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Permalink

<https://escholarship.org/uc/item/62t491nd>

Journal

Journal of the American Helicopter Society, 62(2)

ISSN

0002-8711

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Publication Date

2017-04-01

DOI

10.4050/jahs.62.022007

Peer reviewed

Long-Range and Nonlinear Propagation of Helicopter High-Speed Impulsive Noise



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This paper presents a new approach to predict the nonlinear propagation of helicopter high-speed impulsive noise. The local nonlinear source terms and cumulative nonlinear propagation effects are considered separately. The local nonlinear sources are captured in Euler computational fluid dynamics (CFD), and the cumulative nonlinear steepening is predicted by the frequency-domain Burgers equation. The geometric transformation of sound propagation between two different coordinates is explained. In the Burgers equation, the atmospheric absorption effect is also included. The Burgers equation model is validated against an analytic solution for a sinusoidal wave and then applied to a model-scale rotor noise. The Euler CFD data are used to extract the starting signal and to validate the prediction obtained by the Burgers equation. The sensitivities to the starting signal location and the propagation path that are used in the Burgers equation are studied. The nonlinear propagation is considered for a full-scale UH-1H helicopter. It is shown that the nonlinear propagation significantly distorts the waveform, resulting in a reduction of the pressure amplitude and a broadening of the waveform. However, the atmospheric absorption becomes equally important for large distances as it attenuates high-frequency energy and makes the waveform smoother. The nonlinear propagation and atmospheric attenuation are balanced and checked over long-range propagation. Finally, a nonlinearity indicator that is suitable for rotor noise is proposed to identify the energy transfer due to nonlinear propagation in the frequency domain. It is found that the energy transfer occurs in low frequency and generates scalloped sound pressure level distributions for nonlinear propagation, which is an interesting feature of helicopter nonlinear sound propagation.

Nomenclature

A	amplitude of the Fourier transform pressure, Pa s
B_n	n th harmonic amplitude
c	blade chord length, m
c_0	speed of sound, m s ⁻¹
f	frequency of the acoustic signal, Hz
f_c	cutoff frequency, Hz
f_0	fundamental frequency of the sinusoidal signal, Hz
k	wave number, m ⁻¹
M	Mach number
M_a	Mach number of a peak particle of the sinusoidal wave, $p_a / \rho_0 c_0^2$
m	parameter for plane, cylindrical, and spherical waves
n	index of harmonic
P_{MS}	mean square of pressure, Pa ²
p	acoustic pressure, Pa

p_a	initial pressure amplitude of the sinusoidal wave, Pa
p_{rms}	root mean square of pressure, Pa
$\tilde{p}$	Fourier transform of the pressure, Pa s
Q	amplitude of the Fourier transform of the pressure square, Pa ² s
$Q_{p^2 p}$	quad-spectral density between p^2 and p , Pa ³ s ²
q	acoustic pressure squared, Pa ²
$\tilde{q}$	Fourier transform of the pressure square, Pa ² s
R	rotor blade radius, m
r	propagation distance, m
r_f	final distance for the sound propagation, m
r_i	initial or starting distance for the sound propagation, m
S_{pp}	power spectral density, Pa ² s
U	real part of the Fourier transform of the pressure square, Pa ² s
V	imaginary part of the Fourier transform of the pressure square, Pa ² s
X	real part of the Fourier transform of the pressure, Pa s
$\bar{x}$	shock formation distance, m
Y	imaginary part of the Fourier transform of the pressure, Pa s
α	atmospheric absorption, m ⁻¹

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Presented at the American Helicopter Society 66th Annual Forum, Phoenix, AZ, May 11–13, 2010. Manuscript received April 2016; accepted January 2017.

α'	atmospheric absorption and dispersion, $\alpha' = \alpha + i\beta_d$, m^{-1}
β	coefficient of nonlinearity
β_d	atmospheric dispersion, m^{-1}
γ	ratio of the specific heats
δ	diffusivity of sound, $v \left(\frac{4}{3} + \frac{\mu_B}{\mu} + \frac{\gamma-1}{Pr} \right)$, $\text{m}^2 \text{s}^{-1}$
ε	nonlinear coefficients, $\beta/\rho_0 c_0^3$, $\text{kg}^{-1} \text{s}$
ρ_0	density, kg m^{-3}
σ	dimensionless propagation distance, $x/\bar{x}$
τ	retarded time, s
ϕ	phase of the Fourier transform of the pressure, rad
ψ	phase of the Fourier transform of the pressure square, rad
ω	angular velocity, rad s^{-1}

Introduction

High-speed impulsive (HSI) noise is a particularly intense and annoying noise generated by helicopter rotors in high-speed forward flight (Ref. 1). HSI noise is closely associated with the appearance of shocks and transonic flows around advancing rotor blades. At the onset of HSI noise, the intensity of the noise increases rapidly with increasing advancing-tip Mach number. This shock-associated noise is an important source for annoyance and detection when it is present.

The characteristics of HSI noise can be explained in the context of nonlinear acoustics (Refs. 2,3). The amplitude of the sound generated in the proximity of blade tip is sufficiently high enough so that the sound propagates nonlinearly. In nonlinear acoustics, the travel speed of the waveform depends on the amplitude of the pressure. The zero amplitude points on the wave travel at the freestream speed of sound. The positive and negative amplitude points travel faster and slower, respectively, resulting in a distortion of the waveform and involving the dissipation of sound energy due to the shock wave. In linear wave equations, this distortion of the waveform with the small amplitude can be ignored. As a result, the waveform and amplitude resulting from nonlinear propagation can be noticeably different from sound propagated in a linear manner. In addition, the nonlinear effect is cumulative and its importance increases for long distances.

HSI noise has been mainly investigated in the framework of computational fluid dynamics (CFD) and the acoustic analogy approach embodied the Ffowcs Williams and Hawkings (FW-H) equation (Refs. 4–6). The acoustic analogy uses the CFD data as input and finds the far-field solution of acoustics using a free-space Green's function. In the traditional use of FW-H equation, the quadrupole source describes both the localized nonlinear transonic effects and cumulative nonlinear propagation effects. This quadrupole source is included inside acoustic data surfaces. Outside the acoustic data surfaces, the sound propagation is assumed to be linear. This approach is not appropriate to solve for the long-range propagation of HSI noise since the quadrupole source is required up to distant observer locations where the cumulative nonlinear propagation effect is dominant. The extended use of the quadrupole source simply means that high-quality CFD data need to be computed out to large distances, which is impractical. Furthermore, if such data were computed to large distances, the solution would already be available and the analogy would be unnecessary.

In this paper, we propose a new approach to solve HSI noise by separating cumulative nonlinear distortion from the local nonlinear sound sources that are associated with shocks. The local nonlinear sources are captured in Euler CFD solutions, and then the cumulative nonlinear effect is solved by using the Burgers equation. This enables us to confine the CFD region only near the rotating blades where local nonlinear sources are dominant, and it also allows us to efficiently predict long-range propagation of nonlinear sound.

