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Integration of Gray System Theory with AHP Decision-Making for Wastewater Reuse Decision-Making

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Abstract: The existence of uncertainties in decision-making has given rise to multicriteria decision-making to achieve results that weigh in the effects of uncertainty. This study presents a method for selecting the best alternative for treated wastewater reuse based on gray system theory (GST). Alternatives and appropriate decision criteria for wastewater reuse are determined by experts. The criteria are weighed with the analytical hierarchy process (AHP) method, and the best wastewater-reuse alternative is then selected based on the evaluation based on distance from average solution for gray water (EDAS-G). Results of an example demonstrate wastewater reuse in the environmental sector receives the highest priority among several alternative uses. **DOI: 10.1061/(ASCE)HZ.2153-5515.0000619**. *© 2021 American Society of Civil Engineers*.

Author keywords: Gray system theory (GST); Wastewater reuse; Multicriteria decision-making; Average solution method; Hierarchical analysis process.

Introduction

Water scarcity threatens many world regions, in some cases exacerbated by climate change (Golfam et al. 2021; Ashofteh et al. 2020). Wastewater reuse is an effective strategy for augmenting the water supply in the agricultural, industrial, and municipal sectors (Keshavarz Ghorabaee et al. 2015). The complexities introduced by the type of wastewater, the choice of purification (treatment) method, the preferences of stakeholders and consumers, the place of wastewater reuse, and the acceptance of wastewater reuse render wastewater reuse a multicriteria decision-making problem.

Fatta and Kythreotou (2005) stated the advantages of wastewater reuse, including conservation of ecosystems, reducing and preventing pollution, and decreasing the environmental stress on surface water and groundwater resources. Jaber and Mohsen (2001) developed a decision-support system to assess and select the potential of nonconventional water supply in Jordan, including the desalination of brackish and seawater, treated wastewater, importation of water from boundaries, and harvesting of water. Results by AHP showed water desalination was the best alternative. Almasri and McNeill (2009) selected the best watershed in Palestine's West Bank for wastewater reuse, applying a sustainability approach and Geographic Information System (GIS). They applied the importance order of criteria (IOC) method to assess the results. Bottero et al. (2011) applied the AHP and the analytic network process (ANP) methods for prioritizing and selecting the sustainable wastewater treatment technology, considering aspects of environmental, factors of technological, and costs of economic for a cheese factory in the Italian–Swiss Alps region. The AHP model identified composting as the best alternative, while the simple ANP selected composting and phytoremediation as the best options. The complex ANP selected phytoremediation as the superior wastewater technology. Chen et al. (2012) investigated reuse alternatives for wastewater from washing machines in Sydney, Australia, using the preference ranking organization method for enrichment evaluation (PROMETHEE). Anane et al. (2012) presented a new methodology to rank appropriate sites for irrigation treated wastewater using fuzzy-AHP in an aquifer (Tunisia). The main criteria defined in five groups including irrigation land suitability, conflicts of resources, effectiveness of cost, acceptance of social, and impact of environmental. Kim et al. (2013) implemented the fuzzy-technique for order of preference by similarity (TOPSIS) to choose the best site for treated wastewater reuse. They identified basic criteria based on the framework of driver-pressure-state-impact-response (DPSIR), which that regarded technical, social, economic, and environmental criteria. Hocaoglu (2017) considered using treated wastewater for solving water supply in a tourist region. Two hotels in a Mediterranean region were assessed for water balance between water demands and wastewater resources. A decision tree was developed to optimize wastewater reuse. Arroyo et al. (2018) applied the choosing by advantage (CBA) approach to choose the best alternative among seven wastewater treatment technologies. Several researches have been reported on the ranking of the reusing treated wastewater alternatives using multicriteria decision-making methods. A few examples are: spatial data analysis using compromise programming (CoPr), and the Geographical Information System (GIS) to choose the best scenario of the delimitation of the irrigated area in northern Tunisia (Neji and Turki 2015); Hesitant Fuzzy Criteria Importance Through Inter-criteria Correlation (HF-CRITIC), and Hesitant Fuzzy Multi-Attribute Utility Theory (HF-MAUT) methods to select the best alternative for using reclaimed water in India (Narayanamoorthy et al. 2019); evidential reasoning (ER) to assess the sustainability of alternatives of wastewater reuse in the southern Tehran, Iran, (Akhoundi and Nazif 2018); the social choice-based method for reusing the treated wastewater in Tehran (Mahjouri and Pourmand 2017); application of the AHP-GIS in order to use the urban treated wastewater in the agricultural sector in the Golestan

