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Authors
Jelley, N.A.
Cerny, Joseph
Stahel, D.P.
et al.

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N. A. Jelley, Joseph Cerny, D. P. Stahel, and K. H. Wilcox

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PREDICTIONS OF THE MASSES OF VERY NEUTRON-EXCESS LIGHT NUCLEI*

N. A. Jelley†, Joseph Cerny, D. P. Stahel, and K. H. Wilcox

Lawrence Berkeley Laboratory
and Department of Chemistry
University of California
Berkeley, California 94720

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ABSTRACT

A simple mass equation is derived and is shown to provide a good description of the masses of the recently measured $T_Z = 5/2$ nuclei in the s-d shell. A comparison is made with the method of Garvey et al. and predictions of masses and of the stability of neutron-excess light nuclei are given for both methods.

I. INTRODUCTION

Over the last few years many very neutron-rich light nuclei[1] ($T_Z \geq 5/2$, $A < 50$) have been shown to be particle stable and the masses of several of these have also been determined. On comparing these results with theory, though good agreement with regard to predictions of stability based on the transverse relation of Garvey-Kelson[2,3] is observed, as in the recent correct prediction that ^{17}B should be particle stable[4], poorer agreement is generally found for the measured mass-excess. For example, in the s-d shell there are several $T_Z = 5/2$ nuclei for which there is a significant discrepancy (> 500 keV) between the experimental and the calculated mass-excess.

In this report an alternative scheme, similar in approach to the method of Garvey et al.[2] but taking more explicit account of shell effects, will be described which more successfully accounts for many of the observed masses of neutron-excess light nuclei.

II. MASS EQUATIONS

Following the simple shell model approach of Goldstein and Talmi[5], the mass of a neutron-excess doubly closed shell nucleus, M_o, is related to that of a nucleus with additional m j protons and n j' neutrons in a higher shell, $M(\pi j^m \nu j'^n)$, by the equation:

$$M(\pi j^m \nu j'^n) = M_o + V(\pi j^m) + V(\nu j'^n) + V(j^m, j'^n). \qquad (1)$$

In this expression $V(\pi j^m)$ represents the kinetic energy, interaction with the closed shells and mutual interaction of the m j protons, $V(\nu j'^n)$ that of the n j' neutrons, and $V(j^m, j'^n)$ the interaction between the j protons and j' neutrons.

Simplification is possible in Eq. (1) since the values of $V(\pi j^m)$ and $V(\nu j'^n)$ can each be expressed in terms of just three parameters (as noted below). However, since other configurations besides that of the simple shell model are generally important in describing the ground state wave function of a nucleus, some allowance for such configuration mixing can be made by regarding $V(\pi j^m)$ and $V(\nu j'^n)$ as separate parameters for each value of m and n. This is equivalent to replacing $M_o + V(\pi j^m) + V(\nu j'^n)$ by the sum of arbitrary functions, U(Z) and W(N), of the number of protons and neutrons, respectively. Furthermore, if no odd-odd nuclei are considered, then $V(j^m,j'^n)$ depends only on an average interaction potential, V(jj'), through the relation[2,6] $V(j^m,j'^n) = mn\,V(jj')$. Hence, with the restriction ($\equiv$ mn even) of no odd-odd nuclei and rewriting $M(\pi j^m \nu j'^n)$ as M(Z,N), Eq. (1) is then equivalent to:

$$M(Z,N) = U(Z) + W(N) + mn\,V(jj') \quad [mn \text{ even}]. \tag{2}$$

This mass equation can be generalized to include neutron-excess nuclei from several configurations $\pi j_i \nu j_k$, though still with the requirement that the neutron shell νj_k lie higher than the proton shell πj_i. In this more general case the mass, M(Z,N), is given by what will be denoted the <u>modified</u> shell model mass equation:

$$M(Z,N) = U(Z) + W(N) + \sum_{ik} m_i n_k\, V(j_i j_k) \quad [m_i n_k \text{ even}]. \tag{3}$$

The m_i and n_k are the number of protons and neutrons in the shells πj_i and νj_k, respectively, and the sum $\sum_{ik}$ is over the neutron-proton interaction parameters $V(j_i j_k)$.

For comparison, in the simple shell model M(Z,N) is given by:

$$M(Z,N) = M_o + \sum_i V(j_i{}^{m}i) + \sum_k V(j_k{}^{n}k) + \sum_{ik} m_i n_k V(j_i j_k) \quad [m_i n_k \text{ even}]. \tag{4}$$

Each function of the form $V(j^q)$ represents the interaction energy of q identical nucleons and assuming minimum seniority can be expressed as:

$$V(j^q) = qe_j + q(q-1)a_j/2 + [q/2]b_j$$

where [q/2] is the integer less than or equal to q/2, and e_j, a_j and b_j are three interaction parameters.[6]

Equation (3) is similar to the Garvey-Kelson transverse mass equation[2]:

$$M(Z,N) = F(Z) + G(N) + H(A) \tag{5}$$

where F, G and H are arbitrary functions of the number of protons, neutrons and nucleons, respectively. Comparison of these equations shows that the two methods differ mainly in their parameterization of the residual neutron-proton interaction. In the method of Garvey <u>et al.</u>[2] much of this interaction is given by the function H(A), while in Eq. (3) more explicit account is taken of shell structure by the term $\sum_{ik} m_i n_k V(j_i j_k)$. Also, implicit in Eq. (5) is the assumption that the residual neutron-proton interaction is independent[7] of T_Z.

The differences in assumptions allow the transverse mass equation to be more general than the modified mass equation, both in predicting masses of odd-odd nuclei and in being able to predict masses farther from stability. In both cases predictions are carried out by determining the parameters of the mass equations by a least squares fit to known masses. (For Eq. (5) all known masses of $N \geq Z$ nuclei can be included (except N = Z = odd), while for Eq. (3) only those which possess configurations $\pi j_i \nu j_k$ and which are not odd-odd can be used.)

