 2005 house price bubble and its burst.

The consistent increase in house prices above the trend started in the first quarter of 2003. This date also coincided with the beginning of the rise in persistence in house prices' beliefs. Moreover, the persistence continues to be relatively high, a result consistent with the idea that house price expectations do not comply with the rationality assumption.

To see the feedback effects of the beliefs' evolution over the economy, I estimated the impulse response to a housing preference shock (Figure 3). The I-85 should be seen as close to Rational Expectations IRF since, by assumption, I initialize the learning beliefs as model consistent. The IV-05 coincides with the peak of the house prices cycle, while the IV-08 is close to the start of the financial crisis. Finally, IV-18 gives us a picture of the most recent state of responses.

The impulse response functions suggest a small effect of housing preference shocks on inflation and interest rates. As one should expect, a housing preference shock affects house prices, and given the persistence in the beliefs, the effect does not die out. Finally, the impact on

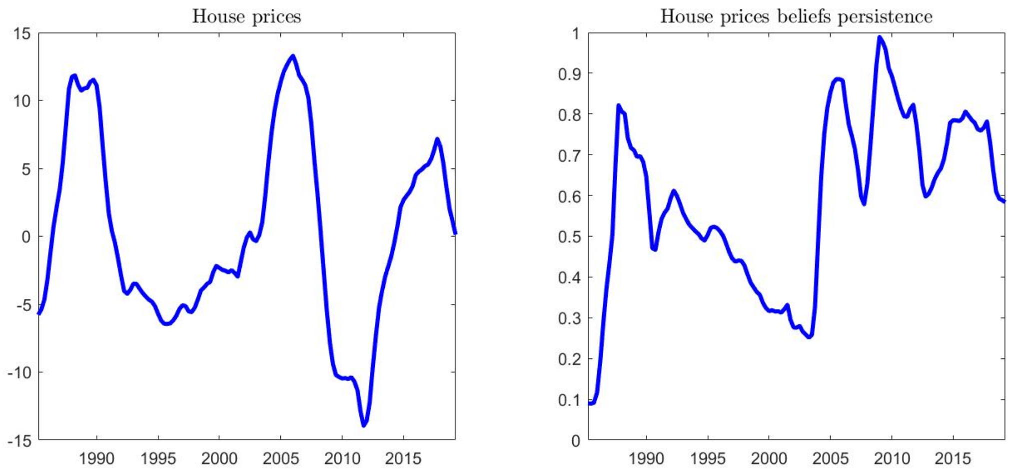

Figure 1.2: Real house prices and the adaptive persistence beliefs for house prices.

output is positive, which comes from the response of entrepreneurs to increase their housing holdings (not shown), which is an input in their production function. In general, we can see that the feedback from the shocks is higher at the peak of the bubble (IV-05).

The higher impact around the bubble is also observable in the IRF of consumption (Figure 4). Here, the model suggests that while patient and impatient households reduce their consumption, given a higher substitution effect, entrepreneurs will increase their consumption. This result can be explained through an income effect, given the increase in production. Since the response of patient and impatient households is small relative to the rise in entrepreneurs' consumption, the IRF suggests a positive response in aggregate consumption.

It is also interesting to see the response to a monetary policy shock (Figures 5 and 6), where we can still observe that the most prominent response is estimated for the peak of the house price bubble (IV-05). This behavior is consistent with the idea that we would have experienced abrupt reactions to house prices, consumption, and output by bursting a bubble with a monetary policy shock. The model and evolution of beliefs for that period suggest an increase in inflation, which, combined with the fall in output, implies a stagflation scenario.

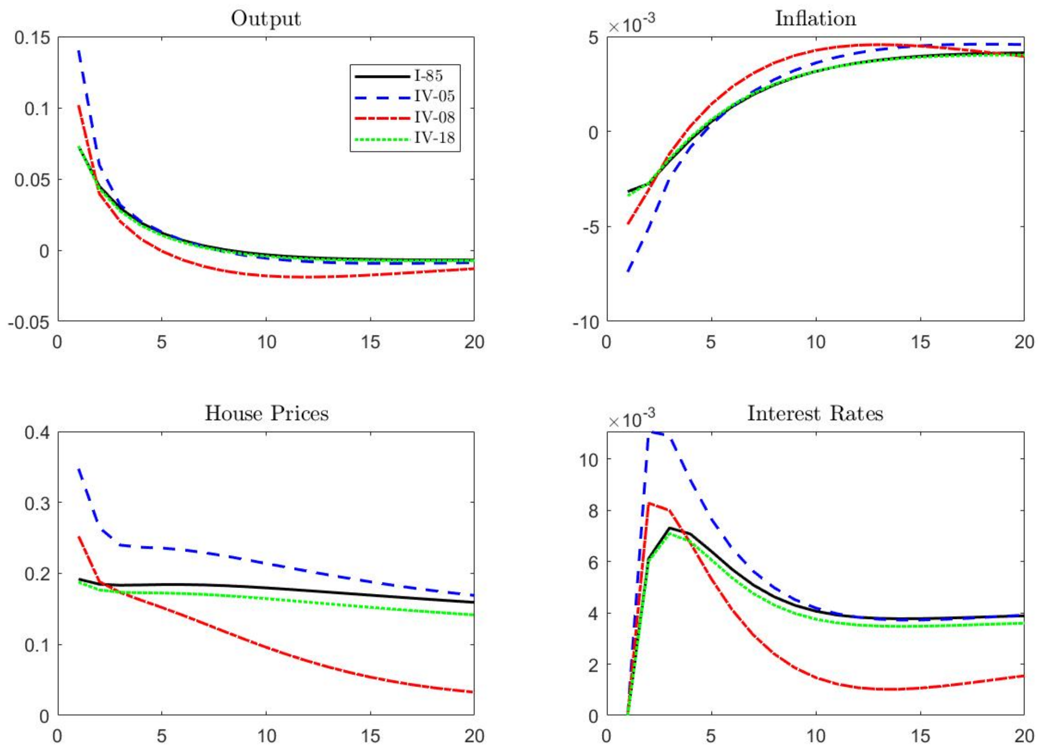

Figure 1.3: Impulse Response Function to a Housing Preference Shock.

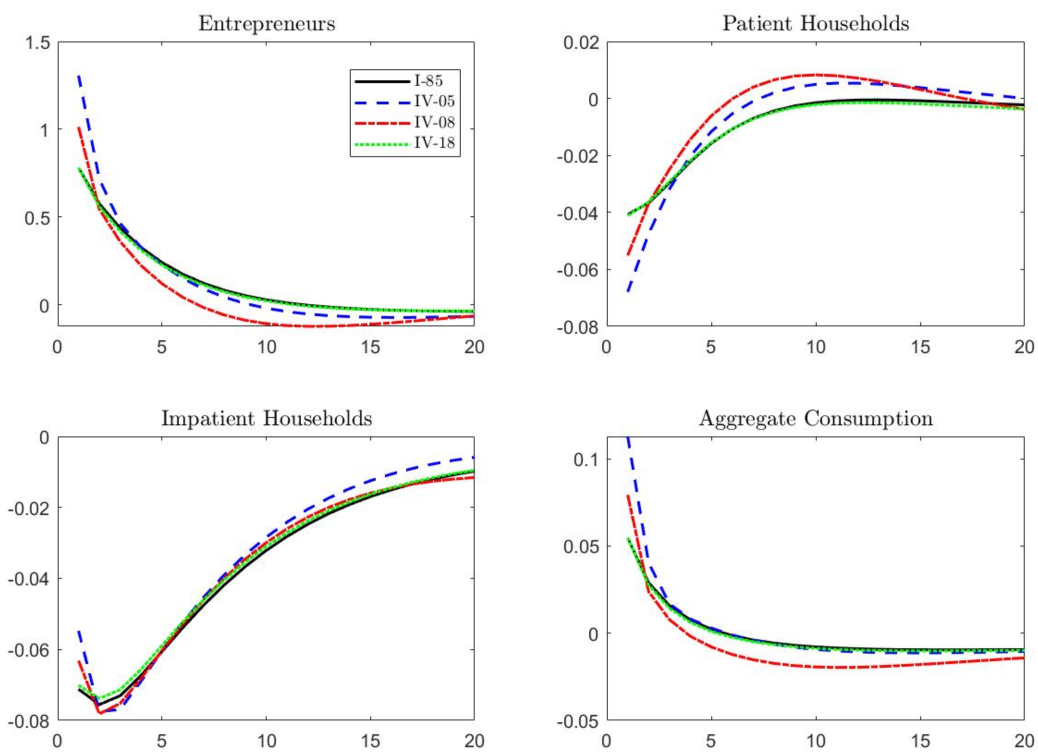

Figure 1.4: Consumption's IRF to a Housing Preference Shock.

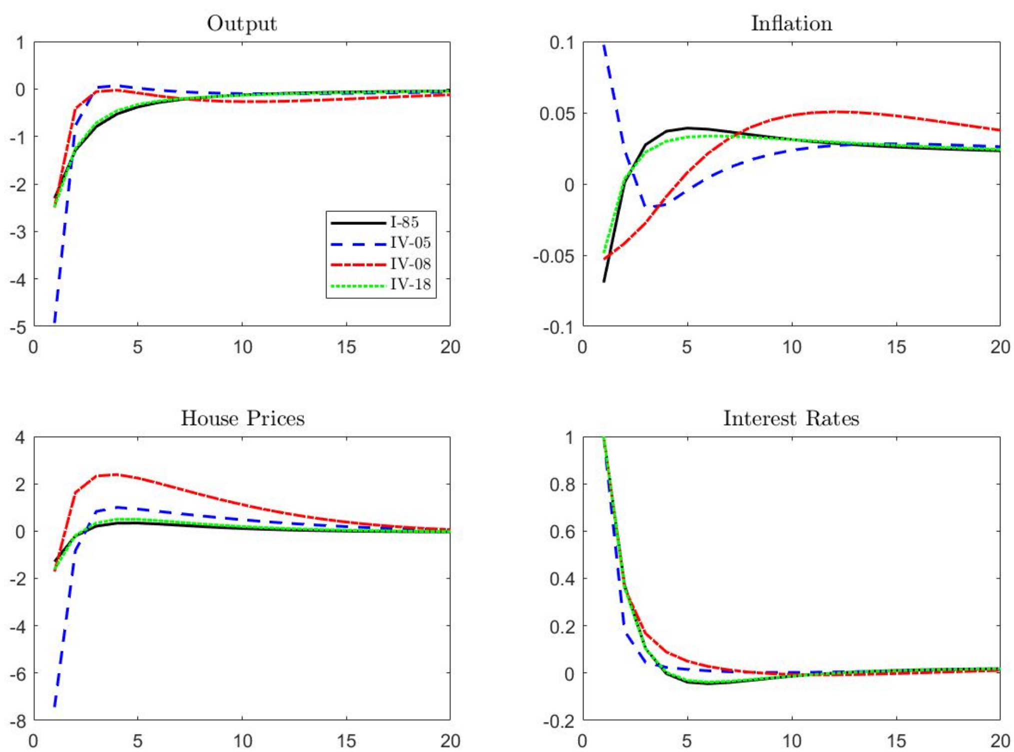

Figure 1.5: IRF to a Monetary Policy Shock.

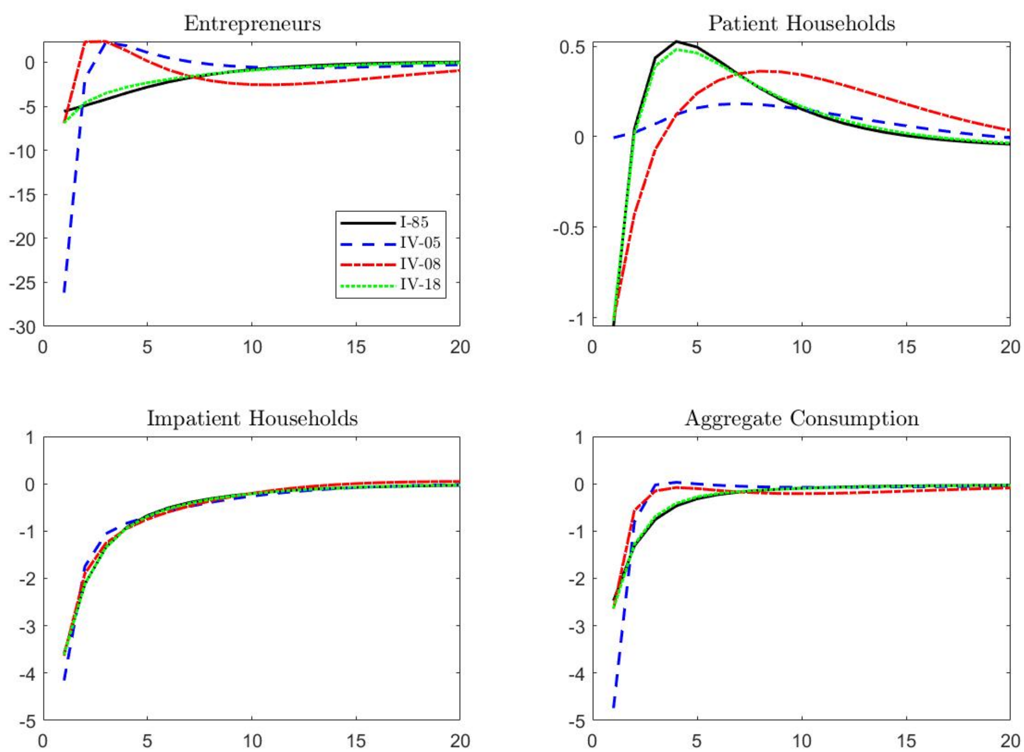

Figure 1.6: Consumption's IRF to a Monetary Policy Shock.

1.6 Conclusion

In this paper, I have explored the role that expectations, formed under a Bayesian adaptive learning mechanism, have in an economy where housing can be used as collateral for borrowing. Moreover, adaptive learning allows me to identify the evolution of responses to shocks, which is not possible under the Rational Expectations assumption.

The posterior odds results favor the adaptive learning model with house prices as a state variable. This specification allowed me to analyze scenarios of the potential feedback effects of the house price bubble in the economy. The IRFs suggest that most of the impact of housing preference shocks goes into output and house prices. Given the estimated model, the increase in aggregate consumption is driven by an important response of entrepreneurs' consumption, which results from a positive income effect.

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Chapter 2

Housing Price Bubbles and Macroprudential Policies, an Adaptive Learning Approach

2.1 Introduction

Credit conditions in the economy are affected by the expectations of asset prices used as collateral by economic agents. In this regard, the amount of borrowing capacity for mortgages depends on expectations of future house prices. Therefore, it is crucial to understand how agents form these predictions and the broader implications of this process.

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In this context, the evidence suggests that house price expectations do not adhere to the rationality assumption and are prone to bubbles.[1,2] For instance, with a dynamic model for housing under rational expectations, Glaeser and Gyourko (2006) have difficulties explaining the positive serial correlation in house price changes observed in the data. Additionally, Piazzesi and Schneider (2009) analyzed data from the Michigan Survey of Consumers and observed that "*starting in 2004, more and more households became optimistic after having watched house prices increase for several years*". Moreover, Gelain and Lansing (2014), employing a Lucas-type asset pricing model, demonstrate that it can approximately match the volatility of the price-rent ratio in the data using near-rational agents. Finally, Branch et al. (2016) constructed a two-sector model with search frictions, a housing market, and a goods market with explicit financial frictions and calibrated it for the US economy from 1996 to 2010. They consider both the Rational Expectations and Adaptive Learning expectations assumptions, where only the latter can generate a house price boom of the same magnitude as exhibited in the data.

Moreover, it is well established that agents' sentiments have a crucial role in explaining the boom-bust cycles of housing markets. Previous evidence suggests that during the 2000s period beliefs related to the housing market were shared by different types of agents, and this sentiment played a significant role in creating the housing bubble. In this regard, Cheng et al. (2014) found that mid-level managers in securitized finance did not exhibit awareness of problems in overall housing markets, while certain groups continued to aggressively increase their exposure to housing during the 2004-2006 period. Furthermore, Soo (2015) created an index of sentiment by measuring the tone of housing news in local newspapers and noted that sentiment forecasts the boom and bust of housing markets by a significant lead, peaking two years before house prices began to decline in 2006. More recently, Kaplan et al. (2020)

[1]Rational agents would expect lower returns after a sustained rise in prices, while irrational agents would continue to predict higher returns.

[2]Selection of papers has been based on closest congruence to the particular point being made. We acknowledge that there is a strand of literature focused on rational bubbles, but this is outside the scope of this paper.

observed that shifts in expectations about house price growth caused the housing boom-bust of the 2000s, where households, investors, and lenders shared these beliefs.

Finally, previous studies document that asset price bubbles can affect credit markets and, in turn, have a clear relevance from a financial stability perspective. Indeed, bubbles could lead to relaxed credit conditions and induce households to increase borrowing. The latter results from agents' beliefs, which suggests that prices would continue to grow, amplifying the effects of credit frictions on the economy. This type of feedback effect has been confirmed by empirical studies done by Berlinghieri (2010) for the US and Anundsen and Jansen (2013) for Norway. Additionally, Brueckner et al. (2012) generate a theoretical model and show that lenders ease their default concerns when house price expectations become more favorable. This perception increases their willingness to extend loans to risky borrowers. Since the housing demand created by subprime lending feeds back onto house prices, such lending also helps to fuel an emerging housing bubble. They find tentative support in empirical work for this connection. Anundsen et al. (2016) found that global housing market developments have predicting power for domestic financial stability risks. Further, they observed that the probability of a crisis increases when bubble-like behavior in house prices coincides with high household leverage. Finally, Jordà et al. (2015) found that credit-financed housing price bubbles have high financial crisis risks.

Motivated by this evidence, our paper studies the effects of housing price bubbles on credit markets' conditions, the banking sector, and the real economy. With this goal in mind, we develop a medium-scale DSGE model with three main ingredients. First, households' borrowing capacity depends on house price expectations, as developed by Iacoviello (2005). This feature introduces a feedback loop between agents' expectations, housing prices, and credit in the model. When households expect housing prices to grow, their borrowing capacity increases, stimulating a credit build-up.

Second, we posit that agents form expectations using an adaptive learning scheme. This assumption provides sufficient flexibility in beliefs to match the documented behavior of housing price expectations. Our model captures the dynamics of a house price bubble in credit supply via the expectations channel with this approach. We should note that adaptive learning allowed us to identify the evolving response to shocks during different stages of the housing price bubble, as opposed to Rational Expectations, which assume a constant response across the sample.

Third, we introduced a stylized banking sector subject to capital requirements along the lines of Gerali et al. (2010) and Angelini et al. (2014). The presence of a banking system in an environment with adaptive learning is crucial to evaluate the interactions between expectations formation and the supply side of credit markets. In addition, this will allow us to assess if macroprudential policies are effective in countering the excessive credit expansion triggered by a housing bubble. While several previous contributions have studied the effects of macroprudential policies in a similar DSGE framework (Angelini et al. (2010), Gambacorta and Karmakar (2018), Hinterschweiger et al. (2021), and Acosta-Smith, Bassanin, Sabuga (2021)), to the best of our knowledge, this is the first paper considering an environment with financial intermediaries and adaptive learning.

The model is estimated for the US economy using Bayesian techniques. This approach allows us to estimate the adaptive learning parameters jointly with the structural parameters of the economy. The estimation results suggest that the introduction of learning improves the model's ability to fit historical data. We observed a clear improvement in the marginal likelihood compared to the model under rational expectations. Moreover, the results show that the model captures an increase in the inertia present in house prices, consistent with the 2000s housing bubble.

Our analysis produces two main results. First, we find that the feedback loop between housing prices and credit is more severe around periods of housing bubbles. The stronger and more persistent rise in housing prices, driven by agents' expectations, generates a more considerable expansion of households' borrowing capacity and, in turn, a more substantial build-up of lending volumes. A positive housing preference shock induces a credit expansion four times larger during a bubble than in regular times. This increase in credit supply fuels a more persistent response of investment and output. However, this reaction comes at the cost of a more extensive deterioration of banks' risk-weighted Capital ratio due to the amplification effects of agents' expectations on banks' lending activities.

Second, we find that capital requirements are more effective in taming the credit cycle during periods of exuberance. During the bubble, capital requirements effectively stabilize the economy by smoothing the increase of housing prices, total credit, and investment, magnified by the learning dynamics. This stabilization effect results from a more significant deviation of banks' risk-weighted capital ratio from the regulatory target during the bubble. Indeed, capital requirements impose proportional costs on banks that deviate from the regulatory capital ratio. The banking system does transfer those higher costs to the real economy by rising lending rates, and this, in turn, mitigates credit expansion. Notably, capital requirements also effectively reduce the deterioration of banks' risk-weighted capital ratio triggered by the shock, improving the banking system's resilience. This result suggests that benefits from capital requirements are more extensive in episodes of euphoria, reaffirming the importance of tightening capital requirements during booms.

We organized the rest of the paper as follows. Section 2.2 describes the model of the economy and the expectations formation. In Section 2.3 we detail the estimation procedure, the data description, and report our results. Section 2.4 provides a comparative analysis of the effects of the macro-prudential policy during the housing bubble, under both Rational Expectations and Adaptive Learning. Section 2.5 concludes.

2.2 Model

We build on the model by Gerali et al. (2010). The economy is populated by patient households, impatient households, and entrepreneurs. Households consume, accumulate housing stock, and work. The two types of households differ in terms of their degree of impatience. The discount factor of patient households is higher than that of impatient households ($\beta > \beta'$). Households' heterogeneity generates positive financial flows in equilibrium, as patient households save, and impatient ones borrow against the value of their housing stock. The housing supply is fixed (its price varies endogenously), but housing reallocation takes place across "patient" and "impatient" households in response to an array of shocks.

Entrepreneurs produce homogeneous intermediate goods using capital and labor. Both types of households supply the latter. Entrepreneurs borrow from banks using their capital stock as collateral to finance their capital purchases. Analogously to impatient households, the discount factor of entrepreneurs is also lower than the one of patient households ($\beta > \beta_E$), such that the borrowing constraint becomes binding. Additionally, capital producers are a model's device to introduce the price of capital.

