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INVESTMENT IRREVERSIBILITY AND PRECAUTIONARY SAVINGS IN
GENERAL EQUILIBRIUM

BY

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Investment Irreversibility and Precautionary Savings in General Equilibrium

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Abstract

Partial equilibrium models suggest that when uncertainty increases, agents increase savings and at the same time reduce investment in irreversible goods. This paper characterizes this problem in general equilibrium with technology shocks, additive output shocks and shocks to the marginal efficiency of investment. Uncertainty is associated with the variance of these random variables, and irreversibility is introduced by a non negativity constraint on investment. I find that irreversibility and changes in uncertainty can be responsible for sizeable movements in aggregate consumption and investment only if the shocks affect the marginal efficiency of investment. For all types of shocks, when concavity of the utility function is moderate or high, the irreversibility constraint never binds and the increase in variance has a negligible impact. Persistence in the shock process induces precautionary savings rather than irreversibility effects. If shocks are idiosyncratic and affect a cross section of agents over capital, an increase in their variance may induce an increase in aggregate investment even if all agents have an incentive to invest less, because zero investment is now an active lower bound for part of the cross section distribution.

1 Introduction

Uncertainty is an important element in the economic decisions of agents. The precautionary savings literature, going back to Leland (1968), indicates that in

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partial equilibrium agents increase their savings rate when uncertainty about future income increases, given a positive third derivative of the utility function.¹ The irreversible investment literature, notably Bernanke (1983) and Dixit and Pindyck (1994), suggests agents delay lumpy irreversible investments when the variance of future returns increases.² These two ideas have opposite implications for aggregate investment in a closed economy general equilibrium model. This paper uses a dynamic stochastic general equilibrium model to characterize the impact of an increase in variance on aggregate investment subject to irreversibility.

Uncertainty is introduced in several forms: technology shocks (multiplicative shocks to output), additive shocks to output, and shocks to the marginal efficiency of investment (shocks to the relative price between consumption and investment, hereafter investment shocks). An increase in uncertainty is introduced through a mean preserving spread in the shock distributions. The response of savings to an increase in variance changes with the type of shock considered, as implied by the different results of Leland, who uses additive shocks to income, and of Levhari and Srinivasan (1969), who use random interest rates in partial equilibrium.³ As we will see, investment tends to respond more to increases in the variance of interest rates than to increases in the variance of output. This response is dependent on the characteristics of the utility function, another long standing issue investigated here. Intuitively, because concavity induces consumption smoothing, it directly counters the impact of investment shocks since they induce a negative correlation between consumption and investment.

Irreversibility is introduced through a non-negativity constraint on investment. A deterministic version of the problem is studied by Arrow and Kurz (1970). More recently Christiano and Fischer (1994) use the stochastic model with technology shocks to develop numerical solution methods for models with constraints. The investment literature uses partial equilibrium S,s models to study uncertainty and irreversibility. Fixed costs are a pervasive feature in these models. We will see that incorporating fixed costs induces large responses of consumption and investment to an increase in variance, whereas with only the non negativity constraint the response is much smaller and often negligible.

Several related papers look at irreversibility with multiple capital goods. Faig (1997) generalizes Abel and Eberly's (1995) partial equilibrium exercise.⁴

¹A good modern reference is Carrol (1997) and the literature reviewed therein. From the Euler equation $1 = R\beta E(C_{t+1}/C_t)^{-\rho}$, with log normal income shocks we derive $E\Delta \log(C_{t+1}) = (\rho/2)var(\Delta \log(C_{t+1})) + \text{constant}$, implying savings increase with a higher variance of consumption growth.

²Gorton and Pennacchi (1990) have a related model, where agents adjust their portfolios towards more liquid assets when uncertainty about the date of death increases.

³Technology and additive shocks both move consumption and investment in the same direction. Investment shocks move consumption and investment in opposite directions.

⁴Their insight is that the irreversibility constraint raises the user cost of capital and reduces investment, but at the same time it implies that over the cycle the constraint may bind and thus raise investment. In the long run the overall impact is ambiguous. Faig's contribution is

Veracierto (1997) looks at business cycle statistics for models with and without irreversibility under idiosyncratic shocks and concludes that reallocation and irreversibility are not significant issues over the cycle. Coleman (1997) looks at a two sector model with negatively correlated idiosyncratic shocks and reaches the opposite conclusion.⁵ In Ejarque (1997), different degrees of irreversibility for different capital goods make uncertainty matter over the cycle both for total and relative investment expenditures.⁶ The present paper provides a wider treatment of the impact of preferences, stochastic processes, types of shocks and types of irreversibility than the above literature and provides a benchmark for understanding the results obtained by the previous authors.

Section II introduces the basic model where the capital stock is the irreversible good. For the irreversibility constraint to bind, low concavity of the utility function, large potential shocks, and low depreciation of capital are necessary. Technology shocks and additive shocks to output generate little or no movement in consumption and investment in response to an increase in variance. However, high variance of investment shocks may have large effects on investment expenditure if utility is not too concave.⁷ Finally, persistence in the shock process has mainly precautionary savings effects rather than contributing to the effect of irreversibility.

Section III follows Cooper, Haltiwanger and Power (1995) and introduces a cross section distribution over capital by adding idiosyncratic shocks to the model. Intuitively, if the cross section distribution is skewed towards one tail, many agents may face the irreversibility constraint, and therefore increased uncertainty may have a big impact on aggregate investment. Two experiments are performed with investment shocks. Increasing the variance of idiosyncratic shocks I find that aggregate investment increases, even if every agent wants to invest less. This occurs because zero investment is an active lower bound for some agents. Increasing the variance of aggregate shocks decreases aggregate investment although generally by a small amount.

Section IV concludes the paper. The model suggests that increased uncertainty and investment irreversibility can be responsible for large movements in real variables only if uncertainty is associated with the slope of the intertemporal budget constraint rather than with its level. The model with heterogeneous agents also suggests that aggregate investment is unlikely to fall significantly with a mean preserving spread.

a general equilibrium characterization of these effects with multiple capital goods.

⁵Coleman's and Veracierto's models are essentially the same. The difference lies in the nature of the stochastic processes and the size of shocks used by the authors.

⁶That paper looks at the Great Depression, introducing large shocks and large increases in variance, which help to obtain an impact of uncertainty and irreversibility.

⁷With additive shocks, 0.5% is an upper bound on investment changes in response to extreme increases in variance over large shocks. With technology shocks 5% is a reasonable value. In the case of investment shocks these effects easily exceed 50% of investment.

2 Model

Consider a standard closed economy, representative agent, dynamic model without government. Production in this economy is divided between consumption and investment. Technology is Cobb-Douglas with capital share equal to α , and is subject to additive (z) and multiplicative (A) shocks:

$$Y = C + I = F(k, A, z) = z + Ak^\alpha$$

Labor supply is assumed to be inelastic and normalized to 1, imposing the burden of adjustment to shocks on the division of output between consumption and investment.⁸ The capital stock depreciates at the rate δ and has the law of motion:

$$k' = (1 - \delta)k + qI$$

where q is a random variable.⁹ Investment will have a non negativity constraint with an associated Lagrange multiplier $\lambda : I \geq 0$ or $k' \geq (1 - \delta)k$. This is the irreversibility constraint. It implies that agents cannot transform capital goods into consumption goods once they have created them through investment. The only way capital goods can disappear is through depreciation.¹⁰

Technology shocks (A), additive shocks to output (z) and investment shocks (q) are the sources of fluctuations in the model and follow Markov processes with transition probability matrices π_A , π_z and π_q . The variance of these processes is also a random variable which follows a two state Markov process with matrix π_v . This translates into different matrices for low and high variance states, for example π_{A_L} and π_{A_H} .

