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MEASUREMENT OF THE WWZ AND ZH CROSS SECTIONS IN THE FOUR-LEPTON CHANNEL AT CENTER-OF-MASS ENERGIES OF 13 TEV AND 13.6 TEV WITH THE CMS DETECTOR AT THE CERN LHC AND DESIGN OF A NOVEL BEAM DELIVERY SYSTEM FOR A FUTURE 10 TEV PLASMA WAKEFIELD COLLIDER

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DESIGN OF A NOVEL BEAM DELIVERY SYSTEM FOR A FUTURE 10 TEV
PLASMA WAKEFIELD COLLIDER

A DISSERTATION SUBMITTED IN PARTIAL SATISFACTION OF THE
REQUIREMENTS FOR THE DEGREE DOCTOR OF PHILOSOPHY IN
PHYSICS

DEPARTMENT OF PHYSICS

BY

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JUNE 2026

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May 2026

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This thesis is dedicated to my parents, Roy Downham II and Amy Bruss (nee Salm), for their constant support and guidance through times good and bad.

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ABSTRACT

"Measurement of the WWZ and ZH cross sections in the four-lepton channel at center-of-mass energies of 13 TeV and 13.6 TeV with the CMS detector at the CERN LHC and design of a novel beam delivery system for a future 10 TeV plasma wakefield collider"

by

Keegan Everett Downham

This thesis presents a measurement of the cross section for the production of two W bosons and one Z boson. The measurement is performed using data collected by the CMS experiment at the CERN LHC at center-of-mass energies $\sqrt{s} = 13$ and 13.6 TeV, corresponding to an integrated luminosity of 200 fb^{-1} . Events with four charged leptons (electrons or muons) are selected for this analysis. Both nonresonant WWZ production and ZH production, with the Higgs boson decaying into two W bosons, are reported. For the first time, the two processes are measured separately in a simultaneous fit. Combining the two modes, signal strengths relative to the standard model (SM) predictions of $0.75^{+0.34}_{-0.29}$ and $1.74^{+0.71}_{-0.60}$ are measured for $\sqrt{s} = 13$ and 13.6 TeV, respectively. The observed (expected) significance for the triboson signal is 3.8 (2.5) standard deviations for $\sqrt{s} = 13.6$ TeV, thus providing the first evidence for triboson production at this center-of-mass energy. Combining the two modes and the two center-of-mass energies, the inclusive signal strength relative to the SM prediction is measured to be $1.03^{+0.31}_{-0.28}$, with an observed (expected) significance of 4.5 (5.0) standard deviations.

The focusing of particle beams for collider experiments is crucial for maximizing the luminosity and thus the discovery potential of these machines. In recent years, plasma wakefield acceleration has emerged as a leading candidate for achieving higher energy collisions with smaller facility footprints due to the large accelerating gradients in the plasma. This higher beam energy poses significant challenges for the final focusing system of the collider. Here, I discuss the various challenges of final focusing for TeV-scale plasma accelerators and propose possible solutions. Finally, I present the first design of a final focusing system for a 10 TeV linear wakefield collider, evaluate its performance, and discuss its shortcomings as well as improvements for future designs.

Part I

Measurement of WWZ and ZH Cross Sections at $\sqrt{s} = 13$ and 13.6 TeV with the CMS detector

CHAPTER 1

THEORETICAL FOUNDATIONS

"I think that it is a relatively good approximation to truth, which is much too complicated to allow anything but approximations, that mathematical ideas originate in empirics."

John von Neumann

1.1 A Brief Overview

"Science is made up of so many things
that appear obvious after they are
explained"

Frank Herbert

Dune (1965)

At present, there is substantial experimental evidence for the existence of four, distinct fundamental forces: the electromagnetic force, the strong nuclear force, the weak nuclear force, and the gravitational force. Colloquially, the electromagnetic and gravitational forces are well understood, explaining the attraction (repulsion) of objects with opposite (same) electric charge and the attractive force between massive bodies, respectively. Less commonly understood are the strong and weak nuclear forces, which are responsible for holding the atomic nucleus together and nuclear decay, respectively.

The fundamental forces describe the available interactions between the known fundamental particles, which are shown in Figure 1.1; each particle can be described by a minimum set of requirements that specify its properties, such as its charge, mass, and spin. Each fundamental particle can be categorized into two broad categories (with one notable exception) based on its spin:

- half-integer spin (spin- $\frac{1}{2}$) particles are referred to as *fermions*, which constitute ordinary matter. These particles obey Fermi-Dirac statistics [1, 2] as a result of the Pauli Exclusion Principle [3], which stipulates that fermions cannot occupy the same energy state.
- whole-integer spin (spin-1) particles are referred to as *bosons*, which are the carriers of the fundamental forces; the fundamental forces can be thought to

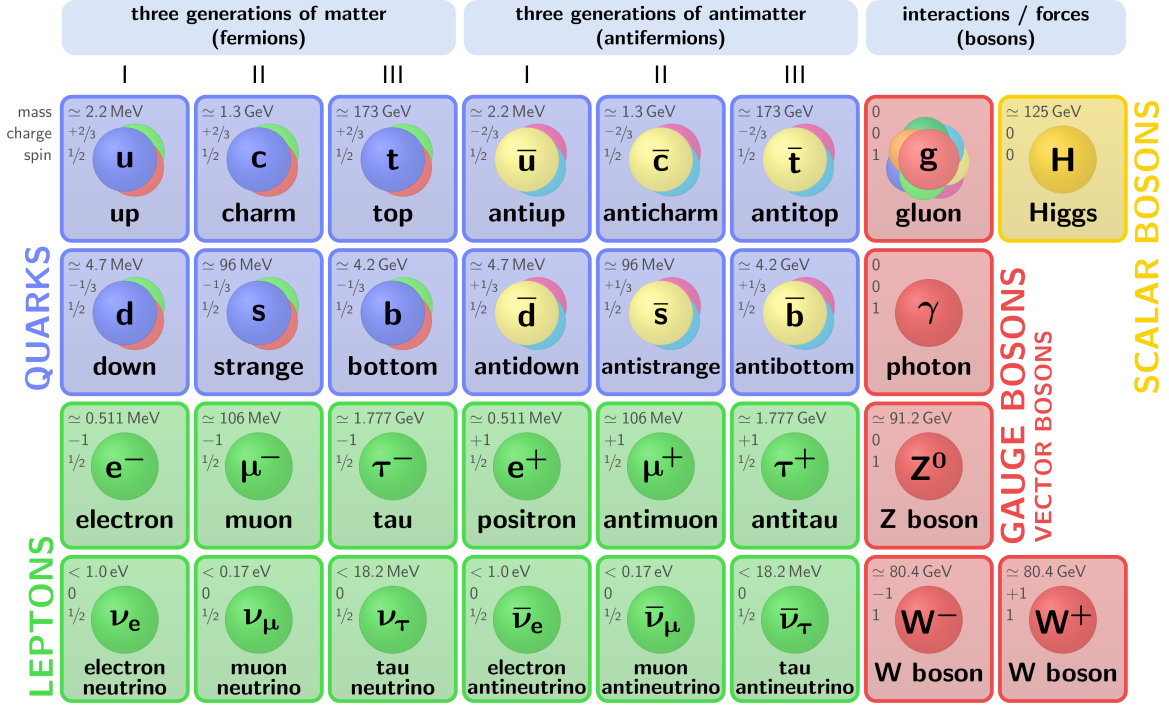


Figure 1.1: Particle content of the Standard Model. Figure taken from https://tikz.net/sm_particles/.

arise from the exchange of the corresponding boson between fermions or bosons themselves. Bosons obey Bose-Einstein statistics [4], which allow for degenerate energy states.

The one notable exception to this classification is the Higgs boson, which is the only fundamental scalar (spin-0) in the Standard Model. As will be discussed in Section 1.3.1, the Higgs mechanism is responsible for the generation of the masses of the fermions and the massive bosons (W and Z).

The fermion content of the SM can be further divided into two categories based on the types of interactions that the particles participate in:

- **quarks**: can interact via the strong, weak, electromagnetic, and gravitational interactions.
- **leptons**: can interact only via the weak, electromagnetic (if electrically charged),

and gravitational interactions.

Interaction	Constant	Value
Electroweak	$SU(2)_L$ coupling: g	0.652
	$U(1)_Y$ coupling: g'	0.357
	EM coupling: α	0.0078
	Weak coupling: α_W	0.0338
Strong	$SU(3)_c$ coupling: g_s	1.221
	Strong coupling: α_s	0.1184

Table 1.1: Coupling constants for the electroweak and strong interactions. Values calculated at the Z mass ($m_Z = 91.1876$ GeV), and obtained from https://math.ucr.edu/home/baez/constants_table.pdf.

The relative strength of the different interactions are parameterized by so-called *coupling constants*, which are shown in Table 1.1; the coupling constants are dependent on the energy scale at which the interaction takes place. Figure 1.2 shows the different measurements for the strong coupling constant α_s . The evolution of the coupling constants are calculated using a procedure called *renormalization*. A detailed discussion of renormalization theory is beyond the scope of this thesis, and I refer the reader to one of the various QFT textbooks cited here [5–7]. The relevant takeaway from the renormalization procedure is the prediction of the coupling constant evolution as a function of the energy scale; the strong and weak coupling constants decrease with energy while the electromagnetic coupling constant grows with energy.

For energies near the weak scale ($E \approx 100$ GeV), the electromagnetic and weak interactions become unified via the process of *electroweak unification* (see Section 1.3),

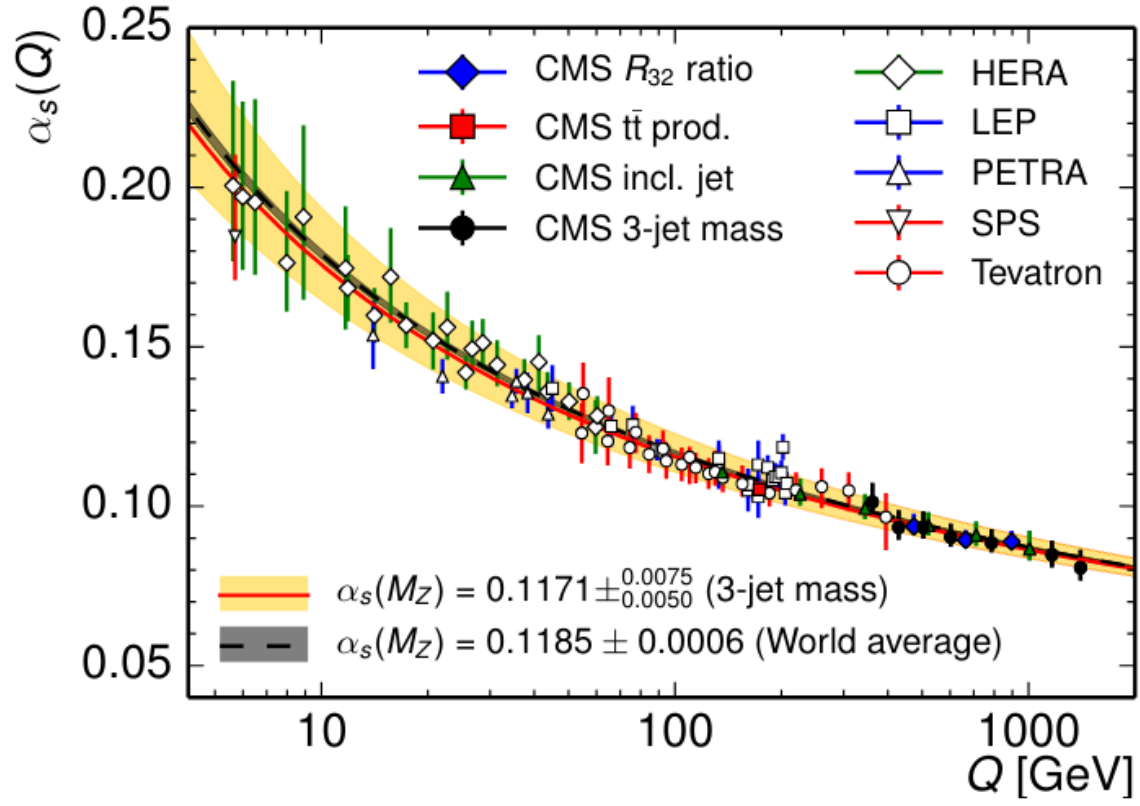


Figure 1.2: Measurements of the strong coupling constant from different experiments as a function of the energy scale Q . Figure taken from ref. [8].

whereby the couplings of the weak and electromagnetic forces can be written in terms of each other. Due to the relative size of the gravitational coupling constant and the scale of particle masses, gravitational effects are negligible compared to strong and electroweak effects in regimes probed by particle physics experiments. Furthermore, a quantum theory of gravity has yet to be established and is thus not included in the framework of the SM.

At the high energies associated with collider experiments, particles produced in collisions can be traveling close to the speed of light in vacuum c . In this regime, Einstein's special relativity describes the particle kinematics, which is based on two principles:

- **Principle of relativity:** the laws of physics are the same in all reference frames.
- **Constant speed of light:** the speed of light in vacuum is constant in all reference frames.

These two principles specify a set of transformations, called Lorentz transformations, that relate the coordinates of events occurring in two reference frames O_1 and O_2 moving with some speed $v \lesssim c$ relative to each other. For simplicity, let us assume $\vec{v} = v\hat{x}$. An event occurs, and we denote the set of space-time coordinates corresponding to the event for O_1 as (x, y, z, t) while for O_2 we write (x', y', z', t') . In the O_2 frame, the two sets of coordinates are then related to each other via:

$$x' = \gamma(x - vt), \quad y' = y, \quad z' = z, \quad ct' = \gamma(ct - \frac{v}{c}x) \quad (1.1)$$

where γ is the relativistic Lorentz factor:

$$\gamma = \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} \quad (1.2)$$

A consequence of the time component transformation in Eq. 1.1 is the concept of time dilation, whereby objects moving at relativistic speeds with respect to an observer experience time moving slower than the observer in the reference frame of the observer. The desired relation, obtained by inverting the first and fourth equalities in Eq. 1.1, is (after setting $x' = 0$):

$$t = \gamma t' \quad (1.3)$$

This has important implications for particle decays, as the decay time in the rest frame of the particle can be extended considerably if the particle travels at relativistic speeds. An example of this is in the case of cosmic muons; solar γ -rays and protons from cosmic sources interact with air molecules in the atmosphere, producing charged pions and kaons, which decay into muons, as shown in Figure 1.3. The muons are produced tens of kilometers above the surface of the earth and have a lifetime (in their rest frame) of $\tau_\mu = 2.2 \times 10^{-6}$ s, however most of the muons make it to the surface of the earth before decaying. This is due partially to the fact that the decay time is enhanced by the relativistic Lorentz factor, resulting from the large kinetic energy the muons have when they are produced. Without time dilation, some muons would still reach the earth, however many of them would undergo decays in the atmosphere.

Two important concepts for the fermions are helicity and chirality, the latter of which will be important when considering the weak interaction. Helicity is simply the projection of the fermion spin $\mathbf{S}$ onto its momentum $\mathbf{p}$:

$$h = \frac{\mathbf{S} \cdot \mathbf{p}}{|\mathbf{p}|} \quad (1.4)$$

For fermions having their spin aligned with their momentum, we refer to them as being "right-handed", while for fermions with spin opposite their momentum, we denote these as being "left-handed", as illustrated in Figure 1.4. A related quantity,

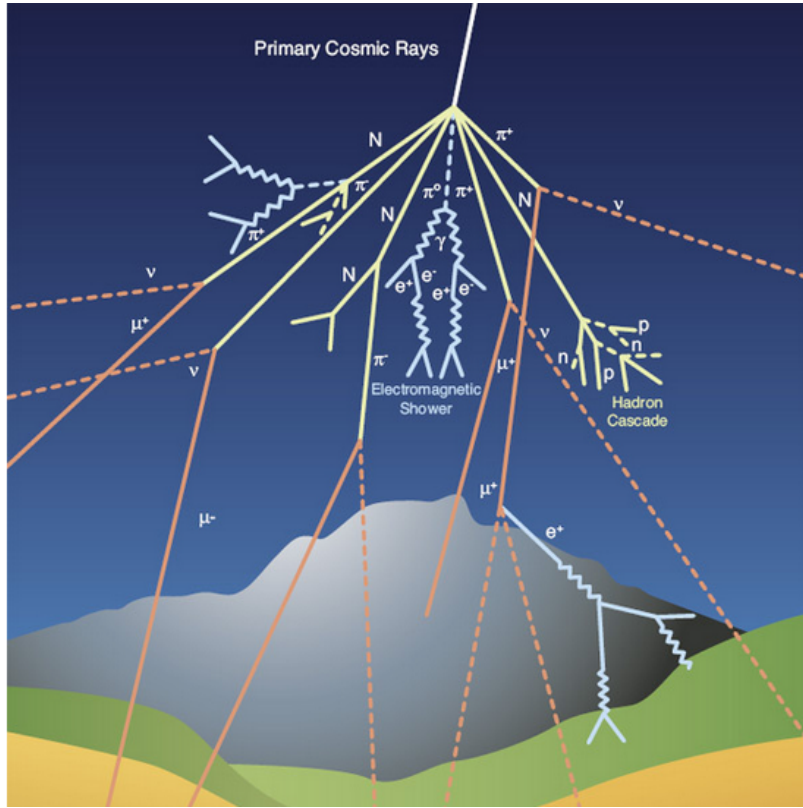


Figure 1.3: Diagram showing the mechanism of muon production in the atmosphere by cosmic rays. Figure taken from <https://cds.cern.ch/images/CMS-PHO-GEN-2017-008-1>.

called chirality, is an intrinsic property of particles (and thus has no dependence on momentum). In the limit of massless fermions, helicity and chirality become equivalent to each other. Chirality is a central feature defining the interactions of the fermions via the weak interaction, and is core to the formulation of the Dirac equation; the equation of motion for fermions. Similar to helicity, chirality can be expressed in terms of right- and left-handed projections. A more comprehensive discussion of chirality and helicity can be found in refs. [5, 9].

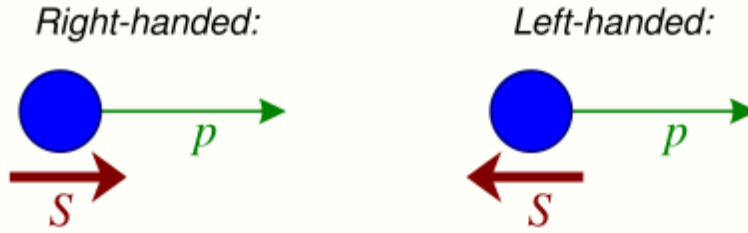


Figure 1.4: Diagram illustrating the concept of helicity, where $\mathbf{S}$ and $\mathbf{p}$ refer to the particle spin and momentum, respectively. Figure taken from <https://encyclopedia.pub/entry/31474>.

The equation of motion for freely propagating fermions is given by the Dirac equation [10], a wave equation uniting special relativity and the Schrodinger equation:

$$(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0 \quad (1.5)$$

where ψ is the 4-component Dirac spinor (which is comprised of right- and left-handed Weyl spinors), $\hbar$ is the reduced Planck constant, γ^μ are the gamma matrices, ∂_μ is the 4-derivative, m is the fermion mass, and c is the speed of light. While revolutionary for the time, the Dirac equation has since become incorporated into the formalism of Quantum Field Theory (QFT) to describe the interactions and dynamics of the fundamental particles. A consequence of the Dirac equation is the existence of *anti-fermions*; each fermion has a corresponding anti-fermion with the same mass and spin but opposite charge, as shown in the middle panel in Figure 1.1. The fermions are

organized into three generations (first, second, and third) based on their mass (and the order in which they were discovered). For each generation, the quarks and leptons are arranged into *doublets* (e.g. up and down, or electron and electron neutrino), which correspond to the weak isospin doublets governing the behavior of the fermions with respect to the weak charged-current interaction (see Section 1.3).

1.2 The Strong Interaction

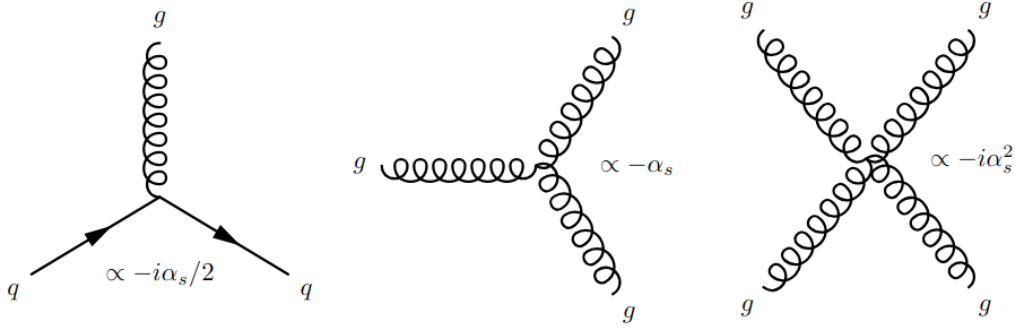


Figure 1.5: Interaction vertices in Quantum Chromodynamics; quark-gluon interaction (left), 3-gluon interaction (center) and 4-gluon interaction (right).

The strong interaction involves the exchange of a massless gluon (g) between quarks, which carry the "color" charge. Additionally, gluons themselves carry the color charge of the strong interaction, which enables gluon-gluon interaction vertices. Diagrams showing the available strong interaction vertices and their corresponding vertex factors are shown in Figure 1.5. The central feature of the strong interaction is the property of *asymptotic freedom*; the coupling of the strong interaction, α_s , decreases at higher energies. As a result, the attraction between quarks due to the exchange of gluons *increases* as the quarks become further apart, as illustrated in the diagram in Figure 1.6; if the quarks are pulled apart far enough, the gluon field between the quarks will have enough energy to produce a quark-antiquark pair.

A natural consequence of asymptotic freedom is that at low energies the cou-

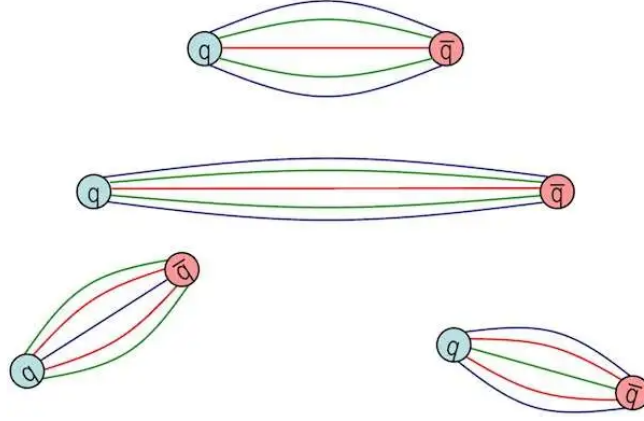


Figure 1.6: Illustration of the interaction between quarks through the gluon field; as quarks are pulled apart, the gluon field becomes more collimated into a "tube", resulting in a stronger force between the quarks. Once enough energy is used to pull the quarks apart, another quark-antiquark pair is created from the vacuum, which hadronizes with the original quarks. Figure taken from Ref. [11].

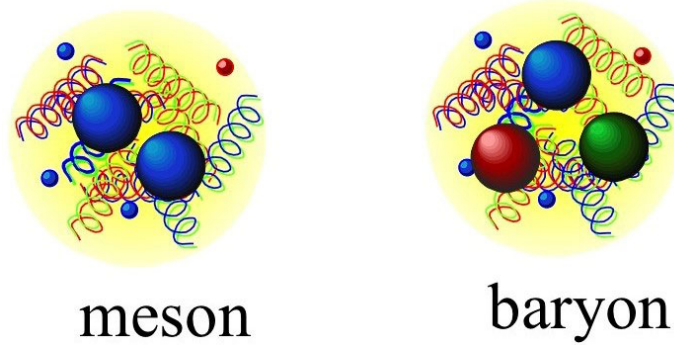


Figure 1.7: Diagrams representing the internal structure of mesons (left) and baryons (right) as two or three valence quarks (large circles) bound together via gluon exchange (squiggly lines). Figure taken from Ref. [12].

pling constant becomes so large that the colored quarks become confined to colorless bound states called *hadrons*. Thus, any quark-antiquark pairs created from gluon fields will recombine with other quarks through a process called *hadronization*. Typically, hadrons are comprised of two (mesons) or three (baryons) valence quarks with a "sea" of gluons being exchanged, as shown in Figure 1.7. While the process of hadronization is not well understood quantitatively due to the size of α_s at low energies (thus causing perturbation theory to no longer apply), the structure of some hadrons can be probed by scattering experiments. From this, one can measure the fraction of momentum carried by the valence quarks, gluons, and other quarks that are produced briefly due to quark pair-production, which are parameterized in *parton distribution functions* (see Section 2.2.6). Experimentally, the properties of QCD (namely asymptotic freedom, color confinement, and hadronization) do not allow for the observation of free quarks; rather, the process of quark pair-production and hadronization result in narrow cones of hadrons commonly referred to as "jets".

It was shown by David Gross, Frank Wilczek, and David Politzer that asymptotic freedom is an intrinsic property non-Abelian gauge theories [13, 14]. Additionally, non-Abelian gauge theories predict that the bosons mediating the interaction carry the fundamental charge [5], thus allowing for self-interactions. These two properties were sufficient motivation to choose a non-Abelian gauge structure to describe the theory of the strong interaction. This theory, called Quantum Chromodynamics (QCD), takes the form of a $SU(3)_c$ local gauge symmetry associated with the color charge carried by quarks and gluons. In QCD, each quark can carry one of the three colors (red, blue, or green), while the antiquarks can carry one of the corresponding "anti-colors" (anti-red, anti-blue, or anti-green). The gauge structure of QCD also implies that there are a total of eight different gluons consisting of pairs of the available colors and anti-colors. For a more complete description of the gauge structure and properties of the strong interaction, I refer the reader to Refs. [5–7, 9].

1.3 The Electroweak Interaction

The theory of the electromagnetic interaction was first elucidated in 1873 by James Clerk Maxwell via the eponymous set of equations [15]:

$$\begin{aligned}
 \nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\
 \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
 \nabla \cdot \mathbf{B} &= 0 \\
 \nabla \times \mathbf{B} &= \mu_0 \mathbf{j} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}
 \end{aligned} \tag{1.6}$$

where $\mathbf{E}$ is the electric field vector, $\mathbf{B}$ is the magnetic field vector, ρ is the charge density, ϵ_0 is the vacuum permittivity, μ_0 is the vacuum permeability, $\mathbf{j}$ is the current density, and c is the speed of light. Maxwell's theory was later refined by Richard Feynman, Julian Schwinger, and Sin-Itiro Tomonaga (among others) using the tools of QFT to produce Quantum Electrodynamics (QED). In the framework of QED, the electromagnetic interaction arises due to the exchange of the neutral, massless photon (γ) between charged particles.

Figure 1.8 shows the available QED interaction vertices; the annihilation (or pair production) process on the left involves a fermion and an antifermion of the same type, while the emission (absorption) process on the right does not change the type of fermion. The strength of the interaction is parameterized by the *fine structure constant* α , where:

$$\alpha = \frac{1}{4\pi\epsilon_0} \frac{e^2}{\hbar c} \approx \frac{1}{137} \tag{1.7}$$

The weak interaction involves two primary types of interactions: *charged-current* (CC) interactions, mediated by the charged $W^\pm$ boson, and *neutral-current* (NC)

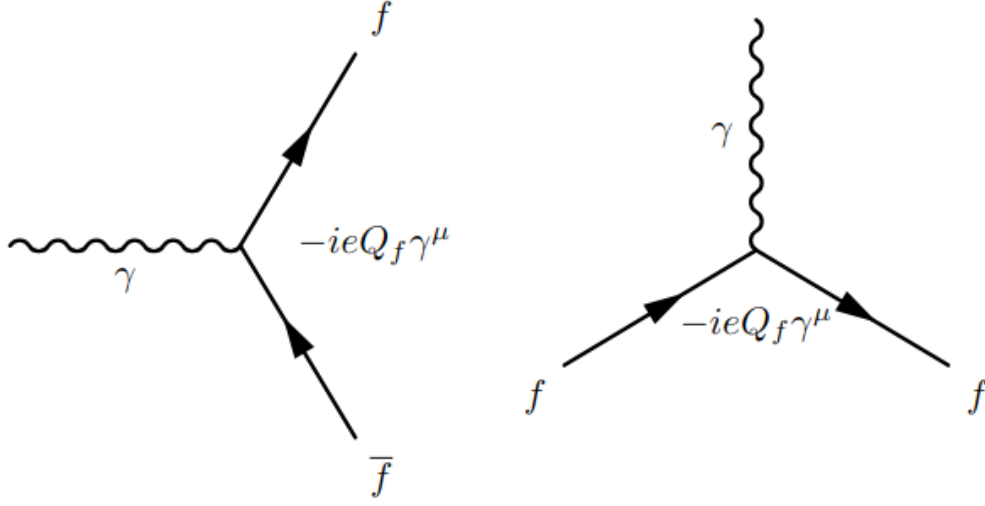


Figure 1.8: Interaction vertices of Quantum Electrodynamics for annihilation/pair production (left) and absorption/emission (left). Fermion (antifermion) lines denoted by f ($\bar{f}$).

interactions mediated by the neutral Z boson. Due to the charge of the W boson, the CC interaction links fermions differing by one unit of electric charge, as shown by the vertices in Figure 1.9. Additionally, the weak CC interaction was observed to violate parity conservation [16] as it only couples to left-handed (chiral) fermions. The NC interaction occurs between fermions of the same or opposite charge, as shown in Figure 1.10, and acts on both right- and left-handed chiral fermions, but in different ways.

The electromagnetic and weak (CC and NC) interactions are described by the Glashow-Salam-Weinberg (GSW) unified electroweak theory in which the couplings of the weak and electromagnetic interactions are related. GSW theory stipulates that the electroweak interaction behaves according to a local $SU(2)_L \times U(1)_Y$ gauge symmetry; the $SU(2)_L$ portion motivates a "doublet" structure of the fermions labeled by quantum numbers t and t_3 , which are known as "weak isospin". For the leptons, the doublets contain leptons of the same generation:

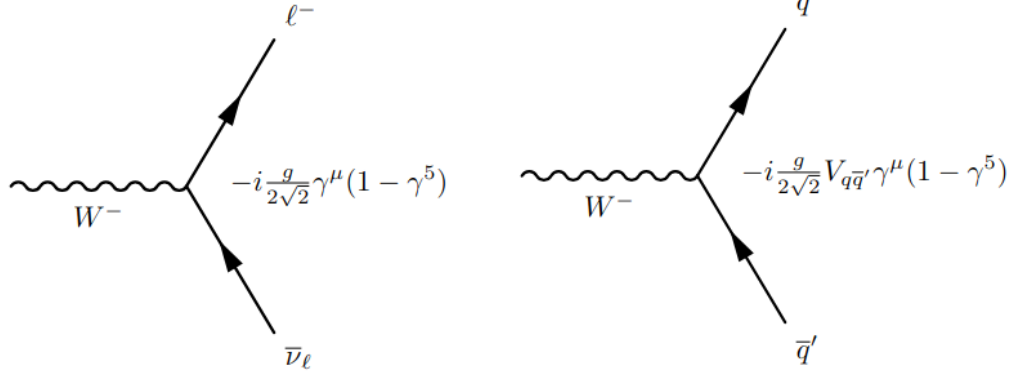


Figure 1.9: Interaction vertices for the weak charged-current interaction for leptons (left) and quarks (right). The factor $V_{q\bar{q}'}$ in the quark diagram is the CKM matrix element for the quark-antiquark pair participating in the interaction.

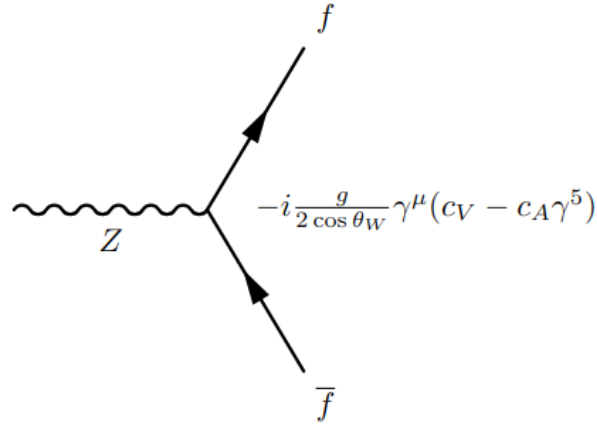


Figure 1.10: Interaction vertex for the weak neutral-current interaction for fermions. The factors c_V and c_A are the vector and axial vector couplings for the fermion participating in the interaction, respectively.

$$t = \frac{1}{2}, \quad \left\{ \begin{matrix} t_3 = +\frac{1}{2} \\ t_3 = -\frac{1}{2} \end{matrix} \right\} \rightarrow \begin{pmatrix} \nu_e' \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu' \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_\tau' \\ \tau^- \end{pmatrix}_L \quad (1.8)$$

where the subscript L refers to the fact that the $SU(2)$ portion of the electroweak interaction only acts on left-handed chiral fermion fields. The quarks are arranged into a similar doublet structure based on the different generations:

$$t = \frac{1}{2}, \quad \left\{ \begin{matrix} t_3 = +\frac{1}{2} \\ t_3 = -\frac{1}{2} \end{matrix} \right\} \rightarrow \begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L \quad (1.9)$$

The primed neutrino (quark) fields in Equation 1.8 (1.9) denote the flavor eigenstates, which are related to the mass eigenstates via the PMNS (CKM) matrix [17] ([18, 19]).

Gauging the $SU(2)_L$ portion results in three gauge fields (bosons) transforming according to a $t = 1$ triplet, which is commonly denoted as $\hat{W}^\mu$:

$$\hat{W}^\mu = \begin{pmatrix} \hat{W}_1^\mu \\ \hat{W}_2^\mu \\ \hat{W}_3^\mu \end{pmatrix} \quad (1.10)$$

The physical W^- and W^+ bosons are constructed via a linear combination of the two gauge fields $\hat{W}_1^\mu$ and $\hat{W}_2^\mu$:

$$\hat{W}_\pm^\mu = \frac{1}{\sqrt{2}}(\hat{W}_1^\mu \mp \hat{W}_2^\mu) \quad (1.11)$$

A natural choice for the third gauge field $\hat{W}_3^\mu$ would be to assign it to the Z boson; however, the $SU(2)_L$ gauge fields only interact with left-handed chiral fermions, while the Z boson interacts with both left-handed and right-handed chiral fermions. This is remedied by introducing an additional $U(1)_Y$ gauge field $\hat{B}^\mu$ with quantum number y (called "weak hypercharge"). The introduction of the additional gauge field is

the essence of electroweak unification; the photon and Z boson fields $\hat{A}^\mu$ and $\hat{Z}^\mu$, respectively, are constructed from linear combinations of two remaining gauge fields $\hat{W}_3^\mu$ and $\hat{B}^\mu$:

$$\begin{aligned}\hat{Z}^\mu &= \cos \theta_W \hat{W}_3^\mu - \sin \theta_W \hat{B}^\mu \\ \hat{A}^\mu &= \sin \theta_W \hat{W}_3^\mu + \cos \theta_W \hat{B}^\mu\end{aligned}\tag{1.12}$$

where θ_W is the *Weinberg angle*, which can be defined in terms of the $SU(2)_L \times U(1)_Y$ gauge couplings g and g' :

$$\cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}}\tag{1.13}$$

Furthermore, via the Higgs mechanism, the Weinberg angle can also be expressed in terms of the ratio of the W and Z masses:

$$\cos \theta_W = \frac{m_W}{m_Z}\tag{1.14}$$

Electroweak unification also tells us that the weak isospin (t_3) and weak hypercharge (y) quantum numbers can be used to express the electric charge number Q of a given particle via the relation:

$$eQ = e(t_3 + \frac{y}{2})\tag{1.15}$$

where e is the electron charge. Similarly, the axial and vector couplings for the neutral-current weak interaction can be written in terms of the fundamental charges and the Weinberg angle [9]:

$$c_V = t_3 - 2Q \sin^2 \theta_W, \quad c_A = t_3\tag{1.16}$$

Electroweak charges for fermions				
Particle	t_3	y	Q	c_V
$(e^-)_L, (\mu^-)_L, (\tau^-)_L$	-1/2	-1	-1	-0.04
$(\nu_e)_L, (\nu_\mu)_L, (\nu_\tau)_L$	+1/2	-1	0	+1/2
$(u)_L, (c)_L, (t)_L$	+1/2	1/3	+2/3	+0.19
$(d)_L, (s)_L, (b)_L$	-1/2	1/3	-1/3	-0.35
$(e^-)_R, (\mu^-)_R, (\tau^-)_R$	0	-2	-1	-0.04
$(\nu_e)_R, (\nu_\mu)_R, (\nu_\tau)_R$	0	0	0	+1/2
$(u)_R, (c)_R, (t)_R$	0	4/3	+2/3	+0.19
$(d)_R, (s)_R, (b)_R$	0	-2/3	-1/3	-0.35

Table 1.2: Weak isospin (t_3), weak hypercharge (y), electric charge (Q), and vector coupling to the Z boson (c_V) for the fermions. Subscripts L and R denote the left-handed and right-handed chiral states for the fermions, respectively. Vector couplings computed using the value of $\sin^2 \theta_W$ shown in Eq. 1.17.

The value of $\sin^2 \theta_W$ calculated in the on-shell scheme [20] is given by [21]:

$$\sin^2 \theta_W = 0.22342 \pm 0.00009 \quad (1.17)$$

Table 1.2 summarizes the weak isospin, weak hypercharge, and vector coupling values for the fermions. Two things can be pointed out immediately: the right handed fields have no weak isospin components, reflecting the fact that the charged weak interaction only acts on left handed fermion fields, and the right-handed neutrinos have no charge assignments, meaning that they do not interact at all within the electroweak theory. The existence of right handed neutrinos is an open question in experimental particle physics.

The $SU(2)_L \times U(1)_Y$ gauge theory is, like QCD, a non-abelian gauge theory. Thus, the gauge bosons are predicted to interact with each other; the structure of these interactions are parameterized by the gauge term in the electroweak Lagrangian. The gauge sector of the electroweak Lagrangian is given by [5]:

$$\begin{aligned}
 \mathcal{L}_{\text{ewk}} = & \underbrace{(\hat{D}_\mu \hat{\phi})^\dagger (\hat{D}^\mu \hat{\phi})}_{\text{Higgs field interaction term}} + \underbrace{\mu^2 \hat{\phi}^\dagger \hat{\phi}}_{\text{Higgs mass term}} - \underbrace{\frac{\lambda}{4} (\hat{\phi}^\dagger \hat{\phi})^2}_{\text{Higgs self interactions}} \\
 & - \underbrace{\frac{1}{4} \hat{\mathbf{F}}_{\mu\nu} \cdot \hat{\mathbf{F}}^{\mu\nu}}_{\text{SU}(2)_L \text{ field term}} - \underbrace{\frac{1}{4} \hat{G}_{\mu\nu} \cdot \hat{G}^{\mu\nu}}_{\text{U}(1)_Y \text{ field term}}
 \end{aligned} \tag{1.18}$$

The fourth and fifth terms in the Lagrangian of Equation 1.18 represent the interaction terms between the electroweak bosons. Expanding these terms in the unitary gauge (i.e. in terms of the physical boson fields), one obtains the following form for the interaction terms [7]:

$$\begin{aligned}
 \mathcal{L}_{\text{int,V}} = & \underbrace{-D^{\dagger\mu} W^{-\nu} D_\mu W_\nu^+ - D^{\dagger\mu} W^{-\nu} D_\nu W_\mu^+}_{\gamma\gamma WW \text{ and } ZZWW \text{ vertices}} \\
 & + \underbrace{ie(F^{\mu\nu} + \cot \theta_W Z^{\mu\nu}) W_\mu^+ W_\nu^-}_{\gamma WW \text{ and } ZWW \text{ vertices}} \\
 & - \underbrace{\frac{1}{2} \left(\frac{e^2}{\sin^2 \theta_W} \right) (W^{+\mu} W_\mu^- W^{+\nu} W_\nu^- - W^{+\mu} W_\mu^+ W^{-\nu} W_\nu^-)}_{\text{four-W vertices}}
 \end{aligned} \tag{1.19}$$

where

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu, \quad A^\mu = \text{electromagnetic vector potential}$$

$$Z^{\mu\nu} = \partial^\mu Z^\nu - \partial^\nu Z^\mu, \quad Z^\mu = \text{Z boson field}$$

$$D_\mu = \partial_\mu - ie(A_\mu + \cot \theta_W Z_\mu)$$

The terms in Equation 1.19 result in the multi-boson vertices shown in Figure 1.11 with their corresponding vertex factors.

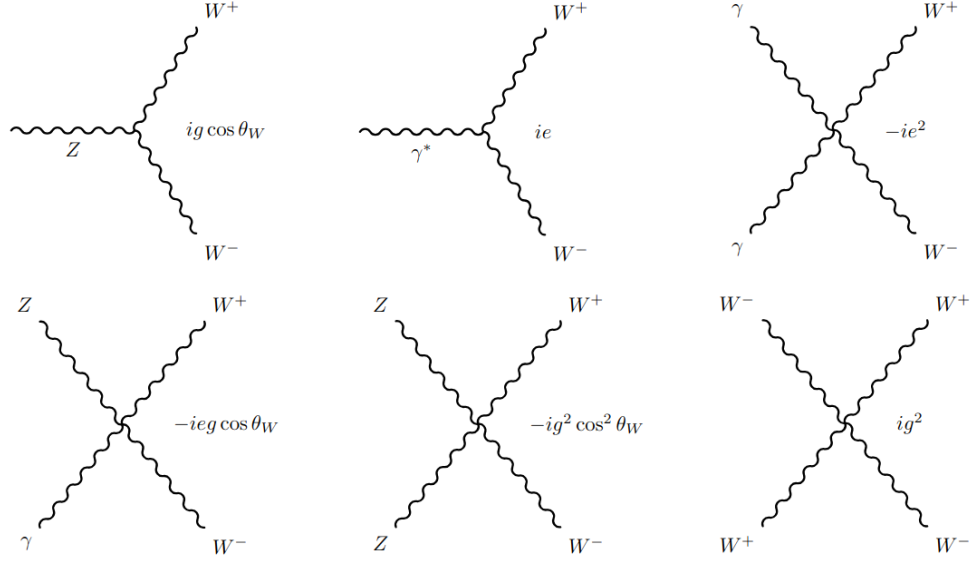


Figure 1.11: Interaction vertices between the massive vector bosons (W and Z) as predicted by the GSW theory.

Due to the Higgs mechanism, additional interactions between the massive gauge bosons and the Higgs boson are also possible. After electroweak symmetry breaking, the terms in the Lagrangian describing the interactions between the Higgs and electroweak bosons (in unitary gauge) is given by [7]:

$$\begin{aligned}
 \mathcal{L}_{\text{int,HV}} &= -(M_W^2 W^{+\mu} W_\mu^- + \frac{1}{2} M_Z^2 Z^\mu Z_\mu) (1 - \frac{H}{v})^2 \\
 &= - \underbrace{(M_W^2 W^{+\mu} W_\mu^- + \frac{1}{2} M_Z^2 Z^\mu Z_\mu)}_{\text{W and Z mass terms}} \\
 &\quad + \underbrace{(\frac{2M_W^2}{v} W^{+\mu} W_\mu^- H)}_{\text{HWW vertex}} + \underbrace{(\frac{M_Z^2}{v} Z^\mu Z_\mu H)}_{\text{HZZ vertex}} \\
 &\quad - \underbrace{(\frac{M_W^2}{v^2} W^{+\mu} W_\mu^- H^2)}_{\text{HHWW vertex}} + \underbrace{(\frac{M_Z^2}{2v^2} Z^\mu Z_\mu H^2)}_{\text{HHZZ vertex}}
 \end{aligned} \tag{1.20}$$

The first two terms in Equation 1.20 are the mass terms for the W and Z bosons,

which are the result of spontaneous symmetry breaking (see Section 1.3.1). The remaining terms produce vertices between the Higgs boson and the W and Z bosons, as shown in Figure 1.12.

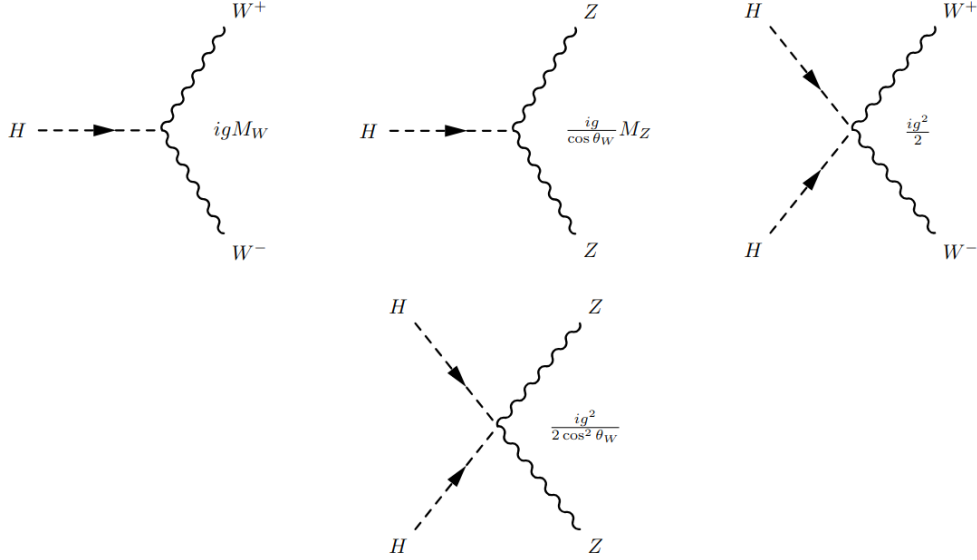


Figure 1.12: Interaction vertices between the Higgs boson and the massive vector bosons (W and Z) as predicted by the GSW theory.

1.3.1 Spontaneous Symmetry Breaking and The Higgs Mechanism

The Higgs mechanism addresses a fundamental problem with the formulation of the SM, namely, the method by which the fermions and bosons can acquire mass while maintaining gauge invariance. In the framework of the Higgs mechanism, the symmetry associated with the local $SU(2)_L \times U(1)_Y$ gauge structure of the electroweak theory is broken spontaneously. This arises as a natural extension to Goldstone's theorem, which addresses the spontaneous breaking of a global symmetry. Goldstone's theorem results from introducing a self-interacting complex scalar field ϕ that is composed of two real fields φ_1 and φ_2 [22]:

$$\phi = (\varphi_1 + i\varphi_2)/\sqrt{2} \quad (1.21)$$

Since ϕ is a scalar field, it obeys the Klein-Gordon equation. The associated Lagrangian for ϕ is a sum of two components; one is the free-particle Klein-Gordon Lagrangian while the other is a term specifying the self-interaction of a complex scalar field:

$$\mathcal{L} = \underbrace{\partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi}_{\text{free K.G. Lagrangian}} - \underbrace{\frac{\lambda}{4} (\phi^\dagger \phi)^2}_{\text{interaction term}} \quad (1.22)$$

where the potential $V(\phi)$ can be identified as:

$$V(\phi) = m^2 \phi^\dagger \phi + \frac{\lambda}{4} (\phi^\dagger \phi)^2 \quad (1.23)$$

Equation 1.23 can be rewritten in terms of φ_1 and φ_2 as:

$$V(\varphi_1, \varphi_2) = \frac{m^2}{2} (\varphi_1^2 + \varphi_2^2) + \frac{\lambda}{4} (\varphi_1^2 + \varphi_2^2)^2 \quad (1.24)$$

For positive m^2 and λ , the potential has a unique minimum at the origin, and the symmetry remains unbroken. However, when choosing $m^2 = -\mu^2 < 0$ and $\lambda > 0$, the potential no longer has a minimum at the origin, and the minimum now exhibits an $O(2)$ symmetry, as shown in Figure 1.13. This choice of m^2 and λ are the defining features of what we will call the Higgs potential, and the minimization of V with respect to ϕ gives a minimum value $\phi_{\min}$ at:

$$|\phi_{\min}| = \sqrt{\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}} \quad (1.25)$$

The rotational symmetry of this minima (or vacuum) allows one to make a choice for the values of φ_1 and φ_2 , thereby fixing the value of the vacuum. A convenient choice is:

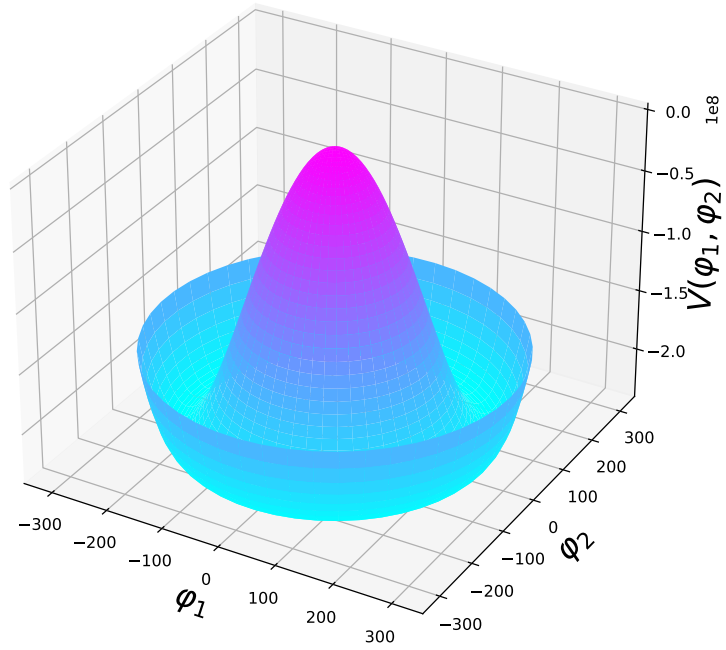


Figure 1.13: Potential V as a function of the scalar fields φ_1 and φ_2 , from equation 1.24 with $m^2 < 0$. Values obtained by using measured values of Higgs boson mass ($m_H = 125$ GeV) and Higgs vacuum expectation value ($v = 246$ GeV).

$$\varphi_{1,\min} = v, \quad \varphi_{2,\min} = 0 \quad (1.26)$$

which allows one to define new fields $\eta(x)$ and $\xi(x)$ that are centered about the vacuum:

$$\eta(x) = \varphi_1(x) - \varphi_{1,\min} = \varphi_1(x) - v \quad (1.27)$$

$$\xi(x) = \varphi_2 - \varphi_{2,\min} = \varphi_2 \quad (1.28)$$

Rewriting the Lagrangian in Eq. 1.22 in terms of these shifted fields, one finds that the η and ξ fields represent massive and massless fields, respectively, and the new Lagrangian no longer exhibits an $O(2)$ symmetry about the origin, as shown in Figure 1.14; this is what is known as *spontaneous symmetry breaking*.

Applying the Goldstone theorem to the local $SU(2)_L \times U(1)_Y$ gauge structure of the electroweak interaction is the basis of the Higgs mechanism. This is done by writing ϕ as a weak isospin doublet comprised of two complex scalar fields, denoted by a charged component (ϕ^+) and a neutral component (ϕ^0) [9]:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \quad (1.29)$$

Both ϕ^+ and ϕ^0 are necessary in order to generate the masses (and thus the longitudinal polarization states) for the $W^\pm$ and Z bosons. Taking the form of the doublet in Eq. 1.29 and plugging it in to the potential in Eq. 1.23 (with $-\mu^2 = m^2 < 0$), the minimum of the potential is satisfied by the condition:

$$\phi^\dagger \phi = \frac{1}{2}(|\phi^+|^2 + |\phi^0|^2) = \frac{v^2}{2} \quad (1.30)$$

The value of ϕ^0 is then chosen such that it has a non-zero vacuum expectation value,

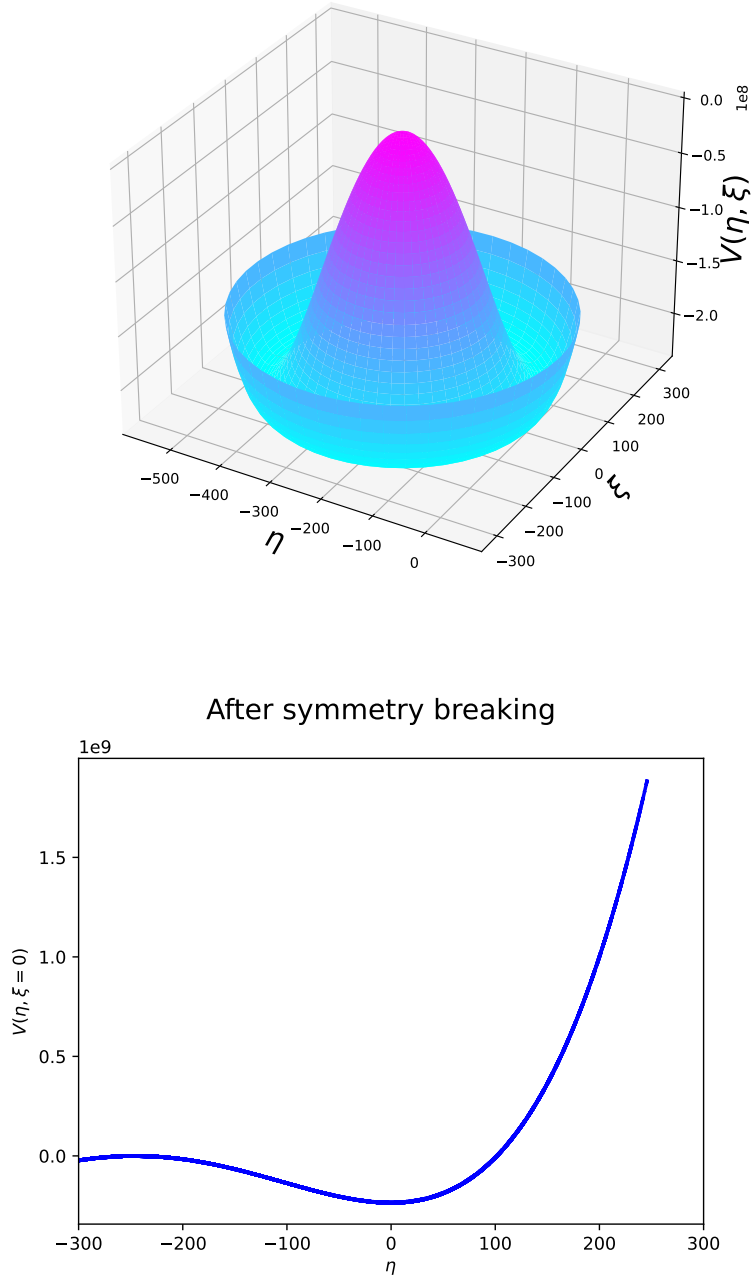


Figure 1.14: Potential V in terms of the shifted fields η and ξ in two dimensions (left) and in a 1D projection with $\xi = 0$ (right).

while the ϕ^+ component vanishes, resulting in the minimum given by:

$$\phi_{\min} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad (1.31)$$

Expanding about the minimum (e.g. introducing the η and ξ fields) breaks the symmetry, resulting in four fields (one for each real field ϕ_i in Eq. 1.29); one massive field $H(x)$, representing the physical Higgs boson, and three massless fields (Goldstone bosons), which give longitudinal polarization states to the $W^\pm$ and Z bosons. Rewriting ϕ in terms of $H(x)$ in the unitary gauge gives:

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad (1.32)$$

The masses of the gauge bosons result from applying the Higgs doublet in Eq. 1.32 to the first term in Eq. 1.18 and isolating the terms quadratic in the field (see Eq. 1.20). This allows one to express the $W^\pm$ mass in terms of the Higgs vacuum expectation value v and the $SU(2)$ gauge coupling g :

$$m_W = \frac{1}{2}gv \quad (1.33)$$

Using Eq. 1.14, it then follows that the Z mass can be written as:

$$m_Z = \frac{1}{2} \frac{gv}{\cos \theta_W} \quad (1.34)$$

The fermion masses are also generated through the spontaneous symmetry breaking process as well, via interactions of the right-handed and left-handed chiral components with the Higgs field due to the non-zero vacuum expectation value v . A more complete derivation of the fermion mass generation is given in ref. [9] (Section 17.5.5), however the main takeaway is stated here for posterity. The coupling of a fermion f to the

Higgs field is parameterized by a Yukawa coupling y_f , which is related to the fermion mass m_f via the relation:

$$y_f = \sqrt{2} \frac{m_f}{v} \quad (1.35)$$

The Higgs boson is thus unique in that its couplings to the particles of the Standard Model all depend on the masses of the particles themselves. Experimentally, the mass of the Higgs boson is measured to be [21]:

$$m_H = 125.20 \pm 0.11 \quad (1.36)$$

Kinematically, an on-shell Higgs can decay to a particular species of particle i if the following condition holds:

$$m_H \geq 2m_i \quad (1.37)$$

The decay modes of the Higgs boson are shown in the right panel of Figure 2.6; for the W and Z bosons, Eq. 1.37 does not hold, but there are still nonzero branching fractions for the decays $H \rightarrow W^+ W^-$ and $H \rightarrow ZZ$. These decays can occur if one or both of the W or Z bosons are off-shell, meaning that they do not satisfy Einstein's energy-momentum relation:

$$E^2 - p^2 \neq m^2 \quad (1.38)$$

(If you're wondering why the factors of c are absent, please refer to Section 2.1.1). Of particular interest for this thesis is the Higgs decay to W bosons, where one of the W bosons is off-shell, denoted by W^* :

$$H \rightarrow W W^* \quad (1.39)$$

which occurs $\approx 21\%$ of the time [21]. The importance of this decay mode will be further discussed in Section 1.5.

