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Dynamics versus Symmetrization in Hadron Interferometry*

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Abstract:

The small angle correlation function of two indistinguishable particles produced in hadronic processes is shown in general to be unrestricted by Bose-Einstein or Fermi-Dirac statistics. The analogy of hadron interferometry to Hanbury-Brown and Twiss intensity interferometry can break down because of dynamical correlations between observed and unobserved fragments. Two physical examples in the context of relativistic nuclear collisions illustrate this point.

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Hadron interferometry¹⁻⁵ is similar to the photon intensity interferometry technique⁶ developed by Hanbury-Brown and Twiss to measure stellar radii. The normalized one and two particle inclusive distributions, $P_1(\underline{k})$ and $P_2(\underline{k}_1, \underline{k}_2)$, are measured for indistinguishable particles produced in a particular reaction. The correlation function, $R(\underline{k}_1, \underline{k}_2) = P_2(\underline{k}_1, \underline{k}_2)/P_1(\underline{k}_1)P_1(\underline{k}_2)$, is then analyzed in order to deduce the space-time history of that reaction. There have been several recent applications of this technique to hadron-hadron⁷⁻⁸ and nucleus-nucleus collisions.⁹⁻¹² In this letter I show, however, that such a space-time interpretation of hadron correlation data cannot be taken for granted. Specific dynamical features as well as the inherent ensemble averaging implicit in inclusive measurements can lead to unexpected correlations at small relative momentum that have nothing to do with the Hanbury-Brown and Twiss effect. This point is illustrated by two examples, final state shadowing and coherent jet production, in the context of nuclear collisions. In addition, a new formula for $R(\underline{k}_1, \underline{k}_2)$ is derived for partially coherent boson fields.

A space-time interpretation of $R(\underline{k}_1, \underline{k}_2)$ is usually based on the following considerations¹⁻⁵: Assume that the particles are produced at space-time points $x_i = (\underline{x}_i, t_i)$, which are distributed according to a normalized density, $\rho(x)$. Let $\psi_{\underline{k}}(x)$ be the single particle incoming scattering wavefunction⁵, incorporating possible final state interactions in an optical potential $V(x)$. The probability that two indistinguishable particles are observed with momenta $\underline{k}_1$ and $\underline{k}_2$ is then

$$P_2(\underline{k}_1, \underline{k}_2) \propto \int d^4x d^4y \rho(x) \rho(y) |\psi_{\underline{k}_1}(x) \psi_{\underline{k}_2}(y) \pm \psi_{\underline{k}_1}(y) \psi_{\underline{k}_2}(x)|^2 \quad (1)$$

The crucial feature in eq. (1) is the interference term due to Bose (+) or Fermi (-) statistics. That term reflects the impossibility of determining whether the pair with momentum $\underline{k}_1$ and $\underline{k}_2$ was produced at x and y , or at y and x , respectively. The resulting intensity interference pattern leads to⁵

$$R(\underline{k}_1, \underline{k}_2) = 1 \pm |\rho_V(\underline{k}_1, \underline{k}_2)|^2 / \rho_V(\underline{k}_1, \underline{k}_1) \rho_V(\underline{k}_2, \underline{k}_2) \quad , \quad (2)$$

where $\rho_V(\underline{k}_1, \underline{k}_2) = \int d^4x \rho(x) \psi_{\underline{k}_1}^*(x) \psi_{\underline{k}_2}(x)$. In the absence of final state interaction ρ_V reduces to the space-time Fourier transform of $\rho(x)$. When $V \neq 0$, R measures a distorted transform of $\rho(x)$. An important property of eq. (2) is that $R(\underline{k}, \underline{k}) = 2$ for any V in the case of boson pairs. Furthermore, since $P_2 \geq 0$, R is bounded for any V . For boson and spin singlet fermion pairs,

$$1 \leq R(\underline{k}_1, \underline{k}_2) \leq 2 \quad (3)$$

For spin triplet fermion pairs, $0 \leq R \leq 1$.

While the derivation of eqs. (2,3) is plausible and widely used, it neglects many sources of correlation besides quantum statistics. One obvious source of additional correlations is two body final state interactions.^{4,5} For example, Coulomb interactions modify eq. (2) by a multiplicative Gamow factor⁵ that makes $R(\underline{k}, \underline{k}) = 0$ for $\pi^\pm \pi^\pm$ pairs. However, such two body effects can be calculated in many cases and unfolded from the observed $R(\underline{k}_1, \underline{k}_2)$. The source of correlations I focus on in this letter is the average over unobserved final states involved in inclusive measurements. If the observed particles are correlated dynamically with the unobserved ones, then this average can alter significantly the form and interpretation of $R(\underline{k}_1, \underline{k}_2)$. I refer to such correlations as ensemble correlations.

