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Authors

Zhou, Xinlin
Chen, Chuansheng
Chen, Lan
[et al.](#)

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Brief article

Holistic or compositional representation of
two-digit numbers? Evidence from the distance,
magnitude, and SNARC effects
in a number-matching task ☆,☆☆

Xinlin Zhou ^{a,b}, Chuansheng Chen ^c, Lan Chen ^a, Qi Dong ^{a,*}

^a *State Key Laboratory of Cognitive Neuroscience and Learning, Beijing Normal University,
Beijing 100875, China*

^b *Institute of Cognitive Neuroscience, University College London, London WC1N 3AR, UK*

^c *Department of Psychology and Social Behavior, University of California, Irvine, CA 92697-7085, USA*

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Abstract

Whether two-digit numbers are represented holistically (each digit pair processed as one number) or compositionally (each digit pair processed separately as a decade digit and a unit digit) remains unresolved. Two experiments were conducted to examine the distance, magnitude, and SNARC effects in a number-matching task involving two-digit numbers. Forty undergraduates were asked to judge whether two two-digit numbers (presented serially in Experiment 1 and simultaneously in Experiment 2) were the same or not. Results showed that, when numbers were presented serially, unit digits did not make unique contributions to the magnitude and distance effects, supporting the holistic model. When numbers were presented simultaneously, unit digits made unique contributions, supporting the compositional model. The SNARC (Spatial-Numerical Association of Response Codes) effect was evident for the whole numbers and the decade digits, but not for the unit digits in both experiments, which indicates that two-digit numbers are represented on one mental number line. Taken together,

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* Corresponding author. Tel.: +8610 58806154; fax: +8610 58807615.
E-mail address: dongqi@bnu.edu.cn (Q. Dong).

these results suggested that the representation of two-digit numbers is on a single mental number line, but it depends on the stage of processing whether they are processed holistically or compositionally.

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Keywords: SNARC effect; Number cognition; Representation; Two-digit number processing

1. Introduction

One of the most important issues in the area of numerical processing is how numbers are represented in the memory. Up to now, three effects have been extensively investigated to reveal the nature of mental representation of numbers: the distance effect, the problem-size or magnitude effect, and the SNARC (Spatial-Numerical Association of Response Codes) effect (Brybaert, 1995; Dehaene, Bossini, & Giraux, 1993; Dehaene, Dupoux, & Mehler, 1990; Fias, Brybaert, Geypens, & d'Ydewalle, 1996). The distance effect (e.g., Moyer & Landauer, 1967) is revealed when subjects take longer and/or make more errors to process (e.g., to compare) two numbers close to each other in magnitude (e.g., 5 vs. 6) than to process two numbers farther away from each other (e.g., 3 vs. 8). The problem-size or magnitude effect means that subjects have more difficulty processing larger numbers than smaller ones (e.g., Brybaert, 1995). Because these two effects rely on the magnitude or semantic representation of numbers, they have been used as an index of semantic processing of numbers. That is, their existence in a particular task would be considered as evidence of semantic processing, whereas their absence has been interpreted as a lack of semantic processing (Zhou et al., *in press*).

In contrast with the distance and problem-size effects, the SNARC effect reveals not only semantic processing, but also the shape of mental representation of numbers. The SNARC effect is revealed when subjects respond more quickly to smaller numbers with the left hand, and more quickly to larger numbers with the right hand on a number processing task (e.g., parity-judgment – whether a number is odd or even) (Dehaene et al., 1993). The SNARC effect suggests that numbers are stored in some type of a mental number line with smaller numbers on the left side and larger numbers on the right side (e.g., Dehaene et al., 1993; Restle, 1970; Seron, Pesenti, Noel, Deloche, & Cornet, 1992; Zorzi, Priftis, & Umiltà, 2002).