The model features nominal wage and price rigidities. In the case of salaries, we assume that workers supply their differentiated labor services through unions, which set wages to maximize members' utility. On the other hand, the final goods market operates under monopolistic competition, where retailers buy intermediate goods from entrepreneurs, differentiate them at no cost, and sell them with a markup.

The banking sector consists of a wholesale branch and a retail branch. The wholesale branch manages the capital-asset position of the bank as it accumulates bank capital out of retained earnings, collects deposits from patient households, and pays some quadratic costs whenever it deviates from the capital requirements. Retail branches lend to impatient households and entrepreneurs. They have market power in setting lending rates, while we assume, for simplicity, perfect competition in the deposits market.

Additionally, a central bank sets the policy rate according to a standard Taylor-type reaction function. Finally, we depart from the full information assumption and suppose that agents form expectations with an adaptive learning scheme. To analyze the effects of learning in this DSGE model, we consider a step-by-step approach. Therefore, as Milani and Park (2019) suggested, we started by assuming that only house price expectations are formed using an adaptive learning scheme, which will later be expanded to include the rest of the forward variables of the model.

2.2.1 Households

We consider a model with two types of households: patient and impatient. Patient households choose consumption (c_t), housing services (h_t), labor hours (n_t), and savings through deposits (d_t), to maximize their lifetime utility, given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t z_t \left(\Gamma_c \log(c_t - \varepsilon_c c_{t-1}) + j_t \Gamma_h \log(h_t - \varepsilon_h h_{t-1}) - \frac{1}{1+\eta} n_t^{1+\eta} \right) \tag{2.1}$$

and are subject to the following budget constraint:

$$c_t + q_t h_t + d_t = \frac{w_t n_t}{\chi_{w,t}} + q_t h_{t-1} + \frac{R_{t-1} d_{t-1}}{\pi_t} + div_t \tag{2.2}$$

where the patient's discount factor is β, while ε_c and ε_h measure habits in consumption and housing services, respectively. Γ_c and Γ_h are scaling factor that ensure that the marginal utilities of consumption and housing are independent of habits at steady-state[3]. The parameter η denotes the inverse Frisch elasticity of labour supply. Let $q_t \equiv Q_t/P_t$ be the real housing price, $w_t \equiv W_t/P_t$ are real wages, $\pi_t \equiv P_t/P_{t-1}$ is gross inflation, R_{t-1} is gross nominal inter-

[3]They are defined as $\Gamma_c \equiv (1 - \varepsilon_c)/(1 - \beta\varepsilon_c)$ and $\Gamma_h \equiv (1 - \varepsilon_h)/(1 - \beta\varepsilon_h)$.

est rate between $t-1$ and t, and div_t are lump-sum profits received from labor unions, and firms and banks owned only by patient households. The term $\chi_{w,t}$ denotes the markup (due to monopolistic competition in the labor market) between the wage paid by intermediate goods firms and the wage paid to households, which accrues to the labor unions. The term j_t is a housing preferences shock, such that an increase in these variable shifts preferences towards housing, while the term z_t captures an inter-temporal preference shock, such that a positive shock increases households' willingness to spent today, acting as a consumption demand shock. We assume that the log of the shocks follow AR(1) processes:

$$\log j_t = (1-\rho_J)\log \bar{j} + \rho_J \log j_{t-1} + u_{j,t}$$
$$\log z_t = \rho_z \log z_{t-1} + u_{z,t}$$

where $u_{j,t} \sim N(0,\sigma_J^2)$ and $u_{z,t} \sim N(0,\sigma_z^2)$.

The relevant first-order conditions for patient households are:

$$q_t = \frac{\mathcal{U}_{h,t}}{\lambda_t^P} + \beta\,\mathbb{E}_t \frac{\lambda_{t+1}^P\, q_{t+1}}{\lambda_t^P} \tag{2.3}$$
$$\frac{w_t}{\chi_{w,t}} = \frac{z_t n_t^{\eta}}{\lambda_t^P} \tag{2.4}$$
$$\lambda_t^P = \beta\mathbb{E}_t\, \frac{\lambda_{t+1}^P\, R_t}{\pi_{t+1}} \tag{2.5}$$

where λ_t^P denotes the budget constraint Lagrange multiplier for patient households[4], and $\mathcal{U}_{h,t}$ is the patient household marginal utility of housing services.

Similarly, impatient households choose consumption (c_t'), housing services (h_t'), labor hours

[4]The Lagrange multiplier is equivalent to the patient household marginal utility of consumption: $\lambda_t^P \equiv \mathcal{U}_{c,t} = \frac{\Gamma_c z_t}{c_t - \varepsilon_c c_{t-1}} - \beta\varepsilon_c \mathbb{E}_t \frac{\Gamma_c z_{t+1}}{c_{t+1}-\varepsilon_c c_t}$.

(n'_t), and real loans (b^I_t), to maximize their lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta'^t z_t \left(\Gamma'_c \log(c'_t - \varepsilon_c c'_{t-1}) + j_t \Gamma'_h \log(h'_t - \varepsilon_h h'_{t-1}) - \frac{1}{1+\eta} n'^{1+\eta}_t \right) \tag{2.6}$$

and are subject to the following budget and borrowing constraints:

$$c'_t + q_t h'_t + \frac{R^{bI}_{t-1} b^I_{t-1}}{\pi_t} = \frac{w'_t n'_t}{\chi'_{w,t}} + q_t h'_{t-1} + b^I_t + div'_t \tag{2.7}$$

$$R^{bI}_t b^I_t \leq m^I_t \mathbb{E}_t q_{t+1} h'_t \pi_{t+1} \tag{2.8}$$

where β' is the impatient households discount factor, Γ'_c and Γ'_h are their corresponding scaling factors[5], R^{bI}_t is the gross nominal interest rate that impatient households have to pay on the loans they receive, and m^I_t represents an stochastic "loan-to-value" ratio for loans to impatient households, such that the maximum amount b^I_t that an impatient household can borrow is bound by the expression in equation (2.8). It is assumed that the stochastic loan-to-value ratio follows an AR(1) process: $\log m^I_t = (1-\rho^I_m)\log m^I + \rho^I_m \log m^I_{t-1} + u_{mI,t}$ with $u_{mI,t} \sim N(0, \sigma^2_{mI})$. The relevant first-order conditions for impatient households are:

$$q_t = \frac{\mathcal{U}'_{h,t}}{\lambda^I_t} + \mathbb{E}_t \frac{m^I_t q_{t+1} \pi_{t+1}}{R^{bI}_t} + \beta'(1-m^I_t)\mathbb{E}_t \frac{\lambda^I_{t+1}\, q_{t+1}}{\lambda^I_t} \tag{2.9}$$

$$\frac{w'_t}{\chi'_{w,t}} = \frac{z_t n'^{\eta}_t}{\lambda^I_t} \tag{2.10}$$

where λ^I_t is the impatient household's budget constraint Lagrange multiplier and $\mathcal{U}'_{h,t}$ its marginal utility of housing services.

[5] They are defined as $\Gamma'_c \equiv (1-\varepsilon_c)/(1-\beta'\varepsilon_c)$ and $\Gamma'_h \equiv (1-\varepsilon_h)/(1-\beta'\varepsilon_h)$.

2.2.2 Entrepreneurs and Capital Producers

In this model, entrepreneurs or non-financial firms produce intermediate goods and operate in a competitive market. Each entrepreneur chooses consumption (c_t^E), real loans (b_t^E), capital (K_t), and labor (n_t,n_t') to maximize its lifetime utility, subject to its production function, a budget constraint and a borrowing constraint, represented by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_E^t \Gamma_c^E \log(c_t^E - \varepsilon_c c_{t-1}^E) \tag{2.11}$$

$$\text{s.t. } y_t = K_{t-1}^{\alpha} n_t^{(1-\sigma)(1-\alpha)} n_t'^{\sigma(1-\alpha)} \tag{2.12}$$

$$c_t^E + w_t n_t + w_t' n_t' + q_t^k K_t + \frac{R_{t-1}^{bE} b_{t-1}^E}{\pi_t} = \frac{y_t}{\chi_{p,t}} + b_t^E + q_t^k (1-\delta) K_{t-1} \tag{2.13}$$

$$R_t^{bE} b_t^E \leq m_t^E \mathbb{E}_t[\pi_{t+1} q_{t+1}^k (1-\delta) K_t] \tag{2.14}$$

where β_E is the entrepreneurs discount factor, Γ_c^E is a scaling factor[6], y_t is the entrepreneurs' production of intermediate goods, $\chi_{p,t} = P_t/P_t^W$ is the retailers gross markup over the intermediate goods price, capital K_t depreciates at rate δ and has a real price equal to q_t^k, R_{t-1}^{bE} is the gross interest rate on loans to entrepreneurs between $t-1$ and t, b_t^E is the entrepreneurs' borrowing expressed in real terms, and m_t^E is a stochastic loan-to-value ratio associated with loans to entrepreneurs, that follows: $\log m_t^E = (1-\rho_m^E) \log m^E + \rho_m^E \log m_{t-1}^E + u_{mE,t}$ with $u_{mE,t} \sim N(0, \sigma_{mE}^2)$.

[6]It is defined as $\Gamma_c^E \equiv (1-\varepsilon_c)/(1-\beta_E \varepsilon_c)$, and as for the previous cases ensures that the marginal utility of consumption is independent of habits in the non-stochastic steady-state.

The first order conditions are given by:

$$q_t^k = (1-\delta)m_t^E \mathbb{E}_t \frac{q_{t+1}^k \pi_{t+1}}{R_t^{bE}} + \mathbb{E}_t \frac{\lambda_{t+1}^E \alpha y_{t+1}}{\lambda_t^E K_t \chi_{p,t+1}} + (1-\delta)\beta_E (1-m_t^E)\mathbb{E}_t \frac{\lambda_{t+1}^E q_{t+1}^k}{\lambda_t^E} \tag{2.15}$$

$$(1-\sigma)(1-\alpha)y_t = \chi_{p,t} w_t n_t \tag{2.16}$$

$$\sigma(1-\alpha)y_t = \chi_{p,t} w_t' n_t' \tag{2.17}$$

where λ_t^E represents the entrepreneurs' budget constraint Lagrange multiplier.

Turning our attention to capital producers, they also operate in a perfectly competitive market. They use two inputs to produce new capital: previous-period undepreciated capital $(1-\delta)K_{t-1}$, bought from entrepreneurs at nominal price Q_t^k, and i_t units of final goods, bought from retailers at price P_t. The new stock of effective capital is sold back to entrepreneurs at the price Q_t^k. Their optimization problem, expressed in real terms, is to choose the amount of investment, i_t, to maximize their flow of profits subject to the capital's law of motion:

$$\max_{i_t} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^E \left[q_t^k a_t^k \left(i_t - \frac{\phi(i_t - i_{t-1})^2}{2\bar{i}} \right) - i_t \right] \tag{2.18}$$

$$\text{s.t.} \quad K_t = a_t^k \left(i_t - \frac{\phi(i_t - i_{t-1})^2}{2\bar{i}} \right) + (1-\delta)K_{t-1} \tag{2.19}$$

The optimization above yields the following condition:

$$q_t^k a_t^k \left(1 - \frac{\phi \Delta i_t}{\bar{i}} \right) = 1 - \phi \beta_E \mathbb{E}_t \frac{\lambda_{t+1}^E q_{t+1}^k a_{t+1}^k \Delta i_{t+1}}{\bar{i} \lambda_t^E} \tag{2.20}$$

where $\bar{i}$ is the investment non-stochastic steady-state, ϕ is a parameter linked to investment adjustment costs, $\Delta i_t \equiv i_t - i_{t-1}$, and the stochastic discount factor is given by $\Lambda_{t,t+1}^E \equiv \frac{\beta_E \lambda_{t+1}^E}{\lambda_t^E}$. Finally, a_t^k is an investment-specific technology that follows an AR(1) process: $\log a_t^k = \rho_K \log a_{t-1}^k + u_{k,t}$, with $u_{k,t} \sim N(0, \sigma_K^2)$.

2.2.3 Final goods sector and nominal rigidities

There is a final goods sector with Calvo-style price rigidities. These firms (which are owned by patient households) buy wholesale goods from wholesale firms in a competitive market, differentiate the goods at no cost, and sell them with a markup $\chi_{p,t}$ over the marginal cost. The CES aggregates of these goods are converted back into homogeneous consumption and investment goods by households. Each period, a fraction $1 - \theta_\pi$ of final good firms set prices optimally, while a fraction θ_π cannot do so and index prices to the steady-state inflation $\bar{\pi}$. Combining the final good firms' optimal pricing decision with the equation for the evolution of the aggregate price level results in a forward-looking Phillips curve, which in its log-linearized form can be written as:

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} - \varepsilon_\pi \hat{\chi}_{p,t} + u_{p,t} \tag{2.21}$$

where the hat represents the deviation of the variable from its steady-state in log terms, $\varepsilon_\pi = (1 - \theta_\pi)(1 - \beta\theta_\pi)/\theta_\pi$ measures the sensitivity of inflation to changes in the markup, $\hat{\chi}_{p,t}$, whereas the term $u_{p,t}$ denotes an i.i.d. price markup shock, $u_{p,t} \sim N(0, \sigma_\pi^2)$.

Wage setting is modeled analogously. Households supply homogeneous labor services to unions. The unions differentiate labor services, set wages subject to a Calvo scheme, and offer labor services to labor packers who reassemble these services into the homogeneous labor composites n_c and n_c', which non-financial firms hire. The pricing rules set by the union imply, after linearization, the following wage Phillips curves:

$$\hat{\omega}_t = \beta \mathbb{E}_t \hat{\omega}_{t+1} - \varepsilon_w \hat{\chi}_{w,t} + u_{w,t} \tag{2.22}$$

$$\hat{\omega}'_t = \beta' \mathbb{E}_t \hat{\omega}'_{t+1} - \varepsilon'_w \hat{\chi}'_{w,t} + u_{w,t} \tag{2.23}$$

where $\omega_t \equiv \frac{w_t \pi_t}{w_{t-1}}$ and $\omega'_t \equiv \frac{w'_t \pi_t}{w'_{t-1}}$ denote wage inflation for each household type, and $u_{w,t} \sim N(0, \sigma_W^2)$ denotes an i.i.d. wage markup shock.

2.2.4 Banking Sector

Following Gerali et al. (2010) and Gambacorta and Signoretti (2014), we assumed there is a banking sector in which each bank (j) consists of two units: a wholesale branch and a retail branch.

The wholesale unit operates in a perfectly competitive market. This unit collects deposits, d_t, from households on which it pays the net interest rate[7] set by the central bank, r_t, and issues wholesale loans B_t^I and B_t^E to retail branches, on which it earns the wholesale loan rates, r_t^{BI} and r_t^{BE} respectively. The two sources of funding, K_t^b and d_t, are perfect substitutes. Bank capital is accumulated out of reinvested profits:

$$\pi_t K_t^b(j) = (1 - \delta_b) K_{t-1}^b(j) + \Pi_{t-1}^b(j) \tag{2.24}$$

where δ_b are the costs of managing bank capital and Π_{t-1}^b denotes the realized overall profits of the bank, namely the profits of the wholesale unit and the retail unit.

The wholesale branches are subject to capital requirements, meaning that they pay a cost when their capital ratio - i.e., the proportion of bank capital (K_t^b) to risk-weighted assets

[7] We denote gross interest rates with uppercase letter, R_t, while net interest rates are represented by lowercase letter, r_t. Therefore, $R_t = 1 + r_t$.

(B_t^*) - deviates from the regulatory level (ν^b). This penalty for deviating from ν^b implies that a bank's capital ratio affects loan interest rates, generating a feedback loop between bank capital, borrowers financing conditions, and the real economy.

More in detail, the bank j wholesale unit's problem is choosing B_t^I, B_t^E, and d_t so as to maximize its profits subject to a balance sheet constraint:

$$\max_{B_t^I(j),B_t^E(j),d_t(j)} \mathbb{E}_0 \sum_{t=0}^{\infty} \Big[R_t^{BI} B_t^I(j) + R_t^{BE} B_t^E(j) - R_t d_t(j) - \left(B_{t+1}^I(j) + B_{t+1}^E(j)\right) \pi_{t+1} \\ + d_{t+1}(j)\pi_{t+1} + (\pi_{t+1} K_{t+1}^b(j) - K_t^b(j)) - \frac{\kappa_{kb}}{2} \left(\frac{K_t^b(j)}{B_t^*(j)} - \nu^b\right)^2 K_t^b(j)\Big] \tag{2.25}$$

$$\text{s.t.} \quad B_t(j) \equiv B_t^I(j) + B_t^E(j) = d_t(j) + K_t^b(j) \tag{2.26}$$

where $\frac{K_t^b}{B_t^*}$ is the bank's capital ratio and $B_t^* \equiv \omega_t^{bI} B_t^I + \omega_t^{bE} B_t^E$ represents the risk weighted assets (RWA) of the bank. The risk weights associated to households' loans, ω_t^{bI}, and to firms' loans, ω_t^{bE}, are sketched along the lines of Angelini et al. (2010) as we describe it later. The first order conditions are:

$$R_t^{BS} = R_t - \kappa_{kb} \left(\frac{K_t^b}{B_t^*} - \nu^b\right) \left(\frac{K_t^b}{B_t^*}\right)^2 \omega_t^{bS} \tag{2.27}$$

For $S = \{I, E\}$. The above equation implies that the loan rate equals the policy rate plus an endogenous spread, positively related to the number of loans and its corresponding perception of riskiness.

As we mentioned before, we assume a stylized law of motion for the bank's risk weights in the spirit of Angelini et al.'s (2010) framework. However, for the impatient households, we add an extra component related to house prices:

$$\omega_t^{bI} = \rho_{wbI}\omega_{t-1}^{bI} + (1-\rho_{wbI})\chi_y^I \Delta^A y_t + (1-\rho_{wbI})\chi_q^I \mathbb{E}_t \Delta^A q_{t+1} + (1-\rho_{wbI})\omega^{bI} \tag{2.28}$$

$$\omega_t^{bE} = \rho_{wbE}\omega_{t-1}^{bE} + (1-\rho_{wbE})\chi_y^E \Delta^A y_t + (1-\rho_{wbE})\omega^{bE} \tag{2.29}$$

where $\Delta^A v_t$ represents the annual growth of variable v_t, i.e. $\Delta^A v_t \equiv \log V_t - \log V_{t-4}$.

The retail loan branch is assumed to operate in a regime of monopolistic competition. This unit buys wholesale loans, differentiates them at no cost, and resells them with a markup to final borrowers. These banks face quadratic adjustment costs for changing over time the rates it charges on loans; these costs are parameterized by κ_{rbS}, for $S = \{I, E\}$, and are proportional to aggregate returns on loans. Retail loan bank j maximizes:

$$\begin{aligned} \max_{r_t^{bI}(j), r_t^{bE}(j)} \mathbb{E}_t \sum_{t=0}^{\infty} \Lambda_{0,t}^P \Big[& r_t^{bI}(j) b_t^I(j) + r_t^{bE}(j) b_t^E(j) - r_t^{BI}(j) B_t^I(j) - r_t^{BE}(j) B_t^E(j) \\ & - \frac{\kappa_{rbI}}{2}\left(\frac{r_t^{bI}(j)}{r_{t-1}^{bI}(j)} - 1\right)^2 r_t^{bI} b_t^I - \frac{\kappa_{rbE}}{2}\left(\frac{r_t^{bE}(j)}{r_{t-1}^{bE}(j)} - 1\right)^2 r_t^{bE} b_t^E \Big] \end{aligned} \tag{2.30}$$

$$\text{s.t. } b_t^S(j) = \left(\frac{r_t^{bS}(j)}{r_t^{bS}}\right)^{-\varepsilon_t^{bS}} b_t^S; \quad \text{for} \quad S = \{I, E\} \tag{2.31}$$

with $B_t^I(j) = b_t^I(j)$ and $B_t^E(j) = b_t^E(j)$. The first two terms are simply the returns from lending to households and entrepreneurs. The next terms reflect the cost of remunerating funds received from the wholesale branch. The last two terms are the costs of adjusting the interest rates. Finally, note that the loans' elasticity terms, ε_t^{bS}, are stochastic, and follow AR(1) processes[8]. After imposing a symmetric equilibrium, the first-order conditions for

[8] From the relation between the elasticity and the markup on loans, given by $\mu_t^S = \frac{\varepsilon_t^{bS}}{\varepsilon_t^{bS}-1}$, we define a shock process for the loans markups. We present the processes in deviation from its steady-state at Appendix B.1

interest rates yield:

$$1 - \varepsilon_t^{bS} + \varepsilon_t^{bS} \frac{r_t^{BS}}{r_t^{bS}} - \kappa_{rbS} \left(\frac{r_t^{bS}}{r_{t-1}^{bS}} - 1 \right) \frac{r_t^{bS}}{r_{t-1}^{bS}} + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}^P}{\lambda_t^P} \kappa_{rbS} \left(\frac{r_{t+1}^{bS}}{r_t^{bS}} - 1 \right) \left(\frac{r_{t+1}^{bS}}{r_t^{bS}} \right)^2 \frac{b_{t+1}^S}{b_t^S} \right] = 0 \tag{2.32}$$

We use the patient households' discount factor because they own the banks. Note that the retail rates depend on the markup and the wholesale rate (the marginal cost for the banks), which depends on the bank's capital position and the policy rate.