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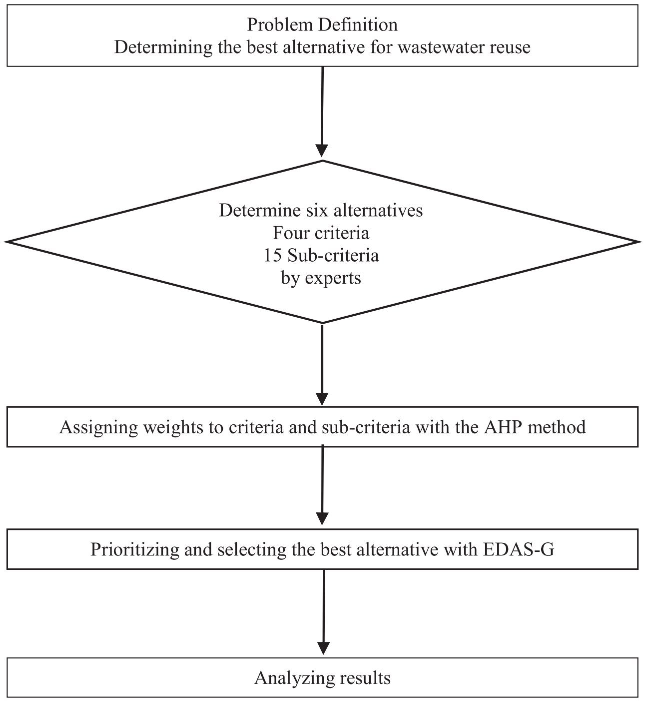

Fig. 1. Flowchart of methodology.

province (Iran) (Zolfaghary et al. 2021); the Elimination et Choice in Translating to Reality (ELECTRE) technique to select the optimal treated wastewater alternative in the eastern part of Tehran province (Ghorbani Mooselu et al. 2020); and application of AHP to evaluate the potential of the reclaimed water reuse in the agricultural irrigation sector in California (Paul et al. 2020).

This study extends the gray system theory (GST) to optimize wastewater reuse as a multicriteria decision-making problem beset by uncertainties. Specifically, this paper combines the GST with the EDAS multicriteria decision-making (MCDM) method to select the best alternative for treated wastewater allocation. The key feature of this method is employing the GST approach to deal with the uncertainties arising from imperfect data. The uncertainties that beset the MCDM methods include: uncertainty in decision-makers (DM) preference and knowledge, and model uncertainty (Mosadeghi et al. 2013). This paper deals with uncertainty stemming from DM imperfect knowledge about the performance of each alternative relative to each criteria. This uncertainty is overcome by applying gray numbers instead of crisp numbers to consider probability factors to cope with uncertainty. An example illustrates the capability of the proposed methodology.

Methods and Materials

This section presents a summary of GST, followed by a brief explanation of the EDAS method, and a full description of the EDAS-G method. The logic of this study's methodology is depicted in Fig. 1.

Foundations of Gray System Theory

Ju-Long (1982) introduced GST, which assigns color, varying from white to black, to information according to how explicit and transparent it is. Information about a phenomenon is assigned the color white if it is fully explicit and transparent (i.e., exact), it is assigned black if it is useless in content, and it is assigned gray if the information content is neither between exact or useless. Gray represents

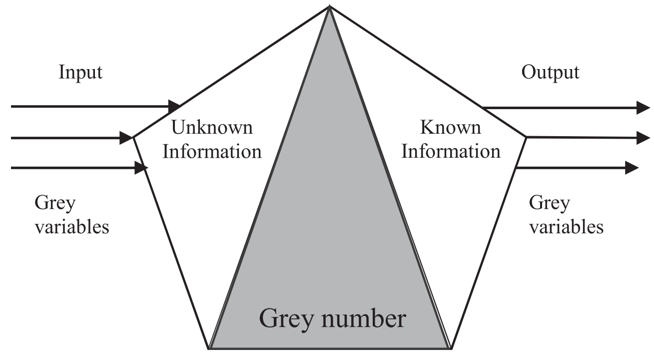

Fig. 2. Schematic of the gray system.

inadequate knowledge about a phenomenon. The gray number does not have an exact value, yet its upper and lower bounds are known. A schematic of gray theory is shown in Fig. 2.

Let $\otimes g$ denote an interval gray number. Its definite upper and lower borders are specified as a value g'' in the range defined by the lower and upper borders $\underline{g}$ and $\bar{g}$, respectively:

$$\otimes g = [\underline{g}, \bar{g}] = [g'' \in g | \underline{g} \leq g'' \leq \bar{g}] \quad (1)$$

The basic operator laws for gray numbers $\otimes g_1 = [\underline{g}_1, \bar{g}_1]$ and $\otimes g_2 = [\underline{g}_2, \bar{g}_2]$ are expressed as follows:

Summation:

$$\otimes g_1 + \otimes g_2 = [\underline{g}_1 + \underline{g}_2, \bar{g}_1 + \bar{g}_2] \quad (2)$$