III. RESULTS AND DISCUSSION

As a means of comparing these two approaches when applied to light neutron-rich nuclei, the masses of the $T_Z = 5/2$ nuclei in the s-d shell have been predicted, and their relative agreement with the experimental values is shown in Fig. 1. For these nuclei the predicted values arising from the transverse mass equation were taken from the calculations of Thibault and Klapisch[3], who included as input from the s-d shell only known $T_Z \leqslant 2$ nuclei. For the other predictions the modified mass equation was used except for the values for ^{21}O and ^{23}F where Eq. (4) from the simple shell model was employed, since insufficient masses are known for Eq. (3) to be used. Only known non-odd-odd $T_Z \leqslant 2$ nuclei, together with ^{29}Na, with configurations $\pi p_{1/2}\ \nu d_{5/2}$, $\pi d_{5/2}\ \nu s_{1/2}$, $\pi d_{5/2}\ \nu d_{3/2}$ and $\pi s_{1/2}\ \nu d_{3/2}$ were included as input. (The mass of ^{29}Na determines the interaction parameter $V(\pi d_{5/2}\ \nu d_{3/2})$). As seen in Fig. 1, considerably better agreement was obtained with the approach of this work than with the transverse mass equation; quantitatively the rms deviations between experiment and calculation are 260 keV and 620 keV, respectively (excluding the mass of ^{21}O because of its large error). A further comparison is afforded using the simple shell model, Eq. (4), alone. This yields a rms deviation of 390 keV, illustrating the importance of configuration mixing, which to some extent is allowed for in Eq. (3).

Another example is discussed in Ref. 8 where the masses of the argon isotopes $^{43-46}Ar$ are compared with the predictions of Eqs. (3) and (5); better agreement is also found using Eq. (3). For these isotopes the predictions of Zeldes et al.[9], which are based on a generalization of an independent particle model, are also in good agreement with experiment. However, for lighter nuclei these latter predictions are less successful, probably due to charge-dependent terms in their mass formula.[2] Comparisons between different methods are made difficult, however, by differences in the input masses that were used and, as has been noted,[4] predictions can be quite sensitive to changes in the masses of only a few nuclei. Bassichis and Ali[10] have recently accounted for the observed mass-excesses of the $T_Z = 5/2$ nuclei ^{25}Ne, ^{29}Mg and ^{33}Si by employing a literal interpretation of the Garvey-Kelson mass formula to relate deviations from the simple predictions for certain sextuplets of nuclei. References to other recent work on mass relations and equations can be found in the paper of Jänecke and Behrens.[7]

Table I presents predictions of mass excesses and one- and two-neutron binding energies of selected neutron-excess nuclei at or just beyond the limits of current investigation obtained through a recalculation with Eq. (5), the transverse mass equation, as well as with Eq. (3), the modifed mass equation, denoted T and M, respectively. Experimental values are given when available (see Ref. 11 and 12 and those cited in the table) and those nuclei only known to be bound or unbound are indicated by the symbol "B" or "U". A complete tabulation of the results is given in Ref. 13.

Calculated T and M values in Table I arise from a least squares fitting program which employed with equal weight the appropriate particle-stable nuclei[14] with $N \geqslant Z$ whose mass-excesses are known to $\leqslant$ 200 keV; those known with less accuracy were not used in these calculations and are shown in the table enclosed in parentheses. All known nuclei (271) with $2 \leqslant Z \leqslant 35$ and $4 \leqslant N \leqslant 50$ were used in obtaining the transverse mass equation values. Compared to the recent calculation,[3] the ten known s-d shell, $T_Z \geqslant 5/2$ nuclei given in Table I were the additional nuclei included. For Eq. (3) the known non-odd-odd nuclei (74) with configurations $\pi p_{3/2}\ \nu p_{1/2}$, $\pi p_{3/2}\ \nu d_{5/2}$, $\pi p_{1/2}\ \nu d_{5/2}$, $\pi d_{5/2}\ \nu s_{1/2}$, $\pi d_{5/2}\ \nu d_{3/2}$, $\pi s_{1/2}\ \nu d_{3/2}$ and $\pi d_{3/2}\ \nu f_{7/2}$ were employed. In Eq. (3) lack of sufficient known masses required assumed values for the mass-excesses of ^{21}O, ^{22}O and ^{14}Be: for ^{21}O and ^{22}O the values from Eq. (4) were used and (to determine the interaction parameter $V(\pi p_{3/2} \nu d_{5/2})$) the mass-excess of ^{14}Be (known to be bound)[15] was taken to equal $^{12}Be + 2n = 41.09$ MeV, close to the value obtained with the transverse equation of 40.72 MeV.

In order to compare how well these two approaches account for known masses,[6] one can evaluate the rms deviation defined as $[\sum_i \Delta_i^2/(N-P)]^{1/2}$ where the Δ_i are differences between the calculated and experimental masses, and N and P are the number of known nuclei and parameters, respectively. For nuclei with $2 \leqslant Z \leqslant 17$ the transverse mass equation yields an rms deviation of 220 keV (N = 82, P = 66) and the modified mass equation 200 keV (N = 51, P = 36). Though these values are very similar it does not necessarily follow that the predictive validity of the two approaches will be the same (compare the results in Fig. 1).

Several comments on nuclei at or near the current limit of experimental accessibility can be made from Table I. It appears that the differences between the T and M approaches observed in the s-d shell for the $T_Z = 5/2$ nuclei persist to lighter nuclei, since the predictions for ^{9}He, ^{13}Be, ^{15}B and ^{19}N differ by more than 750 keV. In the s-d shell the reported ^{22}O mass-excess[12] is much less bound (by 2.1 MeV) than is calculated by either the transverse or the simple shell model mass equations (however, the experimental approach employed by Artukh et al.[12] could possibly suffer from a systematic error in this direction, so that an additional measurement of the mass-excess of ^{22}O is of great interest). For ^{31}Na and ^{32}Na it has been pointed out from recent experimental measurements[11] that their mass-excesses imply a shell closure effect (N = 20) larger than would be deduced from the properties of nuclei closer to stability. Experimentally, the mass-excesses of ^{31}Na and ^{32}Na are 10.6 ± .8 MeV and 16.4 ± 1.1 MeV, respectively,[11] while the transverse mass equation predicts 12.7 MeV and 21.0 MeV. This enhanced closure effect for ^{31}Na is also strikingly seen when comparing with experiment the predicted mass-excess of 14.4 MeV, arising from the modified mass equation.