Most studies of semantic or spatial representation of numbers have focused on one-digit numbers. Recently, researchers have debated about the nature of mental representation of two-digit numbers (see a review by Nuerk & Willmes, 2005). There are two representative models. The first model is the holistic model: That is, two-digit numbers are represented holistically (Brybaert, 1995; Dehaene et al., 1990; Reynvoet & Brybaert, 1999). For example, Reynvoet and Brybaert (1999) found that the priming effect is constant for a given numerical distance regardless of whether the priming number is single- or two-digit number. For example, the priming effect of 7 on the target 9 is the same as the priming effect of 11 on the target 9, suggesting that 11 is indeed as close to 9 as 7 is on the mental number line. If 11 had

been represented compositionally, either the decade or the unit digit (1) would reduce its priming effect because of its large distance from 9.

The second model is the compositional model in which the decade and unit digits of two-digit numbers are represented separately, perhaps with an additional holistic representation (Nuerk, Weger, & Willmes, 2001). The key evidence in support of this model comes from the congruence effect during the comparison of two-digit numbers (Nuerk et al., 2001). A number pair is defined as congruent or compatible when both the decade and unit digits of one number are greater than those of the other number (e.g., 67 and 52). The incongruent pairs have one number with a greater decade digit and the other number with a greater unit digit (e.g., 62 and 47). The congruence effect occurs when it is more difficult for subjects to respond to incongruent number pairs than to congruent pairs after controlling for overall numerical distance and size. The existence of the congruence effect suggests that subjects pay attention to the value of the compositions, not just the whole number. Further support for the compositional model came from two recent studies (Verguts & De Moor, 2005; Zhang & Wang, 2005). Verguts and De Moor (2005) found that the numerical distance effect was evident for unit digits when the decade digits were the same (e.g., 54 vs. 55, 51 vs. 59). When the decade digits differed by 1 (e.g., 59 vs. 61, 57 vs. 64), which was the only other condition, there was no distance effect for the whole numbers. Finally, Zhou et al. (in press) found that reaction times in a number-comparison task were a function of both the whole numbers and the unit digits.

Given the conflicting evidence as reviewed above, it remains unresolved whether two-digit numbers are processed holistically or compositionally (Fias & Fischer, 2005; Gevers & Lammertyn, 2005). The present study aimed to clarify the nature of mental representation of two-digit numbers by making three improvements on the previous research. First, most previous studies used either number-comparison (Dehaene et al., 1990; Nuerk et al., 2001; Verguts & De Moor, 2005; Zhang & Wang, 2005) or parity-judgment tasks (Dehaene, 1993) to investigate the nature of representation of two-digit numbers. Such tasks focus the subjects' attention on the compositions, rather than the entirety, of the two-digit numbers. In a parity-judgment task, subjects would only need to focus their attention on the unit digits. In a number-comparison task, subjects are likely to pay most attention to the decade digits. Only when the decade digits were the same, do the subjects pay attention to the unit digits. In our study, to ensure that subjects pay attention to both decade and unit digits, we used a number-matching paradigm. When the two two-digit numbers were matched or the same (e.g., 12 vs. 12), subjects had to match both decade and unit digits.

The second improvement of our study over the previous research is that we examined several effects. In addition to the distance and magnitude effects examined in previous research, we also examined the SNARC effect. Researchers have demonstrated the SNARC effect for two-digit numbers, but have not used it to investigate whether the numbers are holistically or compositionally represented. For example, Dehaene et al. (1990) asked their participants to press a left or right key to indicate whether visually presented two-digit numbers (from 31 to 99, excluding 65) were smaller or larger than a fixed reference number (65). They found a significant

SNARC effect. That is, if the target number was smaller than 65, it was faster for subjects to press the left key than the right key. Conversely, when the target number was larger than 65, it was to press the right key than the left key. These results were also confirmed (although only at a trend level) with a parity-judgment task (Dehaene et al., 1993) and a simultaneously-presented, number-comparison task (Brybaert, 1995). Brybaert (1995) asked subjects to compare two simultaneously presented two-digit numbers and found that subjects responded more quickly when the smaller number of the number pair was on the left side and the larger one on the right side than when it was the other way around.