Finally, the total profits of the banking group, j, can be written as:

$$\Pi_t^b(j) = r_t^{bI}(j) b_t^I(j) + r_t^{bE}(j) b_t^E(j) - r_t d_t(j) - \frac{\kappa_{kb}}{2} \left(\frac{K_t^b(j)}{B_t^*(j)} - \nu^b \right)^2 K_t^b(j) - \frac{\kappa_{rbI}}{2} \left(\frac{r_t^{bI}(j)}{r_{t-1}^{bI}(j)} - 1 \right)^2 r_t^{bI} b_t^I - \frac{\kappa_{rbE}}{2} \left(\frac{r_t^{bE}(j)}{r_{t-1}^{bE}(j)} - 1 \right)^2 r_t^{bE} b_t^E \tag{2.33}$$

2.2.5 Monetary policy and market clearing

The Monetary policy follows an operational Taylor rule that allows for interest rate smoothing and responds to output and inflation.

$$R_t = R_{t-1}^{r_R} \left(\frac{\pi_{t-1}}{\pi} \right)^{(1-r_R) r_\pi} \left(\frac{y_{t-1}}{y} \right)^{(1-r_R) r_y} R^{1-r_R} e_t \tag{2.34}$$

where r_π and r_y denote the sensibility of the monetary response to inflation and output, respectively, and e_t is assumed to be an auto-regressive monetary policy shock.

The market clearing condition in the housing market is given by:

$$h_t + h_t' = 1 \tag{2.35}$$

while for the final goods market is:

$$y_t = C_t + i_t + \delta_b \frac{K^b_{t-1}}{\pi_t} + adj^b_t \tag{2.36}$$

where $C_t = c_t + c'_t + c^E_t$ is the aggregate consumption, i_t is the total investment, K^b_t is the bank capital, and Adj_t includes all the adjustment costs. The model has a unique stationary equilibrium in which entrepreneurs and impatient households hit the borrowing constraint and borrow up to the limit. The model can be reduced to a linearized system (see Appendix B.1), which summarized in matrix form can be expressed as:

$$\mathcal{A}\mathbb{E}_t\xi_{t+1} + \mathcal{B}\xi_t + \mathcal{C}\xi_{t-1} + \mathcal{E}\varepsilon_t = const. \tag{2.37}$$

2.2.6 Expectations formation

We depart from the usual full information assumption and suppose that agents use a limited information set to form their expectations and learn about economic relationships over time. For this, we assume that agents have the following perceived law of motion (PLM)[9]:

$$\xi^f_t = X_t\phi_t + v_t \tag{2.38}$$

where ϕ_t is a vector containing the learning beliefs parameters, and the regressors matrix X_t is constructed in SUR form[10]. Therefore, the equation above can be described with more

[9]The set of forward variables in the model are: $\xi^f_t = [\hat{q}_t, \hat{R}^{bI}_t, \hat{R}^{bE}_t, \hat{y}_t, \hat{c}_t, \hat{c}'_t, \hat{c}^E_t, \hat{\imath}_t, \hat{h}'_t, \hat{\pi}_t, \hat{\omega}_t, \hat{\omega}'_t, \hat{\lambda}^P_t, \hat{\lambda}^I_t, \hat{\lambda}^E_t, \hat{q}^k_t, \hat{\chi}_{p,t}]$. When we assume that agents form their expectations with an adaptive learning scheme only for house prices, the rest of the variables are set to be consistent with the rational expectations equilibrium.

[10]SUR is the acronym for seemingly unrelated regressions.

detail as[11]:

$$\begin{bmatrix} \xi^f_{1,t} \\ \xi^f_{2,t} \\ \vdots \\ \xi^f_{m,t} \end{bmatrix} = \begin{bmatrix} X_{1,t} & & & \\ & X_{2,t} & & \\ & & \ddots & \\ & & & X_{m,t} \end{bmatrix} \cdot \begin{bmatrix} \phi_{1,t} \\ \phi_{2,t} \\ \vdots \\ \phi_{m,t} \end{bmatrix} + \begin{bmatrix} v_{1,t} \\ v_{2,t} \\ \vdots \\ v_{m,t} \end{bmatrix}.$$

Under this PLM, the forward variables forecasts are:

$$\hat{E}_{t-1}\xi^f_{t+1} = \hat{E}_{t-1}X_{t+1}\phi_{t-1} \tag{2.39}$$

where $\hat{E}_{t-1}X_{t+1}$ is a function of $\hat{E}_{t-1}\xi^f_t = X_t\phi_{t-1}$. By substituting these expectations in the linearized model described in equation (2.37), we can obtain the Actual Law of Motion (ALM).

Agents are assumed not to know the relevant model parameters and they use historical data to learn them over time. Each period, they update their estimates according to the constant-gain learning formula:

$$\phi_t = \phi_{t-1} + \bar{g}R_t^{-1}X_t^T(\xi^f_t - X_t\phi_{t-1}) \tag{2.40}$$

$$R_t = R_{t-1} + \bar{g}(X_t^T X_t - R_{t-1}) \tag{2.41}$$

where (2.40) describes the updating of the learning rule coefficients and (2.41) characterizes the updating of the precision matrix R_t, while $\bar{g}$ denotes the constant gain coefficient. In order to initialize the learning process, we assume that the initial beliefs are model consistent, by using the theoretical moment matrices implied by the REE evaluated under the corresponding structural parameter vector θ.

[11] Each forward variable $\xi^f_{i,t}$ have their own information set $X_{i,t}$, and their corresponding beliefs $\phi_{i,t-1}$.

Along the lines of Slobodyan and Wouters (2012), we assume that economic agents use a simple univariate AR(2) model to form expectations. Additionally, to focus on the effects of housing market expectations on credit and the broader economy, we first posit that only house price expectations deviate from the complete information assumption, as Milani and Park (2019). We then relax both assumptions so that agents form expectations on all forward variables with an adaptive learning scheme. We also allow the PLM to include a set of macro variables considered to affect agents' economic perspectives.

Finally, we have that the state-space model is given by the following measurement and state transition equations:

$$Observations_t = H\xi_t \tag{2.42}$$

$$\xi_t = \gamma_t + F_t\xi_{t-1} + G_t\epsilon_t \tag{2.43}$$

where $\epsilon_t \sim N(0, \Omega)$, H is a matrix that maps the observables from ξ_t, and γ_t, F_t, and G_t are matrices of coefficients, which are convolutions of structural parameters of the economy and the best estimates of agents' beliefs, ϕ_{t-1}.

2.3 Estimation

The model is estimated using Bayesian methods. This approach allows us to jointly evaluate the coefficients describing agents' learning, such as the gain coefficient (indicating their learning speed), together with the structural parameters of the economy [12]. The parameter

[12] This strategy responds to potential criticism of models with learning, in which the results might depend on the parameters that need to be chosen by the researcher.

vector (θ) is:

$$\theta = \{\beta', \varepsilon_c, \varepsilon_h, \phi, \sigma, r_\pi, r_R, r_y, \theta_\pi, \theta_w, \kappa_{rbI}, \kappa_{rbE}, \rho_j, \rho_K, \rho_R, \rho_z, \rho_{\mu I}, \\ \rho_{\mu E}, \rho_{mI}, \rho_{mE}, \sigma_J, \sigma_K, \sigma_\pi, \sigma_R, \sigma_W, \sigma_Z, \sigma_{\mu I}, \sigma_{\mu E}, \sigma_{mI}, \sigma_{mE}, \bar{g}\} \tag{2.44}$$

where we can see that the parameter vector contains the structural parameters describing the dynamics of the economy, the monetary policy rule coefficients, the autocorrelation and standard deviation parameters of the shocks, and the constant gain coefficient $\bar{g}$.

The likelihood is recursively evaluated with the Kalman filter, using equations (2.42) and (2.43). At each moment in time, the beliefs are updated using equations (2.40) and (2.41). We present the priors used for the parameters and their description in Table 2.1. We calibrate the rest of the parameters of the model following the literature, with the values exhibited in Table 2.2.

2.3.1 Data

For the estimation, we use observations for ten series: total real household consumption, price (GDP deflator) inflation, wage inflation, real investment, real households mortgage debt, real non-financial corporations' debt, real house prices, the Federal Funds Rate, mortgage rates, and corporate bond yields. The observations span the period from Q1-1985 to Q4-2007 (Appendix B.2 describes the data in more detail). We present the evolution of the variables in Figure 2.1. The model features ten shocks: investment-specific shocks, wage markup, price markup, monetary policy, inter-temporal preferences, housing preferences, mortgage markup, corporate loans markup, and loan-to-value shocks to mortgages and firms' loans.

Before the estimation, we remove the low-frequency components of consumption, investment, mortgage debt, corporate debt, and house prices using the unobserved components model with a second-order Markov process for the trend, developed in Grant and Chan (2017).

Table 2.1: Prior distributions.

	Description	Distrib.	Mean	SD
β'	Impatient's discount factor	B	0.985	0.003
ε_c	Habit in consumption	B	0.7	0.1
ε_h	Habit in housing	B	0.7	0.1
ϕ	Investment adjustment cost	Γ	5	2
σ	Impatient's wage share	B	0.35	0.05
r_π	Taylor rule response to inflation	N	2	0.33
r_R	Taylor rule inertia	B	0.5	0.1
r_y	Taylor rule response to output	B	0.125	0.025
θ_π	Calvo parameter, Prices	B	0.5	0.075
θ_w	Calvo parameter, wages	B	0.5	0.075
κ_{rbI}	H.H. Loan rate adjustment cost	Γ	6	2.5
κ_{rbE}	Firms Loan rate adjustment cost	Γ	3	2.5
ρ_J	AR coefficient - Housing shock	B	0.50	0.1
ρ_K	AR coefficient - investment shock	B	0.50	0.1
ρ_R	AR coefficient - monetary shock	B	0.50	0.1
ρ_Z	AR coefficient - intertemporal shock	B	0.50	0.1
ρ_μ^I	AR coefficient - h.h. Loan rate markup shock	B	0.50	0.1
ρ_μ^E	AR coefficient - Firms Loan rate markup shock	B	0.50	0.1
ρ_m^I	AR coefficient - h.h. LTV ratio shock	B	0.50	0.1
ρ_m^E	AR coefficient - Firms LTV ratio shock	B	0.50	0.1
σ_J	SD - Housing shock	Γ^{-1}	0.01	1
σ_K	SD - investment shock	Γ^{-1}	0.01	1
σ_π	SD - Price markup shock	Γ^{-1}	0.01	1
σ_R	SD - interest rate shock	Γ^{-1}	0.01	1
σ_W	SD - wage markup shock	Γ^{-1}	0.01	1
σ_Z	SD - intertemporal shock	Γ^{-1}	0.01	1
σ_μ^I	SD - h.h. Loan rate markup shock	Γ^{-1}	0.01	1
σ_μ^E	SD - Firms Loan rate markup shock	Γ^{-1}	0.01	1
σ_m^I	SD - h.h. LTV ratio shock	Γ^{-1}	0.01	1
σ_m^E	SD - Firms LTV ratio shock	Γ^{-1}	0.01	1
$\bar{g}$	Constant Gain Parameter	Γ	0.025	0.01

They showed that the HP filter is a particular case of their framework, but the new model can accommodate salient features of the data by having a more flexible approach.

Table 2.2: Calibrated parameters.

	Description	Value	Based on*
β	Patient's discount rate	0.995	G & IA (2017)
β^E	Entrepreneur's discount rate	0.97	IA & N (2010)
α	Capital share in production	0.35	IA & N (2010)
δ_k	Capital depreciation rate	0.025	G & IA (2017)
η	Labor disutility	1	G & IA (2017)
π	Steady-state gross inflation rate	1.005	G & IA (2017)
j	Housing weight in utility	0.04	G & IA (2017)
χ_p	Steady-state price markup	1.2	G & IA (2017)
χ_w	Steady-state wage markup	1.2	G & IA (2017)
m^I	Steady-state h.h. LTV ratio	0.70	GNSS (2010)
m^E	Steady-state firms LTV ratio	0.20	N & T (2017)
ω^{bI}	Steady-state IRB weight risk	0.37	G & K (2018)
ω^{bE}	Steady-state IRB weight risk	0.92	G & K (2018)
ν^b	S.S. capital requirement	0.085	G & K (2018)
κ_{kb}	Bank's capital penalty cost	8	G & K (2018)
ε^{bI}	S.S. loans elasticity of subs.	2.79	GNSS (2010)
ε^{bE}	S.S. loans elasticity of subs.	3.12	GNSS (2010)
δ_b	Bank's capital depreciation	0.11	G & K (2018)
ρ_{wbE}	Firms IRB AR coeff	0.92	AENPQ (2010)
χ_y^E	Firms IRB output sensitivity	-14	AENPQ (2010)
ρ_{wbI}	H.H. IRB AR coeff	0.82	Own est.
χ_y^I	H.H. IRB output sensitivity	-1.53	Own est.
χ_q^I	H.H. IRB house prices sensitivity	-1.64	Own est.

* G & IA: Guerrieri and Iacoviello; IA & N: Iacoviello and Neri; N & T: Nookhwun and Tsomocos; G & K: Gambacorta and Karmakar; GNSS: Gerali, Neri, Sessa and Signoretti; AENPQ: Angelini, Enria, Neri, Panetta, Quagliariello.

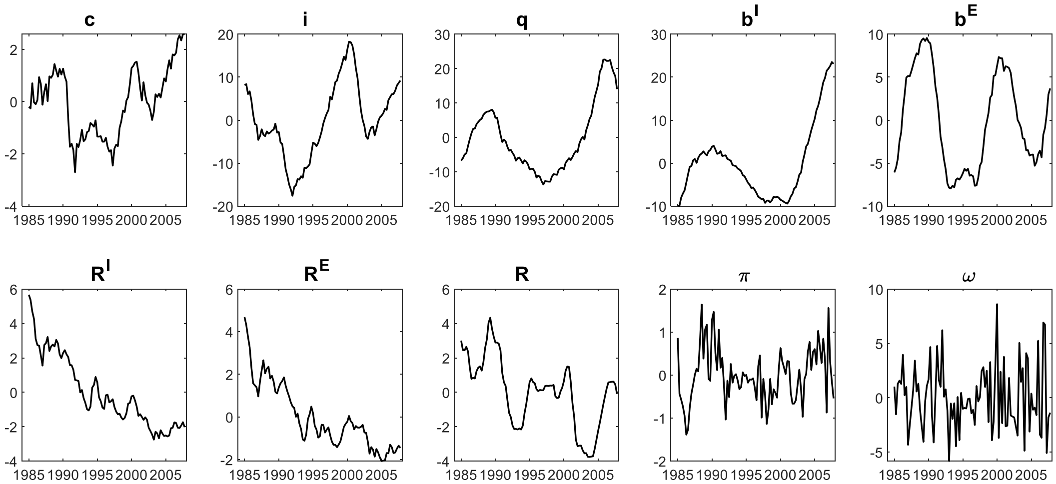
2.3.2 Results

Rational expectations

We present the structural parameters and the house price constant gain estimation in Table 2.3, while the parameters related to the shock process are in Table 2.4. As a first step, we estimated the model under Rational Expectations, providing a benchmark case to compare our results with others in the literature.

Our results under Rational Expectations are consistent with other estimations in the liter-

Figure 2.1: Observed variables - Sample I-1985 to IV-2007.



ature. Concerning the housing parameters, it is noteworthy that both the habit parameter (ε_h) and the autocorrelation of the shock (ρ_J) indicate the high persistence present in house prices. Next, we will see how the expectations formation affects these estimated parameters as learning becomes another source of persistence in the economy.

Adaptive learning expectations in house prices

As a next step, we further assume that the agents hold near-rational expectations by updating their beliefs of house price expectations. The information set available to the agents in this scenario is: $X_{q,t} = [\hat{q}_{t-1}, \hat{q}_{t-2}]$. We assume that agents already know the intercept value and only have uncertainty in the other parameters of their PLM (i.e., there is no intercept in the estimation of the beliefs).

The estimation results for this adaptive learning specification are denoted as $AL-1$ at Tables 2.3 and 2.4. We observed a clear improvement in the marginal likelihood compared with the estimation under rational expectations. Note that there is a reduction in the housing

Table 2.3: Structural Parameters and House Prices' Constant Gain Posterior draws.

	REE			**AL-1**			**AL-2**		
	Mode	**5%**	**95%**	**Mode**	**5%**	**95%**	**Mode**	**5%**	**95%**
β'	0.9741	0.964	0.981	0.9671	0.957	0.976	0.9760	0.966	0.983
ε_c	0.6119	0.542	0.663	0.5653	0.498	0.628	0.5439	0.478	0.613
ε_h	0.9891	0.986	0.993	0.9866	0.984	0.989	0.9815	0.977	0.986
ϕ_K	2.7090	1.260	8.688	3.2735	1.643	8.429	2.1639	0.802	3.366
σ	0.1949	0.154	0.240	0.1627	0.126	0.204	0.1787	0.142	0.213
r_π	2.1406	1.799	2.541	2.1333	1.718	2.563	1.2669	1.131	1.426
r_R	0.8074	0.764	0.840	0.8311	0.799	0.871	0.7856	0.725	0.846
r_y	0.0956	0.072	0.131	0.1056	0.079	0.138	0.1338	0.094	0.171
θ_π	0.8933	0.865	0.916	0.9069	0.879	0.930	0.8302	0.803	0.856
θ_W	0.8580	0.822	0.888	0.8624	0.827	0.893	0.7666	0.730	0.805
κ_{rbI}	6.5828	3.848	10.81	6.9304	4.124	12.18	5.7093	2.886	12.32
κ_{rbE}	11.176	7.089	19.03	12.544	7.585	20.70	6.5910	4.153	9.520
$\bar{g}$				0.0053	0.003	0.010	0.0059	0.003	0.011
Log-MgL		-469.9			-419.3			-369.5	

Note: Based on 200,000 posterior draws, after a burn-in of 20 percent.

preference shock persistence, ρ_J, which is in line with the idea that the propagation of these shocks under learning is captured by the expectations' mechanism and by the internal dynamics of the decision rules.

We can observe that the introduction of learning also affected the estimation of habits in consumption (ε_c) and the income share of impatient agents (σ). The lower value of σ under this specification might indicate that the model no longer requires the volatility generated by impatient households. This outcome might result from the effects that learning has in terms of the volatility of the model.

Adaptive learning in all forward variables

Finally, we extend the assumption of adaptive learning expectations formation to all the forward variables in the model. The information set available to the agents in this scenario

Table 2.4: Structural Shock Parameters Posterior draws.

	REE			**AL-1**			**AL-2**		
	Mode	**5%**	**95%**	**Mode**	**5%**	**95%**	**Mode**	**5%**	**95%**
ρ_J	0.9261	0.893	0.949	0.8256	0.785	0.858	0.7714	0.736	0.805
ρ_K	0.4854	0.384	0.608	0.5180	0.399	0.625	0.4434	0.354	0.525
ρ_R	0.5103	0.409	0.586	0.5339	0.438	0.616	0.8675	0.779	0.902
ρ_Z	0.7912	0.703	0.847	0.8840	0.826	0.924	0.6959	0.570	0.791
ρ_μ^I	0.7970	0.695	0.878	0.7821	0.683	0.871	0.6549	0.532	0.780
ρ_μ^E	0.8097	0.716	0.893	0.8092	0.707	0.894	0.7613	0.668	0.837
ρ_m^I	0.9268	0.889	0.963	0.9551	0.919	0.977	0.9598	0.923	0.979
ρ_m^E	0.9487	0.921	0.973	0.9486	0.918	0.974	0.9491	0.916	0.974
σ_J	0.0490	0.036	0.063	0.1462	0.112	0.178	0.2155	0.165	0.271
σ_K	0.0225	0.010	0.075	0.0249	0.013	0.069	0.0645	0.023	0.101
σ_π	0.0021	0.002	0.002	0.0019	0.002	0.002	0.0016	0.001	0.002
σ_R	0.0011	0.001	0.001	0.0010	0.001	0.001	0.0010	0.001	0.001
σ_W	0.0085	0.008	0.010	0.0086	0.008	0.010	0.0076	0.007	0.009
σ_Z	0.0177	0.015	0.021	0.0205	0.017	0.026	0.0222	0.017	0.031
σ_μ^I	0.0034	0.003	0.005	0.0035	0.003	0.005	0.0072	0.004	0.015
σ_μ^E	0.0033	0.003	0.005	0.0034	0.003	0.005	0.0063	0.004	0.009
σ_m^I	0.0109	0.010	0.013	0.0076	0.007	0.009	0.0084	0.007	0.010
σ_m^E	0.0104	0.009	0.012	0.0101	0.009	0.012	0.0105	0.009	0.012

Note: Based on 200,000 posterior draws, after a burn-in of 20 percent.

is:

$$X_{s,t} = [\hat{R}_{t-1}, \hat{q}_{t-1}, \hat{y}_{t-1}, \hat{\pi}_{t-1}, \hat{q}^k_{t-1}, s_{t-1}, s_{t-2}] \tag{2.45}$$

The addition of macro variables in the PLM, like output, inflation, interest rate, and the asset prices considered in the model, is done to include variables that economic agents usually follow to have a broad sense of economic conditions. In this estimation, we also allow agents to have different learning gains. This assumption would be appropriate in cases where variables have different rates of structural change (Branch and Evans, 2006). Given the plausibility that agents generate expectations for other variables at different rates, we explore this specification so that now $\bar{g}$ will be a vector, rather than a scalar, with the same number of gains as forwarding variables in the model. We present the estimation results for the vector of the constant gains parameters in Table B.2 located at Appendix B.3.

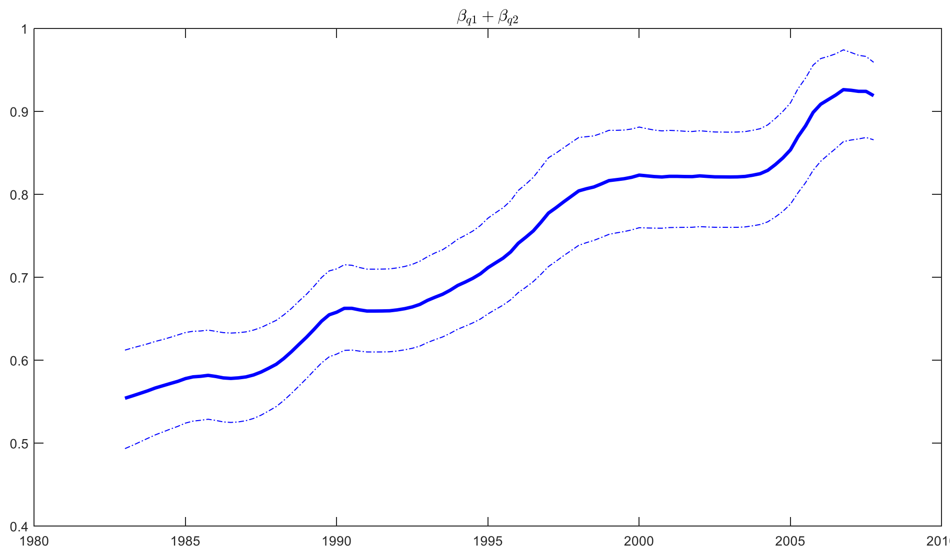
The results of this model are presented as $AL-2$ in Tables 2.3 and 2.4, where we can observe that this specification delivers the highest marginal likelihood of our estimated models. The extra flexibility under this specification allowed the model to fit the inertia observed in the other observed variables, besides house prices, which the previous models could not do.

