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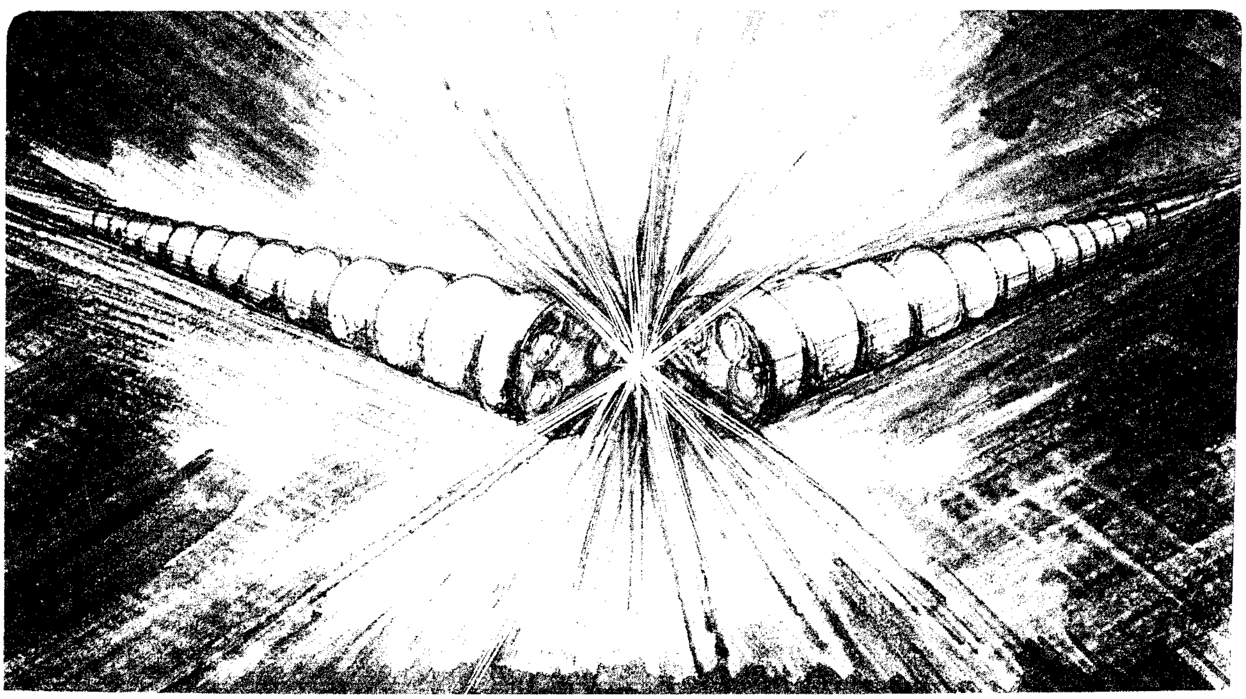

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* On leave from National Laboratory for High Energy Physics in Japan (KEK).

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Free-electron laser operation is formulated using a macroparticle approach based on a universal gain equation. Microwave excitation in a single cavity is derived analytically and is given in the form of analytic recursion equations for a multi-cavity system driven by a sequence of electron bunches. Qualitative and quantitative insights into the basic excitation and saturation mechanisms are provided. Stability analysis on a test particle moving around a macroparticle shows the importance of precise control of bunch spacing.

* On leave from National Laboratory for High Energy Physics in Japan (KEK).

1. Introduction

A free-electron laser (FEL) in the oscillator mode is a well-known concept in the optical frequency range [1]. Recently, a collaboration of LBL, LLNL and MIT has developed by theoretical [2] and numerical simulation [2,3], a microwave FEL operating in the oscillator mode as a power source for a future linear collider. They call the FEL a Standing-Wave FEL (SW/FEL) and have noted advantageous features of the SW/FEL, as distinguished from that of a FEL operating in the amplifier mode. Those features, discussed in detail in Ref. 2, consist of improved microwave aspects as compared to that of earlier version of the Two-Beam Accelerator.

It is well-known that the motion of electrons and spatial evolution of the signal wave and phase in a FEL are described by the KMR equations [4]. Takayama [5] and Sessler et al. [6] have shown that in the case of well bunched beams this evolution can be described by the KMR equations for a single particle which represents a bunch center, namely, a macroparticle. In Ref. 5, Takayama has developed an idea which, with the aid of some approximations [7], allows the KMR equations to be reduced to a universal gain equation (UGE) whose solution can be obtained in a universal gain function (UGF). We have found an exact analytic solution of the UGE, which will be presented in Section 2. In Section 3 we will apply this approach to the microwave FEL. The analytic model is useful for various purposes such as design of the SW/FEL, parameter search for better performance of the SW/FEL, and understanding of the over-all characteristics of the SW/FEL. Use of the model for some studies is made in Sections 4 and 5, but many further uses of the model will surely be made in further papers and by other workers.

2. Single Cavity Theory

We consider a single-stage FEL operating in an oscillator mode in an oversized waveguide with microwave reflectors at both ends. Namely, we consider a cavity which is excited in the TE_{01} mode with a sequence of transversely wiggling bunches. The reflectors may simply be metal plates with small holes which allow the electron beam to pass through. The cavity is taken as rectangular, with dimensions $a^* \times b^*$. We assume a standing wave of angular frequency ω_s, wave number $k_s = [(\omega_s/c)^2 - (\pi/b^*)^2]^{1/2}$ and a cavity length L_c satisfying $k_s L_c = 2n\pi$ (n is an integer). A beam bunched with a slightly different frequency $\omega_b = \omega_s + \Delta\omega$, enters the cavity through the end-plate hole. Each bunch, performing wiggle motion caused by a planar wiggler, starts to couple with the small signal waves. The signal is amplified while arriving at the far end. There the signal is reflected back to the front end, and since the cavity length is appropriately chosen, the signal begins FEL interaction with the next electron bunch. This process is repeated for many macroparticles.

For simplicity, we assume 100% reflection at both ends and neglect wall-losses and wakes caused by the electron bunches. The bunched beam considered here is an approximation to a bunched beam of averaged current I, with each bunch having a small spread in phase.

Following the macroparticle approach, after the FEL interaction within the cavity, the normalized signal amplitude e_s and rf phase advance $\Delta\varphi$ at cavity end ($z = L_c$) are given by

$$e_s(L_c) = \frac{\kappa}{a|b|} e^{y(s_c)}, \tag{1}$$

$$\Delta\varphi = \int_0^{s_c(\equiv |b| L_c)} \left\{ e^{-2y(s)} - [y'(s)]^2 \right\}^{1/2} ds, \tag{2}$$

with

$$a = \frac{2mc^2\gamma}{eZ_0 J a_w},$$

$$b = k_w - \delta k_s - \frac{1}{2}\left(\frac{\omega_s}{c}\right)\left(\frac{a_w}{\gamma}\right)^2,$$

$$s = |b| z, \quad (' \equiv d/ds),$$

where z is the axial coordinate measured from the cavity front-end, $\kappa = \frac{\sin \Delta\psi}{\Delta\psi} \simeq 1$, mc^2 is the electron mass energy, γmc^2 the injection energy, e the electron charge, $Z_0 = 377\Omega$, $J = 2I/a^*b^*$ the averaged current density, a_w wiggler-field strength normalized by wiggler wave-number k_w, $\delta k_s = \frac{\omega_s}{c} - k_s$ the difference of k_s from that in vacuum, and $y(s)$ the UGF. The universal gain equation determines $y(s)$:

$$y'' = \pm\sqrt{e^{-2y} - (y')^2} + e^{-2y} - 2(y')^2, \tag{3}$$

with initial conditions,

$$y(0) = \log \frac{a|b|e_s(0)}{\kappa}, \tag{4}$$

$$y'(0) = \sin[\psi(0)]e^{-y(0)}, \tag{5}$$

where $e_s(0)$ is the initial normalized signal amplitude and $\psi(0)$ is the initial ponderomotive phase. It is noted that the positive and negative signs of eq. (3) correspond to the rf phase's negative and positive advance regimes, respectively. Equation (3) admits an analytic solution which is valid for both cases,

$$y(s) = \frac{1}{2}\log\left\{e^{2y(0)} + 2\left[1 - e^{y(0)}\cos\psi(0)\right]\left[1 - \cos(s)\right] + 2e^{y(0)}\sin\psi(0)\sin(s)\right\}. \tag{6}$$

Using (6), the integration (2) is analytically performed to obtain the change in signal phase over the length of the cavity:

$$\Delta\varphi = \frac{s_c}{2} - \frac{u}{|u|}\left[\tan^{-1}\left(\frac{w\tan(s_c/2) + 2e^{y(0)}\sin\psi(0)}{|u|}\right) - \tan^{-1}\left(\frac{2e^{y(0)}\sin\psi(0)}{|u|}\right)\right], \tag{7}$$

where $u = e^{2y(0)} - 2e^{y(0)}\cos\psi(0)$ and $w = e^{2y(0)} - 4e^{y(0)}\cos\psi(0) + 4$.

Let us consider the next FEL interaction for the same signal where the next bunch is displaced in time by $2\tau = 2L_c/v_p$ (where $v_p = \omega_s/k_s$) from the first bunch. The second bunch sees the initial value of the signal amplitude, $[e_s(0)]_2 = [e_s(L_c)]_1$. Here the subscript stands for the order of passing of the wave packet through the cavity. From the definition of ponderomotive phase, $\psi = (k_s + k_w)z - \omega_s t + \varphi(z)$, we can write the initial value of ponderomotive phase for the macroparticle representing the second bunch by

$$\psi_2 = \left\{-\Delta\omega(2\tau) + \varphi_1(0) + (\Delta\varphi)_1\right\}_{mod\ 2\pi}, \tag{8}$$

where $\Delta\omega$ is the difference between bunching frequency and synchronous frequency. Combining these considerations, we obtain a recursion form for succeeding FEL interactions,

$$y_{i+1} = \frac{1}{2}\log\left\{e^{2y_i} + 2[1 - e^{y_i}\cos\psi_i][1 - \cos(s_c)] + 2e^{y_i}\sin\psi_i\sin(s_c)\right\}, \tag{9}$$

where

$$\psi_i = \{-(i-1)(2\Delta\omega\tau) + \varphi_1(0) + \sum_{k=1}^{i-1}(\Delta\varphi)_k\}_{mod\ 2\pi}, \tag{10}$$

$$(\Delta\varphi)_k = \frac{s_c}{2} - \frac{u_k}{|u_k|}\left[\tan^{-1}\left(\frac{w_k\tan(s_c/2) + 2e^{y_k}\sin\psi_k}{|u_k|}\right) - \tan^{-1}\left(\frac{2e^{y_k}\sin\psi_k}{|u_k|}\right)\right], \tag{11}$$

$$u_k = e^{2y_k} - 2e^{y_k}\cos\psi_k,\ w_k = e^{2y_k} - 4e^{y_k}\cos\psi_k + 4.$$

Here y_i and ψ_i stand for the values of UGF and ponderomotive phase, respectively, at the beginning of the FEL interaction with i-th bunch. From the above results, one can easily determine how the TE_{01} mode in a single cavity is excited due to succeeding FEL interactions; the normalized signal amplitude is given by

$$[e_s(L_c)]_i = \frac{\kappa}{a|b|}e^{y_{i+1}}, \tag{12}$$

while the energy accumulated per unit length after i-th FEL interaction is given by

$$W_i = \frac{a^*b^*}{2cZ_0}\left(\frac{mc^2}{e}\right)^2\left[e_s(L_c)\right]_i^2,$$

$$W_i = (Z_0I^2)\frac{\kappa^2a_w^2}{2c\gamma^2a^*b^*[k_w - \delta k_s - \frac{1}{2}(\frac{\omega_s}{c})(\frac{a_w}{\gamma})^2]^2}e^{2y_{i+1}}. \tag{13}$$

For a typical example, using the parameters listed in table 1, the formulas after eq. (7) give $a = 0.0238m^2$ and $|b| = 5.4407m^{-1}$. For these values we calculate $y_1 = -4.693$, and the analytic model estimates the microwave accumulated in the single cavity as depicted in fig. 1a. The evolution of related parameters is shown in fig. 1b. One may observe two aspects from figs. 1a and b; namely, (i) saturation in the excitation and, (ii) almost uniform phase advance for $i \geq 2$.

To understand these aspects, we proceed to further qualitative discussions on the recursion eq. (9) by expressing it as,

$$e^{2y_{i+1}} = e^{2y_i} + 4\sin^2(s_c/2) + 4\sin(s_c/2)\cdot e^{y_i}\cdot\sin(\psi_i - s_c/2). \tag{14}$$

From eq. (14), we identify a saturation condition beyond which the microwave can not grow in the cavity and microwave energy goes back into beam energy:

$$\sin(\psi_{i_{sat}} - s_c/2) = -e^{-y_{i_{sat}}}\sin s_c/2. \tag{15}$$

Saturation means that a new incoming bunch stays in the accelerating phase. This is an unavoidable result which occurs due to phase shifting and is one of the notable features of the SW/FEL. If the magnitude of the right-hand side in eq. (15) is much smaller than unity, that is, if the signal amplitude at saturation is large, the condition reduces to

$$\psi_{i_{sat}} \sim \frac{s_c}{2}. \tag{16}$$

From the assumption of small initial power, namely $e^{y_1} \ll 1$, we have

$$e^{y_2} = 2\sin(s_c/2), \qquad (\Delta\varphi)_1 = (s_c + \pi)/2 - \psi_1.$$

Using this we obtain an expression for $(\Delta\varphi)_i$ which is valid for $i \geq 2$:

$$(\Delta\varphi)_i = \frac{s_c}{2} - \tan^{-1}\left[\frac{(e^{y_i} - 2\cos\psi_i)\tan(s_c/2)}{e^{y_i} + 2\tan(s_c/2)\sin\psi_i}\right]. \tag{17}$$

From eq. (8) we know that $\psi_2 = -(2\Delta\omega\tau) + \psi_1 + (\Delta\varphi)_1 = -(2\Delta\omega\tau) + (s_c + \pi)/2$. This means that the initial ponderomotive phase for the second bunch does not depend on the initial ponderomotive phase for the first bunch. Then,

$$(\Delta\varphi)_2 = \frac{s_c}{2} - \tan^{-1}\left[\frac{\tan(s_c/2) + \tan(-\Delta\omega\tau)}{1 - \tan(s_c/2)\tan(-\Delta\omega\tau)}\right] = \Delta\omega\tau = -\alpha, \tag{18}$$

where $\alpha = -\Delta\omega\tau$. As mentioned earlier, numerical iterations of the recursion equation demonstrate that $(\Delta\varphi)_i$ is almost constant for $i \geq 2$, and is equal to $-\alpha$. It seems difficult to prove this for larger values of i by a mathematically simple approach because y_i and ψ_i are strongly correlated with each other through eqs. (9) and (10). Nevertheless, we çan ratify these features in an approximate way.