Although these results demonstrated the SNARC effect for two-digit numbers, they did not examine whether two-digit numbers were processed holistically or compositionally. To address that issue, we need to decompose the SNARC effect into those parts that are associated with the whole number, the decade digits, and the unit digits.

Third, we also included the stage of processing as a variable. It is possible that representation of two-digit numbers may depend on the stage of processing.¹ In our first experiment, we presented the two two-digit numbers serially to ensure that the subjects processed the first two-digit number and kept it in their short-term memory before they attempted to match it with the second number. In our second experiment, two two-digit numbers were presented simultaneously so that both numbers are processed simultaneously in the working memory.

Regression analyses were used to examine the distance, magnitude, and SNARC effects (Fias et al., 1996). For the distance effect, reaction times for the non-matched numbers were regressed on the numerical distance based on either the decade digits or the unit digits. For the magnitude effect, the matching trials were used to ensure that subjects paid attention to the entire two-digit numbers. Reaction times for these matching trials were regressed on the whole numbers, decade digits, and unit digits. For the SNARC effect, the differences between the left and the right hands were regressed on the whole numbers, decade digits, and unit digits. Again only trials with matched number pairs were used in these analyses to ensure that subjects paid attention to both decade and unit digits and to avoid other potential confounding effects introduced by number pairs of different sizes. Based on the previous research that showed the distance, magnitude, and SNARC effects (see above for references), we expected the distance, magnitude, and SNARC effects to be significant when regressed on the whole numbers. Because decade digits and the whole numbers had very high shared variance ($r = .995$), decade digits are expected to show the same effects. Thus the critical results involved only the unit digits. According to the compositional model, unit digits would be a significant and unique predictor (after controlling for either the whole numbers or the decade digits) of the three types of number effects, whereas, according to the holistic model, they would not be.

¹ We thank Dr. Marc Brybaert (a reviewer of an earlier version of this manuscript) for this suggestion.

2. Experiment 1

2.1. Methods

2.1.1. Subjects

Twenty undergraduates (ten males and ten females) from Beijing Normal University were recruited. The average age was 21.5 years, ranging from 18.3 to 23.6 years. All participants had normal or corrected-to-normal eyesight. They did not participate in any experiments with number tasks during the past half a year prior to this study. They gave written informed consent before the experiment. After the experiment, each participant was paid RMB 20 yuan (about US\$2.5).

2.1.2. Materials

Seventy-two two-digit numbers from 12 to 98 (excluding numbers with repeated digits such as 22, 33, and 44 and those with zero as the unit digit such as 20, 30, and 40) were used to create matched and non-matched number pairs. For the non-matched number pairs, the two numbers differed only in one of the digits (e.g., 12 vs. 62, 12 vs. 18). There were 72 matched number pairs and 72 non-matched number pairs. Each number pair was presented six times (three times for each of two sessions, see below).

2.1.3. Procedure

Each subject performed the number-matching tasks in two different sessions. In one session, subjects used their left hand to make “Matched” response and their right hand to make “Non-matched” response. In the other session, the bimanual response pattern was reversed. The order of the response pattern was counterbalanced across subjects. Within each session, the trials were split into two four-minute blocks with a one-minute break in between. A ten-minute break was provided between the two sessions.

Subjects were seated in a sound-attenuated room, facing a screen 60 cm away. All the stimuli were presented visually in black against grayish background at the center of the screen. For each trial, a two-digit number (3.5×3 cm in size) was first presented for 150 ms, followed by a 1350 ms blank screen. The second two-digit number was then presented in the same position on the screen as the first number. It remained on the screen until subjects made a judgment of “Matched” or “Non-matched”. After the response and a 1000 ms blank, the next trial began. Participants were encouraged to respond as quickly and accurately as possible.

Before the formal experiment, 20 practice trials (randomly selected from the 144 number pairs) were presented. During the practice, participants would be given a feedback if they committed too many response errors or took too long to respond. Within each block, there were also three warm-up trials that were not included in the analysis.