Milani (2007) noted that learning could endogenously generate persistence in the economy and improve the fit of current monetary DSGE models. By introducing learning, other sources of persistence may lessen their influence on the model. In the results of $AL - 2$ we can observe that there is a reduction in consumption habits (ε_c), price stickiness (θ_π), wage stickiness (θ_w), the interest rate adjustment cost of both loans to impatient households (κ_{rbI}) and entrepreneurs (κ_{rbE}), and the auto-correlation coefficients associated with housing preference shock (ρ_J), inter-temporal preference shock (ρ_Z), impatient loan's markup shock (ρ_μ^I) and firms loan's markup shock (ρ_μ^E).

Given the previous results, it seems appropriate to look at the evolution of the persistence associated with house prices. We present these results in Figure 2.2. Since in the PLM we have both q_{t-1} and q_{t-2}, the persistence is the sum of the coefficients of these two regressors. The figure shows the increase in the inertia present in house prices, consistent with the 2000s housing bubble.

To analyze the belief's feedback effects on the economy, we estimated the impulse response to a housing preference shock (Figure 2.3), where we compare the results under Rational Expectations (REE) and adaptive learning (AL-2). For this exercise, we choose a period before any influence from the housing bubble (I-85) and a period within the effects of the bubble (IV-07) to make a clear contrast of the impact of the bubble on the economy. The transmission mechanism of this shock under the rational expectations hypothesis (black line)

Figure 2.2: Persistence of house prices beliefs.

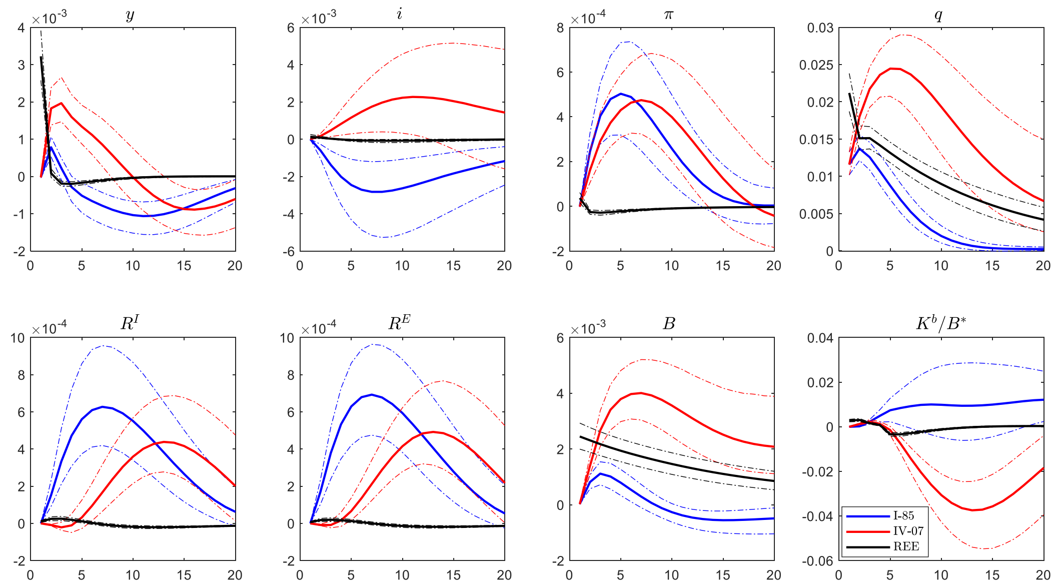


Note: the dotted lines represent a 90 percent credible interval.

works as follows. After a positive housing preference shock, households increase their demand for housing, inflating the corresponding price (q). The rise in housing prices expands the borrowing capacity of the debtors, allowing them to consume more. Meanwhile, the lending expansion (B) and the rise of lending rates (R^I and R^E) propel an increase in banks' profits. Accordingly, financial intermediaries can further expand lending to households and firms. The rise in consumption and investment (i) generates an increase in output (y).

During the bubble (IV-07, red line), the adapting learning scheme amplifies the transmission mechanism of the housing preference shock to lending and the real economy. First, the shock stimulates a stronger and more persistent increase in housing prices (q). This reaction is due to higher inertia from agents' beliefs, at the bubble stage, compared to the case of rational expectations. The more considerable magnitude of the housing prices response generates a more significant expansion of the households' borrowing capacity and, in turn, triggers a massive build-up of credit. As a result, the increase in investment and output is more persistent. However, these effects come at the cost of a more significant decline in the banks'

Figure 2.3: IRF to a housing preference shock, REE vs AL-2.

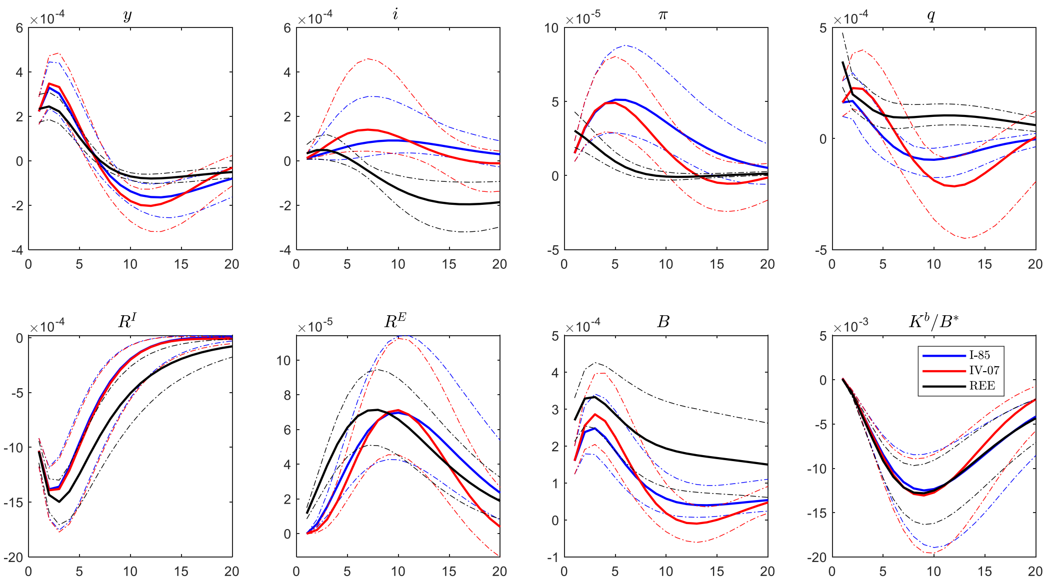


Note: the dotted lines represent a 90 percent credible interval.

Capital-to-RWA ratio (K^b/B^*), which means a material deterioration of the resilience of the banking system.

Interestingly, adaptive learning allows us to isolate the effect of the housing bubble by comparing the impulse response functions (IRFs) during the exuberance episode (IV-07, red line) and regular times (IV-85, blue line). The results document a more robust feedback loop between housing prices and lending during housing bubbles. The same housing preference shock increases lending (B) by a factor of four during the bubble compared to regular times. It is worth noting two additional results. First, the shock produces a deterioration of banks' Capital-to-RWA ratio (K^b/B^*) during the bubble, while the same ratio improves in normal times. The deterioration during the bubble results from the combination of two factors - the more substantial increase in lending and the smoother increase in lending rates. In the model, banks' capital - the numerator of the capital-to-RWA ratio - is accumulated by retained earnings only (see equation 2.24) and, therefore, the smoother path of lending rates mitigates the increase of banks' capital. Instead, the denominator of the risk-weighted

Figure 2.4: IRF to an expansionary impatient loans markup shock, REE vs AL-2.



Note: the dotted lines represent a 90 percent credible interval.

capital ratio is directly affected by the lending volumes, which increase more during the bubble. This result suggests that there might be a more prominent role for macroprudential policies in stabilizing the banking system in periods of exuberance. Second, investment declines after the shock in standard times, while the opposite happens during the bubble. The housing preference shock has more significant spillover effects on consumption during exuberant times. In turn, firms' incentives to borrow to support higher goods production are strong enough to compensate for the increase in lending rates, resulting in a positive investment response. On the contrary, during standard times, the lower propagation of the shock makes firms' incentives to expand production and demand credit weaker.

We further explore the effects of the financial sector on the economy and its interactions with the learning environment. For this, we estimated the impulse response to a reduction in the impatient loans interest rate (Figure 2.4). We can see that, in general, there are modest effects from the evolution of beliefs for this shock, in contrast to what we observed for the housing preference shock.[13] This result may suggest that changes in lending rates are not the main driver for movements in house prices but instead shifts in housing market beliefs, consistent with the findings of Kaplan et al. (2020).

2.4 Macroprudential Policy Analysis

After introducing adaptive learning to the DSGE model with a banking sector, we analyze the effects of macroprudential policies, in the form of banks' capital requirements, under this environment. Although we are not assuming a time-varying regulatory capital-to-RWA ratio (ν^b), how capital requirements are modeled (see equation 2.25) captures some features of a leaning-against-the-wind policy. Indeed, the size of the banks' costs when deviating from the regulatory target increases proportionally to the magnitude of the deviation. Therefore, during a boom, a larger expansion of banks' lending implies a more substantial effect of capital requirements on banks and, in turn, on the real economy.[14] This allows us to characterize capital requirements as set in the model as a macroprudential policy.

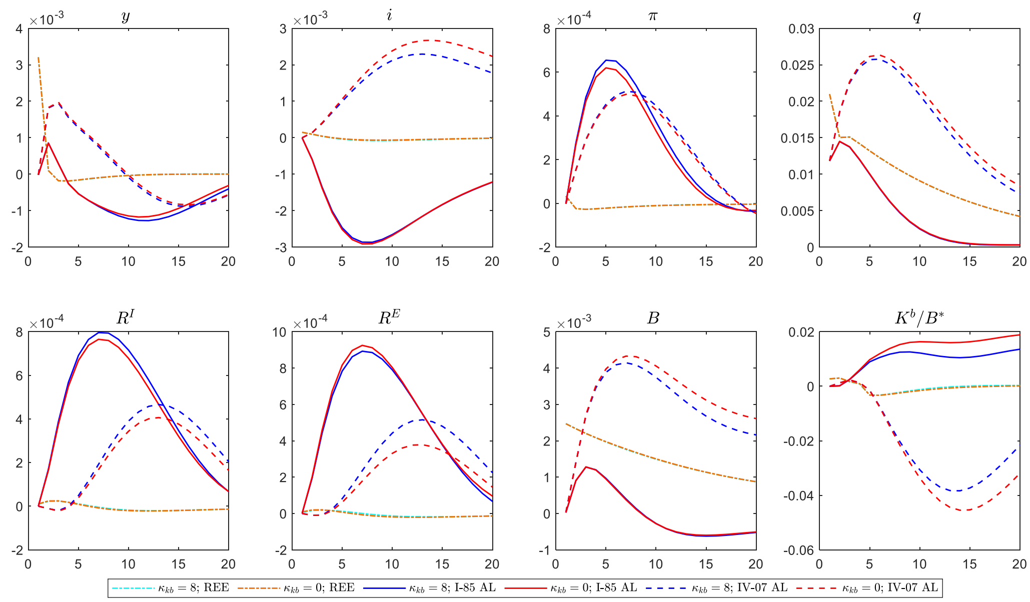
We consider two model's specifications: 1) a benchmark model in which banks are subject to capital requirements ($\kappa_{kb} = 8$); and 2) an alternative model in which capital requirements are not in place ($\kappa_{kb} = 0$). We evaluated the capital requirements performance under the full information assumption and learning, for a standard period (I-85, solid lines) and during the bubble stage (IV-07, dotted lines).

Figure 2.5 shows the effects of capital requirements under a shock to housing preferences. The positive housing preference shock determines an increase in house prices (q), a relaxation of the credit constraint of borrowers, and, in turn, a lending expansion (B). Capital

[13] These subtle differences in the results are also observed in the other credit supply shocks (i.e., firms' loans interest rate, impatients' LTV ratio, and firms' LTV ratio shocks), which we present in Appendix B.3

[14] From a policy perspective, this specification can capture the crucial features of capital buffers (e.g., capital conservation buffer and systemic buffers). Instead, we are not considering the counter-cyclical capital buffer (CCyB), which is an explicit time-varying requirement.

Figure 2.5: IRF to a housing preference shock.



requirements aim to mitigate the credit build-up by imposing costs on banks that deviate from the regulatory capital ratio. The banking system transfers those costs to the real economy by increasing lending rates. From Figure 2.5 we can see the stabilization process in progress, where the model with capital requirements (bluish lines) shows a smoother response of variables like credit and investment compared with the model without capital requirements (reddish lines).

Moreover, Figure 2.5 shows that capital requirements are more effective in taming the credit cycle during the bubble. Under the complete information assumption and under learning for the period pre-bubble (I-85, solid lines), the difference in the model's responses to the shock is similar in both models. On the contrary, during the bubble, macroprudential policies effectively stabilize the economy by smoothing the effect on house prices, total credit, and investment. More robust stabilization is associated with the more significant deviation of banks' capital ratio from the regulatory target during the bubble and the associated costs imposed on banks by capital requirements. Notably, capital requirements also effectively mitigate the decline in banks' capital-to-RWA triggered by the shock, improving the banking system's resilience. This result suggests more significant benefits from capital requirements to stabilize the economy in episodes of exuberance. From a policy perspective, this reaffirms the importance of tightening capital requirements during booms.

2.5 Conclusions

In this paper, we had two main questions: i) what the effects of the housing bubble are, generated via expectations formation, in the credit supply; and ii) how effective is the macroprudential policy in place under this environment with learning. We found that housing shocks have a higher impact on other economic variables under learning, especially in periods of a housing bubble. Additionally, we have evidence that capital requirements may play a role in lessening the effects of housing shocks under the influence of the bubble. Future research must analyze how other macro-prudential policies, such as the leverage ratio and output floor, might impact the credit supply in this environment.

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Chapter 3

Revisiting the Mixing in the Occasionally Binding Constraints estimation with a Gibbs Sampler

3.1 Introduction

There is a well-documented asymmetry in business cycles, in the sense that economic activity presents violent drops during recessions while expansions are moderate. In recent work by Adrian et al. (2019), using quantile regressions, the authors provided time series estimates of the conditional distribution of predicted GDP growth. They found that financial conditions are mainly correlated with higher moments of the estimated distribution. This result suggests that nonlinear approximations to the solutions of DSGE models may be more appropriate.

In terms of the nonlinear approximations under a DSGE framework, I can mention the work done by Guerrieri and Iacoviello (2015, 2017), in which occasionally binding collateral constraints on housing wealth drive an asymmetry in the link between housing prices and economic activity. Additionally, among the literature with estimates involving inequality constraints, I find interesting the work done by Chan and Strachan (2014) and Chan, Koop, and Potter (2016), which use precision-based methods proposed by Chan and Jeliazkov (2009).

Considering the findings of Adrian et al. (2019), it should be important to introduce these nonlinearities in the estimation of DSGE models. As shown in Guerrieri and Iacoviello (2017), one way to do it is by allowing that the collateral constraints in the model are not always binding.

The "occasionally binding constraint" (OBC) model is related to the literature on regime-switching and, in a broader sense, to mixture models. In this OBC model, the classification of the regime is done by defining a latent variable related to the constraint of interest (i.e., a Lagrange multiplier). It assumes that the other variables of the model are contingent on the state of that constraint. At the same time, the structural parameters remain invariant.[1] Using this framework, I simulated data and proceeded to do an estimation. For this, I develop an algorithm where the definition of the blocks of latent variables and parameters has to be done so that the full conditional distributions are tractable. In this context, the sampling of the variable that captures the regime of the model is done marginally from the rest of the latent variables of the model, with the objective of reducing the inefficiency of its draws. For the case of the latent variables, other than the Lagrange multiplier, I use the precision-based method proposed by Chan and Jeliazkov (2009), which provides the draws of these variables at a low computational cost. The results suggest that this procedure offers an efficient simulation of the objective distribution, although there are some caveats and future work.

I organize my paper as follows. In section 3.2, I present a model with the characteristics of

[1] This contrast with a simple Markov Switching model, in which it is assumed a different set of reduced-form parameters for each regime.

the "occasionally binding constraint" and simulate data from it. In section 3.3, I describe my estimation strategy and deliver my results. In section 3.4, I provide my conclusions.

3.2 Model (DGP)

Following Guerrieri and Iacoviello (2015), I consider a model with an impatient consumer that maximizes the following lifetime utility:[2]

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t Z_{c,t} \Gamma_c \frac{(C_t - hC_{t-1})^{1-\gamma} - 1}{1-\gamma} \tag{3.1}$$

subject to a budget constraint and a borrowing constraint:

$$C_t + RB_{t-1} = Y_t + B_t \tag{3.2}$$

$$B_t \leq \delta B_{t-1} + (1-\delta) Z_{b,t} m Y_t \tag{3.3}$$

where C_t is the consumer's consumption, B_t the level of debt, Y_t her income, β is the consumer's discount factor, h captures the consumption habits, γ is the inverse elasticity of substitution, $\Gamma_c \equiv \frac{(1-h)^\gamma}{1-\beta h}$ is a scaling factor that ensures that the marginal utility of consumption is independent of habits in the steady state, R is the steady state gross interest rate, δ is a parameter that measures the borrowing inertia in the borrowing limit, m is the steady state loan-to-value ratio, $Z_{c,t}$ is a preference shock and $Z_{b,t}$ a credit shock, and $\mathbb{E}_t$ represents the rational expectations operator at period t, i.e. the model assumes Rational Expectations. It is assumed that $\beta R < 1$, so in steady state the credit constraint (3.3) is binding.

For simplicity, the consumer's income is exogenously determined and that its logarithm, y_t,

[2] The model is based on what is described in the appendix of Guerrieri and Iacoviello (2015).

follows the following process:

$$y_t = (1-\rho)\bar{y} + \rho y_{t-1} + \varepsilon_{y,t} \tag{3.4}$$

Additionally, the logarithm of the shocks follow AR(1) processes:

$$log(Z_{c,t}) = \rho_{Zc} log(Z_{c,t-1}) + \varepsilon_{Zc,t} \tag{3.5}$$

$$log(Z_{b,t}) = \rho_{Zb} log(Z_{b,t-1}) + \varepsilon_{Zb,t} \tag{3.6}$$

where the parameters ρ, ρ_{Zc}, and ρ_{Zb} represent the persistence of its respective process.

Therefore, the consumer's problem is to choose the level of consumption, C_t, and debt, B_t, to maximize her lifetime utility (3.1) subject to the constraints (3.2) and (3.3). The Lagrangian of the described optimization problem is given by:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ Z_{c,t} \Gamma_c \frac{(C_t - hC_{t-1})^{1-\gamma} - 1}{1-\gamma} \\ + \mu_t (Y_t + B_t - C_t - RB_{t-1}) \\ + \lambda_t (\delta B_{t-1} + (1-\delta) Z_{b,t} m Y_t - B_t) \}\end{aligned}$$

where μ_t is the budget constraint's Lagrange multiplier and λ_t the borrowing constraint's Lagrange multiplier. From the optimization for the consumption path we get a system of seven necessary conditions and seven variables: $\{Y_t, Z_{c,t}, Z_{b,t}, B_t, C_t, \mu_t, \lambda_t\}$.