These mass-excess calculations permit predictions of those nuclei lying on the edge of stability. The limits yielded by this recalculation with the transverse equation differ from those of Ref. 3, which did not employ any $T_Z \geqslant 5/2$ nuclei from the s-d shell, in that a) ^{23}N, ^{26}O, ^{40}Mg, ^{43}Al and ^{48}Si are predicted to be the last nucleon stable isotopes, compared[3] to ^{25}N, ^{28}O, ^{42}Mg, ^{45}Al and ^{46}Si; and b) ^{28}F, ^{29}Ne and ^{37}Mg are predicted to be the first unbound isotopes, compared[3] to ^{30}F, ^{31}Ne and ^{41}Mg. Results from the modified mass equation are less extensive than those from the transverse equation, generally not predicting the edge of stability; however, for the lighter nuclei ^{26}O is calculated by Eq. (3) to be unbound by 240 keV, predicting ^{24}O as the last stable oxygen isotope. Also ^{29}F is calculated to be unbound to 2n decay by 910 keV, compared to the prediction of the transverse equation that it is bound by 770 keV.

Kelson and Garvey[16] have also employed relations based on the charge symmetry of nuclear forces to predict quite successfully the masses of neutron-deficient nuclei through the titanium isotopes. An appendix of Ref. 13 tabulates results for mass-excesses and one- and two-proton binding energies from a similar recalculation of these nuclei which employed current known masses and predictions for neutron-excess nuclei from the transverse equation where necessary. Although many masses change considerably in this recalculation, the only revision[17] in their predictions of the onset of nuclear instability is that ^{31}Ar is now expected to be unbound.

The approach employing the modified mass equation described above appears to be a useful alternate predictive scheme for the masses of very neutron-excess light nuclei. Further mass measurements of nuclei far from stability such as, for example, the nucleon-stable isotopes ^{15}B and ^{19}N will afford particularly interesting new comparisons of this method and that of Garvey _et al._[2] with experiment.

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†Present address: Nuclear Physics Laboratory, University of Oxford, England.

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17. Fig. 16 in Ref. 15 presents those neutron-deficient nuclei expected to lie on the edge of stability, based on minor updating of Ref. 16; ^{33}K and ^{34}K are incorrectly shown therein[15] as nucleon stable due to an error in the proof of Ref. 16.

Table I. Comparisons with experiment of the predictions of the transverse (T) and the modified (M) mass-equations.

Z	N	EL	A	Mass Excess (MeV±MeV) Experimental	Mass Excess Calculated T	Mass Excess Calculated M	Binding Energy (MeV) 1 Neutron Experimental	1 Neutron Calculated T	1 Neutron Calculated M	2 Neutron Experimental	2 Neutron Calculated T	2 Neutron Calculated M
2	6	HE	8	31.57±.03[a]	31.57	31.57	2.61			2.17		
2	7	HE	9	U ≡ unbound	42.61	43.49		-2.97	-3.85		- .36	
2	8	HE	10	U	51.00	52.34		- .32	- .78		-3.29	-4.62
3	7	LI	10	U	33.25			- .21			3.84	
3	8	LI	11	40.94±.08[b]	40.94	41.14				.17		
3	9	LI	12	U	52.94			-3.93			-3.55	
3	10	LI	13		61.56	60.34		- .55			-4.48	-3.05
4	8	BE	12	25.03±.05[c]	25.02	24.75	3.22			3.72		
4	9	BE	13	U	35.39	34.60		-2.31	-1.77		.94	1.84
4	10	BE	14	B ≡ bound	40.72	41.09*		2.74	1.58		.44	- .20
5	9	B	14	23.66 ±.03[d]	23.66		.98			5.86		
5	10	B	15	B	28.75	29.89		2.97			3.97	2.66
5	11	B	16	U	37.97			-1.14			1.83	
5	12	B	17	B	44.36	43.62		1.67			.53	2.41
6	11	C	17	B	21.27	20.86		.50	.90		4.75	5.34
6	12	C	18	B	25.50	24.57		3.84	4.36		4.34	5.27
6	13	C	19	B	33.47	32.41		.10	.24		3.94	4.60
7	12	N	19	B	16.27	15.32		5.07			7.74	8.51
7	13	N	20	B	21.60			2.75			7.82	
7	14	N	21	B	24.50	24.11		5.17			7.92	7.35
8	13	O	21	$\left(9.3^{+.3}_{-.7}\right)$[e]	8.74	8.39+		3.08	3.48		10.80	11.09
8	14	O	22	$\left(11.5^{+.2}_{-.5}\right)$[e]	9.42	9.35+		7.39	7.11		10.47	10.59
8	15	O	23	B	15.48	15.40		2.01	2.02		9.40	9.13
8	16	O	24	B	19.70	19.44		3.85	4.04		5.87	6.06

(continued)

Table I (continued)