2.2. Results and discussion

One subject’s data were discarded because this subject pressed the incorrect keys during the second session. The reaction times of correct trials for each subject were

trimmed according to the three-standard-deviations rule. That is, trials (about 1.5%) were discarded because their reaction time was three standard deviations above or below individual subjects' mean reaction time. The reaction times for each matched number pair were averaged across the three trials, separately for each hand. Data were then averaged across subjects. The gross error rates were low (4.4%) and therefore, not analyzed further.

2.2.1. Non-matched number pairs (distance effect)

Fig. 1 shows the mean reaction times by numerical distance (ranging from 1 to 7) between the two numbers. Results showed no numerical distance effects for either decade ($r = -.30, p = .295$, one-tailed) or unit digits ($r = -.15, p = .376$, one-tailed). These results were further confirmed by grouping the number pairs into “large” distance (greater than 4) and “small” distance (less than 4). The mean reaction times (and standard deviations) for the large- and small-distance number pairs based on decade digits were 555 (23) and 551 (23) ms, respectively. The corresponding numbers based on unit digits were 556 (23) and 551 (22) ms. ANOVA revealed no significant distance effects. Because non-matched numbers differed only in one of the digits, not both, we examined distance effects only for the decade or unit digits, not for the whole numbers.

2.2.2. Matched number pairs (magnitude and SNARC effects)

In term of the magnitude effect, Fig. 2 shows the mean reaction times by hand and by the magnitude of the whole numbers, the decade, and unit digits. With increasing magnitude of the numbers, the reaction time increased for the left hand, but decreased for the right hand. This pattern was clear for both the whole number and the decade digits, but non-existent for the unit digits. Following the procedure proposed by Fias et al. (1996) to calculate the individual SNARC slopes, we conducted linear regression analysis on reaction times for the decade digits (from 1 to 9) and for the unit digits (from 1 to 9) for each subject. We then averaged the regres-

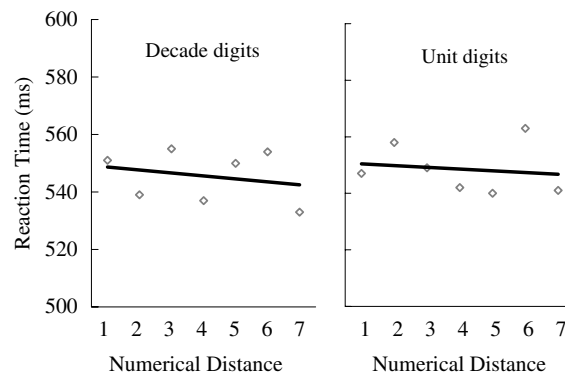


Fig. 1. Reaction times by numerical distance based on decade and unit digits of non-matched number pairs: Experiment 1 (serial presentation).

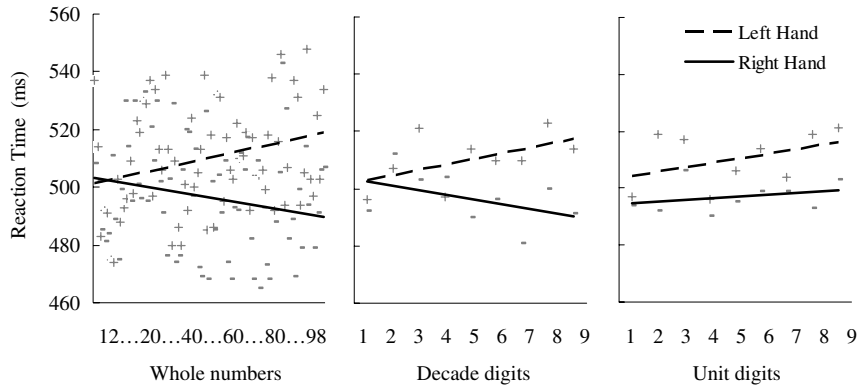


Fig. 2. Reaction times for each hand by the whole numbers, decade digits, and unit digits of matched number pairs: Experiment 1 (serial presentation).

sion slopes across subjects. The averaged regression slopes for decade and unit across two hands were .10 (.53) and 1.00 (.54). One-sample *T* test (Test value = 0) showed no any significant effects. There was also no difference between the two slopes. In multiple regression analysis, unit digits did not make a unique contribution to the variance of reaction times after controlling for the whole numbers.