The Burgers equation (Ref. 2) describes one-dimensional nonlinear sound propagation in the presence of atmospheric absorption. Recently, an efficient frequency-domain algorithm for the Burgers equation, referred to as the split method, has been developed (Ref. 7). It has been demonstrated that the split method provides rapidly convergent predictions and eliminates high-frequency errors so that it results in accurate predictions of propagation for a sinusoidal signal, sonic boom, broadband jet engine noise, and model helicopter noise. In the current paper, the split method is applied to predict the long-range and nonlinear propagation of helicopter HSI noise in the presence of atmospheric absorption.

The goal of this paper is to investigate the validity of the Burgers equation to predict HSI noise for the replacement of the acoustic analogy when it is used with CFD data and to assess the nonlinear propagation of HSI noise quantitatively in the presence of atmospheric absorption. We will propose a new nonlinearity indicator that is suitable for rotor noise to investigate an energy transfer in the frequency content due to nonlinear propagation effects.

Numerical Method

Lee et al. (Ref. 7) developed a new numerical algorithm to solve nonlinear sound propagation using the frequency-domain Burgers equation. The method is briefly reviewed in this section.

The generalized Burgers equation is written as (Ref. 2)

$$\frac{\partial p}{\partial r} + \frac{m}{r} p - \frac{\delta}{2c_0^3} \frac{\partial^2 p}{\partial \tau^2} = \frac{\beta p}{\rho_0 c_0^3} \frac{\partial p}{\partial \tau} \quad (1)$$

where p is the acoustic pressure, β is the coefficient of nonlinearity, and δ is the diffusivity of sound. The parameter m is equal to 0, 1/2, and 1 for plane, cylindrical, and spherical waves, respectively. For perfect gases, $\beta = (\gamma + 1)/2$, and for diatomic gases such as air $\gamma = 1.4$ and therefore $\beta = 1.2$. Note that τ in Eq. (1) is the retarded time, $\tau = t - r/c_0$. Thus the time history of the pressure at each range step is observed in a coordinate system moving with the speed of sound.

Equation (1) can be rewritten as

$$\frac{\partial p}{\partial r} + m \frac{p}{r} - \frac{\varepsilon}{2} \frac{\partial q}{\partial \tau} = \frac{\delta}{2c_0^3} \frac{\partial^2 p}{\partial \tau^2} \quad (2)$$

where $\varepsilon = \beta/\rho_0 c_0^3$, and the square of the pressure is represented by $q(r, \tau)$. The Fourier transform of Eq. (2) yields

$$\frac{d\tilde{p}}{dr} + m \frac{\tilde{p}}{r} + \alpha' \tilde{p} = \frac{i\omega\varepsilon}{2} \tilde{q} \quad (3)$$

where $\tilde{p}$ and $\tilde{q}$ are the Fourier transforms of the pressure and pressure squared, respectively. $\omega^2 \delta / 2c_0^3$ has been replaced with α' , the atmospheric absorption. When dispersion effects are important, they are included in α' such that the absorption becomes a complex variable, $\alpha' = \alpha + i\beta_d$. The real part of the complex coefficient α' is associated with attenuation and can be obtained by the method presented by Bass et al. (Refs. 8,9). A formula for the imaginary part describing the dispersion effects and their variation with frequency are given in Refs. 10 and 11.

The Fourier transform of the pressure and pressure squared are expressed in the form of complex variables,

$$\tilde{p}(r, \omega) = X + iY \quad (4)$$

and

$$\tilde{q}(r, \omega) = U + iV \quad (5)$$

Then, Eq. (3) can be expressed as

$$\frac{d(X + iY)}{dr} + m \frac{(X + iY)}{r} + (\alpha + i\beta_d)(X + iY) = \frac{i\omega\varepsilon}{2}(U + iV) \quad (6)$$

The separation of Eq. (6) into its real and imaginary parts gives

$$\frac{dX}{dr} = -m \frac{X}{r} - (\alpha X - \beta_d Y) - \frac{\omega\varepsilon}{2} V \quad (7a)$$

$$\frac{dY}{dr} = -m \frac{Y}{r} - (\beta_d X + \alpha Y) + \frac{\omega\varepsilon}{2} U \quad (7b)$$

At every range step, U and V are computed from the pressure squared.

Pishchal'nikov et al. (Ref. 12) and Lee et al. (Ref. 7) showed that numerical errors occur at high frequencies and propagate toward lower frequencies when the Burgers equation is solved. This nonphysical growth of energy at high frequencies is due to the limitation of the resolved frequency range in the numerical simulations. Energy cannot flow beyond the maximum frequency range, and, instead, it is reflected toward lower frequencies. To minimize the effect of this numerical error on the time signal, a very large number of harmonics would be needed, which increases the computational cost. Pishchal'nikov et al. (Ref. 12) used an asymptotic solution at high frequencies and combined the solution with the series solutions of the Burgers equation. Lee et al. (Ref. 7) applied a similar asymptotic analytic expression to their method at high frequencies that is coupled with the solution of the Burgers equation to eliminate the nonphysical growth of energy at high frequencies.

In the sawtooth region of a periodic signal, the waveform can be represented by the following Fourier series:

$$p = \frac{2p_0}{1 + \sigma} \sum_{n=1}^{\infty} \frac{1}{n} \sin n\omega\tau \quad (8)$$

That is, the harmonic amplitudes are $B_n = 2/n(1 + \sigma)$, where n is the harmonic index and σ is a dimensionless propagation distance. The ratio of the amplitudes of adjacent frequency components is given by

$$B_n = B_{n-1} \times \frac{n-1}{n} \quad (9)$$

or

$$\left| \frac{\tilde{p}(n)\omega_0}{\tilde{p}((n-1)\omega_0)} \right| = \frac{n-1}{n} \quad (10)$$

where $|\tilde{p}(\omega)|$ is the amplitude of the pressure at frequency of ω . The pressure amplitude should asymptotically decay with a ratio of $(n-1)/n$ at a high frequency, which is defined as a cutoff frequency, f_c . In the present implementation, a cutoff frequency f_c is chosen such that the frequency content of interest is much lower than f_c and the choice of f_c should not affect the results at the interesting frequency region.

Instead of solving for the amplitude at all frequencies by using the frequency-domain Burgers equation, Eq. (10) is used to find the amplitude of the frequencies beyond the cutoff frequency recursively when the amplitude does not decrease with frequency. This split frequency-domain method is based on the assumption that energy should decrease as the frequency increases beyond a given cutoff frequency. By doing this, nonphysical growth at high frequencies can be successfully eliminated. It has been found that the method is not sensitive to the value of the cutoff frequency, f_c , as long as a suitably large value of f_c is selected. Although this technique has been formulated for a periodic wave, its validity for real applications, such as broadband jet noise, sonic boom, and helicopter noise, was demonstrated (Ref. 7).

Since a shock wave consists of many harmonics of nonzero amplitudes, the truncation of harmonics causes the Gibbs phenomenon, or

oscillations at the peaks. A large number of harmonics can avoid the Gibbs phenomenon. However, computation far beyond the audible frequency range causes excessive computational cost. The Lanczos smoothing filter (Ref. 13) is employed at the last range step as a second process to cure high-frequency errors associated with this Gibbs phenomenon. This process effectively eliminates the Gibbs phenomenon and achieves an accurate prediction using fewer harmonics.