Z	N	EL	A	Mass Excess (MeV±MeV) Experimental	Calculated T	Calculated M	Binding Energy (MeV) 1 Neutron Experimental	Calculated T	Calculated M	2 Neutron Experimental	Calculated T	Calculated M
9	14	F	23	3.36±.17[f]	3.40	3.36	7.54			12.74		
9	15	F	24	B	8.04			3.44			10.89	
9	16	F	25	B	11.75	11.26		4.36			7.80	8.24
10	15	NE	25	-2.16 ±.10[g]	-1.95	-2.12	4.28			13.15		
10	16	NE	26	B	.17	- .27		5.95	6.23		9.89	10.43
10	17	NE	27	-	6.52	6.58		1.73	1.21		7.68	7.44
11	15	NA	26	-6.90 ±.02[b,h]	-6.94		5.62			14.63		
11	16	NA	27	-5.62 ±.06[b,i,j]	-5.71	-5.73	6.79			12.41		
11	17	NA	28	-1.14 ±.08[b,i]	-1.02		3.59			10.38		
11	18	NA	29	2.65 ±.10[b,i]	2.32	2.66	4.28			7.87		
11	19	NA	30	8.37 ±.20[b,i]	8.50		2.35			6.63		
11	20	NA	31	(10.6 ±.8)[b]	12.70	14.38		3.87			5.76	4.42
11	21	NA	32	(16.4 ±1.1)[b]	21.02			- .25			3.62	
11	22	NA	33	B	26.90			2.19			1.94	
11	23	NA	34		35.08			- .11			2.08	
12	17	MG	29	-10.75±.05[k]	-10.70	-10.75	3.80			12.31		
12	18	MG	30	B	- 9.37	- 9.21		6.75	6.54		10.56	10.42
12	19	MG	31		- 3.73	- 3.17		2.43	2.03		9.18	8.56
13	18	AL	31	-15.01±.10[l]	-15.00	-15.05	7.19			12.94		
13	19	AL	32	B	-11.14			4.21			11.50	
13	20	AL	33	B	- 9.34	- 8.65		6.27			10.49	9.75
14	19	SI	33	-20.57 ±.05[m]	-20.71	-20.67	4.55			13.76		
14	20	SI	34	B	-20.57	-20.32		7.93	7.72		12.77	12.42
14	21	SI	35	B	-15.02			2.51			10.45	
15	20	P	35	-24.94 ±.08[n]	-24.90	-24.81	8.46			14.74		
15	21	P	36	B	-20.88			4.05			12.46	
15	22	P	37	B	-18.98			6.17			10.22	

(continued)

Table I (continued)

				Mass Excess (MeV±MeV)			Binding Energy (MeV)					
							1 Neutron			2 Neutron		
				Experimental	Calculated		Experimental	Calculated		Experimental	Calculated	
Z	N	EL	A		T	M		T	M		T	M
16	23	S	39	B	-23.07	-23.21		4.33	4.35		12.24	12.31
16	24	S	40	B	-22.50	-22.64		7.50	7.51		11.83	11.85
16	25	S	41		-18.31	-18.42		3.88	3.85		11.38	11.36
17	24	CL	41	B	-27.43	-27.39		7.84			13.65	13.67
17	25	CL	42	B	-24.68			5.32			13.16	
17	26	CL	43		-23.64	-23.61		7.03			12.35	12.36

*Assumed value, see text

+Calculated using equation 4

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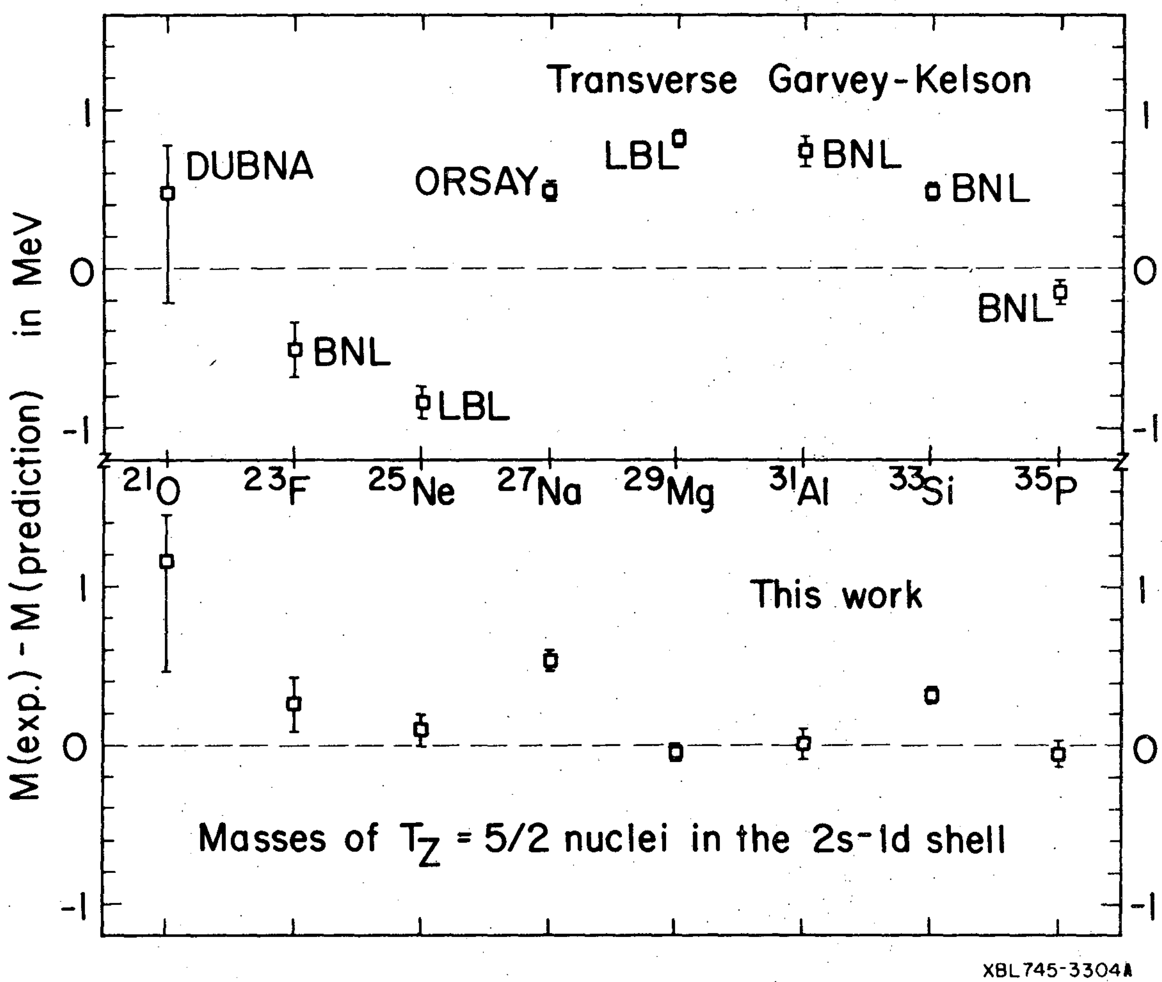
FIGURE CAPTIONS

Fig. 1. Two comparisons of the differences between experimental and predicted mass-excesses for the $T_Z = 5/2$ nuclei in the 2s-1d shell. See text.