From eq. (10) it turns out that the ponderomotive phase ψ_i is a uniformely varying step function of i for $i \geq 2$:

$$\psi_i = -(i-1)(2\Delta\omega\tau) + \varphi_1(0) + (\Delta\varphi)_1 + (i-2)(\Delta\omega\tau),$$

$$\psi_i = -i|\alpha| + \frac{s_c}{2} + \frac{\pi}{2}. \tag{19}$$

Here, the case of $\alpha \geq 0$ is ruled out using the stability analysis discussed in Section 4. Accordingly, we understand that i_{sat}, corresponding to the saturation condition eq. (16), satisfies

$$i_{sat} = Int\left[\frac{\pi}{2|\alpha|}\right]. \tag{20}$$

In addition, it is possible to evaluate approximate forms for $y_{i_{sat}}$ or $e^{y_{i_{sat}}}$, which is proportional to the signal amplitude. For large signal amplitude eq. (14) can be simplified to:

$$e^{y_{i+1}} = e^{y_i} + 2\sin(s_c/2)\cos(|\alpha|i). \tag{21}$$

The solution of eq. (21), which is valid for $i \geq 2$, is given by

$$e^{y_{i+1}} = \sin(s_c/2)\left\{\csc\frac{|\alpha|}{2}\sin\left[(i+1/2)|\alpha|\right] + 1 + 2\cos|\alpha| - 4\cos^2|\alpha|\right\}, \tag{22}$$

where $e^{y_2} = 2\sin(s_c/2)$ and $e^{y_3} = 4\sin(s_c/2)\cos|\alpha|$ are used. Using $\sin|\alpha|/2 \sim |\alpha|/2$, we have

$$e^{y_{i_{sat}+1}} \simeq 2\sin(s_c/2)\left[\frac{1}{|\alpha|} - \frac{1}{2}\right]. \tag{23}$$

Equations (20) and (23) indicate that the saturation depends on the magnitude of α. For the example, for the typical case we know that $y_2 = -.706$ and $|\alpha| = 6.0°$. Accordingly, equations (20) and (23) tell us that $i_{sat} = 15$ and $e^{2y_{i_{sat}}} = 20.0$. These estimates are in fairly good agreement with the exact solution shown in figures 1a and 1b.

3. Multi-Cavity Theory

The single cavity theory can be extended to the multi-cavity system of the SW/FEL Two-Beam Accelerator in a straightforward manner. Recursion formulas describing the spatial evolution of energy and ponderomotive phase for macroparticles are essentially the same as those in a multi-stage klystron-like FEL (or a multi-stage FEL in the amplifier mode) which have been already given in Ref. 4. Accordingly, the recursion formulae in the multi-cavity system which meet the requirement of energy conservation and continuity of beam phase, $\theta = (k_s+k_w)z - \omega_s t$, are written as

$$\gamma_{n+1,i} = \gamma_{n,i} + \frac{mc^2}{eZ_0 J\kappa}\left\{\left([e_s(0)]_{n,i}\right)^2 - \left([e_s(L_c)]_{n,i}\right)^2\right\} + (\Delta\gamma)_{n,i}, \tag{24a}$$

$$\psi_{n+1,i} = \psi_{n,i} - |b(\gamma_{n,i})|L_c + [\varphi(0)]_{n+1,i} - [\varphi(0)]_{n,i}, \tag{24b}$$

$$[\varphi(0)]_{n,i} = [\varphi(0)]_{n,i-1} + (\Delta\varphi)_{n,i-1}, \tag{24c}$$

$$[e_s(0)]_{n,i} = [e_s(L_c)]_{n,i-1}, \tag{24d}$$

where n stands for the stage or cavity number. Using the definition of $y(s)$, eq. (24d) reduces to the initial condition of the UGE,

$$y_{n,i} = \log\left\{\frac{a(\gamma_{n,i})|b(\gamma_{n,i})|}{a(\gamma_{n,i-1})|b(\gamma_{n,i-1})|}\right\} + y_{n,i-1}. \tag{25}$$

Here, energy replenishment $mc^2(\Delta\gamma)_{n,i}$ at the end of each stage is an externally controlable parameter; for instance, in Ref. 3, it has been chosen to be a constant value of $e\sum_{k=1}^{i_{max}} W_{1,k}/(Ii_{max})$.

For the purpose of investigating spatial and temporal evolution of the beam and excited microwaves, a perfect energy replenishment is assumed; each bunch (or macroparticle) enters into the next cavity with the initially assumed energy γmc^2. This simplification elminates eq. (24a) and the logarithmic term of eq. (25) from the recursion relations. For a typical example with $i_{max} = 20$ and $n_{max} = 20$, the remaining recursion formulas give very interesting results. Except for the first bunch, the spacial and temporal evolution of FEL parameters is almost identical in each cavity, as seen in figs. 2a and b. This feature is a characteristic of microwave FELs with low input power.

4. Stability Analysis

Stability of a beam bunch has been a key issue in multistage FELs because bunches propagate with a periodic transient process such as rapidly increasing rf capturing. We are concerned about whether or not a bunch can maintain tight bunching over many stages. To analyze the stability in a multistage klystron-like

FEL, the stability equation [8] has been derived based on the macroparticle approach. The equation is still valid for the present FEL in the oscillator mode and is given by

$$\xi''_{n,i} + \left(\frac{\omega_s}{c}\right)\frac{a_w^3}{\gamma^4}[e_s]_{n,i}\cos\psi_{n,i}(s)\xi_{n,i} = 0, \tag{26}$$

where $\xi_{n,i}$ stands for the oscillation amplitude of an electron moving around the i-th macroparticle (or bunch center) in the n-th cavity. Unlike the case of a FEL in the amplifier mode, here $\psi_{n,i}(s)$ does not change by a large amount within a cavity except for $i = 1$; in fact, $|\Delta\psi_{n,i}| = -s_c + (\Delta\varphi)_{n,i} \ll \pi/2$ for $i \geq 2$.

The stability is uniquely determined by the restoring coefficient which is proportional to $e_s \cos\psi$. As discussed in Sections 2 and 3, $[e_s]_{n,i}$ and $\psi_{n,i}$ do not depend on n but i alone; $[e_s]_{n,i}$ is always positive and sinusoidally changing with i; meanwhile, $\psi_{n,i}$ is a linearly varying step function of i. The macroparticle can be expected to be stable for $|\psi_{n,i}| < \pi/2$. This condition is equivalent to requiring $|i\alpha + \frac{s_c}{2} + \frac{\pi}{2}| \leq \frac{\pi}{2}$. If α is a negative number then stability ocurrs only for $i \geq |\frac{s_c}{2\alpha}|$. By choosing an appopriate α, we can have stability even for the first bunch. In contrast, the case of $\alpha \geq 0$ yields instability, even in the head of the beam, since $e_s \cos\psi < 0$. Thus it turns out that bunch spacing with a slightly different frequency from ω_s, that is, a non-zero α is crucial for bunch stability.