Overall, this algorithm was found to be free of amplitude and phase errors which are present in other numerical algorithms (Ref. 11).

Validation Problem

In this section, an initially sinusoidal wave is used to validate the numerical method. The initial sine wave is given by

$$p(r, \tau) = p_a \sin(2\pi f_0 \tau) \quad (11)$$

The parameters p_a and f_0 are chosen to be 140 dB with reference pressure 20 μ Pa (or 282.84 Pa) and 1000 Hz, respectively.

The shock formation distance for a plane sinusoidal wave is given by

$$\bar{x} = \frac{1}{\beta M_a k} \quad (12)$$

where $\beta = 1.2$ is the coefficient of nonlinearity, M_a is u_a/c_0 , u_a is the peak particle velocity at the source, and k is the wave number. M_a is given by $p_a/\rho_0 c_0^2$ (Ref. 2). The speed of sound and density are chosen to be 343 m/s and 1.213 kg/m³, respectively. A dimensionless parameter $\sigma = x/\bar{x}$ is used to define the propagation distance. The shock formation distance for the sine wave corresponds to $\sigma = 1$ (22.90 m for the current simulation). $\sigma = 3$ is chosen for a benchmark problem in this paper. An analytic solution for the transition region between the preshock and sawtooth regions, i.e., $1 < \sigma < 3$, can be obtained from the Blackstock Bridging Function (BBF) solution (Ref. 2).

Figure 1 shows the predictions obtained by the split method at $\sigma = 3$. The cutoff frequency for the split method is selected as 90% of the Nyquist frequency for each harmonic case. It can be seen that the predictions provide very good agreement with the BBF solution for the pressure waveform. The BBF solution was obtained with 256 harmonics. It can be seen that 30–50 harmonics are sufficient to describe the shock waveform accurately for this waveform, so a calculation beyond these harmonics is simply an additional computational cost. The Lanczos filter was used for both the BBF solution and the predictions to eliminate the Gibbs phenomenon.

Nonlinear Propagation of HSI Noise

In this section, nonlinear propagation of HSI noise is addressed. A model-scale rotor test, conducted by Boxwell et al. (Ref. 14) in 1978 and later repeated by Purcell (Ref. 15) in 1988, has been selected for the validation of the present analysis and code. The rotor was a one-seventh scale model of a UH-1H main rotor with straight, untwisted blades. The model rotor had an NACA 0012 airfoil section. The rotor radius R was 1.045 m with a chord of 7.62 cm. The model was run at several high-speed hover conditions with low thrust. For the current paper, a hovering Mach number of 0.95 is selected for the test case. An Euler CFD solution reported by Baeder et al. (Ref. 17) is used as the starting signal for the Burgers equation. This CFD data capture the local nonlinear source terms accurately. Details of the Euler calculations can be found in Refs. 16,17). In this paper, the temperature and relative humidity used for the atmospheric absorption and dispersion in the Burgers equation are assumed to be 293.15 K and 20%.

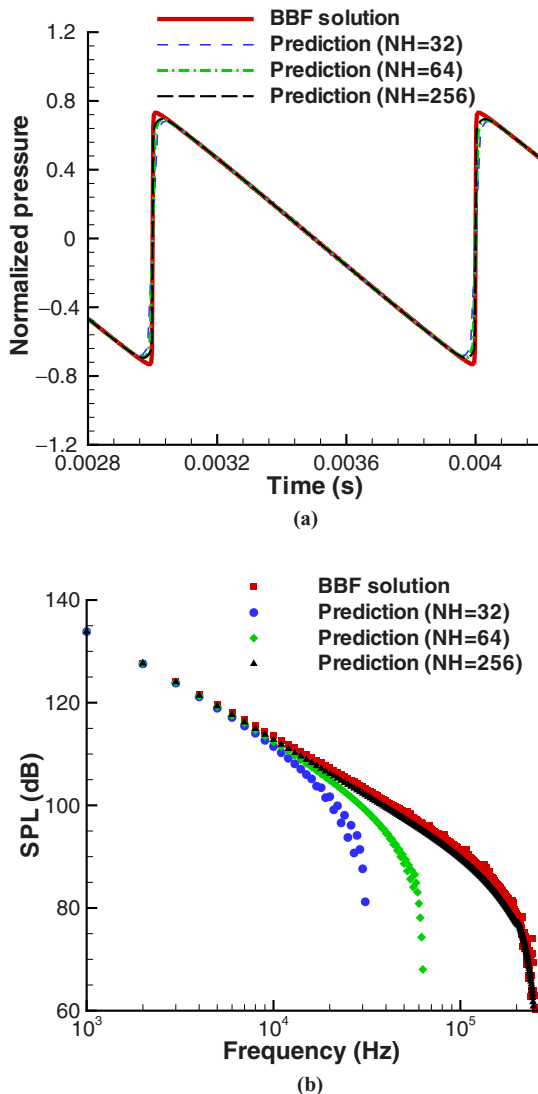


Fig. 1. Comparison between the BBF solution and predictions obtained by the split method for various numbers of harmonics (NH): (a) the normalized pressure time history and (b) sound pressure level. For the BBF solution, only the 256 harmonic result is shown.

The sound propagation occurs and the Burgers equation is solved in the inertial coordinate. However, the CFD data are given in the blade-fixed coordinate. A careful geometric transformation should be considered to use the CFD data for the starting signal in the Burgers equation at the starting distance, r_i , and to validate the results predicted by the Burgers equation against the CFD data at the final distance, r_f , where the CFD data still exist. Figure 2 shows the geometric transformation between the two coordinates. The sound is assumed to be generated at the blade tip (A). The initial data for the Burgers equation are extracted at the starting point (B), and then the signal is propagated to an observer (C) using the Burgers equation model in the inertial frame. Note that the initial signal should include the local nonlinear source terms that are not explicitly considered in the Burgers equation. The selection of the starting point (B) determines the propagation path in the inertial frame. The propagation path $\overline{BC}$ in the inertial frame is perpendicular to the tangential line of the characteristic curve at the starting point (B) and connected to the blade circle A. The characteristic curve is determined by CFD in the blade-fixed frame, and it is a collection of wave fronts.

Then the pressure time history obtained by the Burgers equation at the observer (C) in the inertial frame should be equal to that obtained by the CFD at an observer (D) in the blade-fixed frame. The points C and D are on the same circle with the radius of $\overline{OC}$ where O is the rotor center. The different position is due to the blade rotation or the transformation between the inertial frame and blade-fixed frame. The position D lies on the characteristic curve, or it can be simply found by the intersection of the characteristic curve and a circle with the radius of $\overline{OC}$ where O is the rotor center. The pressure time history at D extracted from the CFD is compared with the prediction by the Burgers equation at C.

