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Shell model analysis of the $B(E2, 2^+ \rightarrow 0^+)$ values in the $A = 70$, $T = 1$ triplet ^{70}Kr, ^{70}Br, and ^{70}Se

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The $B(E2, 2^+ \rightarrow 0^+)$ transition strengths of the $T = 1$ isobaric triplet ^{70}Kr, ^{70}Br, ^{70}Se, recently measured at the RIKEN Radioactive Isotope Beam Factory (RIBF), are discussed in terms of state-of-the-art large scale shell model calculations using the JUN45 and JUN45+LNPS plus Coulomb interactions. In this Letter we argue that, depending on the effective charges used, the calculations are either in line with the experimental data within statistical uncertainties, or the anomaly happens in ^{70}Br, rather than ^{70}Kr. In the latter case, we suggest that it can be due to the presence of a hitherto undetected 1^+ $T = 0$ state below the yrast 2^+ $T = 1$ state. Our results do not support a shape change of ^{70}Kr with respect to the other members of the isobaric multiplet.

Introduction. In the limit of strict isospin symmetry, the matrix elements $M_p(E2) = \sqrt{B(E2, 0^+ \rightarrow 2^+)}$ in a $T = 1$ triplet must vary linearly with the T_z of its members. Nuclear structure details determine the slope of the line and the absolute value of the $M_p(E2)$'s. Isospin symmetry breaking (ISB) effects are known to produce differences in the binding energies and in the excitation energies of the members of an isospin multiplet; these are dubbed Coulomb energy differences (CEDs), mirror energy differences (MEDs), and triplet energy differences (TEDs), respectively [1–3]. Although the Coulomb repulsion among the protons is the main source of these effects, it has been shown that additional ISB terms are needed to explain the available experimental data [4,5]. However, as the values of the MEDs and TEDs are quite small, one should expect the isospin breaking effects in the $M_p(E2)$'s to be even smaller and that linearity should be preserved to a large extent. Notice, however, that sometimes even a small MED can produce quite prominent effects, as in the case reported in Ref. [6], where the ground state spins of the mirror pair ^{73}Sr-^{73}Br were found to be different. In Ref. [7] we have shown that standard ISB effects suffice to explain the inversion of two close lying $5/2^-$ and $1/2^-$ levels (see also [8]).

In a recent experiment carried out at the RIKEN Radioactive Isotope Beam Factory (RIBF) [9], the $B(E2, 0^+ \rightarrow 2^+)$'s of the $T = 1$ triplet ^{70}Kr, ^{70}Br, ^{70}Se were measured. The authors of this work argue that these values, and the extracted $M_p(E2)$'s shown in Fig. 1, are inconsistent with isospin symmetry conservation and suggest that the shape of ^{70}Kr may be different from that of the other members of the multiplet. The systematic shell model studies of Refs. [10,11] show good agreement with the experimental displacement energies in this mass region, with the conclusion that the ISB effects do not influence the bulk properties such as deformation.

In this work we also approach the problem in the framework of the shell model with configuration interaction to study both energies and transition matrix elements in the $A = 70$ triplet and discuss an alternative scenario for ^{70}Br.

Large scale shell model calculations. Two valence spaces are adopted. The first includes the orbits $1p_{3/2}$, $1p_{1/2}$, $0f_{5/2}$, and $0g_{9/2}$ for both protons and neutrons, and we use the effective interaction JUN45[1] [12]. The second set of calculations is made in an extended space which includes the $1d_{5/2}$ orbit, with the JUN45 interaction supplemented with the necessary matrix elements from the LNPS [13] interaction, and we refer to it as JUN45+LNPS. The calculations were performed with the code ANTOINE [14] and involve dimensions of $O(10^9)$, allowing up to 10p-10h excitations from the $p_{1/2}$ and $f_{5/2}$ orbits to the $g_{9/2}$ and $d_{5/2}$, whereas the jumps from the $p_{3/2}$ are restricted to 4p-4h.