To examine the SNARC effect, a difference score was created by subtracting the reaction time for each number for the left hand from that for the right hand. The difference scores were then averaged across the subjects. Fig. 3 shows the scatter plots and regression lines of the averaged difference scores by the two-digit numbers and their compositions. The SNARC effect (indicated by a significant negative correlation) was evident for the whole numbers ($r = -.35, p = .002$) as well as for the

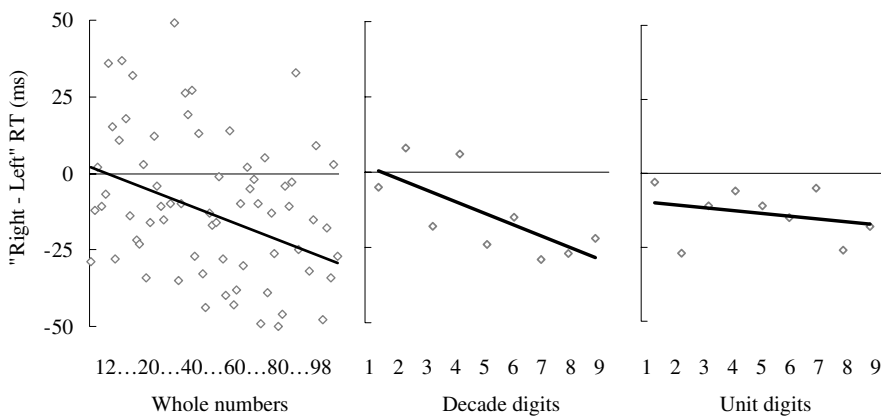


Fig. 3. “Right hand–left hand” differences in reaction times by the whole numbers, decade digits, and unit digits of matched number pairs: Experiment 1 (serial presentation).

decade digits ($r = -.34$, $p = .003$), but not for the unit digits ($r = -.08$, $p = .466$). After partially out the whole numbers, the unit digits did not show a unique contribution to the variance of the difference scores.

2.2.3. Summary

This experiment showed that there was little evidence for the compositional model because there were no significant distance, magnitude, and SNARC effects for unit digits. It appears that, under the condition of this study, two-digit numbers were represented holistically. A crucial aspect of this study was its serial presentation of the number pairs. Because the number pairs were presented serially with an SOA of 1.5 s, the first number was presumably already stored in the short-term memory. Based on the results of Experiment 1, we can tentatively conclude that two-digit numbers appear to be stored as whole numbers, rather than as their compositional decade and unit digits. The question remains, however, whether the two-digit numbers were initially processed as compositions or as the whole numbers. In Experiment 2, we presented the number pairs simultaneously.

3. Experiment 2

3.1. Methods

3.1.1. Subjects

Twenty undergraduates (ten males and ten females) from Beijing Normal University were recruited. The average age was 20.1 years, ranging from 18.0 to 21.9 years. The other characteristics for these subjects matched those for the subjects in Experiment 1.

3.1.2. Materials

The materials were the same as those in Experiment 1.

3.1.3. Procedure

The procedure used in present experiment was the same as that used in Experiment 1 with the following exceptions. The number pairs were presented simultaneously for 800 ms in this experiment. Subjects were asked to judge whether the two numbers matched or not. After subjects responded, there was a blank screen for 1500 ms before the next trial began. An auditory cue of 500 Hz lasting 300 ms and a blank of 200 ms preceded the next trial.

3.2. Results and discussion

The reaction times of correct trials for each subject were trimmed (about 1.4% trials) with the three-standard-deviations rule. The gross error rates (incorrect responses and slow responses) were low (5.9%) and therefore, not analyzed further.