Preferences are CRRA: $u(c) = \frac{1}{1-\rho}C^{1-\rho}$ or $u(c) = \log(c)$ if $\rho = 1$. Consumption must always be positive and carries a Lagrange multiplier λ_c . The agent maximizes the discounted sum of utility flows with discount factor β . The dynamic programming problem, where $V = V(k, A, z, q, \sigma_A, \sigma_q, \sigma_z)$ is the value of entering the current period with capital stock k , and facing technology A , additive shock z , investment shock q , and their respective volatilities is:

$$V = \max_{k'} u \left(z + Af(k) - \frac{k' - (1 - \delta)k}{q} \right) + \beta EV'$$

subject to $C > 0$ and $I \geq 0$, where $V' = V(k', A', z', q', \sigma'_A, \sigma'_q, \sigma'_z)$. The Euler equation for this problem is:

$$u_c - (\lambda - \lambda_c) = q\beta E \left\{ u_{c'} \left[\frac{1 - \delta}{q'} + A' f_k(k') \right] - (1 - \delta) \frac{\lambda' - \lambda'_c}{q'} \right\}$$

⁸This assumption is discussed in the conclusion.

⁹These investment shocks encompass a wide range of models, from financial intermediation as in Bernanke and Gertler (1988) and Cooper and Ejarque (1995), to capital embodied technological change as in Greenwood, Hercowitz and Krusell (1995), or simply shocks to the cost of capital.

¹⁰A less extreme version of this constraint is the relation $k' = (1 - \delta)k + q\eta I$, with $\eta < 1$ if $I < 0$, and $\eta = 1$ if $I \geq 0$. Abel and Eberly (1995) report that estimated policy functions for investment at the firm level are close to the case of complete irreversibility.

and the Lagrange multipliers are either positive or zero according to whether the respective constraints bind.

To understand this first order condition, suppose the agent wants to invest one less unit into capital. He gains marginal utility in consumption today but loses λ if he meets the current constraint on non negative investment. Tomorrow he loses the value of the marginal product of capital in marginal utility, but this loss is not as big overall since the constraint on nonnegative investment tomorrow is relaxed, given *any* k'' he may want to achieve. By exercising the current option of waiting, the agent relaxes the irreversibility constraint tomorrow.

Also, introducing the constraint in the stochastic problem does not increase investment relative to the unconstrained case if the constraint is not currently binding. The irreversibility constraint is a cost associated with excessive levels of capital which can only be avoided by reducing investment. As is clear from the Euler equation, the probability of a binding irreversibility constraint and its associated Lagrange multiplier reduce the future return on capital, and so the marginal product must be higher to compensate. This is the higher rental cost or user cost effect described in Abel and Eberly (1995) and in Faig (1997). Obviously, a currently binding constraint implies zero investment which is more than the desired amount. That is the hangover effect also examined by the same authors.

2.1 Policy Functions

The solution to the model is a policy function $k'(k, A, z, q, \sigma_A, \sigma_q, \sigma_z)$, where the arguments are the state variables of the system.¹¹ Figure 1 shows the optimal decision for this problem under *certainty*. In case the agent holds too much capital today (K_0) relative to the steady state (K_1), he may be limited (distance [a b]) by the irreversibility constraint, $k' = (1 - \delta)k$. Since this deterministic problem has a unique steady state where the constraint never binds, near that point the decision with or without the constraint is the same.¹²

Figure 2 illustrates the impact of increasing concavity in the utility function. If utility is linear the policy function will be horizontal over the range where the constraints are not binding. If utility is concave it will be upward sloping.¹³ The policy function rotates upward and moves closer to the diagonal when concavity increases. In the interval [b1,b2] the constraint is relaxed. This occurs because high concavity of utility implies that, when current capital is high,

¹¹The model is solved by numerical value function iteration on a grid for the capital stock, and imposes a large penalty on any decision that has either negative consumption or investment. Christiano and Fischer (1994) study this model with technology shocks and a constant variance and their algorithm solves for the Lagrange multiplier.

¹²A tempting corollary is that irreversibility only matters under uncertainty. However, the constraint matters in a deterministic problem with a perfectly anticipated future shock.

¹³Different values of ρ can be interpreted as reflecting a necessity or luxury characteristic. A high degree of concavity, because it implies the agent strongly dislikes fluctuations in consumption, indicates a necessity.

consumption is high and the marginal utility cost of investment is low, and vice versa. Therefore, relative to the linear utility case, the agent will invest more under high current capital and less under low current capital.

Figure 3 shows a stochastic problem with two states, where the random variable can be either A , z or q , and the variance is held constant. If the agent is at the "high steady state" - defined as the point where the high policy function crosses the diagonal - and is hit with a bad shock, he wants to reduce his capital stock. If the depreciation rate is very low, he cannot rely on it to reduce the capital stock enough, and so the zero investment constraint binds. Therefore, low depreciation increases the likelihood of a binding constraint, and so do large shocks (large distance between policy functions). The capital stock lives inside a closed interval - an "active range" - defined by the crossing of the lowest (low state) and highest (high state) policy functions with the diagonal.

There is another important effect of concavity described in figures 4 and 5. For additive and technology shocks, linear utility and i.i.d. shocks imply the current state is irrelevant and so all policy functions cross the diagonal horizontally in the same place. Concavity will then *separate* the policy functions from each other. Nevertheless, the irreversibility constraint is at best marginally binding since just when the policy functions start to separate they also rotate towards the diagonal. For investment shocks the current state affects the policy function and thus they do not coincide as they cross the diagonal. The policy functions respond to an increase in concavity by moving *towards* each other, narrowing the active capital range. The irreversibility constraint is binding under linear utility and implies large effects of the increase in variance. However, the movement of the policy functions towards the inside as they rotate makes the irreversibility constraint irrelevant very quickly. These different effects of concavity are a result of the different expenditure switching effects of the different shocks.¹⁴

2.2 Mean preserving spread experiment

The goal is to know whether changes in uncertainty alone substantially change investment decisions. To that purpose I start by looking at i.i.d. shocks. A mean preserving spread is imposed keeping the support of the distribution constant,

¹⁴These are descriptions of simulation results. The intuition is the following: A low value of q' implies high consumption and low marginal utility tomorrow. In the high current state, the occurrence of a low q' changes more expected marginal utility the more convex it is. The relative weight of a state of high consumption tomorrow is higher and so current investment falls. The policy function contracts towards the inside. Here, concavity (consumption smoothing) works directly against the shock on q' (the relative price between consumption and investment), making the irreversibility constraint less binding.