Transverse Garvey-Kelson
DUBNA
ORSAY
LBL
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LBL
M(exp.) - M(prediction) in MeV
1
0
-1
^{21}O
^{23}F
^{25}Ne
^{27}Na
^{29}Mg
^{31}Al
^{33}Si
^{35}P
This work
Masses of T_Z = 5/2 nuclei in the 2s-1d shell
XBL745-3304A

Fig. 1

Appendix I

Experimental and predicted mass excesses and one- and two- neutron binding energies (in MeV) for neutron-excess isotopes of the elements from helium through chlorine. T and M refer to predictions from the transverse and modified mass equations, respectively.

References to masses employed in these calculations but not explicitly cited in the text are:

^{34}P: D. R. Goosman, C. N. Davids and D. E. Alburger, Phys. Rev. C8, 1324 (1973).

^{38}S: G. A. P. Engelbertink and J. W. Olness, Phys. Rev. C3, 180 (1971).

$^{43-46}$Ar: Reference 8.

^{58}Mn: N. C. Dyer and J. H. Hamilton, Nucl. Phys. A173, 393 (1971).

					MASS EXCESS				BINDING ENERGY			
				EXPERIMENTAL	CALCULATED		EXP.-CALC.		1 NEUTRON		2 NEUTRON	
EL	A	Z	N		T	M	T	M	T	M	T	M
HE	6	2	4	17.60	17.60		.00					
	7	2	5	26.11	26.11		.00		-.44			
	8	2	6	31.57	31.57	31.57	.00	.00	2.61		2.17	
	9	2	7	U	42.61	43.49			-2.97	-3.85	-.36	
	10	2	8	U	51.00	52.34			-.32	-.78	-3.29	-4.62
	11	2	9		65.87	65.04			-6.79	-4.63	-7.11	-5.41
	12	2	10		76.11	74.39			-2.17	-1.28	-8.96	-5.91
LI	7	3	4	14.91	14.91		.00					
	8	3	5	20.95	20.95		.00		2.03			
	9	3	6	24.97	24.97	24.77	.00	.20	4.05		6.09	
	10	3	7	U	33.25				-.21		3.84	
	11	3	8	40.94	40.94	41.14	.00	-.20	.38		.17	-.23
	12	3	9	U	52.94				-3.93		-3.55	
	13	3	10		61.56	60.34			-.55		-4.48	-3.05
	14	3	11		72.28				-2.64		-3.19	
	15	3	12		81.59	79.78			-1.24		-3.88	-3.30
BE	8	4	4	(4.94)	6.75							
	9	4	5	11.35	11.35		.00		3.47			
	10	4	6	12.61	12.61	12.77	.00	-.16	6.81		10.29	
	11	4	7	20.18	20.19	20.29	-.01	-.12	.49	.55	7.30	
	12	4	8	25.03	25.02	24.75	.01	.28	3.25	3.62	3.74	4.17
	13	4	9	U	35.39	34.60			-2.31	-1.77	.94	1.84
	14	4	10	B	40.72	41.09*			2.74	1.58	.44	-.20
	15	4	11		51.21	51.11			-2.41	-1.95	.33	-.37
	16	4	12		59.25	57.68			.03	1.51	-2.38	-.45
	17	4	13		70.61	68.37			-3.29	-2.62	-3.26	-1.11
	18	4	14		77.72	75.43			.96	1.01	-2.33	-1.60
B	11	5	6	8.67	8.65	8.82	.01	-.16				
	12	5	7	13.37	13.37		.00		3.36			
	13	5	8	16.56	16.58	16.41	-.01	.16	4.87		8.22	8.56
	14	5	9	23.66	23.66		.00		.99		5.86	
	15	5	10	B	28.75	29.89			2.97		3.97	2.66
	16	5	11	U	37.97				-1.14		1.83	
	17	5	12	B	44.36	43.62			1.67		.53	2.41
	18	5	13		53.57				-1.13		.54	
	19	5	14		59.44	58.51			2.19		1.06	1.25
	20	5	15		68.12				-.60		1.59	
	21	5	16		76.04				.14		-.46	
C	12	6	6	0.	.01	-.12	-.01	.12				
	13	6	7	3.13	3.11	3.01	.01	.12	4.97	4.94		