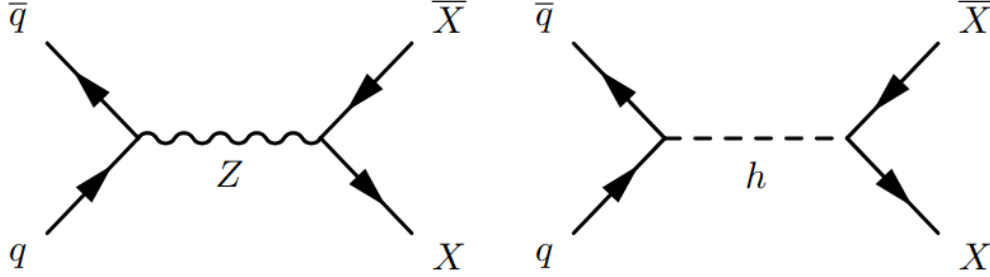


Figure 1.15: Tree-level Feynman diagrams for the hypothetical production process of a pair of X particles through Z production (left) or Higgs production (right).

Within the framework of the SM, the Higgs boson also restores unitarity for certain scattering processes; through destructive interference, Feynman diagrams involving the Higgs can interfere with other diagrams to keep the overall scattering probability bounded (recall, nothing can have a probability greater than one!). To illustrate this point, let's consider a simple example where we are interested in a fictitious scattering process:

$$pp \rightarrow X\bar{X}$$

where X is some hypothetical particle that couples to the Z boson and the Higgs boson only. Now, at leading order, say there are two Feynman diagrams for this process, which are shown in Figure 1.15; one mediated by the Z boson and one mediated by the Higgs boson. The cross section for this process (see Section 2.2.1) depends on the absolute square of the sum of all of the matrix elements contributing to the same final state, which are calculated directly from the Feynman diagrams. If we denote the matrix elements for the Z and Higgs mediated processes as $\mathcal{M}_Z$ and $\mathcal{M}_H$, respectively, the total squared matrix element $\mathcal{M}$ is given by:

$$|\mathcal{M}|^2 = |\mathcal{M}_Z + \mathcal{M}_H|^2 = |\mathcal{M}_Z|^2 + |\mathcal{M}_H|^2 + 2\text{Re}(\mathcal{M}_Z \mathcal{M}_H^*) \quad (1.40)$$

where the third term is the *interference* term. If the interference term is negative then the interference is said to be destructive. For the Higgs boson, this destructive interference can arise from the relative minus sign of the Higgs propagator with respect to the photon, gluon, and W and Z propagators.

1.4 The Standard Model

The culmination of the theories describing the strong and electroweak interactions as well as the observed particles (see Figure 1.1) is what is known as the Standard Model (SM) of particle physics. In group-theory representation, it can be written in the compact form:

$$\text{SM} = \underbrace{SU(3)_c}_{\text{strong}} \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{electroweak}} \quad (1.41)$$

The Lagrangian for the SM, which is considerably less pleasing to look at, is put in the back of this thesis in Appendix B as a matter of principle. In total, the SM contains a total of 25 free parameters; 12 are masses of the fermions, 2 are parameters associated with the Higgs boson (the Higgs mass and vacuum expectation value), 3 are couplings associated with the fundamental forces, 3 are the CKM mixing angles, 3 are the neutrino mixing angles, and the remaining 2 are the QCD CP-violating phase Θ and the CKM CP-violating phase δ . Despite this seemingly ad-hoc formulation, the SM is a remarkably accurate theory over a relatively broad energy range. Nevertheless, it still has its shortcomings, being unable to describe certain phenomena including, but not limited to:

- The origin and makeup of dark matter and dark energy.

- The Baryon-Antibaryon asymmetry.
- The hierarchy problem(s).
- Neutrino mass generation.

A review article describing each of these open problems is presented in ref. [23] (note: this is pre-Higgs discovery). Because of these shortcomings, it is crucial to continue testing the SM across a wide variety of energy scales and in different regimes to pinpoint the directions in which new particles and phenomena might lie.

1.5 Triboson Production

The discovery of the W and Z bosons at UA1 [24, 25] (and subsequent validation at UA2 [26, 27]) was a great step towards proving the accuracy of the GSW theory, however considerable experimental efforts were (and still are) needed to verify consistency between its predictions and the fundamental character of electroweak interactions. First steps included precision measurements of the properties of the W boson at the Large Electron Positron collider (LEP) [28] and Z bosons at LEP and the Stanford Linear Collider (SLC) [29], while successive experiments at the Tevatron at Fermilab [30] and the CERN Large Hadron Collider (LHC) [31] looked to accurately measure the production of two vector bosons (referred to as VV or "diboson production"). As early as the 1990s, there was interest in measuring the production of three vector bosons ("triboson" or VVV) [32], however due to the small cross sections and high energies required for triboson production, it was not until the LHC came online that this frontier could be explored in a statistically significant way. Measurements of the total VVV process by the CMS experiment [33] and ATLAS experiment [34, 35] using Run 2 (2016-2018) data were significant enough to claim an observation, however the individual processes comprising VVV (WWW , WWZ , WZZ , and ZZZ)

are still yet to be measured precisely.

The final piece of the SM, the Higgs boson, was discovered in 2012 by the CMS [36] and ATLAS [37] collaborations, signaling another triumph of the electroweak theory. However, like the W and Z discoveries before it, considerable experimental effort was, and still is, needed to precisely understand the properties and interactions of the Higgs boson. As discussed in Section 1.3, interactions between the Higgs boson and the massive gauge bosons also occur in the SM, resulting in contributions to multi-boson production from diagrams involving the Higgs boson.

The gauge and higgs sectors of the SM are particularly crucial for testing its structure; additional contributions to the SM, in the form of new particles or interactions, would result in deviations from the expected values as predicted by the theory. Triboson production, in particular, is highly sensitive to the gauge structure of the electroweak interaction; figure 1.16 shows tree-level Feynman diagrams for a generic triboson production process, which include the electroweak vertices shown in Figures 1.11 and 1.12.

1.5.1 Signal Processes

The process of interest chosen to test the SM gauge structure in this thesis is the production of two opposite-charge W bosons and one Z boson, henceforth referred to as WWZ production:

$$pp \rightarrow W^\pm W^\mp Z \quad (1.42)$$

This process is comprised of two primary contributions: the *non-resonant* WWZ process, shown by the diagrams in Figure 1.17, which involve no intermediate resonance, and the *resonant* ZH ($H \rightarrow WW^*$) process, shown in Figure 1.18, which involves the intermediate state of a Higgs boson with a Z boson. Furthermore, the

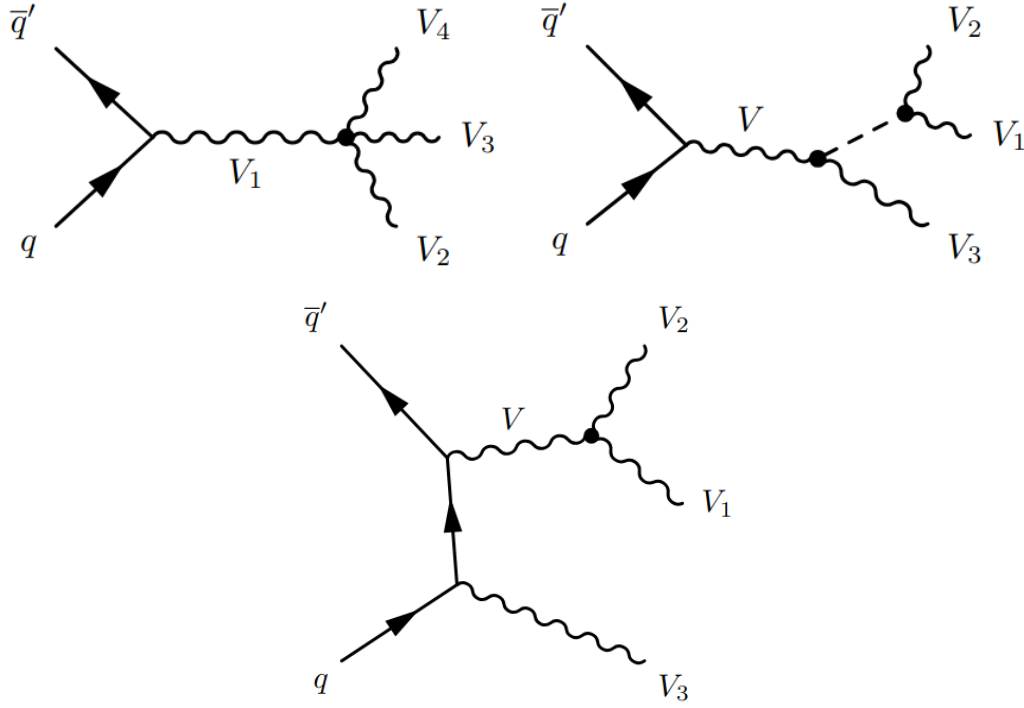


Figure 1.16: Tree-level Feynman diagrams for triboson production with multi-boson vertices illustrated as black dots.

non-resonant process is sensitive to both the quartic ($WWZZ$) and triple (WWZ) gauge boson vertices, while the resonant Higgs process is sensitive to both the HZZ and HWW vertices. This fact motivates the desire to separate these processes to effectively measure multiple gauge boson couplings simultaneously, thereby "killing two birds with one stone". The main phenomenological difference between the two processes is the presence of an off-shell W boson in the resonant ZH process, which is exploited in order to separate the two contributions from each other and perform a simultaneous measurement, which, to date, has never been done prior to the work described in this thesis. The interference between the two processes is estimated to be small enough that it is not relevant for this analysis.

The W and Z bosons have lifetimes of $\mathcal{O}(10^{-25}\text{s})$ [21], and thus they decay over time scales much shorter than can be detected in modern detector experiments (see Section 2.2.2). This means that the presence of the W and Z bosons must be inferred

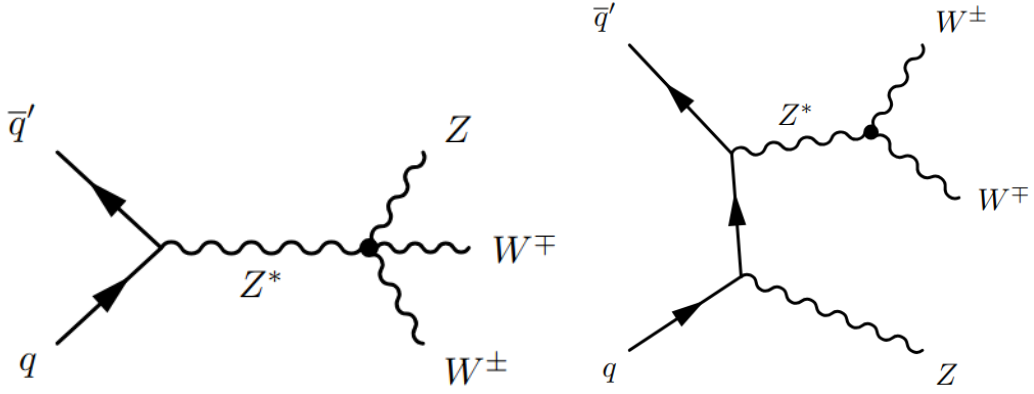


Figure 1.17: Tree-level Feynman diagrams for non-resonant WWZ production with quadratic and triple gauge couplings shown as black dots on the left and right diagrams, respectively.

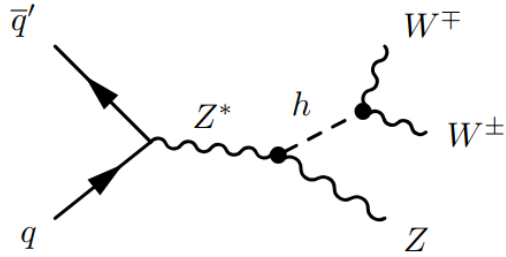


Figure 1.18: Tree-level Feynman diagram for resonant ZH ($H \rightarrow WW^*$) production with black dots illustrating HVV couplings.

from their decay products. In this analysis, we are interested in looking for the WWZ process by identifying *leptonic* decays of the W and Z bosons:

$$W^\pm \rightarrow \ell^\pm + \nu_\ell \quad (\mathcal{BR} \approx 11\%) \quad (1.43)$$

$$Z \rightarrow \ell^+ + \ell^- \quad (\mathcal{BR} \approx 3\%) \quad (1.44)$$

and thus we look for the signature:

$$pp \rightarrow W^\pm W^\mp Z \rightarrow 4\ell + \text{missing energy} \quad (1.45)$$

where ℓ is either an electron (e) or muon (μ) and "missing energy" is in reference to the fact that neutrinos do not show up in the detector (and thus can only be inferred by the presence of missing energy). There are various reasons why choosing leptonic decays of the W and Z is advantageous in searching for our signal processes:

- Leptons are easier to reconstruct experimentally than jets, leading to "cleaner" signatures and better energy resolution, making it more efficient to determine the origin of the lepton.
- Processes producing multiple leptons are rarer than those producing multiple jets, and thus signatures with more leptons will have fewer backgrounds.

Having described the motivations and the means for searching for the WWZ process, I am now equipped to define the objective of this thesis: **The objective of this thesis is to provide measurements of the non-resonant WWZ and resonant $ZH(H \rightarrow WW^*)$ cross sections in events with four leptons at center-of-mass energies of 13 and 13.6 TeV.** This will be done through the analysis of data collected by the Compact Muon Solenoid (CMS) experiment at the CERN Large Hadron Collider.

1.5.2 Background Processes

With a process of interest (or *signal* process) in mind and a particular final state defined (e.g. four leptons with some missing energy), we can now specify the types of processes that can produce the same (or similar) final states to our signal, that arise from different types of interactions. These processes that are not of interest to us but mimic the final state of the signal processes are referred to as *backgrounds*, and considerable effort and care is taken to isolate the signals from the backgrounds (see chapters 6 and 8). The main sources of background in this analysis arise from the production of two Z bosons that decay to four leptons, and the production of a top quark pair with a Z boson, with smaller contributions from other processes.

1.6 Roadmap

In the following chapters, I will systematically guide the reader through the methods of measurement, introducing the relevant background topics where necessary. The rest of Part I is laid out as follows:

- Chapter 2 familiarizes the reader with some necessary background information for understanding how particle physics searches are done.
- Chapter 3 describes the apparatus utilized for this search: the Large Hadron Collider, which accelerates protons up to the 13 and 13.6 TeV energies, and the Compact Muon Solenoid experiment, which detects the particles produced by the proton-proton collisions. A discussion of the reconstruction algorithms for particle identification is also presented here.
- Chapter 4 introduces the format and definition of the data collected by CMS and the simulated Monte-Carlo datasets utilized for comparison with the data.

- Chapter 5 defines the useful physics quantities from the detector that are useful in identifying the particles of interest in this analysis.
- Chapter 6 elucidates the set of selection criteria, based on the physics objects, that is useful for identifying the signal processes and separating them from the sources of background.
- Chapter 7 characterizes the main sources of background in this analysis and discusses the methods for accurately estimating their contributions to the measurement.
- Chapter 8 describes machine-learning methods for separating the signal processes from each other and the sources of background.
- Chapter 9 discusses the statistical framework and methods for presenting the results. The various sources of systematic uncertainty, and their modeling, are also presented.
- Chapter 10 details the obtained results for the cross section measurements and statistical significances of the signal processes.

CHAPTER 2
PHYSICS, CONVENTIONS, AND OTHER NECESSARY
INGREDIENTS

Any discussion of particle physics is not complete without an explanation of the physics that enable measurements of the properties of the Standard Model, the physics relevant for particle detection, and the conventions (i.e. units, coordinate systems) used commonly throughout particle physics experiments. This chapter attempts to address each of these necessary ingredients in a compact and easily digestible manner, however, it is not a fully comprehensive discussion of the details of each of these (very extensive) topics. The author urges the reader to explore the references for more details.

2.1 Conventions

2.1.1 Natural Units

Particle physics deals with the study of very small objects, typically at speeds that are very close to the speed of light. It is therefore convenient to utilize a system of units that reflect the size and velocities of the phenomena we are interested in. Enter *natural units*, which stipulates:

$$\hbar = c = 1 \tag{2.1}$$

A rather convenient consequence of this choice is that energy, momentum, and mass are now all expressed in units of giga-electronvolts ($1 \text{ GeV} = 10^9 \text{ eV}$):

$$[E] = [p] = [m] = \text{GeV} \tag{2.2}$$

which allows us to write Einstein's energy-momentum relation in the following form:

$$E^2 = p^2 c^2 + m^2 c^4 \rightarrow E^2 = p^2 + m^2 \tag{2.3}$$

Furthermore, natural units also allows for other quantities (such as time and length) to be expressed in powers of GeV. The conversion back to S.I. units can be obtained by the following relation:

$$\hbar c = 0.197 \text{ GeV} \cdot 10^{-15} \text{ m} \quad (2.4)$$

A notable exception to the use of natural units in particle physics occurs when considering cross sections for a particular interaction (see Section 2.2.1), which are expressed in units of area. Rather than in powers of GeV, cross sections are expressed in *barns*, where:

$$1 \text{ barn} = 10^{-24} \text{ cm}^2 \quad (2.5)$$

As an interesting historical aside: the term "barn" was coined by M. G. Holloway and C. P. Baker while working on the Manhattan Project [38] in order to describe the interaction cross sections of certain nuclear interactions involving Uranium.

2.1.2 Coordinate System

The CMS experiment (and various other experiments at colliding-beam facilities) utilize the cylindrical coordinate system, as shown in Figure 2.1, with some modifications to account for the relativistic speeds of the final state particles. In this, the z-axis is taken to be the axis along which the incoming beams travel, and ϕ is the azimuthal angle. In a traditional cylindrical coordinate system, the polar angle θ is utilized, which denotes the angle above the z-axis. However, due to the highly boosted nature of the final state particles resulting from particle collisions along the z-axis, it is convenient to replace the polar angle θ with the *rapidity* (y), which is defined as:

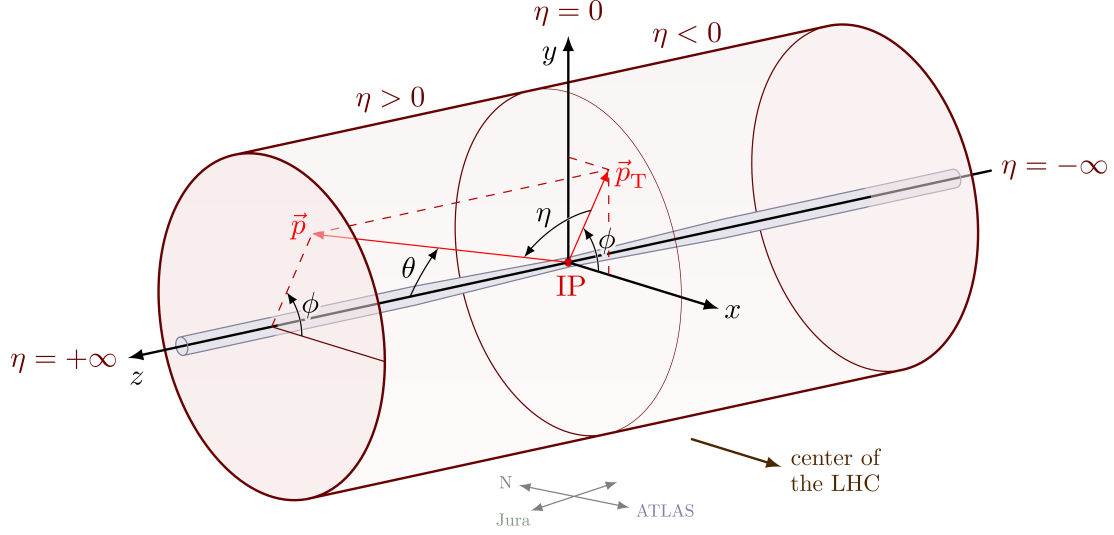


Figure 2.1: Diagram showing the cylindrical coordinate system utilized at CMS; the incoming beams travel along the z axis and collide at the interaction point (IP), highlighted in red. Figure taken from https://tikz.net/axis3d_cms/.

$$y = \frac{1}{2} \log \left(\frac{E + p_z}{E - p_z} \right) \quad (2.6)$$

where E is the particle energy and p_z is the particle momentum along the z (longitudinal) direction. The attractive feature of the rapidity is that differences in rapidity are Lorentz invariant under longitudinal boosts. A related quantity utilized at colliders where the final state particles are highly relativistic (and the mass can be ignored) is the *pseudorapidity* (η), which is defined in terms of the azimuthal angle θ as:

$$\eta = -\log \left[\tan\left(\frac{\theta}{2}\right) \right] \quad (2.7)$$

Thus, as $\theta \rightarrow 0$, η approaches infinity, as shown in Figure 2.2.

With this modified cylindrical coordinate system in mind, one can define a corresponding differential solid angle element ΔR in terms of differentials in ϕ and η :

$$\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} \quad (2.8)$$

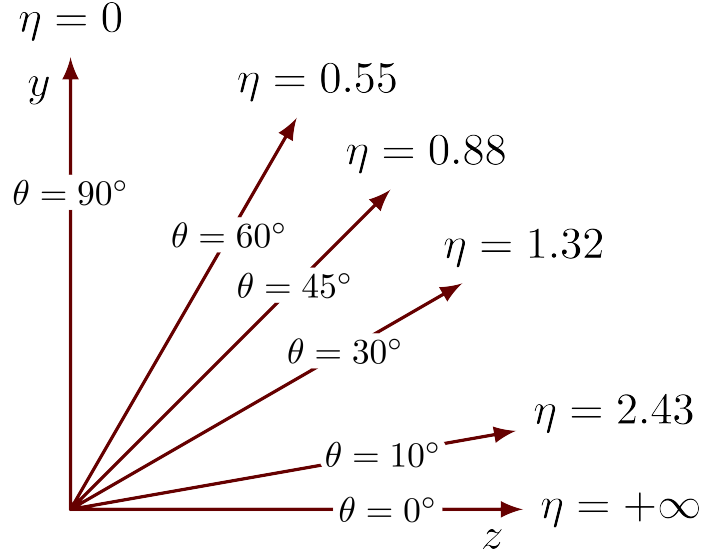


Figure 2.2: Diagram illustrating the relationship between the polar angle θ and the pseudorapidity η in relation to the vertical (y) axis and horizontal (z) axis. Figure taken from https://tikz.net/axis2d_pseudorapidity/

As will be discussed in Chapter 5, ΔR is used extensively for describing the relative (angular) proximity between different final state objects (e.g. particle tracks or calorimeter energy deposits). In particular, it is useful for the clustering of hadronic energy to form jets and for the identification of isolated leptons.

2.2 Physics

In Chapter 1, I discussed the Standard Model: the theory describing the Strong, Electromagnetic, and Weak Interactions in terms of their gauge structures. The development of the Standard Model, in practical terms, resulted from decades of observations from scattering experiments, cosmological data, and neutrino experiments, which are summarized in more detail in Ref. [21]. Such observations require the development of observable quantities that can be predicted by the theory itself, thereby allowing the theory to be tested. The primary observables for testing the Standard Model are:

- **Cross Section:** an area, which, if multiplied by the luminosity, expresses the number of occurrences of a given interaction.
- **Decay Rate:** the rate at which a given particle decays to another type of particle.

These observables (and related quantities) can be used to measure the different free parameters of the Standard Model. In the following sections, I will describe how these values can be calculated from the Standard Model and how they are measured in scattering experiments. Both of these quantities rely on two components; a matrix element ($\mathcal{M}$) that encodes the physics of the given interaction, and a phase-space element representing the final state of the interaction in position and momentum space.

For a given interaction, the total matrix element is a sum over the matrix elements for all Feynman diagrams that contribute to the interaction:

$$\mathcal{M} = \sum_{i=1}^{\infty} \mathcal{M}_i \quad (2.9)$$

The infinite sum in Eq. 2.9 reflects the fact that for any interaction governed by quantum mechanics, there is a contribution from all orders in perturbation theory. Practically, only the contributions from the lowest orders in perturbation theory (the largest contributions to the sum) are kept, as the contributions from higher-order matrix elements are typically quite small. Each Feynman diagram corresponds to a single matrix element, and the matrix element can be calculated directly from the Feynman diagram using the so-called *Feynman Rules* [39]. This topic is covered in great detail in many QFT textbooks (e.g. Refs. [5–7, 9, 40]), and thus I refer the reader to those references rather than attempting to provide a comprehensive description here.

2.2.1 Cross Sections

The behavior and properties of the fundamental particles can be probed by scattering experiments, whereby two beams of particles with known energy are collided with each other or a single beam is collided with a fixed target. The results of these collisions are then measured using detectors, and the probability of a given type of interaction to occur is characterized by its *cross section*, which, as previously noted, is expressed in units of area (typically cm^2 or barns). The cross section σ for a given process is defined as [9]:

$$\sigma = \frac{R}{\Phi \times N_{\text{target}}} \quad (2.10)$$

where R is the rate of interactions (the number of interactions per unit time), Φ is the flux of particles (the number of particles passing through a unit area per unit time), and N_{target} is the number of target particles. To eliminate any dependence on a particular reference frame (and thus allow comparisons between different experiments), the cross section must also exhibit Lorentz invariance. This implies that the rate of interactions and the flux should be formulated in a manifestly Lorentz-invariant manner. This procedure, outlined in Chapter 3 of Ref. [9], results in the general form of the cross section for any two-to-two scattering process ($a + b \rightarrow 1 + 2$), calculated in the center-of-mass (CM) frame:

$$\sigma = \frac{1}{64\pi^2 s} \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|} \int |\mathcal{M}|^2 d\Omega \quad (2.11)$$

where $\mathbf{p}_i$ and $\mathbf{p}_f$ are the 3 momenta of the initial state and final state particles in the CM frame, respectively, $\mathcal{M}$ is the Lorentz-invariant matrix element described in Section 2.2, $d\Omega$ is the differential solid angle element, and s is the Lorentz-invariant *Mandelstam variable* defined as [21]:

$$s = (p_a + p_b)^2 = (p_1 + p_2)^2 = m_a^2 + m_b^2 + 2E_a E_b - 2\mathbf{p}_a \cdot \mathbf{p}_b \quad (2.12)$$

where p_i is the 4-momentum of particle i . Examining Eq. 2.11, one finds that, in general, the cross section (regardless of the process itself) has a strong dependence on the energy of the collision; namely, the cross section shrinks as the square of the collision energy. The physics of the particular interaction is encoded in the matrix element, which, in general, also carries some dependence on the energy scale. Thus, depending on the process of interest, one may find that the cross section increases or decreases as a function of the center of mass collision energy ($\sqrt{s}$). An example of this, relevant to future Higgs factories, is shown in Figure 2.3, which gives the calculated cross sections for associated Higgs boson production processes from electron-positron collisions as a function of the CM collision energy; for energies below approximately 400 GeV, the main contribution to Higgs production is from the ZH production process, while above that energy the main contribution comes from the vector boson fusion (VBF) process.

2.2.2 Particle Decays

"Everything not forbidden is
compulsory"

Murray Gell-Mann (1956)

The fundamental forces enable the decays of massive particles into lighter particles if the process obeys the kinematical constraints (namely, energy conservation) and is not forbidden by the conservation laws of the particular interaction. Furthermore, Quantum Mechanics stipulates that anything not expressly forbidden must occur with some finite probability, and thus the decays of the massive particles is expected to

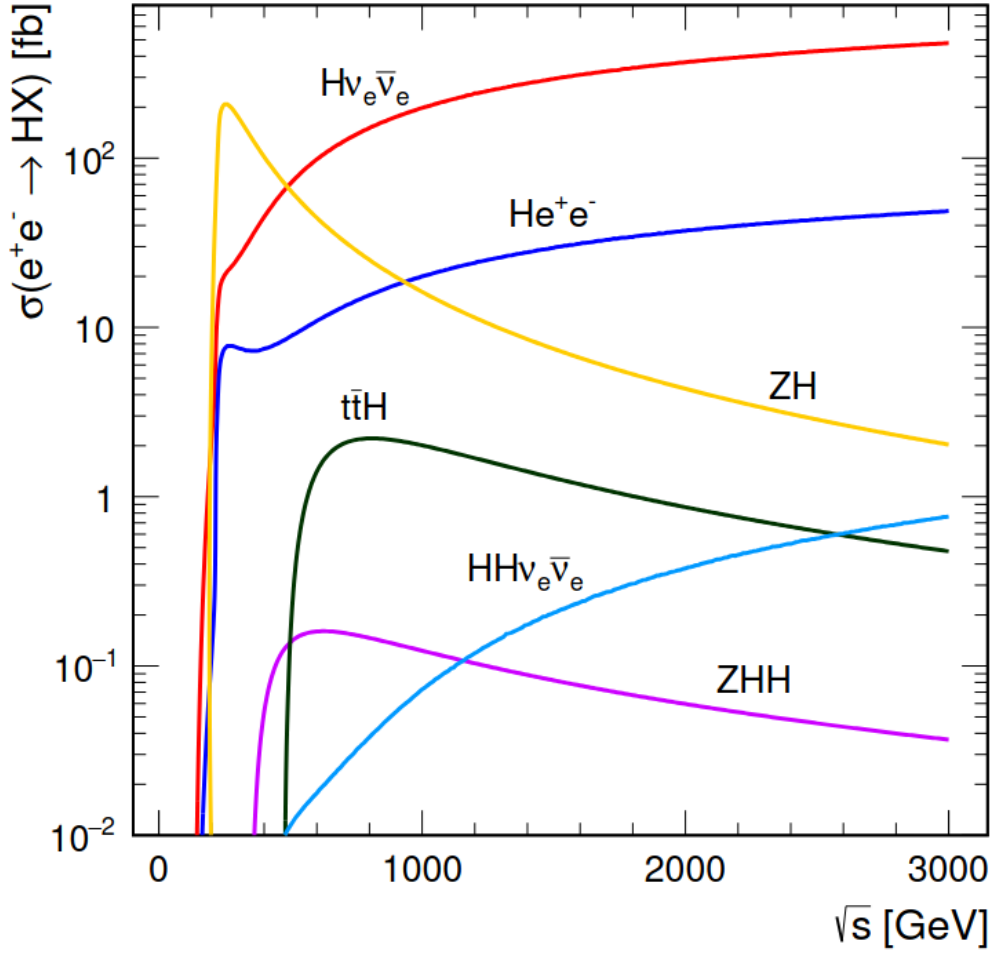


Figure 2.3: Predicted cross sections for associated Higgs production processes from electron-positron collisions as a function of center-of-mass collision energy ($\sqrt{s}$). Figure taken from Ref. [41].

occur. Experimentally, all fundamental and composite particles undergo decays with a few notable exceptions:

- electrons: electron decay is forbidden as there are no lighter states for it to decay to via the electroweak interaction while also conserving electric charge.
- protons: proton decay has yet to be observed experimentally (the lifetime has a lower bound far greater than the age of the universe [21]). Within the Standard Model, the proton is considered a stable bound state.
- neutrinos: neutrinos can only interact via the weak interaction, whose structure prohibits the decays of neutrinos into other neutrino types. Furthermore, neutrino decay via the weak interaction is kinematically forbidden. Neutrinos, however, do undergo neutrino oscillations [42–44], which allows for neutrinos of one type to change to a neutrino of a different type.

The average decay times for various fundamental and composite particles are summarized in Figure 2.4, and their corresponding decay lengths (how far the particle travels before decaying) for a momentum range between 1 and 50 GeV are shown in Figure 2.5. The mean decay time for a given particle is defined to be the inverse of the *total decay width* (Γ):

$$\tau_{\text{decay}} = \frac{1}{\Gamma} \quad (2.13)$$

where Γ is expressed as the sum over all partial decay widths ($d\Gamma$) contributing to the particle decay:

$$\Gamma = \sum_{j \in \text{decays}} d\Gamma_j \quad (2.14)$$

The partial decay width for a given particle decay in its rest frame is given by [21]:

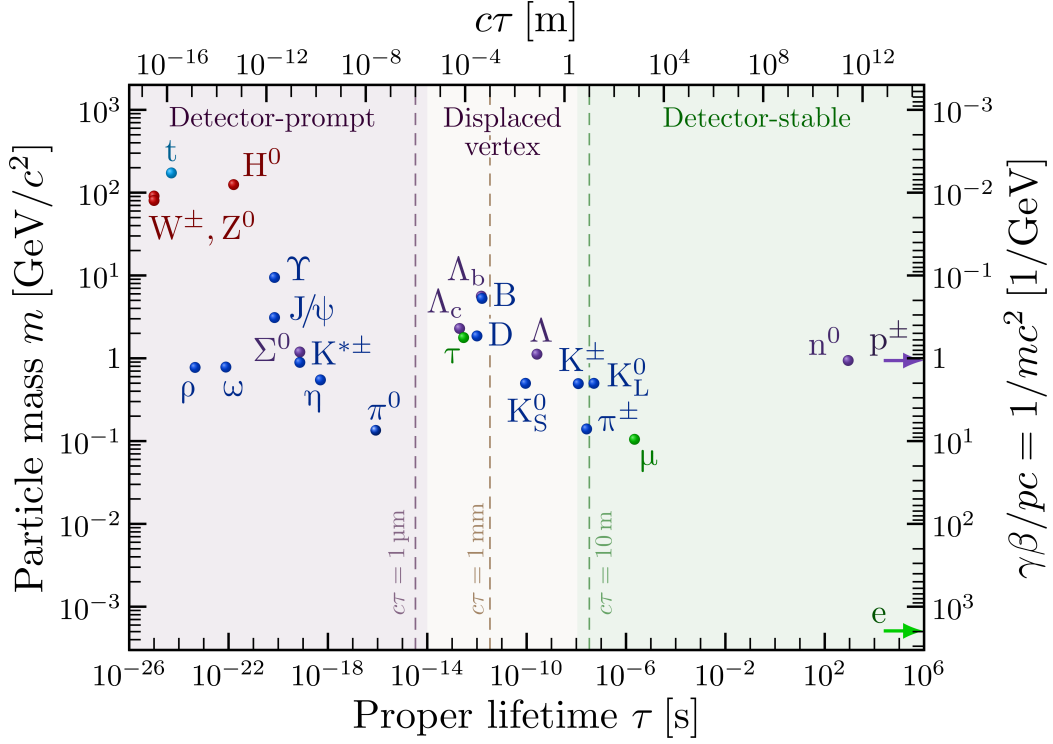


Figure 2.4: Average decay times for SM particles. Figure taken from https://tikz.net/sm_particles_masses/

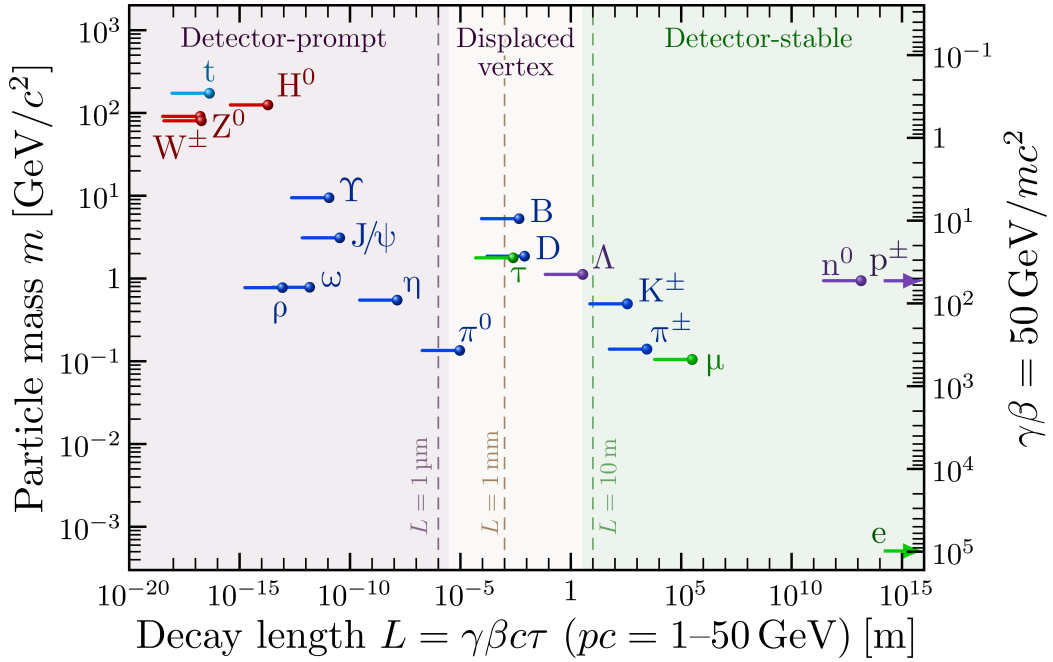


Figure 2.5: Decay lengths for SM particles with momenta between 1 and 50 GeV. Figure taken from https://tikz.net/sm_particles_masses/

$$d\Gamma = \left[\frac{(2\pi)^4}{2M} |\mathcal{M}|^2 \right] \times \left[\delta^4(P - \sum_{i=1}^n p_i) \prod_{i=1}^n \frac{d^3 p_i}{(2\pi)^3 2E_i} \right] \quad (2.15)$$

where M and P is the mass and 4-momentum of the decaying particle, respectively, $\mathcal{M}$ is the Lorentz-invariant matrix element (discussed in Section 2.2) for the decay process, and n is the number of final state particles. The first term in Eq. 2.15 encodes the physics of the decay process, while the second term is the phase space component; the Dirac delta function imposes conservation of 4-momentum between the initial state particle and the decay products, and the product over the phase space elements specify the total available phase space in which the interaction can occur. Integrating over the outgoing particle momenta, for a two-body decay ($X \rightarrow 1 + 2$), Equation 2.15 simplifies to:

$$d\Gamma = \frac{1}{32\pi^2} |\mathcal{M}|^2 \frac{|\mathbf{p}_1|}{M_X^2} d\Omega \quad (2.16)$$

where $\mathbf{p}_1$ is the three-momenta of particle 1, M_X is the mass of the X particle, and $d\Omega$ is the differential solid angle element for particle 1.

Equations 2.15 and 2.16 along with Equations 2.13 and 2.14 can be used to make qualitative statements about the decay times for different types of particles:

- Heavier particles will typically decay faster since there are more kinematically allowed states to decay to.
- Particles that can decay via the strong force will typically decay faster than those that cannot, due to the relative size of the strong coupling constant relative to the electroweak coupling constants.

In this analysis, we are particularly interested in the (leptonic) decay modes of the W boson, the Z boson, the Higgs boson, the top quark, and the τ lepton for

the purposes of isolating our signal processes of interest and rejecting sources of background. As shown in Figure 2.5, the W , Z , and Higgs bosons, along with the top quark, decay over very short lengths and are thus not directly detectable. Thus, the presence of these particles must be inferred solely from their decay products. The branching fractions (the percentage of decays to a given final state) for the W , Z , and H bosons are shown in Figure 2.6. The top-quark branching fractions are the same as the W boson as the top decays $\approx 100\%$ to a W boson and a bottom-quark [21]. The τ lepton decay time is long enough to travel a few millimeters in the detector, which can be measured and used in τ lepton reconstruction algorithms. The major decays of the τ lepton are shown in Table 3.1.

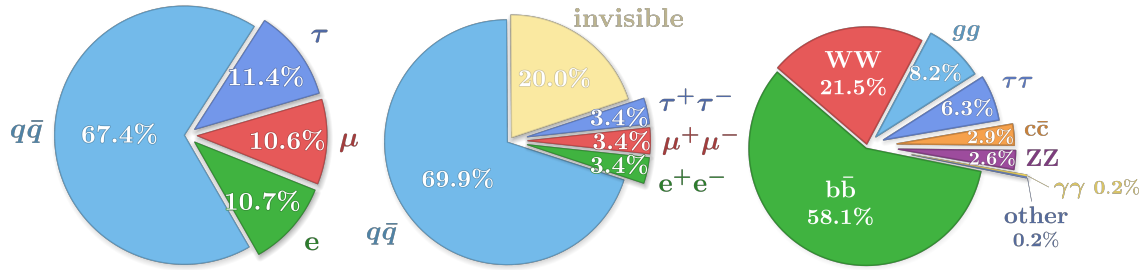


Figure 2.6: Branching fractions for the W boson (left), Z boson (center), and Higgs boson (right). Decays of the W boson to leptons also include the presence of the corresponding neutrino. Figures taken from https://tikz.net/sm_decay_piechart/.

2.2.3 Luminosity and Event Rates

In a collider experiment, the number of events produced per second $\dot{N}$ for a given scattering process is related to the cross section of the process σ (see Section 2.2.1) and the *luminosity* $\mathcal{L}$ of the collider:

$$\dot{N} = \sigma \times \mathcal{L} \quad (2.17)$$

which is expressed in units of $\text{cm}^{-2} \text{s}^{-1}$ or $\text{fb}^{-1} \text{s}^{-1}$. Intuitively, the luminosity can be thought of as the intensity of collisions per second, which depends on the

parameters of the collider itself. For a colliding beam experiment, whereby two beams of particles are shot into each other, the overall luminosity depends on the intensity and structure of the beams themselves as well as their overlap. Representing the beam density distributions as $\rho_1(x, y, s, -s_0)$ and $\rho_2(x, y, s, s_0)$ where s_0 is the time variable ($s_0 = 0$ is the time at which the collision occurs), the luminosity for head-on collisions is given by a 4-dimensional integral [45]:

$$\begin{aligned}\mathcal{L} &= 2N_1N_2fN_b \int \int \int \int_{-\infty}^{+\infty} \rho_1(x, y, s, -s_0)\rho_2(x, y, s, s_0) dx dy ds ds_0 \\ &= 2N_1N_2fN_b \int \int \int \int_{-\infty}^{+\infty} \rho_{1x}(x)\rho_{1y}(y)\rho_{1s}(s-s_0)\rho_{2x}(x)\rho_{2y}(y)\rho_{2s}(s+s_0) dx dy ds ds_0\end{aligned}\tag{2.18}$$

where N_1 and N_2 are the number of particles per bunch, f is the number of collisions per second (the *repetition rate*), and N_b is the number of bunches per pulse. In the second equality, I have assumed that the densities in each direction are uncorrelated, allowing them to be factorized.

Typically, a reasonable approximation is obtained by assuming that both incoming beams have a gaussian profile in both transverse dimensions and the longitudinal dimension and are collided head-on, with perfect overlap between them. In this case, the luminosity is given by compact expression:

$$\mathcal{L} = \frac{N_1N_2fN_b}{4\pi\sigma_x\sigma_y}\tag{2.19}$$

where σ_x (σ_y) is the horizontal (vertical) root-mean-square (RMS) beam size. Here, we have implicitly assumed that both bunches have the same RMS beam sizes and lengths. Equation 2.19 describes what is typically referred to as the *instantaneous luminosity*, e.g. the luminosity per unit time. In particle physics, a more useful

quantity is the *integrated luminosity* $\mathcal{L}_{\text{int}}$:

$$\mathcal{L}_{\text{int}} = \int \mathcal{L}(t) dt \quad (2.20)$$

as it directly relates to the total number of events over some specified time period. At CMS, the total integrated luminosity in Run 2 (2016-2018) was 138 fb^{-1} .

2.2.4 Interactions between charged particles and matter

Charged particles passing through matter will undergo electromagnetic interactions with the electrons orbiting the nucleus, and, at sufficiently high energies, with both the electrons and the nucleus itself. In the former case, the interaction between the charged particle and the electrons kicks the electrons out of orbit, causing ionization in the material and energy loss for the incident particle. In the latter case, the interaction with the atomic nucleus causes a deflection in the trajectory of the charged particle, which causes the particle to lose energy through the emission of *bremsstrahlung* (German for "braking radiation") photons, as shown in Figure 2.7. The characteristic energy scales for ionization and bremsstrahlung for an electron (or positron) in lead are illustrated in Figure 2.8.

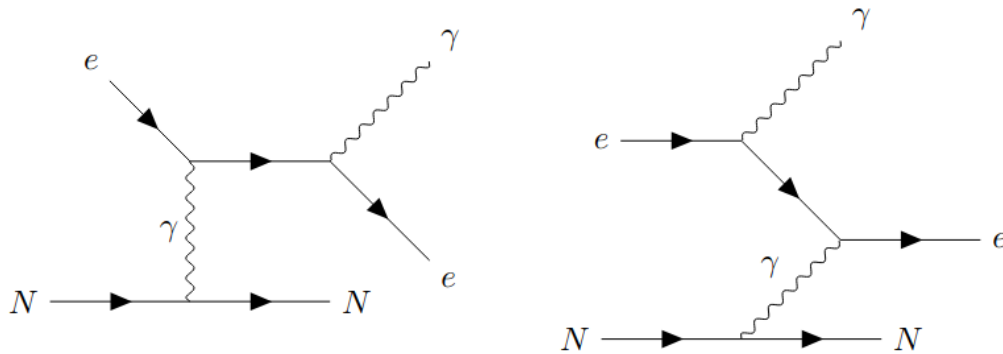


Figure 2.7: Diagrams showing the emission of bremsstrahlung photons due to the interaction of an electron (e) with a nucleus (N).

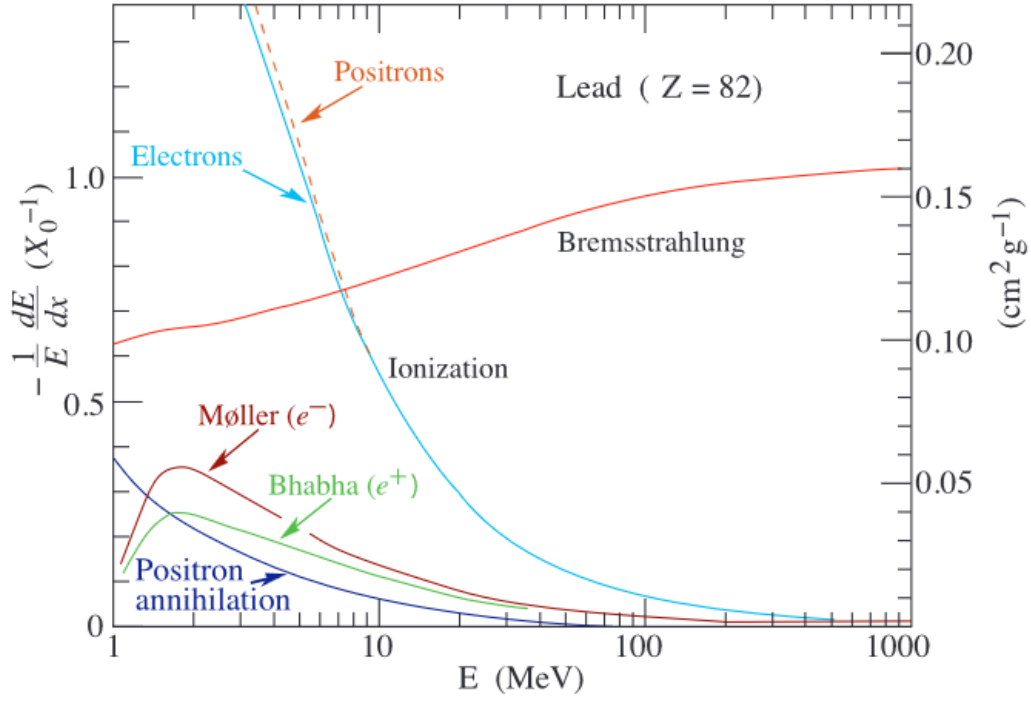


Figure 2.8: Fractional energy loss per radiation length for an electron or positron traversing through lead. Figure taken from Ref. [21].

While single ionization interactions typically result in small energy losses, many of these interactions will occur as the particle traverses a finite length of material. Due to the large number of interactions, the mean energy loss per unit length for a (heavy) charged particle traversing through matter due to ionization is of interest, which is governed (for approximately $0.1 \leq \beta\gamma \leq 1000$) by the Bethe-Bloch equation [21]:

$$-\frac{dE}{dx} = (4\pi N_A r_e^2 m_e c^2) z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \log\left(\frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2}\right) - \beta^2 - \frac{\delta}{2} \right] \quad (2.21)$$

where

- N_A is Avagadro's number ($6.022 \times 10^{23} \text{ mol}^{-1}$).
- r_e is the classical electron radius (2.818 fm).

- m_e is the mass of the electron ($0.511 \text{ MeV}/c^2$).
- z is the charge of the incident particle, in units of the electron charge.
- Z is the atomic number of the material.
- A is the atomic mass of the material.
- $\beta = \frac{v}{c}$ is the velocity of the incident particle relative to the speed of light.
- γ is the Lorentz factor.
- T_{max} is the maximum energy transfer to an electron from a single collision (see Equation 34.4 in Ref. [21]).
- I is the mean excitation energy (on the order of eV).
- δ is an energy-dependent correction that relates to the behavior of the material due to polarization.

For low-energy incident particles ($\beta\gamma \leq 0.1$), Eq. 2.21 breaks down as the particle velocity is comparable with the velocity of the atomic electrons. For very high energy incident particles ($\beta\gamma > 1000$), the contributions from pair production and bremsstrahlung become dominant, and the assumption of constant energy loss used to derive Eq. 2.21 breaks down. The behavior of the Bethe-Bloch equation, as well as its low energy and high energy extensions, is illustrated in Figure 2.9 for positive muons incident on a Copper target.

At very high energies, the contributions to energy loss due to bremsstrahlung and e^+e^- pair production create electromagnetic showers, as illustrated in Figure 2.10. The characteristic length scale of these high-energy showers is parameterized by the *radiation length* (X_0), which is defined as the mean distance over which a charged particle's energy is reduced to $1/e$ of its initial energy. The radiation length is a

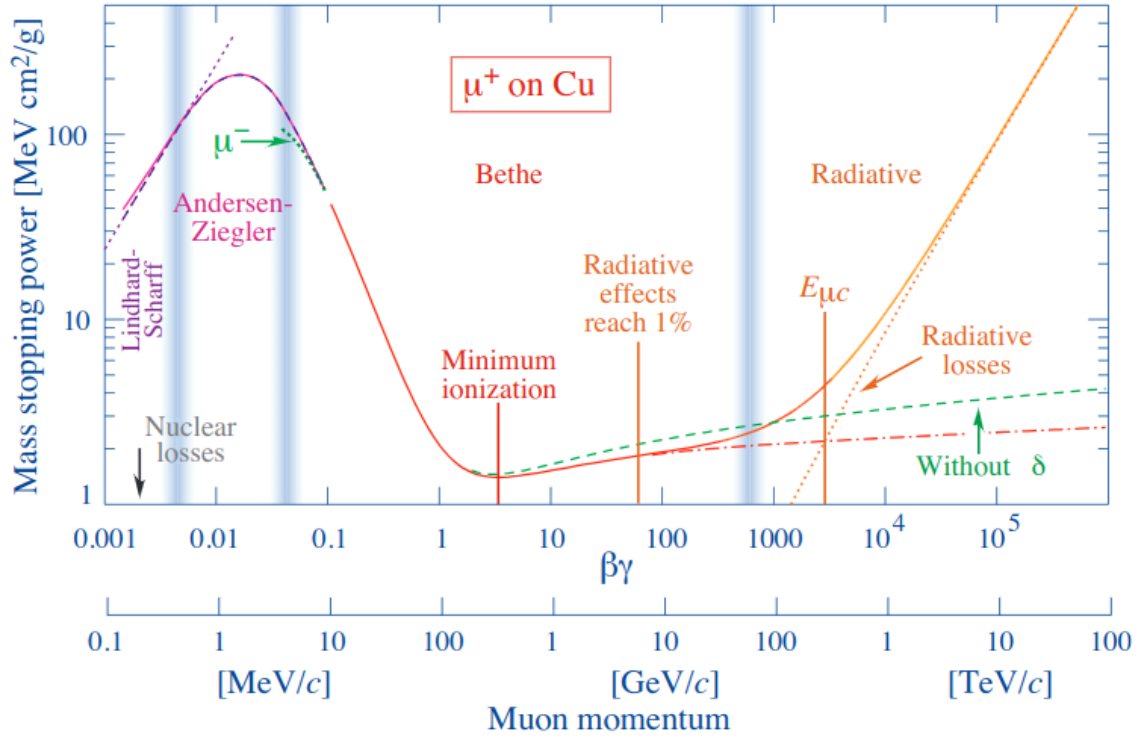


Figure 2.9: Mass stopping power (dE/dx) for positive muons passing through Copper as a function of muon momentum. Figure taken from Ref. [21].

property of the material itself, and lower values are desirable for use in detectors to contain electromagnetic showers in a smaller space.

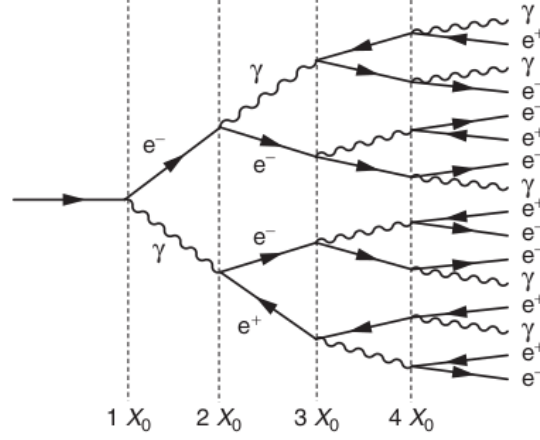


Figure 2.10: Diagram of an electromagnetic shower due to the interaction of a charged particle (an electron) with a material. Vertical lines indicate the number of radiation lengths over which the shower occurs. Figure taken from Ref. [9] (Figure 1.17).

2.2.5 Measurement of Transverse Momenta

Charged particles subject to magnetic fields will experience the Lorentz force (see Eq. 3.1), resulting in a force perpendicular to the velocity of the particle. In the transverse plane, this results in a circular trajectory for the charged particle as it traverses through different detector layers, as shown in Figure 2.11 (left). From this, one can define the *sagitta* (s), as shown in Figure 2.11 (right), as the distance between the midpoint of the arc (created by the particle track) to the chord linking the endpoints of the arc. Geometrically, the sagitta is related to the radius of curvature of the track R_c and the chord length d via:

$$\left(\frac{d}{2}\right)^2 + (R_c - s)^2 = R_c^2 \quad (2.22)$$

Solving Eq. 2.22 for the radius of curvature, one obtains:

$$R_c = \frac{\left(\frac{d}{2}\right)^2 + s^2}{2s} \quad (2.23)$$

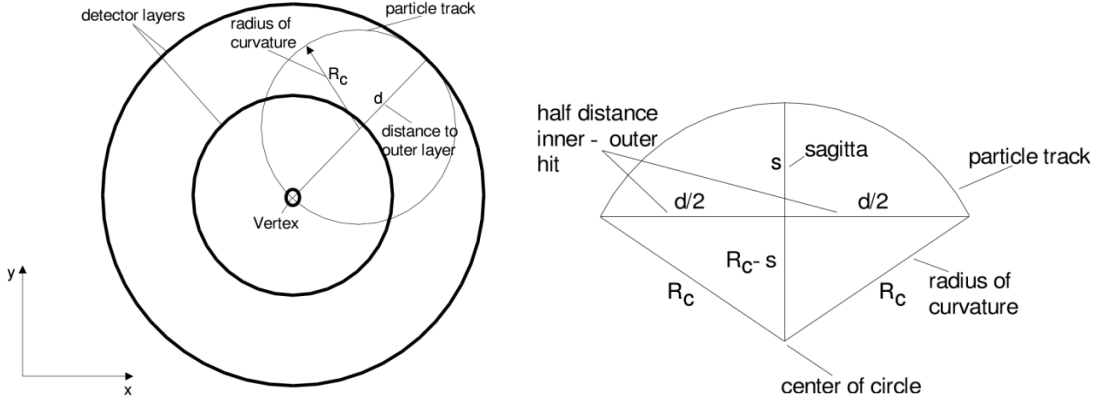


Figure 2.11: Diagram showing the trajectory of a charged particle in a magnetic field in the transverse plane (left) and diagram showing the definition of the sagitta (s) in terms of the radius of curvature (R_c) and the chord linking the two endpoints of the arc (d). Figures taken from Ref. [46] (Figures 3.6 and 3.7).

In a homogeneous magnetic field perpendicular to the particle motion (i.e. along the z direction), the transverse momentum of a charged particle is given by:

$$p_T = q \int B dL \approx q B_z R_c \quad (2.24)$$

where q is the charge of the particle and B_z is the z -component of the magnetic field. Substituting the expression for the radius of curvature obtained in Eq. 2.23, the transverse momentum can be written in terms of the sagitta s and the chord length d :

$$p_T = 0.2998 \times \frac{\left(\frac{d}{2}\right)^2 + s^2}{2s} \times B_z \quad [\text{GeV}] \quad (2.25)$$

This expression provides a simple formula for converting the measured track parameters for a charged particle into a measurement of the transverse momentum.

The transverse momentum is particularly useful in experiments at proton-proton

colliders as, to a very high degree of accuracy, momentum is conserved in the transverse plane. In the longitudinal direction, momentum is still conserved, however it cannot be fully measured as some particles do not escape the beam pipe. This can be leveraged to determine the missing transverse momentum (see Section 5.4), which accounts for the undetectable particles (e.g. neutrinos) produced in a given event. Due to the composite nature of the proton, the longitudinal momentum of each collision is not known precisely (see Section 2.2.6), and thus conservation of momentum along the z axis is not a reliable means of identifying missing energy in proton-proton collisions. As will be seen in Section 2.2.7, the transverse momentum will also be important for the determination of the invariant mass.