On the other hand, a low value of A' tomorrow implies consumption will be low. Thus, at the current high A state, we increase the weight of a state of low consumption tomorrow in the expectation of marginal utility we are forming now, due to the increase in concavity. Thus we invest more and the policy function shifts outwards with concavity. Here, an increase in concavity makes consumption smoothing more desirable and that forces investment to absorb income fluctuations and hit the constraint.

holding all conditional expectations constant and high and low conditional variances equal across states. In this way, only the change in variance drives the results.

The random variables A and q both take values in the set $\{0.8, 1, 1.2\} = \{A_1, A_2, A_3\} = \{q_1, q_2, q_3\}$. Both have unconditional expectation equal to 1. Additive shocks are defined to have the same impact on output as technology shocks.¹⁵ In the numerical implementation only one type of shock is considered at a time.

The following are the Markov matrices used in this experiment:

$$\pi_h = \begin{bmatrix} 0.49 & 0.02 & 0.49 \\ 0.49 & 0.02 & 0.49 \\ 0.49 & 0.02 & 0.49 \end{bmatrix} \quad \pi_L = \begin{bmatrix} 0.01 & 0.98 & 0.01 \\ 0.01 & 0.98 & 0.01 \\ 0.01 & 0.98 & 0.01 \end{bmatrix} \quad \pi_v = \begin{bmatrix} 0.98 & 0.02 \\ 0.02 & 0.98 \end{bmatrix}$$

π_h is the transition matrix for A in the high variance case and π_L is the transition matrix for A in the low variance case. π_v is the matrix governing the transition between π_h and π_L .¹⁶

To put some perspective on the results to come, for i.i.d. shocks the conditional variance increases by a factor of 49 from the low to the high variance state, and the high variance Markov process described above delivers a standard deviation of output of around 20% in the technology/additive shocks case and of around 5% in the investment shocks case.¹⁷

I also want to look at the impact of persistence in this experiment and discretize an AR(1) process into a Markov matrix. For example with technology shocks, $A_t = a + bA_{t-1} + \lambda\varepsilon_t$, with λ as the mean preserving spread factor.¹⁸ The transition between the two AR(1) processes, high and low variance, follows

¹⁵Therefore $z \in \{ -0.2(\bar{k})^\alpha, \quad 0, \quad 0.2(\bar{k})^\alpha \}$, where $\bar{k}$ is the deterministic steady state value of the capital stock. z has unconditional expectation equal to zero.

¹⁶As an example, for technology shocks, $\pi_h(2, 1) = 0.49$ is the probability that $A_{t+1} = A_1$ given $A_t = A_2$. If the agent is currently in state A_2 with high variance, the dynamic programming problem is: $V(A_2, k, \sigma_h) = \max \{u(c) + \beta\pi_h(2, :) [.98V(A', k', \sigma_h) + .02V(A', k', \sigma_L)]\}$ where $\pi_h(2, :)$ indicates the expectation according to the second row of the matrix π_h , $\pi_h(2, :) = E_{A'|\sigma_h, A}$.

¹⁷These economies have very different business cycle properties as investment shocks produce negative correlations between consumption and investment. Nevertheless, underlying the cycle can be a technology shock as well, even if that is not the variable incurring a mean preserving spread, and so we may get all the desired correlations.

¹⁸I consider the value zero for the unconditional mean a , the values 0.05 and 0.95 for the persistence parameter b , and an increase in the conditional standard deviation by a factor of 20, from 0.1 to 2. The support of the distribution had eleven elements evenly spaced from 0.75 to 1.25. The support has to be evenly spaced exactly because of the mean preserving spread and its effect on the conditional densities. In light of the algorithm used, the discretization of this process is not done by the usual quadrature methods since the support of the random variables must remain the same. A caveat of all discretization methods to obtain a Markov matrix from such a continuous process is that an attempt at a mean preserving spread is always polluted by slight changes in the conditional means at the edges of the state space. This turned out not to have an important effect.

the same Markov process described above. Persistence in the AR(1) process is to be distinguished from persistence in the Markov process.

The experiment consists of solving the model and evaluating the difference between the policy functions $k_{t+1}(S_t, k_t, \sigma_h)$ and $k_{t+1}(S_t, k_t, \sigma_L)$, where k_t and S_t are the current states, σ_h and σ_L are indexes of the variance for next period's distribution, and S_t may be a technology, additive or an investment shock.¹⁹ Figure 6 provides a general description of this difference for an increase in the variance of i.i.d. shocks under concave utility. Low variance policy functions are displayed with solid lines, and high variance policy functions with dotted lines. K_L , K_m and K_h represent the low, middle and high stochastic steady states - defined as the point where the respective policy function crosses the diagonal. K_c is the point where the irreversibility constraint first starts to bind. The effects of high variance are reversed if the shock process is an AR(1) with persistence, as illustrated in figure 6.a. Persistence has precautionary savings effects and goes against irreversibility.

2.3 Simulation Results²⁰

It is convenient to start with the linear utility case and then look at concave utility for the different random variables. Linear utility provides a link to section III where heterogeneity is introduced and is somewhat closer to the firm's problem in a decentralized general equilibrium model. The discussion will center around i.i.d. shocks since persistence in the shock process will generate only precautionary savings effects and will not add to the impact of irreversibility.

2.3.1 Additive Output Shocks

Leland (1968) shows that positive precautionary savings arise in a partial equilibrium problem with income shocks.^{21,22} Output here is $y = z + f(k)$. $E(z) =$

¹⁹The numbers mentioned below are averages of sections of the policy functions. Each policy function is divided into 4 segments and the impact of the increase in variance is averaged over all points in each segment and measured in percentage relative to the deterministic steady state capital stock: $(k'_h - k'_L)/k^*$. The exercise in this paper is mainly qualitative in the sense of suggesting the magnitudes of the effect of a change in variance in the different cases.

²⁰The parameters used in the simulations follow loosely the Real Business Cycle literature and have as benchmark: $\alpha = 0.35$, $\beta = 0.95$, $\delta = 0.1$, with ρ being a treatment parameter.

²¹Leland shows in a two period partial equilibrium problem that with a positive third derivative of utility ($\rho > 0$) positive precautionary savings arise, locally increasing with increases in the variance of the stochastic process. For Dr  ze and Modigliani (1972), in the absence of asset and insurance markets (or, as here, in the presence of aggregate undiversifiable risks) savings increase (decrease) relative to the deterministic case if $\rho > (<) 0$.