Concerning the effective charges (see Ref. [15] for a detailed discussion) we have two choices: those of Dufour and Zuker (DZ), $q_\pi = 1.31e$ and $q_\nu = 0.46e$, microscopically derived for harmonic oscillator cores [16], and the standard (ST) ones $q_\pi = 1.5e$ and $q_\nu = 0.5e$. The latter were shown to be adequate for a ^{56}Ni core [15], but for completeness we will present results with both sets. We note that, in the analysis of Ref. [12], the ST effective charges do a better job in this mass region, as compared to the $q_\pi = 1.5e$ and $q_\nu = 1.1e$ values, obtained from a fit to experimental data.

The calculations incorporate the Coulomb interaction between the protons obtained with harmonic oscillator wave functions with the appropriate values of $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$ (MeV). As the effects are perturbative, we have computed them in the JUN45 case and add the same corrections to the JUN45+LNPS nuclear only ones. The results with JUN45 plus Coulomb and ST effective charges are given in Table I. While the absolute excitation energies of the 2^+'s are

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[1] The same valence space and interaction were used in Ref. [9].

TABLE I. $A = 70$ triplet shell model results with the JUN45 interaction plus Coulomb.

Nucleus	$E(2^+)$ (MeV)	$M_p(E2)$ ($e\,\mathrm{fm}^2$)	$\delta M_p(E2)_{\mathrm{coul}}$ ($e\,\mathrm{fm}^2$)
^{70}Kr	0.545	52.2	+3.4
^{70}Br	0.605	47.0	+0.7
^{70}Se	0.648	43.5	−0.3

predicted by JUN45 about 300 keV too low with respect to the experiment, the MED $= -100$ keV and TED $= -17$ keV compare well with the experimental values of −67.0(7.5) keV and −45.2(7.5) keV respectively. The quoted experimental uncertainty corresponds to the root-mean-squared deviation of the ^{70}Kr 2^+ state energies given in [17].

We report in Fig. 1 the calculated $M_p(E2)$'s for JUN45 and JUN45+LNPS with both sets of effective charges. The error bars of the experimental points are taken from Ref. [9] for ^{70}Kr and are the averages of the results of Refs. [9,18,19] for ^{70}Br and ^{70}Se.

The $M_p(E2)$'s obtained with JUN45+LNPS are very similar to those of JUN45, with the added bonus that they produce a better spectroscopy, with the 2^+ energies closer to the experimental values. We see that the Coulomb corrections to the $M_p(E2)$'s are small, but not negligible as claimed in [9], mainly in the case of ^{70}Kr. The induced nonlinearity goes in the direction demanded by the experimental data.

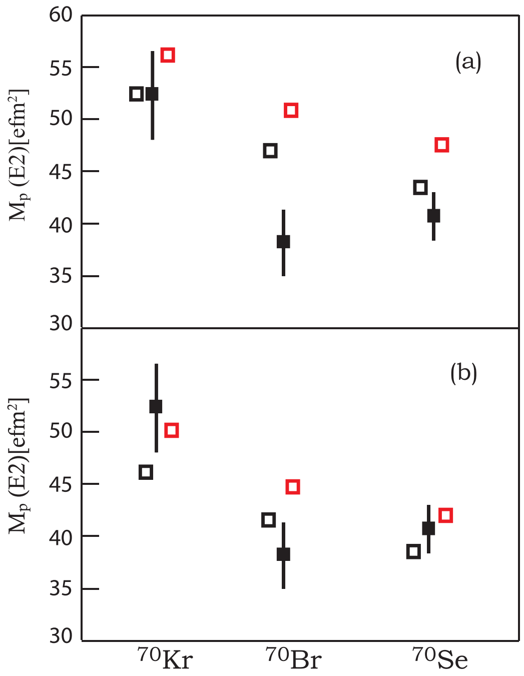

FIG. 1. $M_p(E2)$ in the $A = 70$ isospin triplet (in $e\,\mathrm{fm}^2$). The experimental data are taken from Ref. [9] (see text for a discussion of the error bars). Results of the JUN45 (black squares) and JUN45+LNPS (red squares) calculations. (a) ST effective charges; (b) DZ effective charges.