Figure 2 shows density contours on the rotor plane and propagation path for four different starting positions for a hovering tip Mach number of 0.95. The starting positions are moved closer to the blade tip: $r_B/R = 1.96, 1.65, 1.33$, and 1.03 , where r_B and R are the distance between the starting point and the hub, and the blade radius, respectively. It should be noticed that the circle radius of $\overline{OC}$ or $\overline{OD}$ and the position D are the same for all cases, but the propagation path and the position C are different depending on the starting signal location. In this process, the original source position is assumed to be a single point at the blade tip. This approximation is valid for HSI noise. Table 1 shows the coordinates of A, B, C, and D as well as the starting distance r_i and the final distance r_f . A variable, J , in the table is the index of the CFD mesh.

As discussed earlier, the starting signal at the Burgers equation should include all nonlinear source terms. In other words, only the cumulative nonlinear propagation effect is solved in the Burgers equation. In general, as the starting point is placed further away from the blade tip, the predicted nonlinear propagation becomes more accurate, assuming the CFD data at the starting point is accurate since the loss of the nonlinear source terms is minimized. It is difficult to precisely separate the regions between where the nonlinear source terms are dominant and where the nonlinear propagation is dominant. In the next paragraph, the sensitivity of the prediction to the starting signal location is investigated.

Figure 3 shows a comparison of the acoustic pressure time history for the four starting positions. $r_B/R = 1.96$ provides excellent agreement with the CFD data. As the starting position approaches the blade tip, the first positive peak is slightly overpredicted and the second positive peak immediately following the negative peak is slightly underpredicted. This error is thought to be due to the reason that the nonlinear local effect is not fully captured in the starting signal. But the amplitude of the negative peak is well predicted, even for starting locations close to the blade tip. This result illustrates a sensitivity of the accuracy of the method to the proximity of the source position.

Figure 4 shows a comparison of the nonlinear and linear propagations for the starting data position of $r_B/R = 1.96$. The nonlinear propagation increases the amplitude of the higher frequency components of the signal, resulting in a wave distortion which is visible in the pressure time history. Although the difference occurs at very high frequencies at the short propagation distance, this nonlinear effect increases as the propagation distance increases due to the cumulative effect of nonlinear propagation, as will be shown later. But it should be noted that atmospheric absorption also extracts energy from high frequencies over the propagation, weakening the effect of nonlinear propagation. These two complementary effects are balanced and checked as sound propagates. The relative importance between nonlinear propagation and the atmospheric absorption for long distances will be discussed in the next section.

Figure 5 shows the effect of the propagation path. Menounou and Vitsas (Ref. 18) used the rotor hub for the sound source position for the nonlinear propagation prediction of blade-vortex-interaction noise. It is important to see how the change of the source position affects the result. When the source position is moved to the hub, the propagation

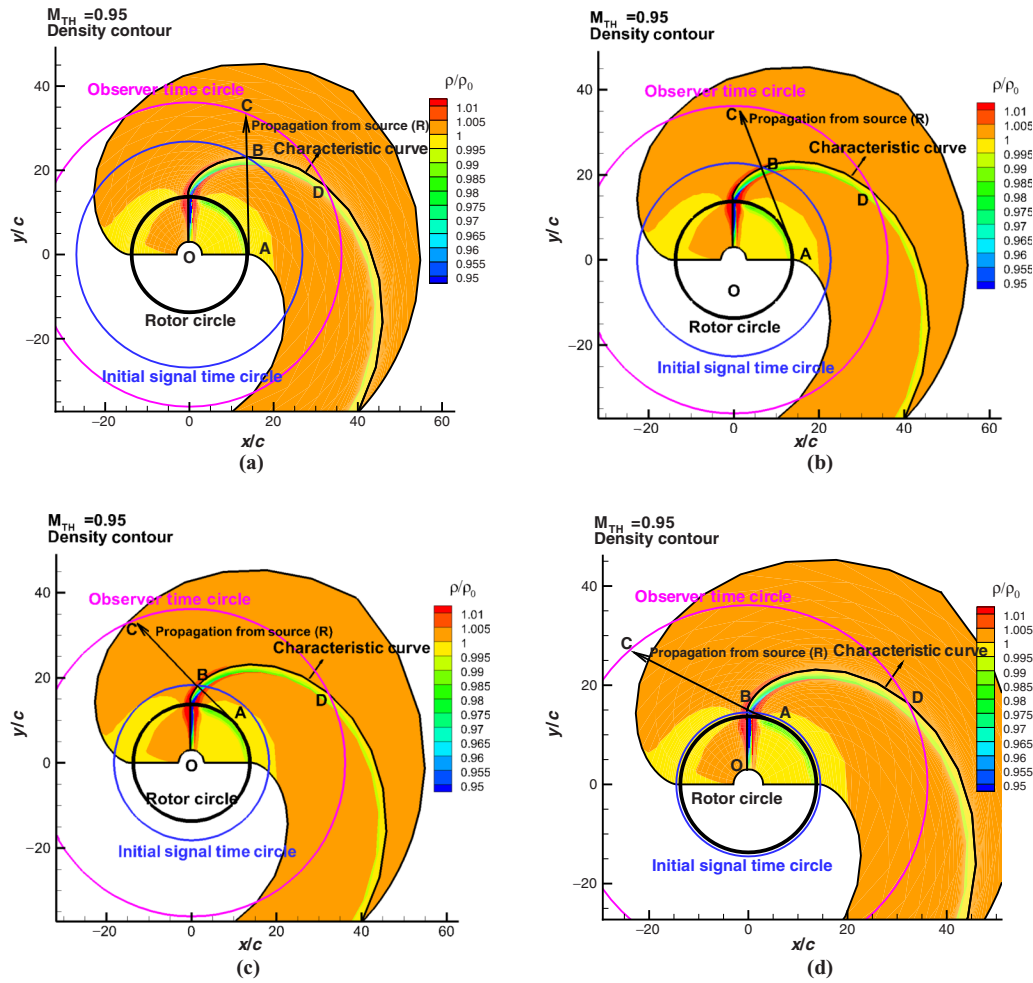


Fig. 2. Propagation path parameters for UH-1H hovering with $M_T = 0.95$: $r_B/R =$ (a) 1.96, (b) 1.65, (c) 1.33, and (d) 1.06.

Table 1. Propagation path parameters for UH-1H hovering with $M_T = 0.95$

r_B/R	A ($x/c, y/c$)	B ($x/c, y/c$)	C ($x/c, y/c$)	D ($x/c, y/c$)	r_i (m)	r_f (m)
1.96 ($J = 36$)	(14.14, 0.11)	(13.62, 23.14)	(13.30, 33.51)	(32.25, 16.23)	1.76	2.55
1.65 ($J = 34$)	(12.60, 5.88)	(6.48, 21.68)	(0.87, 36.12)	(32.25, 16.23)	1.29	2.47
1.33 ($J = 30$)	(11.10, 8.90)	(1.57, 18.12)	(-13.51, 33.41)	(32.25, 16.23)	1.01	2.65
1.06 ($J = 21$)	(3.96, 13.12)	(0.14, 14.90)	(-23.69, 27.08)	(32.25, 16.23)	0.32	2.36

path is no longer perpendicular to the tangential line of the characteristic curve. Therefore, the assumption that the characteristic curve represents a collection of wave fronts is not valid for this case. For the comparison, however, the starting signal position is assigned as the interaction between the propagation path and the characteristic curve, which is denoted as the point B' . Table 2 shows the coordinates of A, B, C, and D as well as the starting distance r_i and the final distance r_f for the hub center noise source. It can be seen that the propagation path from the hub slightly overpredicts the negative peak. This example shows the sensitivity of the prediction to the source location.