2.2.6 Parton Distribution Functions

The proton is a type of baryon; a QCD bound state consisting of two up and one down valence quarks bound together through a "sea" of gluons (as described in Section 1.2). Because of this, the total proton momentum is distributed among the individual proton constituents or *partons*. The fraction of proton momentum carried by each type of parton is parameterized by *parton distribution functions*, which can be measured at collider and fixed-target experiments [47]. These functions are dependent upon the squared momentum transfer μ^2 of the collision; a plot of the measured parton distribution functions at two values of momentum transfer are shown in Figure 2.12. As expected, the majority of the proton momentum is carried by the valence up and down quarks, with a sizeable contribution from gluons at lower momentum. There is also a non-zero contribution from strange, charm, and bottom quarks, which can be pair-produced from gluons.

At proton-proton colliders, such as the Large Hadron Collider, the available energy per collision is limited by the momentum fraction of the interacting partons. Thus,

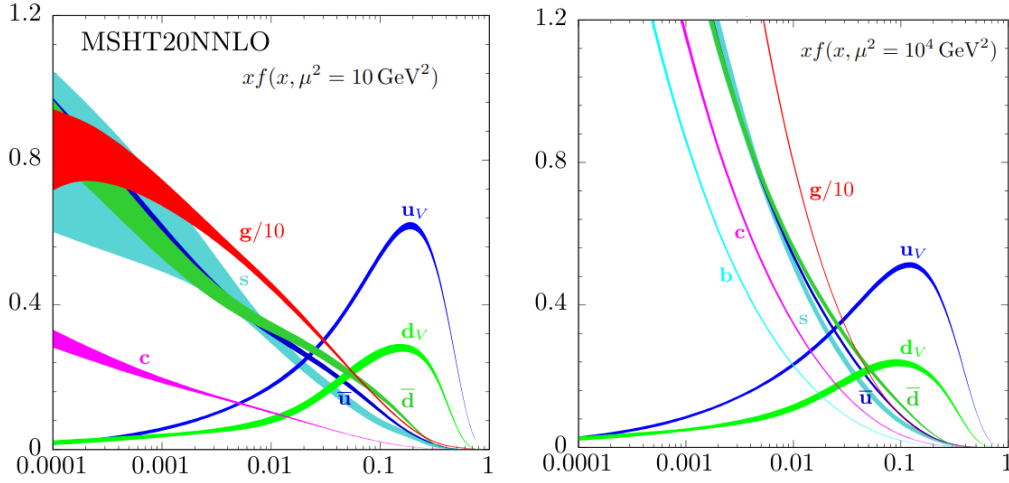


Figure 2.12: Plots of parton distribution functions at $\mu^2 = 10 \text{ GeV}^2$ (left) and $\mu^2 = 10^4 \text{ GeV}^2$ (right) obtained from the NNLO MSHT20 global analysis [48]. In both plots, the x-axis represents the fraction of the total proton momentum, while the y axis represents the parton distribution functions multiplied by the momentum fraction for the given parton. Figure taken from Ref. [21].

for a given center-of-mass collision energy ($\sqrt{s}$), the most probable interactions have a total energy tens to hundreds of times smaller than $\sqrt{s}$.

2.2.7 Invariant Mass

As was discussed in Section 2.2.2, most of the particles in the Standard Model undergo decays. Thus, the presence of particles that decay within the detector must be inferred from the decay products. For particles that have visible decay products (such as charged leptons or hadrons), one can define a Lorentz-invariant quantity known as the *invariant mass* which comes from Einstein's relativistic relation:

$$E^2 = p^2 + m^2 \quad (2.26)$$

where E is the total energy of the particle (or system) and p is the momentum of the particle (or system) in natural units. Equation 2.26 holds in any frame, and so

the invariant mass M for a parent particle can be derived in terms of the masses, energies, and momenta of the decay products:

$$M^2 = (E_1 + E_2)^2 - |\mathbf{p}_1 + \mathbf{p}_2|^2 = m_1^2 + m_2^2 + 2(E_1 E_2 - \mathbf{p}_1 \cdot \mathbf{p}_2) \quad (2.27)$$

where $E_{1(2)}$ is the energy of particle 1 (2), $\mathbf{p}_{1(2)}$ is the three-momentum of particle 1 (2), and $m_{1(2)}$ is the mass of particle 1 (2). In the high energy limit where we can treat the daughter particles as massless (i.e. where $p^2 \gg m^2$), the invariant mass of Eq. 2.27 takes a more simple form:

$$M^2 = 2|\mathbf{p}_1||\mathbf{p}_2|(1 - \cos(\theta)) \quad (2.28)$$

where θ is the scattering angle between the outgoing particles.

Experimentally, the invariant mass can be used for particle identification and the precise measurement of particle masses, as over many collisions the distribution will exhibit a peak (or *resonance*) at the mass of the parent particle. Figure 2.13 shows the invariant mass spectrum for pairs of identified muons at the CMS experiment; the plot exhibits peaks at the different particle resonances, corresponding to the masses of the labeled particles.

The peaks in Figure 2.13 exhibit a shape described by the (relativistic) Breit-Wigner probability distribution function [50]:

$$p(M)dM = \frac{k}{(M^2 - M_0^2)^2 + M_0^2 \Gamma^2} dM \quad (2.29)$$

with

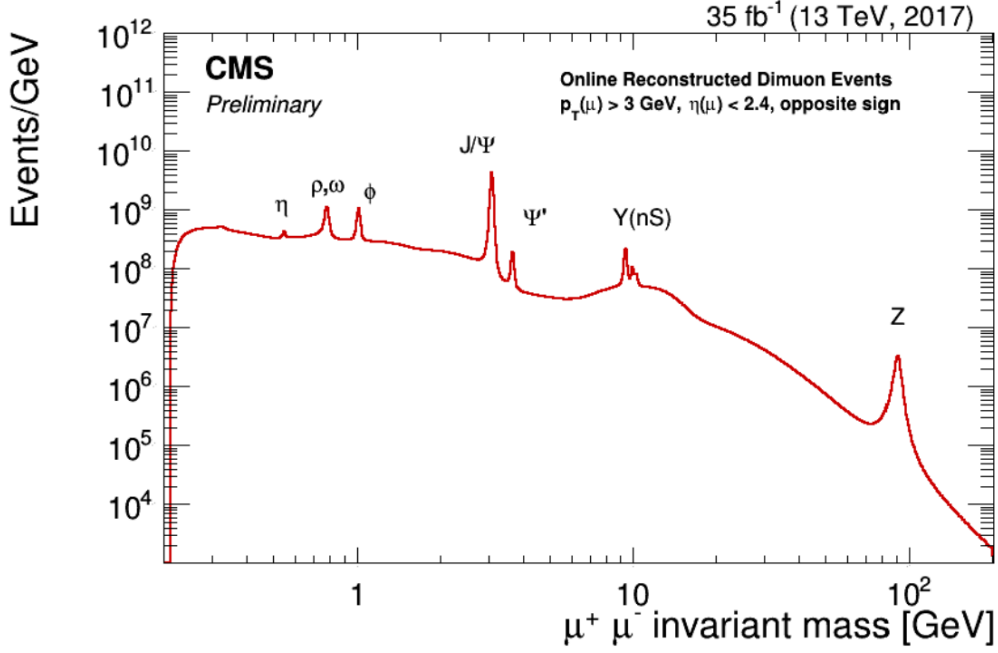


Figure 2.13: Plot showing the reconstructed invariant mass for muon pairs identified with the CMS detector in 2017. The peaks in the distribution correspond to the particle resonances labeled on the plot. Figure taken from Ref. [49].

$$k = \frac{2\sqrt{2}M\Gamma\gamma}{\pi\sqrt{M_0^2 + \gamma}}, \quad \gamma = M_0\sqrt{M_0^2 + \Gamma^2} \quad (2.30)$$

where M_0 is the "true" mass of the particle and Γ is the particle decay width (see Section 2.2.2). This allows one to measure the decay width of a particle from the invariant mass distribution of the decay products, which is illustrated in Figure 2.14 for the Higgs boson.

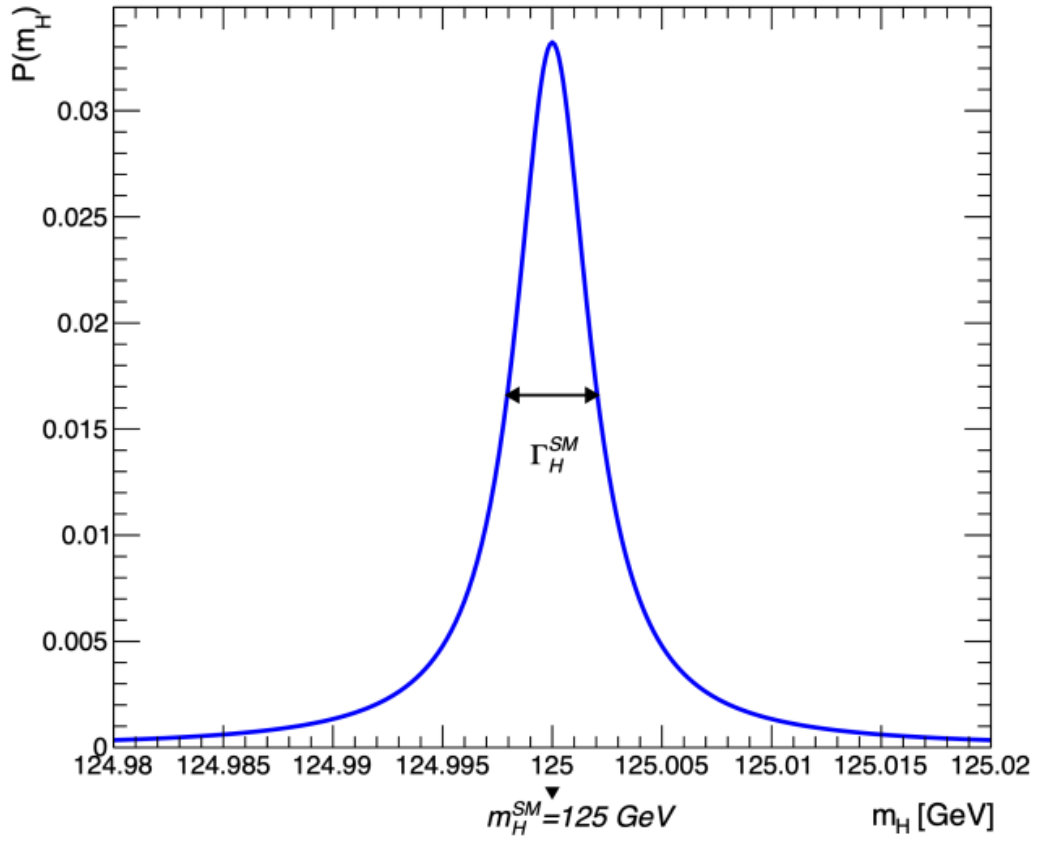


Figure 2.14: Breit-Wigner distribution for the Higgs boson illustrating the total decay width, as measured by the ATLAS collaboration. Figure taken from Ref. [51].

CHAPTER 3

THE CERN LARGE HADRON COLLIDER AND THE CMS DETECTOR

"Heavy machinery unsuitable to be
used by monkeys"

Avi Yagil

CDF Memories (1992)

3.1 The Large Hadron Collider

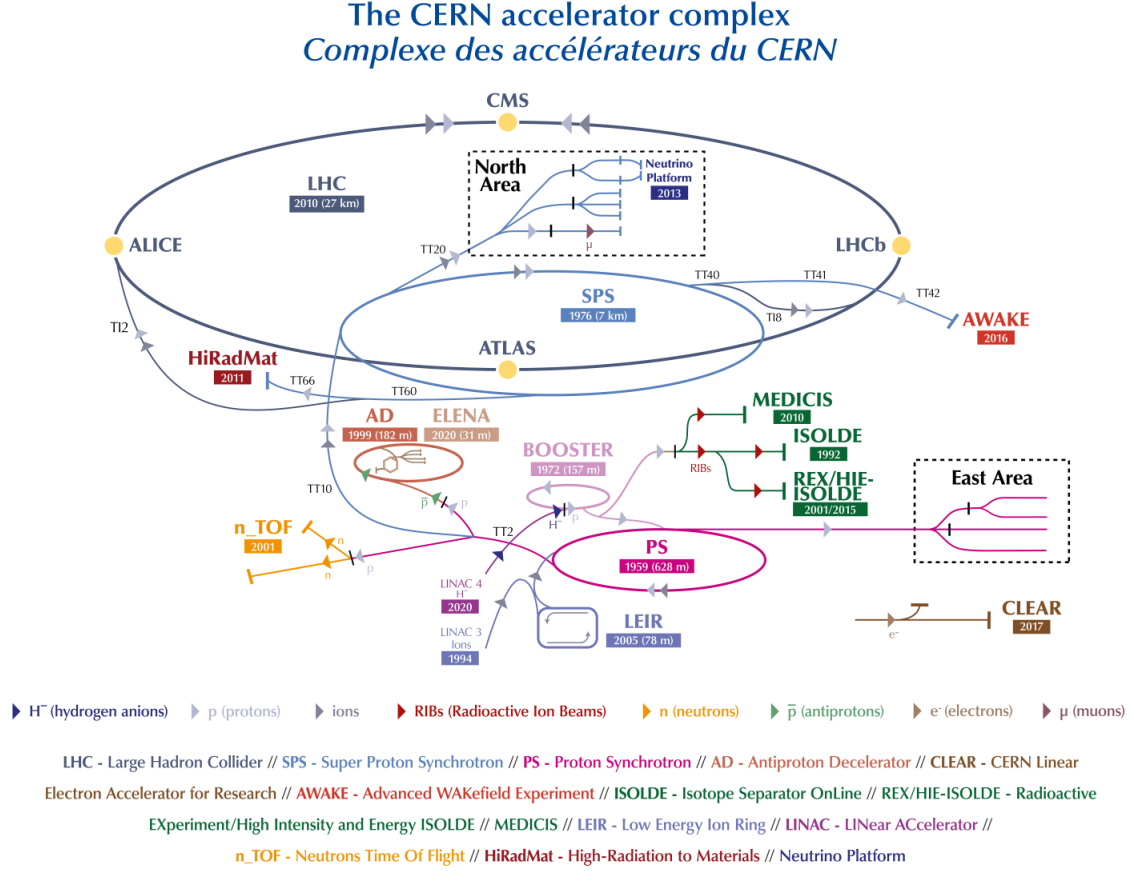


Figure 3.1: The CERN accelerator complex (taken from Ref [52])

At the border of France and Switzerland, near the city of Geneva, lies the European Organization for Nuclear Research (CERN). CERN is home to the world's largest and highest-energy accelerator; the Large Hadron Collider (LHC). Spanning a circumference of 27 km, the LHC comprises the outer ring of a series of accelerators that are hosted at CERN, which is shown in detail in Figure 3.1. In the LHC, counter-rotating bunches of protons are accelerated up to energies of 6.8 TeV (as of Run 3) with an instantaneous luminosity of $\approx 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$. In other words, at the interaction points (i.e. where the proton bunches are collided with each other), the

LHC produces approximately 10^{34} proton-proton collisions per square cm per second at a **center-of-mass energy** ($\sqrt{s}$) of 13.6 TeV. This translates to a bunch collision frequency of 40.0789 MHz, or 1 bunch crossing for every ≈ 25 nanoseconds. These collisions are captured and recorded using highly complex detectors and their readout systems. Along the LHC ring sits four major experiments: ATLAS (Meyrin, CH), ALICE (Saint-Genis-Pouilly, FR), LHCb (Ferney-Voltaire, FR), and CMS (Cessy, FR). Two of these experiments (ATLAS and CMS) are general-purpose detectors used for a variety of physics searches and measurements resulting from proton-proton collisions. The crowning achievement (as of now) of ATLAS and CMS was the 2012 discovery of the Higgs boson [37] [36], resulting in the 2013 Nobel Prize in Physics being awarded to Peter Higgs and Francois Englert.

3.1.1 The Accelerator Complex

The accelerator complex leading up to the LHC begins with a bottle of hydrogen gas, where an electric field is applied to separate the orbiting electrons from the protons. The protons are then sent in groups (or bunches) through the following accelerators within the CERN accelerator complex (in order):

- **Linac 2:** Linear accelerator that accelerates proton bunches to energy of 50 MeV.
- **Proton Synchrotron Booster (PSB):** Synchrotron that accelerates protons to 1.4 GeV.
- **Proton Synchrotron (PS):** Synchrotron that accelerates protons to 25 GeV.
- **Super Proton Synchrotron (SPS):** Synchrotron that accelerates protons to 450 GeV.

After the bunches are accelerated to the target energy in the SPS (450 GeV per beam), the bunches are sent to the LHC ring, where the final acceleration takes place. The beams reach a terminal energy of 6800 GeV (6.8 TeV) prior to collisions.

3.1.2 Acceleration Scheme

The force on a charged particle of velocity $\vec{v}$ and charge q subject to electric and magnetic fields with strengths $\vec{E}$ and $\vec{B}$, respectively, is given by the Lorentz force:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad (3.1)$$

This means that a proton (with an electric charge e) in an electric field will receive a force parallel to the electric field, whereas in a magnetic field it will receive a force perpendicular to both its velocity and the orientation of the magnetic field (and thus does no work on the proton). Thus, the acceleration of protons for energies needed by the LHC requires very large electric fields. These high electric fields are achieved through superconducting radio-frequency (RF) cavities. A depiction of a RF cavity is shown in Figure 3.2.

Typically made of a highly conducting metal, the RF cavity is comprised of successive chambers that, when driven by an external voltage, produce a strong, oscillating electric field that alternates in direction from chamber to chamber. The frequency of the oscillating electric field ω_0 is related to the inductance (L) and capacitance (C) of the driving RLC circuit via the relation:

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (3.2)$$

and its value must be chosen to ensure that the proton bunches are constantly accelerated in the forward direction. In the case of the LHC superconducting RF cavities,

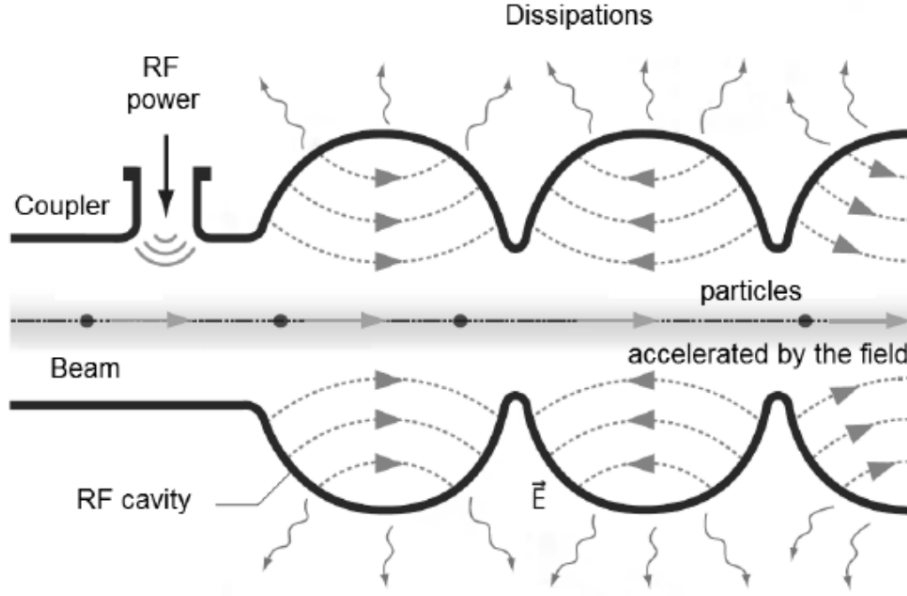


Figure 3.2: Operating principle of a radio-frequency cavity; a supplied RF power drives a series of resonant cavities producing an alternating electric field, which can accelerate (or decelerate) a charged particle depending on the direction of the field and the electric charge of the particle. Figure taken from Ref [53]).

the driving voltage oscillates at $\omega_0 = 400$ MHz. In total, there are 16 superconducting RF cavities in the LHC ring, which together provide a total energy increase of 0.5 MeV per turn to the proton bunches [54].

3.1.3 Focusing Magnets

Assuming proton beams with a Gaussian profile, the luminosity of head-on collisions is proportional to the inverse product of the beam spot sizes at the IP: $\mathcal{L} \propto 1/\sigma_x\sigma_y$ (see Equation 2.19). In general, the purpose of the focusing magnets is to minimize σ_x and σ_y , thus increasing the instantaneous luminosity. Focusing can be achieved through the use of quadrupole electromagnets; a schematic of the quadrupole magnet is shown in Fig. 3.3 (left). In cartesian coordinates, the x and y components of the magnetic field for an (upright) quadrupole electromagnet (to lowest order) is given by [55]:

$$B_x \propto ky, \quad B_y \propto kx \quad (3.3)$$

where k is an energy-independent "focusing strength" (in analogy with the focusing strength of lenses in an optical apparatus) and x and y are transverse to the beam velocity (taken to be the z direction).

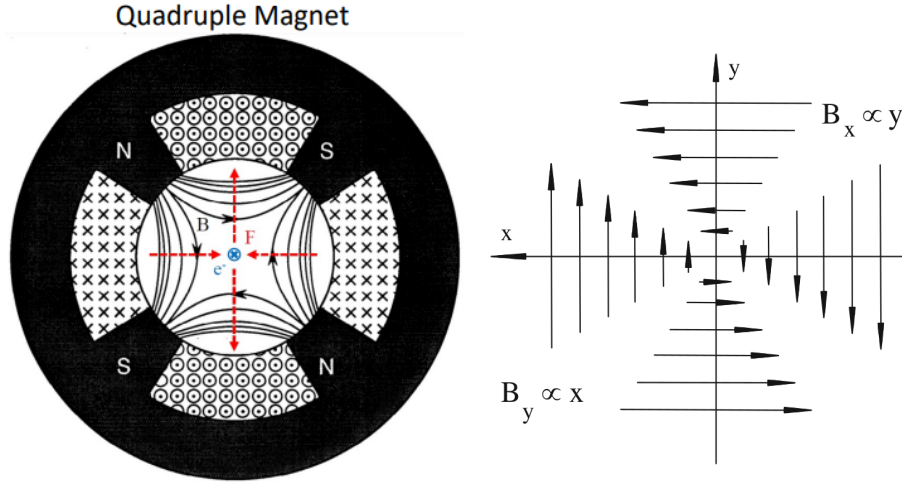


Figure 3.3: Operation principle of a quadrupole magnet (left) and magnetic field lines for a quadrupole magnet (right). Figures taken from Ref. [56] (left) and Figure 5.3 in Ref. [55].

An illustration of the quadrupole field of Eq. 3.3 is shown in Fig. 3.3 (right). It is clear from Eqs. 3.1 and 3.3 that the quadrupole deflects the trajectory of charged particles proportional to its distance from the origin in the x - y plane; in one direction, the deflection results in focusing of the beam while in the other direction the deflection results in defocusing of the beam. Thus, to achieve focusing along both axes transverse to the beam direction, pairs of quadrupole magnets oriented perpendicular to each other are placed in series along the beam trajectory.

In total, the LHC contains 392 quadrupole magnets in the main ring; 360 of these are used around the LHC ring as part of the optical lattice while 32 of these magnets are used for other purposes [57]. The interaction region contains 32 more specialized

large-bore quadrupole magnets for the final focusing of the beam to its minimum transverse size (β^*). For a more complete description of focusing optics, I refer the reader to Part II of this thesis.

3.1.4 Bending Magnets

To bend the proton bunches around the circular orbit of the LHC ring, a uniform magnetic field perpendicular to the velocity of the protons is required (see Eq. 3.1). This uniform field is achieved by dipole magnets, which have:

$$B_x = 0, \quad B_y \propto B_{y,0} \quad (3.4)$$

where $B_{y,0}$ is a constant for the lowest-order dipole field expansion term [55].

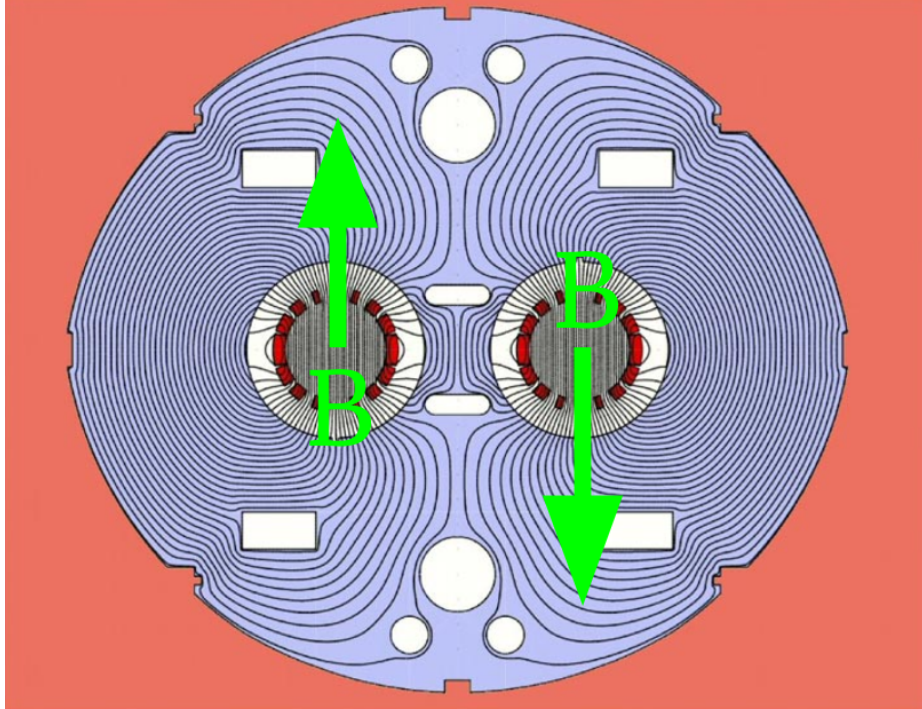


Figure 3.4: Diagram of LHC dipole magnets with magnetic field orientations added (adapted from https://www.bnl.gov/magnets/lhc_ir_dipoles/).

A diagram of the LHC dipole magnets illustrating the magnetic field orientation

is shown in Fig. 3.4; the opposing field orientations for each aperture allows protons to be bent towards the center of the ring while traveling in opposite directions. In general, the magnetic field strength of the dipole magnets is related to the energy of the particle bunches and the total bending radius of the synchrotron via the relation [55]:

$$\frac{1}{\rho}[m^{-1}] = 0.2998 \frac{|B[T]|}{\beta E[GeV]} \quad (3.5)$$

Using Eq. 3.5, one can determine the required dipole magnetic field strength B for the LHC ($\rho = 4243$ m) at $E = 6800$ GeV to be:

$$B_{LHC,dipole} = 5.35 \text{ T} \quad (3.6)$$

In reality, the dipole field strength is considerably higher ($B_{LHC} = 8.33$ T) as the main dipoles only occupy $\approx \frac{2}{3}$ of the full circumference of the LHC ring. In total, the LHC main ring utilizes 1232 dipole magnets; 1104 are used for the optical lattice while 128 are used for other purposes [57]. The interaction region contains 20 specialized dipoles for the purpose of beam separation.

3.2 The CMS Detector

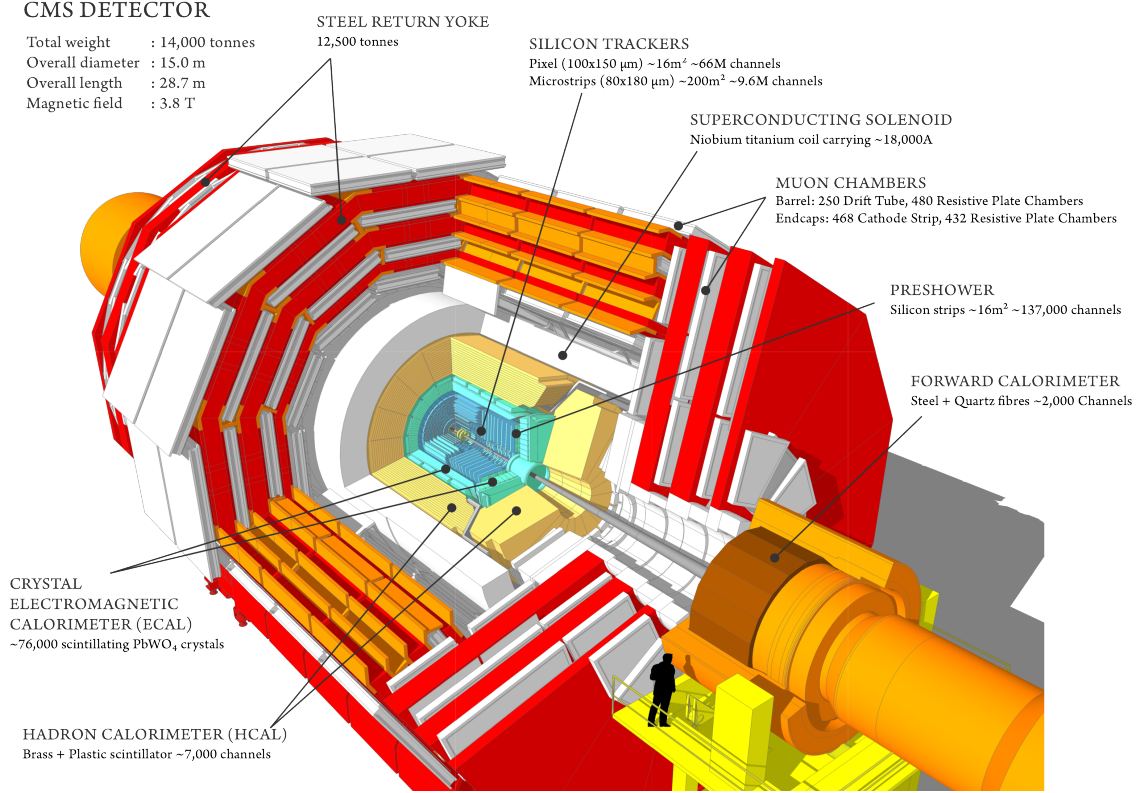


Figure 3.5: Diagram of the CMS detector (taken from Ref [58]).

The Compact Muon Solenoid (CMS) detector is a nearly-**hermetic**, cylindrical apparatus comprised of multiple layers of smaller, specialized detectors (see Fig. 3.5). The innermost detector is the Silicon tracker, which measures the trajectory and momentum of charged tracks. Next is the Electromagnetic Calorimeter (ECAL), which measures electromagnetic energy deposits coming from photons, electrons, and charged hadrons. Beyond the ECAL is the Hadron Calorimeter (HCAL), which measures hadronic energy deposits coming from interactions of charged and neutral hadrons with the detector material. The *piece de resistance* of the CMS detector (to which CMS owes its name), is the massive superconducting solenoid magnet capable of producing a field strength of 3.8 T for the bending of charged particles produced

in the collisions. The outermost layer of CMS is the muon system, responsible for the identification of muons and the measurement of muon momentum and trajectory.

3.2.1 Triggers

As mentioned in Sec. 3.1, the bunch crossing frequency at the interaction points is roughly 40 MHz. Due to data storage constraints, data from all of these collisions cannot be saved for further analysis. This problem is addressed by the implementation of *triggers* that identify "interesting" physics events and save them for offline analysis. CMS employs two types of triggers for this task: a hardware-based "Level 1 Trigger" (L1 trigger) and a software-based "High Level Trigger" (HLT).

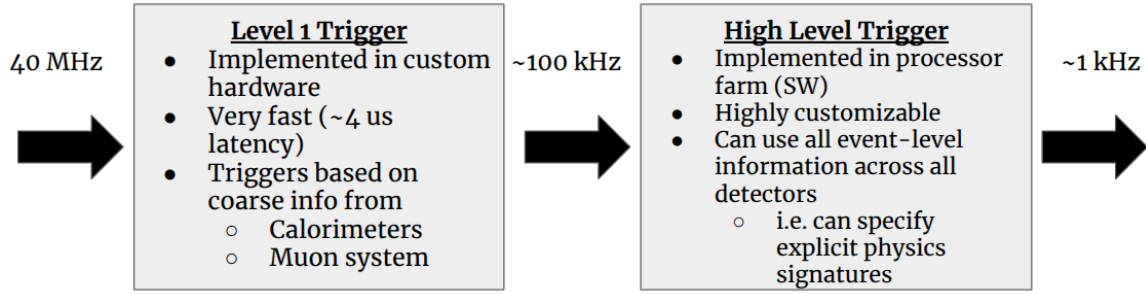


Figure 3.6: Schematic of the CMS trigger system.

The L1 trigger is a fast (fixed latency of $4 \mu\text{s}$) robust trigger system implemented via custom hardware boards [59]. It receives coarse information from the calorimeters (ECAL and HCAL) and the muon system, and executes logic to determine whether an event is worth saving. If so, the L1 trigger generates a "Level-1 accept" (L1A) signal that is distributed to all subsystems of the CMS detector, causing the data from each subsystem to be saved for further analysis by the HLT. Otherwise, the event is discarded. The maximum output rate of the L1 trigger is 100 kHz, a factor of ≈ 400 smaller than the bunch crossing frequency.

The HLT is implemented in a processor farm and utilizes the full event-level information across all detectors to perform an event reconstruction to select explicit

physics signatures [60]. Examples of these physics signatures are discussed in Sec. 4.2. On average, the output rate of the HLT is ≈ 1 kHz; a 100x reduction in the data rate coming from the L1 trigger.

The combination of the L1 trigger and the HLT in CMS is able to reduce the overall data rate from 40 MHz to 1 kHz on average, a factor of approximately 40,000.

3.2.2 Silicon Trackers

The Silicon tracker system sits closest to the LHC beamline at CMS. It is comprised of 2 detectors: a silicon pixel detector and a silicon strip detector. The purpose of this system is twofold:

- To provide measurements of the trajectories of charged particles originating from collisions.
- To provide fast and precise reconstruction of secondary vertices due to particles decaying close to the beam line.

The basic operating principle of the Silicon detectors is as follows: a charged particle passes through the doped Silicon pixel or strip and creates electron-hole pairs. An applied electric field causes the electrons to drift towards electrodes, which results in a current. The current is then amplified, and, if it is above some threshold, infers the presence of a charged particle.

With a total length of 5.8 m and diameter of 2.5 m, the Silicon tracker provides charged particle tracking for pseudorapidity $|\eta| < 2.5$. A diagram of the Silicon tracker system is shown in Fig. 3.7. The pixel detector consists of three barrel layers and two endcap disks, with a total of 1440 Silicon pixel modules. The strip detector consists of 10 barrel layers and 9+3 endcap disks, representing a total of $\approx 15,000$ strip modules.

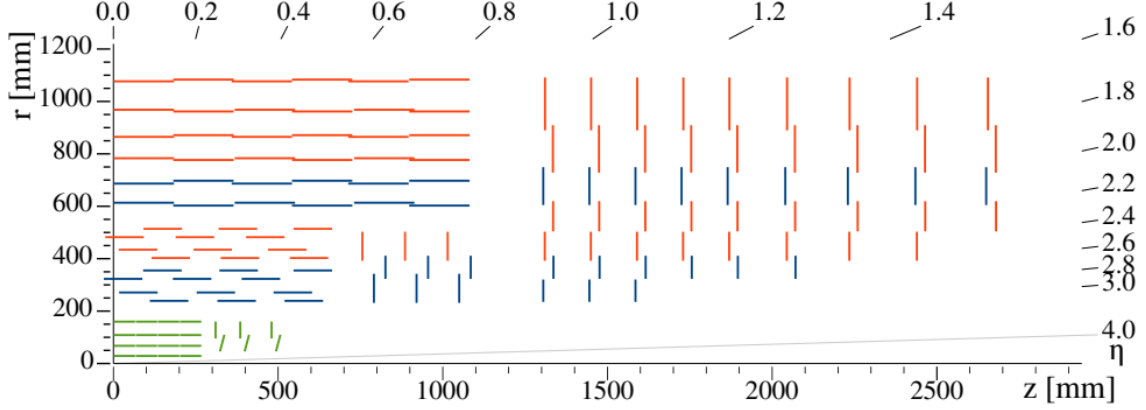


Figure 3.7: Diagram of one quarter of the CMS Silicon tracker system in the r - z plane; the green lines represent the pixel detector modules, while the red and blue lines represent the strip detector modules. Taken from Ref. [61] (Figure 2.1).

To meet the demands for the desired impact parameter resolution, the individual pixels occupy an area of $100 \times 150 \mu\text{m}^2$ in $r - \phi$ and z , respectively. Due to the strip detector sitting further from the beamline, the individual strip size is larger in area: ranging from $10\text{cm} \times 80\mu\text{m}$ at intermediate r up to $25\text{cm} \times 180\mu\text{m}$ at large r (where "intermediate" and "large" are relative to the total radius of the tracker detector).

3.2.3 Electromagnetic Calorimeter

The ECAL measures the energy of photons, electrons, and charged hadrons. The ECAL is composed of lead tungstate (PbWO_4) crystals, which produce scintillation light when a charged particle (or photon) passes through it. Lead tungstate was chosen for the following properties:

- Short **radiation length** ($X_0 = 0.89\text{cm}$)
- Small **Moliere radius** (2.2cm)
- High density ($\rho = 8.28 \text{ g/cm}^3$)

These characteristics allow the ECAL to be very compact with small crystal size

(and thus high granularity).

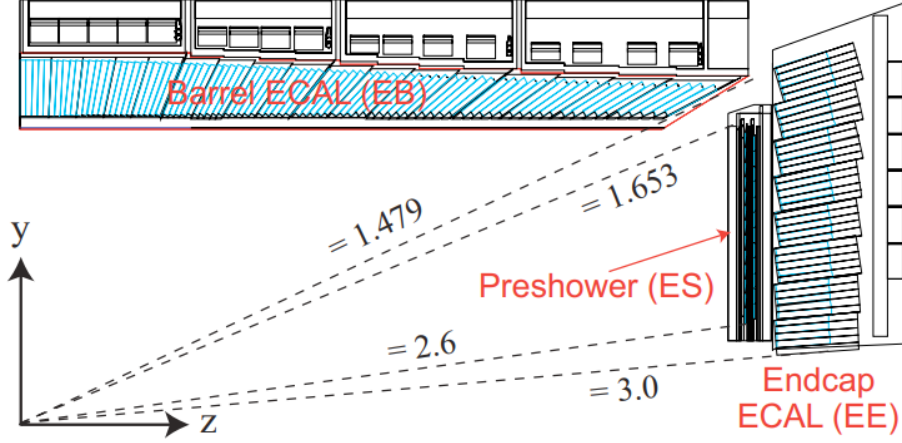


Figure 3.8: Configuration of the CMS ECAL in the transverse direction. Figure taken from Ref. [54] (Figure 4.1).

The barrel region of the ECAL covers pseudorapidity $|\eta| < 1.479$, and includes 61,200 crystals in total. The endcap region is made up of 7324 crystals in each endcap, and covers pseudorapidity $1.479 < |\eta| < 3.0$ (see Fig. 3.8). The individual crystals are shaped like truncated pyramids, with the following dimensions:

- In the barrel ($r = 1.29\text{m}$ at front face): area at front face is $22 \times 22 \text{ mm}^2$, area at rear face is $26 \times 26 \text{ mm}^2$, length is 230 mm ($25.8X_0$).
- In the endcaps ($z = 315.4\text{cm}$ at front face): area at front face is $28.62 \times 28.62 \text{ mm}^2$, area at rear face is $30 \times 30 \text{ mm}^2$, length is 220 mm ($24.7X_0$).

The scintillation light is captured by Avalanche Photodiodes (barrel) or Vacuum Phototriodes (endcap) [54]. The general form of the energy resolution for a calorimeter is given by the formula [62]:

$$\left(\frac{\sigma}{E}\right)^2 = \left(\frac{S}{\sqrt{E}}\right)^2 + \left(\frac{\sigma_n}{E}\right)^2 + c^2 \quad (\text{E in GeV}) \quad (3.7)$$

The S term is called the "stochastic term" and for the ECAL it represents the uncertainty due to fluctuations in the containment of the electromagnetic shower and

photostatics. The σ_n represents the noise contribution to the total uncertainty, and c is a constant term. The values of these parameters for the CMS ECAL were determined in electron test-beam experiments [63] to be

$$S = 2.8\%, \quad \sigma_n = 120\text{MeV}, \quad c = 0.3\% \quad (3.8)$$

Each of these relative contributions to the total energy resolution of the ECAL is illustrated in Fig. 3.9.

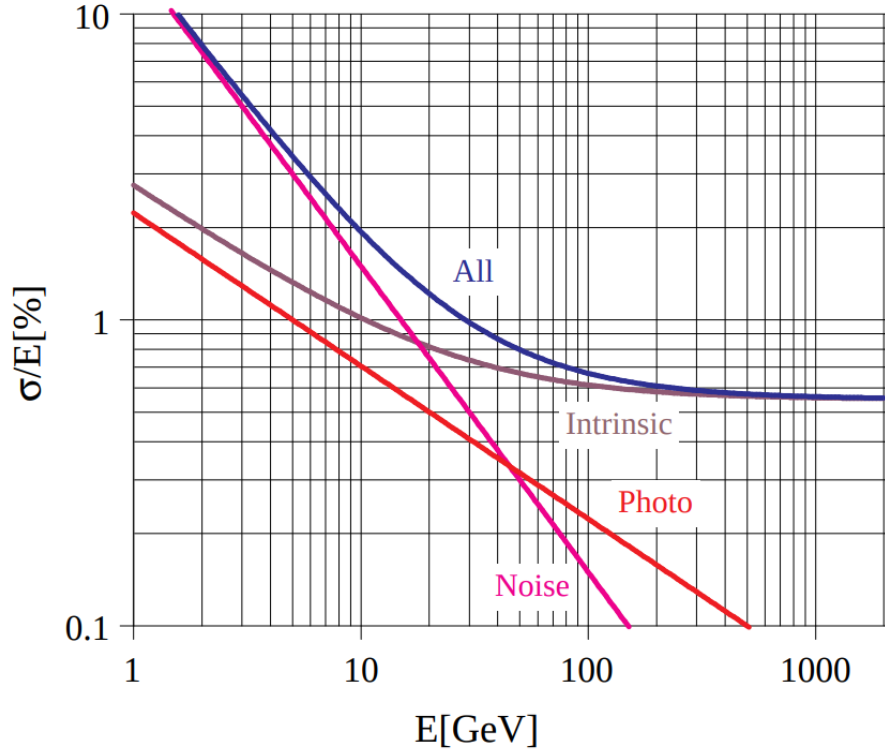


Figure 3.9: Contributions to the total energy resolution (blue line) of the CMS ECAL. The red line corresponds to the contribution from photostatics and the silver line is the contribution from the shower containment fluctuations. The pink line is the noise term. Figure taken from Ref. [62] (Figure 1.3).

3.2.4 Hadron Calorimeter

The CMS hadron calorimeter measures the energy of charged and neutral hadrons, which is crucial for the identification of final states with jets and the precise determination of missing energy within collision events. The HCAL consists of 4 separate detectors: a barrel detector covering $|\eta| < 1.3$ (HB), an endcap detector covering $1.3 < |\eta| < 3.0$ (HE), an outer calorimeter or "tail catcher" used to extend the reach of the barrel detector in the radial direction (HO), and a forward detector that extends the η coverage into the far forward region ($3.0 < |\eta| < 5.2$). The arrangement of the detectors within CMS is shown in Fig. 3.10.

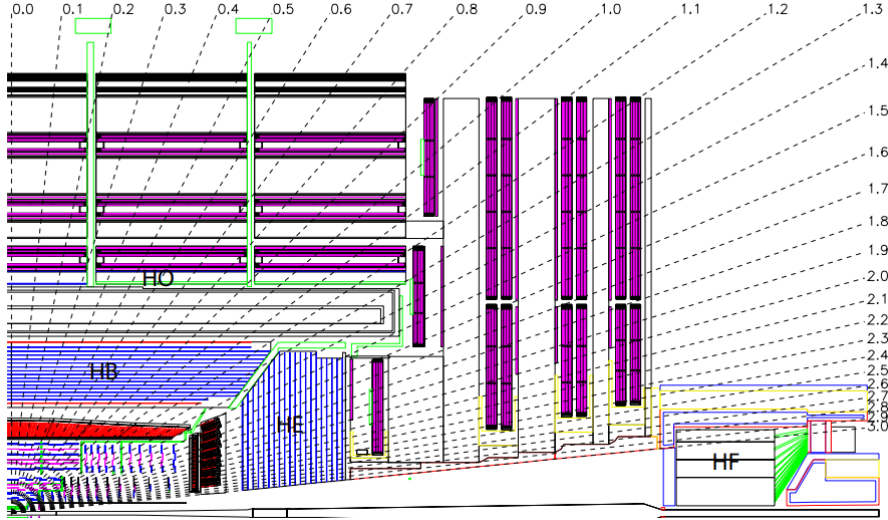


Figure 3.10: Side view of the CMS detector with the locations of the hadron barrel (HB), endcap (HE), outer (HO) and forward (HF) calorimeters highlighted (taken from Ref [64]).

The HB sits between the ECAL and the magnet, occupying a radial range from $1.77\text{m} < R < 2.95\text{m}$. This is somewhat restrictive in terms of calorimeter length, thus motivating the need for the HO outside the magnet to complement the barrel detector in the low- η region.

The HCAL is a *sampling calorimeter*, which consists of alternating layers of a

dense material (cartridge brass) to induce particle showers (due to a low **nuclear interaction length**) and a scintillator material that produces light when a particle shower passes through. The scintillation light is then detected by photosensors, which can be calibrated to determine the energy deposits from the hadronic activity. The combined ECAL + HCAL pion energy resolution was measured in test beams to be [63]:

$$\left(\frac{\sigma(E)}{E}\right)^2 = \left(\frac{84.7\%}{\sqrt{E}}\right)^2 + (7.4\%)^2 \quad (3.9)$$

where E is measured in GeV.

3.2.5 Magnet

The most unique feature of the CMS detector is the presence of a 12.5m long, 3.15m free-bore radius superconducting solenoid magnet [65]. The magnet is capable of delivering a uniform axial magnetic field strength of 3.8 T over its full length for the purpose of bending charged particles, which can then be used to determine the momenta of the charged particles via Eq. 2.25.

The impressive size of the magnet allows for the ECAL and HCAL to sit inside the magnet volume. This eliminates energy losses before the calorimeters due to particle showers in the magnet coil, which improves the measurement accuracy of electromagnetic and hadronic energy deposits.

3.2.6 Muon System

Due to the relatively long muon lifetime ($\tau_\mu = 2.2 \times 10^{-6}$ seconds), the highly-relativistic muons from LHC collisions are capable of traversing long distances before decaying. For this reason, the muon system comprises the outer layer of the CMS detector. The muon system has two primary functions: to measure the trajectory and

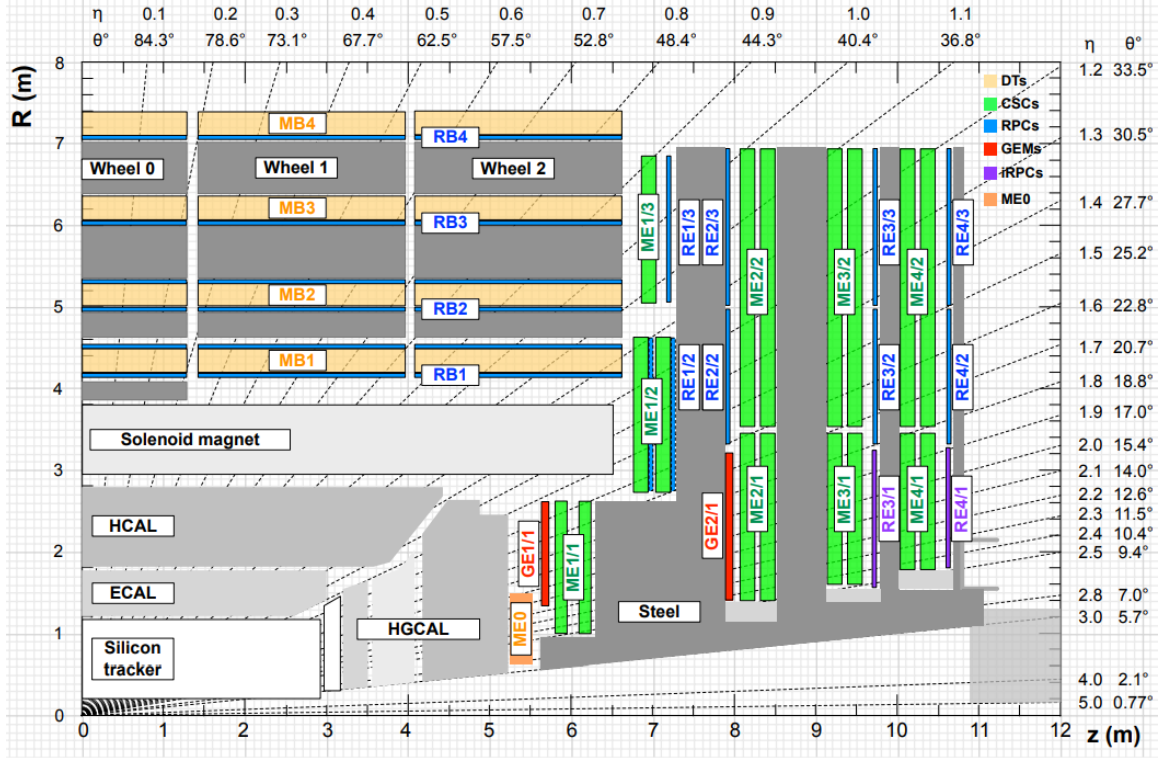


Figure 3.11: Diagram of the CMS muon system in the r - z plane (taken from Ref [66]).

momentum of muons (along with the silicon tracker), and to provide trigger signals (or "primitives") for use in the L1 trigger decision-making algorithm. As of the Run 3 data-taking period, the CMS muon system is comprised of three subsystems: the Drift Tube (DT) chambers, the Resistive Plate Chambers (RPC), and the Cathode Strip Chambers (CSC). A diagram showing the position of these subsystems in relation to the rest of the CMS detector is shown in Fig. 3.11.

The details of the operating principles and characteristics of the subsystems comprising the CMS muon system are as follows:

Drift Tubes: The drift tubes cover the central region of the CMS detector ($|\eta| < 1.2$), and are comprised of individual aluminum cells filled with a gaseous mixture of 85% Ar and 15% CO₂. Within each cell, anode wires run through the center while the cathode sits on the outer edge of each cell, as shown in Fig. 3.12. The operating principle of the DT cells are as follows: muons passing through

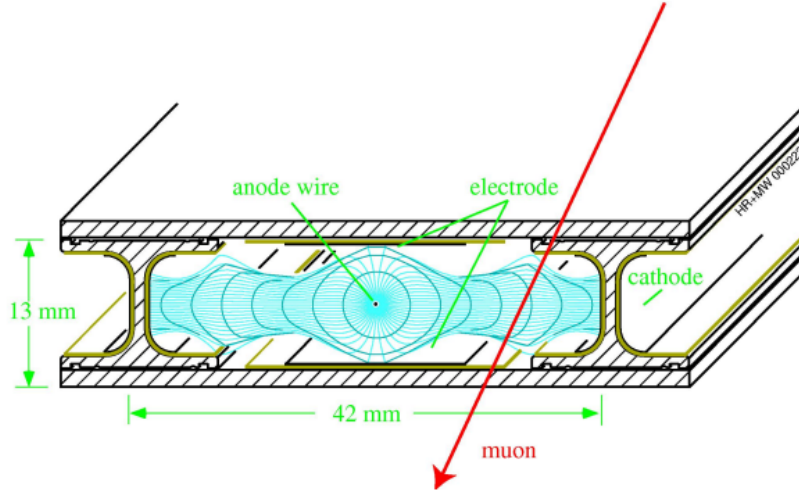


Figure 3.12: Diagram of an individual drift tube cell. The shape of the internal electric field is shown in light blue (taken from Ref [66]).

the chamber ionize the gas, creating free electrons that drift towards the anode wire under an applied electric field. The position of the hit in each cell is determined from the time taken for the electrons to drift to the anode wire. In each cell, the spatial resolution for muon tracks is $\approx 250 \mu\text{m}$. Four cells are stacked parallel to each other into "SuperLayers" by staggering the cells at half-cell intervals. This allows for the determination of the muon positioning and timing by combining hit timing for different cells in a SuperLayer. Three superlayers are combined to form a single chamber: two of the SuperLayers measure the muon position in the bending ($R - \phi$) plane, while the other SuperLayer measures the muon position in the longitudinal ($R - z$) plane.

Resistive Plate Chambers: The RPCs occupy the barrel region into the forward region ($|\eta| < 1.9$), and are designed for accurate timing ($\approx 1.5 \text{ ns}$ resolution) and fast triggering. Two layers of a high-resistance material (known as "bakelite") are separated by a 2 mm gap filled with a gaseous mixture (95.2% $\text{C}_2\text{H}_2\text{F}_4$, 0.3% SF_6 , and 4.5% $i - \text{C}_4\text{H}_{10}$) [66], and the outside of the bakelite are coated with a conductive graphite paint to form the high-voltage and ground electrodes

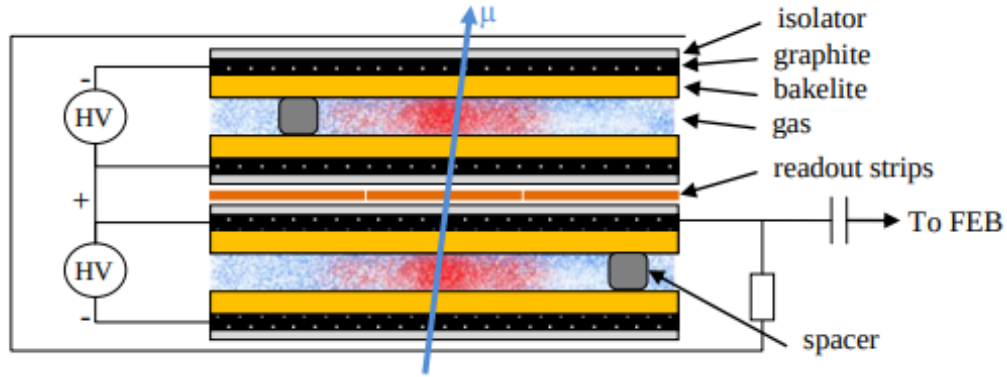


Figure 3.13: Schematic of the resistive plate chambers (taken from Ref [67]).

(see Fig. 3.13) [67]. Two of these bakelite cells are stacked together, separated by an insulator and a metal filament that is used for read-out. Muons passing through an RPC ionize the gas mixture, producing an electron cascade that is amplified by the applied high voltage. The electrons drift towards the anode, and the signal is read out through the metal filament.

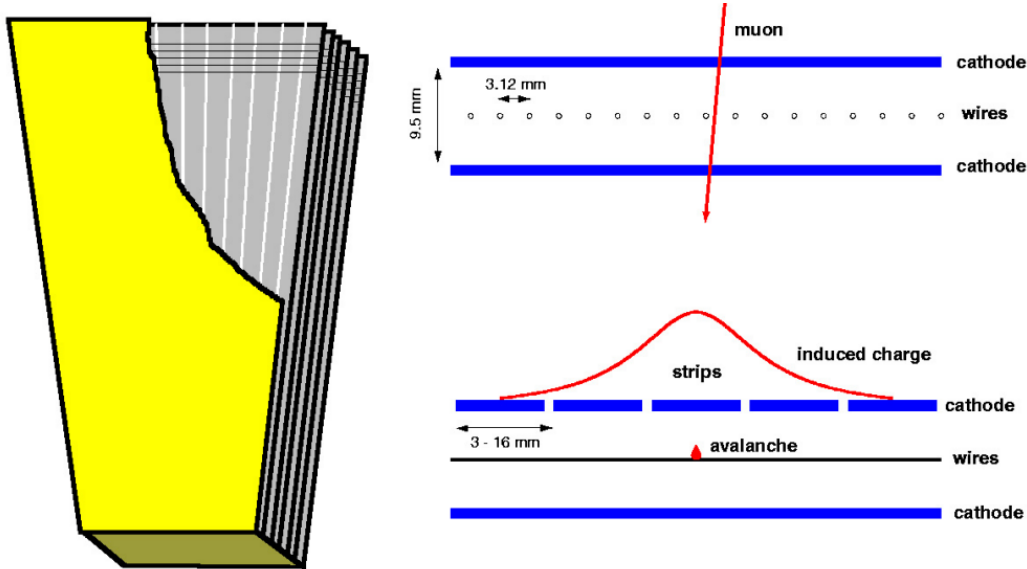


Figure 3.14: Geometry (left) and operating principle (right) of CSC chambers. (taken from Ref [66]).