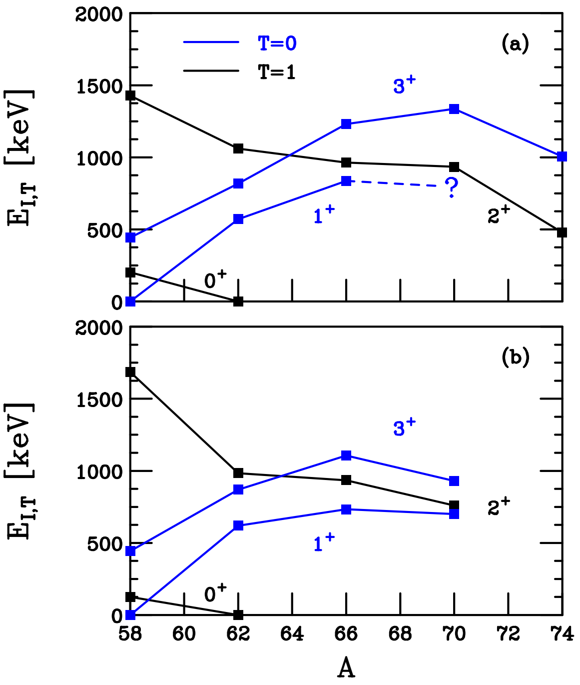

FIG. 2. (a) Systematic of the low-lying $T = 0$ and $T = 1$ states in odd-odd nuclei relevant to the $A = 70$ region. The 1^+ in ^{70}Br, indicated by a question mark, is not known. (b) JUN45+LNPS shell model results.

We now examine in more detail what Fig. 1 tells us. First, let us emphasize that, according to our calculations, none of the members of the triplet can be said to have a well defined shape, the more so in view of the values of the intrinsic shape parameters β and γ and their variances that we obtain from the Kumar invariants [20] as described in Ref. [21]. The value of $\beta = 0.22 \pm 0.05$, means that the fluctuations of Q^2 amount to one-half of its mean value. While the value of $\gamma = 32^\circ$ is suggestive of triaxiality, its fluctuations at 1σ level span the interval 8°–60°. Therefore, the certain degree of quadrupole collectivity they exhibit can be accounted for without resorting to any shape change, in contrast to the conclusions in Ref. [9]. Second, if we examine the lower panel (b) of Fig. 1 we realize that the calculations, although marginally so, are compatible with the data. If we consider the upper panel (a), the anomaly, if it exists, could be due to a dip in the $M_p(E2)$ of ^{70}Br.

We will discuss in the remaining part of the Letter a possible explanation for this apparent behavior.

The structure of ^{70}Br. Despite ^{70}Br being an $N = Z$ odd-odd nucleus, due to the role played in its structure by the $0f_{5/2}$ and $0g_{9/2}$ orbits, isovector pairing dominates over the symmetry energy and its ground state is 0^+, $T = 1$ [22]. The yrast 2^+, $T = 1$ state is located at 934 keV (notice that in well deformed ^{76}Sr it appears at about 200 keV) and the lowest $T = 0$ state, known to date, is a 3^+ at 1336 keV [18,19,23–25]. As shown in the systematics in Fig. 2(a), in other odd-odd $N = Z$ nuclei in the region, the 1^+, $T = 0$ state lies a few hundreds of keV below the 3^+ and also below the 2^+. In Fig. 2(b) we show the results of the JUN45+LNPS interaction (spectroscopically superior to those of JUN45) which remarkably reproduce the experimental trends.

The 1^+ *scenario.* While an inspection of Fig. 2 shows that we should not expect the shell model to predict the excitation energies within $\approx\pm200$ keV, both systematics and theory suggest that it is possible that the 1^+ is lower than the 2^+. In this regard, it is worth noting that the particle plus rotor model and IBM4 calculations discussed in Ref. [23] also predict the 1^+ state lower than the 2^+.

The key ingredient of this scenario is the $B(M1)$ transition probability from the 2^+, $T=1$ state to the 1^+, $T=0$ state, which according to our calculation amounts to $0.22(2)\mu_N^2$.[2] Thus, if the experiments did not have enough sensitivity and the $M1$ transition was not observed, the $B(E2)$ value, extracted from the intensity of the deexcitation of the 2^+ γ ray, would have been underestimated.