Nonlinear Propagation of Full-Scale Rotor Noise

Atmospheric absorption and dispersion depend on the frequency content of the signal. Model-scale rotor noise contains substantially high-frequency content due to the faster rotation speed. To examine realistic

effects of atmospheric absorption and nonlinear propagation, a full-scale rotor and its associated frequency contents are considered. The full-scale rotor rpm is determined to match the same hovering blade tip speed. The propagation distance is also scaled accordingly.

Figure 6 shows a comparison of linear and nonlinear propagations at $r/R = 1000$ for a full-scale UH-1H rotor with the rotor radius of 7.315 m. The atmospheric absorption and dispersion effects are not included in Fig. 6 to isolate the nonlinear propagation effect. The cutoff frequency for the split method is chosen to be as low as 500 Hz to avoid high-frequency errors. It can be seen that the nonlinear propagation reduces the negative amplitude by a half and significantly broadens the width of the signal as compared to the linear propagation due to the distortion of the waveform. The sound pressure level (SPL) distribution shows that the nonlinear propagation has lower noise levels compared to the linear propagation except below 100 Hz. The difference is as high as more than 10 dB around 240 Hz. It is also interesting to note that linear propagation has a smooth distribution of the SPL, but nonlinear

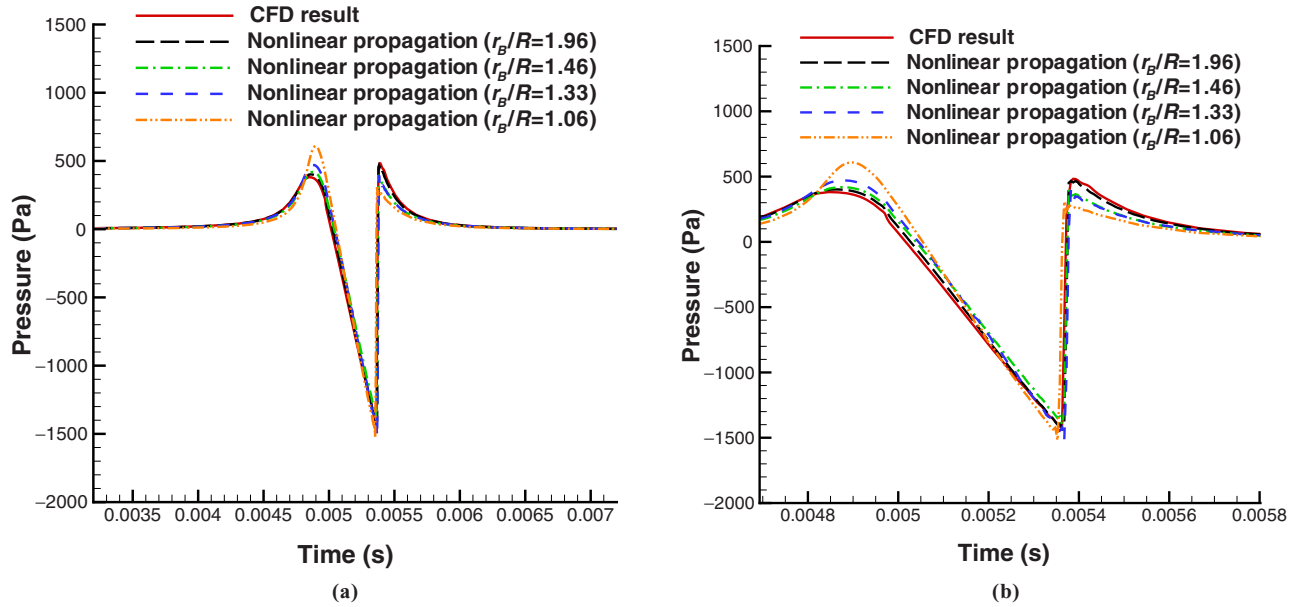


Fig. 3. (a) Acoustic pressure comparison with the Euler CFD result and predictions with different starting points and (b) enlarged figure around the negative peak.

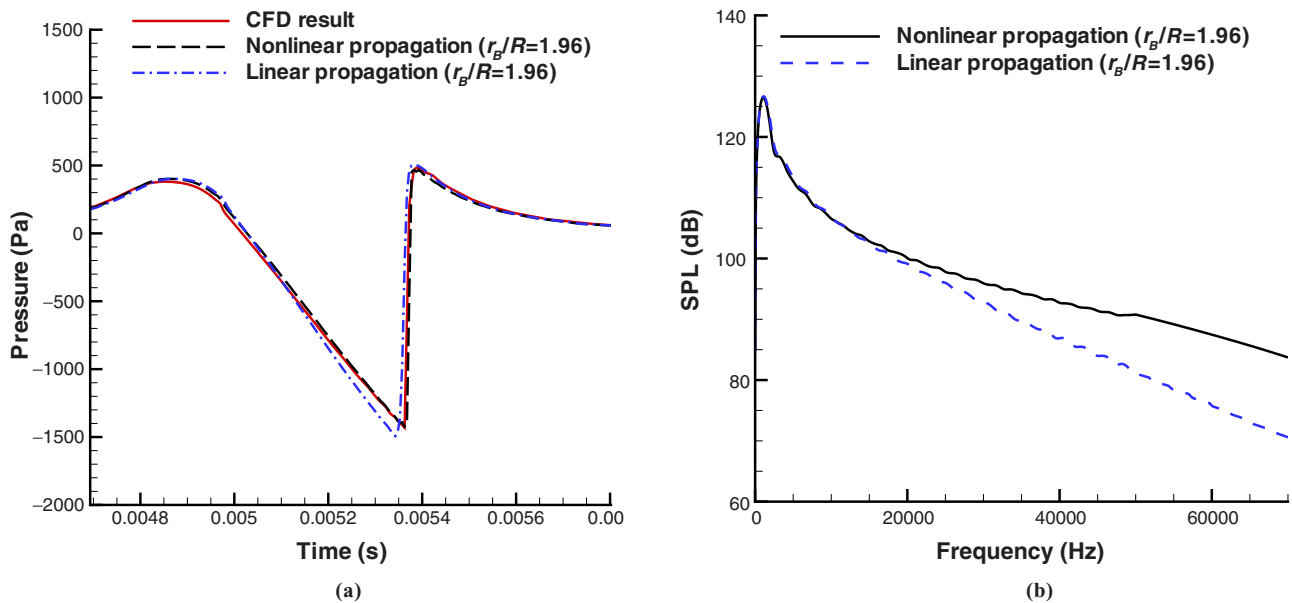


Fig. 4. Comparison of nonlinear and linear propagation with the starting point of $r_B/R = 1.96$ and $r_f = 2.55$: (a) acoustic pressure time history and (b) SPL.

propagation SPL distribution has a scalloped behavior. The source of this scalloping of the SPL distribution for nonlinear propagation will be discussed in more detail in the next section.