Let $\mathcal{U}_{C,t} \equiv \Gamma_c Z_{c,t}(C_t - hC_{t-1})^{-\gamma} - \beta h \Gamma_c \mathbb{E}_t Z_{c,t+1}(C_{t+1} - hC_t)^{-\gamma}$ be the marginal utility. Additionally, let us define $\lambda_{u,t} \equiv \lambda_t / \mathcal{U}_{C,t}$. The conditions for the equilibrium include the budget constraint in equation (3.2), the income process (3.4), the AR(1) exogenous shocks (3.5) and (3.6), together with the first order condition of consumption (3.7), the Euler equation (3.8), and the Kuhn-Tucker condition (3.9):

$$\mu_t = \mathcal{U}_{C,t} \tag{3.7}$$

$$(1-\lambda_{u,t})\mathcal{U}_{C,t} = \beta\,\mathbb{E}_t[(R-\delta\lambda_{u,t+1})\mathcal{U}_{C,t+1}] \tag{3.8}$$

$$\lambda_t(\delta B_{t-1} + (1-\delta)Z_{b,t}mY_t - B_t) = 0 \tag{3.9}$$

Let M_1 be the model under the binding borrowing constraint regime (i.e. $\lambda_t > 0$), which should be the most common given the assumption that the steady-state lies in this regime. On the other hand, let M_2 be the model under the non-binding borrowing constraint regime (i.e. $\lambda_t = 0$).[3]

In the next set of equations, I define the variables in deviation from their steady-state with a hat (i.e. $\hat{x}_t \equiv d\log X_t = \log(X_t/\bar{X})$). In both regimes, M_1 and M_2, the log-linearized equations of the exogenously determined variables are:

$$\hat{y}_t = \rho\hat{y}_{t-1} + \varepsilon_{y,t} \tag{3.10}$$

$$\hat{z}_{c,t} = \rho_{Zc}\hat{z}_{c,t-1} + \varepsilon_{Zc,t} \tag{3.11}$$

$$\hat{z}_{b,t} = \rho_{Zb}\hat{z}_{b,t-1} + \varepsilon_{Zb,t} \tag{3.12}$$

Next, I present the regime contingent equations. In terms of the M_1 regime, the set of

[3] I present the steady-states in Appendix C.1

log-linearized equations are:

$$\hat{b}_t = (1-\delta)m\hat{y}_t + (1-\delta)m\hat{z}_{b,t} + \delta m\hat{b}_{t-1} \tag{3.13}$$

$$\hat{c}_t = \frac{1}{1+(1-R)m}\hat{y}_t + \frac{m}{1+(1-R)m}\hat{b}_t - \frac{Rm}{1+(1-R)m}\hat{b}_{t-1} \tag{3.14}$$

$$\begin{aligned}\hat{\mu}_t = &-\frac{\gamma(1+\beta h^2)}{(1-\beta h)(1-h)}\hat{c}_t + \frac{\gamma h}{(1-\beta h)(1-h)}\hat{c}_{t-1} + \frac{\gamma\beta h}{(1-\beta h)(1-h)}\mathbb{E}_t\hat{c}_{t+1} \\ &+ \frac{1}{1-\beta h}\hat{z}_{c,t} - \frac{\beta h}{1-\beta h}\mathbb{E}_t\hat{z}_{c,t+1}\end{aligned} \tag{3.15}$$

$$\hat{\lambda}_t = \frac{1-\beta\delta}{1-\beta R}\hat{\mu}_t + \delta\beta\ \mathbb{E}_t\hat{\lambda}_{t+1} - \frac{R\beta(1-\beta\delta)}{1-\beta R}\mathbb{E}_t\hat{\mu}_{t+1} \tag{3.16}$$

Meanwhile, the log-linearized equations that are contingent to regime M_2 are:

$$\hat{b}_t = -\frac{1}{m}\hat{y}_t + \frac{1+(1-R)m}{m}\hat{c}_t + R\hat{b}_{t-1} \tag{3.17}$$

$$\begin{aligned}\hat{c}_t = &\frac{\beta h}{1+\beta h^2}\mathbb{E}_t\hat{c}_{t+1} + \frac{h}{1+\beta h^2}\hat{c}_{t-1} + \frac{1-h}{\gamma(1+\beta h^2)}\hat{z}_{c,t} - \frac{\beta h(1-h)}{\gamma(1+\beta h^2)}\mathbb{E}_t\hat{z}_{c,t+1} \\ &- \frac{(1-\beta h)(1-h)}{\gamma(1+\beta h^2)}\hat{\mu}_t\end{aligned} \tag{3.18}$$

$$\hat{\mu}_t = \mathbb{E}_t\hat{\mu}_{t+1} \tag{3.19}$$

$$\hat{\lambda}_t = -1 \tag{3.20}$$

The linearized system of necessary conditions under M_1 and M_2 can be expressed, respectively, as:

$$\mathcal{A}\mathbb{E}_t\xi_{t+1} + \mathcal{B}\xi_t + \mathcal{C}\xi_{t-1} + \mathcal{E}\varepsilon_t = 0 \tag{3.21}$$

$$\mathcal{A}^*\mathbb{E}_t\xi_{t+1} + \mathcal{B}^*\xi_t + \mathcal{C}^*\xi_{t-1} + \mathcal{D}^* + \mathcal{E}^*\varepsilon_t = 0 \tag{3.22}$$

Where $\xi_t = (\hat{y}_t, \hat{z}_{c,t}, \hat{z}_{b,t}, \hat{b}_t, \hat{c}_t, \hat{\mu}_t, \hat{\lambda}_t)'$. The solution for M_1 is done using the Generalized Schur decomposition method proposed by Sims (2001), while for M_2 is obtained using a "guess and verify" approach, as described in Guerrieri and Iacoviello (2015), where the system is pinned down by the household's expectation that the economy will return to the binding constraint regime (since is the steady-state regime). In general, the state-transition can be described by the following equation:

$$\xi_t = J(\xi_{t-1}, \varepsilon_t) + F(\xi_{t-1}, \varepsilon_t)\xi_{t-1} + G(\xi_{t-1}, \varepsilon_t)\varepsilon_t \tag{3.23}$$

3.2.1 Simulated Data

In order to simulate from the described model, I use the parameter values presented in Table 3.1. The data computed in the simulation exercise is presented in Figure 3.1, where we can see that there is a clear change in the behavior of borrowing (B_t) and consumption (C_t) when the credit constraint is either binding ($\lambda_t > 0$) or is not binding ($\lambda_t = 0$).[4]

	Description	**Value**
γ	IES	1
m	Loan-to-value ratio	0.9
δ	Borrowing inertia	0.8
h	Consumption habits	0.5
R	Gross interest rate	1.005
β	Discount factor	0.9870
ρ	Output persistence	0.9
ρ_{zc}	Preference shock persistence	0.8
ρ_{zb}	Credit shock persistence	0.1
σ_y	Output StdDev	0.01
σ_{zc}	Credit shock StdDev	0.015
σ_{zb}	Credit shock StdDev	0.00001

Table 3.1: Model Calibration

[4]In Figure 3.1, the data for the variables, Y_t, B_t, C_t, is standardized.

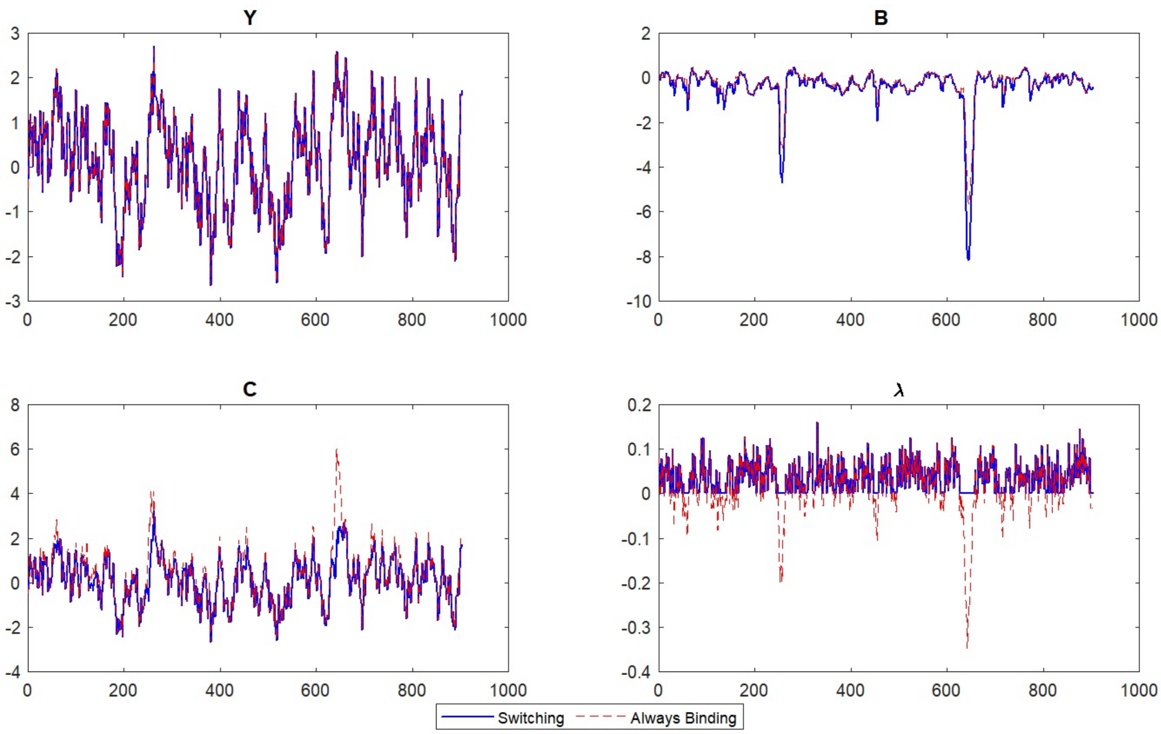

Figure 3.1: Simulated Data from DGP

3.3 Estimation

3.3.1 Proposed algorithm

To define the sampling scheme of my MCMC algorithm, I resort to blocking to construct (when possible) the conditional distributions and be able to proceed with Gibbs sampling. I assume that from the simulated data, only $\mathbf{y}_t = \{\hat{y}_t, \hat{b}_t, \hat{c}_t\}$ will be treated as observed variables.[5] Therefore, the rest of the variables in ξ_t are latent unobserved variables. Finally, the set of parameters to be estimated are represented by $\theta = \{\rho, \rho_{Zc}, \delta, h, \beta, \sigma_Y, \sigma_{Zc}\}$.[6,7]

I start by describing the blocking scheme for the latent variables, $(\lambda_t^*, \xi_{s,t}|\mathbf{y}_t, \theta)$, where $\xi_{s,t} \equiv$

[5]In order to avoid numerical problems, the variables are in logarithm and have been standardized.

[6]The rest of the parameters of the model are assumed to be fixed in the values of the calibration.

[7]Note that since the assumed observed variables have been standardized, the parameters σ_Y, σ_{Zc} should be re-scaled to be comparable with the DGP calibrated parameters.

$\{\hat{z}_{c,t}, \hat{z}_{b,t}, \hat{\mu}_t\}$. I should note that, according to the model in section 3.2, the latent variables are correlated.[8] Therefore, to improve the efficiency of the sampler, these variables should be sampled in a single step. Following the Scheme 2 described in Chan and Jeliazkov (2009), this step is done by sampling $(\lambda_t^* | \mathbf{y}_t, \theta)$, marginally of $\xi_{s,t}$, followed by a draw of $\xi_{s,t}$ from its full conditional distribution, i.e. $(\xi_{s,t} | \mathbf{y}_t, \lambda_t^*, \theta)$. With this blocking scheme, I aim to reduce the inefficiency factors of the sampler and increase its speed of convergence.

The state transition equation for λ_t^* is obtained from the solution of equation (3.21), under regime M_1, scaled to its steady-state level λ_{ss}. I can express such equation as:

$$\lambda_t^* = \lambda_{ss} + F_\lambda \xi_{t-1} + \nu_{\lambda,t}$$

where F_λ is the row of matrix F, from equation (3.23), that corresponds to λ, $\nu_{\lambda,t} \sim N(0, \sigma_\lambda^2)$, and σ_λ^2 is the variance of λ, which will be assumed to be a constant.[9] As I mentioned before, to improve the sampling of λ^*, I integrated out ξ_s, and leave it conditional on the observed data $\mathbf{y}$; i.e.

$$\lambda_t^* = \lambda_{ss} + F_\lambda \mathbf{y}_{t-1} + \nu_{\lambda,t} \tag{3.24}$$

In terms of the measurement equation for λ^* we have the relation with the Lagrange multiplier λ: $\lambda_t = \mathbb{1}\{\lambda_t^* > 0\}$, where λ_t defines the credit constraint regime of the household in period t. This can be represented as a state-transition equation for ξ_t, as equation (3.23),

[8] The variables in $\xi_{s,t}$ depend on the regime, defined by λ_t, while at the same time the solution of equation (3.21) suggest that λ_t will depend on the lagged values of the state variables of the model.

[9] In the estimation I set $\sigma_\lambda = 0.025$, which is close to the 0.0368 model consistent standard deviation with the calibrated parameters used to generate the simulated data.

for each model represented in equations (3.21) and (3.22):

$$\xi_t = F(\lambda_t > 0)\xi_{t-1} + G(\lambda_t > 0)\varepsilon_t \tag{3.25}$$

$$\xi_t = F(\lambda_t = 0)\xi_{t-1} + G(\lambda_t = 0)\varepsilon_t \tag{3.26}$$

where $\{F(\lambda_t > 0), G(\lambda_t > 0)\} \equiv \{F, G\}$ are matrices of reduced form parameters for regime M1 obtained from the solution of equation (3.21), while $\{F(\lambda_t = 0), G(\lambda_t = 0)\} \equiv \{F^*, G^*\}$ are matrices of reduced form parameters for regime M2 solution of equation (3.22).

As stated before, I am conditioning λ^* on the observed variables $\mathbf{y}$. Out of these variables, $\hat{b}_t$ and $\hat{c}_t$ change their structural relation depending on the regime defined by the Lagrange multiplier.[10] Let $\mathbf{y}_{2,t} \equiv \{\hat{b}_t, \hat{c}_t\}$, I express in SUR form the solutions for these variables as:

$$\mathbf{y}_{2,t} = X_{2,t}\phi_{BC} + e_{2,t} \tag{3.27}$$

$$\mathbf{y}_{2,t} = X_{2,t}\phi^*_{BC} + e^*_{2,t} \tag{3.28}$$

where ϕ_{BC} and $e_{2,t} \sim N(0, \Omega_{BC})$ are for the regime M1 ($\lambda^* > 0$), while ϕ^*_{BC} and $e^*_{2,t} \sim N(0, \Omega^*_{BC})$ are for the regime M2 ($\lambda^* \leq 0$), and matrices Ω_{BC} and Ω^*_{BC} are the model consistent co-variances.[11]

From equation (3.24), let $\tilde{\lambda}^*_t = \lambda_{ss} + F_\lambda \mathbf{y}_{t-1}$, and define $\pi_{0,t} \equiv Pr(\lambda^*_t > 0) = 1 - \Phi(-\tilde{\lambda}^*_t/\sigma_\lambda)$, where $\Phi()$ is the CDF of a standard normal distribution. Combining this with equations (3.27) and (3.28), we obtain an expression for the probability that λ^*_t is positive:

$$\begin{aligned} \pi_t &= \frac{\pi_{0,t}\phi(e_{2,t}|\phi_{BC}, \Omega_{BC})}{\pi_{0,t}\phi(e_{2,t}|\phi_{BC}, \Omega_{BC}) + (1 - \pi_{0,t})\phi(e^*_{2,t}|\phi^*_{BC}, \Omega^*_{BC})} \\ &= \frac{1}{1 + \Psi_t} \end{aligned} \tag{3.29}$$

[10] Remember that $\hat{y}_t$ is exogenously determined.

[11] These are sub-matrices, related to variables b_t and c_t, of the Ω matrix that represents the solution to the theoretical second moment: $\Omega = F\Omega F' + G\Sigma G'$.

where $\Psi_t = \frac{(1-\pi_{0,t})\phi(e^*_{2,t}|\phi^*_{BC},\Omega^*_{BC})}{\pi_{0,t}\phi(e_{2,t}|\phi_{BC},\Omega_{BC})}$, and $\phi()$ is the pdf of a Gaussian distribution. Then, the conditional density of λ^*_t is the following 2-component mixture of truncated normal distributions:

$$p(\lambda^*_t|\mathbf{y}_t,\theta) = \pi_t\phi_{(0,\infty)}(\lambda^*_t|\tilde{\lambda}^*_t,\sigma_\lambda) + (1-\pi_t)\phi_{(-\infty,0)}(\lambda^*_t|\tilde{\lambda}^*_t,\sigma_\lambda) \tag{3.30}$$

A draw from the above mixture can be obtained as follows. First, get a Bernoulli draw Z with $\mathbb{P}(Z=1)=\pi_t$. If $Z=1$, sample λ^*_t from the truncated $N_{(0,\infty)}(\lambda^*_t|\tilde{\lambda}^*_t,\sigma_\lambda)$; if $Z=0$, instead sample λ^*_t from $N_{(-\infty,0)}(\lambda^*_t|\tilde{\lambda}^*_t,\sigma_\lambda)$.

Given λ^*, I move on to the sampling of $(\xi_{s,t}|\mathbf{y}_t,\lambda^*_t,\theta)$. It turns out that $G^{-1}F=\mathcal{C}$ and ${G^*}^{-1}F^*=\mathcal{C}^*$, from equations (3.21) and (3.22). I use this result, together with the following two definitions, to rewrite equations (3.25) and (3.26) together. Let
$\mathbf{iG} \equiv \left(G^{-1}\mathbb{1}_{\{\lambda_t>0\}} + {G^*}^{-1}\mathbb{1}_{\{\lambda_t=0\}}\right)$, and $\mathbf{C} \equiv \left(\mathcal{C}\,\mathbb{1}_{\{\lambda_t>0\}} + \mathcal{C}^*\mathbb{1}_{\{\lambda_t=0\}}\right)$. Therefore,

$$\mathbf{iG}\xi_t = \mathbf{C}\xi_{t-1} + \varepsilon_t \tag{3.31}$$

Let

$$H = \begin{pmatrix} \mathbf{iG} & & & & \\ \mathbf{C} & \mathbf{iG} & & & \\ & \mathbf{C} & \ddots & & \\ & & \ddots & \ddots & \\ & & & \mathbf{C} & \mathbf{iG} \end{pmatrix}.$$

Now, I can stack ξ_t and ε_t over time and express the equation above as:

$$H\xi = \varepsilon$$

Out of matrix H, I take the sub-matrices H_{ss}, H_{yy}, and H_{ys}, where the indices ij of H_{ij} represent the rows associated with variable i and columns of variable j. One should keep in mind that matrix H and its sub-matrices are functions of λ. With these matrices, I can express the state-transition and measurement equations, respectively, as:

$$H_{ss}\xi_s = \varepsilon_s \tag{3.32}$$

$$H_{yy}\mathbf{y} = H_{ys}\xi_s + \varepsilon_y \tag{3.33}$$

where $\varepsilon_s \sim N(0, \Sigma_s)$, $\varepsilon_y \sim N(0, \Sigma_y)$, and both Σ_s and Σ_y are part of Σ, a diagonal matrix.[12]

From equation (3.32), the prior for ξ_s can be express as $p(\xi_s|\lambda^*, \theta) \propto \xi_s' H_{ss}'(I_T \otimes \Sigma_s^{-1})H_{ss}\xi_s$. From equation (3.33), one can express the likelihood function as $f(\mathbf{y}|\xi_s, \lambda^*, \theta) \propto (H_{yy}\mathbf{y} - H_{ys}\xi_s)'(I_t \otimes \Sigma_y^{-1})(H_{yy}\mathbf{y} - H_{ys}\xi_s)$. Combining the above expressions, I find that:

$$\begin{aligned} p(\xi_s|\mathbf{y}, \lambda^*, \theta) \propto\ & \xi_s'(H_{ss}'(I_T \otimes \Sigma_s^{-1})H_{ss} + H_{ys}'(I_t \otimes \Sigma_y^{-1})H_{ys})\xi_s \\ & - 2\xi_s' H_{ys}'(I_t \otimes \Sigma_y^{-1})H_{yy}\mathbf{y} \end{aligned}$$

such that $p(\xi_s|\mathbf{y}, \lambda^*, \theta) \sim N(\hat{\xi}_s, K_s^{-1})$, where

$$K_s = H_{ss}'(I_t \otimes \Sigma_s^{-1})H_{ss} + H_{ys}'(I_t \otimes \Sigma_y^{-1})H_{ys} \tag{3.34}$$

$$\hat{\xi}_s = K_s^{-1}(H_{ys}'(I_t \otimes \Sigma_y^{-1})H_{yy}\mathbf{y}) \tag{3.35}$$

To sample $(\xi_s|\mathbf{y}, \lambda, \theta)$ from the above distribution, I follow the efficient estimation of latent states algorithm proposed by Chan and Jeliazkov (2009), which provides draws for ξ_s at low computational costs.

To sample from $(\theta|\mathbf{y}, \xi_s, \lambda)$, most of the steps are standard results with some minor restric-

[12]The elements σ_Y and σ_{Zc} of Σ are part of the θ parameters under estimation. The other elements of Σ are assumed to be known.

tions imposed in the sampling. Only for the case of h and β it was necessary to sample them jointly with an Accept-Reject Metropolis-Hastings (ARMH) algorithm, which I will describe next.

The parameters h and β are observed in the following equation (equivalent to (3.15)):

$$
\begin{aligned}
(1-\beta h)\hat{\mu}_t = & -\frac{\gamma(1+\beta h^2)}{1-h}\hat{c}_t + \frac{\gamma h}{1-h}\hat{c}_{t-1} + \frac{\gamma\beta h}{1-h}\mathbb{E}_t\hat{c}_{t+1} \\
& + \hat{Z}_{c,t} - \beta h\mathbb{E}_t\hat{Z}_{c,t} + \nu^*_{\mu,t}
\end{aligned}
$$

where I introduce a measurement error and it is assumed that $\nu^*_{\mu,t} \sim N(0, \omega^2_{\mu*})$. Additionally, the expectations are estimated from the Rational Expectations Equilibrium under the model in regime M1.

I proceed to define $X_{\mu,t} \equiv \{\hat{c}_t, -\hat{c}_{t-1}, -\mathbb{E}_t\hat{c}_{t+1}, -\hat{Z}_{c,t}, \mathbb{E}_t\hat{Z}_{c,t+1}, \hat{\mu}_t\}$ and vector

$$
\theta_\mu \equiv \left(\frac{\gamma(1+\beta h^2)}{1-h}, \frac{\gamma h}{1-h}, \frac{\gamma\beta h}{1-h}, 1, \beta h, 1-\beta h\right)'
$$

such that

$$
\begin{aligned}
\nu^*_{\mu,t} &= \frac{\gamma(1+\beta h^2)}{1-h}\hat{c}_t - \frac{\gamma h}{1-h}\hat{c}_{t-1} - \frac{\gamma\beta h}{1-h}\mathbb{E}_t\hat{c}_{t+1} - \hat{Z}_{c,t} + \beta h\mathbb{E}_t\hat{Z}_{c,t} + (1-\beta h)\hat{\mu}_t \\
&= X_{\mu,t}\theta_\mu
\end{aligned}
$$

By stacking the variables over time I can express the equation above as $\nu^*_\mu = X_\mu\theta_\mu$. Therefore, the logarithm of the likelihood function is proportional to:

$$
\log f(\nu^*_\mu|h,\beta) \propto -\frac{1}{2}\frac{\theta'_\mu X'_\mu X_\mu\theta_\mu}{\omega^2_{\mu*}}
$$

Given the nonlinear relation between h and β, I proceed to define the approximation of the likelihood function $f(\nu^*_\mu|h,\beta)$.[13] Let $\theta_{hb} \equiv (h,\beta)'$, and allow me to define the gradient as

[13] I am omitting that is also conditional to X_μ to simplify notation.