As the first example of ensemble correlations, I consider final state absorption. Such absorption reduces the single particle wavefunctions, $\psi_k(x)$, by a factor $\exp[-n_a(k)/2]$, where $n_a(k)$ is the average number of absorption mean free paths. In general, n_a depends on the exclusive distribution of final fragments. For example, for a given impact parameter $b > 0$, there is more matter and, hence, more absorption for k in the reaction plane than out of it. The unobserved projectile and target spectators can therefore cast a shadow in the reaction plane. A model of such azimuthally asymmetric absorption is

$$n_a(k) = n(k, \theta) + \Delta n(k, \theta) \cos^2(\phi - \phi^*) , \quad (4)$$

where (θ, ϕ) are the polar and azimuthal angles of k , and ϕ^* specifies the azimuth of the reaction plane. Physically, $\Delta n(k, \theta)$ is the difference of the number of absorption mean free paths in and out of the reaction plane. Assume now that all external variables on which n_a depends except ϕ^* are fixed. For example, b could be determined through specific associated multiplicity triggers¹³. In that case the single and double inclusive distribution must still be averaged over ϕ^* . Because an ensemble average of a product $\langle f_1 f_2 \rangle \neq \langle f_1 \rangle \langle f_2 \rangle$, in general, a correlation is thereby induced. With eq. (4), the ensemble averages of P_1 and P_2 over ϕ^* can be readily carried out. The correlation function simply factorizes as $R(k_1, k_2) = R_a(k_1, k_2) R_v(k_1, k_2)$, where R_v is given by eq. (2), and the absorption correlation function is

$$R_a(k_1, k_2) = I_0 \left\{ \frac{1}{2} [\Delta n_1^2 + \Delta n_2^2 + 2 \Delta n_1 \Delta n_2 \cos 2(\phi_1 - \phi_2)] \right\}^{1/2} / \left\{ I_0 \left(\frac{\Delta n_1}{2} \right) I_0 \left(\frac{\Delta n_2}{2} \right) \right\} , \quad (5)$$

with $\Delta n_i = \Delta n(k_i, \theta_i)$ and $I_0(x)$ being the modified Bessel function. Notice that the intercept for bosons is $R(k, k) = 2 I_0[\Delta n(k)] / I_0^2[\Delta n(k)/2] \approx$

$2 + [\Delta n(k)/2]^2 \geq 2$. Therefore, with eq. (5) the bounds imposed by quantum statistics, eq. (3), can be easily violated.

It is important to emphasize that the enhanced correlation is due entirely to the ensemble average over reaction planes. If the reaction plane were measured event by event and the final momenta were measured always with respect to that plane, then $R_a = 1$, the correlation function would reduce to eq. (2), and a simple space-time interpretation of R would again be possible.

A second interesting example of how correlations are induced by ensemble averaging is provided by coherent jet production. Consider a classical current $J(x)$ that couples to a boson field $\phi(x)$ via $L_{int} = \phi(x)J(x)$. The radiated bosons in this case are described by a coherent state^{5,6}, for which

$$\bar{n}P_1(k) = |J(k)|^2, \quad (6a)$$

$$(\bar{n})^2 P_2(k_1, k_2) = |J(k_1)|^2 |J(k_2)|^2, \quad (6b)$$

$$R(k_1, k_2) = 1, \quad (6c)$$

with $J(k) = \int d^4x J(x)\psi_k^*(x)$ and $\bar{n}$ being the average number of

bosons radiated. The remarkable property of this radiation field is that the bosons are completely uncorrelated, eq. (6c), although Bose symmetrization is properly taken into account.^{5,6,14} Note that eq. (6c) holds in the presence of an arbitrary optical potential⁵. However, even though no space-time interpretation is possible, eq. (6c) satisfies the bounds, eq. (3), imposed by symmetrization alone.