For comparison, typical examples of $\alpha > 0$ and $\alpha < 0$ are shown in fig. 3 as functions of i for the first cavity. One finds that defocusing ocurrs when $\alpha > 0$, as expected. If α is not sufficiently negative, the first few bunches are defocused. Even for a proper choice of α (i.e., sufficiently negative) the focusing is weak for the first bunches since the field e_s is small. In the macroparticle model it is not possible to determine the seriousness of this phenomenon, but in Ref. 2 multiparticle simulations showed that the matter is not serious. This result is quite reasonable, for weak focusing, or even defocusing, of the first few bunches will reduce their contribution to e_s, but soon the increasing e_s will strongly focus the rest of the bunches. Thus, the results obtained with the macroparticle model are applicable to real (multiparticle) bunches even when the first few are defocused.

5. Comparison with Previous Work

The results are now compared with previously reported work in Ref. 2. The continuous model of a standing-wave FEL, which has been developed in the Cartesian form of the signal field $\hat{a} = a_s e^{i\varphi} = (\hat{a_r}, \hat{a_i})$ (where $a_s = e_s\sqrt{2}c/\omega_s$) under the assumptions of continuous energy recovery and slightly detuned bunch spacing, tells us that the beam phase evolves linearly with i, that is, $\theta = \beta(2L_c)i$ (where $\beta = -\Delta\omega/v_p = \alpha/L_c$). The linear dependence of beam phase on i leads to a linearly varying rf phase: $\varphi = \pi/2 - \beta L_c i$. Then, we have

$$\psi = \theta + \varphi = \beta L_c i + \pi/2. \tag{27}$$

The above poderomotive phase expression is in agreement with the corresponding ψ_i in the present paper. Necessarily, this gives the same saturation pulse length, $L_p = \pi/|\beta|$, in both theories.

Meanwhile, substituting the dominant term in eq. (23), $\exp(y_{i_{sat}}) \simeq \frac{|b|L_c}{|\alpha|}$ (using $\sin(s_c/2) \simeq s_c/2$ and $\alpha = L_c\beta$), into eq. (13), we obtain a formula for energy deposited per unit length at saturation,

$$W_{i_{sat}} = \frac{Z_0 I^2 (a_w/\gamma)^2}{2ca^* b^*}\left(\frac{1}{\beta^2}\right). \tag{28}$$

This agrees with the expression for energy deposited per unit length derived from eq. (32) of Ref. 2 where the coupling coefficient D_x is assumed to be 1/2 and a missing factor of 1/16 is introduced [9].

In addition, the result of the stability analysis is consistent with the conclusion reached in Ref. 2. Thus, most of the results which have been obtained in the continuous model are reproduced in the present approach, where we have taken cavities with finite length.

6. Summary and Conclusions

As mentioned earlier, the analytic macroparticle theory relies on two important assumptions, $a_w \gg 1$ and $\delta\gamma/\gamma \ll 1$ where $\delta\gamma mc^2$ is energy loss per cavity. Therefore, its validity depends simply on the reasonability of these assumptions. A large

a_w is in general accompanied with large beam energy for a fixed rf frequency. Such large beam energy is likely to reduce the relative size of energy loss, $\delta\gamma/\gamma$. In addition, the energy conservation law tells us that the change in energy is proportional to the amplified power, namely, I^2. Accordingly, it is expected that the validity of the macroparticle theory improves inversely proportional to I^2, because the drift coefficient in beam phase is an inverse-funtion of γ^2. This is confirmed by comparison with the solution of numerically integrated KMR equations. Figure 4 depicts the ponderomotive phase evolution of the saturation bunch, the 15-th macroparticle, through several cavities for three different values of current. As expected, the agreement between the numerical and analytical results is better for lower beam currents.

We have obtained a fully analytic FEL theory for a well-prebunched beam. Since the theory takes account of basic aspects of discreteness such as the finite size of cavity, the obtained result clearly elucidates the dependence of the FEL performance on these parameters.

Error sensitivity analysis of a multistage FEL, using the macro-particle theory is of great interest. This will be given in a forthcoming paper. For the purpose of further confirming the theory, an experiment in which (say) a single stage is driven by a well-bunched beam is most desirable.

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Table 1 FEL parameters

Injected beam current	I	0.6kA
Injection beam energy (normalized)	γ	24.81
Normalized wiggler amplitude	a_w	6.926
Wiggler wave length	$2\pi/k_w$	$0.26m$
Cavity length	L_c	$9.2cm$
RF frequency	$\omega_s/2\pi$	$17.1GHz$
Bunch spacing parameter	$2\Delta\omega\tau$	$12°$
Initial RF energy per unit lenght	$W(0)$	$1.7 \times 10^{-5} J/m$
Waveguide dimension	$a^* \times b^*$	$0.1 \times 0.03m^2$

References

[1] For instance, P. Luchini and H. Motz, Undulators and Free-Electron Laser (Clarendon Press, Oxford, 1990) 247.

[2] A.M. Sessler, D.H. Whittum, J.S. Wurtele, W.M. Sharp and M.A. Makowski, Nucl. Inst. and Meth. A306 (1991) 592.

[3] G. Rangarajan, A.M. Sessler and W.H. Sharp, to be published in Proceedings of the 13-th Int. FEL Conf. (Santa Fe, 1991).

[4] N.M. Kroll, P.L. Morton and M.N. Rosenbluth, IEEE J. Quantum Electron. 17 (1981) 1436, and D. Prosnitz, A. Szoke and V.K. Neil, Phys. Rev. A24 (1981) 1436.

[5] K. Takayama, Phys. Rev. Lett. 63 (1989) 516, and K. Takayama, Part. Accel. 36 (1992, to be published).

[6] A.M. Sessler, D.H. Whittum and J.S. Wurtele, Part. Accel. 31 (1990) 69.

[7] The approach requires a relatively large vector potential of the wiggler field and an implicitely tapered wiggler. Here, the latter condition should be read as uniform beam energy during FEL interaction. This may be logically justified by the observation that the FEL interaction is not sensitive to a relatively small change in beam energy through a short interaction region.

[8] S. Hiramatsu, K. Ebihara, Y. Kimura, J. Kishiro, T. Ozaki, K. Takayama and H.Kurino, Part. Accel. 31 (1990) 75.

[9] D.H. Whittum, private communication (1992). In Ref. 2, equation 32 has a misprint (the factor $2D_x$ should be $D_x/2$, and consequently, there should be a factor of 1/16 in the given formula). The tables and figures are correct.

Figure Captions

Fig.1a Normalized signal amplitude, e_s, in the first cavity. The analytical result (eq. 6) is shown by a solid line. Numerical simulation result with a macroparticle is indicated by a broken line.