To further explore our conjecture, we assume that ^{70}Br follows isospin symmetry and determine a value of $M_p(E2) = 46.5(4.6)\, e\,\mathrm{fm}^2$ from a linear fit of the ^{70}Kr and ^{70}Se, $|T_z|=1$ pair. Then, for a given energy of the hitherto unknown 1^+ state, E_1^+, we calculate the strength of the transition $2^+ \to 1^+$ in such a way that the $M1/E2$ branching ratio gives a $2^+ \to 0^+ \gamma$-ray intensity that agrees with the measured $M_p(E2) = 38.1(3.1)\, e\,\mathrm{fm}^2$ matrix element [9]. The result is shown in Fig. 3 by the red dashed line and shaded area.

We now consider the conditions of the RIKEN RIBF experiment, in terms of statistics, peak to background, and energy resolution to establish the values excluded by the measurements for the observation of a 3σ peak in the spectrum. Here, we assume the detection of the higher energy γ transition, namely the $2^+ \to 1^+$ or the $1^+ \to 0^+$. The results are presented in the form of an exclusion plot (shaded green area) in Fig. 3. Our $B(M1)$ estimates above and the shell model results lie within the allowed region and a consistent solution exists, indicated by the intersection of the empirical and shell model values.

Furthermore, with the limited information we can assess from Ref. [23], the statistics of the relevant coincidence spectra shown in the paper seems consistent with the nonobservation of a γ-ray peak with the intensities allowed by the exclusion plot. Last but not least, the lifetime measurements of Ref. [18] require some discussion. In contrast to the even-even cases, the line-shape analysis in the odd-odd ^{70}Br could potentially be more susceptible to the unknown feeding of the 2^+ state from $T=0$ states above. Therefore, side-feeding corrections, not fully captured in a singles spectrum, could make the effective lifetime of the 2^+ level appear longer.

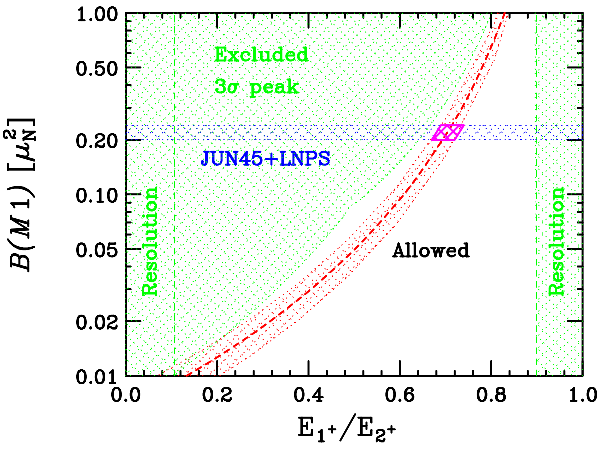

FIG. 3. $B(M1)$ strength from the 2^+ to a hypothetical 1^+ required to explain a missing intensity of the 2^+ γ ray (red dashed line and shaded area) and the excluded regions imposed by the experimental conditions of the setup in Ref. [9] (green shaded area; see text for details). The intersection of the shell model results (blue shaded area) with the empirically required strength determine a possible solution (magenta shaded area).

Conclusions. In summary, we present a large scale shell model analysis of the $B(E2, 2^+ \to 0^+)$'s in the $A=70$, $T=1$ triplet. The calculations were performed using the JUN45 (+LNPS) interactions in the model spaces $1p_{3/2}$, $1p_{1/2}$, $0f_{5/2}$, and $0g_{9/2}$ (+ $1d_{5/2}$) above the ^{56}Ni core. ISB effects due to the Coulomb force were taken into account. Our results suggest alternatives to the shape change proposed in Ref. [9]. On one hand, the calculated $M_p(E2)$ matrix elements, using the DZ effective charges, appear in line with the experimental data, given the statistical uncertainties. On the other hand, the use of ST effective charges may indicate that ^{70}Br, rather than ^{70}Kr, deviates from the isospin symmetry expectations, and we have proposed a scenario which could explain the Coulomb excitation measurements. Given the important ISB effects implied by the experimental data, perhaps further experimental work with a more sensitive γ-ray spectrometer (such as a tracking array) should be considered to probe the scenarios discussed in this work.

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[2] These results are obtained with no g_s quenching. For reference, the theoretical value of $B(M1, 3^+ \to 2^+) = 0.013\mu_N^2$ is compatible with the experimental value $0.027(12)\ \mu_N^2$.

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