Figure 7 includes the linear and nonlinear propagations with the atmospheric absorption at $r/R = 1000$. The nonlinear steepening increases the width of the negative peak duration and shifts the phase to the right, whereas the atmospheric absorption weakens the distortion by attenuating high-frequency energy and shifts the phase to the left. As a result, the phase of the negative peak is different for two results. The nonlinear propagation reduces the peak amplitude compared to the linear propagation. However, when the atmospheric absorption is considered, the

difference of the peak amplitude is small at such a large distance. Again, the SPL distribution has the same scalloping between 200 and 400 Hz for nonlinear propagation. It is also important to note how important the atmospheric absorption is even in the linear propagation. The atmospheric absorption substantially changes the waveform resulting from linear propagation. Therefore, the atmospheric absorption effect should always be included for long-range sound propagation.

Figure 8 shows the evolution of the pressure and SPL for the linear and nonlinear propagations at $r/R = 100, 500$, and 1000 . Nonlinear propagation is important in terms of the amplitude and phase. The SPL can be different by 3–4 dB below 100 Hz and even more when the scallop

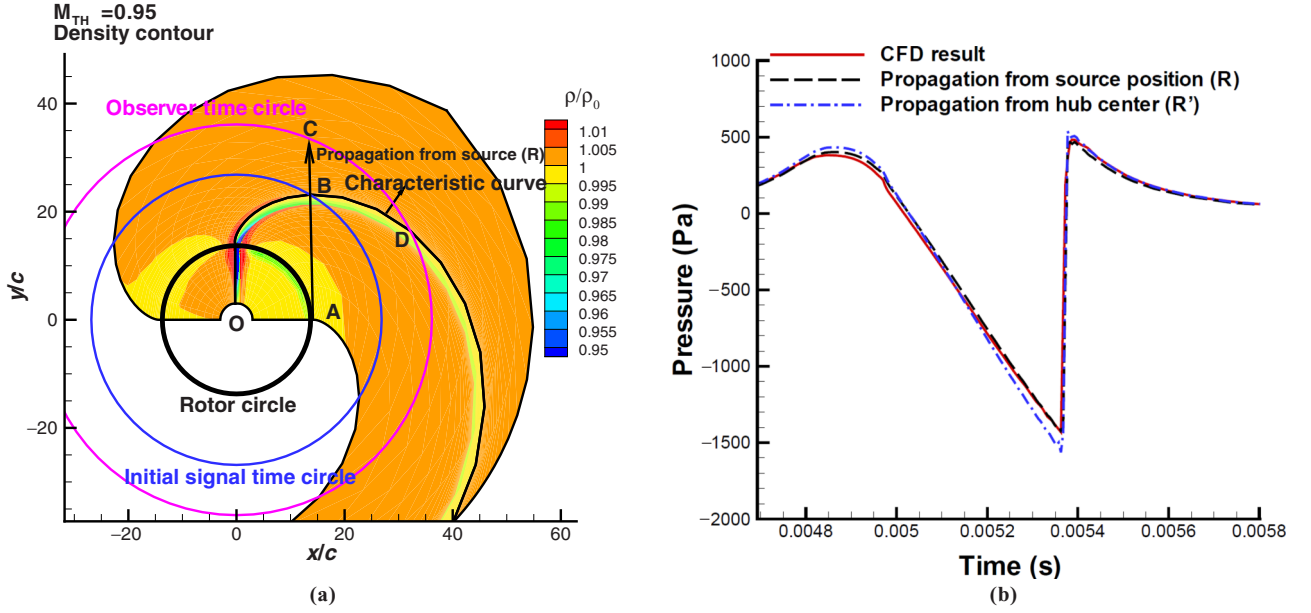


Fig. 5. Effect of the original source location and corresponding propagation path on the acoustic pressure.

Table 2. Propagation path parameters for the original sound source located at the hub for UH-1H hovering with $M_T = 0.95$

r_i/R	A ($x/c, y/c$)	B' ($x/c, y/c$)	C ($x/c, y/c$)	D ($x/c, y/c$)	r'_i (m)	r'_f (m)
1.96 ($J = 36$)	(14.14, 0.11)	(9.85, 25.03)	(13.30, 33.51)	(32.25, 16.23)	2.05	2.75

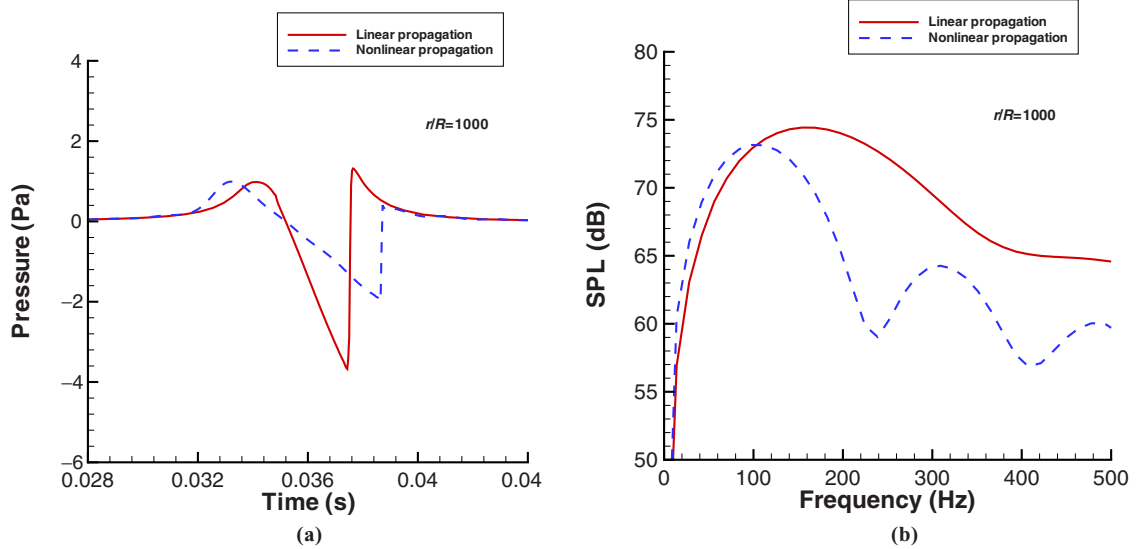


Fig. 6. Comparison of linear and nonlinear propagations at $r/R = 1000$ without atmospheric absorption/dispersion effects for a full-scale rotor: (a) acoustic pressure time history and (b) SPL.

occurs, or at high frequency. As the propagation path increases, the atmospheric absorption effect becomes more important and the difference between the linear and nonlinear propagations becomes smaller.