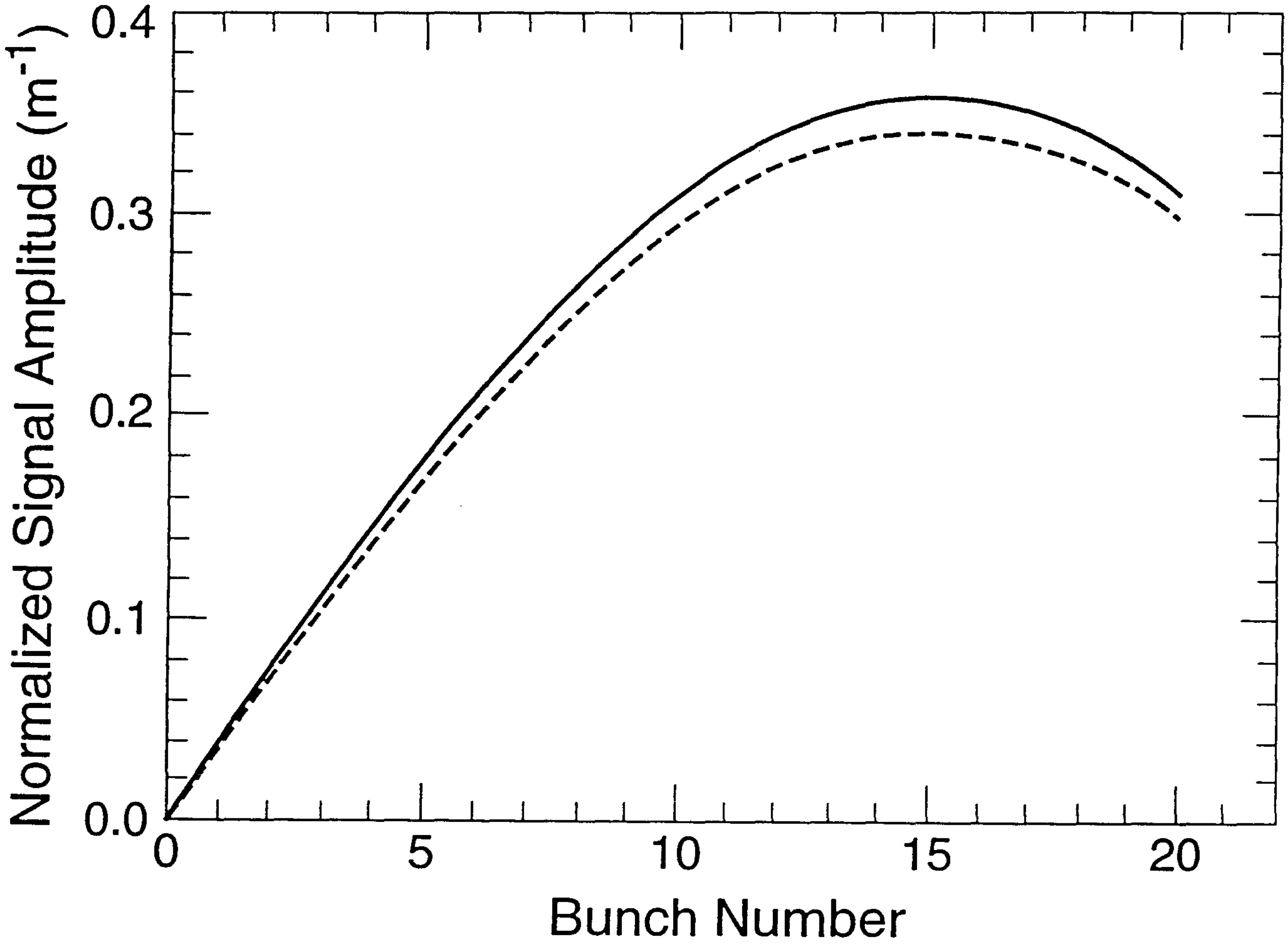
Fig.1b Ponderomotive phase ψ_i,of succeeding macroparticles at the start of the first cavity, and rf phase advance through the cavity, $\Delta\varphi_i$,for succeeding macroparticles. The analytical results (eqs. 10 and 11) are shown by the solid lines and numerical results with a macroparticle are given by the broken lines.

Fig.2a Energy deposited, W_i, and ponderomotive phase, ψ_i, for the 10-th cavity.

Fig.2b Energy deposited, W_i, and ponderomotive phase, ψ_i, for the 20-th cavity.

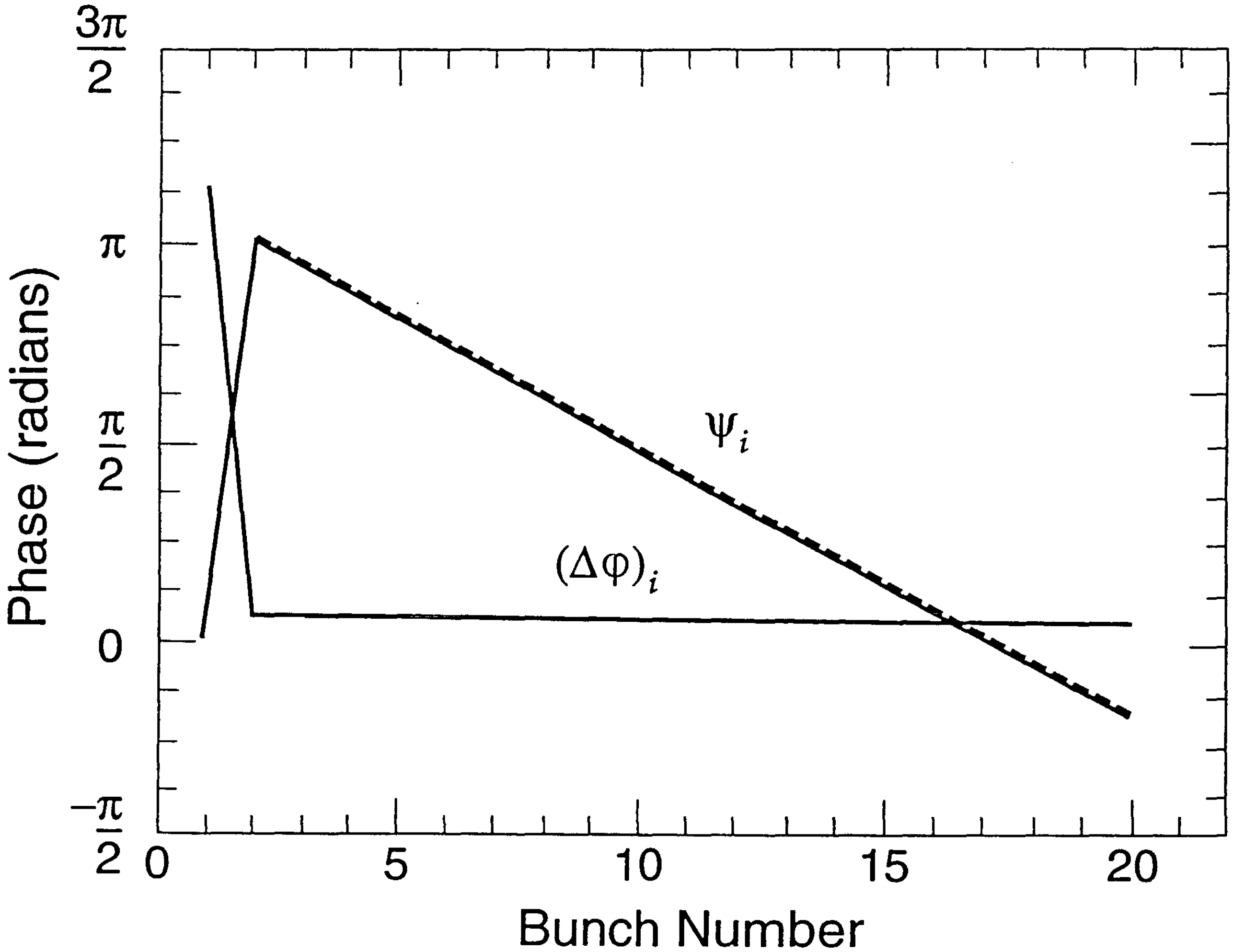
Fig.3 Restoring coefficient, $e^{y_i} \cos\psi_i$, for succeeding macroparticles in the first cavity, for various values of α.