Cathode Strip Chambers: Covering the forward region of the CMS detector ($0.9 < |\eta| < 2.4$), the Cathode Strip Chambers are multi-wire proportional coun-

ters [68] that consist of seven layers of highly segmented cathode strips (radial direction) and six layers of anode wires (azimuthal direction) as shown in Fig. 3.14 (left). Cathode wires measure ϕ , which can be used to calculate the p_T via Eq. 2.25, while anode wires measure η . Each chamber is filled with a mixture of 40% Ar, which is the ionizing agent, 50% CO₂, which acts as a quenching gas, and 10% CF₄, which prevents etching and aging of the anode wires [66]. Muons passing through the gas mixture ionize the Ar gas, leaving a trail of electrons that drift to the anode wires under an applied electric field. This induces a mirror charge on the cathode strips, which is digitized by the Cathode Front-End Board (CFEB). The current on the anode wires is digitized by the Anode Front-End Board (AFEB). This data is relayed along to the Optical Trigger Mother Board (OTMB), which utilizes this information to construct track segments. Segments with hits in four of the six gas layers create trigger primitives (objects to be sent to the L1A algorithm). Upon receiving an L1A trigger, data from the OTMB, ALCT (which collects data from AFEBs), and CFEBs are sent to the Optical Data Acquisition Mother Board (ODMB). The ODMB gathers and processes data from all front-end electronics for multiple chambers and sends the data to the Detector-Dependent Unit (DDU); a VME board that collects data from the 15 ODMBs in a given CSC sector and sends it to the global CMS DAQ output path. Data from the ODMB is sent to the DDU at a rate of 1.6 Gbps (with a target of $12.5 \text{ Gbps} \times 4 \text{ links} = 50 \text{ Gbps}$ for HL-LHC data-taking). The full electronics chain from on-chamber to read-out is summarized in Fig. 3.15.

The full CSC system is comprised of a total of 540 trapezoidal chambers arranged into 4 layers (stations) along the z axis, with 2-3 rings per station (arranged in the ϕ direction) and 18 or 36 chambers per ring, comprising two discs

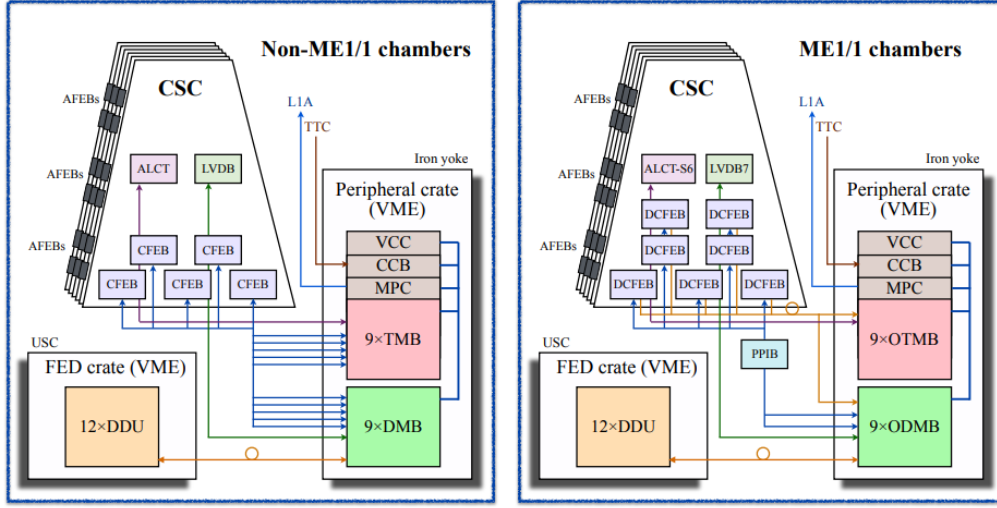


Figure 3.15: Summary of CSC on-chamber and peripheral electronics for ME1/1 (left) and non-ME1/1 chambers (right). Taken from Ref [66].

(endcaps). The spatial resolution of each CSC chamber varies from $50 \mu m$ to $140 \mu m$, and the average timing resolution is ≈ 3 ns.

3.3 Particle Reconstruction at CMS

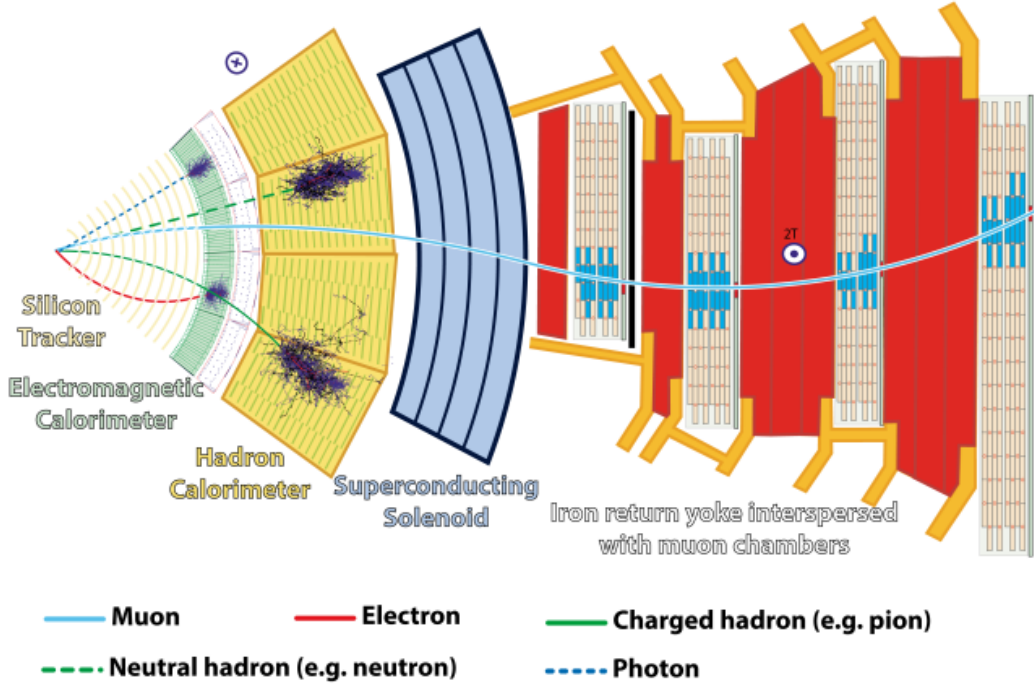


Figure 3.16: A transverse slice of the CMS detector with representative particle trajectories (taken from Ref [69]).

Despite the large number of fundamental and composite particles that can be produced in LHC collisions, only five types of particles are actually detectable by the CMS detector (see Fig. 3.16): photons, electrons, muons, neutral hadrons (such as neutrons), and charged hadrons (such as charged pions). Thus, the presence of other particles, such as a W or Z boson, must be inferred from the decay products that show up in the detector. This requires precise reconstruction of the energy and trajectory of the detectable particles. This is achieved through the so-called "Particle Flow" (PF) algorithm [70], which combines information from different subdetectors. The details of the PF algorithm for different particle species is elucidated in the following sections.

3.3.1 Electron Reconstruction

Electron identification in CMS relies on information from the silicon tracker system and the ECAL. As noted in Sec. 2.2.4, charged particles passing through materials undergo *bremsstrahlung*, resulting in photon energy deposits within the detector. In the case of electrons produced at the typical LHC energies, the energy loss due to bremsstrahlung can be significant in the CMS detector, and must be accounted for in the electron identification algorithm.

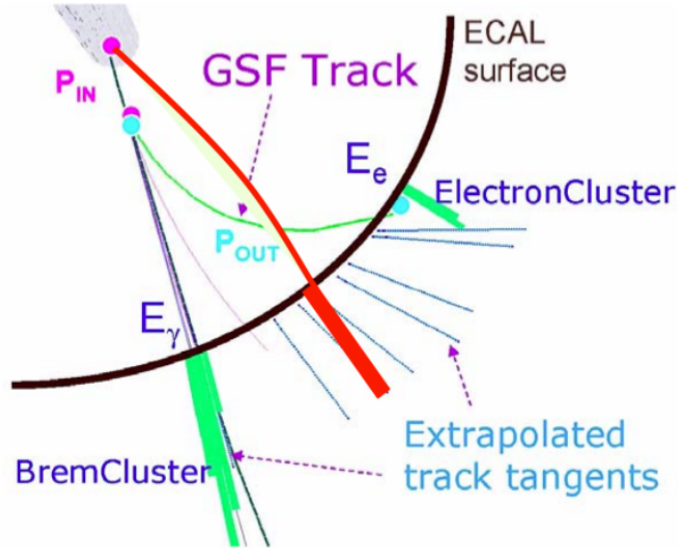


Figure 3.17: Diagram highlighting the basic components of the GSF track finding algorithm for electrons (taken from Ref. [71]).

The reconstruction is seeded by energy deposits within the ECAL crystals, which is then matched to curved tracks within the silicon tracker detector. The matching procedure involves a "Gaussian Sum Filter" (GSF) [72], which accounts for the changing electron trajectory due to bremsstrahlung and energy loss due to photon conversions within the detector material (see Fig. 3.17). Furthermore, the matching requires consistency between the p_T measurement by the tracker and the energy measurement by the ECAL, providing a robust measurement of electron tracks.

3.3.2 Photon Reconstruction

Photon identification in CMS relies on information from the ECAL in tandem with the rest of the identified tracks within the full detector. Since photons are electrically neutral, they do not bend in the magnetic field and also leave no tracks within the tracker detector. Thus, the only direct means of detecting photons is through energy clusters in the ECAL. However, photon energy deposits can arise from electron bremsstrahlung (see Sec. 3.3.1), and thus the electron reconstruction accounts for this by using the GSF to match these clusters to the electron track. The PF algorithm treats the tracks first (electrons, muons, and charged hadrons), and the remaining ECAL energy deposits not matched to tracks are then labeled as isolated photon candidates.

3.3.3 Muon Reconstruction

Muons are reconstructed using hits from the muon system, tracks from the Silicon tracker detector, or a combination of both. Because of this, there are three main types of reconstructed muon objects that are utilized in CMS:

- **Standalone Muons:** hits in the muon system (DT or CSC detector) form track segments. These segments are used in a fit to find all hits in all muon subsystems (CSC, DT, and RPC) consistent with the muon trajectory. The full track formed from this fit is the standalone-muon track.
- **Tracker Muons:** a standalone-muon track is matched to hits in the Silicon tracker detector. If the two tracks, when propagated to a common surface, are compatible with each other, the track from the Silicon tracker is called an "inner track". If the "inner track" has $p_T > 0.5$ GeV and total momentum greater than 2.5 GeV, it is then referred to as a tracker-muon track.

- **Global Muons:** tracks from the "inner track" are fit with the tracks from the standalone-muon track to form a global-muon track.

For high-momentum muons ($p_T > \approx 200$ GeV), the global muon algorithm is highly efficient and improves upon the muon momentum resolution from the tracker-only fit. For low-momentum muons ($p_T < 5$ GeV), many of which undergo multiple scattering in the steel return yoke of the magnet, the global muon algorithm is rather inefficient as the muons are not matched to multiple layers in the muon system. Therefore, these low-momentum muons are best identified using the tracker muon reconstruction.

The PF algorithm utilizes the global and tracker muon tracks to construct muon candidates. The algorithm includes requirements for the muon isolation (i.e. how much "other" energy is near the muon track), which can be used to distinguish muons from hadrons that reach the muon system. Muons that do not fall into this "isolated global muon" category are subject to different algorithms using the tracker muon tracks that attempt to recuperate the overall muon identification efficiency [73].

Muon reconstruction in the muon system is very efficient: very few particles other than muons reach the muon detectors as the ECAL and HCAL are able to absorb them with high efficiency. In the event where hadron-shower remnants reach the muon system, known as "punch-through", information from the ECAL and HCAL can be used with the muon tracks to improve the identification efficiency (such as the isolation requirements described above).

3.3.4 Neutral Hadron Reconstruction

Like photons, neutral hadrons do not bend in the magnetic field of CMS and do not leave tracks in the tracker detectors. The presence of neutral hadrons can only be inferred based on energy deposits in the HCAL that are not matched to tracks. Thus,

in an analogous fashion to what is done for photons, the PF algorithm handles tracks first (electrons, muons, and charged hadrons) and the leftover HCAL deposits are then taken to be neutral hadrons.

3.3.5 Charged Hadron Reconstruction

Charged hadron reconstruction in CMS is done by combining information from the tracker detector, the ECAL, and the HCAL. The PF algorithm starts with the charged tracks and performs a matching procedure to determine whether the track is consistent with any energy deposits in the ECAL or the HCAL. If so, the track is labeled as a charged hadron and its energy and momentum are determined from the momentum of the tracker track (assuming a charged pion mass). This energy is then compared to the deposits within the calorimeters, and, if they are consistent with each other, the energy and momentum is corrected by fitting the values from all three subsystems [74].

3.3.6 Tau Lepton Reconstruction

Tau leptons (τ) decay to leptons and neutrinos ($\approx 35\%$) and to hadrons and neutrinos ($\approx 65\%$). The individual decay modes and their corresponding decay fractions are shown in Table 3.1. The leptonic decays of the τ are difficult to identify, as the only detectable decay products are the charged leptons e and μ (which are nearly indistinguishable from prompt electrons and muons arising from other sources). The hadronic decays can be distinguished from jets arising from quarks and gluons by utilizing information from the multiplicity, collimation, and isolation of the decay products that are reconstructed in the PF algorithm.

The algorithm used to identify hadronic tau (τ_h) decays is known as the "hadrons-

Type	Decay	$\mathcal{BR}$
Leptonic	$\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$	17.8%
	$\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$	17.4%
Hadronic	$\tau^- \rightarrow h^- \nu_\tau$	11.5%
	$\tau^- \rightarrow h^- \pi^0 \nu_\tau$	26.0%
	$\tau^- \rightarrow h^- \pi^0 \pi^0 \nu_\tau$	10.8%
	$\tau^- \rightarrow h^- h^+ h^- \nu_\tau$	9.8%

Table 3.1: Major decays of the τ lepton [75]. The label h^- denotes a charged hadron, pion, or kaon.

plus-strips" (HPS) algorithm. A diagram of the HPS algorithm is shown in Fig. 3.18. The algorithm utilizes charged hadron information from the tracker along with π^0 candidates, which are obtained from electrons and photons from photon conversions, in $\eta \times \phi$ "strips". The jet constituents are then combined into τ_h candidates if they are compatible with one of the main hadronic tau decay modes listed in Table 3.1.

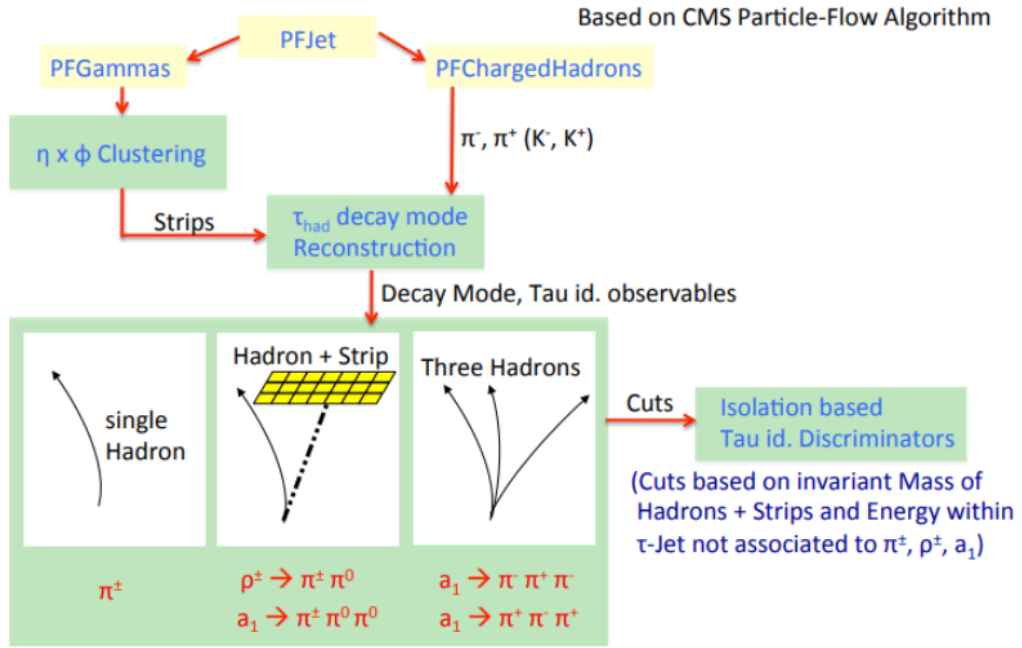


Figure 3.18: Operating principle of the HPS algorithm for the identification of hadronically-decaying τ leptons.

CHAPTER 4

DATA AND SIMULATION

4.1 Overview

To compare data collected by CMS with the SM prediction, one must rely on simulated Monte Carlo (MC) datasets to properly model the signal process(es) of interest and the background processes that mimic the signal(s). The MC dataset creation starts with a generator; a program that models the underlying physics process by calculating scattering amplitudes from Feynman diagrams at a particular order in perturbation theory. Examples of physics generators widely used in CMS analyses are Madgraph [76–78], POWHEG [79–82], and MCFM [83]. Particle decays can also be specified at the generator level, which allows for particular final states to be simulated for further study. Since protons are composite objects, the hard scattering process must be convoluted with the parton distribution function of the proton to account for the different possible initial states. The Born-level differential cross section for a 2-particle scattering process into n final state particles at the generator level with the specified parton distribution functions is of the form [84]:

$$d\sigma_{1,2 \rightarrow n} \propto \int_0^1 dx_1 dx_2 f(x_1, \mu_f) f(x_2, \mu_f) \int d\Phi_n |\mathcal{M}(\Phi_n; \mu_f, \mu_r)|^2 \quad (4.1)$$

where x_1 and x_2 are the momentum fractions of the initial-state particles, f is the parton distribution function, μ_f and μ_r are the **factorization scale** and **renormalization scale**, respectively, $d\Phi_n$ is the n -particle phase-space element, and $\mathcal{M}$ is the scattering amplitude for the underlying physics process.

Depending on the process and the order to which the calculation is performed in perturbation theory, the generator will produce additional partons for the physics process. Due to color confinement, quarks cannot exist as free states, and thus undergo parton showering (red lines in Fig. 4.1) and hadronization (light green ovals in Fig. 4.1), which is modeled with programs such as PYTHIA [85, 86]. These programs, due to the size of α_s , may simulate extra radiative corrections in the scattering process

and thus the partons produced at the generator level may or may not match those produced by the parton showering and hadronization programs. This is resolved by a "matching" procedure, which matches jets from the matrix element calculation to those produced by the parton showering program.

After the simulation of the hard scattering process, parton showering and hadronization (and modeling of other radiative corrections), and the jet matching, the generated events are then fed into a program to simulate the detector response (e.g. GEANT4 [87]).

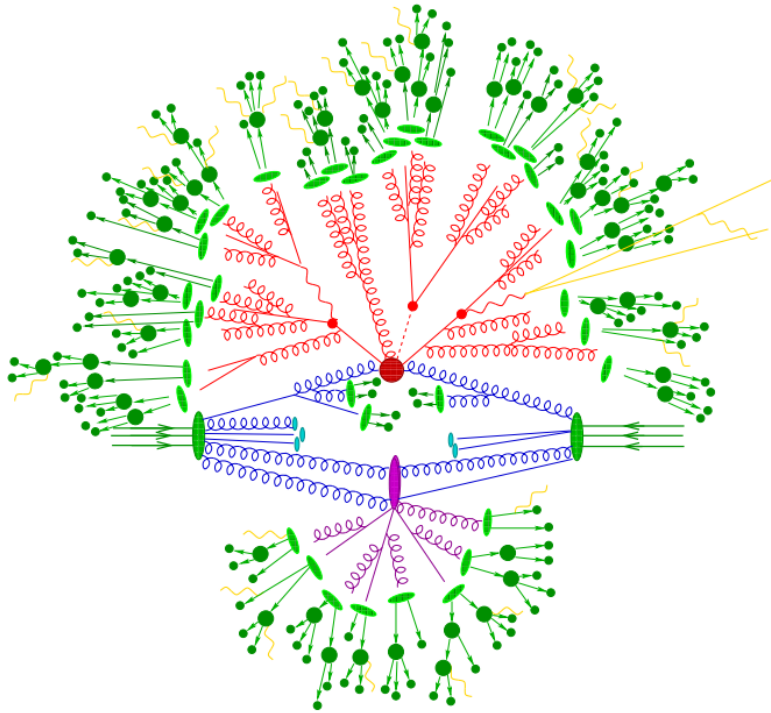


Figure 4.1: Sketch of a proton-proton collision as simulated by a MC event generator. Figure taken from Ref [84] (Figure 3).

4.1.1 Data Tiers at CMS

Data from the CMS detector is a collection of electrical signals from the different detectors, which are then reconstructed into physics objects as described in Sec. 3.3.

The physics objects can be further condensed into so-called "high-level" objects that are useful for condensing the size of individual events while maintaining flexibility for use in a large number of different studies. All of this information is stored in various data formats, or *tiers*, for use in offline analysis. The main data tiers of CMS are summarized here:

RAW: Contains the data sent directly from the detectors in the online data format.

It includes the L1 trigger result as well as the HLT trigger bits and other HLT information. Typical event size is 0.70-0.75 MB/event.

RECO: Contains the reconstructed physics objects, such as: photons, electrons, muons, jets, missing transverse energy, heavy quarks, and decayed taus. It also includes the full information from the RAW tier as well. The typical event size is 1.3-1.4 MB/event.

AOD: Contains mainly high-level physics objects (e.g. electrons, muons, taus, and Z candidates) while summarizing the full RECO information (e.g. tracks with hits, calorimeter clusters with hits, jets, vertices). The typical event size is 0.5 MB/event.

MiniAOD: Contains high-level physics objects and pre-RECO information, which allows for the flexibility to perform re-reconstruction of physics objects using different algorithms. The typical event size is 0.04-0.05 MB/event.

NanoAOD: Only contains the high-level physics objects in the form of a flat ROOT TTree (1 entry = 1 stored event). This provides a minimal set of physics objects for the vast majority of physics analyses. The typical event size is 0.001 MB/event.

The primary data tier used for Run 3 CMS analyses (including this analysis) is the NanoAOD format.

4.2 High Level Trigger Menu

As described in Sec. 3.2.1, the storage of data by CMS relies on triggers (L1A and HLT). Thus, all data read out by CMS corresponds to at least one HLT decision or "bit"; a particular set of requirements based on reconstructed objects, such as electrons, muons, hadronic energy, or missing energy. In total, there are hundreds of HLT bits utilized by CMS, each corresponding to a specific set of requirements on the reconstructed objects.

Year	Trigger	Type
2016	HLT_Ele23_Ele12_CaloIdL_TrackIdL_IsoVL_DZ	ee
	HLT_Mu8_TrkIsoVVL_Ele23_CaloIdL_TrackIdL_IsoVL	$e\mu$
	HLT_Mu8_TrkIsoVVL_Ele23_CaloIdL_TrackIdL_IsoVL_DZ	$e\mu$
	HLT_Mu23_TrkIsoVVL_Ele8_CaloIdL_TrackIdL_IsoVL	$e\mu$
	HLT_Mu23_TrkIsoVVL_Ele8_CaloIdL_TrackIdL_IsoVL_DZ	$e\mu$
	HLT_Mu17_TrkIsoVVL_Mu8_TrkIsoVVL_DZ	$\mu\mu$
	HLT_Mu17_TrkIsoVVL_Mu8_TrkIsoVVL	$\mu\mu$
	HLT_Mu17_TrkIsoVVL_TkMu8_TrkIsoVVL	$\mu\mu$
2017	HLT_Mu17_TrkIsoVVL_TkMu8_TrkIsoVVL_DZ	$\mu\mu$
	HLT_Ele23_Ele12_CaloIdL_TrackIdL_IsoVL	ee
	HLT_Mu23_TrkIsoVVL_Ele12_CaloIdL_TrackIdL_IsoVL_DZ	$e\mu$
	HLT_Mu8_TrkIsoVVL_Ele23_CaloIdL_TrackIdL_IsoVL_DZ	$e\mu$
2018	HLT_Mu17_TrkIsoVVL_Mu8_TrkIsoVVL_DZ_Mass8	$\mu\mu$
	HLT_Ele23_Ele12_CaloIdL_TrackIdL_IsoVL	ee
	HLT_Mu23_TrkIsoVVL_Ele12_CaloIdL_TrackIdL_IsoVL_DZ	$e\mu$
	HLT_Mu8_TrkIsoVVL_Ele23_CaloIdL_TrackIdL_IsoVL_DZ	$e\mu$
2022	HLT_Mu17_TrkIsoVVL_Mu8_TrkIsoVVL_DZ_Mass3p8	$\mu\mu$
	HLT_Ele23_Ele12_CaloIdL_TrackIdL_IsoVL	ee
	HLT_Mu23_TrkIsoVVL_Ele12_CaloIdL_TrackIdL_IsoVL_DZ	$e\mu$
	HLT_Mu8_TrkIsoVVL_Ele23_CaloIdL_TrackIdL_IsoVL_DZ	$e\mu$
2023	HLT_Mu17_TrkIsoVVL_Mu8_TrkIsoVVL_DZ_Mass3p8	$\mu\mu$
	HLT_Ele23_Ele12_CaloIdL_TrackIdL_IsoVL	ee
	HLT_Mu23_TrkIsoVVL_Ele12_CaloIdL_TrackIdL_IsoVL_DZ	$e\mu$
	HLT_Mu8_TrkIsoVVL_Ele23_CaloIdL_TrackIdL_IsoVL_DZ	$e\mu$
2023	HLT_Mu17_TrkIsoVVL_Mu8_TrkIsoVVL_DZ_Mass3p8	$\mu\mu$
	HLT_Mu17_TrkIsoVVL_Mu8_TrkIsoVVL_DZ_Mass3p8	$\mu\mu$

Table 4.1: List of HLT trigger bits for the Run 2 and Run 3 portions of the analysis.

In this analysis, we are interested in multi-lepton final states. Thus, the HLT bits should correspond to events in which there are at least two well-identified leptons (ee , $e\mu$, $\mu\mu$); known as "di-lepton" triggers. Table 4.1 shows the HLT trigger bits utilized in this analysis for Run 2 and Run 3; the names of the HLT bits are representative of the selection requirements. Take, for instance, the HLT bit name

HLT_Mu23_TrkIsoVVL_Ele8_CaloIdL_TrackIdL_IsoVL_DZ

Each of these strings have the following meanings:

- "Mu23": A muon with $p_T > 23$ GeV.
- "TrkIsoVVL": tracker isolation at the very-very-loose working point.
- "Ele8": An electron with $p_T > 8$ GeV.
- "CaloIdL": calorimeter ID at the loose working point.
- "TrackIdL": tracker ID at the loose working point.
- "IsoVL": isolation at the very-loose working point.
- "DZ": require that the d_z of the primary vertex be smaller than some value.

The term "working point" is HEP jargon; it refers to some selection requirement that corresponds to a pre-defined efficiency. To elucidate this, let's consider a simple example: say we have a variable with three different pre-defined working points (loose, medium, and tight). The loose working point may correspond to a signal efficiency (i.e. the fraction of signal events passing the cut divided by the total number of signal events) of, say, 90% while rejecting 20% of the background. The medium (tight) working point will have a lower signal efficiency of, say, 60% (40%) with a larger background rejection percentage of 50% (70%). Thus, there is an inherent

Year	Golden JSON	Int. Lumi (fb^{-1})
2016	Cert_271036-284044_13TeV_Legacy2016_Collisions16_JSON	36.33
2017	Cert_294927-306462_13TeV_UL2017_Collisions17_GoldenJSON	41.48
2018	Cert_314472-325175_13TeV_Legacy2018_Collisions18_JSON	59.83
2022	Cert_Collisions2022_355100_362760_Golden	34.65
2023	Cert_Collisions2023_366442_370790_Golden	27.245

Table 4.2: Integrated luminosity of data-taking year with the corresponding JSON files specifying the certified luminosity blocks.

tradeoff for each working point between the signal efficiency (how much signal you keep) and background rejection (how much background you remove). While working points are a somewhat contrived concept, their adoption is pragmatic; they are mostly utilized when trying to identify certain particles. The discussion of working points will be continued in Sec. 5.1.

4.3 Primary Datasets

Data for this analysis is obtained from the full "Run 2" period of CMS data-taking, corresponding to the years 2016-2018, and the first two years of the "Run 3" period of CMS data-taking, corresponding to the years 2022-2023. The integrated luminosity for each year of data-taking is shown in Table 4.2 and the totals for the Run 2, Run 3, and Combined (Run 2 + Run 3) periods are as follows:

$$\mathcal{L}_{\text{Run2}} = 138 \text{ fb}^{-1}$$

$$\mathcal{L}_{\text{Run3}} = 62 \text{ fb}^{-1}$$

$$\mathcal{L}_{\text{Combined}} = \mathcal{L}_{\text{Run2}} + \mathcal{L}_{\text{Run3}} = 200 \text{ fb}^{-1}$$

The datasets recorded by CMS for analysis are based on the HLT trigger bits;

those corresponding to the di-lepton trigger bits discussed in Sec. 4.2 (see Table 4.1) are shown in Table 4.3 for Run 2 and Table 4.4 for Run 3. The dimuon ($\mu\mu$) datasets are denoted by "DoubleMuon" for Run 2 (and for Run2022C in Run 3), "Muon" for the year 2022 in Run 3, and "Muon0" and "Muon1" for the year 2023 in Run 3. The dielectron (ee) datasets are identified using the string "DoubleEG" for 2016-2017 in Run 2, and "EGamma" for 2018 in Run 2 and both data-taking years in Run 3. The electron-muon ($e\mu$) datasets are denoted by "MuonEG" for all data-taking years in Run 2 and Run 3.

4.4 Simulated (Monte-Carlo) Samples

The full list of the MC simulated samples with corresponding cross sections in the Run 2 and Run 3 portions of the analysis are shown in Tables 4.5 and 4.6, respectively. For each year, the MC samples were produced during a different campaign; a period of sample creation according to the parameters and running conditions for the CMS detector and the reconstruction algorithms. The simulated samples for 2016APV and 2016 were produced in the RunIISummer16NanoAODAPVv9 (106X) and RunIISummer16NanoAODv9 (106X) campaigns, respectively. The samples for 2017 were produced in the RunIISummer20UL17NanoAODv9 (106X) campaign, and the samples for 2018 were produced in the RunIISummer20UL18NanoAODv9 (106X) campaign. In 2022, the samples for the pre (post)-EE period were produced in the Run3Summer22(EE)NanoAODv12 (130X) campaigns, while the 2023 (2023 BPix) samples were produced in the Run3Summer23(BPix)NanoAODv12 (130X) campaign.

For Run 2 (Run 3), the non-resonant WWZ signal sample is simulated using Madgraph [76–78] version 2.6 (2.9) at next-to-leading order (NLO) precision with the FxFx [77] scheme to match jets from the matrix element calculations and parton showers. This sample is simulated with two on-shell W bosons and two additional

same-flavor, opposite-sign leptons (e , μ , or τ) with an invariant mass greater than 30 GeV to remove the γ^* contribution. The cross sections for the non-resonant WWZ sample is normalized to the value obtained from the generator at NLO. The ZH signal process is comprised of two contributions; the $gg \rightarrow ZH$ (gluon-initiated) and $qq \rightarrow ZH$ (quark-initiated) diagrams. These contributions are modeled separately with the POWHEG v2 generator at NLO in perturbative QCD [79–82] in Run 2 and Run 3. Both the gluon-initiated and quark-initiated samples are simulated with an on-shell H boson, and require the invariant mass of the leptons coming from the Z boson to be greater than 10 GeV. The cross sections for both ZH contributions are normalized to the values recommended by the LHC Higgs Working Group [88, 89].

For the signal processes and the major background processes, samples including leptonic decays of heavy particles (i.e. W, Z, t) are utilized to decrease the statistical uncertainty of these contributions in the signal and control regions and also to provide an adequate number of events for the training of the BDTs.

Background processes are also estimated with MC simulations, as well as with dedicated control regions, as discussed in Sections 7.1.1 and 7.1.2. Some of the background processes (WW , ZZ , $t\bar{t}$, single top, and $H \rightarrow ZZ$) are produced with POWHEG to NLO accuracy in QCD, while others (WZ , WWW , WZZ , ZZZ , $t\bar{t}V$, $t\bar{t}H$, WH , tWZ , tZq , and $W + \text{jets}$) are simulated to NLO accuracy in QCD using Madgraph. The $gg \rightarrow ZZ$ process is produced with the MCFM v7.0.1 program [83].

For all samples utilized in this analysis, the parton showering and hadronization are modeled with PYTHIA [85, 86] (version 8.240 for Run 2, and version 8.30), and the NNPDF3.1 NNLO [90] sets of parton distribution functions are employed. In all cases, the detector response is simulated using the GEANT4 program [87].

For the $qq \rightarrow ZZ$ and $gg \rightarrow ZZ$ samples, the cross sections are normalized to the next-to-next-to leading order (NNLO) values. This is done by the use of so-called "k-factors", which represent the ratio of the cross section at some order in perturbation

theory to that at another order in perturbation theory. Here, we care about the NLO to NNLO k-factor:

$$\text{k-factor} = \frac{\sigma_{\text{NNLO}}}{\sigma_{\text{NLO}}} \quad (4.2)$$

Since the ZZ processes are simulated at NLO, we can normalize the samples to NNLO by multiplying the NLO cross sections by the appropriate k-factor. For the $gg \rightarrow ZZ$ processes, we apply a k-factor of 1.7 [91] to the cross sections listed in Tables 4.5 and 4.6. For the $qq \rightarrow ZZ$ processes, a k-factor of 1.1 [91] is applied to the cross sections listed in Tables 4.5 and 4.6.

Type	Dataset Sample Name
Dielectron (ee)	DoubleEG_Run2016B-ver2_HIPM
	DoubleEG_Run2016C-HIPM
	DoubleEG_Run2016D-HIPM
	DoubleEG_Run2016E-HIPM
	DoubleEG_Run2016F-HIPM
	DoubleEG_Run2016F
	DoubleEG_Run2016G
	DoubleEG_Run2016H
	DoubleEG_Run2017B
	DoubleEG_Run2017C
	DoubleEG_Run2017D
	DoubleEG_Run2017E
	DoubleEG_Run2017F
	EGamma_Run2018A
	EGamma_Run2018B
	EGamma_Run2018C
	EGamma_Run2018D
Electron-Muon ($e\mu$)	MuonEG_Run2016B-ver1_HIPM
	MuonEG_Run2016B-ver2_HIPM
	MuonEG_Run2016C-HIPM
	MuonEG_Run2016D-HIPM
	MuonEG_Run2016E-HIPM
	MuonEG_Run2016F-HIPM
	MuonEG_Run2016F
	MuonEG_Run2016G
	MuonEG_Run2016H
	MuonEG_Run2017B
	MuonEG_Run2017C
	MuonEG_Run2017D
	MuonEG_Run2017E
	MuonEG_Run2017F
	MuonEG_Run2018A
	MuonEG_Run2018B
	MuonEG_Run2018C
	MuonEG_Run2018D
Dimuon ($\mu\mu$)	DoubleMuon_Run2016B-ver1_HIPM
	DoubleMuon_Run2016B-ver2_HIPM
	DoubleMuon_Run2016C-HIPM
	DoubleMuon_Run2016D-HIPM
	DoubleMuon_Run2016E-HIPM
	DoubleMuon_Run2016F-HIPM
	DoubleMuon_Run2016F
	DoubleMuon_Run2016G
	DoubleMuon_Run2016H
	DoubleMuon_Run2017B
	DoubleMuon_Run2017C
	DoubleMuon_Run2017D
	DoubleMuon_Run2017E
	DoubleMuon_Run2017F
	DoubleMuon_Run2018A
	DoubleMuon_Run2018B
	DoubleMuon_Run2018C
	DoubleMuon_Run2018D

Table 4.3: List of datasets used in the Run-2 portion of the analysis. Samples from 2016 are completed with the string `_UL2016_MiniAODv2_NanoAODv9-v2`. Samples from 2017 are completed with the string `_UL2017_MiniAODv2_NanoAODv9-v1`. Samples from 2018 are completed with the string `_UL2018_MiniAODv2_NanoAODv9-v1`.

Type	Dataset Sample Name
Dimuon ($\mu\mu$)	DoubleMuon_Run2022C
	Muon_Run2022C
	Muon_Run2022D
	Muon_Run2022E
	Muon_Run2022F
	Muon_Run2022G
	Muon0_Run2023C
	Muon1_Run2023C
	Muon0_Run2023D
	Muon1_Run2023D
Dielectron ($e\mu$)	EGamma_Run2022C
	EGamma_Run2022D
	EGamma_Run2022E
	EGamma_Run2022F
	EGamma_Run2022G
	EGamma0_Run2023C
	EGamma1_Run2023C
	EGamma0_Run2023D
Electron-Muon ($e\mu$)	EGamma1_Run2023D
	MuonEG_Run2022C
	MuonEG_Run2022D
	MuonEG_Run2022E
	MuonEG_Run2022F
	MuonEG_Run2022G
	MuonEG_Run2023C
	MuonEG_Run2023D

Table 4.4: List of datasets used in the Run-3 portion of the analysis. Samples from 2022 and 2023 are completed with the string -22Sep2023-v*.

Process	Sample Name	($\sigma \times \text{BR}$) (pb)
Non-Resonant WWZ	WWZJetsTo4L2Nu_4F_TuneCP5_13TeV-amcatnlo-pythia8	0.002067
ZH ($H \rightarrow WW^*$)	HZJ_HToWWTo2L2Nu_ZTo2L_M-125_TuneCP5_13TeV-powheg-jhugen727-pythia8	0.00177
	GluGluZH_HToWWTo2L2Nu_M-125_TuneCP5_13TeV-powheg-pythia8	0.00282
$qq \rightarrow ZZ$	ZZTo4L_TuneCP5_13TeV-powheg-pythia8	1.3816*
	ZZTo2Q2L_mllmin4p0_TuneCP5_13TeV-amcatnloFFFX-pythia8	0.1757*
$gg \rightarrow ZZ$	GluGluToContInToZZTo(4e/4mu/4tau)_TuneCP5_13TeV-mcfm701-pythia8	0.0045951*
	GluGluToContInToZZTo(2e2mu/2e2tau/2mu2tau)_TuneCP5_13TeV-mcfm701-pythia8	0.009219*
Rare Top + X	TTZToLL_M-1to10_TuneCP5_13TeV-amcatnlo-pythia8	0.0493
	TTZToLLNuNu_M-10_TuneCP5_13TeV-amcatnlo-pythia8	0.2810
	TTZToQQ_TuneCP5_13TeV-amcatnlo-pythia8	0.5297
	TTWJetsToLNU_TuneCP5_13TeV-amcatnloFFFX-madspin-pythia8	0.235
	TTWJetsToQQ_TuneCP5_13TeV-amcatnloFFFX-madspin-pythia8	0.4062
	tZq_ll_4f_ckm_NLO_TuneCP5_13TeV-amcatnlo-pythia8	0.0758
	ttHJetToNonbb_M125_TuneCP5_13TeV_amcatnloFFFX_madspin_pythia8	0.2151
	TWZToLL_tlept_Wlept_5f_DR_TuneCP5_13TeV-amcatnlo-pythia8	0.0015
Diboson	WWTo2L2Nu_TuneCP5_13TeV-powheg-pythia8	12.178
	SSWW_TuneCP5_13TeV-madgraph-pythia8	0.02794
	WZTo3LNU_TuneCP5_13TeV-amcatnloFFFX-pythia8	5.284
Other (non-signal) Triboson	WWW_4F_TuneCP5_13TeV-amcatnlo-pythia8	0.2086
	WZZ_TuneCP5_13TeV-amcatnlo-pythia8	0.05565
	ZZZ_TuneCP5_13TeV-amcatnlo-pythia8	0.01398
tt	TTTo2L2Nu_TuneCP5_13TeV-powheg-pythia8	87.315
	TTToSemiLeptonic_TuneCP5_13TeV-powheg-pythia8	364.35
Higgs	VHToNonbb_M125_TuneCP5_13TeV-amcatnloFFFX_madspin_pythia8	0.952
	GluGluHToZZTo4L_M125_TuneCP5_13TeV_powheg2_JHUGenV7011_pythia8	0.011814
Single Top	ST_tW_top_5f_inclusiveDecays_TuneCP5_13TeV-powheg-pythia8	35.85
	ST_tW_antitop_5f_inclusiveDecays_TuneCP5_13TeV-powheg-pythia8	35.85
	ST_s-channel_4f_leptonDecays_TuneCP5_13TeV-amcatnlo-pythia8	3.6818
	ST_t-channel_top_4f_InclusiveDecays_TuneCP5_13TeV-powheg-madspin-pythia8	136.02
	ST_t-channel_antitop_4f_InclusiveDecays_TuneCP5_13TeV-powheg-madspin-pythia8	80.95
V + Jets	WJetsToLNU_TuneCP5_13TeV-amcatnloFFFX-pythia8	61526.7

Table 4.5: List of simulated event samples for the Run 2 measurement. All samples have the same base name from 2016-2018. Samples from 2016 pre(post)-VFP are completed by the string `RunIISummer20UL16NanoAODv9` (`RunIISummer20UL16NanoAODAPVv9`). Samples from 2017 are completed by the string `RunIISummer20UL17NanoAODv9`. Samples from 2018 are completed by the string `RunIISummer20UL18NanoAODv9`. Cross sections marked with an asterisk have additional k-factors applied, which are described in Section 4.4.

Process	Sample Name	($\sigma \times \text{BR}$) (pb)
Non-Resonant WWZ	WWZto4L2Nu_4F_TuneCP5_13p6TeV_amcatnlo-pythia8	0.002244
ZH ($H \rightarrow WW^*$)	ZH_ZtoAll_Hto2Wto2L2Nu_M-125_TuneCP5_13p6TeV_powheg-minlo-HZJ-jhugenv752-pythia8	0.0186
	GluGluZH_ZtoAll_Hto2Wto2L2Nu_M-125_TuneCP5_13p6TeV_powheg-jhugenv752-pythia8	0.00312
qq $\rightarrow$ ZZ	ZZto4L_TuneCP5_13p6TeV_powheg-pythia8	1.529*
gg $\rightarrow$ ZZ	GluGlutoContinto2Zto4(E/Mu/Tau)_TuneCP5_13p6TeV_mcfm-pythia8	0.0051*
	GluGlutoContinto2Zto(2E2Mu/2E2Tau/2Mu2Tau)_TuneCP5_13p6TeV_mcfm701-pythia8	0.0103955*
Rare Top + X	TTLL_MLL-4to50_TuneCP5_13p6TeV_amcatnlo-pythia8	0.03949
	TTLL_MLL-50_TuneCP5_13p6TeV_amcatnlo-pythia8	0.08646
	TTZ-ZtoQQ-1Jets_TuneCP5_13p6TeV_amcatnloFXFX-pythia8	0.6603
	TZQB-Zto2L-4FS_MLL-30_TuneCP5_13p6TeV_amcatnlo-pythia8	0.08
	TTHtoNon2B_M-125_TuneCP5_13p6TeV_powheg-pythia8	0.24111
	TWZ_TtoLNU_WtoLNU_Zto2L_DR1_TuneCP5_13p6TeV_amcatnlo-pythia8	0.001669
	WWto2L2Nu_TuneCP5_13p6TeV_powheg-pythia8	12.98
Diboson	VBS-SSWW_PolarizationTT_TuneCP5_13p6TeV_madgraph-pythia8	0.01762
	VBS-SSWW_PolarizationTL_TuneCP5_13p6TeV_madgraph-pythia8	0.01152
	VBS-SSWW_PolarizationLL_TuneCP5_13p6TeV_madgraph-pythia8	0.002177
	WZto3LNU_TuneCP5_13p6TeV_powheg-pythia8	5.31
Other triboson	WWW_4F_TuneCP5_13p6TeV_amcatnlo-madspin-pythia8	0.2328
	WZZ_TuneCP5_13p6TeV_amcatnlo-pythia8	0.06
	ZZZ_TuneCP5_13p6TeV_amcatnlo-pythia8	0.02
tt	TTto2L2Nu_TuneCP5_13p6TeV_powheg-pythia8	98.0
Higgs	VH_HtoNonbb_M-125_TuneCP5_13p6TeV_amcatnloFXFX-madspin-pythia8	1.0132
	GluGluHtoZZto4L_M-125_TuneCP5_13p6TeV_powheg2-JHUGenV752-pythia8	0.01434
Single Top	TWminusto2L2Nu_TuneCP5_13p6TeV_powheg-pythia8	4.67
	TbarWplusto2L2Nu_TuneCP5_13p6TeV_powheg-pythia8	4.67
	TWminustoLNU2Q_TuneCP5_13p6TeV_powheg-pythia8	19.36
	TbarWplustoLNU2Q_TuneCP5_13p6TeV_powheg-pythia8	19.36
	TbarBQ_t-channel_4FS_TuneCP5_13p6TeV_powheg-madspin-pythia8	87.2
	TBbartoLplusNuBbar-s-channel-4FS_TuneCP5_13p6TeV_amcatnlo-pythia8	3.7
	TBbarQ_t-channel_4FS_TuneCP5_13p6TeV_powheg-madspin-pythia8	145.0
V + Jets	WtoLNU-2Jets_TuneCP5_13p6TeV_amcatnloFXFX-pythia8	64481.58

Table 4.6: List of simulated event samples for the Run 3 measurement. All samples have the same base name for 2022 pre-,post-EE, 2023, and 2023 BPix. Samples from 2022 pre-EE are completed by the string `Run3Summer22NanoA0Dv12`. Samples from 2022 post-EE are completed by the string `Run3Summer22EENanoA0Dv12`. Samples from 2023 (2023 BPix) are completed by the string `Run3Summer23(BPix)NanoA0Dv12`. Cross sections marked with an asterisk have additional k-factors applied, which are described in Section 4.4.

CHAPTER 5

RECONSTRUCTED OBJECTS

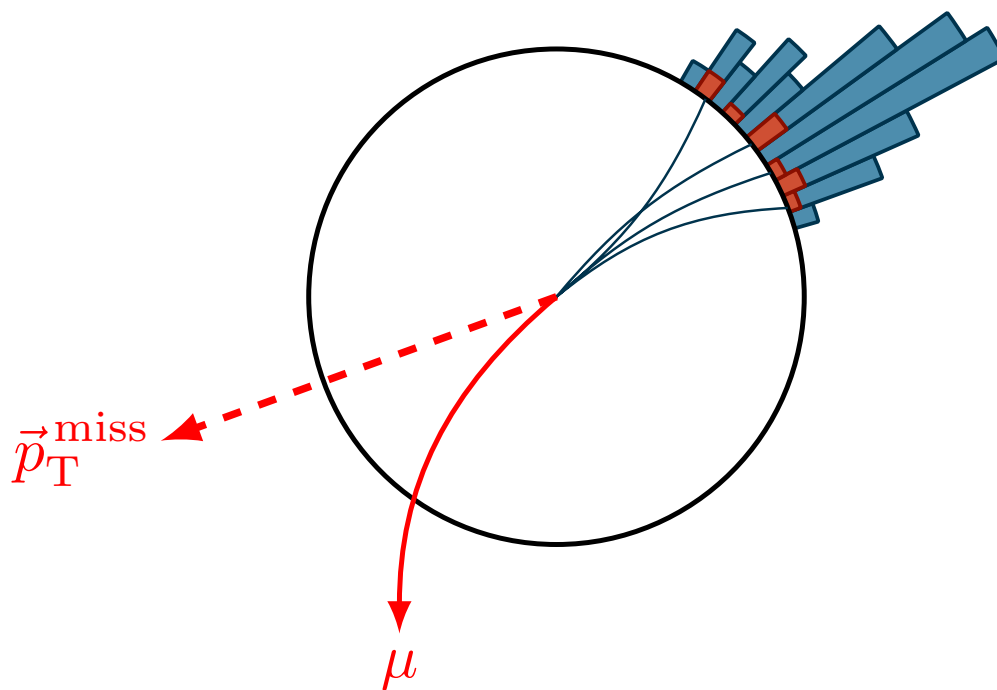


Figure 5.1: Diagram of the transverse plane of CMS showing tracks (dark blue curved lines) and calorimeter deposits (red and blue rectangles) corresponding to jets, a muon track (curved red line), and the missing transverse momentum (red dashed line) for a representative event. Figure drawn with code obtained from https://tikz.net/transverse_plane/

5.1 Overview

There are three primary types of objects utilized in this analysis: leptons, missing transverse momentum (p_T^{miss}), and jets (see Fig. 5.1). The leptons are crucial for inferring the presence of the Z boson and aid in identifying the two W bosons. The presence of p_T^{miss} is important, along with the leptons, for identifying the W bosons and helping to separate the signal processes from the dominant ZZ background. Jets are utilized to estimate and reject the other dominant background processes that feature jets in the final state, such as $t\bar{t}Z$ and tWZ . The following sections detail the different algorithms, methods, and/or criteria for obtaining these objects.

5.2 Leptons

This analysis targets the 4-lepton final state, where $\ell = e/\mu$, and thus selecting leptons in a robust and efficient way is of critical importance. This is done by utilizing dedicated lepton IDs for electrons and muons, which start from PF electron and muon candidates described in Sections 3.3.1 and 3.3.3, respectively. Different IDs are chosen for the Run 2 and Run 3 portions of the analysis, which are described in Sections 5.2.1 and 5.2.2, respectively.

Some of the variables used in lepton identification (e.g. d_{xy}) are defined with respect to a certain primary vertex (PV); a vertex that, ideally, represents the spatial origin of the physics process of interest. The procedure for determining the PVs is described in detail in Ref. [92] and briefly summarized here:

- **Track Selection:** Choose tracks in the Silicon tracker that originate within the primary interaction region.
- **Clustering:** Group the selected tracks into clusters according to their origin.
- **Fitting:** Within each cluster, fit the tracks to determine the position of the vertex from which the tracks originate.

The PV used to determine the variables used in the lepton identification criteria is defined to be the PV with the largest value of the summed squared p_T of all physics objects originating from the vertex.

5.2.1 Run 2

For the Run 2 portion of the analysis, we choose electron and muon candidates according to the "Tight" requirements of the TOP-UL-MVA ID developed for the CMS four top quark observation [93]. The selection requirements for the electron and muon

IDs are summarized in Tables 5.1 and 5.2, respectively; the ID requirements include cuts on common lepton quantities (which are defined in Sec. A.1.1) as well as a MVA score (TOP-UL Lepton MVA) obtained by training on lepton kinematic variables. The input variables for the training of the electron and muon MVAs are shown in Table 5.3. A validation of this ID is performed in a ZZ background-enriched region, which is described in detail in Sec. C.1.1. From this, we conclude that this lepton ID is sufficiently validated. The efficiency for selecting electrons at the "tight" working point is measured to be $\approx 85\%$, while for muons the efficiency is $\approx 88\%$. Further details of the Run 2 lepton ID are presented in Appendix C.

Electrons	
Observable	Tight
p_T	$> 10 \text{ GeV}$
$ \eta $	< 2.5
$ d_{xy} $	$< 0.05 \text{ cm}$
$ d_z $	$< 0.1 \text{ cm}$
$\frac{d}{\sigma_d}$	< 8
I_{rel}	< 0.4
Conversion rejection	✓
Charge consistency	✓
Missing hits	≤ 1
EGamma POG MVA	—
DeepJet of closest jet	—
p_T^{ratio}	—
Top-UL Lepton MVA	> 0.81

Table 5.1: Selection criteria for electrons for the TOP-UL MVA ID. Variable definitions are presented in Sec. A.1.1.

Muons	
Observable	Tight
p_T	$> 10 \text{ GeV}$
$ \eta $	< 2.4
$ d_{xy} $	$< 0.05 \text{ cm}$
$ d_z $	$< 0.1 \text{ cm}$
$\frac{d}{\sigma_d}$	< 8
I_{rel}	< 0.4
Muon POG ID	WP-Medium
DeepJet of closest jet	—
p_T^{ratio}	—
Top-UL Lepton MVA	> 0.64

Table 5.2: Selection criteria for muons for the TOP-UL MVA ID. Variable definitions are presented in Sec. [A.1.1](#).

Variable	Input variables for TOP Lepton MVA			Brief Description
	Electron MVA	Muon MVA		
p_T	✓	✓		Transverse Momentum
η	✓	✓		Lepton η
jetNDauCharged	✓	✓		Number of charged daughters of closest jet
Charged Mini PF Relative Isolation	✓	✓		Sum of charged hadronic PF candidates with p_T -dependent cone size
Neutral Mini PF Relative Isolation	✓	✓		Sum of neutral hadronic PF candidates with p_T -dependent cone size
jetPtRelv2	✓	✓		Relative momentum of lepton to nearest jet after subtracting lepton
jetPtRatio	✓	✓		Ratio between p_T of lepton and nearest jet
PF Relative Isolation	✓	✓		Sum of p_T of PF objects within $\Delta R = 0.3$ cone
btagDeepFlavB score	✓	✓		B-tagging discriminant score of nearest jet to lepton
SIP_{3D}	✓	✓		Ratio of lepton impact parameter to its uncertainty (in 3D space)
$\log(d_{xy})$	✓	✓		logarithm of transverse impact parameter
$\log(d_z)$	✓	✓		logarithm of longitudinal impact parameter
segmentComp		✓		Probability of a muon originating from a given track

Table 5.3: Input variables used in the training of the TOP-UL MVA Lepton ID.

5.2.2 Run 3

In the Run 3 portion of the analysis, electron and muon candidates are selected according to recommendations from the CMS Muon and Electron Physics Object Groups (POGs) for muons and electrons, respectively. The details are as follows:

- **Muons:** The medium- p_T ($15 \text{ GeV} < p_T < 200 \text{ GeV}$) recommendation is used. We choose muons according to the "medium" working point for this ID and impose the "LoosePFIso" working point for muon isolation. A summary of the full requirements is given in Table 5.4, and the variable definitions are discussed in Sec. A.1.1.
- **Electrons:** The MVA-based electron ID with isolation is utilized here. We choose electrons that satisfy the "WP90" working point requirements for this ID. The full set of requirements is shown in Table 5.5, and the variables are described in Sec. A.1.1.

Validation of the Run 3 lepton IDs is performed in dilepton events around the Z boson mass peak, and good agreement is found between data and simulation. Details of the validation (with plots) are presented in Sec. C.1.2. The efficiencies of the lepton IDs for Run 3 are 90% for electrons and 92% for muons.

Muons	
Observable	Cut
p_T	$> 10 \text{ GeV}$
$ \eta $	< 2.4
$ d_{xy} $	$< 0.05 \text{ cm}$
$ d_z $	$< 0.1 \text{ cm}$
$\frac{d}{\sigma_d}$	< 8
Muon POG ID	WP-Medium
pflsolId	≥ 2

Table 5.4: Selection criteria for muons for the Muon POG recommended muon ID for Run 3. Variable definitions are provided in Sec. [A.1.1](#).

Electrons	
Observable	Cut
p_T	$> 10 \text{ GeV}$
$ \eta $	< 2.5
$ d_{xy} $	$< 0.05 \text{ cm}$
$ d_z $	$< 0.1 \text{ cm}$
$\frac{d}{\sigma_d}$	< 8
Missing hits	≤ 1
tight charge	$= 2$
Conversion rejection	$\checkmark$
EGamma POG MVA Iso ID	WP90

Table 5.5: Selection criteria for the EGamma POG recommended `mvaIsoWP90` electron ID for Run 3. Variable definitions are provided in Sec. [A.1.1](#).

5.3 Jets

Two of the major backgrounds in this analysis, $t\bar{t}Z$ and tWZ , involve the production of bottom quark jets (b-jets) from the decay of the top quarks (recall from Sec. 2.2.2 that top quarks decay almost exclusively to bottom quarks and a W boson). Due to the (relatively) long B hadron lifetime ($\mathcal{O}(10^{-12}s)$), B hadrons can travel up to a few millimeters in the detector before decaying and producing a jet (see Fig. 2.5). Thus, jets containing a B hadron will, in addition to the primary jet vertex close to the PV, exhibit a secondary vertex (SV) that is displaced from the PV, as shown in Figure 5.2.

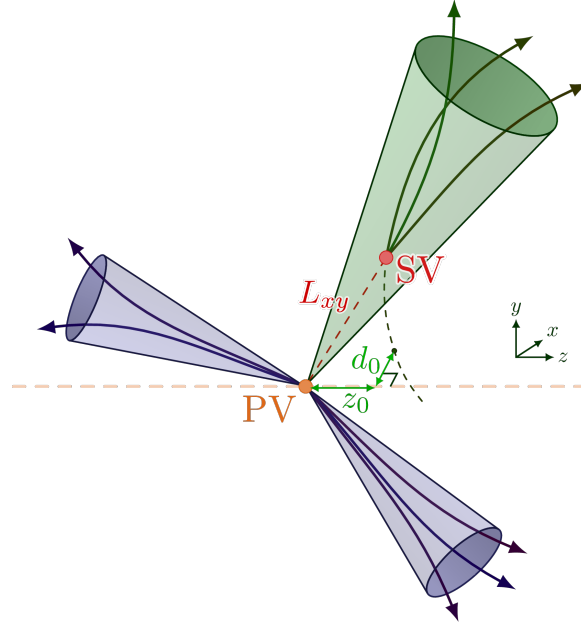


Figure 5.2: Diagram of a three-jet event with two light-flavor jets (purple cones) and a b-jet (green cone). The jets originate from the primary vertex (PV), but the b-jet has a secondary vertex (SV) that is displaced by a distance L_{xy} from the PV. Figure taken from https://tikz.net/jet_btag/

This displacement (d_{xy}) can be measured with modern Silicon tracker detectors. Additionally, the high charge multiplicity of B hadron decays (i.e. how many charged particles are produced in the decay) [21], the presence of an additional vertex in the jet, and whether a lepton is identified within the jet provide complementary infor-

mation that aid in identifying b-quark jets. These variables are used in a series of algorithms called "b-tagging" algorithms, which aim primarily to discriminate between jets originating from bottom quarks and jets originating from the light (up, down, strange) quarks and gluons. Jets containing charm quark hadrons can also exhibit a SV (with smaller displacement) with smaller charge multiplicity [21], and so discrimination between b and c quark jets is also of importance to these algorithms. The performance of three separate b-tagging algorithms employed by CMS is shown in Fig. 5.3.