Negation:

$$-\otimes g_2 = [-\bar{g}_2, -\underline{g}_2] \quad (3)$$

Subtraction:

$$\otimes g_1 - \otimes g_2 = [\underline{g}_1 - \bar{g}_2, \bar{g}_1 - \underline{g}_2] \quad (4)$$

Multiplication:

$$\otimes g_1 \times \otimes g_2 = [\min(\underline{g}_1\underline{g}_2, \underline{g}_1\bar{g}_2, \bar{g}_1\underline{g}_2, \bar{g}_1\bar{g}_2), \max(\underline{g}_1\underline{g}_2, \underline{g}_1\bar{g}_2, \bar{g}_1\underline{g}_2, \bar{g}_1\bar{g}_2)] \quad (5)$$

Inversion:

$$\otimes g_2^{-1} = \left[\frac{1}{\bar{g}_2}, \frac{1}{\underline{g}_2}\right] \quad (6)$$

Division:

$$\otimes g_1 \div \otimes g_2 = [\underline{g}_1, \bar{g}_1] \times \left[\frac{1}{\bar{g}_2}, \frac{1}{\underline{g}_2}\right] \quad (7)$$

Scaling:

$$K \otimes g_1 = K[\underline{g}_1, \bar{g}_1] = [K\underline{g}_1, K\bar{g}_2] \quad (8)$$

where g_1 and g_2 = lower borders of g_1 and g_2, respectively; and g_1 and g_2 = upper borders of g_1 and g_2, respectively.

The grade of grayness equals the distance between the upper and lower borders (i.e., $\bar{g} - \underline{g}$). By increasing the grade of grayness (by increasing the distance between the borders), the interval gray number tends to a black number. In contrast, by decreasing the grade of grayness (the distance between the bounds decreases), the interval gray number tends to a white number. In the event that the upper border equals the lower border, the interval gray number becomes a crisp number. The crisp number is defined as follows:

$$g_{(\zeta)} = (1-\zeta)\underline{g} + \zeta\bar{g} \tag{9}$$

where $g_{(\zeta)}$ = whitened value of an interval gray number; and ζ = whitening coefficient, $\zeta = [0, 1]$. Eq. (9) is as follows when $\zeta = 0.5$:

$$g_{(\zeta=0.5)} = \frac{1}{2}(\underline{g} + \bar{g}) \tag{10}$$

EDAS-GRAY (EDAS-G) Method

The EDAS was introduced by Keshavarz Ghorabaee et al. (2015) and is one the newest multicriteria decision-making methods. The positive distance from average (PDA) and the negative distance from average (NDA) are measured. The ranking of alternatives is done based on the maximum PDA values and minimum NDA values. The steps of the EDAS-G method are as follows:

Step 1. Calculation of the gray decision-making matrix:

$$\otimes G = \begin{bmatrix} [\underline{g}_{11}, \bar{g}_{11}] & [\underline{g}_{12}, \bar{g}_{12}] & \cdots & [\underline{g}_{1n}, \bar{g}_{1n}] \\ [\underline{g}_{21}, \bar{g}_{21}] & [\underline{g}_{22}, \bar{g}_{22}] & \cdots & [\underline{g}_{2n}, \bar{g}_{2n}] \\ \vdots & \vdots & \ddots & \vdots \\ [\underline{g}_{m1}, \bar{g}_{m1}] & [\underline{g}_{m2}, \bar{g}_{m2}] & \cdots & [\underline{g}_{mn}, \bar{g}_{mn}] \end{bmatrix} \tag{11}$$

where $\bar{g}_{mn}$ and g_{mn} = upper and lower border of assessment of alternative m based on criterion n.

Step 2. Calculate the gray average solution with respect to all decision criteria:

$$\otimes g = ([\underline{g}_1^*, \bar{g}_1^*], [\underline{g}_2^*, \bar{g}_2^*], \ldots, [\underline{g}_n^*, \bar{g}_n^*]) \tag{12}$$

where:

The average of minima is:

$$\underline{g}_i^* = \frac{\sum_{i=1}^m \underline{g}_{ij}}{m} \tag{13}$$

The average of maxima equals:

$$\bar{g}_i^* = \frac{\sum_{i=1}^m \bar{g}_{ij}}{m} \tag{14}$$

where $\underline{g}_i^*$ = average solution for lower borders; and $\bar{g}_i^*$ = average solution for upper borders.