				MASS EXCESS					BINDING ENERGY			
				EXPERIMENTAL	CALCULATED		EXP.-CALC.		1 NEUTRON		2 NEUTRON	
EL	A	Z	N		T	M	T	M	T	M	T	M
C	14	6	8	3.02	3.02	3.07	.00	-.05	8.16	8.01	13.14	12.96
	15	6	9	9.87	9.87	10.06	.00	-.18	1.22	1.08	9.38	9.09
	16	6	10	13.69	13.70	13.70	.00	.00	4.25	4.43	5.47	5.52
	17	6	11	B	21.27	20.86			.50	.90	4.75	5.34
	18	6	12	B	25.50	24.57			3.84	4.36	4.34	5.27
	19	6	13	B	33.47	32.41			.10	.24	3.94	4.60
	20	6	14		36.70	36.61			4.84	3.87	4.94	4.11
	21	6	15		45.04				-.27		4.57	
	22	6	16		50.75				2.36		2.09	
	23	6	17		61.51				-2.68		-.32	
	24	6	18		69.92				-.34		-3.02	
N	15	7	8	.10	.11	.29	.00	-.19				
	16	7	9	5.68	5.68		.00		2.49			
	17	7	10	7.87	7.87	7.68	.00	.19	5.89		8.38	8.76
	18	7	11	13.27	13.27		.00		2.67		8.55	
	19	7	12	B	16.27	15.32			5.07		7.74	8.51
	20	7	13	B	21.60				2.75		7.82	
	21	7	14	B	24.50	24.11			5.17		7.92	7.35
	22	7	15		30.62				1.95		7.12	
	23	7	16		36.27				2.43		4.38	
	24	7	17		45.60				-1.26		1.17	
	25	7	18		53.50				.17		-1.09	
	26	7	19		64.67				-3.10		-2.93	
	27	7	20		73.52				-.77		-3.87	
O	16	8	8	-4.74	-4.74	-4.74	.00	.00				
	17	8	9	-.81	-.80	-.99	.00	.18	4.14	4.32		
	18	8	10	-.78	-.78	-.60	.00	-.19	8.05	7.68	12.19	12.00
	19	8	11	3.33	3.39	3.33	-.06	.00	3.90	4.14	11.95	11.82
	20	8	12	3.80	3.74	3.80	.06	.00	7.72	7.60	11.62	11.75
	21	8	13	(9.30)	8.74	8.39+			3.08	3.48	10.80	11.09
	22	8	14	(11.50)	9.42	9.35+			7.39	7.11	10.47	10.59
	23	8	15	B	15.48	15.40			2.01	2.02	9.40	9.13
	24	8	16	B	19.70	19.44			3.85	4.04	5.87	6.06
	25	8	17		28.52	28.96			-.75	-1.45	3.10	2.59
	26	8	18		34.84	35.82			1.75	1.21	1.00	-.24
	27	8	19		45.12	47.20			-2.21	-3.30	-.45	-2.10
	28	8	20		52.30	55.54			.89	-.27	-1.32	-3.57
	29	8	21		64.96				-4.58		-3.70	
	30	8	22		73.78				-.75		-5.33	
F	19	9	10	-1.49	-1.54		.06					
	20	9	11	-.02	-.02		.00		6.55			
	21	9	12	-.05	.01		-.06		8.05		14.59	

EL	A	Z	N	MASS EXCESS EXPERIMENTAL	CALCULATED T	CALCULATED M	EXP.-CALC. T	EXP.-CALC. M	BINDING ENERGY 1 NEUTRON T	1 NEUTRON M	2 NEUTRON T	2 NEUTRON M
F	22	9	13	2.83	2.78		.04		5.30		13.34	
	23	9	14	3.36	3.40	3.36	-.04	.00	7.45		12.75	
	24	9	15	B	8.04				3.44		10.89	
	25	9	16	B	11.75	11.26			4.36		7.80	8.24
	26	9	17		18.99				.84		5.20	
	27	9	18		24.41	24.98			2.65		3.48	2.42
	28	9	19		33.03				-.55		2.10	
	29	9	20		39.79	42.04			1.32		.77	-.91
	30	9	21		50.43				-2.57		-1.25	
	31	9	22		58.71				-.21		-2.78	
	32	9	23		69.18				-2.39		-2.60	
	33	9	24		77.33				-.08		-2.47	
NE	20	10	10	-7.04	-6.99		-.06					
	21	10	11	-5.73	-5.79		.06		6.87			
	22	10	12	-8.02	-7.98		-.04		10.27		17.14	
	23	10	13	-5.15	-5.27		.12		5.36		15.63	
	24	10	14	-5.95	-6.08	-5.99	.13	.04	8.88		14.24	
	25	10	15	-2.16	-1.95	-2.12	-.21	-.04	3.95	4.20	12.82	
	26	10	16	B	.17	-.27			5.95	6.23	9.89	10.43
	27	10	17		6.52	6.58			1.73	1.21	7.68	7.44
	28	10	18		10.29	10.78			4.30	3.87	6.03	5.09
	29	10	19		18.48	19.49			-.12	-.64	4.18	3.23
	30	10	20		23.22	25.17			3.33	2.39	3.21	1.76
	31	10	21		33.32				-2.03		1.30	
	32	10	22		39.82				1.57		-.46	
	33	10	23		49.66				-1.77		-.20	
	34	10	24		56.16				1.58		-.19	
	35	10	25		66.13				-1.90		-.32	
	36	10	26		73.16				1.04		-.86	
NA	23	11	12	-9.53	-9.45		-.08					
	24	11	13	-8.42	-8.17		-.25		6.79			
	25	11	14	-9.36	-9.48	-9.26	.12	-.10	9.38		16.17	
	26	11	15	-6.90	-6.94		.04		5.53		14.92	
	27	11	16	-5.62	-5.71	-5.73	.08	.11	6.84		12.37	12.62
	28	11	17	-1.14	-1.02		-.12		3.38		10.22	
	29	11	18	2.65	2.32	2.66	.33	-.01	4.74		8.12	7.75
	30	11	19	8.37	8.50		-.13		1.89		6.63	
	31	11	20	(10.60)	12.70	14.38			3.87		5.76	4.42
	32	11	21	(16.40)	21.02				-.25		3.62	
	33	11	22	B	26.90				2.19		1.94	
	34	11	23		35.08				-.11		2.08	
	35	11	24		41.09				2.06		1.95	
	36	11	25		49.53				-.36		1.70	
	37	11	26		56.26				1.34		.98	