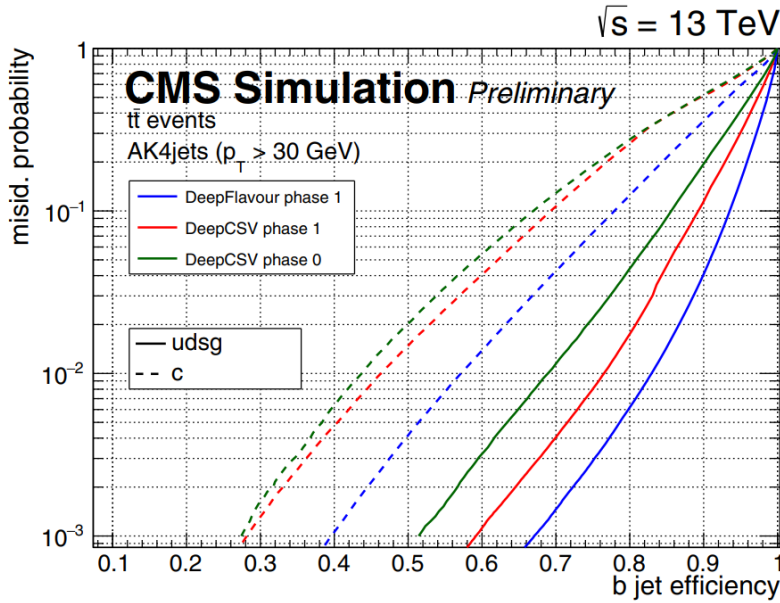


Figure 5.3: Performance of CMS b-tagging algorithms with simulated $t\bar{t}$ events in 2016 for AK4 jets with $p_T > 30$ GeV. Solid lines represent the performance with respect to light flavor (udsg) jets and dotted lines are with respect to charm jets. Figure taken from Ref. [94].

In CMS, jets are formed by clustering groups of PF reconstructed objects using the anti- k_T (AK) algorithm [95]. This algorithm works by, as with previous jet algorithms (e.g. k_T [96] and Cambridge/Aachen [97, 98]), defining two distances: a distance between different PF candidate particles i and j in the event (d_{ij}), and a distance between each PF candidate particle and the beam (d_{iB}). The definitions are

as follows:

$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{\Delta_{ij}^2}{R^2} \quad (5.1)$$

$$d_{iB} = k_{ti}^{2p} \quad (5.2)$$

where k_{ti} is the p_T of particle i , R is a free parameter that represents the angular radius defined in Eq. 2.8, p is a free parameter (which will be discussed shortly), and Δ_{ij} is defined as

$$\Delta_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2 \quad (5.3)$$

where y_i and ϕ_i are the rapidity and azimuthal angle of particle i , respectively. The distinguishing feature of the AK algorithm is that the parameter $p = -1$.

The distances d_{ij} and d_{iB} are calculated for each PF candidate in the event, and the clustering algorithm starts with the smallest distance and iterates over all particles:

- If the smallest distance is a d_{ij} it combines the particles i and j into a single object, which is put back into the list of particles.
- If the smallest distance is a d_{iB} , the particle i is labeled as a "jet" and it is removed from the list of particles.

This process of clustering, re-calculating, and labeling continues until there are no more particles left. The AK algorithm is referred to as a "infrared and collinear safe" algorithm. The term "infrared safe" refers to the fact that the algorithm produces the same jets with and without the inclusion of soft radiation. "Collinear safe" refers to the property of the algorithm that results in the same jets regardless of whether

the jets originate from a single parton or a set of collinear partons with the same total momentum [99]. It is a favorable algorithm for many reasons (as discussed in Ref. [95]), most notably the stable behavior of the jet boundaries in the case of soft radiation.

In this analysis, we consider jets constructed from PF candidates using the AK algorithm with radius parameter $R = 0.4$ (AK4 jets). For the Run 2 (Run 3) period, jets are selected as "Candidates" if they have $p_T > 20$ GeV, $|\eta| < 2.4$, and satisfy the "tight" working point of the Run 2 (Run 3) AK4 CHS [74] (AK4 PUPPI [100]) JetMET POG ID, as is summarized in Table 5.6. Variable definitions are presented in Sec. A.1.2. We require Candidate Jets to be separated from Candidate Leptons (see Sec. 5.2) by $\Delta R > 0.4$. In the Run 3 portion of the analysis, we apply so-called "jet veto maps" [101] that veto events that contain jets falling into specific regions of the CMS detector to avoid spurious p_T^{miss} .

Candidate jet selection		
Variable	Run 2 Cut	Run 3 Cut
p_T	> 20 GeV	> 20 GeV
$ \eta $	< 2.4	< 2.4
$\Delta R_{j,\ell}$	> 0.4	> 0.4
Neutral Hadron Fraction	< 0.90	< 0.90
Neutral EM Fraction	< 0.90	< 0.90
Number of Constituents	> 1	> 1
Muon Fraction	< 0.80	< 0.80
Charged Hadron Fraction	> 0	> 0.01
Charged Multiplicity	> 0	> 0
Charged EM Fraction	< 0.80	< 0.80

Table 5.6: Selection requirements for candidate jets in the Run 2 and Run 3 portions of the analysis.

Jets passing the Candidate selection criteria are subject to the "deepFlavor" discriminator, which is used to tag jets originating from b-quarks. We define Candidate b-tagged jets as Candidate Jets that satisfy the "loose" criteria of the deepFlavor tagger [102]. The efficiency of the b-tagging algorithm is studied in $t\bar{t}Z$ events passing

the preselection requirements defined in Section 6.2. We define the efficiency to be:

$$\epsilon_{btag} = \frac{N_{tagged,gen}}{N_{gen}} \quad (5.4)$$

where $N_{tagged,gen}$ is the number of jets that are tagged according to the "loose" criteria of the deepFlavor tagger and matched to a jet at the generator level containing a B-hadron, and N_{gen} is the number of jets matched to a generator-level jet containing a B-hadron. The efficiencies are measured as a function of the p_T and $|\eta|$ of the jets, and are shown in Figure 5.4 for the Run 2 data-taking years and in Figure 5.5 for the Run 3 data-taking years. The b-tagging efficiency is $\approx 90\%$ in Run 2 and $\approx 92\%$ in Run 3.

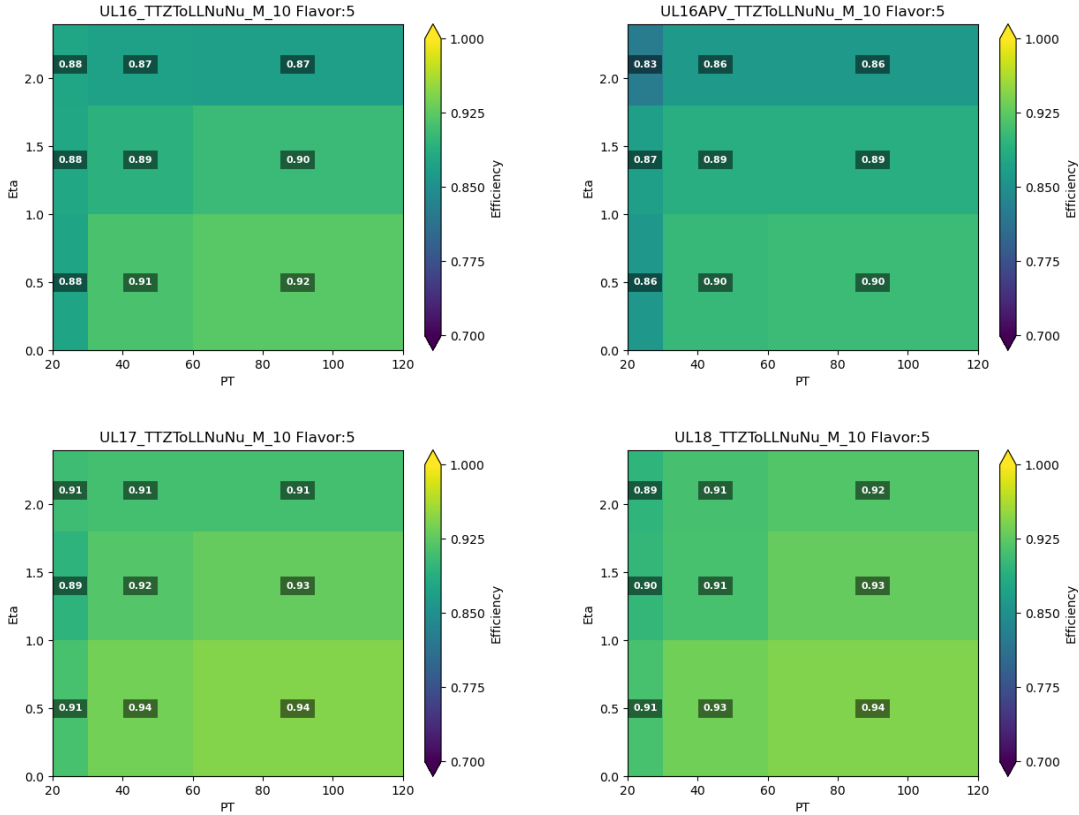


Figure 5.4: Efficiencies for the deepFlavor b-tagging algorithm in $t\bar{t}Z$ events as a function of jet p_T and $|\eta|$ in 2016 (top left), 2016APV (top right), 2017 (bottom left), and 2018 (bottom right).

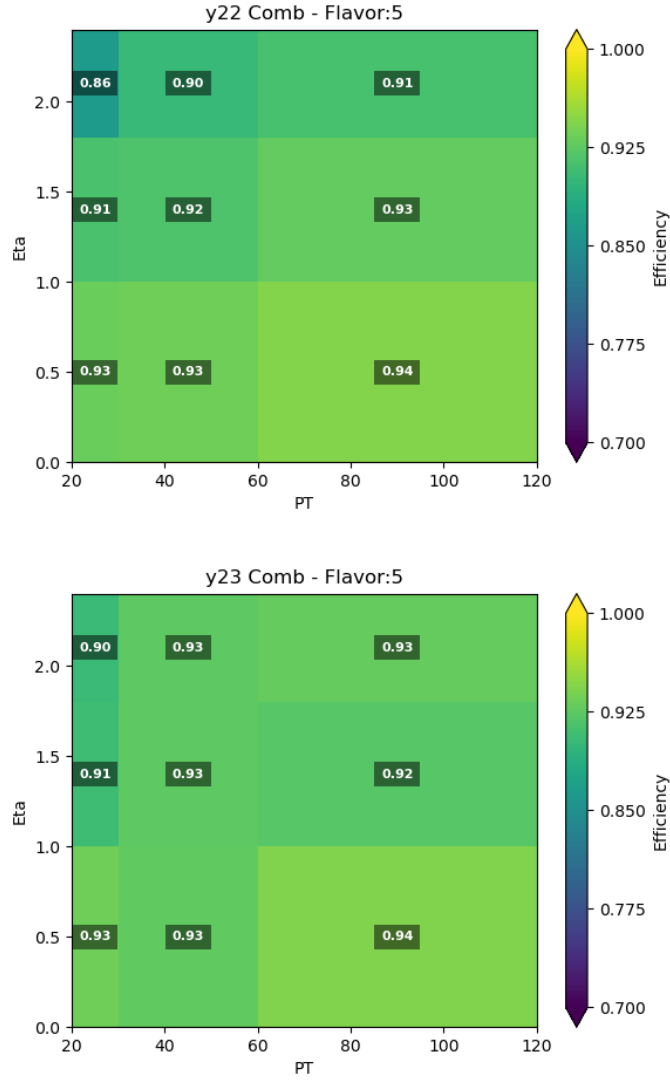


Figure 5.5: Efficiencies for the deepFlavor b-tagging algorithm in $t\bar{t}Z$ events as a function of jet p_T and $|\eta|$ in 2022 (top) and 2023 (bottom).

5.4 Missing Transverse Momentum

Since neutrinos are not detectable by CMS, their presence must be inferred through conservation of momentum in the transverse plane. Thus, one can define the missing transverse momentum to be the negative vector sum of the p_T of all particles in the event [100]:

$$\vec{p}_T^{miss} = - \sum_{i \in \text{event}} \vec{p}_{T,i} \quad (5.5)$$

Of course, this formula assumes that all particle momenta are reconstructed perfectly, which is never the case. Expanding the right hand side of Eq. 5.5, there are contributions to the total transverse momentum from jets, leptons, and unclustered energy deposits:

$$\vec{p}_T^{miss} = - \left[\sum_{i \in \text{jets}} \vec{p}_{T,i} + \sum_{j \in \text{leptons}} \vec{p}_{T,j} + \sum_{k \in \text{unclustered}} \vec{p}_{T,k} \right] \quad (5.6)$$

As discussed in Sec. 3.2, the energy resolution associated with jets is typically much worse than that for leptons. Thus, the first term on the right hand side of Eq. 5.6 is the main contributor to the uncertainty of the p_T^{miss} . To improve the accuracy of the jet momentum contribution, various algorithms and procedures have been developed such as: Pileup Per Particle Identification (PUPPI) [100], Charged Hadron Subtraction (CHS) [74], and various others. For both the Run 2 and Run 3 portions of this analysis, we utilize the PUPPI algorithm to reconstruct p_T^{miss} .

The PUPPI algorithm works at the level of individual particles within a jet or those corresponding to unclustered energy: it defines a local shape α_i for each PF candidate particle. In the region where tracking is available (the central region), α_i takes the form:

$$\alpha_i^C = \log \sum_{j \in \text{Ch, LV}} \frac{p_{T,j}}{\Delta R_{ij}} \times \Theta(R_{\min} \leq \Delta R_{ij} \leq R_0) \quad (5.7)$$

where the sum is over charged particles originating from the leading vertex. In the region where no tracking information is available (the forward region):

$$\alpha_i^F = \log \sum_{j \in \text{event}} \frac{p_{T,j}}{\Delta R_{ij}} \times \Theta(R_{\min} \leq \Delta R_{ij} \leq R_0) \quad (5.8)$$

where the sum is over all PF particles in the event. The variable Θ is the Heaviside step function, while ΔR_{ij} is the angular separation of particles i and j in $\eta - \phi$ space (see Eq. 2.8). The values $R_{\min}$ and R_0 set the minimum and maximum cone size used in the calculation of α_i . They take the following values:

$$R_{\min} = 0.02, \quad R_0 = 0.3$$

Qualitatively, α_i is used to distinguish radiation coming from parton showers from that originating from **pileup** interactions (see Figure 12.3); the parton-shower like radiation has a harder p_T spectrum than pileup-like radiation. Weights w_i are then calculated for each particle by comparing α_i^C to the median α for charged pileup (in the central region) or by comparing α_i^F to the median α for all particles in the event (in the forward region). The weights represent the probability of a particle being consistent with parton-shower like radiation; a weight closer to one is ideally assigned to a particle from the hard scatter, while a weight closer to zero is ideally assigned to a particle from pileup. The weights are then used to re-scale the 4-momentum of each PF particle, and particles with small re-scaled 4-momentum are then discarded. The remaining set of momentum-corrected particles can then be utilized to determine p_T^{miss} . The main advantage of the PUPPI algorithm is that it is agnostic to the physics process that creates the jet; it targets general features of the particle shower

characteristic of hadron jets, resulting in consistent behavior over a large number of signal topologies.

5.5 Other Useful Objects and Quantities

From the basic objects utilized in this analysis we can construct more complicated objects that will further aid in our ability to isolate the signal processes. The following sections describe some particularly useful quantities that are employed in this analysis.

5.5.1 Transverse Mass

As noted in Sec. 2.2.2, the leptonic decays of the W boson are *semi-invisible*: they involve the production of a charged lepton and its corresponding neutrino or antineutrino. The lepton is detectable, while the neutrino is not, and so some of the W boson energy is lost, resulting in p_T^{miss} . To reconstruct the W boson mass with the missing energy requires the introduction of a mass-like quantity called the transverse mass (m_T) [24]:

$$m_T^2 = 2(E_T^\ell E_T^\nu - \mathbf{p}_T^\ell * \mathbf{p}_T^{miss}) \approx 2p_T^\ell p_T^\nu (1 - \cos(\phi_{\nu,\ell})) \quad (5.9)$$

where p_T^ℓ is the transverse momentum of the lepton, p_T^ν is the p_T^{miss} (in the case where this neutrino is the only source of missing energy), and $\phi_{\nu,\ell}$ is the azimuthal angle between the lepton p_T and p_T^{miss} . By construction, the transverse mass is bounded from above by the mass of the parent particle (e.g. the W boson):

$$m_W \geq m_T \quad (5.10)$$

In reality, detector resolution and the finite width of the W boson smears the reconstructed p_T^{miss} and the p_T of the lepton, resulting in a tail that extends beyond

the W mass. Figure 5.6 shows a distribution of the transverse mass from the CMS experiment [103].

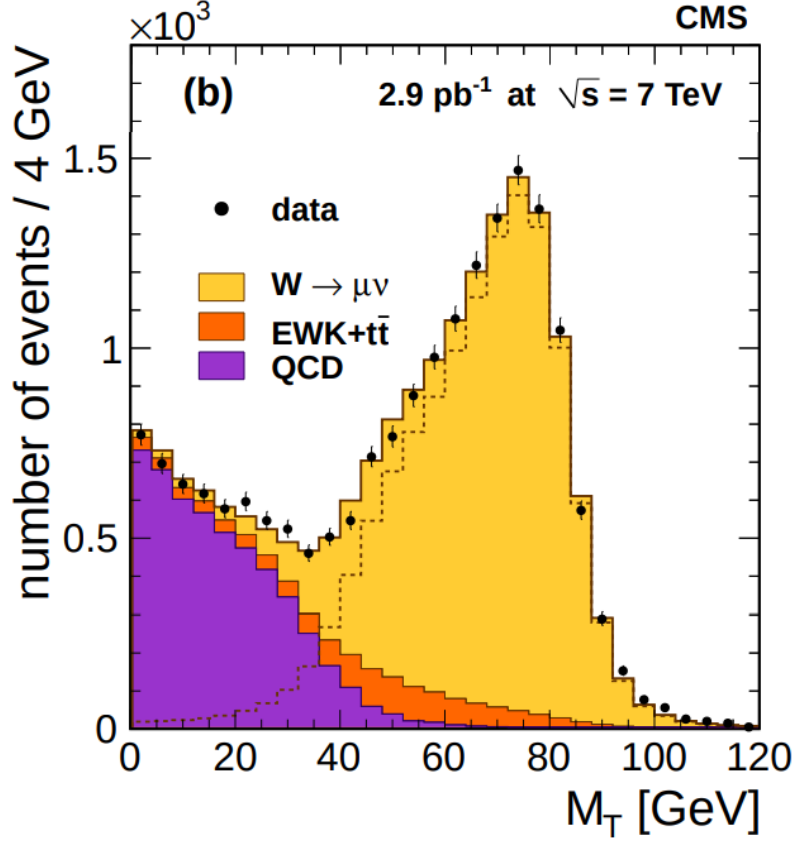


Figure 5.6: Transverse mass distribution for events with a single identified muon and missing transverse energy using the CMS detector at $\sqrt{s} = 7$ TeV. The signal is $W \rightarrow \mu\nu$, which peaks just below the mass of the W boson. Figure taken from Ref. [103].

5.5.2 Double Transverse Mass

The concept of the transverse mass can be extended to a process by which two particles are produced and decay semi-invisibly, as shown in Fig. 5.7.

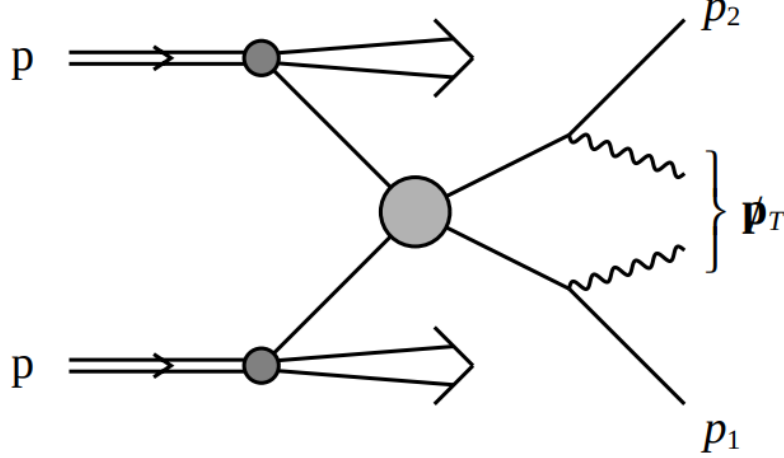


Figure 5.7: Diagram of a generic proton-proton collision resulting in two particles that decay semi-invisibly; two of the decay products are detectable, with momenta p_1 and p_2 , while the others are not and their momentum sum is equal to p_T^{miss} . Figure taken from ref. [104].

In this case, the measurable quantities are the momenta of the outgoing visible particles and the total p_T^{miss} , however the relative fraction of p_T^{miss} carried by each of the neutrinos is not known. Hence, one can define the double transverse mass or "stransverse mass" (m_{T2}) using the known information [104]:

$$m_{T2}^2 = \min_{p_1^\nu + p_2^\nu = p_T^{miss}} \left[\max(m_T^2(p_T^{\ell 1}, p_1^\nu), m_T^2(p_T^{\ell 2}, p_2^\nu)) \right] \quad (5.11)$$

where m_T is the transverse mass given by Eq. 5.9, and p_1^ν and p_2^ν are the undetermined individual neutrino momenta (which must equal p_T^{miss}) over which the minimization is performed. In analogy with m_T , m_{T2} is also bounded from above by the mass of the parent particle X :

$$m_X \geq m_{T2} \tag{5.12}$$

Since we are interested in measuring the ZH process where the Higgs boson decays via two W bosons:

$$H \rightarrow W W^* \rightarrow \ell \nu \ell \nu$$

the quantity m_{T2} is particularly useful in this analysis for reconstructing the mass of the Higgs for this signal, which helps to isolate it from the WWZ signal process and the major backgrounds.

5.5.3 Helicity (Decay) Angle

Consider the following scattering process:

$$pp \rightarrow XZ$$

where Z is a Z boson and X is a hypothetical particle that decays to two leptons:

$$X \rightarrow \ell_1 + \ell_2$$

A diagram of the decay of the X in its rest frame is shown in Fig. 5.8:

- p_X is the 4-momenta of the X in the lab frame.
- γ_X is the Lorentz factor for the particle X .
- q_1 and q_2 are the 4-momenta of the final state leptons in the rest frame of the particle X .

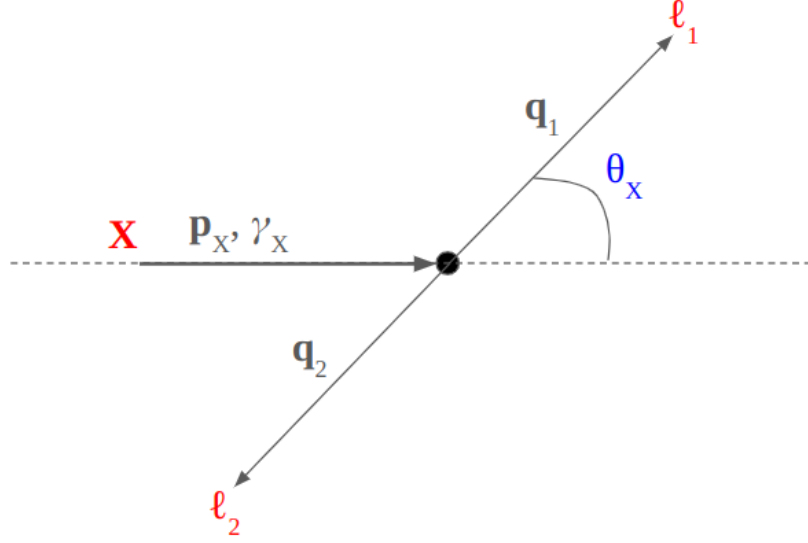


Figure 5.8: Diagram of the X decay to leptons. The 3-momenta of the X (in the lab frame) and the two leptons (in the rest frame of the X) are denoted by $\mathbf{p}_X$, $\mathbf{q}_1$, and $\mathbf{q}_2$, respectively. The helicity angle θ_X is shown in blue.

The helicity angle is the angle between the lepton 3-momentum in the rest frame of the X and the 3-momentum of the X in the lab frame:

$$\cos(\theta_X) = \frac{\mathbf{p}_X \cdot \mathbf{q}_1}{|\mathbf{p}_X| \times |\mathbf{q}_1|} \quad (5.13)$$

I note that there are two choices for the lepton momentum used in Eq. 5.13: in this analysis, we calculate $\cos(\theta_X)$ using the highest p_T lepton (the leading lepton) among the W -candidate leptons (see Sec. 6.2 for definition). In experiments, the lepton momenta in the lab frame is measured, and so the corresponding momenta in the rest frame of the particle X is obtained by boosting to the lab frame using the Lorentz factor of the particle X , γ_X .

This variable is useful in separating the signal processes from each other as well as from the major background processes, as it contains polarization information about the X . Consequently, unpolarized systems will exhibit a uniform distribution in $\cos(\theta_X)$. For the non-resonant WWZ signal process, the particle X represents the

visible part of the WW system (where both W bosons are on-shell) . For the ZH signal process, the particle X represents the visible part of the WW^* system resulting from the decay of the Higgs boson. For the ZZ and $t\bar{t}Z$ backgrounds, X represents the second Z boson and the visible component of the $t\bar{t}$ system, respectively. The distribution of $|\cos(\theta_X)|$ for the signal and background processes is shown in Figure 5.9.

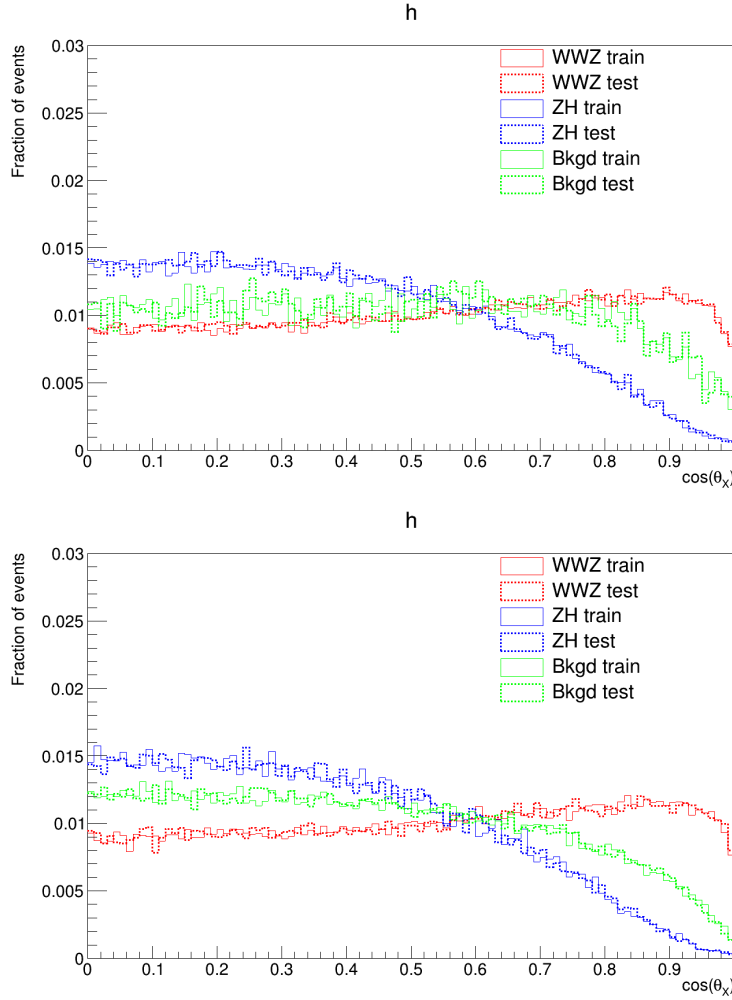


Figure 5.9: Absolute value of the cosine of the helicity angle in the OF channel (top) and SF channel (bottom) for WWZ , ZH , and background (sum of ZZ , $t\bar{t}Z$, and tWZ) events passing the training region selection. All processes are normalized to unity to show shape differences.

CHAPTER 6

EVENT SELECTION

"The sculpture is already complete within the marble block, before I start my work. It is already there, I just have to chisel away the superfluous material."

Michelangelo

6.1 Overview

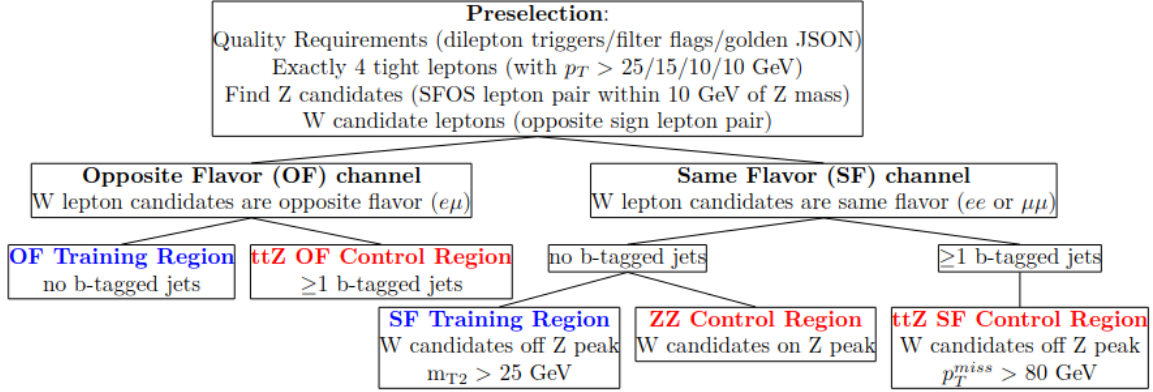


Figure 6.1: Diagram showing the selection requirements for the different categories used in this analysis. Blue labels denote the regions used in the BDT training, and red labels correspond to regions where the background estimation is performed.

Events are first subjected to a series of "pre-selection" requirements to isolate the 4-lepton final state and establish the presence of a Z boson. The remaining two leptons (i.e. those not consistent with the Z boson) are identified as "W-candidates" and are used to categorize events into one of two channels:

- Opposite Flavor (OF) channel: the W candidates have a different flavor (e.g. $e\mu$).
- Same Flavor (SF) channel: the W candidates have the same flavor (e.g. ee or $\mu\mu$).

The reason for this split is motivated by the different background contributions in each channel, as will be discussed in Sections 6.2.1 and 6.2.2. A combination of jets, p_T^{miss} , and lepton information is then used to further split events into training regions (used to train the boosted decision tree multiclassifier) and control regions (used to estimate the normalizations for the major background contributions). Figure 6.1 summarizes the selection criteria for the different regions employed in this analysis. The details of the selection are described in the following sections.

6.2 Pre-selection

To select multi-lepton events, each event is required to pass at least one of the dilepton triggers listed in Table 4.1. If an event in data passes triggers according to multiple datasets (i.e. $e\mu$ and $\mu\mu$), an overlap removal procedure is utilized to prevent double counting of events. Each event is further subject to noise filters, which are filters applied to different detector components to suppress noise arising during data-taking. The noise filters utilized in this analysis are listed in Table 6.1 for both eras.

Era	Noise Filters
Run 2	Flag_goodVertices Flag_globalSuperTightHalo2016Filter Flag_HBHENoiseFilter Flag_HBHENoiseIsoFilter Flag_EcalDeadCellTriggerPrimitiveFilter Flag_BadPFMuonFilter Flag_BadPFMuonDzFilter Flag_eeBadScFilter* Flag_ecalBadCalibFilter
Run 3	Flag_goodVertices Flag_globalSuperTightHalo2016Filter Flag_EcalDeadCellTriggerPrimitiveFilter Flag_BadPFMuonFilter Flag_BadPFMuonDzFilter Flag_eeBadScFilter Flag_ecalBadCalibFilter Flag_hfNoisyHitsFilter

Table 6.1: Noise filters used for Run 2 (top) and Run 3 (bottom). Filters marked with * are not used for 2016 or 2016APV.

Each event is required to have exactly four light charged lepton candidates (e or μ) that pass the Candidate Lepton requirements described in Section 5.2.1 for Run 2 and Section 5.2.2 for Run 3. The candidate lepton with the highest p_T (the *leading* lepton) is required to have $p_T > 25$ GeV, while the candidate lepton with the second highest p_T (the *subleading* lepton) is required to have $p_T > 15$ GeV. The other two candidate leptons in the event are required to have $p_T > 10$ GeV. These leptons are

then tagged according to their origin; leptons arising from the decay of a Z boson, which are referred to as Z-candidate leptons, and leptons arising from the decay of a W boson, which are referred to as W-candidate leptons. The Z-candidate leptons are chosen to be the same-flavor (ee or $\mu\mu$) opposite charge pair of leptons with invariant mass ($m_{\ell\ell}$) closest to the Z mass. To ensure the presence of an on-shell Z boson (and reduce the background contribution coming from off-shell Z production), the Z-candidate leptons are required to have $|m_{\ell\ell} - m_Z| < 10$ GeV. Once the Z-candidate leptons are chosen, the remaining two leptons are tagged as the W-candidate leptons. To ensure that the W-candidate leptons originate from an opposite charge pair of W bosons, we require that the W-candidate leptons have opposite charge. To reduce the contribution from the ZZ background, the W-candidate leptons are required to have $|m_{\ell\ell} - m_Z| > 10$ GeV.

Further requirements are imposed on leptons to discriminate against leptonic τ decays and contributions from "fake" leptons (i.e. objects, such as jets, that are incorrectly identified as leptons); muons (electrons) in the event are required to have $d/\sigma_d < 4$ (and $I_{\text{rel},03,\text{all}} < 0.2$). Backgrounds resulting from the production of light mesons are rejected by requiring that any opposite charge pair of leptons has an invariant mass greater than 12 GeV.

After selecting the Z-candidate and W-candidate leptons, events are separated according to the flavor of the W-candidate leptons. The following sections describe this split.

6.2.1 Opposite-Flavor (OF) Channel

The opposite flavor (OF) channel is designed to target decays of the W bosons to opposite-flavor leptons:

$$W W \rightarrow e\mu + p_T^{miss}$$

where p_T^{miss} denotes the presence of neutrinos from the W decays. We thus define the OF channel to be events in which the W-candidate leptons have different flavor (i.e. one muon and one electron). The major background contributions in the OF channel are from $ZZ \rightarrow 4\tau$ events where the τ leptons decay leptonically to electrons or muons, and $t\bar{t}Z$ and tWZ where the top-quarks and vector bosons decay to leptons. The $t\bar{t}Z$ and tWZ processes can be reduced considerably by vetoing b-tagged jets resulting from the top-quark decays. In the OF channel, events having one or more b-tagged jet consistent with the Candidate b-tagged jet defined in Section 5.3 are vetoed. Table 6.2 shows the relative contributions of different processes in the OF channel for the Run 2 portion of the analysis.

The major backgrounds ZZ and $t\bar{t}Z$ can be distinguished from the signal processes by exploiting the kinematics of the W-candidate leptons, namely through the m_{T2} variable (see Section 5.5.2) and the invariant mass of the W-candidate leptons, which will be denoted as $m_{\ell\ell}^{Wcands}$. Figure 6.2 (right) shows the distribution of m_{T2} for the signal and background processes in Run 2. For the ZH process, the m_{T2} distribution drops sharply above $m_H/2$ as the W bosons arise from an on-shell higgs boson. In the case of the non-resonant WWZ process, both W bosons are on-shell and do not arise from a resonance, and hence the distribution is broader compared to the ZH process. The ZZ^* process tends to have small values of m_{T2} due to the fact that the τ leptons from the Z decay are generally collinear, which forces the leptons from the τ decays to be approximately collinear with the p_T^{miss} (see Equations 5.9 and 5.11). The $t\bar{t}Z$ process tends to have similar m_{T2} to the non-resonant WWZ process, as the W-candidate leptons originate from two on-shell W bosons produced from the top-quark decays.

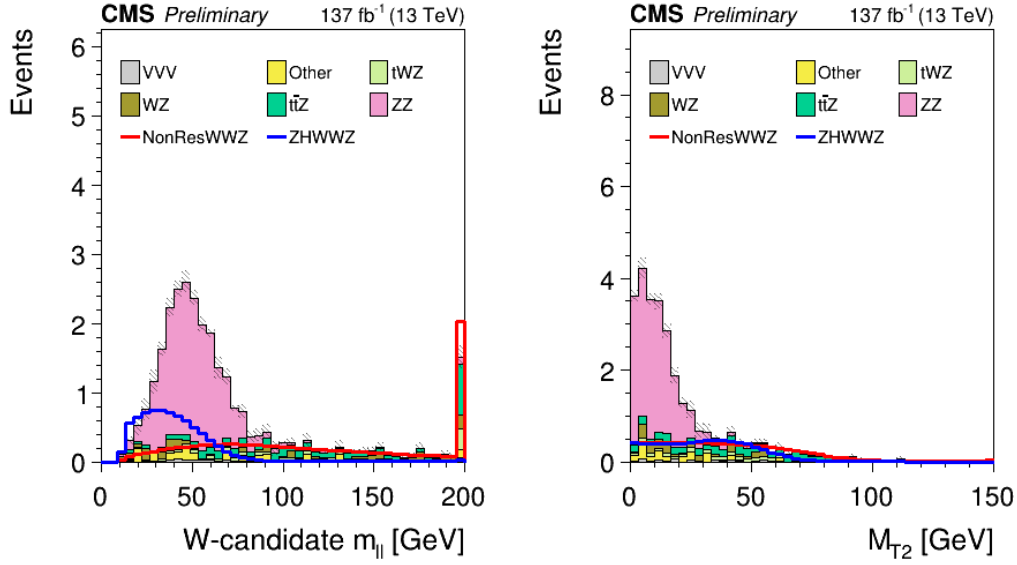


Figure 6.2: Dilepton mass distribution of W-candidate leptons in opposite-flavor channel (left) and m_{T2} variable computed with the W-candidate lepton momenta and p_T^{miss} in the opposite-flavor channel (right) for Run 2.

Figure 6.2 (left) shows the distribution of $m_{\ell\ell}^{Wcands}$ for Run 2; below about 100 GeV, the main background contribution is from the ZZ^* process, while above 100 GeV the $t\bar{t}Z$ process is dominant. For ZZ^* , $m_{\ell\ell}^{Wcands}$ is constructed from leptons arising from an off-shell Z boson that then decays to τ leptons, and thus one would expect the invariant mass to be below the mass of the Z and there to be some p_T^{miss} from the leptonic τ decays. For $t\bar{t}Z$, the invariant mass is constructed from the leptons coming from the top-quark pair decay to W bosons (and b-quark jets). Due to the large top-quark mass ($m_t = 172.5$ GeV), the resulting leptons (which are comparatively light) will have large momentum, resulting in a large invariant mass.

These variables (along with many others) are utilized in the training of the OF BDT multiclassifier described in Chapter 8. A description of the other input variables for the training is provided in Section 8.4.

6.2.2 Same-Flavor (SF) Channel

The same flavor (SF) channel is designed to target decays of the W bosons to same-flavor leptons:

$$W W \rightarrow e e + p_T^{miss}$$

or

$$W W \rightarrow \mu \mu + p_T^{miss}$$

where p_T^{miss} is used to represent the neutrinos. Because of this, the SF channel is defined as a collection of events in which the W-candidate leptons are the same flavor (i.e. either two muons or two electrons). The SF channel is dominated by the $ZZ^* \rightarrow 4\ell$ background process. The subleading $t\bar{t}Z$ and tWZ backgrounds are heavily reduced by vetoing events with one or more b-tagged jets satisfying the Candidate b-tagged jet criteria described in Section 5.3. Figure 6.2 shows the relative contributions of the different processes in the SF channel for the Run 2 portion of the analysis.

At tree level, the $ZZ^* \rightarrow 4\ell$ process has no p_T^{miss} (except for cases where the Z decays to a pair of τ leptons), while the signal processes contain non-zero real p_T^{miss} due to the semi-invisible decays of the W bosons. Figure 6.3 (left) shows the p_T^{miss} distribution in the SF channel for Run 2. Additionally, separation between the signals and the ZZ^* background can be obtained through the vector sum of lepton p_T in the event, $p_{T,4\ell}$. For signal events, the presence of neutrinos tends to increase the value of $p_{T,4\ell}$ on average. For the ZZ^* process, $p_{T,4\ell}$ tends to be lower than the signal process due to the lower p_T of the leptons resulting from the decay of the off-shell Z boson. This quantity only depends on the momenta of the

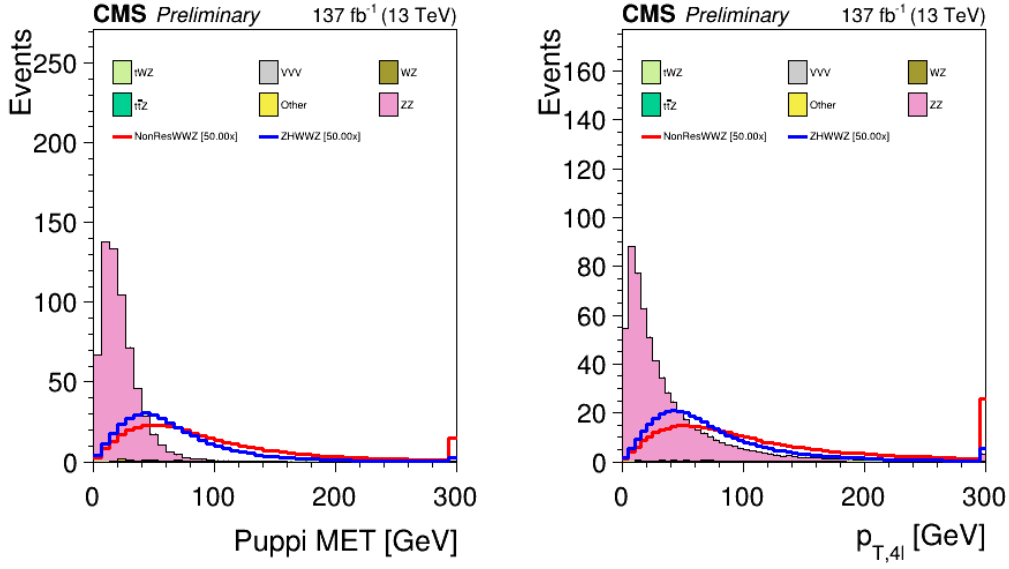


Figure 6.3: Missing transverse momentum in the same-flavor channel (left) and $p_{T,4\ell}$ variable in the same-flavor channel (right) for Run 2. Signal processes are scaled up by 50x to show shape comparison with ZZ background.

4-lepton system, making it robust against mismeasurement errors associated with p_T^{miss} due to hadronic energy contributions. Thus, $p_{T,4\ell}$ can provide complementary information to p_T^{miss} . The $p_{T,4\ell}$ distribution in the SF channel for Run 2 is shown in Figure 6.3 (right). Additionally, the m_{T2} variable also provides good discrimination power between the signal processes and the ZZ^* background due to the small p_T^{miss} for the ZZ^* process.

The p_T^{miss} and $p_{T,4\ell}$ are utilized (along with other variables) in the training of the SF BDT multiclassifier (see Chapter 8), while the m_{T2} is used in the selection for the SF training region described in Section 6.3 as well as in the BDT training itself. Other variables utilized in the BDT training are described in Section 8.4.

6.3 Training Region

To better distinguish the signal processes from the background processes and to distinguish each signal process from each other, a BDT multiclassifier is utilized to

	WWZ	ZH	ZZ	ttZ	tWZ	WZ	other
all events	47.31 ± 0.04	39.44 ± 0.05	6401.93 ± 2.14	531.63 ± 1.62	48.04 ± 0.12	805.43 ± 8.01	7500.86 ± 19.27
Preselection	22.85 ± 0.03	16.96 ± 0.03	2894.79 ± 1.31	138.67 ± 0.69	23.87 ± 0.08	8.72 ± 0.69	67.86 ± 1.49
OF Channel	7.29 ± 0.01	5.8 ± 0.02	16.45 ± 0.09	3.28 ± 0.1	1.49 ± 0.02	1.61 ± 0.28	1.32 ± 0.33
SF Channel	6.69 ± 0.01	5.84 ± 0.02	550.84 ± 0.54	3.08 ± 0.1	1.38 ± 0.02	1.55 ± 0.26	27.13 ± 0.69
OF TR	7.29 ± 0.01	5.8 ± 0.02	16.45 ± 0.09	3.28 ± 0.1	1.49 ± 0.02	1.61 ± 0.28	1.32 ± 0.33
SF TR	4.38 ± 0.01	3.52 ± 0.01	83.06 ± 0.21	2.08 ± 0.08	0.97 ± 0.01	0.76 ± 0.18	4.09 ± 0.4

Table 6.2: Yields for signal and background processes for different event selection categories in the Run 2 portion of the analysis. The uncertainties shown are statistical only.

construct signal regions. Events passing the preselection requirements (see Sec. 6.2) for the OF channel (see Sec. 6.2.1) and SF channel (see Sec. 6.2.2) are subject to additional requirements to construct the Training Region (TR), which is used for the training, testing, and evaluation of the BDT multiclassifier.

In the SF channel, we construct the SF TR by requiring events passing the SF preselection requirements to also satisfy $m_{T2} > 25$ GeV. This is done to reduce the number of ZZ events entering the SF TR. The m_{T2} distribution for events passing the SF preselection requirements in Run 2 is shown in Figure 6.4; the cut on m_{T2} reduces the ZZ background by a factor of approximately six. In the OF channel, the OF TR is constructed by taking events that pass the OF preselection requirements; no additional requirements are imposed. The contributions of signal and background processes in the OF and SF TRs are summarized in Table 8.4.

6.4 Signal Regions

Events passing the TR selection in the SF and OF channels are then used for the evaluation of the BDT multiclassifier, which is described in detail in Chapter 8. This assigns three different BDT scores to each event: a WWZ score (S_{WWZ}), a ZH score (S_{ZH}), and a background score (S_{BKG}) that are transformed using the *softmax* transformation [105]:

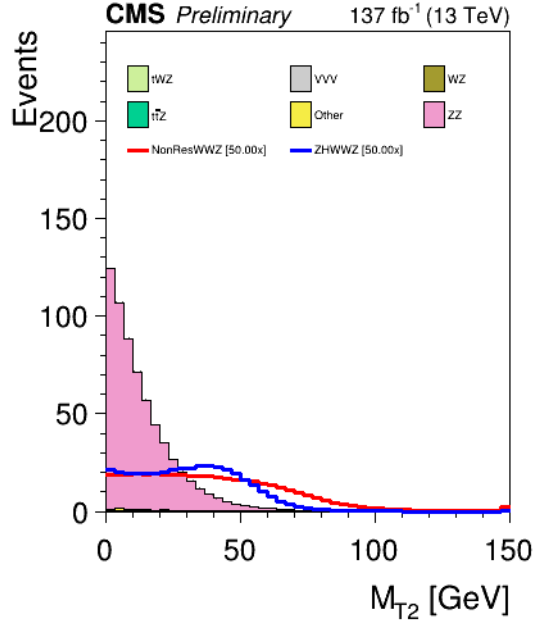


Figure 6.4: m_{T2} variable computed with the W-candidate lepton momentum and p_T^{miss} in the same-flavor channel for Run 2. Signal processes are scaled up by 50x to show shape comparison with ZZ background.

$$\sigma(\mathbf{S})_i = \frac{\exp(S_i)}{\sum_{j=1}^N \exp(S_j)} \quad (6.1)$$

where $\mathbf{S} = \{S_{WWZ}, S_{ZH}, S_{BKG}\}$ is the vector of BDT scores and $N = 3$. Before the transformation, each BDT score has $S_i \in (-1, 1)$, while after the transformation the scores take the values $\sigma(\mathbf{S})_i \in (0, 1)$. By definition, this mapping imposes that the sum of the transformed scores is equal to one:

$$\sigma(\mathbf{S})_{WWZ} + \sigma(\mathbf{S})_{ZH} + \sigma(\mathbf{S})_{BKG} = 1 \quad (6.2)$$

The distributions of the three BDT scores for the signal and background processes in the OF channel are shown in Figure 6.5 for Run 2 and Figure 6.7 for Run 3. In the SF channel, the BDT scores are shown in Figure 6.6 for Run 2 and Figure 6.8 for Run 3. The constraint in Eq. 6.2 allows us to plot all three scores for each event

simultaneously in a two-dimensional *ternary* plot. The top (bottom) of Figure 6.9 shows the ternary plot for simulated MC samples in the OF (SF) channel for Run 2. The same plots for Run 3 in the OF (SF) channel are shown in Figure 6.10 (bottom). Ternary plots for Run 2 and Run 3 with data included are displayed in Figures 6.13 and 6.14, respectively.

The three BDT scores are used to construct *Signal Regions* (SRs), which are regions that are (ideally) pure in the signal processes of interest used to perform measurements. The bin boundaries are determined by a brute-force scan over different bin configurations, and the scans are performed independently for the SF and OF categories. For each category, we first split the full region into two: one enriched in WWZ events ($\sigma(\mathbf{S})_{WWZ} \geq \sigma(\mathbf{S})_{ZH}$) and one enriched in ZH events ($\sigma(\mathbf{S})_{ZH} > \sigma(\mathbf{S})_{WWZ}$). Visually, this corresponds to the vertical line on the ternary plot shown in, for example, Figure 6.11. In Run 2, we choose a total of 4 bins in each of these sub-regions by varying the background score in steps of 0.01 for the three different boundaries. The sensitivity of the bin configurations to the signal processes is assessed by computing the so-called "Z-metric", which represents the Asymptotic form of the statistical significance [106]:

$$Z = \sqrt{2((S + B) \log(1 + \frac{S}{B}) - S)} \quad (6.3)$$

where S is the number of signal events in the bin and B is the number of background events in the bin. The Z-metric is computed for all 8 bins in each channel, and the values are summed in quadrature to assess the total Z-score. The "optimal" binning configuration for each channel is chosen to be the bin boundaries that maximize the total Z-score with "reasonable" signal MC yields in each bin. In total, there are 16 total SR bins (8 OF, 8 SF) used in the Run 2 portion of the analysis. The definitions of the bin boundaries determined by the optimization scan are shown in Table 6.3

and visualized in the ternary plots shown in Figure 6.11.

For Run 3, the same method is utilized. However, due to the lower number of events in Run 3, we utilize only 4 total bins in the OF channel and 4 bins in the SF channel, for a total of 8 SR bins. The definitions of the bin boundaries from the scan are shown in Table 6.4 and visualized in the ternary plots shown in Figure 6.12.

Each of the SRs defined in Tables 6.3 and 6.4 are used (along with the control regions, described in Chapter 7) for the extraction of the results, the details of which are described in Chapter 9. The data and MC yields in the SRs are shown in Figure 10.1.

Bin	OF Bins		SF Bins
	S_{WWZ}	S_{BKG}	S_{BKG}
1	$> S_{ZH}$	< 0.08	< 0.10
2	$> S_{ZH}$	$\in [0.08, 0.21)$	$\in [0.10, 0.15)$
3	$> S_{ZH}$	$\in [0.21, 0.41)$	$\in [0.15, 0.25)$
4	$> S_{ZH}$	≥ 0.41	≥ 0.25
5	$\leq S_{ZH}$	< 0.01	< 0.02
6	$\leq S_{ZH}$	$\in [0.01, 0.11)$	$\in [0.02, 0.10)$
7	$\leq S_{ZH}$	$\in [0.11, 0.38)$	$\in [0.10, 0.20)$
8	$\leq S_{ZH}$	≥ 0.38	≥ 0.20

Table 6.3: Definitions of the BDT Signal Regions for the Run 2 portion of the analysis.

Bin	OF Bins		SF Bins
	S_{WWZ}	S_{BKG}	S_{BKG}
1	$> S_{ZH}$	< 0.16	< 0.04
2	$> S_{ZH}$	≥ 0.16	≥ 0.04
3	$\leq S_{ZH}$	< 0.17	< 0.04
4	$\leq S_{ZH}$	≥ 0.17	≥ 0.04

Table 6.4: Definitions of the BDT Signal Regions for the Run 3 portion of the analysis.

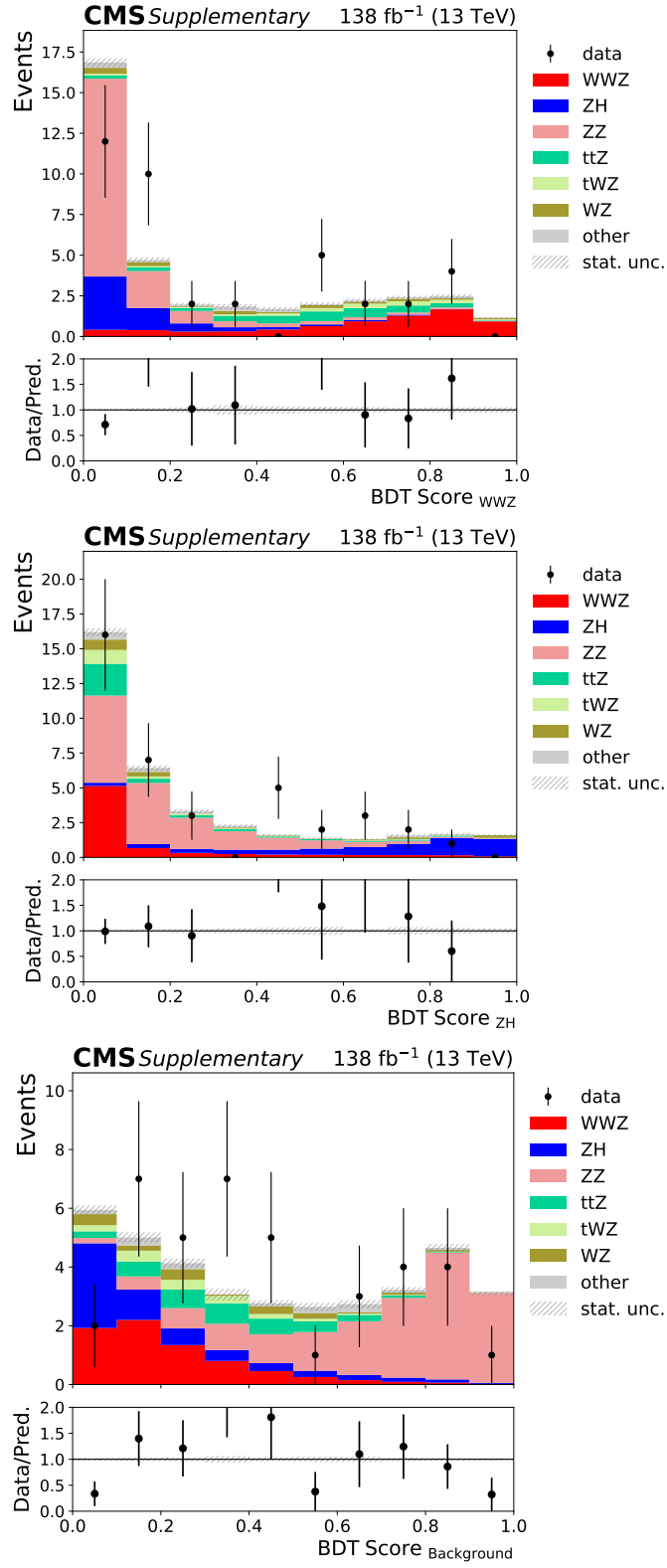


Figure 6.5: Distributions of the WWZ (top), ZH (middle), and background (bottom) BDT scores for MC and data events in the OF channel for the Run 2 portion of the analysis.