Step 3. Calculate the PDA, $\otimes d_{ij}^* = [\underline{d}_{ij}^+, \bar{d}_{ij}^+]$ for benefit criteria, and the NDA, $\otimes d_{ij}^- = [\underline{d}_{ij}^-, \bar{d}_{ij}^-]$ for cost criteria. The following definitions apply [the PDA (NDA) value is given by the top (bottom) formula in each of the following equations]:

$$\underline{d}_{ij}^+ = \begin{cases} \dfrac{\max(0, (\underline{g}_{ij} - \bar{g}_j^*))}{0.5(\underline{g}_j^* + \bar{g}_j^*)} \\ \dfrac{\max(0, (\underline{g}_j^* - \bar{g}_{ij}))}{0.5(\underline{g}_j^* + \bar{g}_j^*)} \end{cases} \tag{15}$$

$$\bar{d}_{ij}^+ = \begin{cases} \dfrac{\max(0, (\bar{g}_{ij} - \underline{g}_j^*))}{0.5(\underline{g}_j^* + \bar{g}_j^*)} \\ \dfrac{\max(0, (\bar{g}_j^* - \underline{g}_{ij}))}{0.5(\underline{g}_j^* + \bar{g}_j^*)} \end{cases} \tag{16}$$

$$\underline{d}_{ij}^- = \begin{cases} \dfrac{\max(0, (\underline{g}_j^* - \bar{g}_{ij}))}{0.5(\underline{g}_j^* + \bar{g}_j^*)} \\ \dfrac{\max(0, (\underline{g}_{ij} - \bar{g}_j^*))}{0.5(\underline{g}_j^* + \bar{g}_j^*)} \end{cases} \tag{17}$$

$$\bar{d}_{ij}^- = \begin{cases} \dfrac{\max(0, (\bar{g}_j^* - \underline{g}_{ij}))}{0.5(\underline{g}_j^* + \bar{g}_j^*)} \\ \dfrac{\max(0, (\bar{g}_{ij} - \underline{g}_j^*))}{0.5(\underline{g}_j^* + \bar{g}_j^*)} \end{cases} \tag{18}$$

where $\underline{d}_{ij}^+$ and $\bar{d}_{ij}^+$ = lower and upper borders of gray PDA, respectively; and $\underline{d}_{ij}^-$ and $\bar{d}_{ij}^-$ = lower and upper borders of gray NDA, respectively.

Step 4. Calculate the weighted sum of the gray PDA and gray NDA as follows:

$$\underline{Q}_i^+ = \sum_{j=1}^n w_j \underline{d}_{ij}^+ \text{ (lower values of PDA)} \tag{19}$$

$$\bar{Q}_i^+ = \sum_{j=1}^n w_j \bar{d}_{ij}^+ \text{ (upper values of PDA)} \tag{20}$$

$$\underline{Q}_i^- = \sum_{j=1}^n w_j \underline{d}_{ij}^- \text{ (lower values of NDA)} \tag{21}$$

$$\bar{Q}_i^- = \sum_{j=1}^n w_j \bar{d}_{ij}^- \text{(upper values of NDA)} \tag{22}$$

where w_j = j-th weight; the operator $\otimes Q_i^+ = [\underline{Q}_i^+, \bar{Q}_i^+]$ = weighted sum of gray PDA; and the operator $\otimes Q_i^- = [\underline{Q}_i^-, \bar{Q}_i^-]$ = weighted sum of gray NDA.

Step 5. Compute the values normalized of the weighted sum of the gray PDA and gray NDA:

$$\underline{S}_i^+ = \frac{\underline{Q}_i^+}{\max_k \bar{Q}_k^+} \text{ (for lower sum of PDA)} \tag{23}$$

$$\bar{S}_i^+ = \frac{\bar{Q}_i^+}{\max_k \bar{Q}_k^+} \text{ (for upper sum of PDA)} \tag{24}$$

$$\underline{S}_i^- = 1 - \frac{\bar{Q}_i^+}{\max_k \bar{Q}_k^+} \text{ (for lower sum of NDA)} \tag{25}$$

$$\bar{S}_i^- = 1 - \frac{\underline{Q}_i^-}{\max_k \bar{Q}_k^+} \text{ (for upper sum of PDA)} \tag{26}$$

where the operator $\otimes S_i^+ = [\underline{S}_i^+, \bar{S}_i^+]$ = normalized weighted sum of the gray PDA; and the operator $\otimes S_i^+ = [\underline{S}_i^+, \bar{S}_i^+]$ = normalized weighted sum of the gray NDA.

Step 6. Compute the appraisal score for alternatives as follows:

$$S_i = \frac{1}{2}[(1-\alpha)(\underline{S}_i^- + \underline{S}_i^+) + \alpha(\bar{S}_i^- + \bar{S}_i^+)] \tag{27}$$

where S_i = appraisal score; and α = variable between 0 and 1, being 0, respectively, if experts assign maximum importance to the lower bound, and 1 if the experts assign maximum importance to the

Table 1. Weights of decision criteria and the consistency ratio (CR)

Criteria	Weight
Environmental (max)	0.043
Economic (min)	0.312
Technical (max)	0.542
Cultural–Social (max)	0.103
CR = 0.08	

Table 2. Weights of subcriteria corresponding to the environmental criteria and consistency ratio (CR)