EL	A	Z	N	MASS EXCESS EXPERIMENTAL	CALCULATED T	CALCULATED M	EXP.-CALC. T	EXP.-CALC. M	BINDING ENERGY 1 NEUTRON T	1 NEUTRON M	2 NEUTRON T	2 NEUTRON M
NA	38	11	27		64.75				-.42		.92	
	39	11	28		71.33				1.49		1.07	
	40	11	29		81.07				-1.66		-.17	
	41	11	30		89.81				-.67		-2.34	
MG	24	12	12	-13.93	-14.05		.12					
	25	12	13	-13.19	-13.28		.08		7.30			
	26	12	14	-16.21	-16.18	-16.28	-.04	.07	10.97		18.27	
	27	12	15	-14.58	-14.53	-14.60	-.06	.01	6.43	6.39	17.40	
	28	12	16	-15.02	-14.95	-14.94	-.06	-.08	8.50	8.41	14.92	14.80
	29	12	17	-10.75	-10.70	-10.75	-.05	.00	3.82	3.88	12.31	12.29
	30	12	18	B	-9.37	-9.21			6.75	6.54	10.56	10.42
	31	12	19		-3.73	-3.17			2.43	2.03	9.18	8.56
	32	12	20		-1.31	-.15			5.65	5.06	8.08	7.09
	33	12	21		6.39				.37		6.02	
	34	12	22		10.61				3.85		4.22	
	35	12	23		18.31				.37		4.22	
	36	12	24		22.78				3.60		3.97	
	37	12	25		30.92				-.06		3.54	
	38	12	26		35.92				3.07		3.01	
	39	12	27		44.01				-.03		3.05	
	40	12	28		49.12				2.97		2.94	
	41	12	29		58.51				-1.32		1.64	
	42	12	30		65.82				.77		-.56	
AL	27	13	14	-17.20	-17.17	-17.26	-.03	.07				
	28	13	15	-16.85	-17.17		.33		8.08			
	29	13	16	-18.21	-18.03	-18.11	-.18	-.10	8.93		17.01	16.99
	30	13	17	-15.89	-15.79		-.10		5.83		14.76	
	31	13	18	-15.01	-15.00	-15.05	-.01	.04	7.28		13.11	13.08
	32	13	19	B	-11.14				4.21		11.50	
	33	13	20	B	-9.34	-8.65			6.27		10.49	9.75
	34	13	21		-3.30				2.03		8.30	
	35	13	22		.44				4.33		6.37	
	36	13	23		6.60				1.91		6.24	
	37	13	24		10.77				3.90		5.81	
	38	13	25		17.17				1.67		5.57	
	39	13	26		21.78				3.46		5.14	
	40	13	27		28.40				1.45		4.92	
	41	13	28		33.17				3.31		4.76	
	42	13	29		41.12				.12		3.43	
	43	13	30		48.12				1.07		1.19	
	44	13	31		56.96				-.76		.31	
	45	13	32		64.44				.59		-.17	
SI	28	14	14	-21.49	-21.35	-21.41	-.14	-.08				

				MASS EXCESS					BINDING ENERGY			
				EXPERIMENTAL	CALCULATED		EXP.-CALC.		1 NEUTRON		2 NEUTRON	
EL	A	Z	N		T	M	T	M	T	M	T	M
SI	29	14	15	-21.89	-21.79	-21.92	-.10	.03	8.51	8.58		
	30	14	16	-24.43	-24.66	-24.45	.23	.02	10.94	10.60	19.45	19.18
	31	14	17	-22.95	-22.95	-22.92	.01	-.03	6.37	6.54	17.31	17.15
	32	14	18	-24.09	-23.95	-24.05	-.14	-.04	9.07	9.20	15.43	15.75
	33	14	19	-20.57	-20.71	-20.67	.15	.10	4.84	4.69	13.90	13.89
	34	14	20	B	-20.57	-20.32			7.93	7.72	12.77	12.42
	35	14	21	B	-15.02				2.51		10.45	
	36	14	22	B	-12.81				5.87		8.38	
	37	14	23		-6.95				2.21		8.08	
	38	14	24		-4.51				5.63		7.84	
	39	14	25		1.50				2.06		7.69	
	40	14	26		4.63				4.94		7.00	
	41	14	27		10.91				1.79		6.73	
	42	14	28		14.23				4.75		6.54	
	43	14	29		21.88				.42		5.17	
	44	14	30		27.24				2.71		3.13	
	45	14	31		35.92				-.61		2.11	
	46	14	32		41.90				2.09		1.49	
	47	14	33		51.29				-1.32		.78	
	48	14	34		58.03				1.33		.01	
	49	14	35		68.59				-2.48		-1.15	
P	31	15	16	-24.44	-24.44	-24.56	.00	.12				
	32	15	17	-24.30	-24.52		.22		8.15			
	33	15	18	-26.34	-26.14	-26.35	-.20	.01	9.69		17.84	17.94
	34	15	19	-24.55	-24.56		.01		6.49		16.18	
	35	15	20	-24.94	-24.90	-24.81	-.03	-.13	8.42		14.91	14.61
	36	15	21	B	-20.88				4.05		12.46	
	37	15	22	B	-18.98				6.17		10.22	
	38	15	23	B	-14.85				3.94		10.11	
	39	15	24		-12.80				6.02		9.96	
	40	15	25		-8.27				3.54		9.56	
	41	15	26		-5.48				5.28		8.82	
	42	15	27		-.64				3.23		8.52	
	43	15	28		2.38				5.05		8.29	
	44	15	29		8.39				2.06		7.12	
	45	15	30		13.59				2.87		4.93	
	46	15	31		20.77				.90		3.76	
	47	15	32		26.53				2.31		3.21	
	48	15	33		34.70				-.10		2.21	
	49	15	34		41.39				1.38		1.29	
	50	15	35		51.03				-1.57		-.18	
	51	15	36		58.57				.53		-1.04	
S	32	16	16	-26.01	-25.94	-25.95	-.07	-.06				
	33	16	17	-26.59	-26.64	-26.62	.05	.03	8.77	8.73		