²²Consider a two period problem where an individual must decide how much to save (θ), given random income in period 2. $c_1 = (1 - \theta)I_1$, $c_2 = I_2 + (1 + r(\theta))\theta I_1$. $I_2 \sim D(\mu, \sigma^2)$. $U = U(c_1, c_2)$. The deterministic Euler equation is $U_1 = (1 + r + \theta \frac{\partial r}{\partial \theta})U_2$. A Taylor approximation delivers $E(U_1) - (1 + r + \theta \frac{\partial r}{\partial \theta})E(U_2) \approx (\sigma^2/2)[U_{122} - (1 + r + \theta \frac{\partial r}{\partial \theta})U_{222}]$. $U_{222} < 0$ implies positive precautionary savings. $r(\theta)$ is a proxy for a general equilibrium effect. If $\theta \frac{\partial r}{\partial \theta}$ is not

0, and in the i.i.d. case $E(z'|z) = 0$, for any z . The Euler equation with linear utility is:

$$1 - \lambda + \lambda_c = \beta \{f_k(k') + (1 - \delta) - (1 - \delta)E(\lambda' - \lambda'_c)\}$$

The current value of z has no impact on this first order condition unless one of the *current* constraints is binding. With i.i.d. shocks all policy functions $k'(k)$ cross the diagonal at the same point, independently of (σ, z) . The constraints never bind in the neighborhood of this point. The mean preserving spread and the introduction of the irreversibility constraint cannot affect the optimal decision as long as shocks do not come close to totally destroying output. With $\rho < 0.5$ and $\delta > 0.075$ the simulations show virtually no impact of the increase in variance or of the introduction of the irreversibility constraint. The variance of additive i.i.d. shocks can only matter if utility is concave. With $\rho > 0$ we have $u_c - \lambda = \beta [(1 - \delta) + f_k(k')] E(u_{c'}) - (1 - \delta)\beta E(\lambda')$. Here there was virtually no effect of the increase in variance for at least $1 < \rho < 2$ and $0.075 < \rho < 0.1$. So, for additive i.i.d shocks, neither irreversibility nor precautionary savings are relevant.

2.3.2 Technology Shocks

The Euler equation with linear utility and positive consumption is:

$$1 - \lambda + \lambda_c = \beta \{E(A')f_k(k') + (1 - \delta) - (1 - \delta)E(\lambda' - \lambda'_c)\}.$$

Again, i.i.d. shocks imply that all policy functions $k'(k)$ cross the diagonal at the same point. The mean preserving spread and the irreversibility constraint do not affect the problem once the agent is in this region.²³ Also, even at moderate levels of concavity the policy functions are close to the diagonal and the irreversibility constraint does not bind. In the simulations for roughly $\delta > 0.075$ and $\rho > 1.4$ the irreversibility constraint is never binding over the active capital range.

i.i.d. shocks. For $0 < \rho < 2$ and $\delta > 0.075$, in the low state with low capital, investment increased less than 1% with temporary high variance and less than 2% if the variance states are very permanent. With the irreversibility constraint the maximum negative impact on investment of the increase in variance was around 4% with high capital and in the high state of nature with persistent variance, and 2% with temporary variance.²⁴

The idea that a temporary increase in uncertainty carries a higher premium for waiting does not find support in this framework because capital has an insurance component. When it is low it insures the agent against the irreversibility

too big Leland's result *should* hold.

²³Even if $E(A'|A) \neq E(A')$, the variance only matters through the Lagrange multipliers.

²⁴With $\delta = 0.1$, the non negativity constraint never binds for any ρ . The shock must take away more than 20% of output in order for this constraint to bind.

constraint and when it is high it insures him against low consumption. This insurance value is higher the more persistent the high variance state.

Persistence is introduced in the form of the AR(1) process discussed above with the following model specification: $\alpha = 0.35$, $\beta = 0.95$, $\delta = 0.075$, $\rho = 0.5$, $a = 0$, and $b = 0.05$, or $b = 0.95$, and the conditional standard deviation increasing 20 times from 0.1 to 2. The variance states were permanent. This yields the following results in the presence of the irreversibility constraint:

i) $b = 0.05$: at the high state with a high capital stock, 5% is an upper bound on the investment reduction due to the increase in variance. Virtually everywhere else the impact is zero.

ii) $b = 0.95$: at the high state with high capital, investment actually *increases* around 4%. At the low state with low capital investment can decrease up to 3%. In a large (A, k) neighborhood of the deterministic steady state the change in investment does not exceed 2%.

Persistence and a minimum level of consumption induces very large movements in investment in this model. I simulated a version of this problem with utility given by $u = \frac{1}{1-\rho} [c_t - \bar{c}]^{1-\rho}$ with $\bar{c}$ equal to 50% of deterministic steady state consumption.²⁵ I also increased the degree of concavity to $\rho = 1.3$. With $b = 0.05$ the increase in variance failed to produced more than 2% changes in investment. With persistence, $b = 0.95$, the scenario is dramatically different. At high levels of capital and high states investment can increase as much as 70%, and at low levels of capital investment can fall by as much as 45%. More importantly, here it is possible to generate sizeable movements (10-15%) in investment reasonably near the deterministic steady state. Finally, with $b = 0.95$ and $\rho = 0.5$ we get a 30% increase in investment with high capital and at the high state of nature, and a 15% decrease at the opposite end. In a *very* small neighborhood of the deterministic steady state the effect is around zero, but in a small neighborhood the variation in investment can reach $\pm 10\%$.

2.3.3 Investment Shocks

Levhari and Srinivasan (1969) show that for the CRRA utility function and a log normal distribution, savings increase with a mean preserving spread if $\rho > 1$ and decrease if $\rho < 1$.²⁶ The Euler equation with linear utility and positive

²⁵Note that increasing $\bar{c}$ imposes a tighter lower bound on consumption and therefore a tighter *higher* bound on investment. Irreversibility is a *lower* bound on investment.

²⁶They examine the optimal savings decision in a dynamic infinite horizon partial equilibrium model with an exogenous random interest rate. Eaton (1980) shows that if the agent can hedge against uncertainty by buying today, and storing at no cost, a substitute to the good that has a random price, an increase in variance of a lognormal relative price with CRRA utility raises savings for high concavity of utility, $\rho > 1$, and lowers savings otherwise (if $\rho < 1$). In general there is some value of ρ above (below) which savings increase (decrease) with an increase in variance.

consumption is:

$$1 - \lambda + \lambda_c = q\beta f_k(k') + q\beta(1 - \delta)E\left(\frac{1}{q'}\right) - (1 - \delta)E\left(\frac{\lambda' - \lambda'_c}{q'}\right)$$

The current q changes the agent's decision and therefore there are different stochastic steady states *even with i.i.d. shocks and linear utility*, unlike the previous additive and technology shocks. This is because $1/q$ is the price of investment in consumption units, and even in the linear case, marginal utility of consumption is weighed by this relative price.^{27,28} There is no impact of the increase in variance on the return on investment, other than through the Lagrange multipliers. At the low current q state, if the current constraints do not bind, an increase in variance will not affect investment unless future shocks can make the consumption constraint bind.²⁹ In the high q state, the negative effect of the irreversibility constraint will generate large investment reductions following the increase in variance.

Concave utility yields $u_c - \lambda = q\beta E u_{c'} \left[\frac{1-\delta}{q'} + f_k(k') \right] - q\beta(1 - \delta)E\left(\frac{\lambda'}{q'}\right)$, and we know that the constraint quickly ceases to matter. In the simulations with $\delta > 0.075$ and $\rho > 1.4$ the irreversibility constraint never binds.

i.i.d. shocks. With linear utility, in the low q state with persistent variance, investment does not change with the mean preserving spread (mps).³⁰ In the high q state and at high levels of capital, investment decreases 30 to 50%.³¹ So, for the (near) linear model the irreversibility constraint always binds with a negative shock for sufficiently high levels of capital. There is a strong reduction in investment at high levels of capital and in the high q state, in the presence of the constraint and with persistent variance. Without the constraint the mean preserving spread has virtually zero impact. For the concave model the irreversibility constraint generally does not bind: any large effects of the linear model disappear when ρ rises above 1.2 to 1.4. Finally, if the high variance state is temporary the reduction in investment with the increase in variance is substantially smaller.