Nonlinear Indicator for Rotor Noise

In this section, a nonlinearity indicator is investigated to determine the reason for the changes in the SPL distribution for nonlinear propagation

as compared to linear propagation. The nonlinearity indicator shows the energy transfer from a certain frequency range to another. First, the quad-spectral density Q_{p^2p} is defined as the imaginary part of the cross-spectral density between the square of the acoustic pressure and the pressure, which is given by

$$Q_{p^2p}(f) = \text{Im}[\text{FFT}\{p^2(t)\}\text{FFT}^*\{p(t)\}] \quad (13)$$

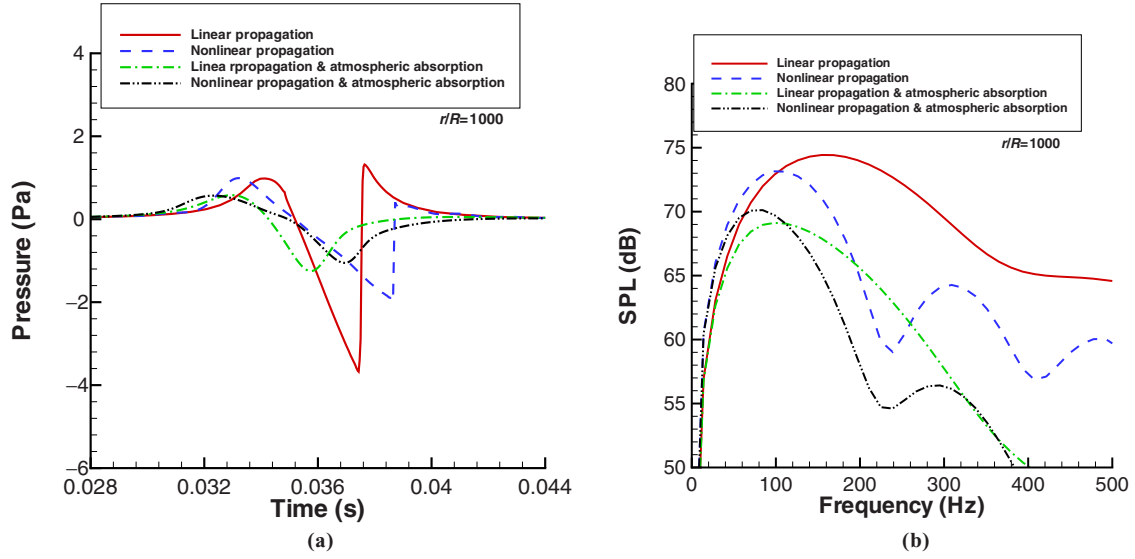


Fig. 7. Comparison of linear and nonlinear propagations at $r/R = 1000$ with atmospheric absorption/dispersion effects for a full-scale rotor: (a) acoustic pressure time history and (b) SPL.

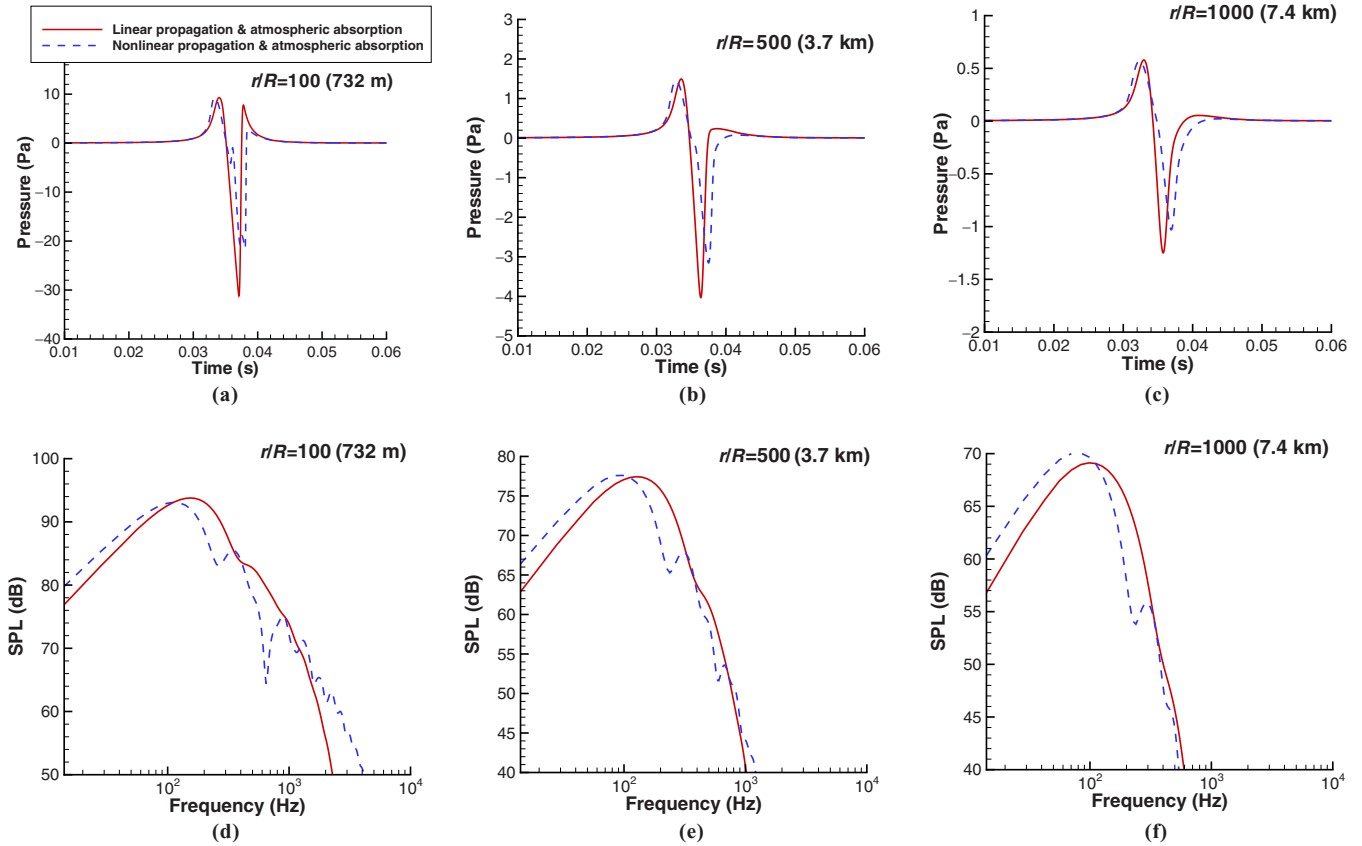


Fig. 8. Prediction of linear and nonlinear propagation over distances: pressure at $r/R =$ (a) 100, (b) 500, and (c) 1000, SPL at $r/R =$ (d) 100, (e) 500, and (f) 1000.