Fig.4 Evolution of the ponderomotive phase for the 15-th macroparticle, $\psi_{n,15}$, as a function of cavity number for three different values of current, namely, 100 Amps, 600 Amps (the typical case), and 2 kAmps. The numerical results with a macroparticle are given by the broken lines.



XBL 922-5305

Fig. 1a



XBL 922-5302

Fig. 1b

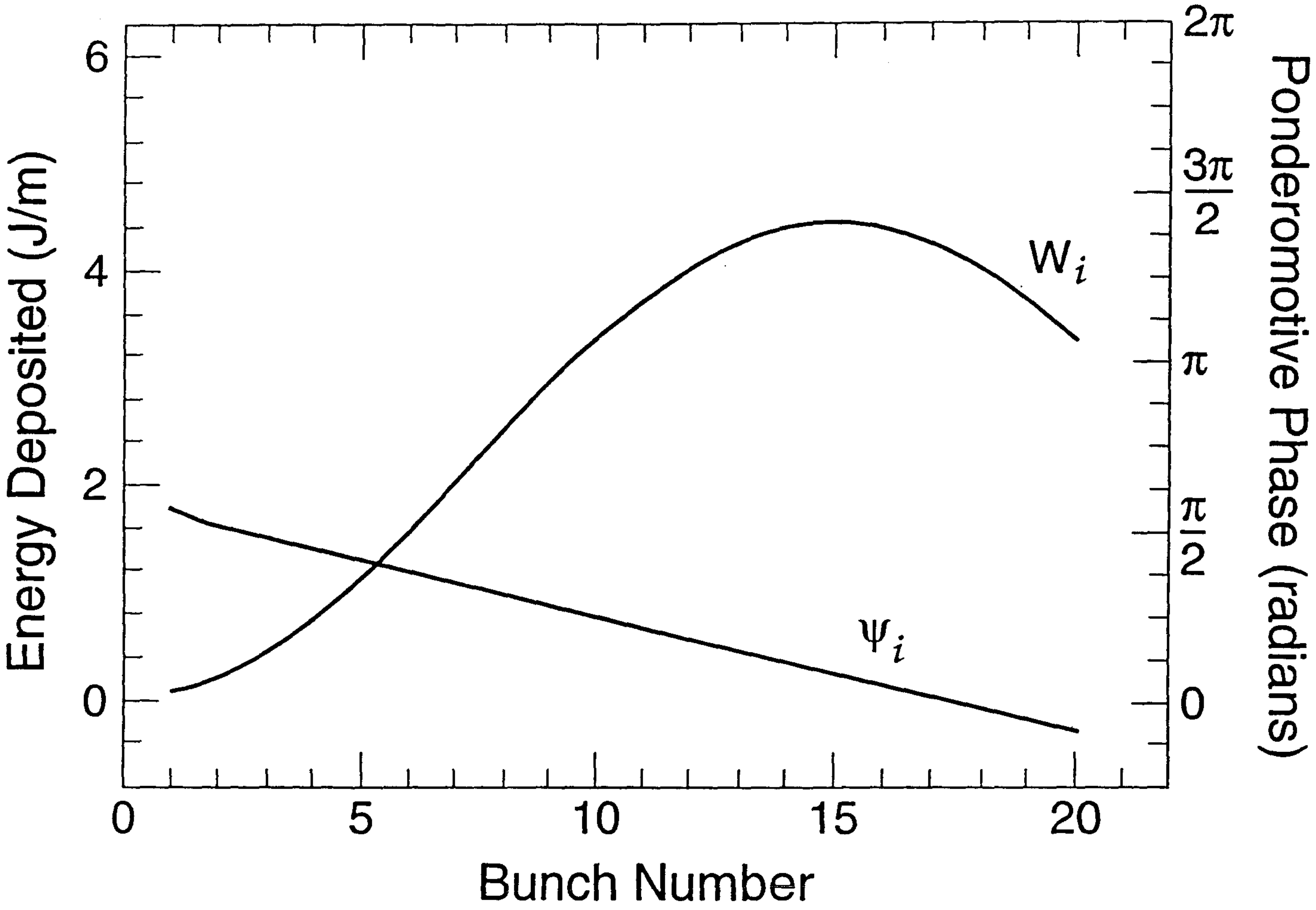


Fig. 2a

XBL 922-5304

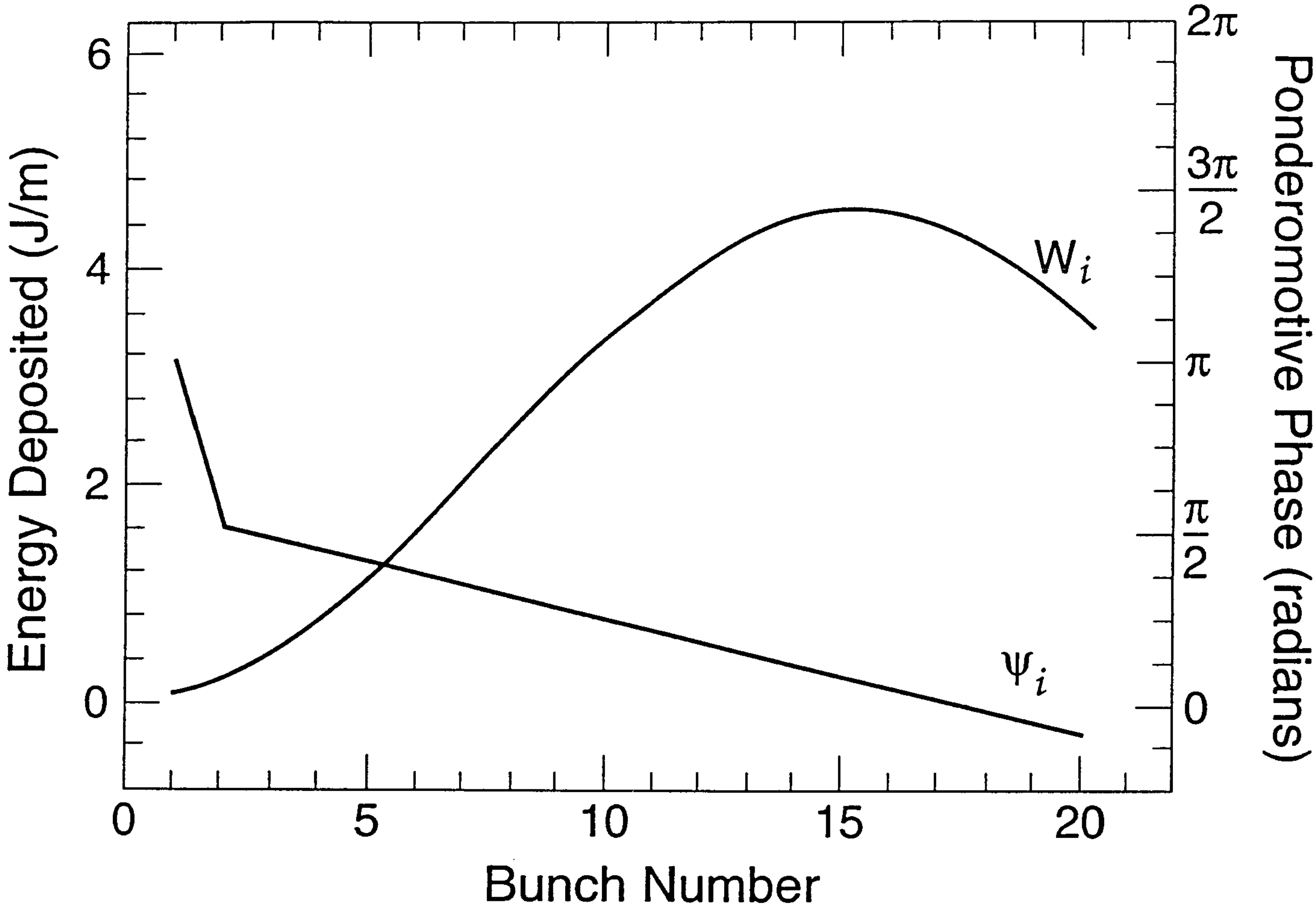


Fig. 2b

XBL 922-5303

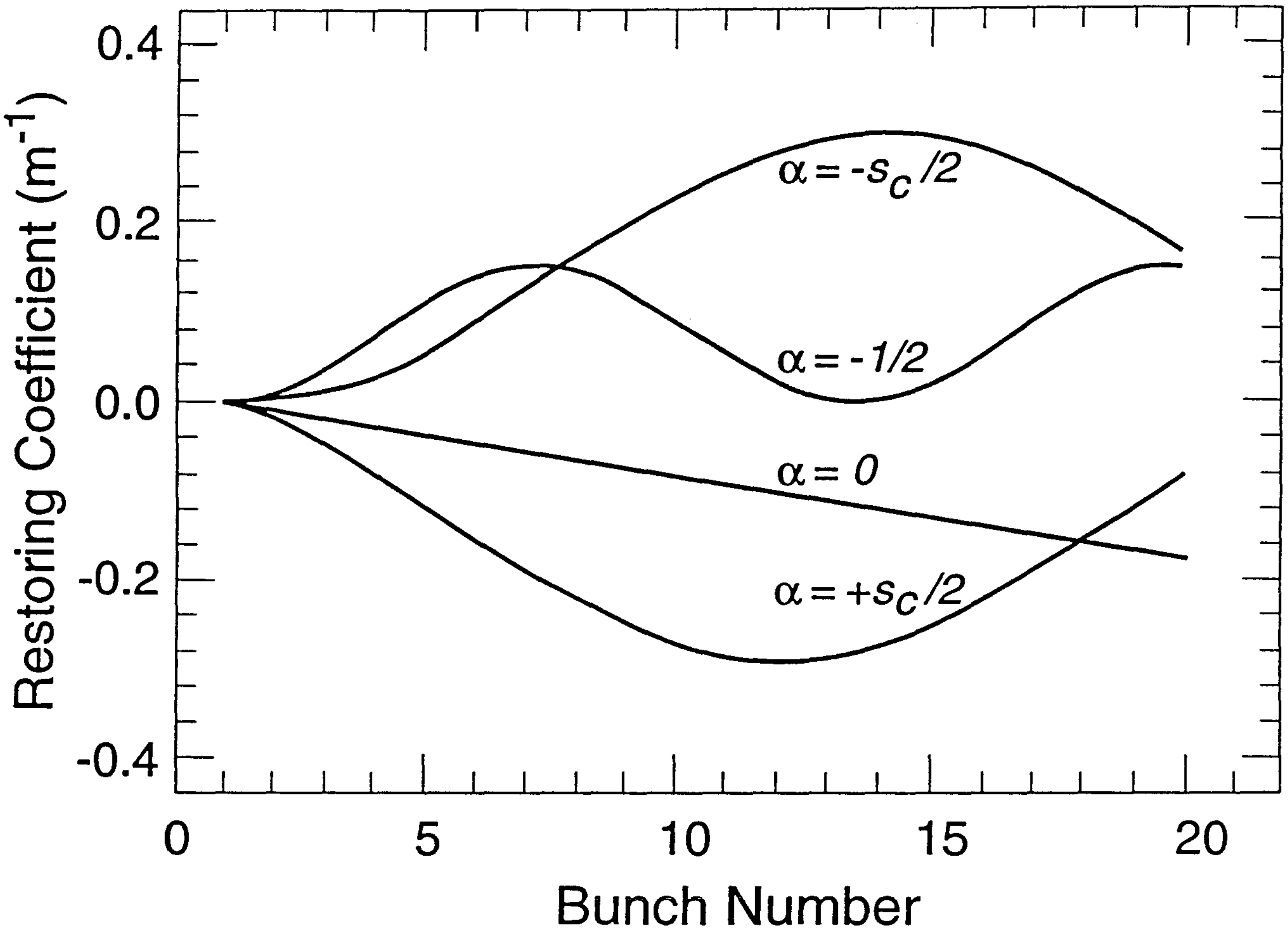
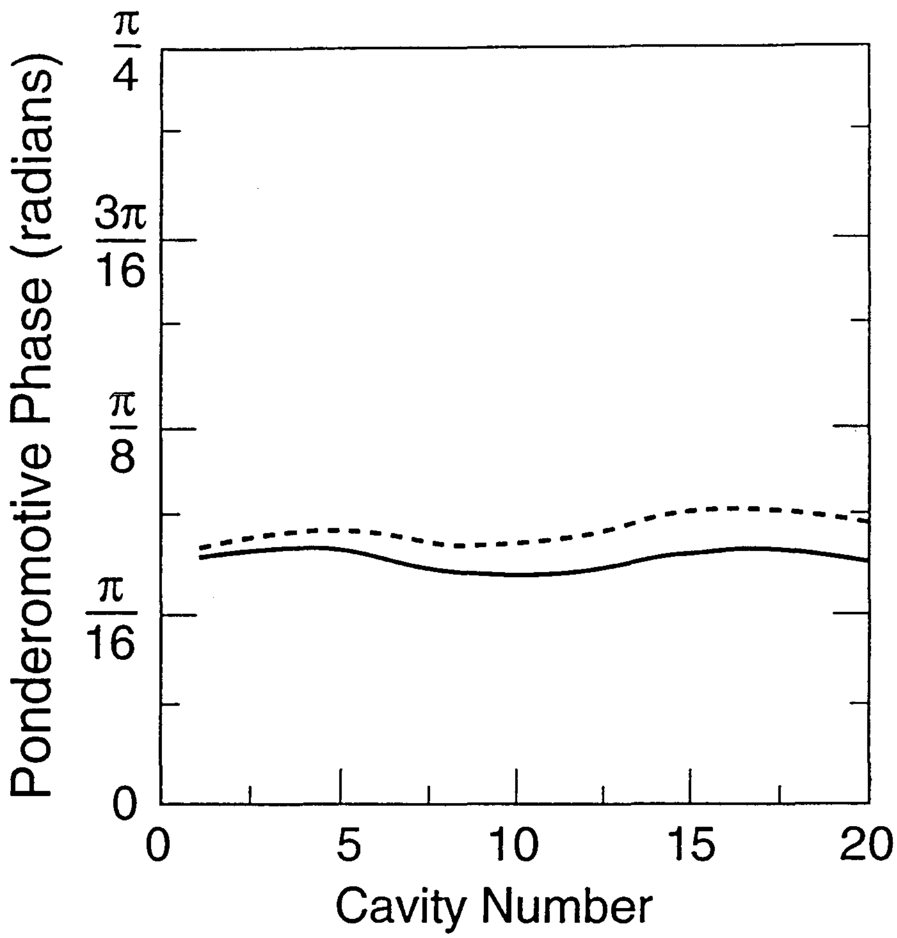