Persistence. Without the irreversibility constraint we obtain:³²

²⁷ $\beta(1 - \delta)E(1/q')$ is the expected discounted value of one unit of the remaining capital evaluated at replacement cost in tomorrow's consumption units.

²⁸Also here, $E(q) = 1$ implies via Jensen's inequality that $E(1/q) > 1$, and this non linearity will imply "non-neutrality" of the variance as $E(1/q)$ is increasing in the variance of q . To avoid this problem the m.p.s. is performed on $1/q$.

²⁹The "steady state" associated with a high q , must be high enough so that the agent will invest all his income to get there, if that state occurs.

³⁰Persistent variance means 0.98 probability of staying in the current variance state. Simulations with $\rho = 0.5$ and $\delta = 0.1$. The irrev. constraint binds for non drastic shocks.

³¹If the low variance state is persistent (0.98) but the high variance state is temporary (0.02), investment decreases between 10 and 15% with the m.p.s..

³²Here we have: $\alpha = 0.35$, $\beta = 0.95$, $\delta = 0.075$, $\rho = 0.5$, $a = 0$, $b = 0.05$, and $b = 0.95$.

1a) $b = 0.05$: Shocks are close to i.i.d. Without the constraint there is virtually no effect of the mean preserving spread. This replicates the result from the three state process above.

1b) $b = 0.95$: At the high state the agent *increases* investment substantially (as much as 90%) with the high variance. At the low state the agent decreases investment substantially (as much as -66%). These "overinvestment" and "underinvestment" reactions are exacerbated as capital increases. Close to the deterministic steady state the effect is about zero.³³

With the irreversibility constraint we obtain:

2a) $b = 0.05$: Here, the higher the state of nature, and the more capital the agent has, the more he reduces investment, up to -25%. The agent never increases investment more than 0.5% at the very lowest state and with as little capital as allowed in the model.

2b) $b = 0.95$: the results from point 1b) above are basically replicated. At high values of capital and for the high states, the increase in investment is well above 50% as the variance increases. Thus, persistence induces precautionary savings and does not compound on the irreversibility effect.³⁴

2.3.4 Summary

The model implies that a near linear utility function, a low depreciation rate, and large investment shocks are necessary to make the irreversibility constraint bind in the stochastic problem. This is intuitive since with low concavity the agent is willing to make larger shifts between consumption and investment, and investment shocks (shocks on the relative price between consumption and investment) affect exactly this decision.

Concavity also highlights the difference between the two types of shocks. Under technology or additive shocks, an increase in concavity implies a higher probability of a binding irreversibility constraint because consumption smoothing is now desirable and that forces investment to absorb income fluctuations and maybe hit the constraint. This effect is nevertheless negligible. Under investment shocks, concavity and consumption smoothing work directly against the shock, making the irreversibility constraint less binding.

The effects are more pronounced when the high variance state is persistent. This is due to the fact that when capital is low it *insures* the agent against the irreversibility constraint and when it is high it does so against low consumption.

With an AR(1) process with persistence, an increase in variance can induce large movements in investment of a precautionary savings nature, irrespective of the type of shock. Investment will increase (decrease) in the high (low) states

³³This is in the spirit of Abel and Eberly's exercise.

³⁴This is also the case where we can see the largest effect of irreversibility. Comparing cases 1b) and 2b) there is a large drop in investment by introducing the constraint. It is still the case, however, that investment increases with the mean preserving spread so the conclusion is slightly misleading.

of nature. Investment will only fall significantly (after the increase in variance) because of irreversibility with i.i.d. investment shocks and low concavity of utility.

This exercise indicates that the definition of the stochastic process and the shape of preferences are essential for the results. Also, the length of the time period is important as it determines the depreciation and interest rates. If we believe that large shocks to the economy are like the oil price shocks and occur in a short period of time, then irreversibility can have a larger impact than in some of the experiments performed here. Nevertheless, a substantial degree of adjustment can occur over the course of a quarter or a year. In any case, the qualitative nature of the results does not change.

2.4 Fixed Costs

It is useful here to discuss the notion of irreversibility further. S,s models of investment typically include fixed costs of adjustment. These are generally present independently of the direction of adjustment. That is also a common feature of other investment models like the machine replacement model of Cooper, Haltiwanger and Power (1995). The zero investment constraint is a *future* cost associated with the probability of making the *wrong* current decision. The fixed cost is a *current* cost associated with *any* decision. While the results of Ramey and Shapiro (1997) suggest the non-negativity constraint is an important aspect of irreversibility, fixed costs can have a large impact on the behavior of the model used in this paper.³⁵ The dynamic problem with a fixed cost of investing (F) and technology shocks is:

$$V(k, A, \sigma) = \max \left\{ \begin{array}{l} \max_{k'} u(Af(k) - I - F) + \beta EV(k', A', \sigma') \\ u(Af(k)) + \beta EV((1 - \delta)k, A', \sigma') \end{array} \right\}$$

with $k' = (1 - \delta)k + I$, and $C > 0$, $I \geq 0$. I define the fixed cost to be small enough such that the deterministic model has a steady state with positive investment. Even so, the fixed cost can be a powerful investment deterrent and should induce large effects of the increase in variance.³⁶

Figure 7 illustrates the policy functions for this model with $\beta = 0.95$, $\alpha = 0.35$, $\delta = 0.1$, and $\rho = 0.5$. The fixed cost is set at 4% of the deterministic steady state output, and shocks are i.i.d.³⁷ We can see that the policy functions are substantially altered relative to the standard model. In particular, as the agent

³⁵Ramey and Shapiro find large wedges between the purchase and resale price of capital goods.

³⁶In the deterministic case, with large fixed costs the agent never reaches a steady state and instead lives in a cycle. "Steady state" investment is not large enough to make it worth incurring the fixed cost.

³⁷4% of output is a very large value for F. If F is set at 4% of steady state *investment* we get similar results (a 20% reduction in investment following the m.p.s. at high levels of capital). They are not as strong but are still much bigger than in the standard model.

approaches the region where the zero investment decision is dominant, the policy function starts to reflect that discontinuity. In addition, high variance induces significant changes in the policy functions. Note that for $\rho = 0.5$ the standard model delivered no impact of the variance increase. Here investment decreases substantially, about 40%, when the capital stock is high and the inaction option dominates. At low values of capital the marginal product is so high that the investment option always dominates and the increase in variance increases investment (5 to 7%). Therefore the variance has a significant impact on this problem. The crucial feature of this model is that inaction has two benefits. First it saves the fixed cost, and second, it moves the agent away from the other irreversibility constraint.

Finally, while the representative agent framework can be regarded as inadequate for this exercise, it is still the case that even if we consider heterogeneity there is an interest rate that clears the market. This means that not all agents will be constrained since the interest rate would fall and some investment would occur. The effect must be on a group of marginal agents and this is captured in the representative agent framework. Nevertheless, it is useful to look at heterogeneity, which I do next.