Subcriteria	Weight
Effects on humans	0.039
Effects on vegetation	0.168
Effects on soil	0.296
Effects on water resources	0.096
Effects on natural ecosystems	0.401
CR = 0.06	

Table 3. Weights of subcriteria corresponding to the economic criteria and the consistency ratio (CR)

Subcriteria	Weight
Investment cost	0.098
Operation and maintenance cost	0.312
Energy cost	0.510
Revenue from wastewater reuse	0.081
CR = 0.06	

Table 4. Weights of subcriteria associated with the technical criteria and the consistency ratio (CR)

Subcriteria	Weight
Facilities and equipment	0.251
Feasibility	0.103
Ease of operation	0.488
Quality of reused wastewater relative to the type of consumption	0.157
CR = 0.05	

Table 5. Weights of subcriteria corresponding to cultural-social criteria and consistency ratio (CR)

Subcriteria	Weight
General acceptance	0.250
Wastewater reuse by customers	0.750
CR = 0.00001	

upper bound of the gray interval number. Eq. (27) takes the following form when $\alpha = 0.5$:

$$S_i = \frac{1}{4}(\underline{S}_i^+ + \bar{S}_i^+ + \underline{S}_i^- + \bar{S}_i^-) \tag{28}$$

Step 7. Prioritize the alternatives with regard to the appraisal score (the alternative with the highest appraisal score is the best alternative).

Definition of the Alternatives, Criteria, and Subcriteria

The definition the wastewater reuse alternatives, criteria, and subcriteria are based on facts, consistent with actual conditions. This study provides a methodology for selecting the best alternative for the use of wastewater. The alternatives are implementing water reuse for (1) the industrial sector, (2) artificial groundwater recharge, (3) agricultural irrigation, (4) nonagricultural irrigation, (5) meeting the environmental water demand, and (6) the recreational sector.

The first alternative, that is, the industrial sector, would reuse the treated wastewater for cooling systems, such as cooling towers or cooling pools. It would also be used for boiler water supply.

Table 6. Final weights of the subcriteria

Criteria	Subcriteria	Final weight
Environmental	Effects on humans	0.0017
	Effects on vegetation	0.0072
	Effects on soil	0.0127
	Effects on water resources	0.0041
	Effects on natural ecosystems	0.0172
Economic	Investment cost	0.306
	Operation and maintenance cost	0.0973
	Energy cost	0.1591
	Revenue from wastewater reuse	0.0253
Technical	Facilities and equipment	0.1360
	Feasibility	0.0558
	Ease of operation	0.2645
	Quality of reused wastewater relative to the type of consumption	0.0851
Cultural-Social	General acceptance	0.0258
	Wastewater reuse by consumers	0.0773

Table 7. Gray decision-making matrix

	Alternatives					
Criteria	First	Second	Third	Fourth	Fifth	Sixth
Environmental						
Humans	35	47	45	59	57	60
	68	80	55	94	85	65
Vegetation	45	52	27	56	61	70
	55	55	58	62	70	73
Soil	56	62	55	55	54	45
	75	74	60	70	66	54
Water resources	65	73	84	85	65	35
	70	75	90	90	75	38
Natural ecosystems	23	56	50	32	48	50
	35	60	60	36	50	55
Economic						
Investment cost	45	30	54	43	30	40
	55	40	67	56	40	45
Operation and maintenance cost	60	50	67	79	30	65
	65	55	69	90	35	68
Energy cost	80	66	58	44	30	50
	85	76	65	64	35	60
Revenue from wastewater reuse	30	75	47	50	70	55
	36	86	78	60	77	79
Technical						
Facilities and equipment	40	40	55	45	80	83
	50	50	58	55	83	85
Applicability	67	52	49	40	43	40
	97	58	58	81	54	54
Ease of operation	47	51	56	30	37	20
	50	55	68	39	45	30
Revenue from wastewater reuse	80	60	63	75	82	50
	90	67	73	77	86	60
Cultural-Social						
General acceptance	76	30	55	41	56	59
	80	35	65	49	76	65
Wastewater reuse by customers	45	40	30	69	51	90
	49	50	47	75	59	95

Table 8. Range of the average solution and the sum of lower and upper values

Criteria	The average solution	Sum
Environmental		
Humans	50.500	123.667
	73.167	
Vegetation	51.833	114
	62.167	
Soil	54.500	121
	66.500	
Water resources	67.833	140.833
	73	
Natural ecosystems	43.167	92.500
	49.333	
Economic		
Investment cost	40.333	90.833
	50.500	
Operation and maintenance cost	58.500	122.167
	63.667	
Energy cost	54.667	118.83
	64.167	
Revenue from wastewater reuse	54.500	123.833
	69.333	
Technical		
Facilities and equipment	57.167	120.667
	63.500	
Applicability	48.500	115.500
	67.000	
Ease of operation	40.167	88
	47.833	
Revenue from wastewater reuse	68.333	143.833
	75.500	
Cultural-Social		
General acceptance	52.833	114.500
	61.667	
Wastewater reuse by customers	54.167	116.667
	62.500	