To describe a coherent jet, consider for simplicity

$$|J(\underline{k})|^2 = \bar{n}_0 (2\pi\sigma_0^2)^{-3/2} \exp[-(\underline{k} - \underline{k}_0)^2 / 2\sigma_0^2] , \quad (7)$$

where $\bar{n}_0$ is the average number of bosons in the jet, $\underline{k}_0$ and σ_0 are the average momentum and momentum dispersion in the jet. Such a current could arise in nuclear collisions, for example, as a result of pion condensation.¹⁵

In that case, $J(x)$ corresponds to the divergence of the spin-isospin density, $\underline{k}_0$ is the condensate wavenumber ($k_0 \sim 2m_\pi$), and $(2\sigma_0^2)^{-1/2} \sim r_0 A^{1/3}$ is the radius of the A particle system. For large A, $\sigma_0 \ll k_0$, and an almost monochromatic beam of pions is produced. Nevertheless, from eq. (6c) the pions are still completely uncorrelated as measured by R! This shows that R measures dynamical rather than kinematical (momentum space) correlations. Ensemble correlations are induced in this example, if $\underline{k}_0$ varies from event to event according to a normalized distribution, $\gamma(\underline{k}_0)$. The ensemble average over $\underline{k}_0$ then leads to

$$P_1(\underline{k}) = \int d^3k_0 \gamma(\underline{k}_0) (2\pi\sigma_0^2)^{-3/2} e^{-(\underline{k}-\underline{k}_0)^2 / 2\sigma_0^2} , \quad (8a)$$

$$R(\underline{k}_1, \underline{k}_2) = \frac{e^{-(\underline{k}_1 - \underline{k}_2)^2 / 4\sigma_0^2}}{(4\pi\sigma_0^2)^{3/2}} \frac{P_1[(\underline{k}_1 + \underline{k}_2)/2; \sigma_0/\sqrt{2}]}{P_1(\underline{k}_1)P_1(\underline{k}_2)} , \quad (8b)$$

From the normalization condition on γ , the intercept can be shown to satisfy $R(\underline{k}, \underline{k}) \geq 1$. In particular, $R(\underline{k}, \underline{k})$ can again be arbitrarily large. If $\gamma(\underline{k})$ is a gaussian of width $\sigma_c \gg \sigma_0$, then $R(\underline{k}, \underline{k}) \approx (\sigma_c^2 / 2\sigma_0^2)^{3/2} \exp(+k^2 / 2\sigma_c^2) \gg 1$. The coherent limit $R = 1$ is recovered, on the other hand, if $\sigma_c \ll \sigma_0$.

Thus far, only pure chaotic or pure coherent field ensemble correlations were considered. To treat the more general partially coherent case, the source current can be written as⁵

$$J(\underline{k}) = J_0(\underline{k}) + J_\Delta(\underline{k}) \sum_{i=1}^N e^{i\phi_i} \psi_k^*(x_i) , \quad (9)$$

where $J_0(\underline{k})$ is the collective current, eq. (7), and $J_\Delta(\underline{k})$ characterizes the pion source in an isolated inelastic NN scattering. The x_i specify N inelastic scattering centers that are assumed to be distributed randomly in a space-time region according to the density $\rho(x)$. The random phases ϕ_i vary between 0 and 2π . The single and double inclusive distributions are then given by an ensemble average of eqs. (6a,6b) over $\{N, \phi_i, x_i\}$ and over $\gamma(\underline{k})$. The ensemble average over the chaotic field parameters has been evaluated in Ref. (5) (see eqs. (4.60,4.61)). The new average over the coherent component leads to

$$R(\underline{k}_1, \underline{k}_2) = R_a(\underline{k}_1, \underline{k}_2) \left\{ 1 + [R_0(\underline{k}_1, \underline{k}_2) - 1] D(\underline{k}_1) D(\underline{k}_2) + [R_v(\underline{k}_1, \underline{k}_2) - 1] [1 - D(\underline{k}_1)] [1 - D(\underline{k}_2)] + 2 \text{Re} R_I(\underline{k}_1, \underline{k}_2) (D(\underline{k}_1) [1 - D(\underline{k}_1)] D(\underline{k}_2) [1 - D(\underline{k}_2)])^{1/2} \right\} , \quad (10)$$

where R_0 is given by eq. (8b), R_v is given by eq. (2), and the interference factor, R_I , is given by

$$R_I(\underline{k}_1, \underline{k}_2) = \frac{\langle J_0^*(\underline{k}_1) J_0(\underline{k}_2) \rangle}{[n_0(\underline{k}_1) n_0(\underline{k}_2)]^{1/2}} \frac{\rho_v(\underline{k}_1, \underline{k}_2)}{[\rho_v(\underline{k}_1, \underline{k}_1) \rho_v(\underline{k}_2, \underline{k}_2)]^{1/2}} . \quad (11)$$