3 Cross Section Analysis

The previous section developed the individual decision problem and mapped it into the aggregate economy via the representative agent assumption. This section takes a step away from that interpretation and introduces idiosyncratic shocks and a cross section distribution over capital. Intuitively, if many agents are located on the tail of the capital distribution where the irreversibility constraint binds, aggregate effects of uncertainty and irreversibility may be large.

Idiosyncratic technology shocks affect the individual production process once the factors of production are in place. Idiosyncratic investment shocks may pertain to specific investment projects like their probability of success, installation costs or specific relations with financial intermediaries. This heterogeneity requires keeping track of the cross section distribution over capital, implying that the capital stock of each agent is a state variable. Here I follow Cooper, Haltiwanger and Power (1995) and rule out the possibility of smoothing consumption by trading away the idiosyncratic uncertainty.³⁸ The only characteristics that differentiate agents are the realization of the idiosyncratic shock and the current capital stock. I solve the previous representative agent problem and use the policy functions to iterate on an initially exogenous cross section distribution.

³⁸If the different firms can cross subsidize each other, a firm with a negative and persistent idiosyncratic shock would see its capital stock *totally* disappear because it would always pay to subsidize a firm with a positive persistent shock. This may imply a larger role for irreversibility but it does not necessarily imply that aggregate investment should change with a m.p.s. more or less than in the exercise that follows.

This allows me to highlight some characteristics of the more general problem, at the expense of the general equilibrium interpretation.

This section addresses the problem of the firm rather than the consumer. The exercise is therefore comparable to that performed in the machine replacement problem and in the standard S,s literature. Finally, given the previous section result that changes in the variance of i.i.d. investment shocks deliver the largest impact of irreversibility in the model, this section focuses on those shocks in the quantitative analysis.³⁹

3.1 The Model with Investment Shocks

ξ_t is the realization of an idiosyncratic investment shock, with density $f(\xi_t|\xi_{t-1})$. As before, each agent solves the usual dynamic programming problem. But now there is a continuum of agents/firms with mass equal to 1, each of whom has a given amount of capital. They are distributed in the capital stock state space with density $g_t(k)$. For every point in the k state space there is a continuum of firms holding that amount of capital. Each agent knows the current realization of the aggregate investment state, q_t , and his idiosyncratic "investment ability", ξ_t . They also know the process governing q_{t+1} and ξ_{t+1} , in particular the variance, (σ) , of the relevant distribution. Individual capital accumulation obeys:

$$k_{t+1} = (1 - \delta)k_t + \xi_t q_t I_t$$

The cross section density evolves through time according to:

$$g_{t+1}(k_j) = \int \int \hat{I}_k(k_{t+1}(k_i, q_t, \sigma_t, \xi) = k_j) f(\xi_t|\xi_{t-1}, \sigma_{t-1}) g_t(k_i) d\xi dk_i$$

where $\hat{I}_k$ is an indicator function that assumes the value 1 if the policy function is such that $k_{t+1} = k_j$, and zero otherwise. If the $f(\xi_t|\xi_{t-1}, \sigma_{t-1})$ distribution is never degenerate there will exist a stationary distribution $g(k)$ for each value of q and σ .⁴⁰ In the absence of the idiosyncratic distribution, all agents would converge to the same point in the k state space. Aggregate investment is given by:

$$I_t = \int \int \left[\frac{k_{t+1}(k_i, q_t, \sigma_t, \xi_{jt}) - (1 - \delta)k_i}{q_t \xi_{jt}} \right] f(\xi_{jt}|\xi_{t-1}, \sigma_{t-1}) g_t(k_i) d\xi_{jt} dk_i$$

The integrand is total investment of agents located at k_i .

³⁹Utility is linear since that produced the largest effects of variance in the model with investment shocks. Linear utility eliminates precautionary savings from the problem and isolates the effect of irreversibility. The value of ρ actually used is 0.1.

⁴⁰This is stated as a simulation result. In any case, the policy functions are a mapping from the "active" range of capital onto itself, and this is a closed and convex set that includes a minimum and a maximum and is everywhere dense. Also the nature of the probability distribution guarantees it, since it is a Markov matrix that rules out any partial infinite cycles, implying the agent will always go through every state at some point in time.

A timing issue arises here. When idiosyncratic uncertainty increases, current shocks (aggregate and idiosyncratic) have already occurred, and so, with $k_{t+1}(k_i, q_t, \sigma_v, \xi_{jt}) \equiv k_{t+1}(*, \sigma_v)$, the impact change in investment is given by

$$I_{t,h} - I_{t,L} = \int \int \left[\frac{k_{t+1}(*, \sigma_h) - k_{t+1}(*, \sigma_L)}{q_t \xi_{jt}} \right] f(\xi_{jt} | \xi_{t-1}, \sigma_{t-1}) g_t(k_i) d\xi_{jt} dk_i$$

At this point the current (low variance) distribution $f(\xi_{jt} | \xi_{t-1}, \sigma_L)$ has no impact on the problem. Aggregate investment will decrease at impact. It is one period later, when ξ actually follows the high variance distribution $f(\xi_{jt} | \xi_{t-1}, \sigma_h)$, that this component affects investment. In this next period, a larger proportion of agents is incurring good or bad shocks. The agents that incur positive shocks will increase their investments, but the agents that incur bad shocks may not be able to cut investment by as much as their desired amount due to the irreversibility constraint. Figure 8 illustrates this point.⁴¹ Aggregate investment will then increase.

Finally, an increase in the depreciation rate will relax the *binding* irreversibility constraint for some agents and may actually *reduce* aggregate investment, whereas for a representative agent located at point b an increase in the depreciation rate would actually increase investment since the agent would have a non binding constraint relaxed.

3.2 Simulations

In the first experiment I increase the variance of idiosyncratic shocks. I simulate a time series of the model under π_{ξ_h} , and another under π_{ξ_L} , with the same aggregate shocks. The policy functions are obtained solving the dynamic programming problem where the state vectors are $\hat{\xi} = [0.8 \ 1 \ 1.2]$ and $\hat{q} = [0.9 \ 1 \ 1.1]$. Note that the volatility transition applies only to idiosyncratic shocks (which are aggregated out). The sequence of aggregate shocks corresponds to one given draw from their Markov process. The transition matrices used, π_{ξ_h} and π_{ξ_L} , are the same as in the three state mean preserve experiments performed earlier with i.i.d. shocks. The transition matrix for the variance and the one for aggregate shocks are:

$$\pi_q = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \quad \pi_v = \begin{bmatrix} 0.98 & 0.02 \\ 0.02 & 0.98 \end{bmatrix}$$

Figure 9 shows the difference between time series of consumption and investment under low and high volatility, generically $dx = (x_h - x_L)/x_L$. The

⁴¹When all firms are at point b they all invest the same. With the mean preserving spread, firms that suffer a bad shock and move to point c on the irreversibility constraint must invest zero, which represents an excess of d over their otherwise desired capital stock. Therefore the investment reduction of these agents may not be enough to overcome the increase in investment of agents who jump to point a .

parameters used are $\alpha = 0.35$, $\beta = 0.95$, $\delta = 0.1$, $\rho = 0.1$. There are three graphs corresponding to this experiment. In all of them the horizontal axis shows the time series of aggregate shocks. In this experiment, after the effect of shocks disappears, consumption increases around 10% for all 3 aggregate states. We have also for investment, $di \approx [15\% \ 10\% \ 5\%]$ for states [low middle high]. In the low aggregate state investment increases more with the high idiosyncratic variance because there is always the probability that an aggregate shock will send the economy up and therefore relax the binding constraint on the bottom of the cross section distribution.