Table 9. Positive gray distance from the average solution

	Alternatives					
Criteria	First	Second	Third	Fourth	Fifth	Sixth
Environmental						
Humans	0	0	0	0	0	0
	0.1536	0.4771	0.0728	0.7035	0.5580	0.2345
Vegetation	0	0	0	0	0	0.1374
	0.0556	0.0556	0.1082	0.1784	0.3187	0.3713
Soil	0	0	0	0	0	0
	0.3388	0.3223	0.0909	0.2562	0.1901	0
Water resources	0	0	0.1562	0.1704	0	0
	0.0308	0.1018	0.3148	0.3148	0.1018	0
Natural ecosystems	0	0.1441	0.0144	0	0	0.0144
	0	0.3640	0.3640	0	0.1477	0.2559
Economic						
Investment cost	0	0.0073	0	0	0.0073	0
	0.1211	0.4514	0	0.1651	0.4514	0.2312
Operation and	0	0.0573	0	0	0.3847	0
maintenance cost	0.0600	0.2273	0	0	0.5512	0
Energy cost	0	0	0	0	0.3310	0
	0	0	0.1038	0.3394	0.5750	0.2384
Revenue from	0	0.0915	0	0	0.0108	0
wastewater reuse	0	0.5087	0.3795	0.0888	0.3634	0.3957
Technical						
Facilities and	0	0	0	0	0.2735	0.3232
equipment	0	0	0.0138	0	0.4282	0.4613
Applicability	0	0	0	0	0	0
	0.8398	0.1645	0.1645	0.5628	0.0925	0.0925
Ease of operation	0	0.0720	0.1856	0	0	0
	0.2235	0.3371	0.6326	0	0.1098	0
Revenue from	0.0626	0	0	0	0.0904	0
wastewater reuse	0.0313	0	0.0649	0.1205	0.2475	0
Cultural–Social						
General acceptance	0.2504	0	0	0	0	0
	0.4745	0	0.2125	0	0.4047	0.2125
Wastewater reuse by	0	0	0	0.1140	0	0.4714
customers	0	0	0	0.3571	0.0829	0.7000

The second alternative, that is, the artificial groundwater recharge sector, would inject the treated wastewater in aquifers to reduce groundwater level drawdown caused by groundwater withdrawal. The third alternative, that is, the agricultural irrigation sector (as the largest freshwater consumer), would dedicate the treated wastewater to irrigation in the agricultural sector, which would save freshwater resources and serve as fertilizer. The fourth alternative, that is, the nonagricultural irrigation sector, would dedicate the treated wastewater to pastureland irrigation, forestry, and fodder crops irrigation. The fifth alternative, that is, supplying the environmental water demand, would dedicate the treated wastewater to aquaculture. The sixth alternative, that is, the recreational sector, would dedicate the treated wastewater to green-space irrigation in parks, golf courts, and water recreational centers, and filling artificial ponds and lakes.

The decision criteria account for environmental, economic, technical, and cultural–social factors. Several subcriteria are defined to cope with the complexity of wastewater reuse, and for better allocating appropriate scores by decision-makers for the decision criteria. The subcriteria corresponding to the environmental criteria are the effects of water reuse on humans, on vegetation, on soils, on water resources, and on natural ecosystems. The subcriteria associated with economic criteria are the investment cost, operation and maintenance cost, the energy costs (for wastewater treatment), and the revenue from wastewater reuse. The subcriteria corresponding to the technical criteria are facilities and equipment, applicability, ease of operation, and feasibility of water reuse based on water quality. The subcriteria associated with the cultural–social criteria are general acceptance and wastewater reuse by consumers.

Determination of the Criteria and Subcriteria Weights with the AHP Model

Saaty (1989) introduced the AHP model based on the four principles of: reciprocity, homogeneity, dependency, and expectations. The AHP model is a powerful algorithm for solving complex problems by mimicking human reasoning (a type of artificial intelligence algorithm when implement in computational software). The AHP determines the criteria weights by model by first defining the goal level, criteria level, and subcriteria level, followed by the aggregation of the decision-makers' opinions based on pairwise comparisons of relative importance of criteria. Relative importance is assigned as follows: 1 means criteria j and k are equally important, 3 means j is slightly more important than k, 5 means j is more important than k, 7 means j is strongly more important than k, and 9 means j is absolutely more important than k.