The effect of shadowing has been included in eq. (10) via the absorption correlation factor R_a , given by eq. (5). In addition, the degree of coherence⁵ is defined by $D(\underline{k}) = n_0(\underline{k}) / [n_0(\underline{k}) + n_\Delta(\underline{k})]$, where the number of coherent and incoherent pions is $n_0(\underline{k}) = \langle |J_0(\underline{k})|^2 \rangle$ and $n_\Delta(\underline{k}) = \langle N | J_\Delta(\underline{k}) |^2 \rangle \rho_v(\underline{k}, \underline{k})$. Note that the intercept value of R is given by

$$R(\underline{k}, \underline{k}) = R_a(\underline{k}, \underline{k}) \left\{ 2 + [R_0(\underline{k}, \underline{k}) - 2] D^2(\underline{k}) \right\} , \quad (12)$$

Equations (10-12) are much more general formulas for boson interferometry than were proposed up to now.^{5,14} Note the special cases included in eq. (10). For $D(\underline{k}) = 1$, the pure coherent field form is recovered. The less general partially coherent field form^{5,14} with $R(\underline{k}, \underline{k}) = 2 - D^2(\underline{k})$ is recovered when both $\Delta n = 0$ and $\sigma_c \ll \sigma_0$, i.e., $R_a = R_0 = 1$. Finally, the familiar Hanbury-Brown and Twiss form, eq. (2), follows when $D(\underline{k}) = 0$ and $R_a(\underline{k}_1, \underline{k}_2) = 1$.

To illustrate eq. 10, consider the partially coherent jet produced if pionic instabilities would occur in nuclear collisions. The coherent jet component can be parametrized by a toroidal distribution¹⁵

$$\gamma(\underline{k}) = (2\pi\sigma_c^2)^{-3/2} I_0(k_\perp k_c / \sigma_c^2) e^{-(k^2 + k_c^2)/2\sigma_c^2} \quad (13)$$

with $k_c \sim 2m_\pi$, $\sigma_c \sim m_\pi$, and $k_\perp$ being the magnitude of the component of $\underline{k}$ with perpendicular to the beam axis. Since $(\sigma_c/\sigma_0)^2 \sim A^{2/3} \gg 1$, it follows from eq. (8) that $n_0(\underline{k}) \approx \bar{n}_0 \gamma(\underline{k})$ and that for $q^2 \lesssim \sigma_0^2 \ll \sigma_c^2$,

$$R_0(\underline{k}, \underline{k} + \underline{q}) \approx e^{-q^2/4\sigma_0^2} [(4\pi\sigma_0^2)^{3/2} \gamma(\underline{k})]^{-1} \quad (14)$$

Note that for $(k_\parallel, k_\perp) \approx (0, 2m_\pi)$, $R_0(\underline{k}, \underline{k}) \sim A \gg 1$. Therefore, the coherent jet correlations differ dramatically from the Hanbury-Brown and Twiss form. The chaotic field component can be described by the density $\rho(q) = \exp(-q^2 \langle r^2 \rangle)$, in terms of the rms radius of the interaction region. Since the degree of coherence $D(\underline{k})$ is expected to be very small¹⁵, only the R_v and $R_0 D^2$ terms contribute significantly in eq. (10). It is the large amplification ($R_0 \gg 1$) due to ensemble correlations in eq. (14) that allows $\chi_{jet}(\underline{k}) \equiv R_0(\underline{k}, \underline{k}) D^2(\underline{k}) \sim 1$ in

spite of $D \ll 1$. For example, for $A \sim 100$, $D \sim 0.1$, $X_{\text{jet}} \sim 1$. Defining now a jet radius $r_{\text{jet}}^2 = (4\sigma_0^2)^{-1}$ we obtain finally for $D(k) \ll 1$

$$R(\underline{k}, \underline{k} + \underline{q}) \approx R_a(\underline{k}, \underline{k} + \underline{q}) \left\{ 1 + e^{-q^2 \langle r^2 \rangle} + X_{\text{jet}}(\underline{k}) e^{-q^2 r_{\text{jet}}^2} \right\}. \quad (15)$$

Equation (15) illustrates that a small coherent jet component may be easier to detect via $R(\underline{k}_1, \underline{k}_2)$ than via the single inclusive distribution because of the amplification factor R_0 . However, it is also clear that absorption or other chaotic ensemble correlations could lead to the same distortion of R when $X_{\text{jet}} = 0$. The point is that when $R(\underline{k}_1, \underline{k}_2)$ differs from the Hanbury Brown and Twiss form due to dynamical correlations, it will always be necessary to measure more exclusive properties of the reaction (e.g., the reaction plane) in order to distinguish between competing mechanisms.

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