A mean preserving spread on the distribution of idiosyncratic shocks can have substantial *positive* effects on aggregate investment. When we aggregate the cross section distribution, agents that remain in the middle state for ξ do not change their decision sizeably, and the investment of agents that do suffer shocks may aggregate to a higher value than before, since agents that suffer negative shocks are not able to disinvest due to the non negativity constraint.⁴²

Second, I increase the variance of aggregate shocks. The π_v matrix now governs the transition between high and low variance of aggregate investment shocks. With the appropriate relabeling π_{qh} , π_{ql} , π_ξ , π_v are just as in the previous experiment. For the percentage indicator $dx = (x_h - x_L)/x_L$, the impact on aggregate variables of a higher variance of aggregate shocks is virtually zero. This is due to the narrow support of aggregate shocks. With larger shocks aggregate investment will fall with the mean preserving spread, but not by much.

If the distribution of idiosyncratic shocks remains constant and aggregate shocks suffer a mean preserving spread, the impact on aggregate investment is likely to be negative but small.

4 Conclusion

This paper uses a dynamic stochastic general equilibrium model to characterize the relative importance of precautionary savings incentives and of irreversibility constraints in response to an increase in uncertainty. Uncertainty is associated with the variance of the random variable driving the model. Additive shocks to output, technology shocks and shocks to the marginal efficiency of investment are investigated. Irreversibility is introduced by a non negativity constraint on investment.

Model simulations show that it is difficult to make this constraint bind for the technology and additive shocks cases. For the investment shocks model low concavity of utility is enough to generate a binding constraint. This is

⁴²Faig (1997) shows that an economy where $k' > \mu k$ does not necessarily have lower investment than an economy where $k' > 0$. His experiment relaxes the constraint but has 100% implicit depreciation. If the depreciation rate is low, irreversibility becomes more of an issue. Abel and Eberly (1995) have zero depreciation.

intuitive since with low concavity the agent is willing to make larger shifts between consumption and investment, and the shock affects exactly this decision. An increase in concavity, by imposing consumption smoothing, works directly against the irreversibility constraint and reduces the impact of the mean preserving spread very quickly. In all versions of the model the effects are more pronounced when the high variance state is persistent. This is due to the fact that capital has an insurance value against low consumption (when it is high) and against the irreversibility constraint (when it is low). When the shocks themselves are persistent, an increase in variance induces a precautionary savings reaction. Investment increases in the high states of nature and decreases in the low states.

A variant of the model with investment shocks introduces a cross section distribution of agents over capital and adds idiosyncratic shocks. Aggregate investment increases following a mean preserving spread on idiosyncratic shocks because zero investment is an active lower bound for some agents. This effect is enhanced the bigger idiosyncratic shocks are. If we are willing to accept larger shocks for individual agents that may make irreversibility matter more, we should also expect a larger increase in aggregate investment following an increase in idiosyncratic variance. When the increase in uncertainty occurs in the distribution of aggregate shocks, aggregate investment is likely to decrease following the mean preserving spread, although by a small amount.

The results with investment shocks suggest that an increase in variance may have sizeable effects mainly if it is associated with a relative price, inducing agents to change relative purchases. A fixed labor supply may bias the results since the agent does not change its labor supply in response to the relative price of leisure (wage). However, the empirical evidence of a low labor supply elasticity suggests this may not be a very strong limitation of the model. A bigger limitation is the existence of only one asset with only one depreciation rate. Ejarque (1997) finds significant effects of a mean preserving spread on technology shocks exactly because different irreversibility characteristics imply a change in the relative price between different assets.

The definition of irreversibility is also important. If we consider a model with the additional feature of a fixed cost of undertaking any nonzero level of investment, the increase in variance may have large effects in the representative agent economy, even with technology shocks. Such a feature is pervasive in the investment literature, namely in S,s type models, and it suggests a transaction cost problem rather than a second hand market notion of irreversibility. This subtle difference produces large differences in the behavior of the models. In the standard model with a non negativity constraint, investment only falls with an increase in variance if utility is close to linear; if shocks affect the relative price between consumption and investment; if the high variance state is persistent, and if the shock process conditional on the variance is close to i.i.d..