$f = (f_h, f_\beta)'$, where:

$$f_h \equiv \frac{\partial}{\partial h} \log f(\nu_\mu^*|\theta_{hb})\bigg|_{\theta_{hb}=\tilde{\theta}_{hb}} = -\frac{\theta_\mu' X_\mu' X_\mu}{\omega_{\mu*}^2} \frac{\partial \theta_\mu}{\partial h}\bigg|_{\theta_{hb}=\tilde{\theta}_{hb}}$$
$$f_\beta \equiv \frac{\partial}{\partial \beta} \log f(\nu_\mu^*|\theta_{hb})\bigg|_{\theta_{hb}=\tilde{\theta}_{hb}} = -\frac{\theta_\mu' X_\mu' X_\mu}{\omega_{\mu*}^2} \frac{\partial \theta_\mu}{\partial \beta}\bigg|_{\theta_{hb}=\tilde{\theta}_{hb}}$$

and let the elements of the negative Hessian G be express as $G_{i,j}$, for $i, j = h, \beta$, where:

$$\begin{aligned} G_{i,j} &\equiv -\frac{\partial^2}{\partial i \partial j} \log f(\nu_\mu^*|\theta_{hb})\bigg|_{\theta_{hb}=\tilde{\theta}_{hb}} \\ &= \frac{\theta_\mu' X_\mu' X_\mu}{\omega_{\mu*}^2} \frac{\partial^2 \theta_\mu}{\partial i \partial j}\bigg|_{\theta_{hb}=\tilde{\theta}_{hb}} + \left(\frac{\partial \theta_\mu}{\partial i}\bigg|_{\theta_{hb}=\tilde{\theta}_{hb}} \right)' \frac{X_\mu' X_\mu}{\omega_{\mu*}^2} \frac{\partial^2 \theta_\mu}{\partial j}\bigg|_{\theta_{hb}=\tilde{\theta}_{hb}} \end{aligned}$$

With these definitions, I can expand the log-likelihood, $\log f(\nu_\mu^*|\theta_{hb})$, around $\tilde{\theta}_{hb}$ to obtain the expression:

$$\begin{aligned} \log f(\nu_\mu^*|\theta_{hb}) &\approx \log f(\nu_\mu^*|\tilde{\theta}_{hb}) + (\theta_{hb} - \tilde{\theta}_{hb})' f - \frac{1}{2}(\theta_{hb} - \tilde{\theta}_{hb})' G (\theta_{hb} - \tilde{\theta}_{hb}) \\ &= -\frac{1}{2}[\theta_{hb}' G \theta_{hb} - 2\theta_{hb}(f + G\tilde{\theta}_{hb})] + c_f^*, \end{aligned}$$

where c_f^* is a constant independent of θ_{hb}.

I posit a Gaussian prior for $\theta_{hb} \sim N(\theta_{hb,0}, K_{hb,0}^{-1})$. Combined with the approximation of the log-likelihood, I get the following expression for the conditional distribution of θ_{hb}:

$$\begin{aligned} \log p(\theta_{hb}|\nu_\mu^*) &\propto \log f(\nu_\mu^*|\theta_{hb}) + \log p(\theta_{hb}) \\ &\approx -\frac{1}{2}[\theta_{hb}'(G + K_{hb,0})\theta_{hb} - 2\theta_{hb}(f + G\tilde{\theta}_{hb} + K_{hb,0}\theta_{hb,0})] + c_f^{**}, \end{aligned} \tag{3.36}$$

where again c_f^{**} is a constant independent of θ_{hb}. This result imply that the approximating posterior distribution is Gaussian with precision $K_{hb} \equiv G + K_{hb,0}$ and mean vector $K_{hb}^{-1}(f + G\tilde{\theta}_{hb} + K_{hb,0}\theta_{hb,0})$.

I choose that the point $\tilde{\theta}_{hb}$, around which the Taylor expansion is constructed, to be the posterior mode, $\hat{\theta}_{hb}$, which has the advantage that it can be obtained via the Newton-Raphson method. More specifically, it follows from equation (3.36) that the negative Hessian of $\log p(\theta_{hb}|\nu_\mu^*)$ evaluated at $\theta_{hb} = \tilde{\theta}_{hb}$ is K_{hb}, while the gradient at $\theta_{hb} = \tilde{\theta}_{hb}$ is given by

$$\frac{\partial}{\partial \theta_{hb}} \log p(\theta_{hb}|\nu_\mu^*)\bigg|_{\theta_{hb}=\tilde{\theta}_{hb}} = -K_{hb}\tilde{\theta}_{hb} + 2(f + G\tilde{\theta}_{hb} + K_{hb,0}\theta_{hb,0})$$

Hence, we can implement the Newton-Raphson method as follows: initialize with $\theta_{hb} = \theta_{hb}^{(1)}$. For $s = 1, 2, ...$, use $\tilde{\theta}_{hb} = \theta_{hb}^{(s)}$ in the evaluation of f, G, and K_{hb}, and denote them as $f(\theta_{hb}^{(s)})$, $G(\theta_{hb}^{(s)})$, and $K_{hb}(\theta_{hb}^{(s)})$, respectively, where the dependence on $\theta_{hb}^{(s)}$ is made explicit. Compute $\theta_{hb}^{(s+1)}$ as

$$\begin{aligned} \theta_{hb}^{(s+1)} &= \theta_{hb}^{(s)} + K_{hb}(\theta_{hb}^{(s)})^{-1} \frac{\partial}{\partial \theta_{hb}} \log p(\theta_{hb}|\nu_\mu^*)\bigg|_{\theta_{hb}=\tilde{\theta}_{hb}} \\ &= K_{hb}(\theta_{hb}^{(s)})^{-1}(f(\theta_{hb}^{(s)}) + G(\theta_{hb}^{(s)})\theta_{hb}^{(s)} + K_{hb,0}\theta_{hb,0}) \end{aligned} \tag{3.37}$$

If $\|\theta_{hb}^{(s+1)} - \theta_{hb}^{(s)}\| \geq \epsilon$, for some pre-fixed tolerance level ϵ, then continue; otherwise stop and $\hat{\theta}_{hb} = \theta_{hb}^{(s+1)}$. Given the mode $\hat{\theta}_{hb}$, the negative Hessian K_{hb} at $\hat{\theta}_{hb}$ can be easily computed.

To sample from $(h, \beta|\xi, \theta_{-(h,\beta)})$, where θ_{-j} represents any other parameter in θ other than j, I use the approximation of the posterior $N(\tilde{\theta}_{hb}, K_{hb}^{-1})$ to make candidate draws and accept or reject them via an ARMH step as proposed by Chib and Jeliazkov (2005).[14] Using the ARMH algorithm, I can construct a better approximation of the objective distribution, and consequently, the acceptance rate is substantially higher than the baseline MH algorithm.

To sample from $(\rho|\xi, \theta_{-\rho})$, notice that ρ's measurement equation is given by (3.10). For convenience, I re-express that equation as: $\hat{y}_t = X_{\rho,t}\rho + \varepsilon_{y,t}$, and by stacking $\hat{y}_t$ and $X_{\rho,t}$ over

[14]For a more detailed description of the Newton-Raphson method and the ARMH step I used, I direct the reader to Chan and Strachan (2014).

time I can represent it as $\hat{y} = X_\rho \rho + \varepsilon_y$. I assume a prior $N(\rho_0, S^2_{\rho 0})$. Therefore,

$$
\begin{aligned}
p(\rho|\hat{y}) &\propto f(\hat{y}|\rho)\, p(\rho) \\
&\propto \rho'(1/S^2_{\rho 0} + X'_\rho X_\rho/\sigma^2_y)\rho - 2\rho(\rho_0/S^2_{\rho 0} + X'_\rho \hat{y}/\sigma^2_y)
\end{aligned}
$$

such that the conditional distribution is $p(\rho|\hat{y}, \sigma_y) \sim N(\hat{\rho}, K^{-1}_\rho)$, where:

$$K_\rho = 1/S^2_{\rho 0} + X'_\rho X_\rho/\sigma^2_y \tag{3.38}$$

$$\hat{\rho} = K^{-1}_\rho(\rho_0/S^2_{\rho 0} + X'_\rho \hat{y}/\sigma^2_y) \tag{3.39}$$

While sampling from $(\rho|\hat{y}, \sigma_y)$ the algorithm only accept draws that are between $(0, 1)$, since the persistence, ρ, is defined for values in that space.

Analogously, to sample from $(\rho_{Zc}|\xi, \theta_{-\rho_{Zc}})$ we should look at its measurement equation (3.11). Re-expressing and stacking over time we have $\hat{Z}_c = X_{Zc}\rho_{Zc} + \varepsilon_{Zc}$. With an assumed prior $N(\rho_{Zc,0}, S^2_{\rho Zc,0})$, we get that $p(\rho_{Zc}|\hat{Z}_c, \sigma_{Zc}) \sim N(\hat{\rho}_{Zc}, K^{-1}_{\rho Zc})$, where:

$$K_{\rho Zc} = 1/S^2_{\rho Zc,0} + X'_{Zc}X_{Zc}/\sigma^2_{Zc} \tag{3.40}$$

$$\hat{\rho}_{Zc} = K^{-1}_{\rho Zc}(\rho_{Zc,0}/S^2_{\rho Zc,0} + X'_{Zc}\hat{Z}_c/\sigma^2_{Zc}) \tag{3.41}$$

And again, the algorithm accept only draws that are between $(0, 1)$.

For $(\delta|\xi, \theta_{-\delta})$, we observe that its measurement equations are (3.13) and (3.16). I re-express and stack over time equation (3.13) as:

$$b_\delta = X_{b\delta}\delta + e_b$$

where $b_{\delta,t} = \hat{b}_t - m\hat{y}_t - m\hat{Z}_{c,t}$ and $X_{b\delta,t} = m\hat{b}_{t-1} - m\hat{y}_t - m\hat{Z}_{c,t}$ and $e_{\delta,t}$ is a measurement error that is assumed Gaussian $N(0, \omega_b^2)$. Analogously, for equation (3.16),

$$\lambda_\delta = X_{\lambda\delta}\delta + e_\lambda$$

where $\lambda_{\delta,t} = (1-\beta R)\hat{\lambda}_t - \hat{\mu}_t + \beta R \mathbb{E}_t\hat{\mu}_{t+1}$ and $X_{\lambda\delta,t} = \beta(\mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\mu}_t + \beta R\mathbb{E}_t\hat{\mu}_{t+1})$ and $e_{\lambda,t}$ is a measurement error that is assumed Gaussian $N(0, \omega_\lambda^2)$. Finally, I assume a prior $N(\delta_0, S_{\delta 0}^2)$. Therefore, the logarithm of the objective distribution is:

$$\begin{aligned}\log p(\delta|\xi, \theta_{-\delta}) &\propto \log p(\delta) + \log f(\hat{b}|\delta, \theta_{-\delta}) + \log f(\hat{\lambda}|\delta, \theta_{-\delta}) \\ &\propto \delta'(1/S_{\delta 0}^2 + X_{b\delta}'X_{b\delta}/\omega_b^2 + X_{\lambda\delta}'X_{\lambda\delta}/\omega_\lambda^2)\delta \\ &\quad - 2\delta'(\delta_0/S_{\delta 0}^2 + X_{b\delta}'b_\delta/\omega_b^2 + X_{\lambda\delta}'\lambda_\delta/\omega_\lambda^2)\end{aligned}$$

Note that the variables in equation (3.16) are all unobserved, so to improve the sampling of δ, I integrate them out. As a result, the conditional distribution is $p(\delta|\hat{b}, \theta_{-\delta}) \sim N(\hat{\delta}, K_\delta^{-1})$, where:

$$K_\delta = 1/S_{\delta 0}^2 + X_{b\delta}'X_{b\delta}/\omega_b^2 + X_{\lambda\delta}'X_{\lambda\delta}/\omega_\lambda^2 \tag{3.42}$$

$$\hat{\delta} = \delta_0/S_{\delta 0}^2 + X_{b\delta}'b_\delta/\omega_b^2 \tag{3.43}$$

To construct $p(\sigma_Y|\hat{y}, \theta_{-\sigma_Y})$, I remind the reader that the measurement equation for σ_Y is (3.10), where $\varepsilon_{y,t} = \hat{y}_t - X_{\rho,t}\rho$, and it is assumed that $\varepsilon_{y,t} \sim N(0, \sigma_Y^2)$. I consider a prior $p(\sigma_Y) \sim \Gamma^{-1}(\nu_0, S_0)$. From standard results, I have that the conditional distribution is also an inverse Gamma, $p(\sigma_Y|\hat{y}, \theta_{-\sigma_Y}) \sim \Gamma^{-1}(\hat{\nu}, \hat{S})$, with updated parameters:

$$\hat{\nu} = \nu_0 + \frac{T-1}{2} \tag{3.44}$$

$$\hat{S} = S_0 + \sum_{t=1}^{T} \frac{\varepsilon_{y,t}^2}{2} \tag{3.45}$$

Before moving on to define $p(\sigma_{Zc}|\xi, \theta_{-\sigma_{Zc}})$, I must point out that there some issues with the identification of the consumer's preference shock ($\hat{z}_{c,t}$), and here I would provide some intuition behind it. Note that when the consumer is in the binding credit constraint regime, M_1, both her consumption and borrowing decisions are restricted by her budget (3.2) and credit (3.3) constraints, making it difficult to elicit the consumer's preference shocks. On the other hand, when the consumer is in the non-binding credit constraint, M_2, her borrowing decision is contingent on her consumption decision; which in turn is related to the Euler equation (3.8); this implies that in this regime the consumption decision depends on the consumer's preference shock. Consequently, it is in regime M_2 where one could identify the preference shock and its related parameters.

In this context, to construct $p(\sigma_{Zc}|\xi, \theta_{-\sigma_{Zc}})$ I follow the next steps. First, I use the data under the non-binding credit constraint regime ($\lambda_t^* \leq 0$) and employ equation (3.18), to which I add a measurement error. Simplifying the parameters' notation, I have that:

$$\hat{c}_t = \phi_{c,1}\mathbb{E}_t\hat{c}_{t+1} + \phi_{c,2}\hat{c}_{t-1} + \phi_{c,3}\hat{z}_{c,t} - \phi_{c,4}\mathbb{E}_t\hat{z}_{c,t+1} - \phi_{c,5}\hat{\mu}_t + e_{c,t}$$

where $e_{c,t} \sim N(0, \omega_c^2)$, and assume that ω_c^2 have an inverse Gamma prior. Therefore, the posterior for ω_c^2 will also be inverse Gamma with updating parameters:

$$\hat{\nu}_c = \nu_{c,0} + \frac{\tau - 1}{2} \tag{3.46}$$

$$\hat{S}_c = S_{c,0} + \sum_{t=1}^{\tau} \frac{e_{c,t}^2}{2} \tag{3.47}$$

where $\tau = \sum_{t=1}^{T} \mathbb{1}\{\lambda_t^* \leq 0\}$. Next, I assume that the variance of the measurement error is composed primarily from the noise of the latent variables in the simplification of equation (3.18).[15] This give us $\omega_c^2 \approx (\phi_{c,3} + \phi_{c,4}\rho_{Zc})^2\sigma_{Zc}^2 + \phi_{c,5}^2\omega_\mu^2$. From the REE solution I get the

[15] I base this assumption on the notion that consumption is an observed variable.

model consistent variance,[16] ω_μ^2, so that after solving for σ_{Zc}^2 I get:

$$\sigma_{Zc}^2 \approx (\omega_c^2 - \phi_{c,5}^2\omega_\mu^2)/(\phi_{c,3} + \phi_{c,4}\rho_{Zc})^2 \tag{3.48}$$

To sample from $(\sigma_{Zc}^2|\xi, \theta_{-\sigma_{Zc}})$, I make draws from $(\omega_c^2|\xi, \lambda \leq 0, \theta) \sim \Gamma^{-1}(\hat{\nu}_c, \hat{S}_c)$ and afterwards solve the approximation described above for σ_{Zc}^2, rejecting any combination that results in a negative σ_{Zc}^2.

In summary, the steps of my MCMC algorithm are the following:

1. Draw jointly $(\lambda^*, \xi_s|\mathbf{y}, \theta)$ as follows:

   (a) Sample $(\lambda^*|\mathbf{y}, \theta)$, marginally of ξ_s, each period t at a time from equation (3.30), by drawing a Bernoulli Z, where $(Z = 1) \equiv (\lambda_t^* > 0)$ and $(Z = 0) \equiv (\lambda_t^* \leq 0)$, and sample λ_t^* from the corresponding truncated Gaussian distribution;

   (b) Sample $(\xi_s|\mathbf{y}, \lambda^*, \theta)$ by drawing from $N(\hat{\xi}_s, K_s^{-1})$, with precision K_s given by equation (3.34) and mode $\hat{\xi}_s$ by equation (3.35), using the precision sampler in Chan and Jeliazkov (2009).

2. Draw $(\theta|\mathbf{y}, \xi_s, \lambda^*)$ as described for each case:

   (a) Sample jointly $(h, \beta|\xi, \theta_{-(h,\beta)})$, by making candidate draws from the approximation $N(\hat{\theta}_{hb}, K_{hb}^{-1})$, where $\hat{\theta}_{hb}$ is found via the Newton-Raphson method, described in equation (3.37), and accept or reject them via an ARMH step as proposed by Chib and Jeliazkov (2005).

   (b) Sample $(\rho|\hat{y}, \theta_{-\rho}) \sim N(\hat{\rho}, K_\rho^{-1})$, with precision K_ρ and mean $\hat{\rho}$, given by equations (3.38) and (3.39), respectively.

   (c) Sample $(\rho_{Zc}|\hat{Z}c, \theta_{-\rho_{Zc}}) \sim N(\hat{\rho}_{Zc}, K_{\rho Zc}^{-1})$, with precision $K_{\rho Zc}$ and mean $\hat{\rho}_{Zc}$, given by equations (3.40) and (3.41), respectively.

[16] When computing the model consistent variance for μ_t, I constructed without the effects of σ_{Zc}^2. Therefore, the subtraction of ω_c^2 in equation (3.48) takes out the noise not related to σ_{Zc}^2.

(d) Sample $(\delta|\mathbf{y}, \theta_{-\delta}) \sim N(\hat{\delta}, K_\delta^{-1})$, with precision K_δ and mean $\hat{\delta}$, given by equations (3.42) and (3.43), respectively.

(e) Sample $(\sigma_Y^2|\hat{y}, \theta_{-\sigma_Y}) \sim \Gamma^{-1}(\hat{\nu}, \hat{S})$, with degrees of freedom $\hat{\nu}$ and scale parameter $\hat{S}$, given by equations (3.44) and (3.45), respectively.

(f) Sample $(\sigma_{Zc}^2|\xi, \theta_{-\sigma_{Zc}})$ by making draws from $(\omega_c^2|\xi, \lambda \leq 0, \theta) \sim \Gamma^{-1}(\hat{\nu}_c, \hat{S}_c)$, with degrees of freedom $\hat{\nu}_c$ and scale parameter $\hat{S}_c$, given by equations (3.46) and (3.47), respectively, and solve for σ_{Zc}^2 as in equation (3.48).

3.3.2 Results

The estimation is based on a sample of 10,000 draws kept after a burn-in of 1,000 draws. In Table 3.2 I present the calibration value used in the data generating process, the assumed prior distribution,[17] and the results of the posterior draws.[18,19] Additionally, I show the trace plots in Figure 3.2, while in Figure 3.5 I contrast the simulated data from the DGP and the estimated mean of the λ^* latent variable.

Variable	DGP	Prior	Mean	5%	95%	Ineff Factor
δ	0.80	$N(0.70, 0.05^2)$	0.7817	0.7142	0.8506	1.1
h	0.50	$N(0.50, 0.15^2)$	0.4875	0.4635	0.5092	1.2
ρ	0.90	$N(0.70, 0.1^2)$	0.9021	0.8788	0.9256	0.9
ρ_{Zc}	0.80	$N(0.80, 0.02^2)$	0.7985	0.7657	0.8309	2.8
β	0.987	$N(0.985, 0.002^2)$	0.9870	0.9834	0.9906	1.1
σ_Y	0.44	$\Gamma^{-1}(2, 1)$	0.4283	0.4122	0.4453	1.1
σ_{Zc}	0.65	$\Gamma^{-1}(2, 1)$	0.7922	0.6349	0.9854	1.3

Table 3.2: Posterior mean and 90 percent credible interval.

The results show that the posterior mean is a good approximation for most of the parameters,

[17] In Table 3.2, the priors with $N(\mu_0, \sigma_0^2)$, the hyperparameters are the mean and variance. For the priors with $\Gamma^{-1}(\nu_0, S_0)$, the hyperparameters are the degrees of freedom and the scale.

[18] The DGP value of σ_Y and σ_{Zc} are 0.01 and 0.015, respectively. In the Table, these values are scaled by the standard deviation of the observed data used for its standardization.

[19] The inefficiency factors are defined as $1+2\sum_{j=1}^{J}\phi_j$, where ϕ_j is the sample autocorrelation at lag length j, and J is chosen large enough so that the autocorrelation tapers off. I borrow the code from Eisenstat, Chan, and Strachan (2016) for their computation.

and the 90 percent credible interval contains all the "true" values, although there are some caveats. First, I want to remark that because of the identification issues related to the consumer preference shock described in the previous section, I am using a very informative prior for ρ_{Zc}. Even with this tight prior, we can see that the highest inefficiency factor was for the draws of ρ_{Zc} (2.8). Additionally, in Figure 3.3 we can see that the distribution of σ_{Zc} is not centered at the "true" value, and it is skewed to the right, suggesting that as a whole there are still some problems with the identification of this shock.