One of the characteristics of AHP model is calculation of the consistency ratio quantifying the reliability of the calculated weights. The consistency ratio is determined as follows:

$$\mathrm{CR} = \frac{\frac{\lambda_{\max} - n}{n - 1}}{\mathrm{RI}} \tag{29}$$

where CR = consistency ratio; $\lambda_{\max}$ = maximum eigenvalue; n = dimension of the gray matrix; and RI = random index. The random index is calculated as follows: if $n = 1$ then RI = 0; $n = 2$, RI = 0; $n = 3$, RI = 0.52; $n = 4$, RI = 0.9; and $n = 5$, RI = 1.12. Saaty (1996) recommended the CR should be equal or less than 0.1, otherwise if the CR exceeds 0.1 the pairwise comparison must be repeated. The proposed GST approach is exemplified with a case study in a region of Iran where wastewater reuse is considered as a strategy to reduce the stress on natural freshwater resources.

Results

Weights of Criteria Obtained with the AHP Model

The positive (i.e., benefit) and negative (i.e., cost) decision criteria are obtained and their weights are computed with the AHP model.

Table 10. Negative gray distance from the average solution (lower range, upper range)

	Alternatives					
Criteria	First	Second	Third	Fourth	Fifth	Sixth
Environmental						
Humans	0	0	0	0	0	0
	0.6173	0.4232	0.4555	0.2291	0.2615	0.2129
Vegetation	0	0	0	0	0	0
	0.3012	0.1784	0.6170	0.1082	0.0205	0
Soil	0	0	0	0	0	0.0083
	0.1736	0.0774	0.1901	0.1901	0.2066	0.3554
Water resources	0	0	0	0	0	0.4237
	0.1136	0	0	0	0.1136	0.5396
Natural ecosystems	0.1766	0	0	0.1556	0	0
	0.5694	0	0	0.3748	0.0288	0
Economic						
Investment cost	0	0	0.0771	0	0	0
	0.3229	0	0.5872	0.3450	0	0.1028
Operation and	0	0	0.0546	0.2510	0	0.0218
maintenance cost	0.1064	0	0.1719	0.5157	0	0.1555
Energy cost	0.2665	0.0309	0	0	0	0
	0.5105	0.3590	0.1739	0.1571	0	0.0898
Revenue from	0.2988	0	0	0	0	0
wastewater reuse	0.6353	0	0.3607	0.3122	0	0.2315
Technical						
Facilities and	0.1188	0.1188	0	0.0359	0	0
equipment	0.3895	0.3895	0.1409	0.3066	0	0
Applicability	0	0	0	0	0	0
	0	0.2597	0.3117	0.4675	0.4156	0.4675
Ease of operation	0	0	0	0.0265	0	0.2311
	0.0189	0	0	0.4053	0.2462	0.6326
Revenue from	0	0.0185	0	0	0	0.1159
wastewater reuse	0	0.2155	0.1738	0.0070	0	0.3546
Cultural–Social						
General acceptance	0	0.3115	0	0.0607	0	0
	0	0.5531	0.1164	0.3610	0.0990	0.0466
Wastewater reuse by	0.0886	0.0714	0.1229	0	0	0
customers	0.3000	0.3857	0.5571	0	0.1971	0

The benefits, costs, and weights are employed for selecting the best wastewater reuse alternative with the EDAS-G. The results concerning benefits, costs, and weight are presented in Table 1. The consistency ratio is less than 0.1, which reflects the degree of credibility of the experts' judgment. The calculated weights of subcriteria and their consistency ratio are listed in Tables 2–5. The weights of each set of subcriteria were applied to the pertinent criteria to take into account the weights of the subcriteria in the final ranking of the water-reuse alternatives. The calculated weights of the subcriteria corresponding to the decision criteria are presented in Table 6. The weights of the subcriteria are calculated with the EDAS-G method. From the point of views of decision-makers and experts the largest weight associated with the subcriteria corresponds to the ease of operation. The energy cost is the largest among the costs.

Results of the EDAS-G Method

Construction of the Gray Decision Matrix

The calculated gray decision matrix based on expert opinions is displayed in Table 7. The upper and lower member functions of the gray decision matrix are assigned for each criterion associated with each wastewater reuse alternative by the decision makers. Therefore, the gray accounts for the uncertainties in decision-making problems.

Determination of the Average Solution

The calculated average solutions for each criterion and their sum are listed in Table 8.

Calculation of the PDA and NDA

The calculated positive distances from the average solution corresponding to each decision criterion are listed in Table 9. The calculated negative distances from the average solution for each criterion are listed in Table 10.

Calculation of the Normalized Values

The calculated normalized values of the weighted sum of the gray PDA and gray NDA are listed in Table 11.