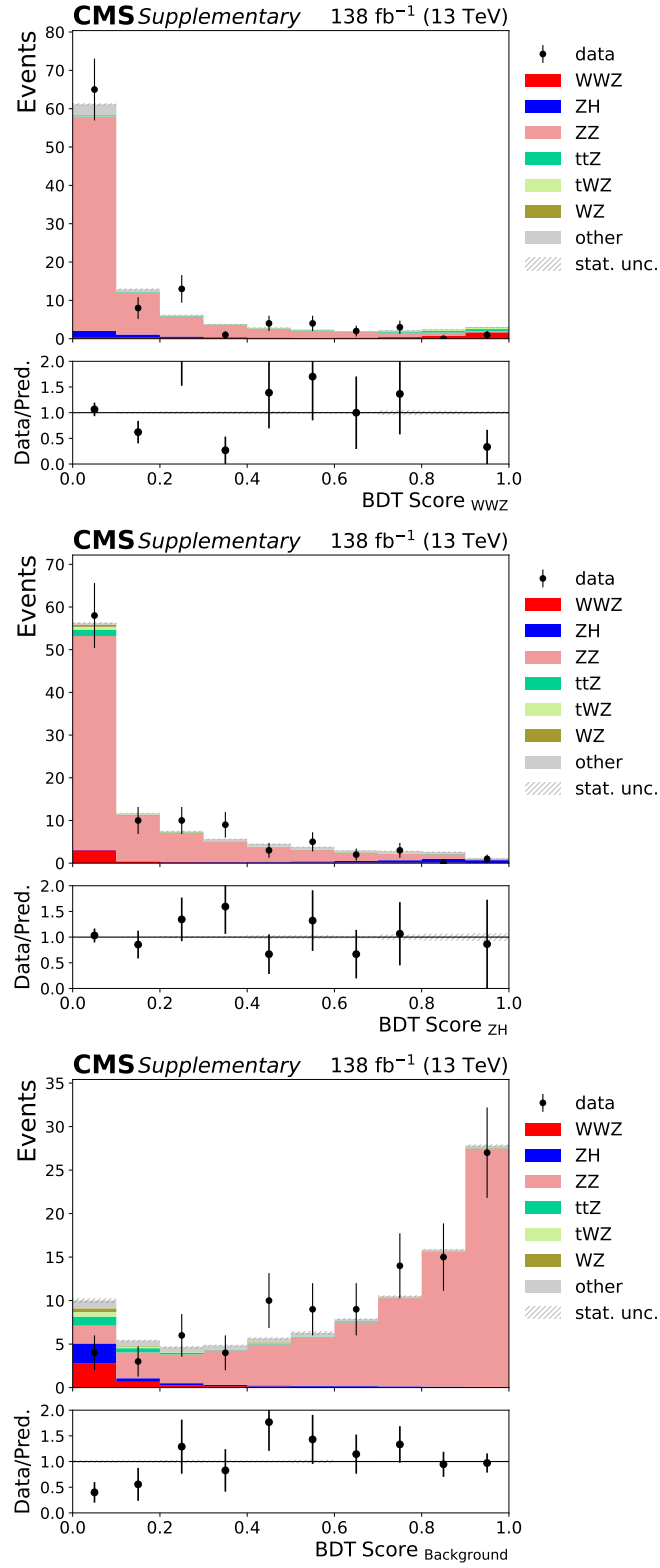


Figure 6.6: Distributions of the WWZ (top), ZH (middle), and background (bottom) BDT scores for MC and data events in the SF channel for the Run 2 portion of the analysis.

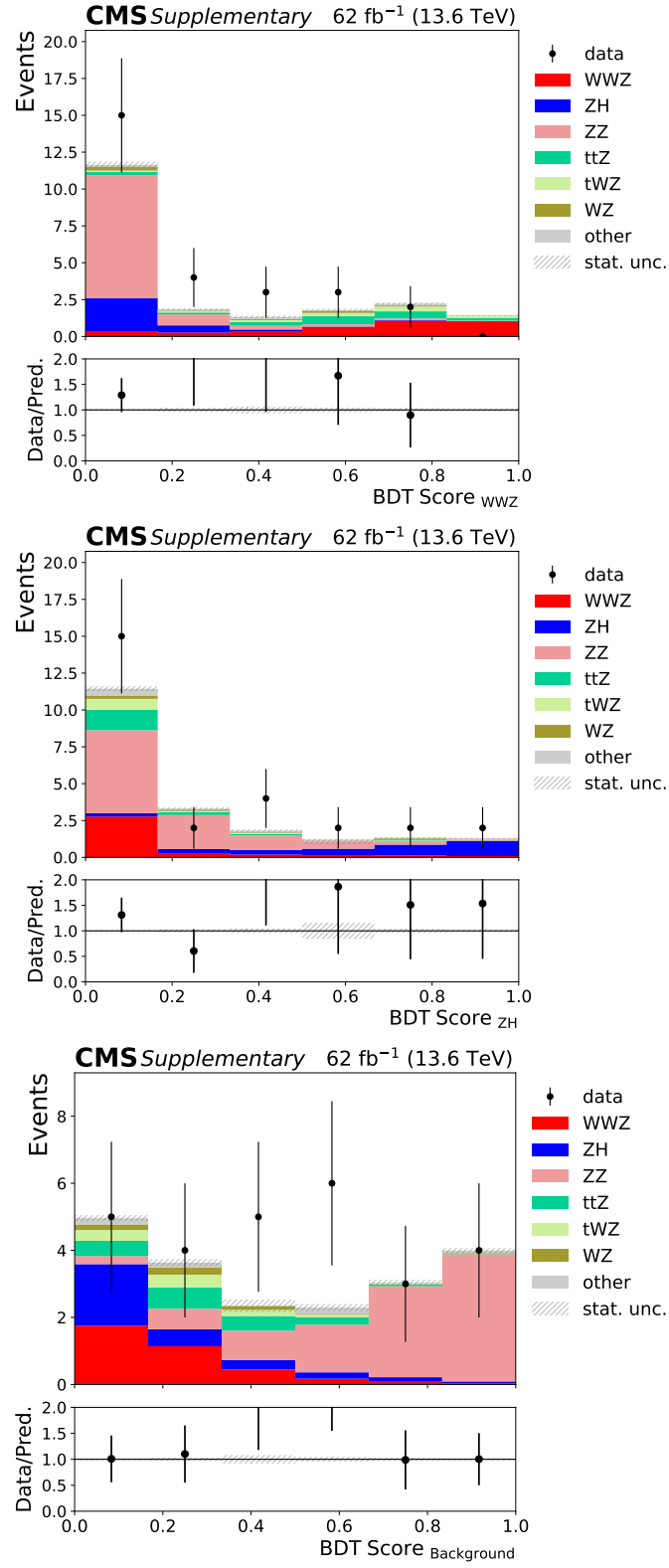


Figure 6.7: Distributions of the WWZ (top), ZH (middle), and background (bottom) BDT scores for MC and data events in the OF channel for the Run 3 portion of the analysis.

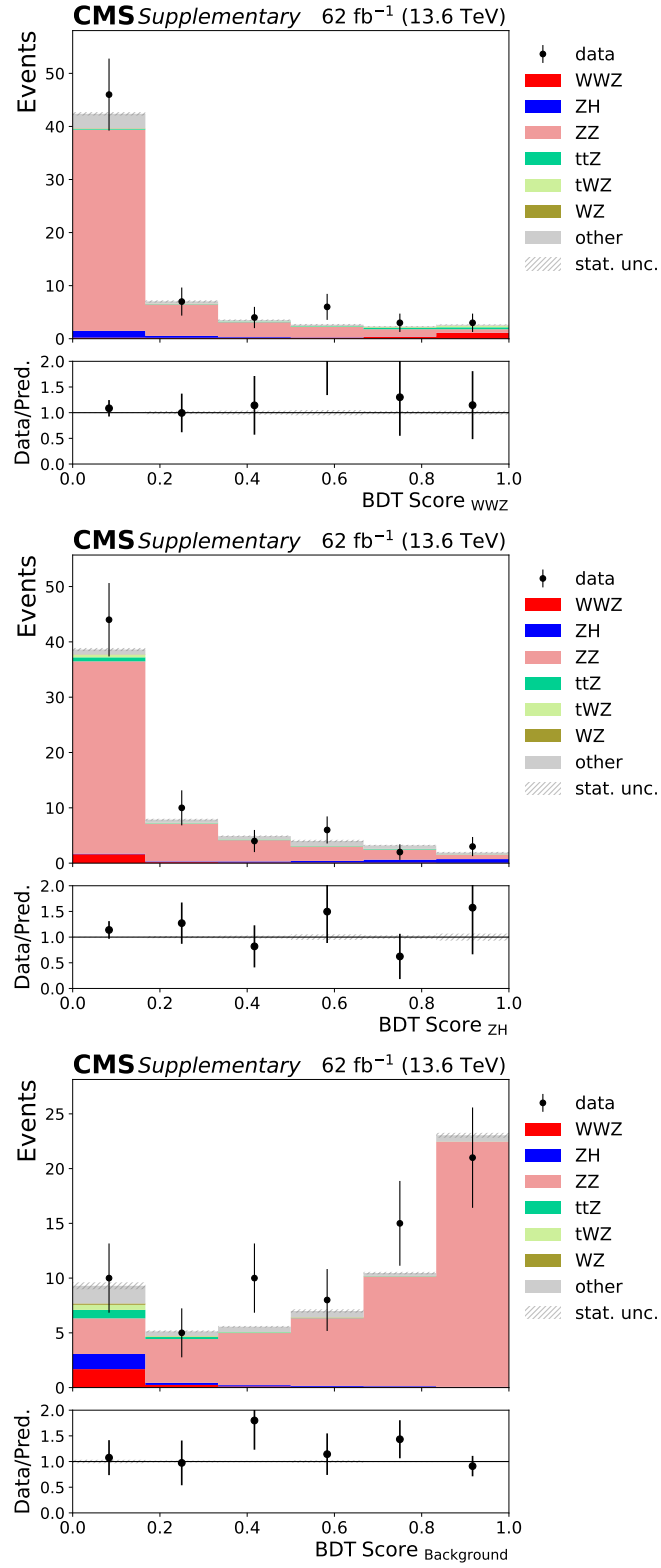


Figure 6.8: Distributions of the WWZ (top), ZH (middle), and background (bottom) BDT scores for MC and data events in the SF channel for the Run 3 portion of the analysis.

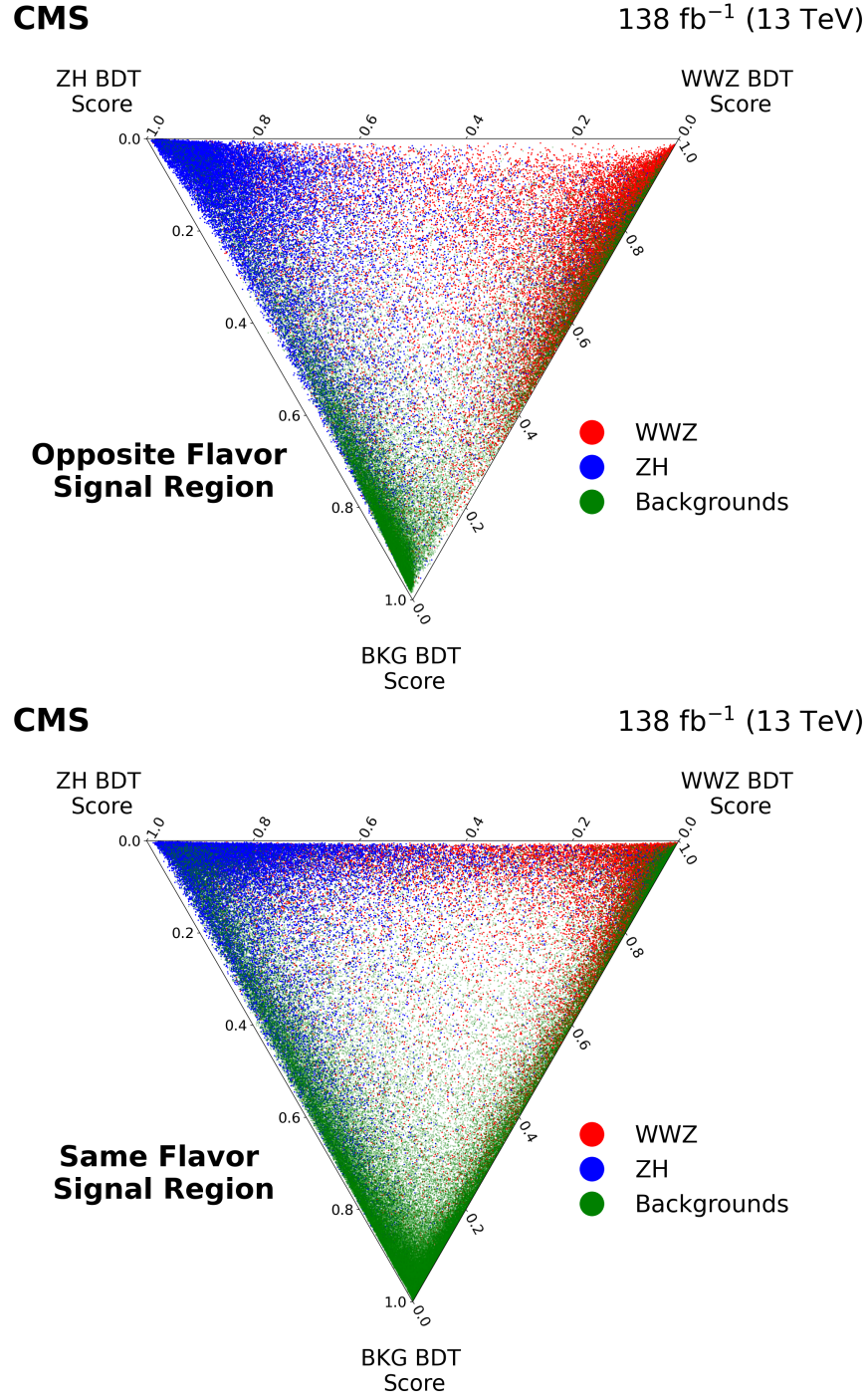


Figure 6.9: Ternary plot showing MC events for WWZ (red), ZH (blue), and background (green) in the OF channel (top) and SF channel (bottom) for Run 2.

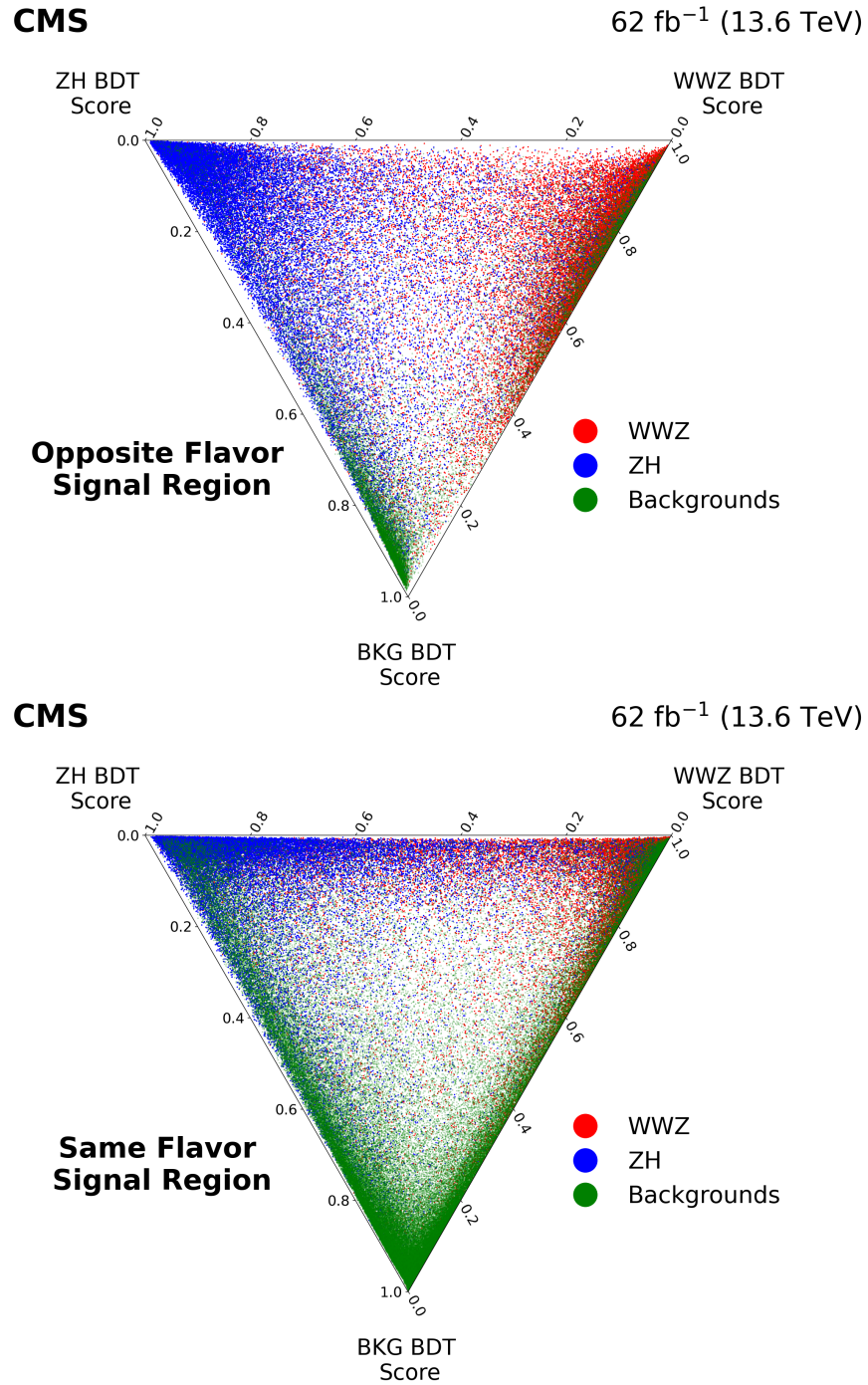


Figure 6.10: Ternary plot showing MC events for WWZ (red), ZH (blue), and background (green) in the OF channel (top) and SF channel (bottom) for Run 3.

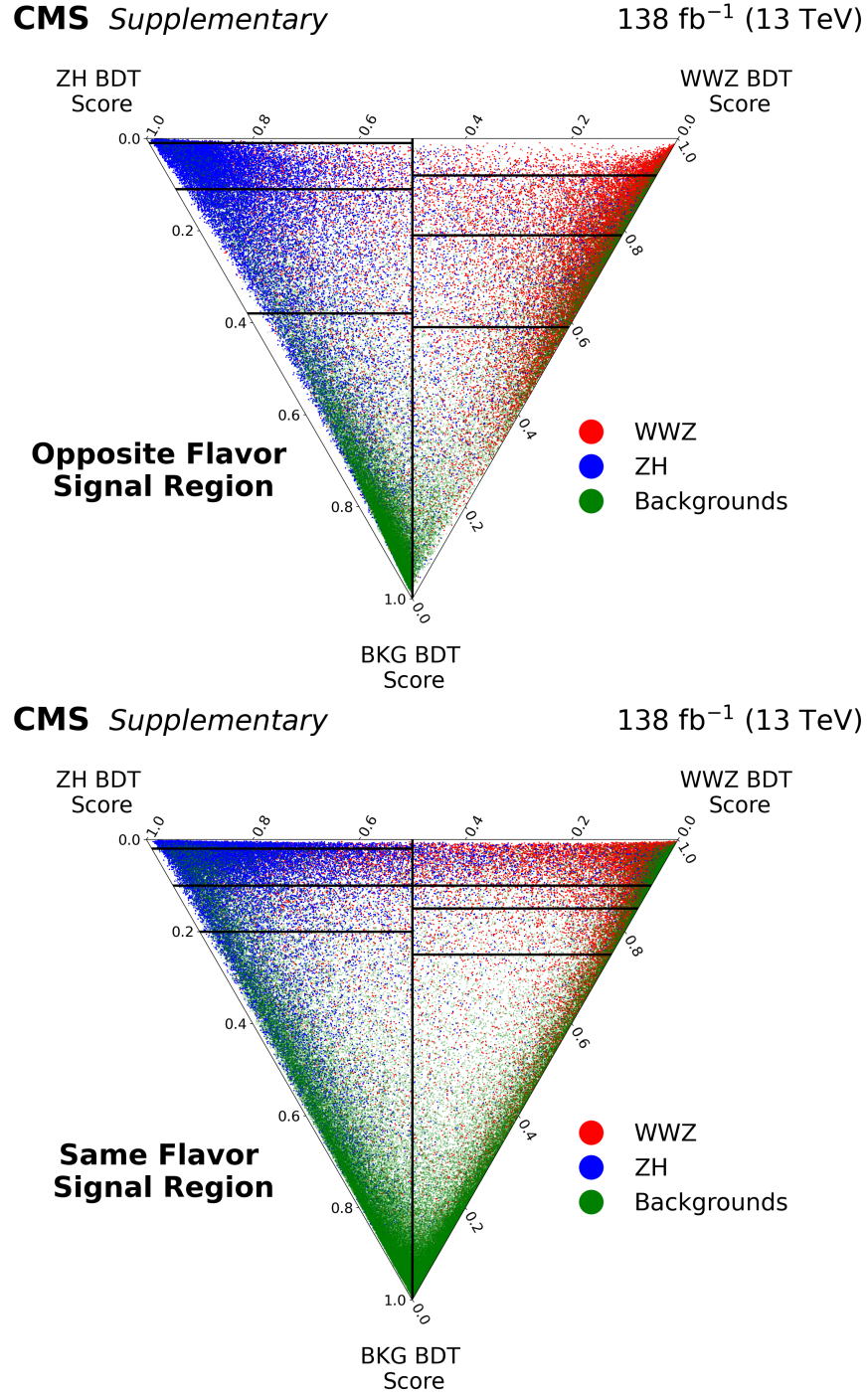


Figure 6.11: Ternary plot showing MC events for WWZ (red), ZH (blue), and background (green) in the OF channel (top) and SF channel (bottom) for Run 2. Bin boundaries defined in Table 6.3 are shown in black.

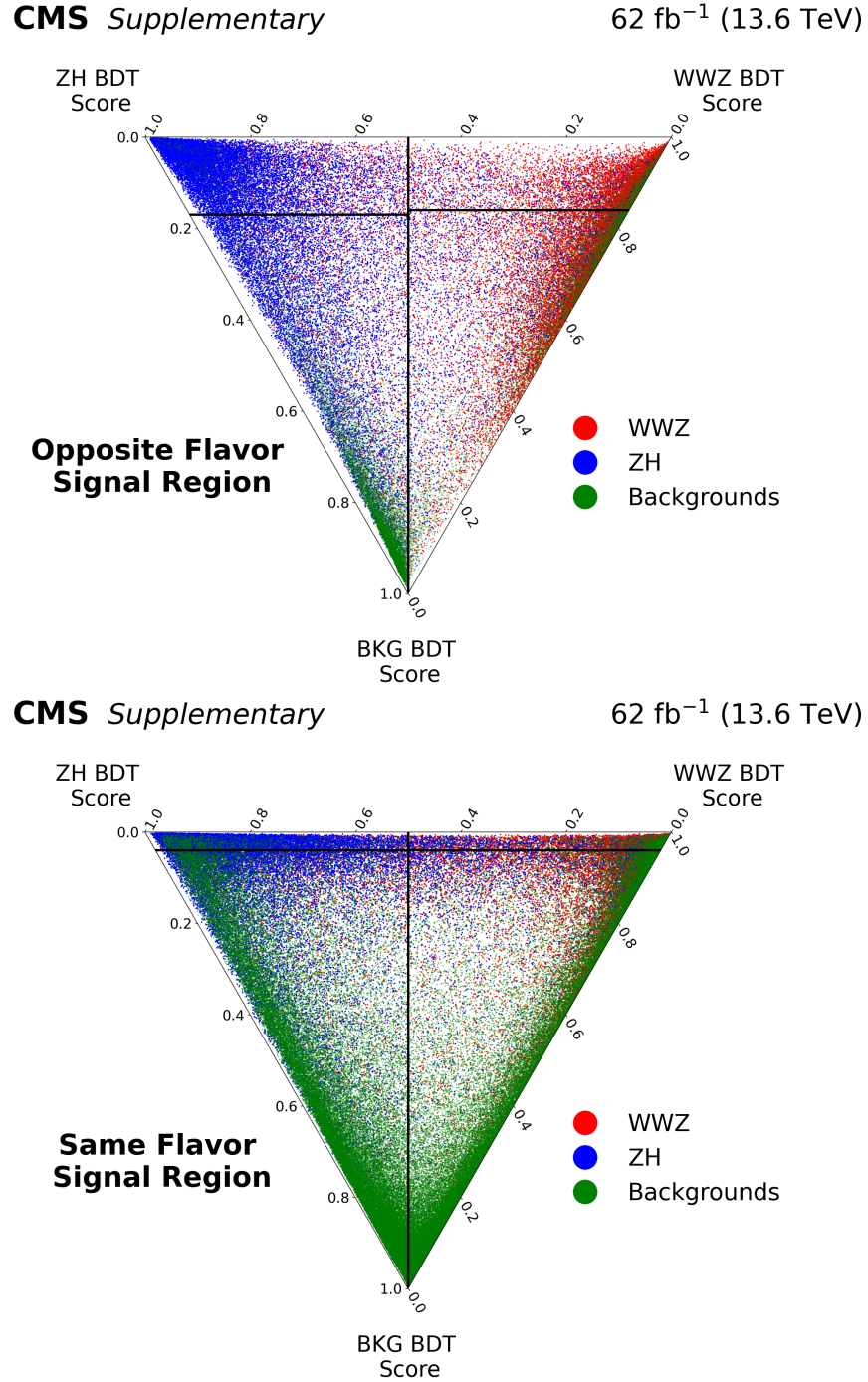


Figure 6.12: Ternary plot showing MC events for WWZ (red), ZH (blue), and background (green) in the OF channel (top) and SF channel (bottom) for Run 3. Bin boundaries defined in Table 6.4 are shown in black.

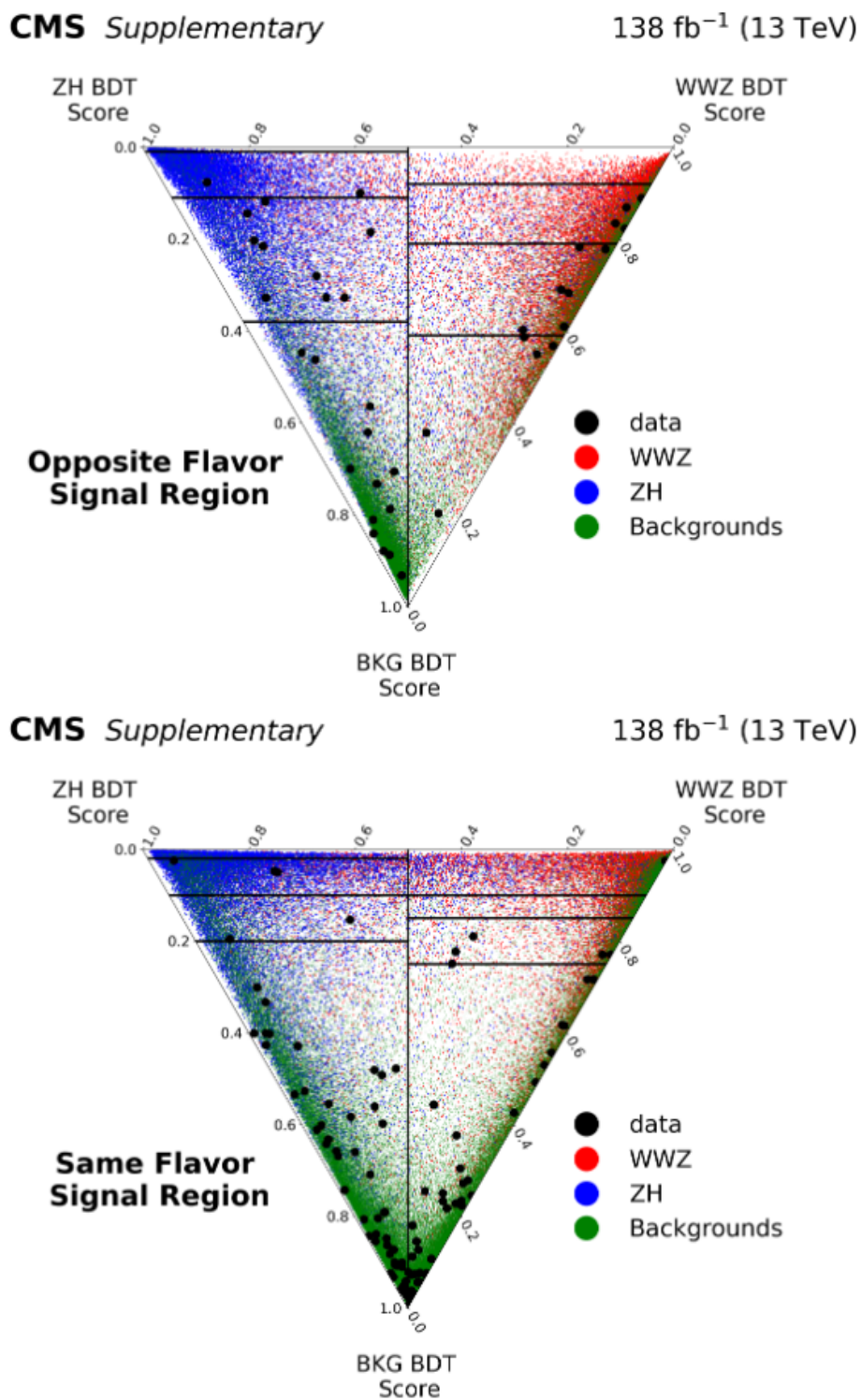


Figure 6.13: Ternary plot showing MC events for WWZ (red), ZH (blue), background (green), and data (black dots) in the OF channel (top) and SF channel (bottom) for Run 2. Bin boundaries defined in Table 6.3 are shown in black.

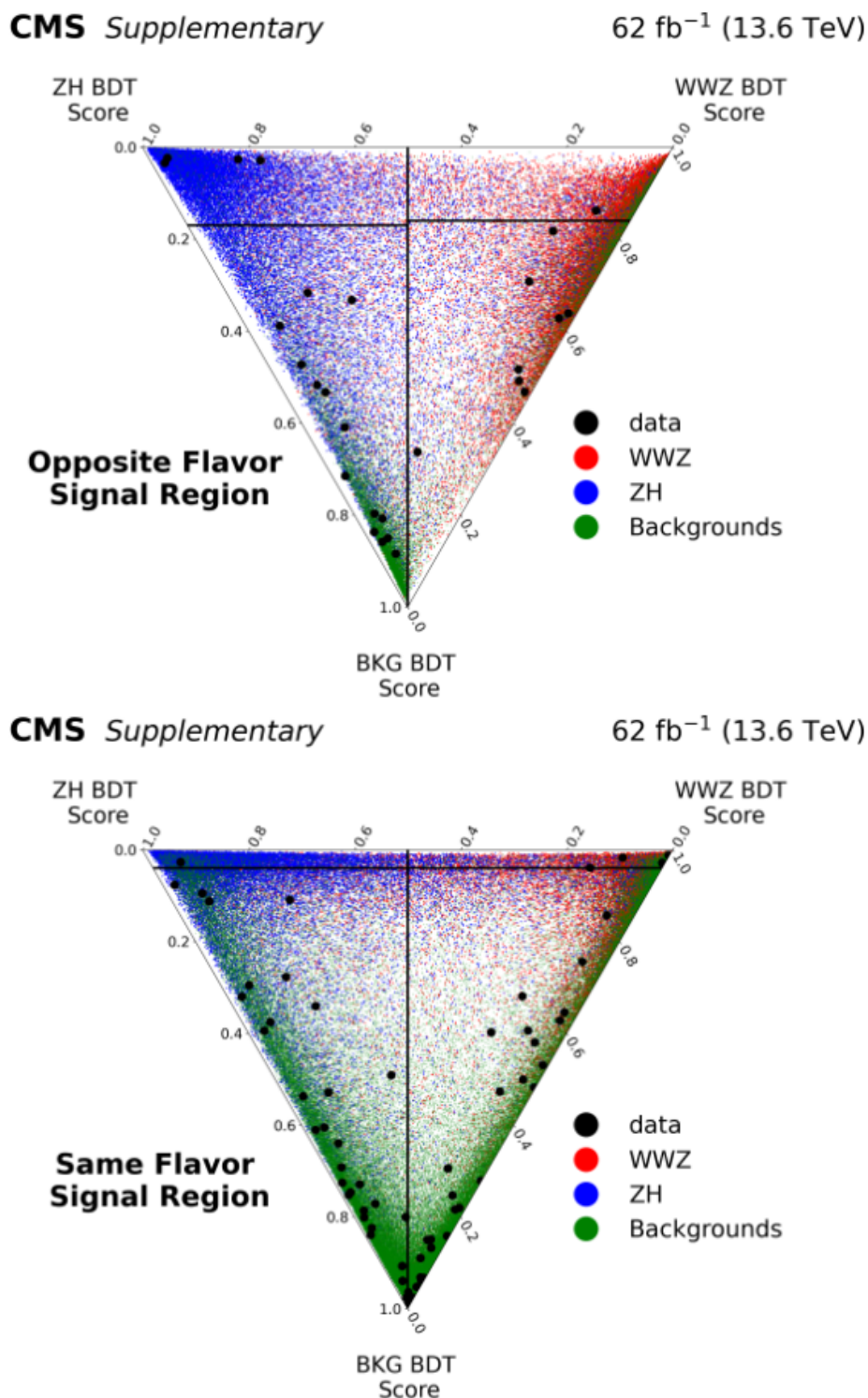


Figure 6.14: Ternary plot showing MC events for WWZ (red), ZH (blue), background (green), and data (black dots) in the OF channel (top) and SF channel (bottom) for Run 3. Bin boundaries defined in Table 6.4 are shown in black.

CHAPTER 7

BACKGROUNDS

7.1 Backgrounds

There are multiple background processes that enter the 4-lepton final state. Each background process can be classified either as *prompt* or *non-prompt*. The prompt backgrounds are those that produce 4 real leptons (e , μ , or τ that decays leptonically) in the hard scattering process. The dominant prompt backgrounds are the ZZ , $t\bar{t}Z$, and tWZ processes in which all decays are to leptons. The non-prompt backgrounds are those that produce fewer than 4 real leptons in the hard scattering process, but enter the 4-lepton final state when an object that is not a true prompt lepton (e.g. a jet) is incorrectly identified as a lepton. These incorrectly identified objects are commonly referred to as "fake" leptons. The dominant non-prompt background arises from the WZ process, in cases where WZ decays to 3 leptons with one additional fake lepton.

The dominant backgrounds are ZZ and $t\bar{t}Z$. The tWZ process represents a sub-leading but non-negligible contribution that has a similar topology to the $t\bar{t}Z$ process. Several *control regions* (CRs) are designed to study these dominant backgrounds. When performing the measurements of the signal strengths and statistical significance, the control regions are fit to data simultaneously with the signal regions; the background normalizations are taken as a parameter in the fit, which helps to properly estimate the background contributions in the signal regions. The details of this data-driven background estimation are presented in Sec. 7.1.1 for the ZZ background and Sec. 7.1.2 for the $t\bar{t}Z$ and tWZ backgrounds.

Smaller contributions to the overall background include the non-prompt background from WZ , same- and opposite-charge WW production, other tri-boson processes (WWW , WZZ , ZZZ), top quark production in association with a W or H boson, WH , single Higgs production (with $H \rightarrow ZZ$), $t\bar{t}$, single top quark production, and $V + \text{jets}$ ($V = W$ or Z). These smaller backgrounds are estimated di-

rectly from MC with cross section uncertainties applied as described in Section 9.5.2. The non-prompt WZ background is described in Section 7.1.3. Additional checks to ensure proper modeling of the simulation are also performed on the single Higgs background, as described in Section 7.1.4.

The contributions of the signal and background processes in the various CRs defined in the following sections are summarized in Table 7.1. Additionally, the cross section times branching ratio to 4-leptons for the major background processes are shown in Table 7.2 for both center-of-mass energies considered in this analysis.

7.1.1 The ZZ background

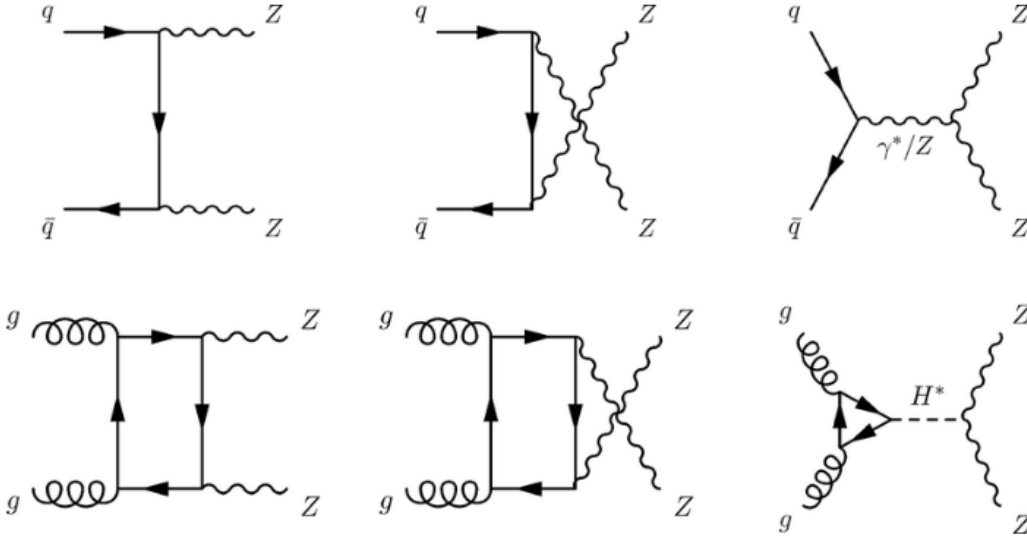


Figure 7.1: Lowest-order Feynman diagrams for quark-initiated (top) and gluon-initiated (bottom) ZZ production. Figure taken from Ref. [111].

With a cross section times branching ratio to 4 leptons of 86 fb at $\sqrt{s} = 13$ TeV, the dominant background contribution is the ZZ production process. The ZZ background includes contributions from the quark-initiated process, shown in the top Feynman diagrams in Fig. 7.1, and the gluon-initiated process, as shown in the bottom Feynman diagrams in Fig. 7.1. This background can be reduced significantly

Era	Control Region	WWZ	ZH	ZZ	ttZ	tWZ	WZ	other	Bkg
Run 2	tXZ OF CR	2.88 ± 0.01	1.99 ± 0.01	4.93 ± 0.06	56.81 ± 0.44	9.22 ± 0.05	0.99 ± 0.26	2.62 ± 0.27	74.57 ± 0.58
	tXZ SF CR	1.45 ± 0.01	0.72 ± 0.01	3.6 ± 0.06	28.03 ± 0.31	4.29 ± 0.03	0.12 ± 0.1	1.72 ± 0.23	37.76 ± 0.4
	ZZ CR	0.83 ± 0.0	0.05 ± 0.0	1495.77 ± 0.89	0.43 ± 0.04	0.17 ± 0.01	0.11 ± 0.09	8.83 ± 0.57	1505.3 ± 1.07
Run 3	tXZ OF CR	0.75 ± 0.01	0.49 ± 0.01	1.48 ± 0.04	18.66 ± 0.39	3.53 ± 0.03	0.16 ± 0.07	0.6 ± 0.35	24.44 ± 0.53
	tXZ SF CR	0.38 ± 0.0	0.18 ± 0.01	1.03 ± 0.03	8.79 ± 0.28	1.64 ± 0.02	0.04 ± 0.03	0.71 ± 0.16	12.21 ± 0.33
	ZZ CR	0.42 ± 0.0	0.03 ± 0.0	800.71 ± 0.84	0.22 ± 0.04	0.13 ± 0.01	0.07 ± 0.03	4.45 ± 0.39	805.58 ± 0.93

Table 7.1: Yields in control regions for Run 2 and Run 3 portions of the analysis. Uncertainties shown are statistical only.

Era	Process	σ_{theo} (fb)	$\mathcal{BR}(4\ell)(\%)$	$\sigma_{\text{theo}} \times \mathcal{BR}(4\ell)$ (fb)
Run 2 ($\sqrt{s} = 13$ TeV)	ZZ	$16900_{-500}^{+600} \pm 200$ [91]	0.51	86
	$t\bar{t}Z$	859_{-84}^{+76} [107]	0.46	4.0
	tWZ	136_{-8}^{+9} [76, 77]	0.46	0.63
	WZ	$42500_{-1400}^{+1600} \pm 600$ [79–82]	0.035	15
Run 3 ($\sqrt{s} = 13.6$ TeV)	ZZ	17000_{-1400}^{+1900} [108–110]	0.51	87
	$t\bar{t}Z$	935_{-91}^{+83}	0.46	4.3
	tWZ	148_{-9}^{+10} [76, 86]	0.46	0.69
	WZ	$50500_{-2100}^{+2600} \pm 1100$ [79–82]	0.035	18

Table 7.2: Cross sections times branching ratios to 4 leptons for the major background contributions. The WZ contribution assumes a fake rate of 1% for objects faking an additional lepton.

by noting that the Z boson decays leptonically to an opposite sign, same-flavor pair of leptons:

$$Z \rightarrow e^+ e^-, \quad Z \rightarrow \mu^+ \mu^-, \quad Z \rightarrow \tau^+ \tau^-$$

In the case where the Z decays to electrons or muons, the ZZ background will produce the following lepton multiplicities in the final state:

$$ZZ \rightarrow 4e, \quad ZZ \rightarrow 4\mu, \quad ZZ \rightarrow 2e2\mu$$

Crucially, there is also no real p_T^{miss} in this final state, which provides a means of reducing the background. In the case where the Z decays to τ leptons, the ZZ background can exhibit the $4e$, 4μ , and $2e2\mu$ final state shown above, or it can produce different final states such as:

$$ZZ \rightarrow 4\tau \rightarrow 3e1\mu, \quad ZZ \rightarrow 4\tau \rightarrow 1e3\mu, \quad ZZ \rightarrow 2e2\tau \rightarrow 3e1\mu, \quad ZZ \rightarrow 2\mu2\tau \rightarrow 1e3\mu$$

However, the τ decays of the Z are suppressed in the 4-lepton final state relative to the $Z \rightarrow ee$ and $Z \rightarrow \mu\mu$ decays due to the additional branching ratio for $\tau \rightarrow e\nu_e$ (17.8% [21]) and $\tau \rightarrow \mu\nu_\mu$ (17.4% [21]). This final state does produce some real p_T^{miss} through the presence of neutrinos.

The lower relative contribution of ZZ to the $3e1\mu$ and $1e3\mu$ final state is the primary motivation for the choice to split events into OF and SF channels (see Chapter 6) as ZZ is greatly reduced in the OF channel. To study the ZZ background, we define a ZZ CR orthogonal to the training region defined in Sec. 6.3. For this, we require events with 4 leptons that are consistent with originating from two Z bosons; we perform the same selection as is described in the SF preselection (see Sec. 6.2.2), but with an inverted requirement on $m_{\ell\ell}^{\text{Wcands}}$:

$$|m_{\ell\ell}^{\text{Wcands}} - m_Z| < 10 \text{ GeV}$$

In the ZZ CR, we find the ZZ process to comprise nearly 100% of all simulated events for both Run 2 and Run 3 portions of the analysis. The yields of various processes in the ZZ CR are summarized in Table 7.1. The agreement between data and simulation is checked in the ZZ CR by looking at various distributions of key kinematic variables:

- Invariant mass of Z -candidate ($m_{\ell\ell}^{\text{Zcands}}$) and W -candidate ($m_{\ell\ell}^{\text{Wcands}}$) leptons: both expected to exhibit a peak near the Z mass due to the on-shell requirement. Z -candidate mass is shown in Fig. 7.2 (7.3) and W -candidate mass is shown in Fig. 7.2 (7.3) for Run 2 (Run 3).

- Four lepton invariant mass ($m_{4\ell}$): expected to have a peak at 2 times the Z mass. Plot shown in Fig. 7.2 (7.3) for Run 2 (Run 3).
- p_T^{miss} : expected to be small since dominant contribution ($Z \rightarrow ee$ or $\mu\mu$) has no real p_T^{miss} . Some real p_T^{miss} comes from $Z \rightarrow \tau\tau$ decays. Shown in Fig. 7.2 (7.3) for Run 2 (Run 3).

Additional distributions showing the modeling of other variables in the ZZ CR are displayed in Sec. E.1.1. The distributions show good agreement between the data and the simulation.

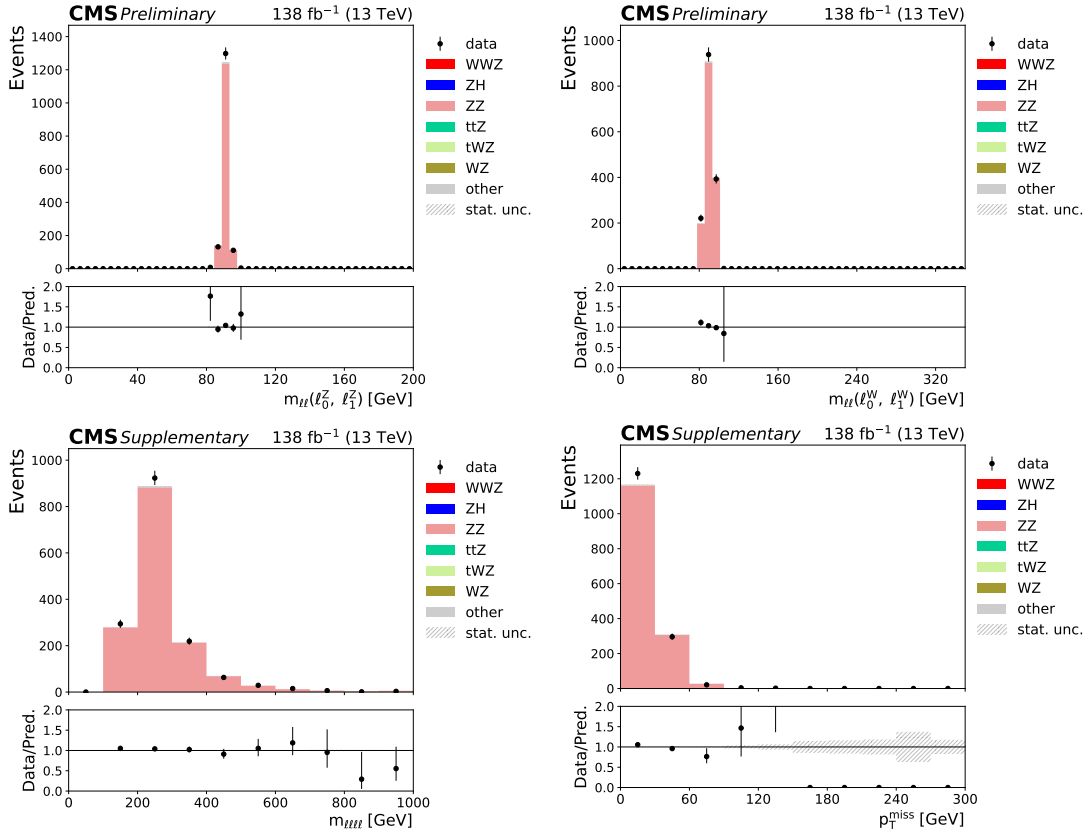


Figure 7.2: Distributions showing the invariant mass of the Z -candidate leptons (top left) and W -candidate leptons (top right), the 4-lepton invariant mass (bottom left) and p_T^{miss} (bottom right) in the ZZ CR for Run 2.

Within the fits performed for the extraction of results, the ZZ CR is included (as a single bin for Run 2 and Run 3) whose normalization is allowed to float freely in

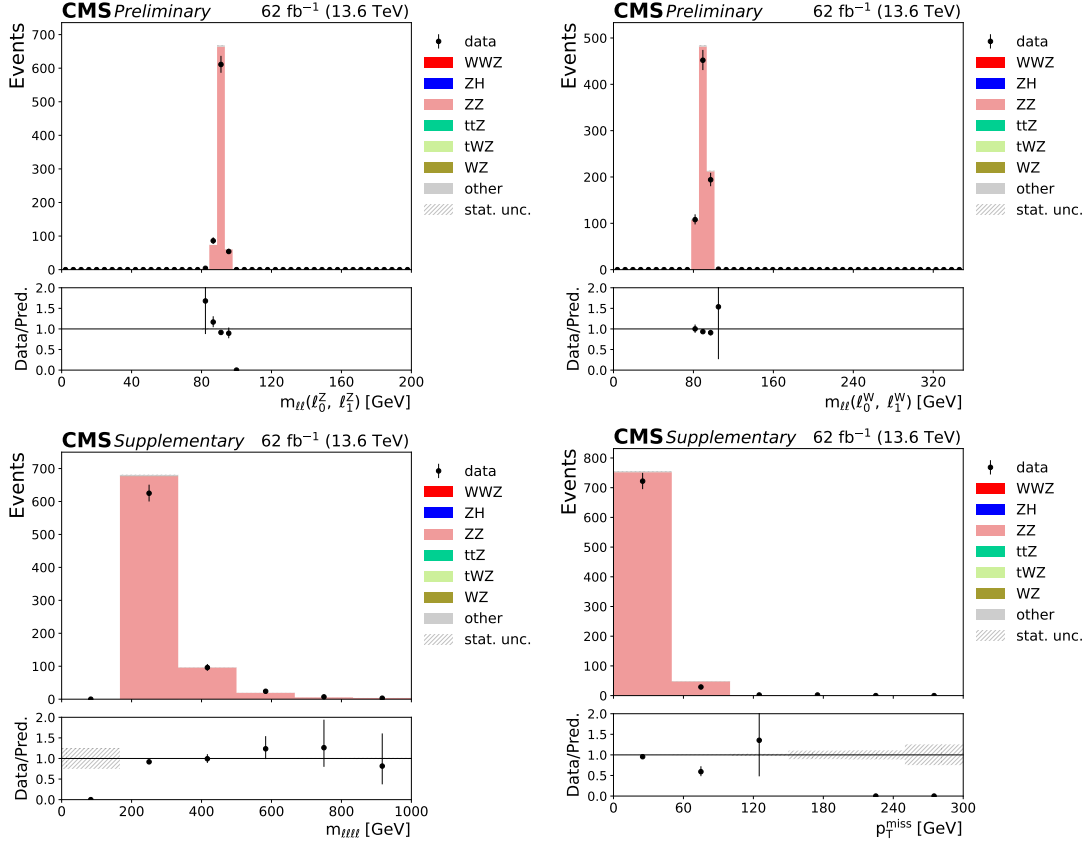


Figure 7.3: Distributions showing the invariant mass of the Z -candidate leptons (top left) and W -candidate leptons (top right), the 4-lepton invariant mass (bottom left) and p_T^{miss} (bottom right) in the ZZ CR for Run 3.

the fit as a nuisance parameter. The fit parameter κ_{ZZ} is expressed as the ratio of the post-fit ZZ normalization to the pre-fit ZZ normalization (across all bins):

$$\kappa_{ZZ} = \frac{N_{ZZ,\text{post-fit}}}{N_{ZZ,\text{pre-fit}}} \quad (7.1)$$

Due to the high statistics and purity, the ZZ CR is the main contributor to the determination of κ_{ZZ} (with lesser contributions from the other bins in the fit). The post-fit values of κ_{ZZ} are summarized in Table 10.3 and shown in the nuisance parameter impact plots presented in Section F.1.

7.1.2 The $t\bar{t}Z$ and tWZ backgrounds

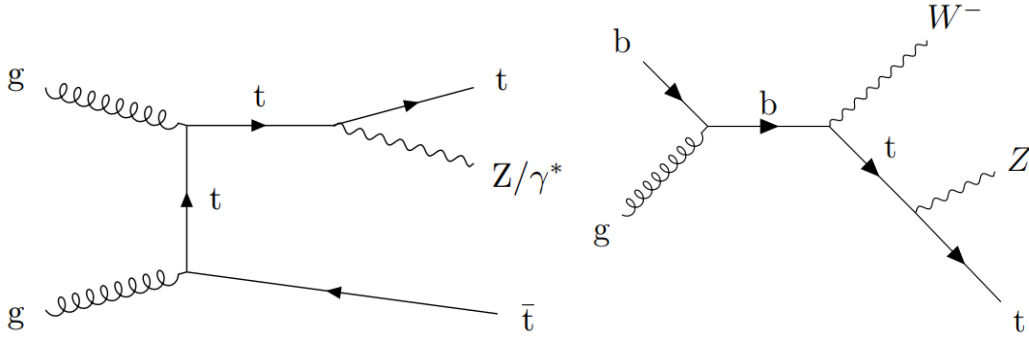


Figure 7.4: Lowest Order Feynman diagrams for $t\bar{t}Z$ production (left) and tWZ production (right). The diagram for $t\bar{t}Z$ is taken from Ref. [112], and the diagram for tWZ is taken from Ref. [113].

The next largest contributions to the 4-lepton final state are the $t\bar{t}Z$ and tWZ processes. Leading order Feynman diagrams for these processes are shown in Figure 7.4. The cross sections times branching ratios for each process are shown in Table 7.2 for 13 TeV and 13.6 TeV center-of-mass energy. Unlike the ZZ process, the $t\bar{t}Z$ and tWZ processes have significant p_T^{miss} arising from the leptonic decays of the W bosons. However, the presence of b-tagged jets from top-quark decays provides a useful means of reducing the overall $t\bar{t}Z$ and tWZ contributions. The $t\bar{t}Z$

and tWZ backgrounds are studied in dedicated control regions (tXZ CRs) in both the OF channel and the SF channel. Both backgrounds are studied together as they exhibit very similar characteristics in nearly all kinematic variables, however the $t\bar{t}Z$ contribution is dominant (see e.g., Fig 7.7).

The tXZ SF CR is constructed by first taking events passing the SF preselection (defined in Sec. 6.2.2) and inverting the requirement on the number of Candidate b-tagged jets (defined in Sec. 5.3):

$$N_{btags} \geq 1$$

To reduce the contribution from ZZ events, we also require the presence of large p_T^{miss} ($p_T^{miss} > 80$ GeV). The contributions from the different signal and background processes in the tXZ SF CR are shown in Table 7.1. The agreement between data and simulation is checked in a number of key distributions, including the number of b-tagged jets, p_T^{miss} , and jet multiplicity (see Figures 7.5 and 7.6). Additional distributions showing the data to MC agreement in other variables are presented in Section E.1.3. From these, we conclude that there is good modeling of the data in simulation for both Run 2 and Run 3.

In the OF channel, the tXZ CR is formed by taking events passing the OF preselection (see Sec. 6.2.1) with the same inverted b-tagged requirement as was used for the tXZ SF CR. The distributions of key variables in the tXZ OF CR are shown in Figures 7.7 (7.8) for Run 2 (Run 3); other variables are shown in Section E.1.2. Good agreement is seen between data and simulation in the tXZ OF CR.

In each channel, 2 bins are utilized in the fit (described in detail in Section 9.4) based on the number of b-tagged jets: $N_{btags} = 1$ and $N_{btags} > 1$. Within the fit, the normalization of the $t\bar{t}Z$ plus tWZ contribution is treated as a floating nuisance parameter, and the relative contributions of each process in each bin are taken from

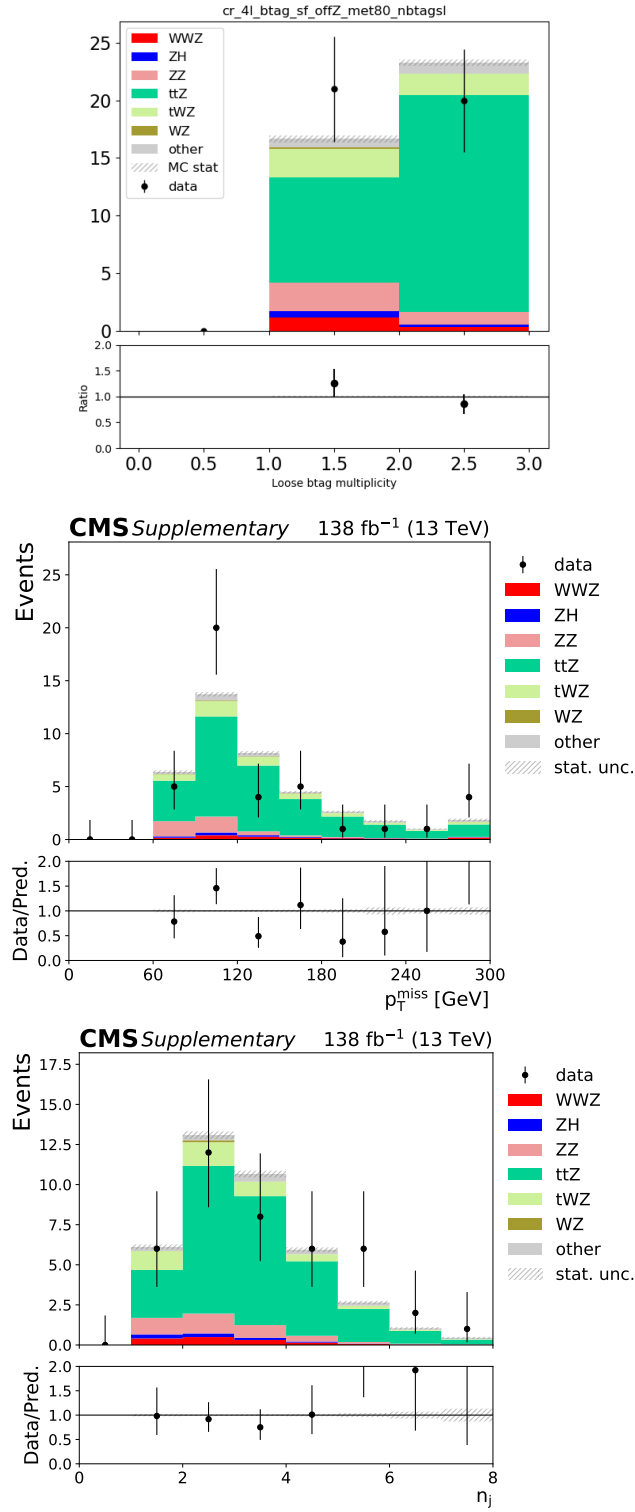


Figure 7.5: Distributions showing the number of b-tagged jets (top), p_T^{miss} (middle), and the number of jets (bottom) in the tXZ SF CR for Run 2.