Morfe and Howell derived the ensemble-averaged version of the generalized Burgers equation that can be written as

$$\frac{\partial}{\partial r} [r^2 e^{2\alpha(f)r} S_{pp}(r, f)] = -2\pi f r^2 \frac{\beta}{\rho_0 c_0^3} e^{2\alpha(f)r} Q_{p^2 p}(r, f) \quad (14)$$

The left-hand side of Eq. (14) represents the spatial rate of change of the power spectral density $S_{pp}(r, f)$, which has been corrected for spherical spreading r^2 and atmospheric absorption $e^{2\alpha(f)r}$. For linear propagation, the right-hand side of Eq. (14) becomes zero. Therefore, the right-hand side of Eq. (14) can be used as an indicator to represent the nonlinearity of the propagation, which accounts for the sum/difference

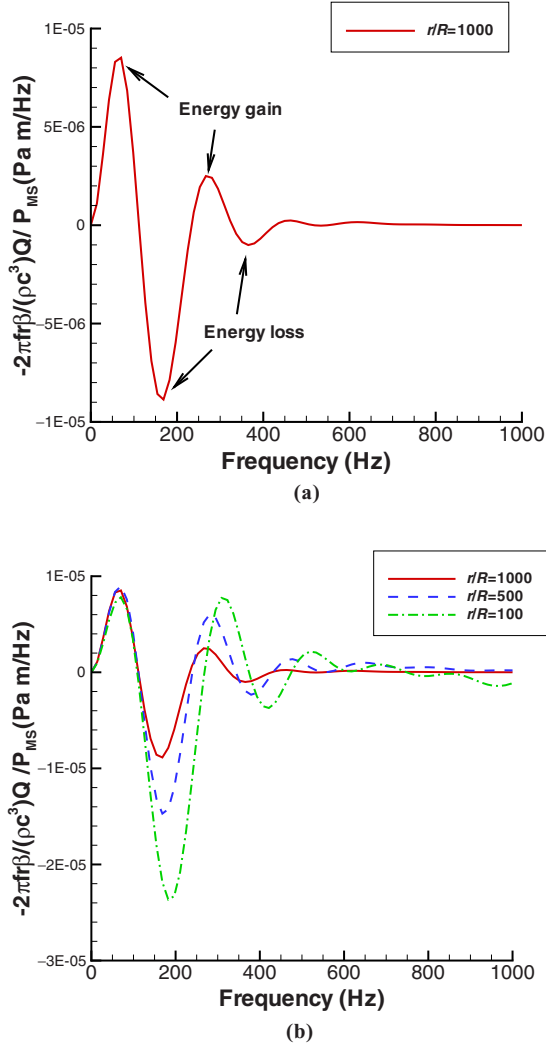


Fig. 9. Nonlinearity indicator with atmospheric absorption/dispersion effects for a full-scale rotor: (a) $r/R = 1000$ and (b) comparison for three propagation distances, $r/R = 100, 500, 1000$.

frequency generation that occurs during the propagation. At frequencies where the right-hand side is negative, there is a net energy loss due to nonlinearity (i.e., energy is being transferred to other frequencies). At frequencies where the right-hand side is positive, there is a net energy gain.

Gee (Ref. 10) and Saxena (Ref. 19) studied the use of nonlinearity indicators in determining the nonlinear behavior of the jet noise. Gee used the right-hand side of Eq. (14), and Saxena used another indicator, Q/S , where S is defined $S_{pp}(f)p_{rms}$. $S(f)$ is the power spectral density, and p_{rms} is the root-mean-square of the pressure. It was found, however, that these two indicators do not clearly show the energy transfer for rotor noise. Therefore, in the present paper, a new indicator is introduced for rotor noise, which is given as

$$-2\pi fr \frac{\beta}{\rho_0 c_0^3} Q_{p^2 p}(r, f) / P_{MS} \quad (\text{Pa m/Hz}) \quad (15)$$

where P_{MS} is the mean square of the pressure.

The indicator is scaled by propagation distance so that it can be compared for different propagation distances. In addition, since it is

normalized by the square of the pressure, the amount of the nonlinearity can be compared for different signals.

Figure 9 shows the nonlinearity indicator for (a) $r/R = 1000$ and (b) three different propagation distances. It is shown that the energy is transferred from frequencies around 200 Hz into frequencies around 100 and 300 Hz. This energy transfer is consistent to the hills and valleys of the scalloped SPL distribution shown in Figs. 6 and 7. The nonlinear indicator clearly explains that the spectrum change is due to the nonlinear propagation rather than the atmospheric absorption. This local energy transfer and the scalloped variations of SPL are a new finding in helicopter noise, and it was not clearly shown in any other problems such as jet noise or sonic boom. However, it is not clear why energy is pushed from 200 Hz to higher and lower frequencies. We suspect that this might be related to the original shape of the signal whose maximum amplitude occurs near 200 Hz. Further investigation into this local energy transfer is needed. Although a high-frequency rise is observed for nonlinear propagation, the amplitude at high frequencies is negligible. As a result, the nonlinearity indicator is almost zero at high frequencies. In addition, Fig. 9(b) shows that the energy transfer is large for short propagation distance and it is reduced with increasing propagation distance. It is thought that the energy is extracted through atmospheric absorption during propagation so that the amount of the net energy change is reduced over propagation distance. However, the frequency bands where the energy transfer occurs remain the same.

Conclusions

The nonlinear propagation of helicopter high-speed impulsive noise was presented in this paper. The CFD solution provided the localized nonlinear sources in the near-field, and the Burgers equation was applied to solve nonlinear propagation over a long distance. In this approach, the nonlinear source and nonlinear propagation effects are handled separately so that it can be more practical and useful for long-range propagation of rotor noise than the acoustic analogy, which requires both the localized nonlinear sources and the nonlinear propagation effect from the CFD data.

The split method for the Burgers equation has been validated against the analytic solution for a sinusoidal wave and model-scale rotor noise. It has been shown that the acoustic pressure obtained by the Burgers equation begins to deviate from the accurate solution as the starting signal approaches the blade tip. This discrepancy is due to the reason that the localized nonlinear effect is not fully captured in the starting signal. For a high-speed flight above the tip Mach number greater than 0.9, the nonlinear wave steepening becomes important up to several kilometers for a full-scale rotor. The nonlinear steepening results in distorting the waveform, lowering the peak amplitude, and delaying the peak time compared to the linear propagation. As the distance increases, however, the atmospheric absorption attenuates high-frequency energy and makes the waveform much more smoother.

Finally, a new nonlinearity indicator that is suitable to rotor noise was proposed to investigate the energy flow from certain frequencies to others due to the nonlinear propagation effect. It has been demonstrated that the energy transfer in nonlinear sound propagation resulted in a scalloped SPL distribution, resulting in differences between linear and nonlinear propagations as large as 10 dB at around 200 Hz. This scalloped SPL variation is a new finding, and an interesting feature in helicopter nonlinear sound propagation.

Overall, this paper provides the quantitative assessment of HSI noise for long-range propagation. However, the test case is limited only to a hovering case. A more detailed investigation of nonlinear propagation of rotorcraft noise—in forward flight and full directivity—is recommended.

Acknowledgments

This research was funded by the U.S. Government under Agreement No. W911W6-06-2-0008 under the task of Vertical Lift Research Center of Excellence (VLRCE).

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