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Figure 1

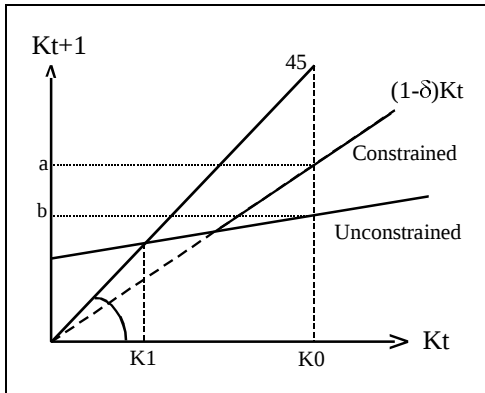


Figure 2

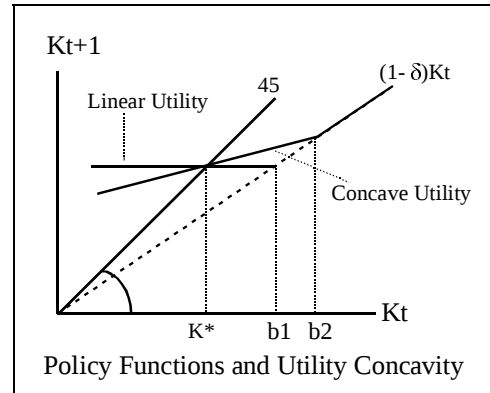


Figure 3

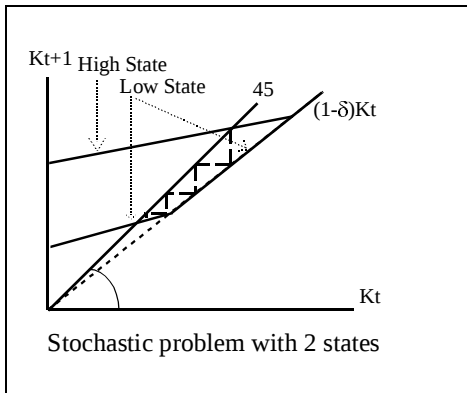


Figure 4

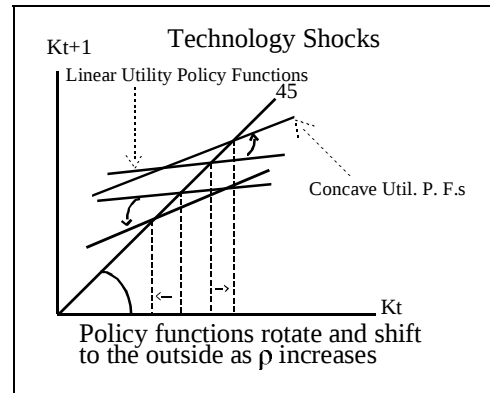


Figure 5

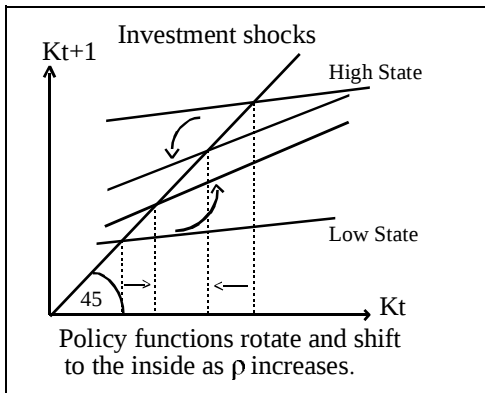


Figure 6

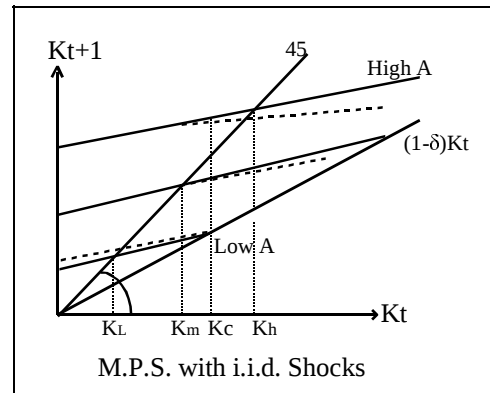


Figure 6.a

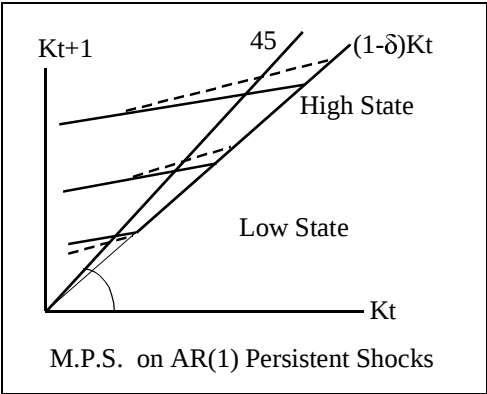


Figure 7

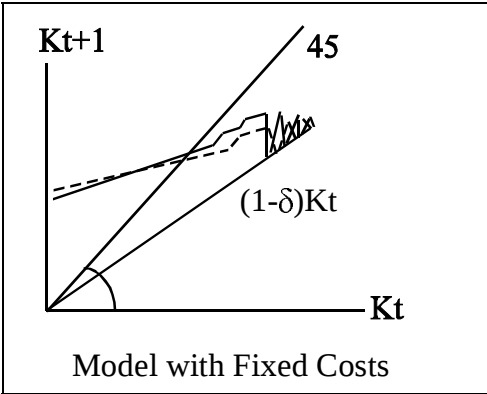


Figure 8

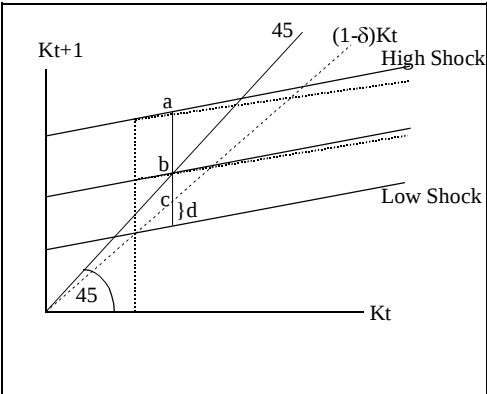


Figure 9.1

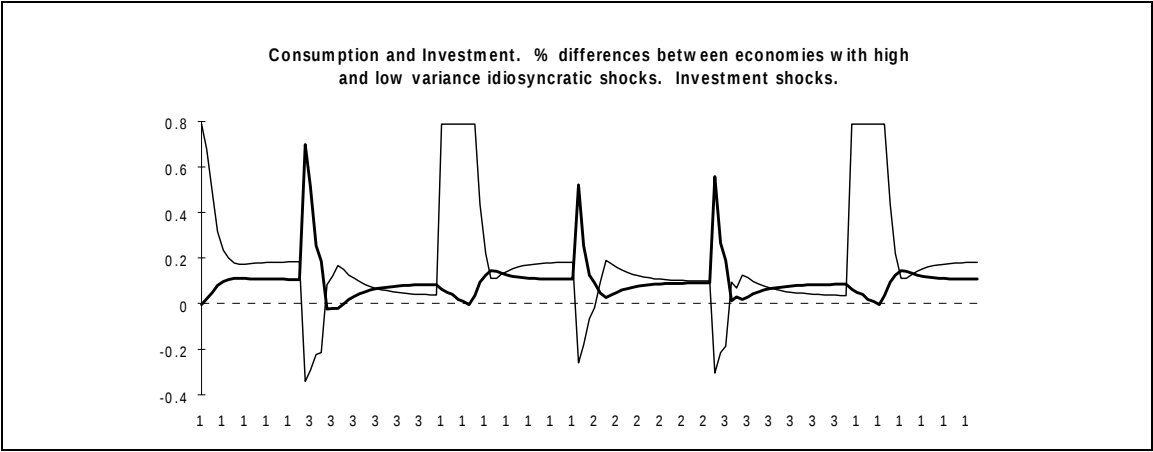


Figure 9.2

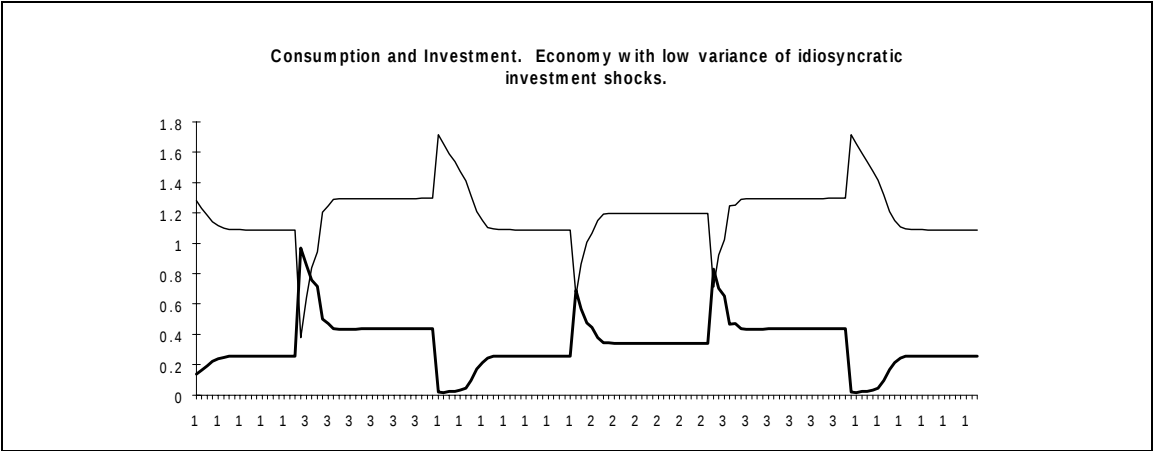


Figure 9.3