In terms of the sensibility of the results to the priors, I can point out that draws for ρ and σ_Y are the least sensible for the choice of prior. On the other hand, δ and β are highly sensitive to the choice of prior, while the draws of h are susceptible to the choice of prior variance but not to the prior mean. When the prior of h is not diffuse enough, the ARMH step for the pair (h, β) stops updating, eroding the efficiency in the sampler intended with this step.[20]

In the marginal density plots, presented in Figure 3.3, we can see that the distributions have a well-defined bell curve shape, although the ones for δ and h are not center at their "true" values. Moreover, the density of parameter h also appears to be a small skewness to the left. In Figure 3.4, I present scatter plots for all the combinations of estimated parameters. We can see some irregularities at the bottom of pairs[21] $(\beta, \delta)_{[1,4]}$ and $(\beta, \rho_{Zc})_{[3,2]}$, where it appears that the sampling of β was not updating a small number of times. The scatter plots related to ρ_{Zc} have an odd shape, which reflects the tight prior used for this parameter, while the upside skewness of σ_{Zc} is also reflected in these plots.

[20] The most common case when the sampler for (h, β) stops updating is when the draw of h is well below its mean. This behavior can be lessened when the prior variance for h is high enough to help the sampler to accept other draws.

[21] The subscript represent the [row,column] location in Figure 3.4.

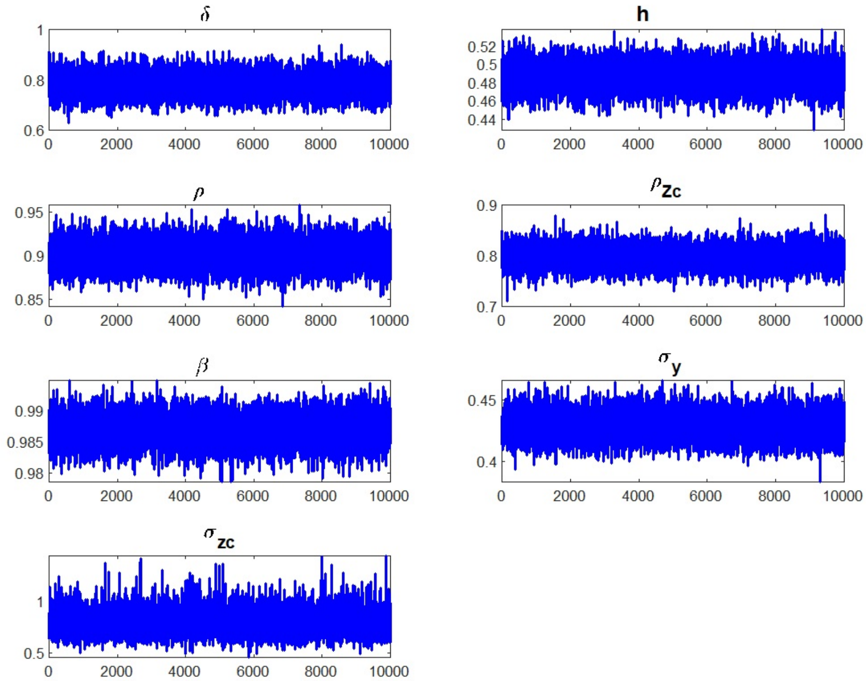

Figure 3.2: Trace plots for each θ.

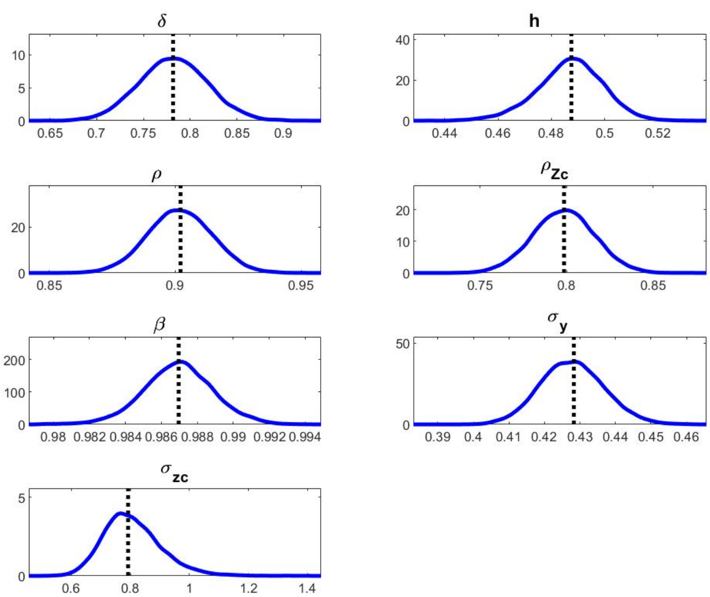

Figure 3.3: Posterior Marginal Densities.

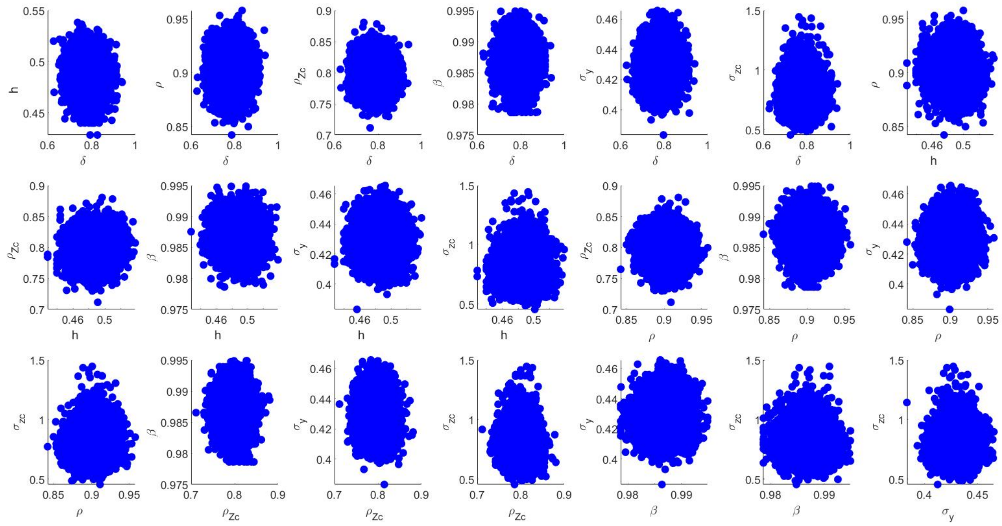

Figure 3.4: Scatter plots of draws.

Finally, in terms of the variable that defines the change of regime, λ^*, in Figure 3.5 I contrast the variable from the DGP simulation (on the left) with the posterior mean estimation (on the center). We can observe that the posterior mean predicts reasonably well the change of regime, especially in periods where the data suggest a substantial dip in λ^*, i.e., its value goes well below zero. These periods with big drops are also distinguishable on the 80 percent credible interval, presented on the right side of Figure 3.5. Additionally, the sampling of λ^* is done efficiently, as is evident from the reasonable low inefficiency factors computed from its draws (see Figure 3.6). This result suggests that the sampling λ^*, done marginally of the other latent variables, is in the right direction of increasing the efficiency of the sampler.

3.4 Conclusions and Future Work

In this paper, I generate data from a model that switches from regime depending on the consumer's credit constraint state. Based on the data from the model simulation, I proposed an MCMC algorithm to recover the latent variable that indicates the change of regime and

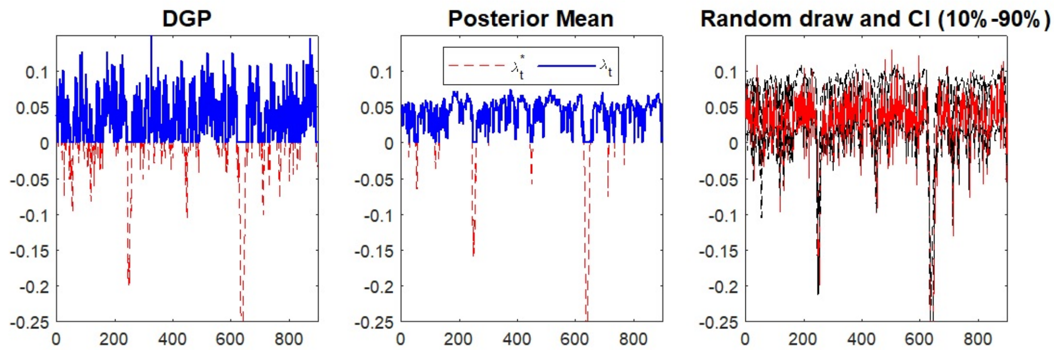

Figure 3.5: Lagrange Multiplier (λ_t) estimation.

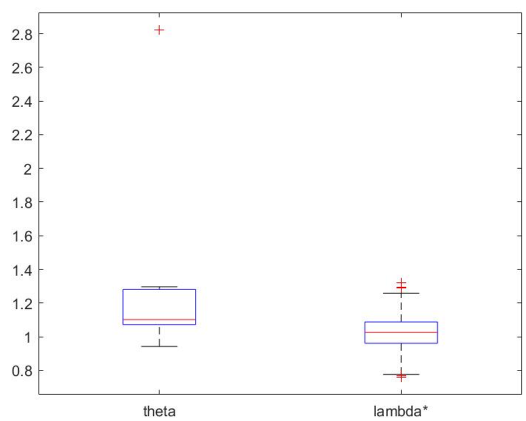

Figure 3.6: Inefficiency factors for θ and λ^* draws

structural parameters of interest. The algorithm relies on drawing the latent variable that signals the regime change, λ^*, from a 2-component mixture of truncated normal distributions, where the distribution is defined marginally from the rest of the latent variables. Next, the algorithm proceeds to draw the rest of the latent variables, ξ_s, from their full conditional distribution, using the precision sampler of Chan and Jeliazkov (2009), which provides the draw at a low marginal computational cost. Given the latent variables, I can construct the conditional posterior distributions for most of the structural parameters θ and draw directly using a Gibbs sampler. Only for the parameters (h, β) I build an approximation of the objective distribution and sample using the ARMH algorithm, which provides an efficient simulation of the parameters.

The estimation results show that the algorithm can adequately identify the variable that defines the regime change while it recovers most of the "true" values of the structural parameters. This estimation is based only on 10,000 posterior draws, and the chain converged with low inefficiency factors. However, I must mention that I used informative priors to overcome identification issues prevalent in the estimation.

I aim to estimate an empirical application using a version of this algorithm in future work. Nonetheless, to achieve the latter, it would be imperative to improve the sampler so that it performs well without relying on tight priors.

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Appendix A

Chapter 1

A.1 Linearized model

Based on Iacoviello (2005). Log-Linearized Model (Appendix 1):

1. Aggregate demand:

$$\hat{Y}_t = \frac{c}{Y}\hat{c}_t + \frac{c'}{Y}\hat{c}'_t + \frac{c''}{Y}\hat{c}''_t + \frac{I}{Y}\hat{I}_t \tag{A.1}$$

$$\hat{c}'_t = \hat{c}'_{t+1} - (\hat{R}_t - \hat{\pi}_{t+1}) \tag{A.2}$$

$$\begin{aligned} \hat{I}_t - \hat{K}_{t-1} = \gamma(\hat{I}_{t+1} - \hat{K}_t) + \frac{1-\gamma(1-\delta)}{\psi}(\hat{Y}_{t+1} - \hat{X}_{t+1} - \hat{K}_t) \\ + \frac{1}{\psi}(\hat{c}_t - \hat{c}_{t+1}) \end{aligned} \tag{A.3}$$

2. Housing/consumption margin:

$$\hat{q}_t = \gamma_e \hat{q}_{t+1} + (1-\gamma_e)(\hat{Y}_{t+1} - \hat{X}_{t+1} - \hat{h}_t) - m\beta(\hat{R}_t - \hat{\pi}_{t+1}) \\ - (1-m\beta)\Delta\hat{c}_{t+1} - \phi_e(\Delta\hat{h}_t - \gamma\Delta\hat{h}_{t+1}) \tag{A.4}$$

$$\hat{q}_t = \gamma_h \hat{q}_{t+1} + (1-\gamma_h)(\hat{j}_t - \hat{h}''_t) - m''\beta(\hat{R}_t - \hat{\pi}_{t+1}) \\ + (1-m''\beta)(\hat{c}''_t - \omega\hat{c}''_{t+1}) - \phi_h(\Delta\hat{h}''_t - \beta''\Delta\hat{h}''_{t+1}) \tag{A.5}$$

$$\hat{q}_t = \beta\hat{q}_{t+1} + (1-\beta)\hat{j}_t + \iota\hat{h}_t + \iota''\hat{h}''_t + \hat{c}'_t - \beta\hat{c}'_{t+1} \\ + \frac{\phi_h}{h'}(h\Delta\hat{h}_t + h''\Delta\hat{h}''_t - \beta h\Delta\hat{h}_{t+1} - \beta h''\Delta\hat{h}''_{t+1}) \tag{A.6}$$

3. Borrowing constraints:

$$\hat{b}_t = \hat{q}_{t+1} + \hat{h}_t - (\hat{R}_t - \hat{\pi}_{t+1}) \tag{A.7}$$

$$\hat{b}''_t = \hat{q}_{t+1} + \hat{h}''_t - (\hat{R}_t - \hat{\pi}_{t+1}) \tag{A.8}$$

4. Aggregate supply:

$$\hat{Y}_t = \frac{\eta}{\eta-(1-\nu-\mu)}(\hat{A}_t + \nu\hat{h}_{t-1} + \mu\hat{K}_{t-1}) \\ - \frac{1-\nu-\mu}{\eta-(1-\nu-\mu)}(\hat{X}_t + \alpha\hat{c}'_t + (1-\alpha)\hat{c}''_t) \tag{A.9}$$

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} - \kappa\hat{X}_t + \hat{u}_t \tag{A.10}$$

5. Flows of funds/evolution of state variables:

$$\hat{K}_t = \delta \hat{I}_t + (1-\delta)\hat{K}_{t-1} \tag{A.11}$$

$$\begin{aligned} \frac{b}{Y}\hat{b}_t = {} & \frac{c}{Y}\hat{c}_t + \frac{qh}{Y}\Delta\hat{h}_t + \frac{I}{Y}\hat{I}_t + \frac{Rb}{Y}(\hat{b}_{t-1} + \hat{R}_{t-1} - \hat{\pi}_t) \\ & - (1 - s' - s'')(\hat{Y}_t - \hat{X}_t) \end{aligned} \tag{A.12}$$

$$\begin{aligned} \frac{b''}{Y}\hat{b}''_t = {} & \frac{c''}{Y}\hat{c}''_t + \frac{qh''}{Y}\Delta\hat{h}''_t + \frac{Rb''}{Y}(\hat{b}''_{t-1} + \hat{R}_{t-1} - \hat{\pi}_t) \\ & - s''(\hat{Y}_t - \hat{X}_t) \end{aligned} \tag{A.13}$$

6. Monetary policy rule and shock processes:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R) r_\pi \hat{\pi}_{t-1} + (1 - r_R) r_Y \hat{Y}_{t-1} + \hat{e}_{R,t} \tag{A.14}$$

$$\hat{j}_t = \rho_j \hat{j}_{t-1} + \hat{e}_{j,t} \tag{A.15}$$

$$\hat{u}_t = \rho_u \hat{u}_{t-1} + \hat{e}_{u,t} \tag{A.16}$$

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \hat{e}_{A,t} \tag{A.17}$$

Note that in the model, the jump or choice variables are: $\hat{c}_t$, $\hat{c}'_t$, $\hat{c}''_t$, $\hat{I}_t$, $\hat{X}_t$, and $\hat{q}_t$. In this sense, the predetermined variables of the model are: $\hat{K}_t$, $\hat{b}_t$, $\hat{b}''_t$, $\hat{R}_t$, $\hat{j}_t$, $\hat{u}_t$, $\hat{A}_t$, $\hat{Y}_t$, $\hat{h}_t$, $\hat{h}''_t$, and $\hat{\pi}_t$. Finally, the forward variables are the variables that appear with expectations in the model and are defined in section 3 of the paper.

Appendix B

Chapter 2

B.1 Log-linearized Model

1. Patient Household:

$$
\begin{aligned}
(1-\beta\varepsilon_c)\,\hat{\lambda}_t^P &= (1-\beta\varepsilon_c\rho_Z)\hat{z}_t + \frac{\beta\varepsilon_c}{1-\varepsilon_c}E_t\hat{c}_{t+1} \\
&\quad - \frac{1+\beta\varepsilon_c^2}{1-\varepsilon_c}\hat{c}_t + \frac{\varepsilon_c}{1-\varepsilon_c}\hat{c}_{t-1};
\end{aligned} \tag{B.1}
$$

$$
\begin{aligned}
(1-\beta\varepsilon_h)\,\hat{\mathcal{U}}_{h,t} &= (1-\beta\varepsilon_h\rho_Z)\hat{z}_t + (1-\beta\varepsilon_h\rho_J)\hat{j}_t + \frac{\beta\varepsilon_h}{1-\varepsilon_h}\mathbb{E}_t\hat{h}_{t+1} \\
&\quad - \frac{1+\beta\varepsilon_h^2}{1-\varepsilon_h}\hat{h}_t + \frac{\varepsilon_h}{1-\varepsilon_h}\hat{h}_{t-1};
\end{aligned} \tag{B.2}
$$

$$\hat{\chi}_{w,t} = \hat{\lambda}_t^P + \hat{w}_t - \hat{z}_t - \eta\hat{n}_t; \tag{B.3}$$

$$\hat{q}_t = (1-\beta)\,\hat{\mathcal{U}}_{h,t} - \hat{\lambda}_t^P + \beta\mathbb{E}_t\,\hat{\lambda}_{t+1}^P + \beta\mathbb{E}_t\hat{q}_{t+1}; \tag{B.4}$$

$$\hat{\lambda}_t^P = \beta\mathbb{E}_t\,\hat{\lambda}_{t+1}^P + \beta\hat{R}_t - \beta\mathbb{E}_t\hat{\pi}_{t+1}; \tag{B.5}$$

2. Impatient Household:

$$\begin{aligned}(1-\beta'\varepsilon_c)\,\hat{\lambda}^I_t &= (1-\beta'\varepsilon_c\rho_Z)\hat{z}_t + \frac{\beta'\varepsilon_c}{1-\varepsilon_c}\mathbb{E}_t\hat{c}'_{t+1} \\ &\quad - \frac{1+\beta'\varepsilon_c^2}{1-\varepsilon_c}\hat{c}'_t + \frac{\varepsilon_c}{1-\varepsilon_c}\hat{c}'_{t-1};\end{aligned} \tag{B.6}$$

$$\begin{aligned}(1-\beta'\varepsilon_h)\,\hat{\mathcal{U}}'_{h,t} &= (1-\beta'\varepsilon_h\rho_Z)\hat{z}_t + (1-\beta'\varepsilon_h\rho_J)\hat{j}_t \\ &\quad + \frac{\beta'\varepsilon_h}{1-\varepsilon_h}\mathbb{E}_t\hat{h}'_{t+1} - \frac{1+\beta'\varepsilon_h^2}{1-\varepsilon_h}\hat{h}'_t + \frac{\varepsilon_h}{1-\varepsilon_h}\hat{h}'_{t-1};\end{aligned} \tag{B.7}$$

$$\begin{aligned}\hat{q}_t &= \theta_1\,\hat{\mathcal{U}}'_{h,t} - \theta_2\hat{\lambda}^I_t + \left(\frac{m^I\pi}{R^{bI}} - m^I\beta'\right)\hat{m}^I_t - \frac{m^I\pi}{R^{bI}}\hat{R}^{bI}_t \\ &\quad + \frac{m^I\pi}{R^{bI}}\mathbb{E}_t\hat{\pi}_{t+1} + \theta_3\mathbb{E}_t\hat{q}_{t+1} + (1-m^I)\beta'\mathbb{E}_t\,\hat{\lambda}^I_{t+1};\end{aligned} \tag{B.8}$$

$$\hat{\chi}'_{w,t} = \hat{\lambda}^I_t + \hat{w}'_t - \hat{z}_t - \eta\hat{n}'_t; \tag{B.9}$$

$$\begin{aligned}&\frac{c'}{Y}\hat{c}'_t + \frac{qh'}{Y}\hat{h}'_t + \frac{R^{bI}b^I}{\pi Y}\hat{R}^{bI}_{t-1} + \frac{R^{bI}b^I}{\pi Y}\hat{b}^I_{t-1} - \frac{R^{bI}b^I}{\pi Y}\hat{\pi}_t \\ &\quad = \frac{w'n'}{Y}\hat{w}'_t + \frac{w'n'}{Y}\hat{n}'_t + \frac{b^I}{Y}\hat{b}^I_t + \frac{qh'}{Y}\hat{h}'_{t-1};\end{aligned} \tag{B.10}$$

$$\hat{b}^I_t = \hat{m}^I_t - \hat{R}^{bI}_t + \mathbb{E}_t\hat{q}_{t+1} + \mathbb{E}_t\hat{\pi}_{t+1} + \hat{h}'_t; \tag{B.11}$$

where

$$\theta_1 \equiv 1 - \frac{m^I\pi}{R^{bI}} - (1-m^I)\beta'; \qquad \theta_2 \equiv 1 - \frac{m^I\pi}{R^{bI}};$$

$$\theta_3 \equiv \frac{m^I\pi}{R^{bI}} + (1-m^I)\beta';$$

3. Entrepreneurs and capital producers:

$$(1-\beta_E\varepsilon_c)\,\hat{\lambda}_t^E = \frac{\beta_E\varepsilon_c}{1-\varepsilon_c}\mathbb{E}_t\hat{c}_{t+1}^E - \frac{1+\beta_E\varepsilon_c^2}{1-\varepsilon_c}\hat{c}_t^E + \frac{\varepsilon_c}{1-\varepsilon_c}\hat{c}_{t-1}^E; \tag{B.12}$$

$$\hat{y}_t = (1-\sigma)(1-\alpha)\hat{n}_t + \sigma(1-\alpha)\hat{n}'_t + \alpha\hat{K}_{t-1}; \tag{B.13}$$

$$(1-\sigma)(1-\alpha)\hat{y}_t = \hat{\chi}_{p,t} + \hat{n}_t + \hat{w}_t; \tag{B.14}$$

$$\sigma(1-\alpha)\hat{y}_t = \hat{\chi}_{p,t} + \hat{n}'_t + \hat{w}'_t; \tag{B.15}$$

$$\begin{aligned} q_t^k = {} & \left(\frac{\pi}{R^{bE}} - \beta_E\right)(1-\delta_k)m^E\,\hat{m}_t^E + \left(\frac{m^E\pi}{R^{bE}} + (1-m^E)\beta_E\right)(1-\delta_k)\mathbb{E}_t\hat{q}_{t+1}^k \\ & + \frac{(1-\delta_k)m^E\pi}{R^{bE}}(\mathbb{E}_t\hat{\pi}_{t+1} - \hat{R}_t^{bE}) + \left(1-(1-\delta_k)\frac{m^E\pi}{R^{bE}}\right)(\mathbb{E}_t\hat{\lambda}_{t+1}^E - \hat{\lambda}_t^E) \\ & + \left(1-(1-\delta_k)\frac{m^E\pi}{R^{bE}} - (1-\delta_k)(1-m^E)\beta_E\right)(\mathbb{E}_t\hat{y}_{t+1} - \mathbb{E}_t\hat{\chi}_{p,t+1} - \hat{K}_t) \end{aligned} \tag{B.16}$$

$$\hat{b}_t^E = -\hat{R}_t^{bE} + \hat{m}_t^E + \mathbb{E}_t\hat{q}_{t+1}^k + \mathbb{E}_t\hat{\pi}_{t+1} + \hat{K}_t; \tag{B.17}$$

$$\begin{aligned} & \frac{c^E}{Y}\hat{c}_t^E + \frac{wn}{Y}\hat{w}_t + \frac{wn}{Y}\hat{n}_t + \frac{w'n'}{Y}\hat{w}'_t + \frac{w'n'}{Y}\hat{n}'_t + \frac{Kq^k}{Y}\hat{q}_t^k + \frac{q^kK}{Y}\hat{K}_t^k \\ & + \frac{R^{bE}b^E}{\pi Y}(\hat{R}_{t-1}^{bE} + \hat{b}_{t-1}^E - \hat{\pi}_t) = \frac{1}{\chi_p}(\hat{y}_t - \hat{\chi}_{p,t}) + \frac{b^E}{Y}\hat{b}_t^E + \frac{q^k(1-\delta_k)K}{Y}\hat{K}_{t-1}; \end{aligned} \tag{B.18}$$

$$\hat{K}_t = \delta_k\hat{i}_t + \delta_k\hat{a}_t + (1-\delta_k)\hat{K}_{t-1}; \tag{B.19}$$

$$\hat{q}_t^k = -\hat{a}_t^k - \phi\beta_E\mathbb{E}_t\hat{i}_{t+1} + (1+\beta_E)\phi\hat{i}_t - \phi\hat{i}_{t-1}; \tag{B.20}$$