3.2.1. Non-matched number pairs (distance effect)

Fig. 4 shows the mean reaction times by numerical distance. As the numerical distance increased, reaction times decreased, for both decade ($r = -0.57, p = .090$, one-tailed) and unit digits ($r = -0.73, p = .032$, one-tailed). When grouped into large- and small-distance number pairs (as in Experiment 1), ANOVA showed that response to the former was shorter than for the latter, $F(1, 19) = 37.05, p < .001$. Reaction times for large- and small-distance number pairs were, respectively, 571 (13) and 558 (11) ms for decade digits, and 584 (13) and 567 (14) ms for unit digits.

3.2.2. Matched number pairs (magnitude and SNARC effects)

Fig. 5 shows the mean reaction times by the whole numbers, decade digits, and unit digits. There was clear evidence of magnitude effects: Reaction times increased

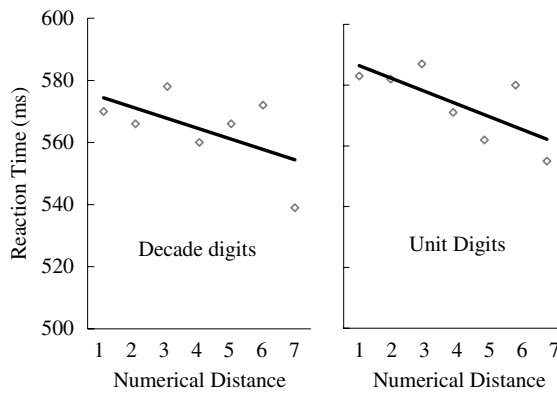


Fig. 4. Reaction times by numerical distance based on decade and unit digits of non-matched number pairs: Experiment 2 (simultaneous presentation).

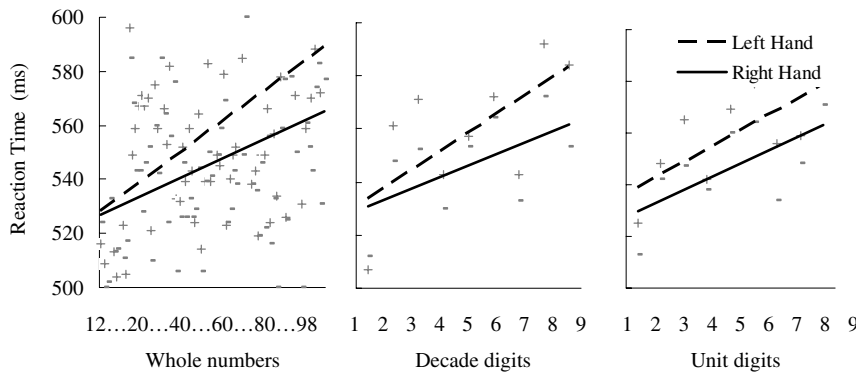


Fig. 5. Reaction times by the whole numbers, decade digits, and unit digits of matched number pairs: Experiment 2 (simultaneous presentation).

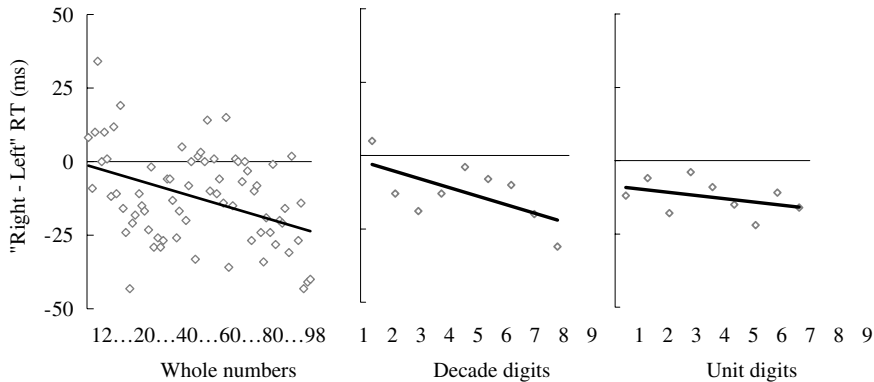


Fig. 6. "Right hand-left hand" differences in reaction times by the whole numbers, decade digits, and unit digits of matched number pairs: Experiment 2 (simultaneous presentation).