Table 12. Appraisal scores (S_i) and final ranking of water-reuse alternatives

Alternatives	S_i	Rank
First	0.387	5
Second	0.457	3
Third	0.556	2
Fourth	0.334	6
Fifth	0.756	1
Sixth	0.442	4

Table 11. Weighted and normalized weighted grey sums of positive and negative distances from the average

	$\otimes Q_i^+$		$\otimes Q_i^-$		$\otimes S_i^+$		$\otimes S_i^-$	
Alternatives	$\otimes \underline{Q}_i^+$	$\otimes \bar{Q}_i^+$	$\otimes \underline{Q}_i^-$	$\otimes \bar{Q}_i^-$	$\otimes \underline{S}_i^+$	$\otimes \bar{S}_i^+$	$\otimes \underline{S}_i^-$	$\otimes \bar{S}_i^-$
First	0.0118	0.1585	0.0760	0.2144	0.0384	0.5161	0.2570	0.7366
Second	0.0296	0.1588	0.0362	0.1900	0.0965	0.5171	0.3417	0.8745
Third	0.0500	0.2251	0.0172	0.1765	0.1628	0.7330	0.3882	0.9405
Fourth	0.0093	0.1376	0.0407	0.2886	0.0303	0.4480	0.0000	0.8589
Fifth	0.1355	0.3071	0.0000	0.1103	0.4412	1.0000	0.6178	1.0000
Sixth	0.0816	0.1901	0.0749	0.2703	0.2658	0.6192	0.0633	0.7403

Table 13. New gray decision-making matrix

Criteria	Alternatives					
	First	Second	Third	Fourth	Fifth	Sixth
Environmental						
Humans	35	47	45	59	57	60
	68	80	55	80	85	65
Vegetation	45	52	27	56	61	56
	55	55	58	62	70	73
Soil	56	62	45	55	54	45
	75	74	78	70	66	54
Water resources	35	73	84	85	65	35
	78	75	90	90	75	38
Natural ecosystems	23	56	50	22	48	50
	35	60	60	49	50	55
Economic						
Investment cost	45	30	54	43	25	40
	55	40	67	56	45	45
Operation and	60	39	67	79	60	65
maintenance cost	65	70	69	90	73	68
Energy cost	80	66	58	34	30	50
	85	76	65	57	35	60
Revenue from	30	75	50	50	70	55
wastewater reuse	36	86	85	60	77	79
Technical						
Facilities and	40	40	55	45	85	83
equipment	50	50	58	55	90	85
Applicability	76	52	49	40	43	40
	97	58	58	81	54	54
Ease of operation	47	51	56	30	37	10
	50	55	68	39	45	35
Revenue from	80	60	60	75	82	50
wastewater reuse	90	67	75	77	86	60
Cultural-Social						
General acceptance	76	30	55	41	50	59
	80	35	65	49	76	65
Wastewater reuse by	45	40	37	69	51	90
customers	49	50	45	75	69	95

Table 14. New appraisal scores (S_i) and final ranking of water-reuse alternatives

Alternatives	S_i	Rank
First	0.434	5
Second	0.504	3
Third	0.597	2
Fourth	0.419	6
Fifth	0.727	1
Sixth	0.439	4

Final Prioritization and Sensitivity Analysis

The ranking of treated wastewater reuse alternatives was calculated by setting $\zeta = 0.5$. Ranking results are listed in Table 12, where it is seen the fifth alternative, namely the allocation of wastewater reuse to the meet environmental flow demand, was best-ranked ranked (rank equal to 1), with the reuse of wastewater to agricultural irrigation being the second best-ranked (rank equal to 2), and reuse for artificial groundwater recharge, recreation, industrial sector, and green space irrigation ranked with third, fourth, fifth, and sixth priorities, respectively. The upper and lower bound of the gray decision-making matrix were changed to address the sensitivity analysis of the proposed model (Table 13). The sensitivity analyses showed that the Si of the alternatives was changed but the ranking of the alternatives was not altered. It means the priority of the alternatives was robust using GST approach (Table 14).

Results and Discussion

This work introduced a gray theory method to select the best alternative for wastewater reuse considering multiple decision criteria. The alternatives for wastewater reuse and the criteria and subcriteria employed to rank them were identified. The AHP method was employed to aggregate experts' opinions and calculate the weights of criteria and subcriteria.

Gray theory is useful in reducing the uncertainty in multicriteria decision-making. This work employed a combination of EDAS with gray theory (EDAS-G) for selecting the best alternative for water reuse. The uncertainties in this instance include quantifying and determining the superiority of some criteria over other criteria. For instance, public acceptance of wastewater reuse in a community is difficult to assess because to the difficulty of questioning all community residents or conducting representative surveys. Gray theory takes into account the effect of uncertainty in decision making by evaluating the criteria for each alternative by means of upper and lower membership functions for each decision criterion. This avoids, and is superior to, the subjective practice of assigning arbitrary weight to decision criteria in ranking management alternatives. Combining the AHP and EDAS-G methods and their application to multicriteria decision problems is effective in ranking mutually exclusive management alternative. The APH/EDAS-G methodology was herein applied to select the best alternative for wastewater reuse in an Iranian region, and in ranking all the alternatives from best to worst.

Data Availability Statement

All data, models, and code generated or used during the study appear in the published article.

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