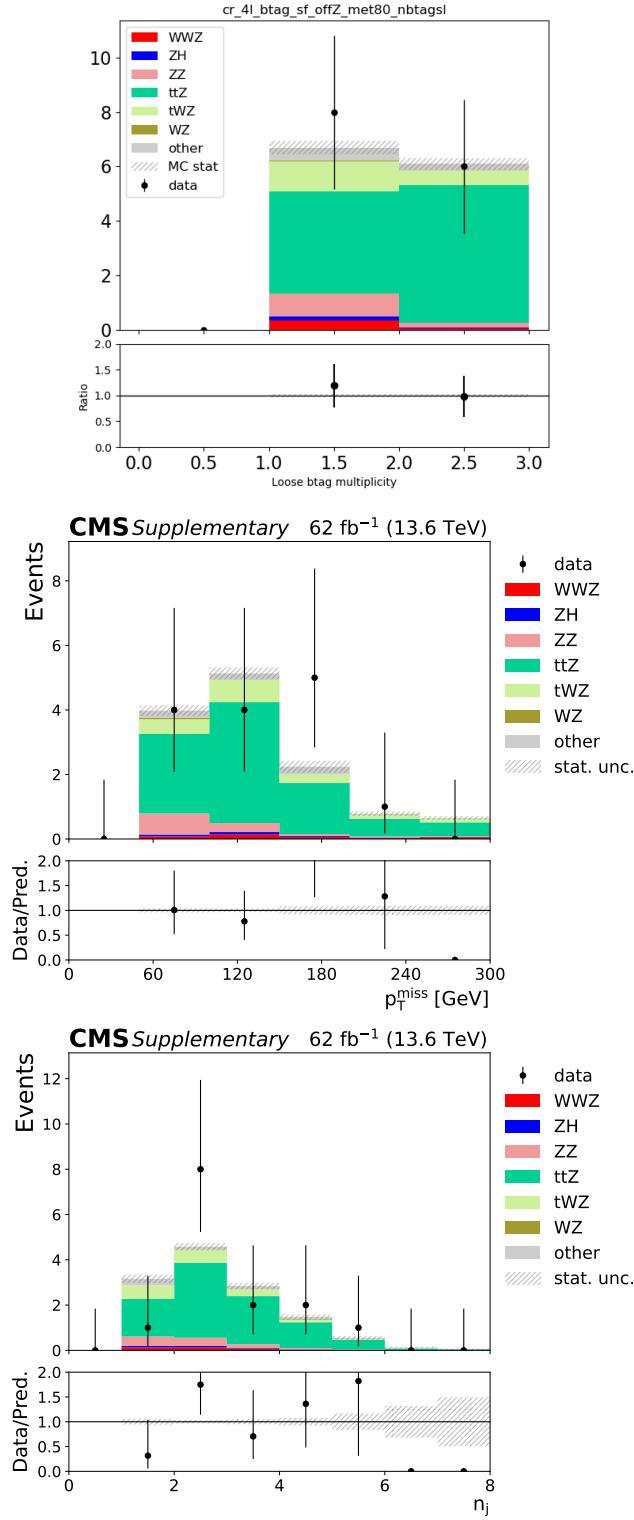


Figure 7.6: Distributions showing the number of b-tagged jets (top), p_T^{miss} (middle), and the number of jets (bottom) in the tXZ SF CR for Run 3.

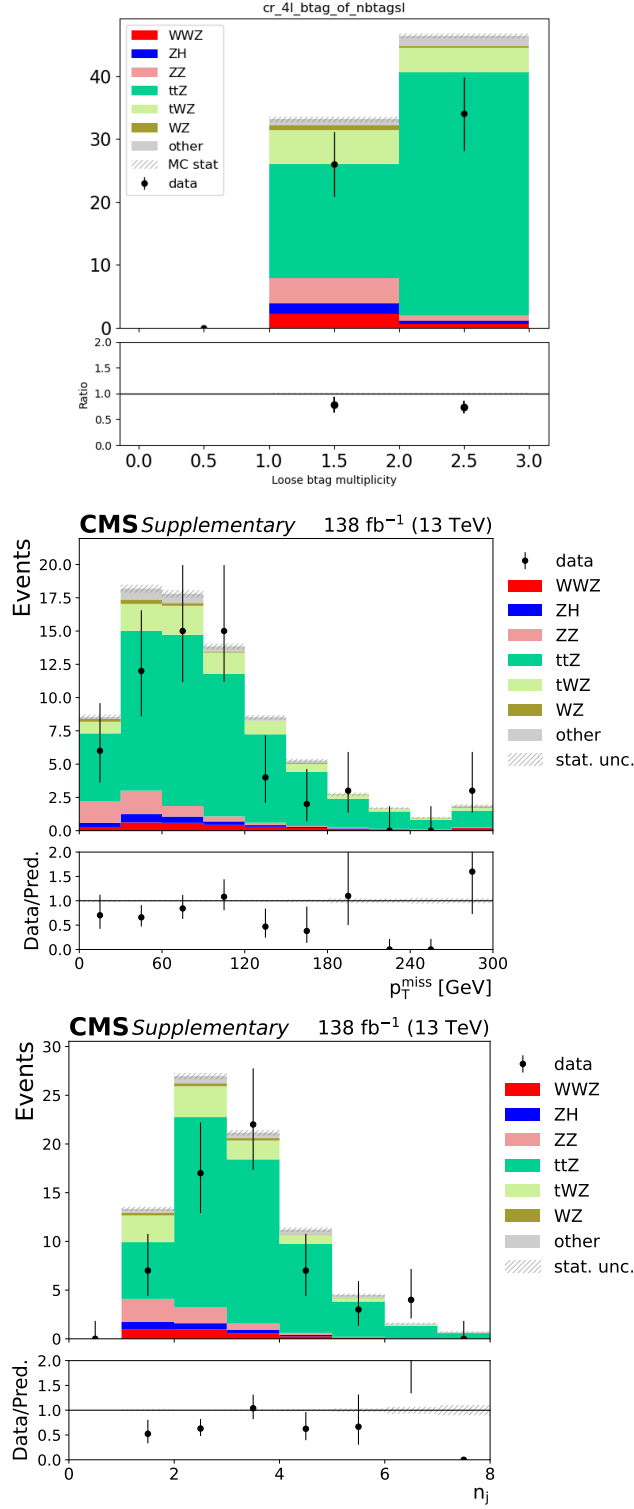


Figure 7.7: Distributions showing the number of b-tagged jets (top), p_T^{miss} (middle), and the number of jets (bottom) in the tXZ OF CR for Run 2.

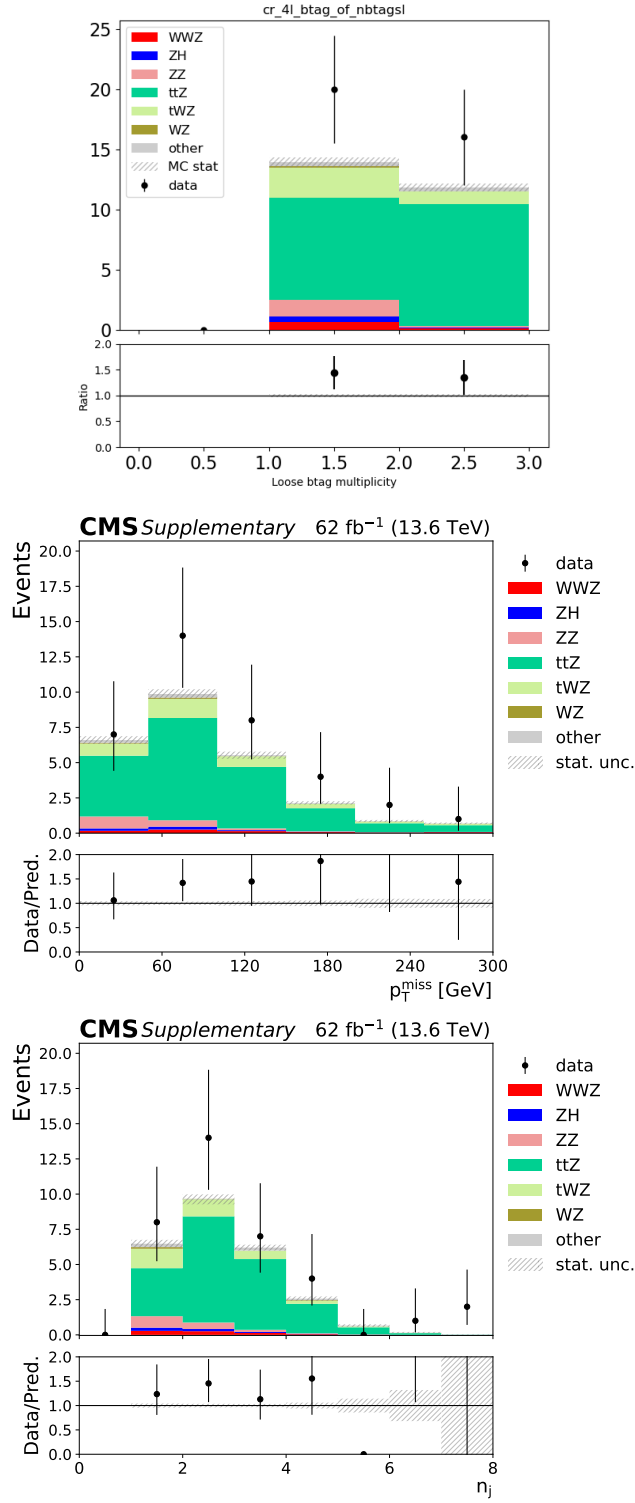


Figure 7.8: Distributions showing the number of b-tagged jets (top), p_T^{miss} (middle), and the number of jets (bottom) in the tXZ OF CR for Run 3.

simulation. The post-fit normalizations are summarized in Table 10.3 and shown in the nuisance parameter impact plots displayed in Section F.1.

7.1.3 The WZ +fake background

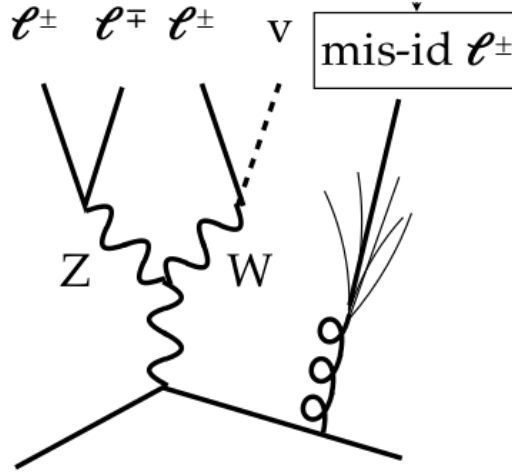


Figure 7.9: Diagram of the WZ process with an additional fake lepton.

The WZ process is expected to enter into the 4-lepton final state through leptonic decays of the W and Z bosons with an additional fake lepton (see Figure 7.9) due to misidentification of a different object (e.g. a jet). Due to the large cross section times branching ratio relative to the signal processes, even a small fake rate (i.e. the probability of a fake lepton passing the selection requirements) can result in a non-negligible contribution from WZ events.

The expected contribution from the non-prompt WZ + fake background is small compared to the main backgrounds (ZZ , $t\bar{t}Z$, and tWZ), making up approximately 2% of the total background in the training region (see Table 8.4). Because of this, the WZ + fake background is estimated directly from simulation with a 30% uncertainty applied to the overall WZ normalization.

7.1.4 Higgs background

The contribution from Higgs-related processes to the overall background in the signal regions is expected to be small. Because of this, we estimate the Higgs background from simulation. The main processes involving the Higgs that enter the 4-lepton final state are ZH events where $H \rightarrow \tau\tau$, and $gg \rightarrow H \rightarrow ZZ \rightarrow 4\ell$. For the former process, isolating the $ZH \rightarrow \ell\ell + \tau\tau$ events is rather difficult due to the prompt τ decays to electrons or muons, which are nearly indistinguishable from ZH signal events. For the latter process, we check the modeling of the simulated background in a region around the Higgs mass.

The $gg \rightarrow H \rightarrow ZZ$ process can be selected with high purity by noting that one of the Z bosons is off-shell, which reduces the overlap with the dominant ZZ background. This enables us to define a Higgs validation region to check the modeling of this process. The Higgs validation region consists of events with two pairs of same-flavor, opposite-charge leptons with one pair of leptons having an invariant mass within 10 GeV of the mass of the Z boson (representing the on-shell Z boson) and the invariant mass of the 4-lepton system to be within 10 GeV of the Higgs mass. Figure 7.10 (7.11) shows the the distributions of the invariant masses for each Z boson as well as for the 4-lepton system in Run 2 (Run 3). These plots show good agreement between data and simulation, which is sufficient to validate the modeling of the $gg \rightarrow H \rightarrow ZZ$ background process.

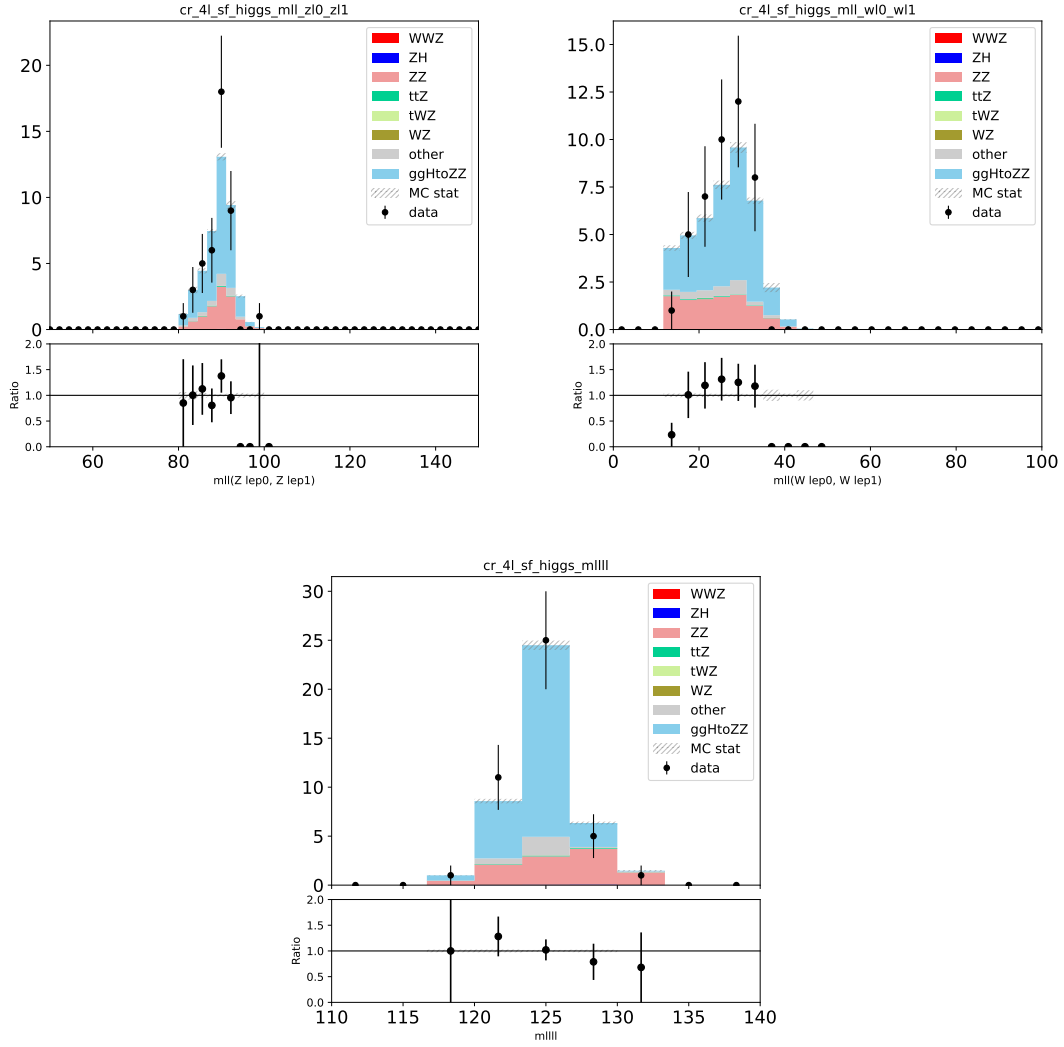


Figure 7.10: Distributions showing the invariant mass of the on-shell (top left) and off-shell (top right) Z boson and of the 4-lepton system (bottom) in the Higgs validation region for Run 2.

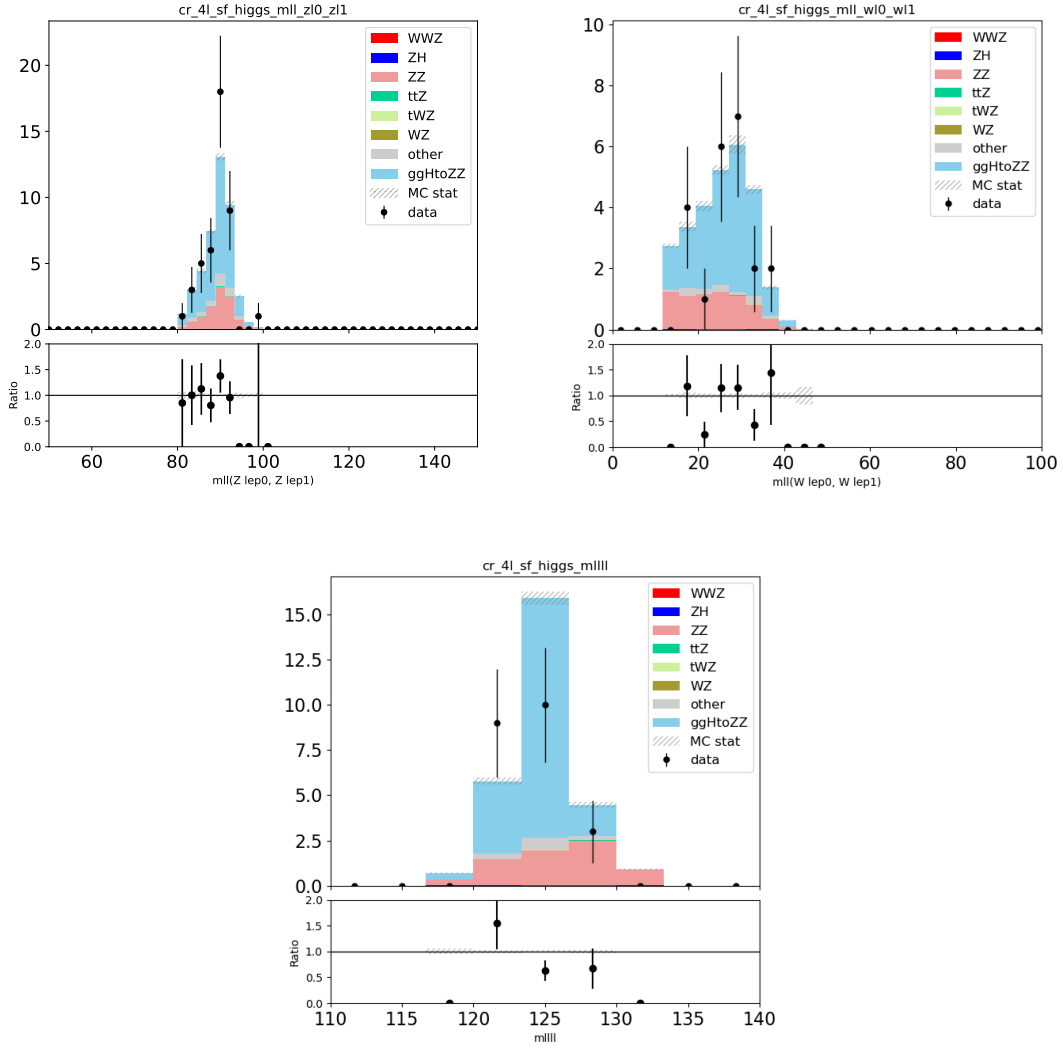


Figure 7.11: Distributions showing the invariant mass of the on-shell (top left) and off-shell (top right) Z boson and of the 4-lepton system (bottom) in the Higgs validation region for Run 3.

CHAPTER 8

MACHINE LEARNING METHODS

"Before I came here, I was confused
about this subject. Having listened to
your lecture, I am still confused – but
on a higher level"

Enrico Fermi

8.1 Overview

To perform measurements of processes of interest in a HEP analysis, one must effectively separate these processes from the sources of background. In essence, this boils down to a simple classification task. Traditionally, this was done by the use of rectangular "cuts" on a number of variables that show good discrimination power between the signal and background processes. An example is illustrated in Fig. 8.1 for an arbitrary variable V : the background exhibits a falling distribution while the signal peaks around $V=2$ and exhibits a long tail. To maintain a reasonable signal efficiency while rejecting a large portion of the background, one could reject events with V greater than, say, 4 to 6. Ultimately, this is not an exact science, and the choice of the cut value is subject to a number of considerations regarding the nature of the measurement itself.

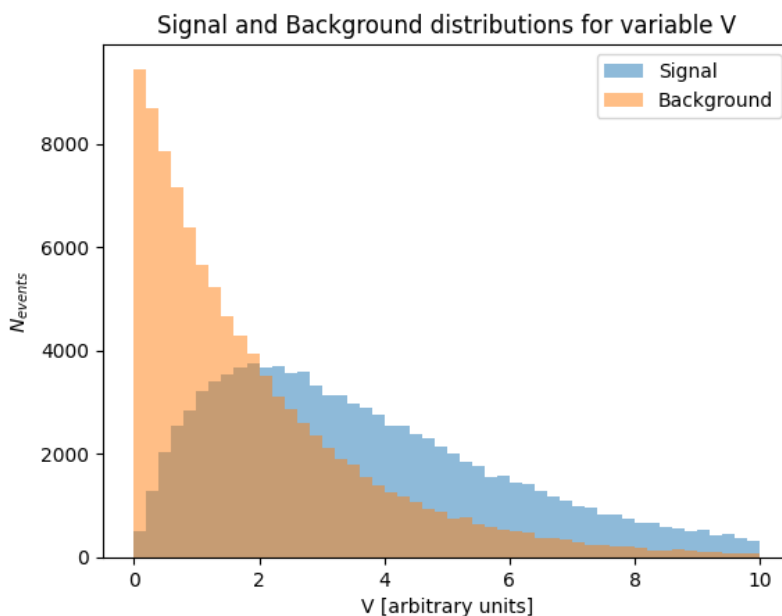


Figure 8.1: Example signal and background distributions for an arbitrary variable V .

This procedure for separating the signal and background processes is employed, to some degree, in all HEP analyses (the event selection described in Sec. 6.2 is comprised

entirely of rectangular cuts). However, the push to search for rarer processes among larger backgrounds has motivated the use of machine-learning (ML) techniques for classification and regression tasks. In this approach, variables showing (ideally, good) separation between signal and background are fed into a network (e.g. a boosted decision tree or neural network) that performs a series of computations to attempt to classify each event as "signal-like" or "background-like" through a single value (a "score"). Most modern analyses (including this one) employ a mixture of rectangular cuts and ML methods; the rectangular cuts are used to "pre-select" signal and background events that can be used for the training and evaluation of the ML method.

The use of ML techniques in HEP should be regarded with a fair level of skepticism for a few reasons:

- Loss in interpretation: the output "score" of a ML method has no physical meaning, and thus is not grounded in any solid logical foundation.
- Reliance on the ML methods to "do the thinking" rather than making careful decisions to help the performance of the ML method. This includes the "kitchen sink" method of throwing any variable into the ML training without thinking about whether the variable is well-modeled or even relevant for the separation task.

Thus, to establish legitimacy in ML techniques for a given HEP analyses, one must perform a sufficient validation of the input variables used in the training and the training itself.

8.2 Boosted Decision Trees

A decision tree is a supervised learning algorithm describing a flow of logic between connected nodes in a hierarchical structure. In supervised learning, labels for the data are supplied to the algorithm (e.g. "signal" and "background") and, through the training process, the model is able to learn the features of each type of class from the set of input variables associated with the data. New data is then fed into the model, which is able to classify the data into a particular category based on the supplied labels, or assigns it a score, which determines the compatibility of the data with a category. Typically, one starts with a set of input data, which is split among nodes in an initial layer, a *root node*, based on a logical condition. The data moves through successive layers of nodes, or *internal nodes*, based on whether further logical conditions are met or not met. These connections between layers, called "branches", are determined via the process of training. In training, nodes are split into different branches according to a splitting criterion, which represents the ability for a node to perform effective classification. There are a number of different metrics that can be used, such as the entropy information gain, the Gini Impurity, and various others described in ref. [114]. In the context of a HEP task where the goal is the separation of signal and background, a useful figure of merit for node splitting is:

$$\frac{S}{\sqrt{S+B}},$$

where S is the number of weighted signal events and B is the number of weighted background events. For complex classification tasks, a decision tree is typically comprised of many layers of internal nodes, which terminate in external (or *leaf*) nodes. The leaf nodes signify the different possible classifications of the dataset.

The training of a decision tree can be made more robust against fluctuations and errors by a technique known as "boosting". In boosting, a small subset of the data is

taken to train a small decision tree, and the performance of the model is evaluated. A series of weights are applied to the model to correct issues in the previous training iteration, which can direct the model to focus on different input parameters. Once the model is updated, it runs another training iteration with a small subset of the data, and the weights can be adjusted again. This process of sequential learning is repeated multiple times. Boosting, applied to a decision tree, is referred to as a *boosted decision tree* (BDT). The performance of a BDT is characterized by a receiver operating characteristic (ROC) curve; the classifier's true positive rate (e.g. the probability of classifying a data point according to its true category) is plotted against its false positive rate (e.g. the probability of classifying a data point incorrectly). An example of ROC curves for different classifiers are shown in Figure 8.2. Typically, the ROC curve with a higher true positive rate per false positive rate has the best sensitivity, and thus the best performance.

In circumstances where there are more than two distinct output classes for a BDT (e.g. signal 1, signal 2, and background), an extension of the BDT classification algorithm, a multiclassifier, can be utilized. In this scheme, each class is trained against the remaining classes, resulting in N different scores for N different classes. There are $N(N - 1)$ comparisons made (and thus $N(N - 1)$ ROC curves) during the training, corresponding to the number of unique pairs of classes.

Prior to training, the user specifies a set of *hyperparameters* that dictate the training configuration. Typical hyperparameters include: the depth of the decision tree (number of layers), the number of trees used in the boosting, and the minimum amount of data used at a given node. The hyperparameters can have a strong impact on the effectiveness of the training, and typically optimization of the hyperparameters, or tuning, is required to determine the best performance. Tuning is also important to prevent *overtraining*, whereby the model learns the training dataset too well. Overtraining can result in the model becoming prone to learning statistical

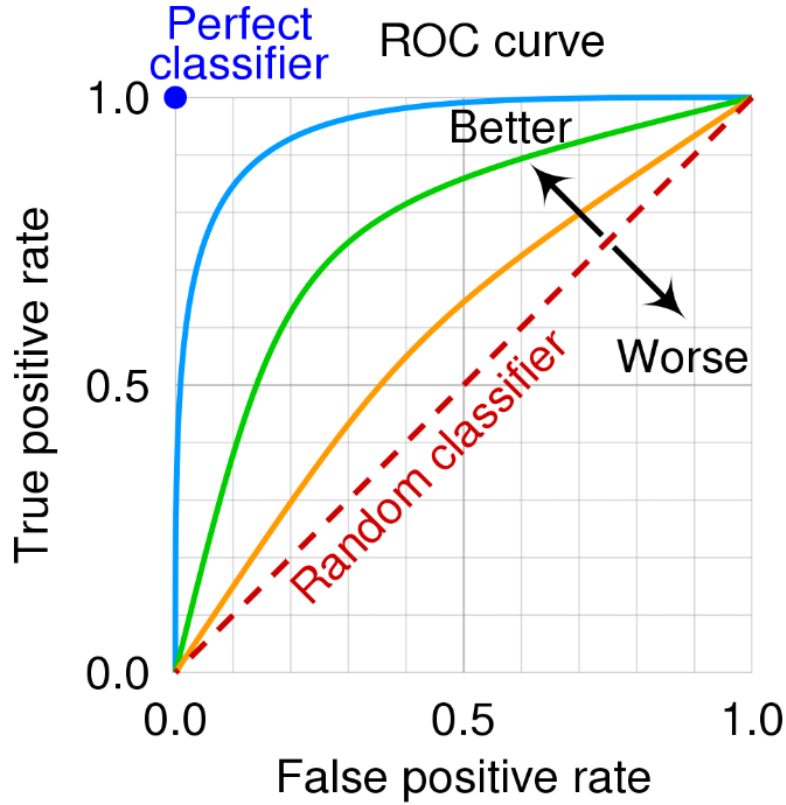


Figure 8.2: Diagram illustrating example ROC curves for different classifiers.

fluctuations in the data, thus eroding its ability to generalize and thus characterize different data. The number of tunable parameters within the model and the size of the training dataset both have a strong impact on whether the model itself is prone to overtraining; if there are more parameters than data, there is a high likelihood that the model will exhibit overtraining.

8.3 Training Details

To separate the signal processes (WWZ , ZH) from each other and from the sum of backgrounds, a BDT multiclassifier is utilized in the opposite-flavor and same-flavor channels. The multiclassifier trains each process (i.e. WWZ , ZH , Backgrounds) against the other two processes, resulting in three different BDT scores assigned to

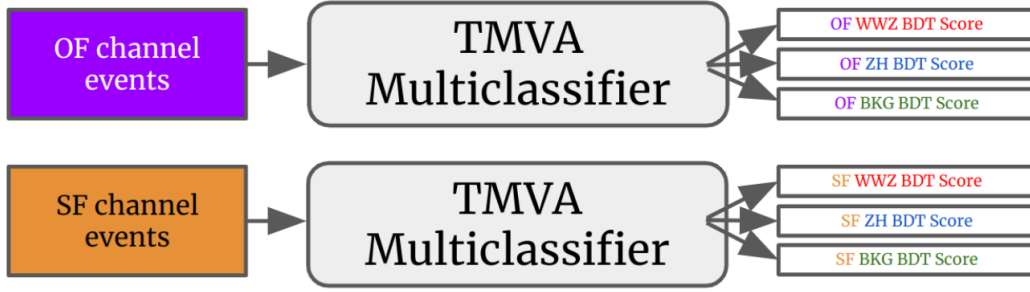


Figure 8.3: Diagram summarizing the BDT multiclass training and outputs.

each event (see Figure 8.3); a WWZ score (S_{WWZ}), a ZH score (S_{ZH}), and a Background score (S_{BKG}). The scores are transformed using a *softmax transformation* that imposes the sum of scores must be equal to one:

$$S_{WWZ} + S_{ZH} + S_{BKG} = 1 \quad (8.1)$$

This transformation allows us to visualize the distribution of BDT scores for events using a "ternary" or "triangle" plot (see Fig. 6.11).

Events passing the training region (TR) selection requirements (see section 6.3) are used for the training, validation, and evaluation of the BDTs. Table 8.1 shows the list of input variables for the SF and OF BDT training, and the distributions of these variables are shown in figures 8.6-8.13 (normalized to unity to show shape differences). The input variable distributions with all processes included and data overlaid, for both Run 2 and Run 3 are shown in Figures D.1-D.20. Many of these variables were chosen to exploit the kinematics of the WW system as well as the p_T^{miss} in order to better discriminate against the dominant ZZ background and separate the signal processes from each other. More details on the input variables are presented in Section 8.4. Jet variables were also included as a means of distinguishing the signal processes from the $t\bar{t}Z$ and tWZ processes. The rankings of the input variables for the training of the BDT multiclassifiers is shown in Table 8.3; this should not be taken too seriously as the variable rankings are determined by the number of times

that a given variable is used for the node splitting. Thus, highly correlated variables (e.g. m_{ll} and m_{4l}) will tend to cluster near each other in the rankings.

Input variables for 4-lepton BDTs			
Variable	OF BDT	SF BDT	Brief Description
m_{ll}	✓	✓	Invariant mass of W candidates
m_{4l}	✓	✓	4-lepton invariant mass
$\Delta\phi(l^{W1}, p_T^{miss})$	✓	✓	Radial distance between leading W candidate lepton and MET
$\Delta\phi(l^{W2}, p_T^{miss})$	✓	✓	Radial distance between subleading W candidate lepton and MET
$\Delta\phi(4lep, p_T^{miss})$	✓	✓	Radial distance between 4-lepton system and MET
$\Delta\phi(Wcands, p_T^{miss})$	✓	✓	Radial distance between W candidate system and MET
$\Delta\phi(Z, p_T^{miss})$	✓	✓	Radial distance between Z boson and MET
$\Delta R(WW, Z)$	✓	✓	Solid angle separation between W candidate system and Z boson
$\Delta R(l^{W1}, l^{W2})$	✓	✓	Solid angle separation between W candidate leptons
$\Delta R(l^{Z1}, l^{Z2})$	✓	✓	Solid angle separation between Z candidate leptons
H_T	✓	✓	Sum of hadronic energy
$S_{T,lep,MET}$	✓	✓	Sum of lepton p_T and p_T^{miss}
p_T^{miss}	✓	✓	Missing transverse energy (Puppi Algorithm)
m_{T2}	✓	✓	Analogue of m_T for 2 semi-invisibly decaying particles
m_T^{4lep}	✓	✓	Transverse mass formed by 4 lepton system and MET
m_T^{Wcands}	✓	✓	Transverse mass formed by W candidate system and MET
$m_T^{l^{W1}}$	✓	✓	Transverse mass formed by leading W candidate and MET
$m_T^{l^{W2}}$	✓	✓	Transverse mass formed by subleading W candidate and MET
p_T^{4l}	✓	✓	Scalar p_T sum of 4 lepton system
p_T^{Z1}	✓	✓	Transverse momentum of leading Z candidate lepton
p_T^{Z2}	✓	✓	Transverse momentum of subleading Z candidate lepton
p_T^{W1}	✓	✓	Transverse momentum of leading W candidate lepton
p_T^{W2}	✓	✓	Transverse momentum of subleading W candidate lepton
$\cos(\theta_X)$	✓	✓	cosine of helicity angle of X particle where $X \rightarrow l^{W1} + l^{W2}$
$\min(\Delta R(l^{W1}, j))$	✓	✓	Minimum solid angle difference between leading W candidate lepton and jet
$\min(\Delta R(l^{W2}, j))$	✓	✓	Minimum solid angle difference between subleading W candidate lepton and jet
N_{jets}	✓	✓	Number of jets in the event

Table 8.1: Input Variables for the 4-lepton BDTs

Yields for the signal and background processes in the SF and OF TR are shown in Table 8.4. In both channels, the ZZ process is the dominant background contribution; in the SF channel this comes from ZZ events with one off-shell Z boson, while in the OF channel the ZZ contribution is due to Z decays to τ leptons which then decay to the light leptons ($\ell = e, \mu$). The WZ , VVV , and Other processes are excluded from the BDT training due to the low number of unweighted events and the low yields in the TR. Thus, the full background contribution used in the BDT training is a sum over the three dominant background contributions: ZZ , $t\bar{t}Z$, and tWZ .

Training is performed using the ROOT Toolkit for Multivariate Analysis (TMVA) [115].

The hyperparameters chosen for the training of the multiclassifier are shown in table 8.2. The definitions of the hyperparameters that were varied in the tuning process are as follows:

- **N_{trees}** : Number of decision trees used in the training.
- **MaxDepth**: The maximum number of nodes that are used to construct a single decision tree.
- **MinNodeSize**: The smallest fraction of the overall training data that is used to construct individual nodes within a decision tree.
- **Boost Type**: This is the algorithm that is used for boosting (e.g. AdaBoost or Gradient Boost)
- **Learning Rate**: Also called the "shrinkage" for the Gradient Boost Algorithm; this parameter is used for regularization by modifying the update rule for the model. Lower values of the learning rate correspond to the model converging more slowly, making it more robust against overfitting.
- **SeparationType**: This is the parameter that is used to determine the splitting criteria for individual nodes. This is generally a function of the training data, and when its value is above or below some threshold in an individual node it will cause the node to split within the tree.

The criteria for choosing the optimal set of hyperparameters for the training is as follows: first, train the multiclassifier using a wide array of hyperparameter configurations, then pick the configurations that do not exhibit obvious signs of overtraining. Of the configurations that do not exhibit overtraining, we then choose the configuration with the highest ROC-Integral value across the 3 different multiclassifiers. The

ROC curves for the different hyperparameter configurations considered in this analysis is shown in Figures 8.4. What should be noted by looking at these distributions is that "fine-tuning" of the hyperparameters, in general, gives diminishing returns; there are a number of different configurations that converge to about the same ROC-integral value, and therefore exhibit the same performance (to within a few percent). Figures 8.14-8.16 and 8.17-8.19 show the BDT scores for the training and testing datasets along with the ratio of the number of training vs testing events in each bin for the OF and SF channels, respectively. These plots show good agreement between the testing and training datasets for the signal processes and background processes for all BDT scores and over both channels, suggesting that no overtraining is present.

Hyperparameters for BDT Training			
Hyperparameter	Description	Opposite-Flavor	Same-Flavor
N_{trees}	Number of trees used in the BDT training	800	800
MinNodeSize	Minimum fraction of the training data used to construct individual nodes	5%	5%
Boost Type	Algorithm used for Boosting	Gradient Boost	Gradient Boost
Shrinkage	Learning rate for the Gradient Boost algorithm	0.1	0.1
MaxDepth	Maximum node depth of decision tree	3	3
SeparationType	Parameter to use to determine the node splitting criterion	$\frac{S}{\sqrt{S+B}}$	$\frac{S}{\sqrt{S+B}}$
NegWeights	What to do with negative weights in training?	Ignore	Ignore

Table 8.2: Hyperparameters for BDT multiclassifier training

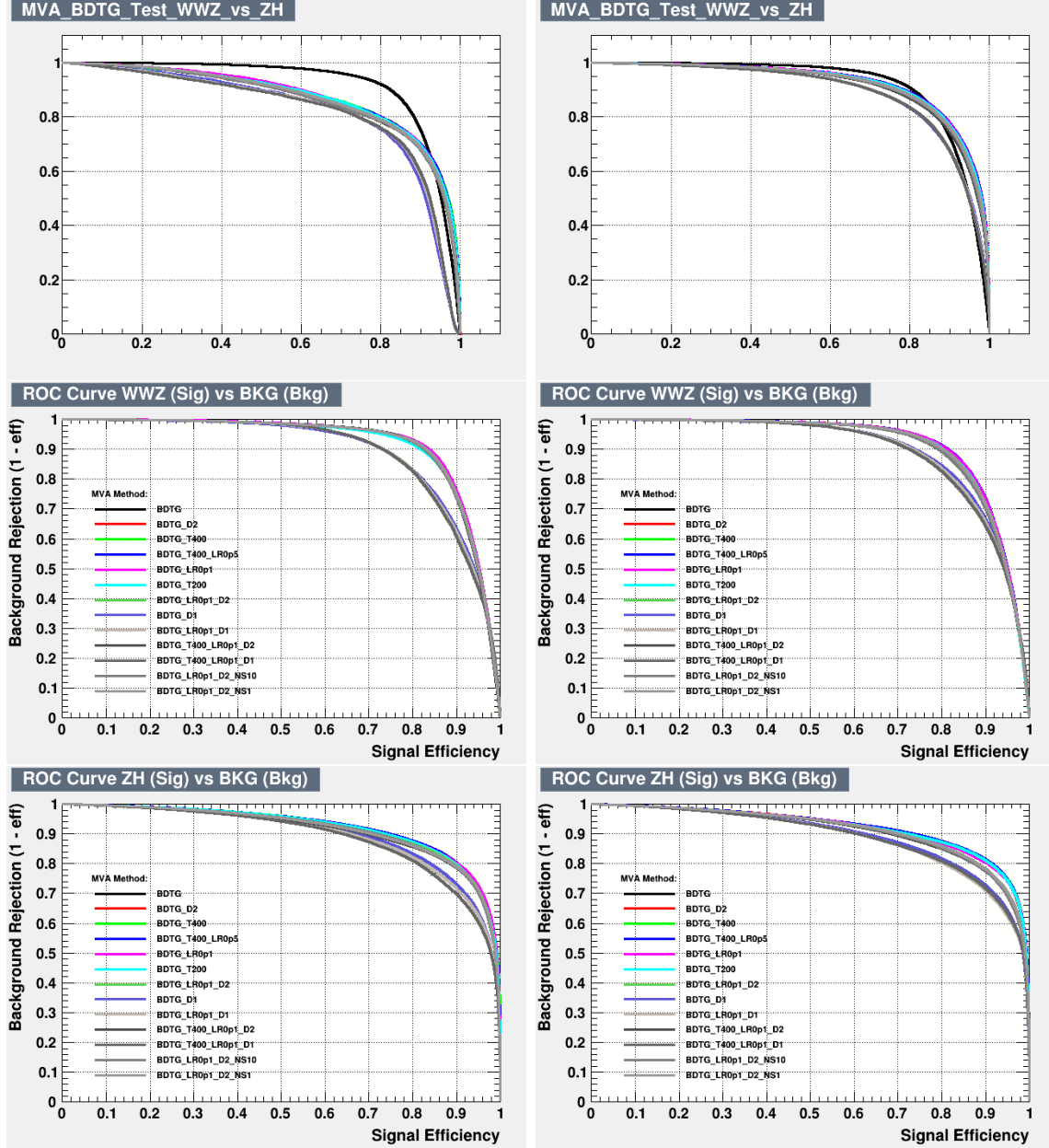


Figure 8.4: ROC curves for different hyperparameter configurations considered in the BDT multiclassifier training.

BDT Variable Rankings		
Ranking	OF BDT Training	SF BDT Training
1	$m_{\ell\ell}$	$m_{\ell\ell}$
2	$m_{4\ell}$	$m_{4\ell}$
3	$\Delta\phi(4\ell, p_T^{miss})$	p_T^{miss}
4	H_T	$\Delta\phi(4\ell, p_T^{miss})$
5	$\Delta R(\ell^{W1}, \ell^{W2})$	$\Delta R(\ell^{W1}, \ell^{W2})$
6	$p_T^{4\ell}$	H_T
7	$\Delta R(\ell^{Z1}, \ell^{Z2})$	$p_T^{4\ell}$
8	$\min(\Delta R(\ell^{W1}, j))$	$m_T^{4\ell}$
9	m_T^{W1}	$\min(\Delta R(\ell^{W1}, j))$
10	m_T^{Wcands}	$\min(\Delta R(\ell^{W2}, j))$
11	p_T^{W1}	p_T^{W1}
12	$\Delta R(WW, Z)$	m_T^{W1}
13	m_T^{W2}	$\Delta\phi(Zcands, p_T^{miss})$
14	$\Delta\phi(\ell^{W2}, p_T^{miss})$	$\Delta R(\ell^{Z1}, \ell^{Z2})$
15	m_{T2}	$\Delta R(WW, Z)$
16	p_T^{miss}	m_T^{Wcands}
17	$S_{T,lep,MET}$	m_T^{W2}
18	p_T^{W2}	$\Delta\phi(\ell^{W2}, p_T^{miss})$
19	$\min(\Delta R(\ell^{W2}, j))$	p_T^{W2}
20	$\Delta\phi(\ell^{W1}, p_T^{miss})$	m_{T2}
21	p_T^{Z2}	$\Delta\phi(WW, p_T^{miss})$
22	p_T^{Z1}	$\Delta\phi(\ell^{W1}, p_T^{miss})$
23	$m_T^{4\ell}$	p_T^{Z1}
24	N_{jets}	$S_{T,lep,MET}$
25	$\cos(\theta_X)$	$\cos(\theta_X)$
26	$\Delta\phi(WW, p_T^{miss})$	p_T^{Z2}
27		N_{jets}

Table 8.3: Input Variable Rankings for the BDT Multiclassifiers.

8.4 Input Variables

The input variables for the BDT multiclassifier were chosen for any number of the following reasons:

- Separation of WWZ from ZH .
- Separation of WWZ or ZH from ZZ .
- Separation of WWZ or ZH from tXZ .

The most important variables in the OF channel are the dilepton mass of the W candidates $m_{\ell\ell}$ and m_{T2} , which are crucial for separating the both the WWZ process from the ZH process and both signals from the ZZ background, as described in Section 6.2.1. In the SF channel, $m_{\ell\ell}$ is complemented by p_T^{miss} and $p_T^{4\ell}$ variables, which are also key in separating the signals from each other and from ZZ , as discussed in Section 6.2.2. The 4-lepton invariant mass is highly correlated with the $m_{\ell\ell}$ variable, but provides additional information about the Z candidate leptons. Similarly, the variants on the m_T variable are also useful to provide complementary information to m_{T2} regarding the relative orientation between the leptons and the missing transverse energy.

Angular variables between the leptons, as well as between leptons and p_T^{miss} are useful in further isolating the signal processes from each other and from ZZ . In the transverse plane, the WWZ and ZH processes balance the momentum of the Z with the momentum of the WW and H , respectively, as shown in Figure 8.5. Thus, angular differences in ϕ between the leptons and the p_T^{miss} can be used to exploit this difference. The ZZ process has, ideally, back-to-back emission of the Z boson in the transverse plane. Also, since the main component of ZZ has no real missing transverse momentum, the angular difference between leptons and p_T^{miss} should be, to first order, isotropic. Additionally, the lepton solid angle separation

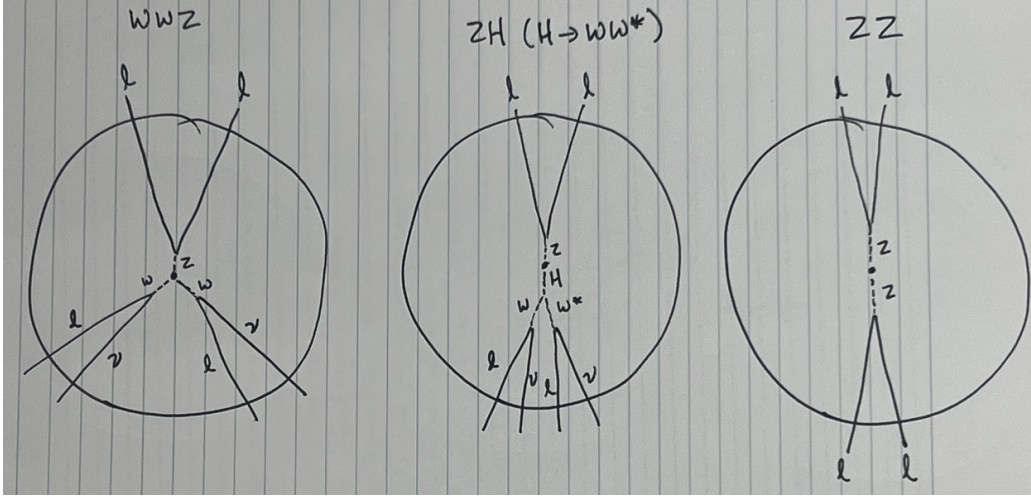


Figure 8.5: Artist rendition of WWZ , ZH , and ZZ signatures in the transverse plane.

and transverse momenta information also provides useful information for separating ZH from WWZ , especially $\Delta R(\ell^{W1}, \ell^{W2})$ and the lepton p_T ; the leptons from the $H \rightarrow WW^*$ decays are, on average, more collinear and lower p_T than those from the non-resonant WW production.

Jet information (N_{jets} , H_T , and $\Delta R(\ell, j)$ variables) is used primarily for the separation of the signal processes from the tXZ backgrounds. The signal processes should predominantly have no real jets, while the tXZ should have at least one real jet (in the case of tWZ) or at least two real jets (in the case of ttZ).

8.5 Validation

Validation of the BDT multiclassifier training is performed by checking the agreement between simulation and data in the CRs for the BDT input variables and the BDT scores themselves. Figure D.21 (Figure D.22) shows the BDT score distributions in the ZZ CR for Run 2 (Run 3). Figures D.25 and D.23 show the BDT score distributions for Run 2 in the SF and OF $t\bar{t}Z$ CRs, respectively. These distributions show good agreement between data and simulation. Corresponding Run 3 plots are

shown in Figures [D.26](#) and [D.24](#). Furthermore, the modeling of the BDT input variables are checked in each of the CRs. Figures [E.1-E.5](#), [E.21-E.25](#), and [E.11-E.15](#) show the Run 2 BDT input variable distributions in the ZZ CR, tXZ SF CR, and tXZ OF CR, respectively. These distributions show good agreement between data and simulation for all of the BDT input variables. The distributions for Run 3 are also included in Appendix [E](#). From both of these checks, it can be concluded that the BDT multiclassifier is sufficiently validated.

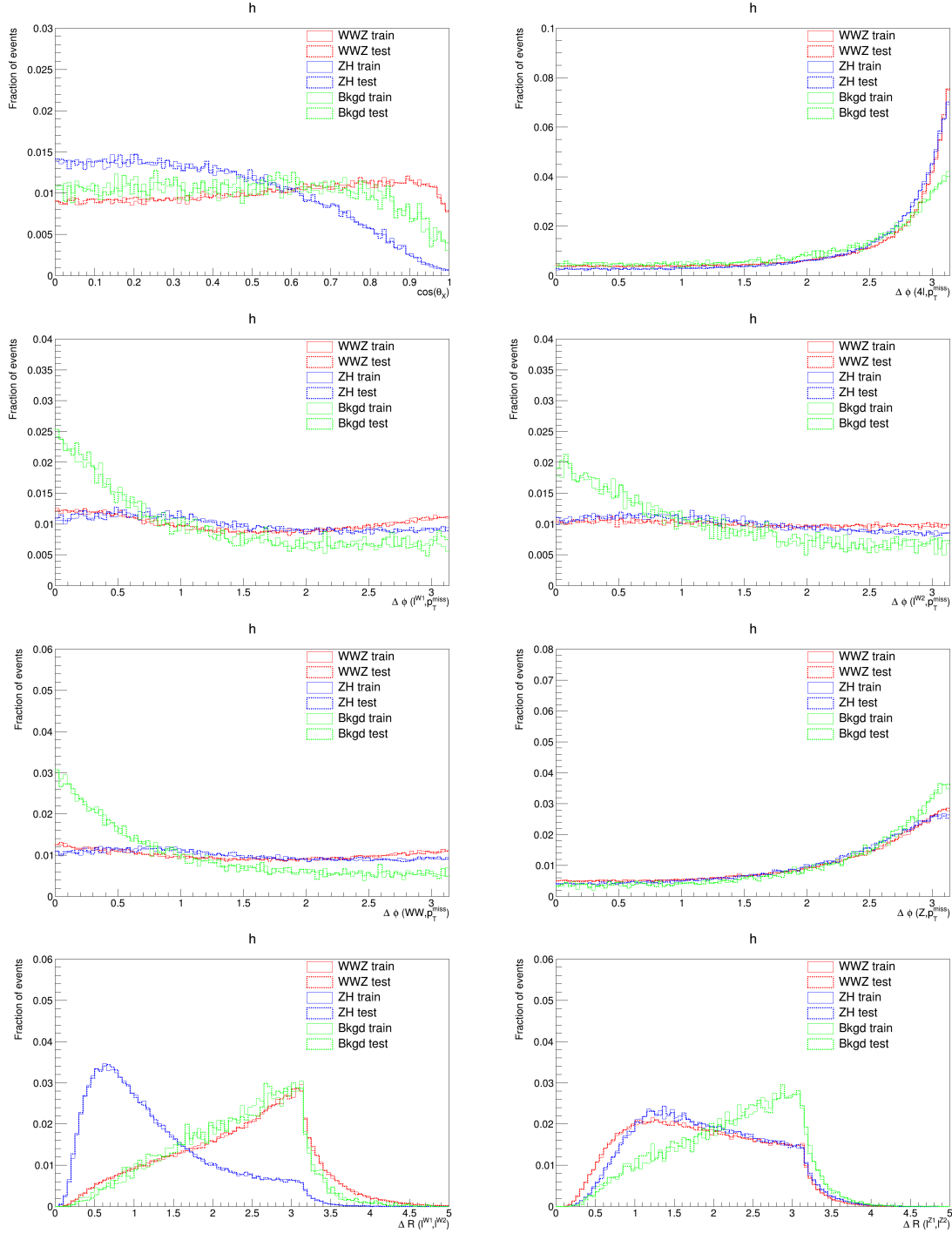


Figure 8.6: BDT input variables in the OF TR for Run 2. All contributions are normalized to unity to show shape differences.

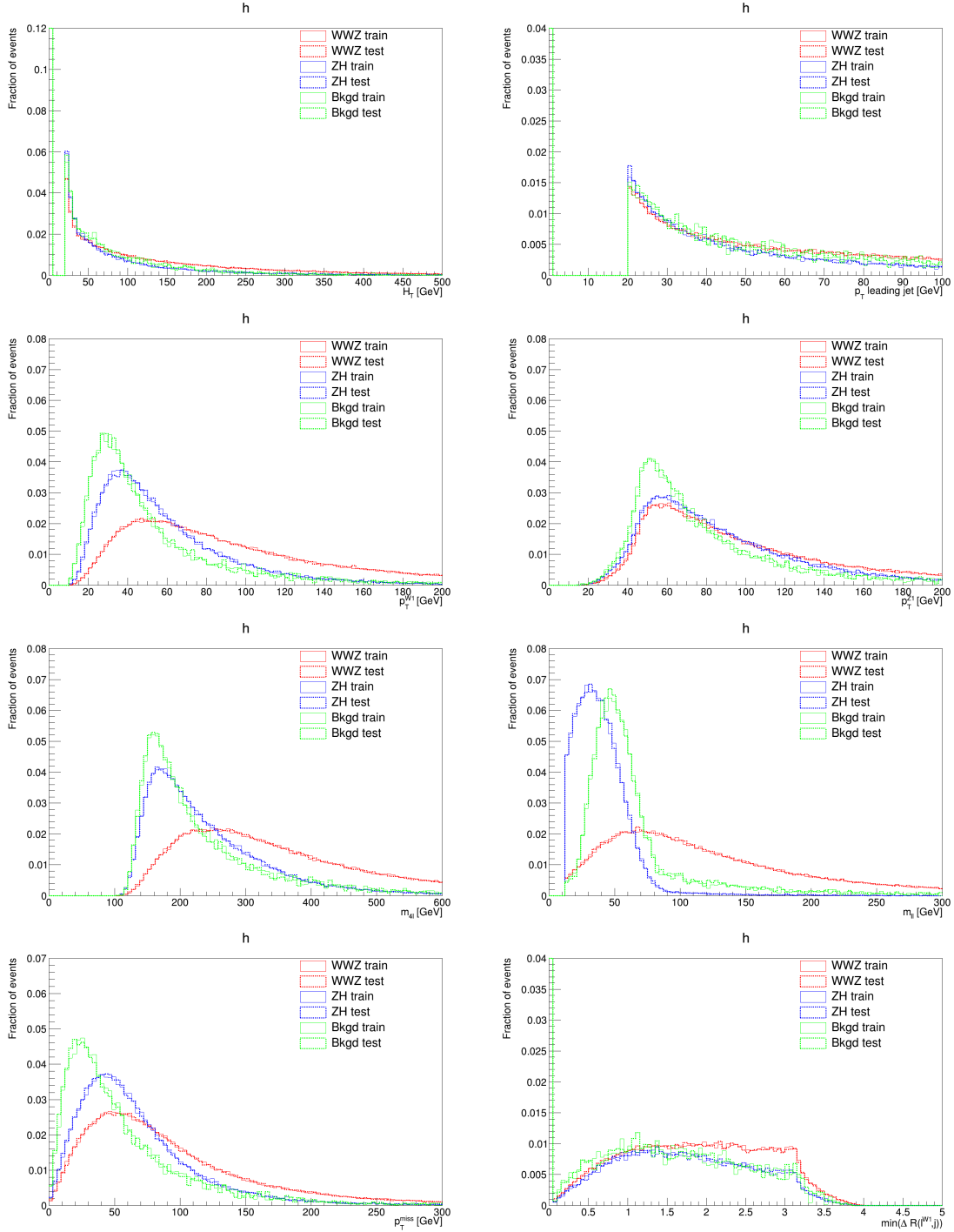


Figure 8.7: BDT input variables in the OF TR for Run 2. All contributions are normalized to unity to show shape differences.