				MASS EXCESS					BINDING ENERGY			
				EXPERIMENTAL	CALCULATED		EXP.-CALC.		1 NEUTRON		2 NEUTRON	
EL	A	Z	N		T	M	T	M	T	M	T	M
S	34	16	18	-29.93	-29.92	-29.93	-.01	.01	11.35	11.39	20.12	20.12
	35	16	19	-28.85	-28.82	-28.74	-.02	-.10	6.98	6.88	18.33	18.27
	36	16	20	-30.67	-30.70	-30.59	.03	-.08	9.95	9.91	16.93	16.79
	37	16	21	-26.91	-26.98	-27.04	.07	.13	4.35	4.53	14.30	14.44
	38	16	22	-26.86	-26.81	-26.93	-.05	.07	7.90	7.96	12.25	12.49
	39	16	23	B	-23.07	-23.21			4.33	4.35	12.24	12.31
	40	16	24	B	-22.50	-22.64			7.50	7.51	11.83	11.85
	41	16	25		-18.31	-18.42			3.88	3.85	11.38	11.36
	42	16	26		-16.96	-17.05			6.73	6.70	10.61	10.55
	43	16	27		-12.43	-12.62			3.54	3.64	10.26	10.34
	44	16	28		-11.05	-10.81			6.70	6.27	10.23	9.91
	45	16	29		-5.20				2.22		8.91	
	46	16	30		-1.49				4.37		6.59	
	47	16	31		5.47				1.11		5.48	
	48	16	32		10.01				3.53		4.64	
	49	16	33		18.12				-.04		3.49	
	50	16	34		23.89				2.30		2.26	
	51	16	35		33.33				-1.36		.94	
	52	16	36		39.82				1.58		.21	
	53	16	37		49.86				-1.96		-.39	
	54	16	38		56.76				1.17		-.79	
CL	35	17	18	-29.01	-29.07		.06					
	36	17	19	-29.52	-29.51		-.01		8.51			
	37	17	20	-31.76	-31.69	-31.70	-.07	-.06	10.25		18.76	
	38	17	21	-29.80	-29.70		-.10		6.08		16.33	
	39	17	22	-29.80	-29.92	-29.86	.12	.06	8.29		14.38	14.30
	40	17	23	(-27.54)	-27.66				5.81		14.11	
	41	17	24	B	-27.43	-27.39			7.84		13.65	13.67
	42	17	25	B	-24.68				5.32		13.16	
	43	17	26		-23.64	-23.61			7.03		12.35	12.36
	44	17	27		-20.74				5.18		12.20	
	45	17	28		-19.53	-19.19			6.85		12.03	11.73
	46	17	29		-15.17				3.72		10.57	
	47	17	30		-11.69				4.59		8.31	
	48	17	31		-5.95				2.33		6.92	
	49	17	32		-1.46				3.58		5.92	
	50	17	33		5.74				.87		4.46	
	51	17	34		11.30				2.51		3.38	
	52	17	35		19.69				-.31		2.20	
	53	17	36		25.81				1.95		1.64	
	54	17	37		34.67				-.79		1.16	
	55	17	38		41.20				1.54		.76	
	56	17	39		50.84				-1.57		-.03	
	57	17	40		58.12				.79		-.78	

Appendix II

Predicted mass excesses and one- and two-proton binding energies (in MeV) of the $T_Z = -2$, $-5/2$, and -3 nuclei through the titanium isotopes. The method of Kelson and Garvey (Ref. 16) was employed. In similar fashion to their work, the letter in parentheses beside the nuclear species specifies the type of information available on the neutron-excess isotope used in the prediction, with D or X signifying a nuclide whose mass is known or unknown, respectively. For the latter, the mass predicted by the transverse mass equation (and listed in Table I) was assumed.

TZ = -2
****

NUC.	MASS EXCESS	BINDING ENERGY 1P	BINDING ENERGY 2P
6C (D)	35.77	-.15	-2.44
10N (X)	39.38	-3.21	-1.92
12O (D)	33.06	-.32	-2.82
14F (D)	33.37	-2.56	-1.22
16NE(D)	24.67	.23	-2.09
18NA(D)	25.57	-1.66	.20
20MG(D)	17.40	2.76	2.39
22AL(D)	17.95	.14	3.40
24SI(D)	10.75	3.28	3.41
26P (D)	11.19	-.11	3.31
28S (D)	4.46	2.28	3.00
30CL(D)	4.89	-.62	2.59
32AR(D)	-2.16	2.47	2.69
34K (D)	-1.41	-.65	2.71
36CA(D)	-6.45	2.59	2.64
38SC(D)	-4.67	-1.17	1.84
40TI(D)	-9.01	2.23	1.55

TZ = -5/2
****

NUC.	MASS EXCESS	BINDING ENERGY 1P	BINDING ENERGY 2P
17NA(X)	35.79	-3.63	-3.60
19MG(X)	32.63	.23	-1.43
21AL(X)	26.62	-1.93	.84
23SI(D)	23.29	1.95	2.08
25P (D)	19.48	-1.44	1.83
27S (D)	17.91	.57	.46
29CL(D)	14.31	-2.56	-.28
31AR(D)	11.73	.45	-.17
33K (D)	7.51	-2.38	.09
35CA(D)	4.73	1.16	.51
37SC(D)	3.81	-2.97	-.38
39TI(D)	2.19	.43	-.74

TZ = -3
****

NUC.	MASS EXCESS	BINDING ENERGY 1P	BINDING ENERGY 2P
18MG(X)	44.08	-1.00	-4.83
20AL(X)	42.23	-2.31	-2.08
22SI(X)	32.11	1.79	-.13
24P (X)	32.91	-2.33	-.38
26S (X)	27.25	-.48	-1.93
28CL(D)	27.98	-2.78	-2.21
30AR(X)	21.65	-.05	-2.61
32K (X)	21.75	-2.73	-2.23
34CA(X)	14.03	.77	-1.61
36SC(X)	15.26	-3.26	-2.10
38TI(D)	10.72	.38	-2.59

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720