with the size of the numbers. This pattern was clear for the whole number, the decade digits and the unit digits. We conducted linear regression analysis on reaction times for decade digits (from 1 to 9) and for unit digits (also from 1 to 9) for each subject and then averaged the regression slopes across subjects. The averaged regression slopes for decade and unit digits across two hands are 5.08 and 4.62, respectively. One-sample T test (Test value = 0) showed that there were significant magnitude effects, $t(19) = 7.54$, $p < .001$ for decade digits; $t(19) = 9.32$, $p < .001$ for unit digits. In multiple regression analysis (regressing reaction times on both the whole numbers and unit digits), unit digits made a unique contribution to the variance of reactions times after controlling for the whole numbers. For the data from the left hand, the regression coefficient for unit digits was $B = 5.22$ ($SE = 1.38$), $\beta = .36$, $t = 3.77$, $p < .001$. That for the whole numbers was $B = .70$ ($SE = .14$), $\beta = .49$, $t = 5.03$, $p < .001$. For the right hand, coefficient for the unit digits was $B = 4.32$ ($SE = 1.32$), $\beta = .35$, $t = 3.28$, $p = .002$ and that for the whole numbers was $B = .45$ ($SE = .13$), $\beta = .35$, $t = 3.36$, $p = .001$.

To examine the SNARC effect, a difference score was created by subtracting the reaction times for the left hand from those for the right hand. Fig. 6 shows the scatter plots and regression lines of difference scores by the two-digit numbers and their compositions. The SNARC effect (indicated by a significant negative correlation) was evident for the whole number ($r = -.40$, $p = .001$) as well as for the decade digits ($r = -.38$, $p = .001$), but not for the unit digits ($r = -.13$, $p = .266$). After partially out the whole number, the unit digits also did not show a unique contribution to accounting for the variance of the difference scores.

3.2.3. Summary

When the number pairs were presented simultaneously, we found significant distance and magnitude effects for unit digits (unique effects after controlling for the whole numbers), indicating that the unit digits were processed separately from the whole numbers. However, the SNARC effect was not significant for unit digits, but it was significant for the whole numbers and the decade digits.

4. General discussion

The present study aimed to investigate whether two-digit numbers showed holistic or compositional representation. By using a number-matching paradigm, this study found that the mental representation of two-digit numbers depended on the stage of processing. The initial processing seemed to involve the compositions of the two-digit numbers as shown by the significant distance and magnitude effects for unit digits in Experiment 2 (simultaneous presentation). Once the numbers are stored in the short-term memory, however, they showed holistic representation, as shown by a lack of distance and magnitude effects for unit digits in Experiment 1 (serial presentation). In both experiments, however, the SNARC effect was significant only for the whole numbers and decade digits, but not for unit digits. This result suggests that two-digit numbers are represented on a single mental number line, not multiple number lines based on unit digits.

Based on the results of the current two experiments, we can also reconcile some of the conflicting results from previous studies. For example, Brysbaert and colleagues (Brysbaert, 1995; Reynvoet & Brysbaert, 1999) used serial presentation (i.e., asking subjects to name one- or two-digit numbers after a priming number was presented) and found evidence for the holistic model. Hinrichs et al. (1981) and Dehaene et al. (1990) asked subjects to compare target numerals with a standard (e.g., 55) that was held in the memory. They found that the reaction time decreased logarithmically with the absolute distance between the target and the standard. In contrast, Nuerk et al. (2001) and Verguts and De Moor (2005) used simultaneous presentation of number pairs and found evidence for the compositional model.

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