4. Final goods and nominal wage rigidities:

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} - \varepsilon_\pi\hat{\chi}_{p,t} + u_{p,t}; \tag{B.21}$$

$$\hat{\omega}_t = \beta\mathbb{E}_t\hat{\omega}_{t+1} - \varepsilon_w\hat{\chi}_{w,t} + u_{w,t}; \tag{B.22}$$

$$\hat{\omega}'_t = \beta'\mathbb{E}_t\hat{\omega}'_{t+1} - \varepsilon'_w\hat{\chi}'_{w,t} + u_{w,t}; \tag{B.23}$$

$$\hat{\omega}_t = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t; \tag{B.24}$$

$$\hat{\omega}'_t = \hat{w}'_t - \hat{w}'_{t-1} + \hat{\pi}_t; \tag{B.25}$$

where

$$\varepsilon_\pi \equiv \frac{(1-\theta_\pi)(1-\beta\theta_\pi)}{\theta_\pi},$$

$$\varepsilon_\omega \equiv \frac{(1-\theta_\omega)(1-\beta\theta_\omega)}{\theta_\omega}, \qquad \varepsilon'_\omega \equiv \frac{(1-\theta_\omega)(1-\beta'\theta_\omega)}{\theta_\omega},$$

5. Banking sector:

$$\hat{B}_t = \frac{b^I}{B}\hat{b}^I_t + \frac{b^E}{B}\hat{b}^E_t; \tag{B.26}$$

$$\hat{B}_t = \frac{d}{B}\hat{d}_t + \frac{K^b}{B}\hat{K}^b_t; \tag{B.27}$$

$$\hat{K}^b_t = -\hat{\pi}_t + \frac{1-\delta_b}{\pi}\hat{K}^b_{t-1} + \frac{\Pi^b}{\pi K^b}\hat{\Pi}^b_{t-1}; \tag{B.28}$$

$$\hat{B}^*_t = \frac{\omega^{bI} b^I}{B^*}(\hat{\omega}^{bI}_t + \hat{b}^I_t) + \frac{\omega^{bE} b^E}{B^*}(\hat{\omega}^{bE}_t + \hat{b}^E_t); \tag{B.29}$$

$$\hat{R}^{BI}_t = \hat{R}_t - \frac{\kappa_{kb}(\nu^b)^3\omega^{bI}}{R}(\hat{K}^b_t - \hat{B}^*_t); \tag{B.30}$$

$$\hat{R}^{BE}_t = \hat{R}_t - \frac{\kappa_{kb}(\nu^b)^3\omega^{bE}}{R}(\hat{K}^b_t - \hat{B}^*_t); \tag{B.31}$$

$$\hat{\omega}^{bI}_t = \rho_{wbI}\hat{\omega}^{bI}_{t-1} + (1-\rho_{wbI})\chi^I_y\Delta^A\hat{y}_t + (1-\rho_{wbI})\chi^I_q\mathbb{E}_t\Delta^A\hat{q}_{t+1}; \tag{B.32}$$

$$\hat{\omega}^{bE}_t = \rho_{wbE}\omega^{bE}_{t-1} + (1-\rho_{wbE})\chi^E_y\Delta^A\hat{y}_t; \tag{B.33}$$

$$\begin{aligned}\hat{R}^{bI}_t = &\frac{\varepsilon^{bI}-1}{\varepsilon^{bI}-1+(1+\beta)\kappa_{rbI}}\hat{R}^{BI}_t + \frac{\kappa_{rbI}}{\varepsilon^{bI}-1+(1+\beta)\kappa_{rbI}}\hat{R}^{bI}_{t-1}\\ &+ \frac{\beta\kappa_{rbI}}{\varepsilon^{bI}-1+(1+\beta)\kappa_{rbI}}\mathbb{E}_t\hat{R}^{bI}_{t+1} + \frac{\varepsilon^{bI}-1}{\varepsilon^{bI}-1+(1+\beta)\kappa_{rbI}}\hat{\mu}^I_t;\end{aligned} \tag{B.34}$$

$$\begin{aligned}\hat{R}^{bE}_t = &\frac{\varepsilon^{b}-1}{\varepsilon^{bE}-1+(1+\beta)\kappa_{rbE}}\hat{R}^{BE}_t + \frac{\kappa_{rbE}}{\varepsilon^{bE}-1+(1+\beta)\kappa_{rbE}}\hat{R}^{bE}_{t-1}\\ &+ \frac{\beta\kappa_{rbE}}{\varepsilon^{bE}-1+(1+\beta)\kappa_{rbE}}\mathbb{E}_t\hat{R}^{bE}_{t+1} + \frac{\varepsilon^{bE}-1}{\varepsilon^{bE}-1+(1+\beta)\kappa_{rbE}}\hat{\mu}^E_t;\end{aligned} \tag{B.35}$$

$$\begin{aligned}\hat{\Pi}^b_t = &\frac{R^{bI}b^I}{\Pi^b}\hat{R}^{bI}_t + \frac{(R^{bI}-1)b^I}{\Pi^b}\hat{b}^I_t + \frac{R^{bE}b^E}{\Pi^b}\hat{R}^{bE}_t + \frac{(R^{bE}-1)b^E}{\Pi^b}\hat{b}^E_t\\ &- \frac{Rd}{\Pi^b}\hat{R}_t - \frac{(R-1)d}{\Pi^b}\hat{d}_t;\end{aligned} \tag{B.36}$$

6. Final goods and housing market clearing:

$$\hat{y}_t = \frac{C}{Y}\hat{C}_t + \frac{i}{Y}\hat{i}_t - \frac{K^b\delta_b}{Y\pi}\hat{\pi}_t + \frac{K^b\delta_b}{Y\pi}\hat{K}^b_{t-1}; \tag{B.37}$$

$$\hat{C}_t = \frac{c}{C}\hat{c}_t + \frac{c'}{C}\hat{c}'_t + \frac{c^E}{C}\hat{c}^E_t; \tag{B.38}$$

$$\hat{h}_t = -\frac{1-h}{h}\hat{h}'_t; \tag{B.39}$$

7. Monetary policy rule and shock processes:

$$\hat{R}_t = r_R\hat{R}_{t-1} + (1-r_R)r_\pi\hat{\pi}_{t-1} + (1-r_R)r_Y\hat{Y}_{t-1} + \hat{e}_t; \tag{B.40}$$

$$\hat{z}_t = \rho_z\hat{z}_{t-1} + \hat{u}_{z,t}; \tag{B.41}$$

$$\hat{j}_t = \rho_j\hat{j}_{t-1} + \hat{u}_{j,t}; \tag{B.42}$$

$$\hat{a}^k_t = \rho_K\hat{a}^k_{t-1} + \hat{u}_{K,t}; \tag{B.43}$$

$$\hat{e}_t = \rho_R\hat{e}_{t-1} + \hat{u}_{R,t}; \tag{B.44}$$

$$\hat{m}^I_t = \rho_{mI}\hat{m}^I_{t-1} + \hat{u}_{mI,t}; \tag{B.45}$$

$$\hat{m}^E_t = \rho_{mE}\hat{m}^E_{t-1} + \hat{u}_{mE,t}; \tag{B.46}$$

$$\hat{\mu}^I_t = \rho_{\mu I}\hat{\mu}^I_{t-1} + \hat{u}_{\mu I,t}; \tag{B.47}$$

$$\hat{\mu}^E_t = \rho_{\mu E}\hat{\mu}^E_{t-1} + \hat{u}_{\mu E,t}; \tag{B.48}$$

B.2 Data description

- Consumption: Real Personal Consumption Expenditures, from Bureau of Economic Analysis (BEA), log-transformed and detrended with the UCUR-2M approach.

- Price Inflation: quarterly change in GDP Implicit Price Deflator, from BEA, and demeaned using the data from the sample under consideration.

- Wage inflation: Real Compensation per Hour in Nonfarm Business Sector, from the US Bureau of Labor Statistics (BLS), log-transformed, detrended with the UCUR-2M approach, first differenced, and expressed in nominal terms adding back price inflation.

- Investment: Real Private Nonresidential Fixed Investment, from BEA, log-transformed and detrended with the UCUR-2M approach.

- House Prices: Real Home Price Index, from Robert Shiller online data webpage: (http://www.econ.yale.edu/ shiller/data.htm); log-transformed and detrended with the UCUR-2M approach.

- Household debt: Households Residential Mortgages [HHMSDODNS], from the Flow of Funds of the Federal Reserve and retrieved from FRED, St. Louis Fed; deflated with the Implicit Price Deflator, log-transformed and detrended with UCUR-2M.

- Firms debt: Non-financial Corporate Debt [BCNSDODNS], from the Flow of Funds of the Federal Reserve and retrieved from FRED, St. Louis Fed; deflated with the Implicit Price Deflator, log-transformed and detrended with the UCUR-2M approach.

- Nominal Interest Rate: Effective Federal Funds Rate, from the Federal Reserve and retrieved from FRED, St. Louis Fed; demeaned and divided by 400 to be expressed in quarterly units.

- Impatient loan interest rate: Freddie Mac, 30-Year Fixed Rate Mortgage Average [MORTGAGE30US], retrieved from FRED, St. Louis Fed; demeaned and divided by 400 to be expressed in quarterly units.

- Entrepreneurs loan interest rate: Moody's Seasoned Baa Corporate Bond Yield [BAA], retrieved from FRED, St. Louis Fed; demeaned and divided by 400 to be expressed in quarterly units.

Above the UCUR-2M approach denotes the unobserved components model with a second-order Markov process for the trend developed in Grant and Chan (2017).

B.3 Tables and other results

B.3.1 Estimation of the risk weights parameters for impatient loans

Along the lines of what is described in Appendix 1 of Angelini et al. (2010), we used US data on the delinquency rates on single-family residential mortgages from the 100th largest US banks by assets, from the Federal Reserve, as a proxy for the probability of default. Together with the same assumptions as Angelini et al. (2010) [1], we used this time series into the Basel II capital requirements formulae to estimate the risk weights for loans to households.

We present the results of Angelini et al. (2010) as case 0 at Table B.1. In addition, we explore adding house prices (2) and house prices expectations (3) in the specification, as shown in the following equations:

$$\hat{\omega}_t^{bI} = \rho_{wbI}\hat{\omega}_{t-1}^{bI} + (1-\rho_{wbI})\chi_y^I \Delta^A \hat{y}_t + (1-\rho_{wbI})\chi_q^I \Delta^A \hat{q}_t;$$

$$\hat{\omega}_t^{bI} = \rho_{wbI}\hat{\omega}_{t-1}^{bI} + (1-\rho_{wbI})\chi_y^I \Delta^A \hat{y}_t + (1-\rho_{wbI})\chi_q^I \mathbb{E}_t \Delta^A \hat{q}_{t+1};$$

As Angelini, et al (2010), we use quarterly data that spans from Q1-1991 to Q4-2007. The results are presented in the next table:

Our estimations were done in R, using the brms package. Following Angelini, et al. (2010), who corrected the standard errors for heteroskedasticity and autocorrelation of residuals, we assumed a model with heterogeneous variances and a moving average term. The assumed priors were $\rho_{wb} \sim N_{(0,1)}(0.5, 0.25)$ and $\chi_v \sim N_{(-\infty,0)}(-10, 5)$ for $v = \{y, q\}$.

[1] Angelini et al. (2010) used the residential mortgage function, with LGD equal to 20 percent, following QIS5. Using the regulatory formulae, we obtained the capital requirements (as a percentage of the exposure at default), which are then multiplied by 12.5 in order to obtain risk weights.

Table B.1: Household Loans Risk Weights Nonlinear Regression Estimation Results

Parameter	**0**	**1**	**2**	**3**
ρ_{wbI}	0.94	0.95	0.83	0.82
	(0.04)	(0.03)	(0.04)	(0.05)
χ_y^I	-10	-6.57	-1.60	-1.53
	(8)	(3.62)	(0.87)	(1.04)
χ_q^I			-1.78	-1.64
			(0.46)	(0.68)
R^2	0.89	0.917	0.942	0.940
obs	67	67	67	67

Looking at the table, we can see a significant effect from house prices in the proxy measure for risk weights in household debt mortgage. Also, note that the results are similar between the regressions with house prices annual growth rate (2) and its expectations (3). In this regard, we decided to use the specification under (3) to calibrate the banks' risk weights for loans to households in the model.

B.3.2 Constant Gain Learning Parameters under AL-2 Specification

Table B.2: AL2 - Constant Gains Posterior draws.

	Mode	**5%**	**95%**
$\bar{g}_{01}$	0.0059	0.0028	0.0113
$\bar{g}_{02}$	0.0225	0.0117	0.0457
$\bar{g}_{03}$	0.0210	0.0097	0.0401
$\bar{g}_{04}$	0.0274	0.0153	0.0427
$\bar{g}_{05}$	0.0219	0.0106	0.0468
$\bar{g}_{06}$	0.0268	0.0131	0.0541
$\bar{g}_{07}$	0.0214	0.0112	0.0402
$\bar{g}_{08}$	0.0155	0.0080	0.0312
$\bar{g}_{09}$	0.0171	0.0082	0.0307
$\bar{g}_{10}$	0.0134	0.0060	0.0286
$\bar{g}_{11}$	0.0219	0.0091	0.0351
$\bar{g}_{12}$	0.0224	0.0110	0.0410
$\bar{g}_{13}$	0.0205	0.0104	0.0390
$\bar{g}_{14}$	0.0246	0.0111	0.0448
$\bar{g}_{15}$	0.0209	0.0099	0.0496
$\bar{g}_{16}$	0.0204	0.0103	0.0457
$\bar{g}_{17}$	0.0223	0.0124	0.0410

Note: Based on 200,000 posterior draws, after a burn-in of 20 percent.

B.3.3 Other IRF for shocks related to credit conditions

Figure B.1: IRF to an expansionary firms loans markup shock, REE vs AL-2.

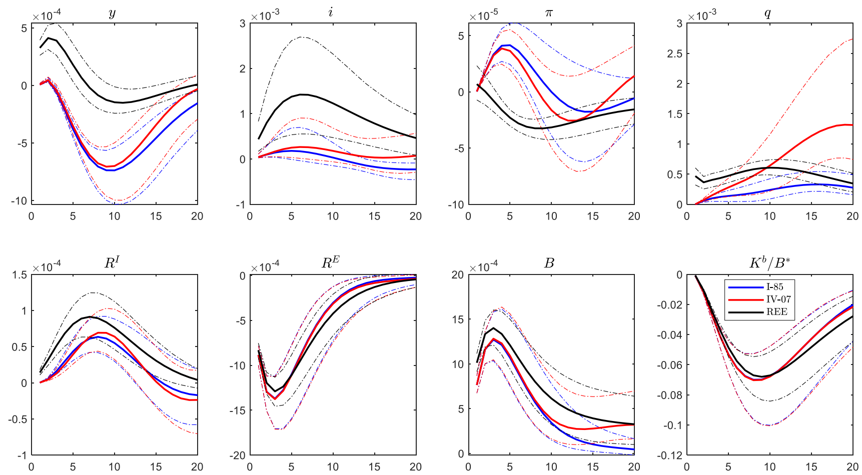


Figure B.2: IRF to an Impatient LTV ratio shock, REE vs AL-2.

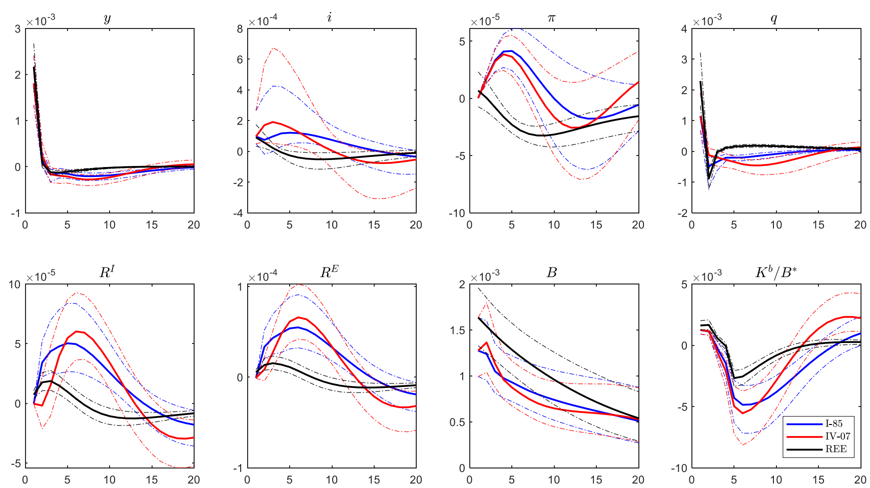
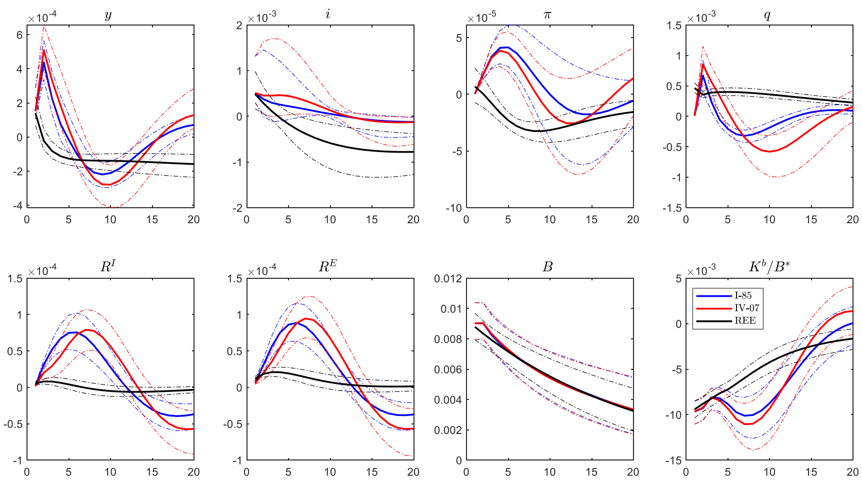


Figure B.3: IRF to an Firms LTV ratio shock, REE vs AL-2.



Note: the dotted lines represent a 90 percent credible interval.

Appendix C

Chapter 3

C.1 Steady-states

The model is based on the description available at the Appendix of Guerrieri and Iacoviello (2015). I assumed that the credit constraint is binding in steady state ($\beta R < 1$).

$$Y = \bar{Y} \qquad = 1 \tag{C.1}$$

$$Z_c = \bar{Z}_c \qquad = 1 \tag{C.2}$$

$$Z_b = \bar{Z}_b \qquad = 1 \tag{C.3}$$

$$B = Z_b m Y \qquad = m \tag{C.4}$$

$$C = Y + (1 - R)B \qquad = 1 + (1 - R)m \tag{C.5}$$

$$\mathcal{U}_C = Z_c C^{-\gamma} \qquad = (1 + (1 - R)m)^{-\gamma} \tag{C.6}$$

$$\lambda = \frac{1 - \beta R}{1 - \beta\delta}\mathcal{U}_C \qquad = \frac{1 - \beta R}{1 - \beta\delta}(1 + (1 - R)m)^{-\gamma} \tag{C.7}$$

Note that in equilibrium the Lagrange multiplier of the budget constraint, μ, is equal to the marginal utility, $\mathcal{U}_C$.