Fig. 3

XBL 922-5301



XBL 922-5306

Fig. 4a

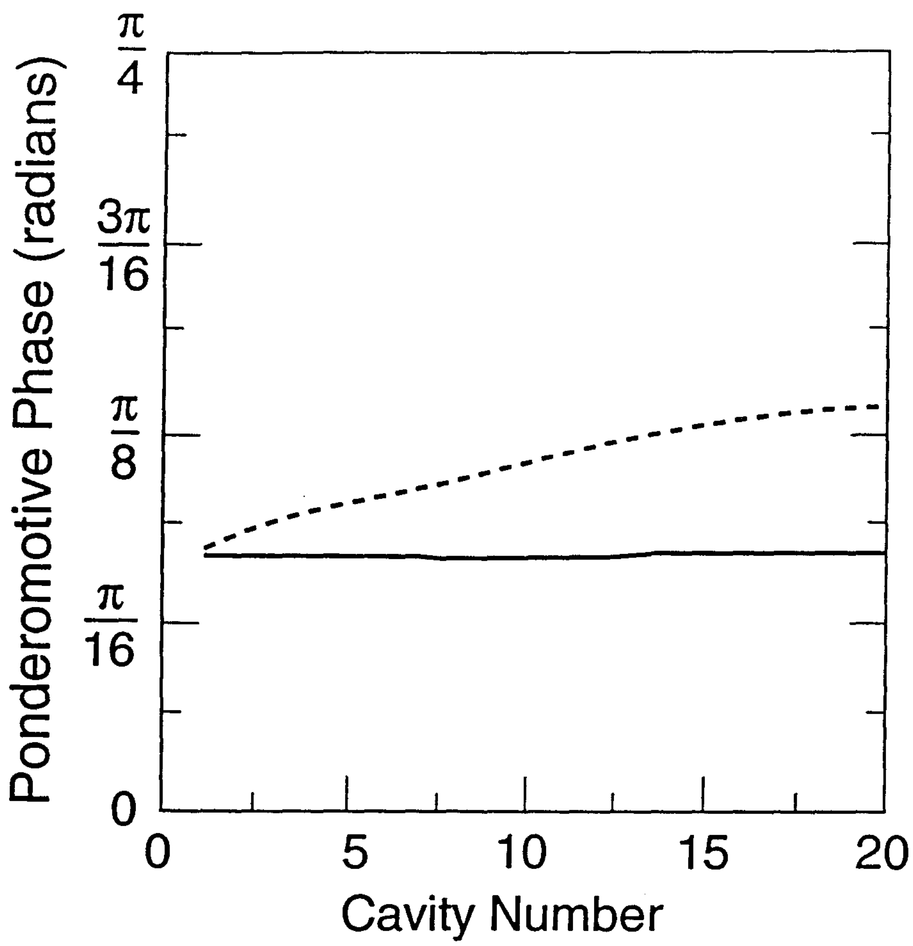


Fig. 4b

XBL 922-5307

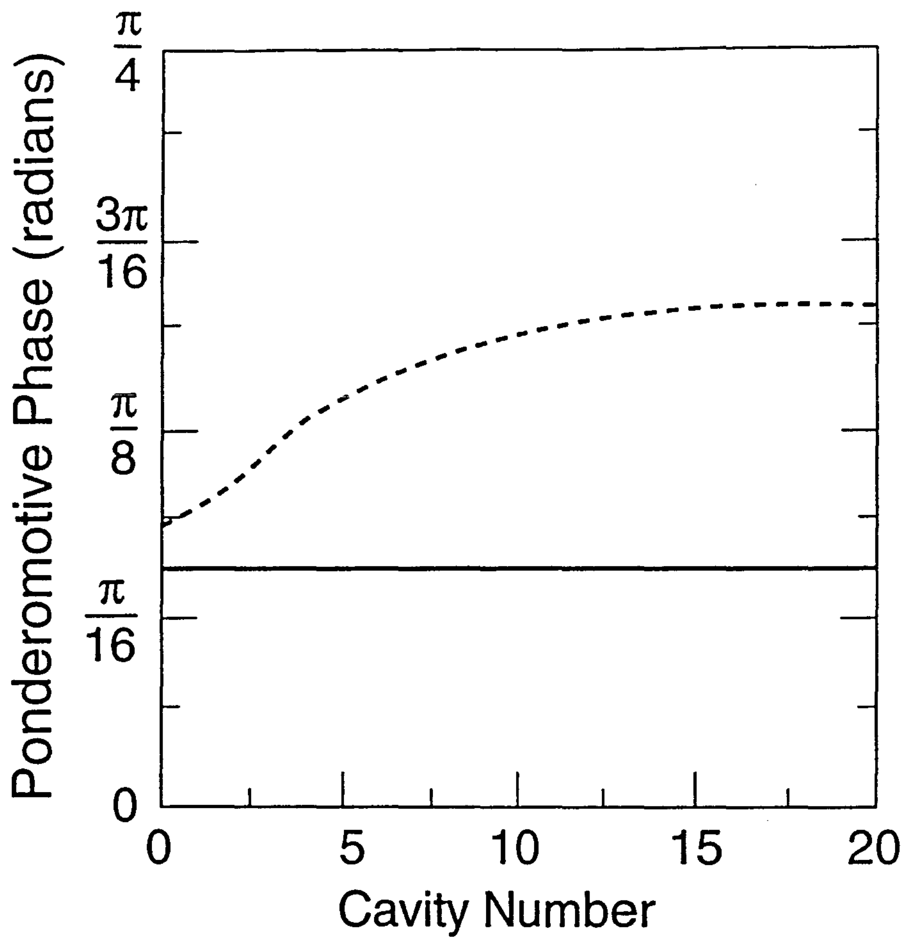


Fig. 4c

XBL 922-5308