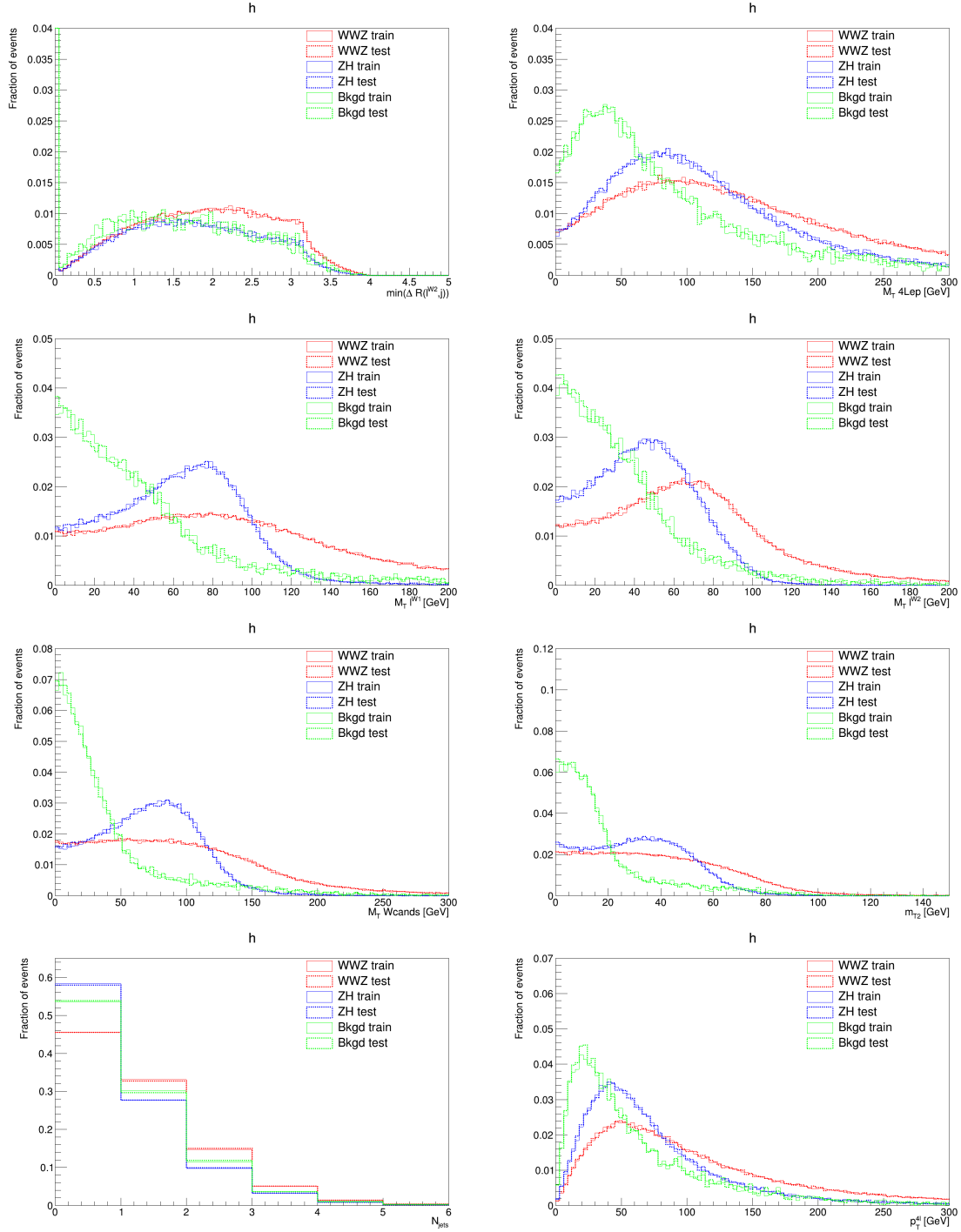


Figure 8.8: BDT input variables in the OF TR for Run 2. All contributions are normalized to unity to show shape differences.

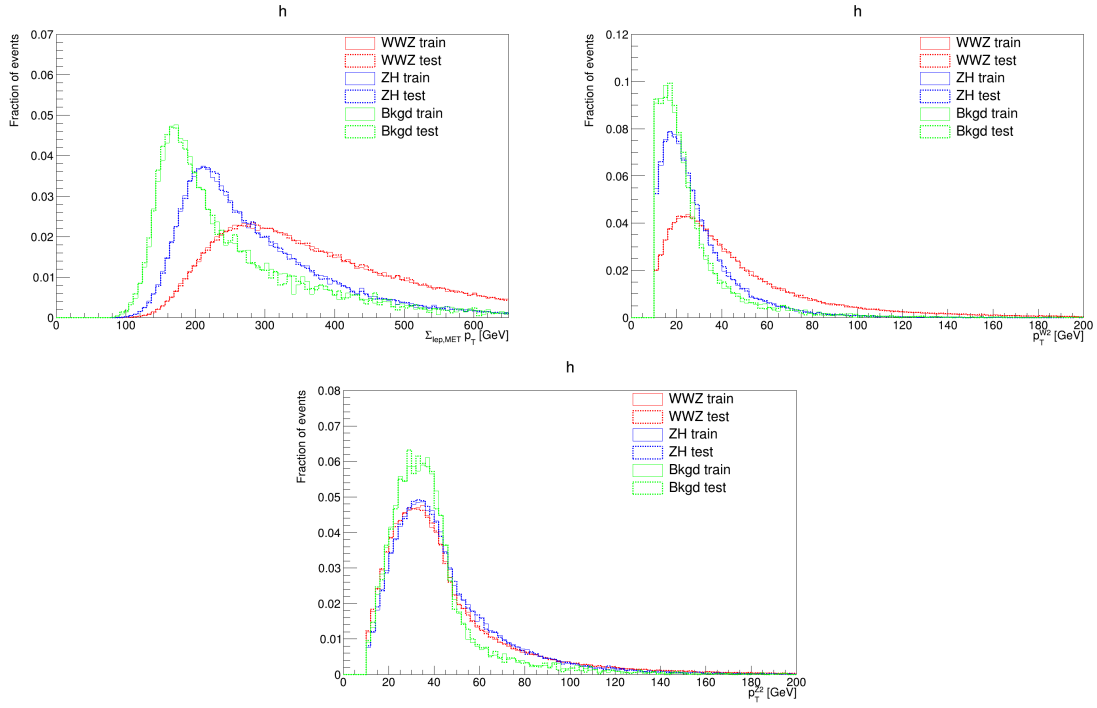


Figure 8.9: BDT input variables in the OF TR for Run 2. All contributions are normalized to unity to show shape differences.

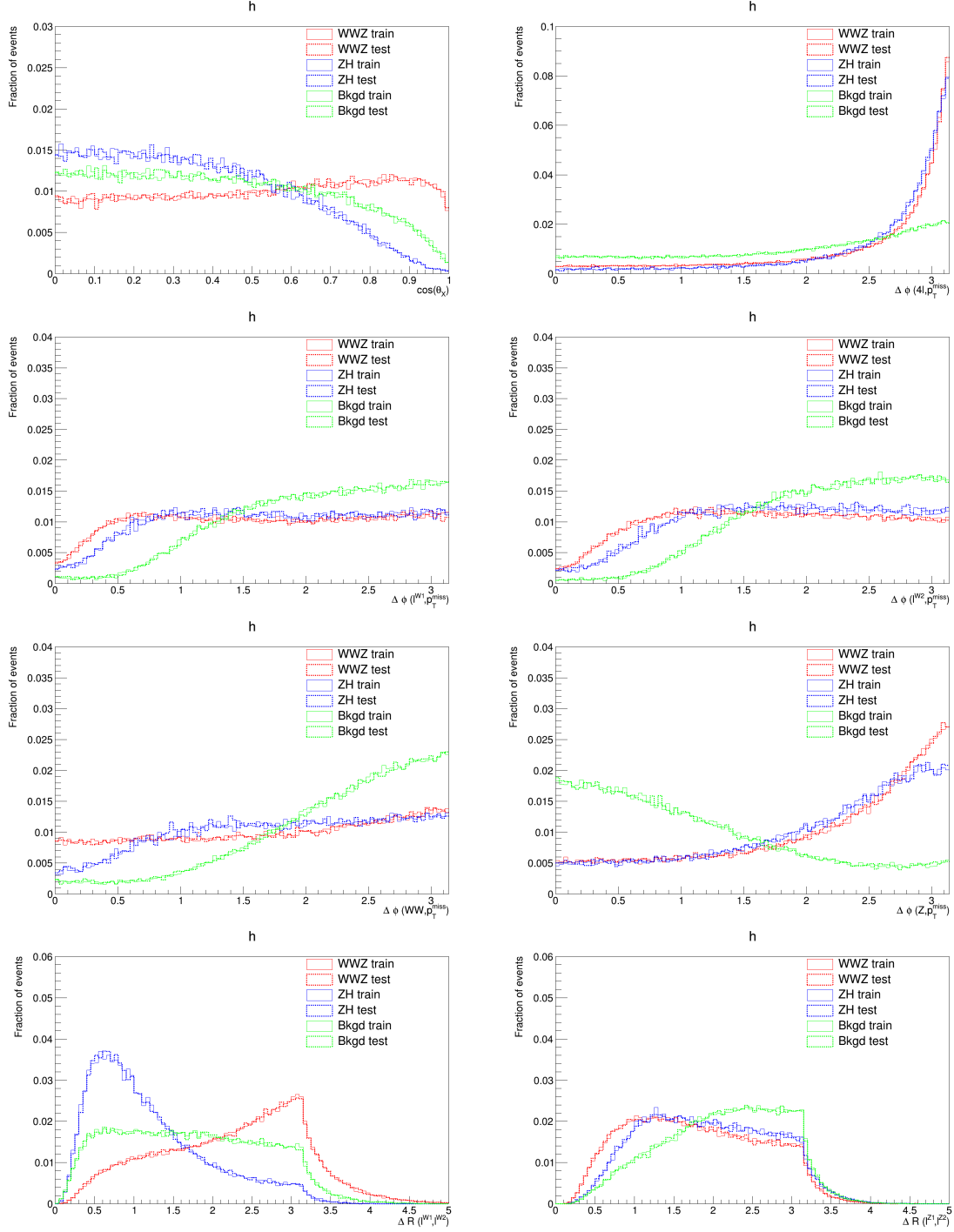


Figure 8.10: BDT input variables in the SF TR for Run 2. All contributions are normalized to unity to show shape differences.

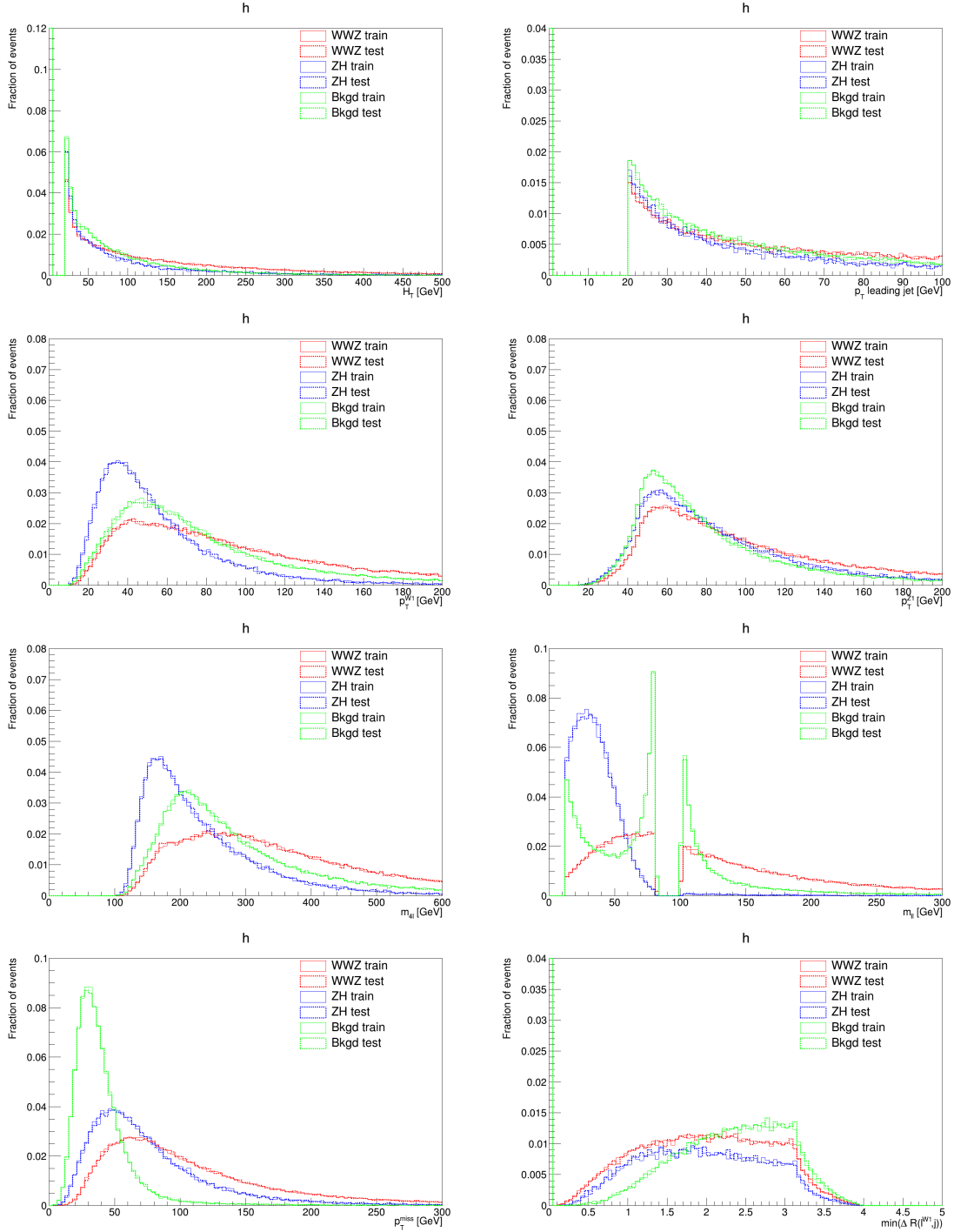


Figure 8.11: BDT input variables in the SF TR for Run 2. All contributions are normalized to unity to show shape differences.

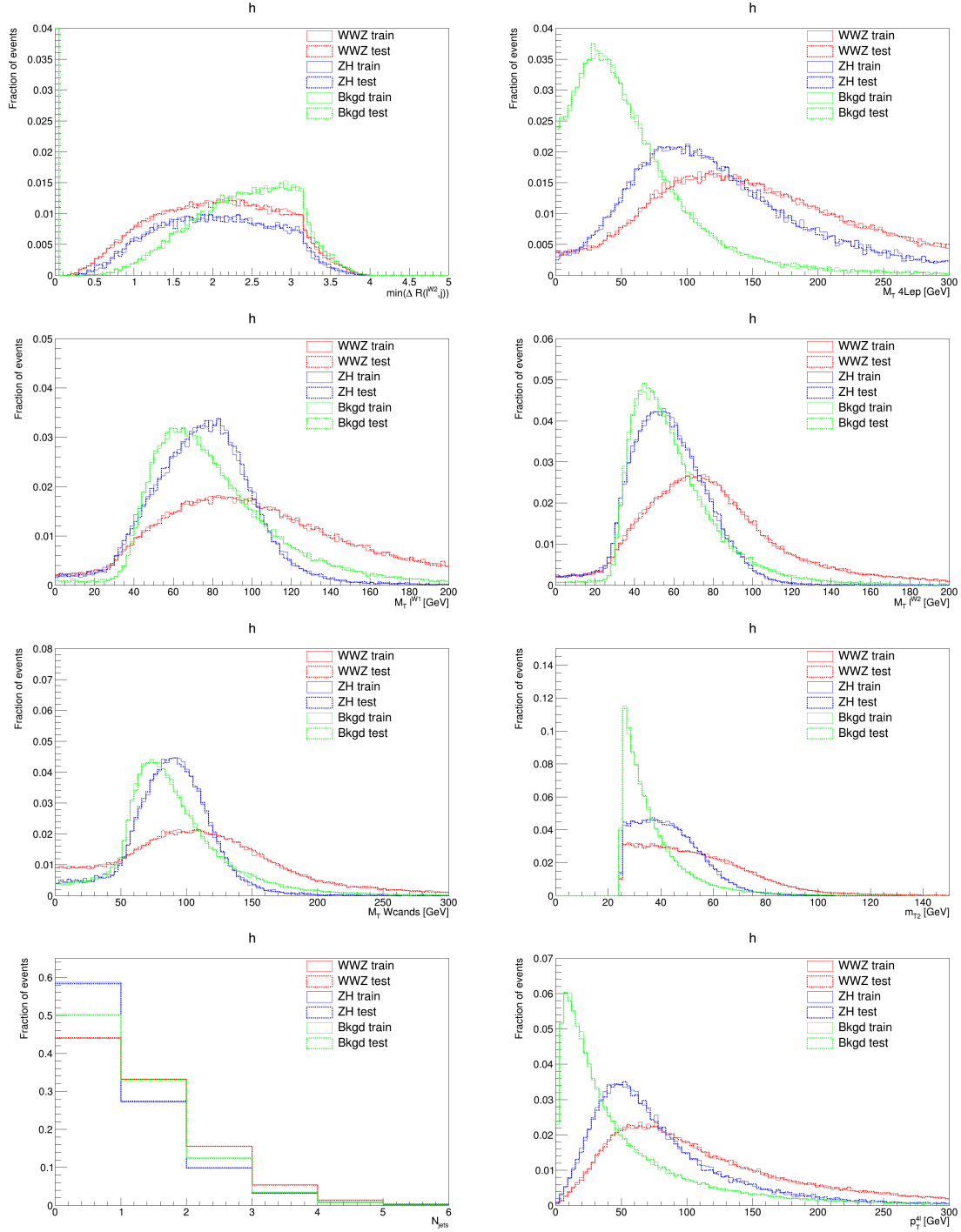


Figure 8.12: BDT input variables in the SF TR for Run 2. All contributions are normalized to unity to show shape differences.

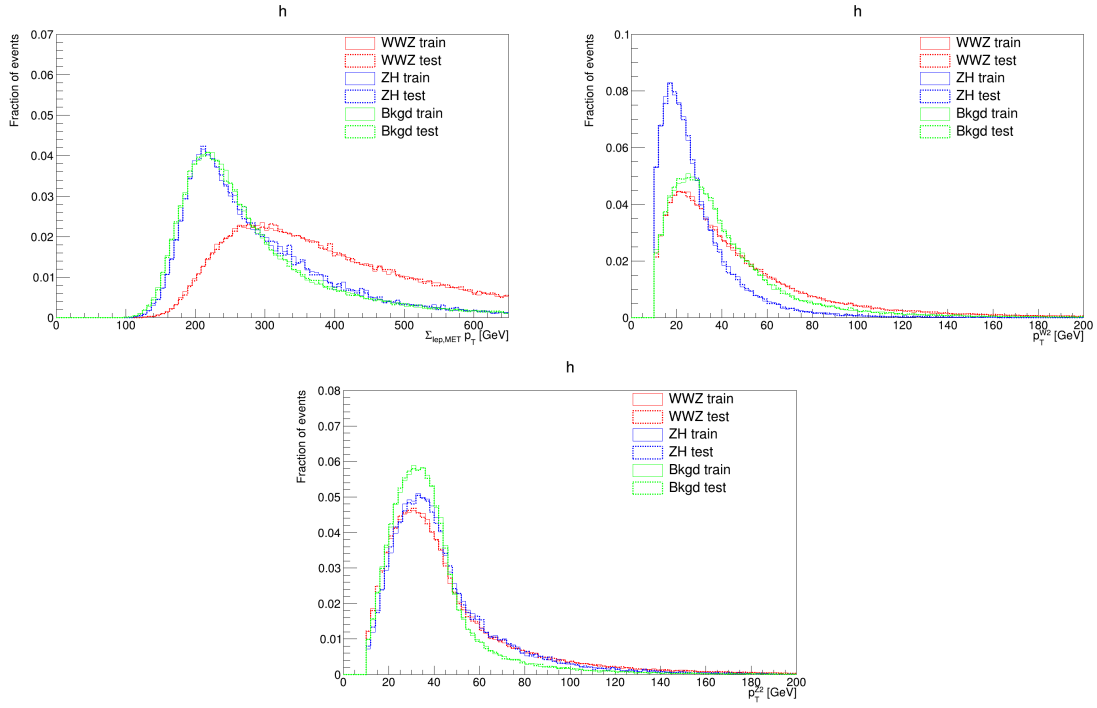


Figure 8.13: BDT input variables in the SF TR for Run 2. All contributions are normalized to unity to show shape differences.

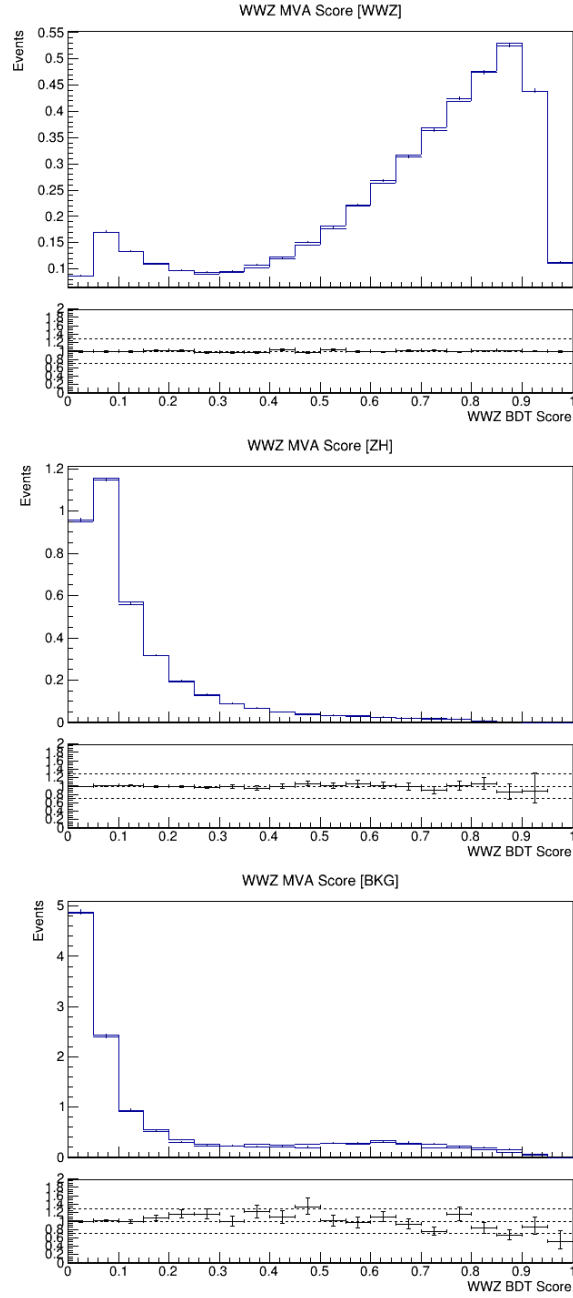


Figure 8.14: Overtraining checks for signal and background processes for the WWZ BDT score in the OF channel. Training data is shown as solid lines, while testing data is shown as points with error bars. The ratio plots show the ratio of the number of training events to testing events per bin.

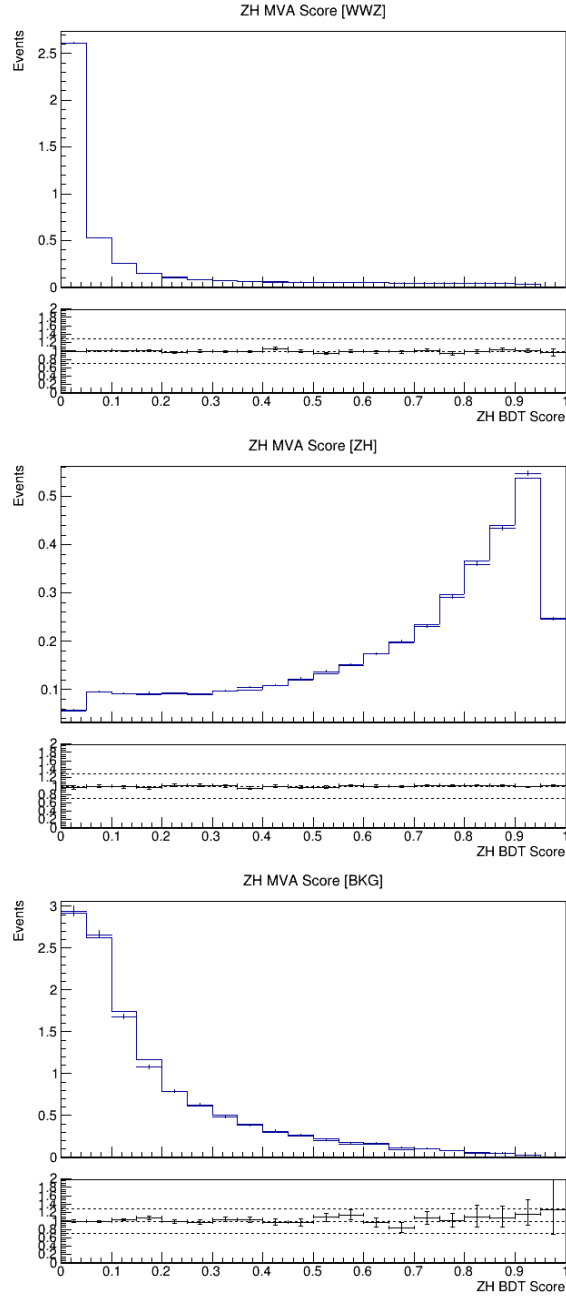


Figure 8.15: Overtraining checks for signal and background processes for the ZH BDT score in the OF channel. Training data is shown as solid lines, while testing data is shown as points with error bars. The ratio plots show the ratio of the number of training events to testing events per bin.

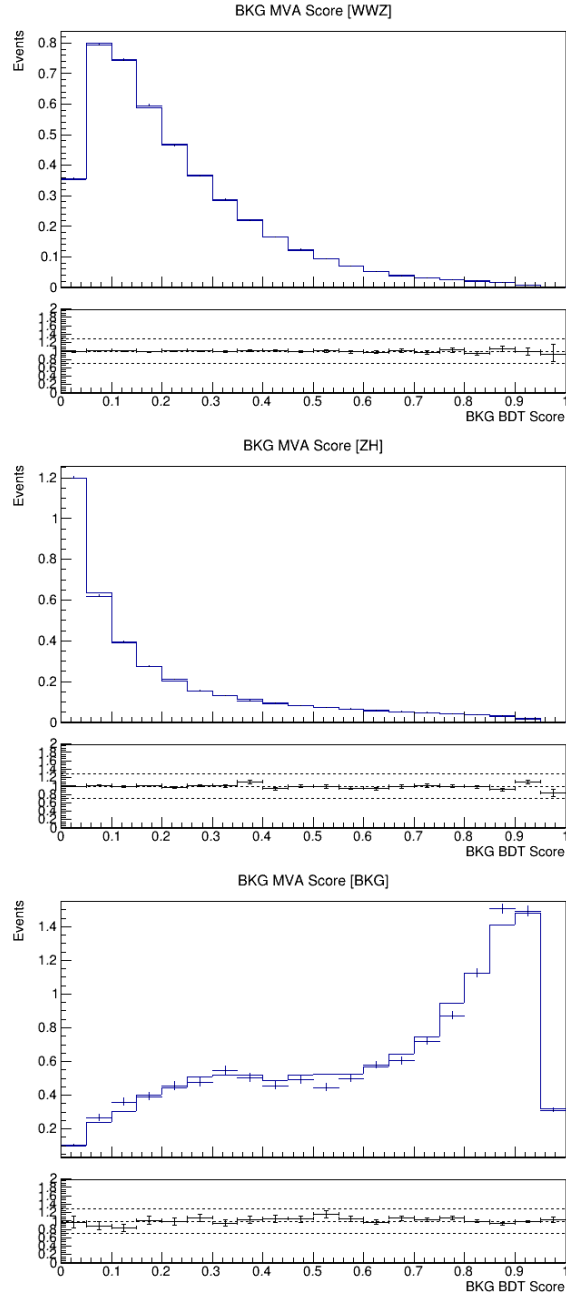


Figure 8.16: Overtraining checks for signal and background processes for the Background BDT score in the OF channel. Training data is shown as solid lines, while testing data is shown as points with error bars. The ratio plots show the ratio of the number of training events to testing events per bin.

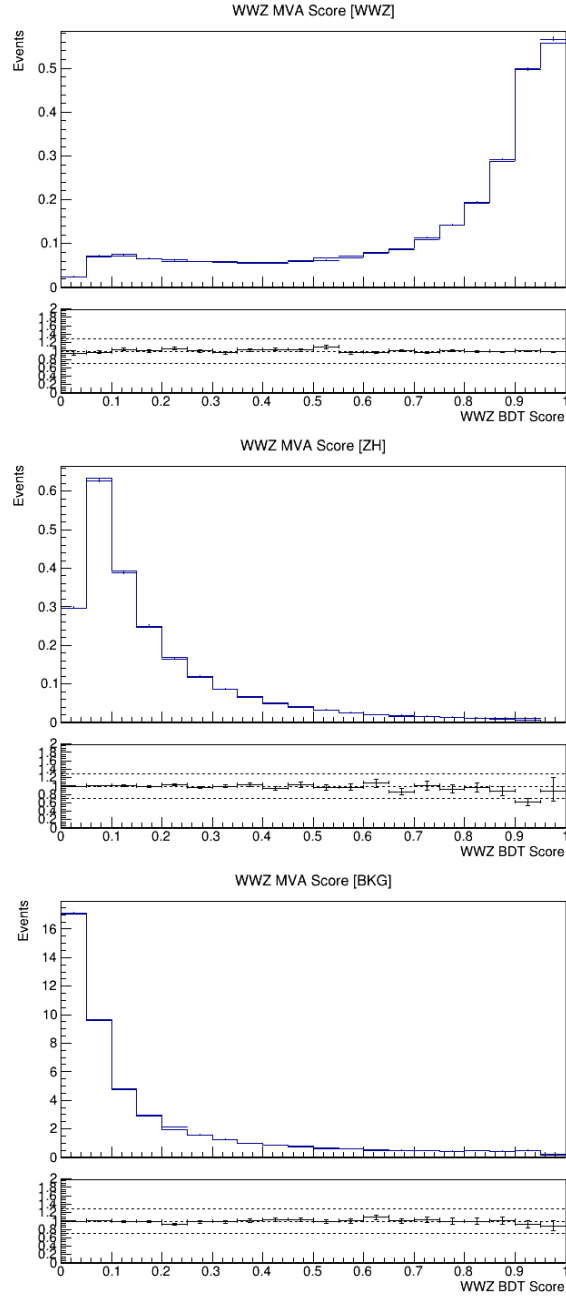


Figure 8.17: Overtraining checks for signal and background processes for the WWZ BDT score in the SF channel. Training data is shown as solid lines, while testing data is shown as points with error bars. The ratio plots show the ratio of the number of training events to testing events per bin.

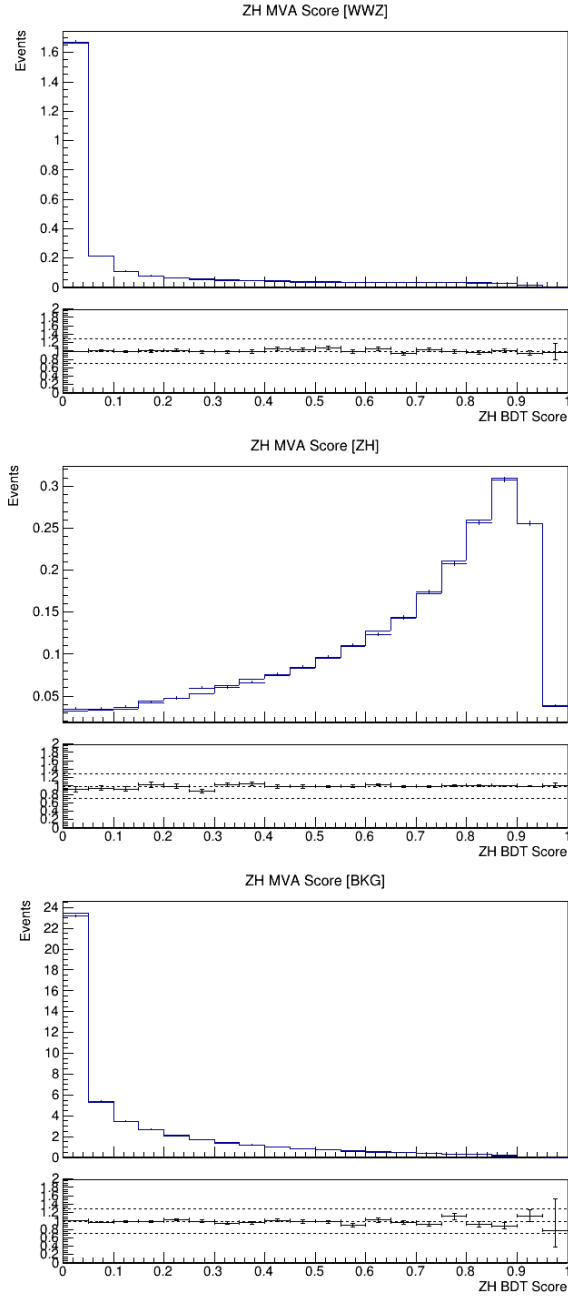


Figure 8.18: Overtraining checks for signal and background processes for the ZH BDT score in the SF channel. Training data is shown as solid lines, while testing data is shown as points with error bars. The ratio plots show the ratio of the number of training events to testing events per bin.

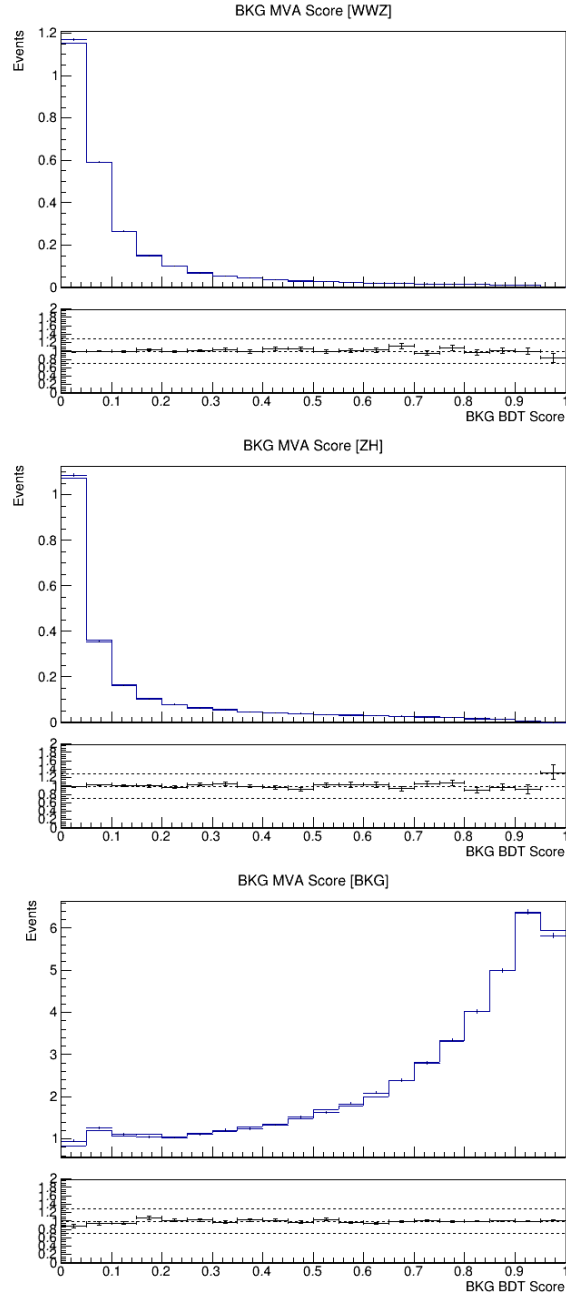


Figure 8.19: Overtraining checks for signal and background processes for the Background BDT score in the SF channel. Training data is shown as solid lines, while testing data is shown as points with error bars. The ratio plots show the ratio of the number of training events to testing events per bin.

CHAPTER 9

STATISTICAL METHODS

"If your experiment needs a
statistician, you need a better
experiment"

Ernest Rutherford

9.1 Overview

At the core of scientific measurement is the statistical analysis of data. In the context of HEP applications, this generally refers to the procedure of statistical hypothesis testing. This involves the formulation of a hypothesis which may represent some particular theory or model (e.g. the Standard Model). An observation or measurement can be compared with this hypothesis through the determination of a p-value. The p-value can be interpreted as the "probability of obtaining the observed result or a more extreme one given the null hypothesis". Thus, smaller values of α would imply that the data is less compatible with the null hypothesis. In many applications outside of HEP, a p-value of $\alpha = 0.05$ is considered "statistically significant" and could be considered grounds for rejecting the null hypothesis. In HEP, the standard for rejecting the null hypothesis is

$$\alpha_{discovery} < 3.0 \times 10^{-7} \quad (9.1)$$

which corresponds to a Z-score of 5σ . While this is incredibly strict (and rather arbitrary), it does almost guarantee that deviations with respect to the null hypothesis are caused by new phenomena rather than statistical fluctuations.

In some cases, it is more convenient to report a Z-score rather than a p-value. The Z-score is the number of standard deviations away from the mean value of the measurement (assuming a Gaussian distribution) for the observed value or data point. It can be obtained from the p-value via the transformation:

$$Z = \Phi^{-1}(1 - \alpha) \quad (9.2)$$

where Φ is the cumulative distribution for a Gaussian with zero mean and unit variance.

There are two distinct statistical approaches that are utilized in this analysis, each representing a different type of measurement:

- **Signal Strength Measurement:** Here, we assume the Standard Model, and perform a measurement of the cross section of the individual processes. This is done by constructing a binned likelihood function and finding the value of the cross section that maximizes the likelihood (or minimizes the negative-log-likelihood). This is discussed in more detail in Sec. 9.4.1.
- **Evaluation of the Significance:** This can be done by invoking Wilks' Theorem on the likelihood to determine the Z-score of the signal process in question. This is discussed in more detail in Sec. 9.4.2.

9.2 Formulating the Likelihood

This analysis involves fitting over statistically-independent bins comprised of observed event counts (N_i) and predictions for the number of expected signal (S_i) and background (B_i) events. Thus, for each bin, the likelihood of observing N_i events given some prediction $P_i = S_i + B_i$ is given by the Poisson distribution:

$$p(N_i|P_i) = \frac{P_i^{N_i} e^{-P_i}}{N_i!} \quad (9.3)$$

In all cases considered here, each bin is statistically independent from all other bins and it follows that the full likelihood is product over the likelihoods for each bin:

$$\mathcal{L}_{tot} = \prod_{i=1}^{N_{bins}} p(N_i|P_i) \quad (9.4)$$

In the absence of nuisance parameters, Eqs. 9.3 and 9.4 can be combined to give the full likelihood for a multi-bin fit. However, this formulation of the likelihood

is simplistic and lacks the following important ingredients for the purposes of this measurement:

- A modifier to the signal normalization, commonly referred to as a *Signal Strength* (SS), that reflects the goal of the fit as a means of comparing the predicted number of events with the observed number of events, or, analogously, the theoretical cross section σ_{SM} with the measured cross section σ .
- A procedure for incorporating systematic uncertainties into the background prediction and the signal efficiency.

To incorporate the first ingredient into the likelihood formulation, I can make the following substitution:

$$S_i \rightarrow \sum_{j=1}^N \mu_j S_i^{(j)} \quad (9.5)$$

where the sum over j indices represents a sum over the N different signal processes that are considered in the fit and μ_j is the SS parameter for the j -th signal process. The second ingredient requires a similar substitution for the background contribution to the number of events in a particular bin:

$$B_i \rightarrow \beta_i B_i \quad (9.6)$$

as well as for the signal strength:

$$\mu_i \rightarrow \kappa_i \mu_i \quad (9.7)$$

where β_i and κ_i are *nuisance parameters* that modify the total background contribution and signal strength, respectively. Unlike the signal strength, we are not particularly interested in the value of β and κ in the fit. However, for the purposes of

arriving at a sensible answer, we do care that β and κ take on reasonable values when fitted for, which can be achieved by imposing constraints on the nuisance parameters.

From a statistical point of view, within the context of a frequentist analysis, the nuisance parameters are likelihoods that should be taken from auxiliary experiments. In practice, a hybrid bayesian-frequentist approach is employed, and thus the nuisances are modeled using a probability density function (PDF), which I will call $w(\beta_j, \sigma_j)$ (background nuisances) or $w(\kappa_j, \delta_j)$ (signal nuisances). In the next section, I will attempt to motivate a particular choice for w . For now, I note that the inclusion of nuisance parameters also requires the modification of the likelihood shown in Eq. 9.4 to account for said nuisance parameters and their constraints:

$$\mathcal{L}_{tot} = \left(\prod_{i=1}^{N_{bins}} p(N_i | P_i(\mu, \kappa_i, \beta_i)) \right) \times \left(\prod_{i=1}^{N_{bins}} w(\beta_i, \sigma_i) \right) \times \left(\prod_{i=1}^{N_{bins}} w(\kappa_i, \delta_i) \right) \quad (9.8)$$

The form of the first term is the same as Eq. 9.3 with the substitutions given by Eqs. 9.5, 9.6 and 9.7:

$$\begin{aligned} \mathcal{L}_{tot} = & \left(\prod_{i=1}^{N_{bins}} \frac{(\sum_{j=1}^k \kappa_i^{(j)} \mu_j S_i^{(j)} + \beta_i B_i)^{N_i} \times e^{-(\sum_{j=1}^k \kappa_i^{(j)} \mu_j S_i^{(j)} + \beta_i B_i)}}{N_i!} \right) \\ & \times \left(\prod_{i=1}^{N_{bins}} w(\beta_i, \sigma_i) \right) \times \left(\prod_{i=1}^{N_{bins}} \left(\sum_{j=1}^k w(\kappa_i^{(j)}, \delta_i^{(j)}) \right) \right) \end{aligned} \quad (9.9)$$

Looking at Eq. 9.9, we can see that the form of the likelihood is already getting pretty ugly, even without a specified PDF for the nuisance parameters. Fortunately, this problem can be navigated by a simple transformation, which will be discussed in Sec. 9.4. For now, we take a brief interlude to discuss the treatment of the systematic uncertainties, which represent the nuisance parameters in this analysis.

9.3 Treatment of Systematic Uncertainties

"The use of lognormal PDFs to represent systematic uncertainties is convenient, and, perhaps, at some level, somehow justified"

Claudio Campagnari

Phys 250 Lecture Notes

As shown in Section 9.2, the binned likelihood accounts for the presence of nuisance parameters β and κ . In a HEP analysis, the nuisance parameters are often efficiencies associated with the experiment and measurement apparatus (e.g. the accelerator complex and the detector), the reconstruction of particles using various identification algorithms, and the theoretical and statistical limitations of the Monte Carlo simulation. A comprehensive discussion of the sources of systematic uncertainty in this analysis is presented in Sec. 9.5. Heuristically, each efficiency associated with a particular nuisance parameter can be thought of as the product of many different independent efficiencies. To illustrate this, let us consider a particular nuisance parameter that is of importance for this analysis: the muon identification efficiency. The total muon identification efficiency ϵ_μ depends on (among other things):

- The efficiency of a muon leaving hits in a number of layers of one of the CMS muon detectors to be reconstructed by the PF algorithm ($\epsilon_{detection}$)
- The efficiency of the tracker detector to detect the muon hits ($\epsilon_{tracker}$)
- The efficiency of the global muon fit to match the track to the hits in the muon detector (ϵ_{fit})
- The efficiency of requirements for muon isolation, impact parameter, p_T , η , and

other variables that are used in muon identification algorithms (ϵ_{ID})

Each of these efficiencies is either a Gaussian random variable itself or a product of multiple Gaussian random variables. It then follows that:

$$\epsilon_{\mu} = \epsilon_{detection} \times \epsilon_{tracker} \times \epsilon_{fit} \times \epsilon_{ID} \times \dots \quad (9.10)$$

is also a product of (many) Gaussian random variables. Another obvious, but important, stipulation is that each of the efficiencies must be positive-definite (i.e. $\epsilon_i > 0, \forall i$), as it does not make physical sense for an efficiency to take on a negative value. These two conditions are sufficient to specify an ideal choice of PDF for ϵ_{μ} : the *Lognormal* distribution.

For a general nuisance parameter β_i with standard deviation σ_{β_i} , the lognormal pdf has the following form:

$$w(\beta_i, \sigma_{\beta_i}) = \frac{1}{\sqrt{2\pi}\alpha_i} \frac{1}{\beta_i} \exp\left(-\frac{1}{2} \frac{(\log \beta_i - a_i)^2}{\alpha_i^2}\right) \quad (9.11)$$

The parameters α_i and a_i are related to the mean of β_i (which I will denote as m_{β_i}) and the standard deviation σ_{β_i} via the following transformations:

$$\text{mean of } \beta_i : m_{\beta_i} = \exp\left(a_i + \frac{\alpha_i^2}{2}\right) \quad (9.12)$$

$$\text{variance of } \beta_i : \sigma_{\beta_i}^2 = m_{\beta_i}^2 (\exp(\alpha_i^2) - 1) \quad (9.13)$$

$$\text{mode of } \beta_i : \overline{\beta_i} = \exp(a_i - \alpha_i^2) \quad (9.14)$$

The maximum-likelihood solution to Eq. 9.11 corresponds to $\beta_i = \overline{\beta_i}$. Since β_i modifies the normalization of the background, it makes sense to choose the maximum-

likelihood solution to correspond to $\overline{\beta_i} = 1$ (i.e. the nominal background prediction).

Imposing this condition leads to the following relations:

$$a_i = \alpha_i^2 \tag{9.15}$$

$$\sigma_{\beta_i}^2 = \exp(3\alpha_i^2)(\exp(\alpha_i^2) - 1) \tag{9.16}$$

Eq. 9.16 must be solved numerically for α_i as it is not invertible. The reasoning above can also be used for nuisances modifying the signal strength(s) as we would also like to stipulate that their maximum-likelihood solutions correspond to unity. Thus, the above equations hold for the signal nuisances.

In addition to the "proper" lognormal shown above, the CMS HiggsCombine Tool (which is used for the statistical treatment of this analysis) includes a more general "double-sided" lognormal distribution for modeling nuisance parameters. This treatment allows for different $\pm 1\sigma$ variations, and its implementation is described in more detail in Ref. [116].

Now that we have a useful PDF choice for the modeling of our nuisance parameters, we can continue with our formulation of the likelihood.

9.4 Extraction of Results

As mentioned in Sec. 9.1, we are primarily interested in performing two types of measurements in this analysis: signal strength measurements and evaluation of the statistical significance of the different processes of interest. In the former case, we want to determine the value of the signal strength parameter that maximizes the likelihood of the "signal-plus-background" hypothesis. However, given the unsavory form of the binned likelihood (Eq.9.9), this can be costly to evaluate even with computational

methods. To alleviate this, rather than evaluating (or maximizing) the likelihood, we can evaluate (or minimize) the negative-log-likelihood (NLL) L :

$$L = -\log \mathcal{L}_{tot} \quad (9.17)$$

From this, we can obtain the general form of the binned NLL with multiple signal strength parameters and lognormal nuisance parameters by utilizing Eqs. 9.9 and 9.11:

$$L = \sum_{i=1}^{N_{bins}} \left(\sum_{j=1}^k \kappa_i^{(j)} \mu_j S_i^{(j)} \right) + \beta_i B_i - N_i \log \left[\left(\sum_{j=1}^k \kappa_i^{(j)} \mu_j S_i^{(j)} \right) + \beta_i B_i \right] \quad (9.18)$$

$$+ \beta_i + \frac{1}{2} \frac{(\log \beta_i - a_i)^2}{\alpha_i^2} + \sum_{j=1}^k \left(\kappa_i^{(j)} + \frac{1}{2} \frac{(\log \kappa_i^{(j)} - c_i^{(j)})^2}{d_i^{(j)2}} \right)$$

where N_{bins} is the number of bins in the fit, k is the number of signal strength parameters, and $c_i^{(j)}$ and $d_i^{(j)}$ are the corresponding lognormal parameters for the nuisances modifying the j -th signal strength parameter. It should be noted that this formulation of the binned negative-log-likelihood accounts for a single background nuisance parameter for each bin: in reality (as will be illustrated in Sec. 9.5), there are many different nuisance parameters for each bin (which can be effectively transformed into a single nuisance parameter) and, in general, different nuisance parameters impacting different processes within each bin. For the sake of brevity, I will restrict myself here to consider the simplified, one-nuisance-per-bin case.

The measurement of the significance, as will be shown in Sec. 9.4.2, can be obtained from the NLL evaluated at the minimum value (from the signal strength measurement) and the NLL at $\mu = 0$ using Wilks' theorem.

In the following subsections, I will recap the measurements that are being performed in this analysis and state the form of the likelihoods that are used for these

measurements using Eq. 9.18 as a reference point.

9.4.1 Signal Strength

The best-fit signal strength value $\hat{\mu}$ is the value of μ that maximizes the likelihood:

$$\mathcal{L}(\hat{\mu}S + B) = \max_{\mu}(\mathcal{L}(\mu S + B)) \quad (9.19)$$

For reasons stated previously, the (negative) log-likelihood is more desirable to work with. In this case, $\hat{\mu}$ corresponds to the value of μ that minimizes the NLL:

$$L(\hat{\mu}S + B) = -\log(\mathcal{L}(\hat{\mu}S + B)) = \min_{\mu}(-\log[\mathcal{L}(\mu S + B)]) \quad (9.20)$$

When performing scans of the NLL, it is customary to plot the delta-log-likelihood (DLL) as a function of μ , which is simply the difference between the NLL and the NLL at $\mu = \hat{\mu}$:

$$DLL(\mu) = L(\mu S + B) - L(\hat{\mu}S + B) = -\log(\mathcal{L}(\mu S + B)) + \log(\mathcal{L}(\hat{\mu}S + B)) \quad (9.21)$$

The DLL will be particularly useful for determining the Significance, which will be discussed in Sec. 9.4.2.

As stated in Sec. 1.5, the main purpose of this analysis is to perform measurements of the cross section for the WWZ , ZH , and inclusive ($WWZ + ZH$) processes to compare with the SM predictions at $\sqrt{s} = 13$ and 13.6 TeV. This is equivalent to measuring the signal strength parameter μ for these processes, which can be defined as the ratio of the measured cross section to the predicted cross section:

$$\mu = \frac{\sigma_{obs}}{\sigma_{SM}} \quad (9.22)$$

In total, 9 measurements of the signal strength are performed in this analysis:

- Inclusive ($WWZ + ZH$) at 13 TeV, 13.6 TeV, and Combined (13 + 13.6 TeV)
- WWZ at 13 TeV, 13.6 TeV, and Combined (13 + 13.6 TeV)
- ZH at 13 TeV, 13.6 TeV, and Combined (13 + 13.6 TeV)

Six of these measurements (those at 13 TeV and 13.6 TeV) correspond to cross section measurements since they are at well-defined center-of-mass energies. The combined measurements, since they include two different center-of-mass energies, are not used for measuring the cross section but rather represent a weighted average of the measurements. This is ultimately useful for assessing the consistency of the measurements with the SM over multiple $\sqrt{s}$ values.

Run 2:

In the Run 2 analysis, corresponding to $\sqrt{s} = 13$ TeV, 21 total bins are fit (16 SRs and 5 CRs). For the inclusive process, a single signal strength $\mu_{tot,13}$ is utilized in the NLL, representing the sum of the WWZ and ZH signal strengths. Using Eq. 9.18, the NLL for the inclusive 13 TeV signal strength measurement is given by:

$$\begin{aligned} L_{inc,13} = & \sum_{i=1}^{21} (\kappa_i \mu_{tot,13} S_i + \beta_i B_i - N_i \log [\kappa_i \mu_{tot,13} S_i + \beta_i B_i]) \\ & + \beta_i + \frac{1}{2} \frac{(\log \beta_i - a_i)^2}{\alpha_i^2} + \kappa_i + \frac{1}{2} \frac{(\log \kappa_i - c_i)^2}{d_i^2} \end{aligned} \quad (9.23)$$

For the individual WWZ and ZH measurements, the corresponding signal strengths are both included in the NLL:

$$\begin{aligned}
 L_{2D,13} = & \sum_{i=1}^{21} (\kappa_i^{(1)} \mu_{1,13} S_i^{(1)} + \kappa_i^{(2)} \mu_{2,13} S_i^{(2)} + \beta_i B_i) \\
 & - N_i \log \left[\kappa_i^{(1)} \mu_{1,13} S_i^{(1)} + \kappa_i^{(2)} \mu_{2,13} S_i^{(2)} + \beta_i B_i \right] \\
 & + \beta_i + \frac{1}{2} \frac{(\log \beta_i - a_i)^2}{\alpha_i^2} + \kappa_i^{(1)} + \frac{1}{2} \frac{(\log \kappa_i^{(1)} - c_i^{(1)})^2}{d_i^{(1)2}} + \kappa_i^{(2)} + \frac{1}{2} \frac{(\log \kappa_i^{(2)} - c_i^{(2)})^2}{d_i^{(2)2}}
 \end{aligned} \tag{9.24}$$

where $\mu_{1,13}$ is the signal strength for the WWZ process at 13 TeV and $\mu_{2,13}$ is the signal strength for the ZH process at 13 TeV. During the minimization of this NLL for a particular process (e.g. WWZ), the other process (e.g. ZH) is also profiled in the fit.

Run 3:

In the Run 3 analysis, corresponding to $\sqrt{s} = 13$ TeV, 13 total bins are fit (8 SRs and 5 CRs). The form of the likelihood for the inclusive measurement is the same as the one shown in Eq. 9.23, but with a sum over 13 bins rather than 21. Similarly, for the individual WWZ and ZH measurements, the likelihood is of the same form as shown in Eq. 9.24 but with the sum occurring over 13 bins rather than 21.

Combined (Run 2 + Run 3):

This analysis also performs a measurement of the Combined (Run 2 + Run 3) signal strengths for all three processes to assess the consistency of the measurements with the SM over multiple center-of-mass energies. The combined signal strength can be thought of as a "weighted average" over the signal strengths at $\sqrt{s} = 13$ and 13.6 TeV. In this case, the likelihood involves a sum over all bins from the Run 2 and Run 3 analysis, corresponding to 34 bins in total. For the inclusive measurement, the likelihood is of the same form as that presented in Eq. 9.23, while for the individual WWZ and ZH measurements the likelihood takes the same form as the one shown

in Eq. 9.24.

9.4.2 Significance

In addition to the signal strength measurements, we are also interested in the statistical significance of the signal processes (inclusive, WWZ , and ZH) with respect to the background-only hypothesis. In the limit of high statistics (i.e. a large number of event counts), the likelihood for μ becomes Gaussian with mean $\hat{\mu}$ (the maximum-likelihood solution) and standard deviation σ due to Wilks' Theorem [117]:

$$\mathcal{L}(\mu) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\mu - \hat{\mu})^2}{2\sigma^2}\right) \quad (9.25)$$

with maximum

$$\mathcal{L}(\hat{\mu}) = \frac{1}{\sqrt{2\pi}\sigma} \quad (9.26)$$

Using Eqs. 9.25 and 9.26, the DLL (see Eq. 9.21) can be written as:

$$DLL(\mu) = \log\left(\frac{\mathcal{L}(\hat{\mu})}{\mathcal{L}(\mu)}\right) = \frac{(\mu - \hat{\mu})^2}{2\sigma^2} \quad (9.27)$$

which is simply just a chi-squared (χ^2) distribution with one degree of freedom. Setting $\mu = 0$ (the background-only hypothesis) in Eq. 9.27, the Z-score of the maximum-likelihood solution can be identified as:

$$\text{Z-score: } \frac{\hat{\mu}}{\sigma} = \sqrt{2 \times DLL(\mu = 0)} \quad (9.28)$$

where $\hat{\mu}$ is expressed as some multiple of standard deviations (σ).

In this analysis, we compute the observed and expected significance using Eq. 9.28 for the inclusive, WWZ , and ZH processes for the Run 2, Run 3, and Combined (Run

2 and Run 3) data-taking periods. When computing the significance for the WWZ and ZH processes, we treat the other process as background whose normalization is profiled in the fit as a nuisance parameter.

9.5 Sources of Systematic Uncertainty

The primary sources of systematic uncertainty, their correlations across the different data-taking periods, and their impact on the analysis are summarized in Table 9.1.

There are two major categories of systematic uncertainties that are considered in this analysis: those arising from experimental sources (described in Sec. 9.5.1) and those arising from the theoretical modeling (described in Sec. 9.5.2). Additionally, a small description of the relevant data-to-MC correction (which the systematic uncertainty is associated with) is provided, if applicable. In general, the uncertainties are treated as correlated across all data-taking years unless otherwise specified. Impact plots are shown in Sec. F.1, where impacts of the various systematic uncertainties on the measured signal strength parameters are shown for fits with **Asimov data** in the SRs (Sec. F.1.1) and for fits with unblinded data (Sec. F.1.2).

In some cases, the minor background processes (e.g. WZ and "other") have yields consistent with zero (predicted yield is smaller than the MC statistical uncertainty) in a particular bin. To address this, we fix the yields for these processes to be equal to zero in the affected bins. The overall impact of this procedure on the total SR MC yields is very small ($\approx 0.2\%$).

9.5.1 Experimental Uncertainties

Luminosity: There are seven major contributions to the overall luminosity calibration uncertainty: length scale, orbit drift, transverse factorizability, beam-beam effects, beam current calibration, ghosts and satellites, and cross-detector con-

Systematic	Status of systematic uncertainties	
	Correlations for R2 and R3	Approximate avg. size
Lumi	Uncorrelated	1.6, 1.4% (R2, R3)
PU	Correlated	2 %
Prefire	N/A (only R2)	< 1%
JER/JES	Uncorrelated	< 1% (per source)
MET	Uncorrelated	1% , < 1%
Ele. efficiency	Uncorrelated	4, 7 % (R2, R3)
Muon efficiency	Uncorrelated	< 1%
B-tagging	See text	< 1% (per source)
Fake Uncertainty	Uncorrelated	30% (on WZ)
μ_R/μ_F	Correlated	< 1%
ISR/FSR	Correlated	< 1%
Background xsec	Correlated	30% (on "other" bkg.)

Table 9.1: Summary of systematic uncertainties. The approximate average size is shown (i.e., size of the effect on yields), averaging across signal region categories. For systematics that have multiple components (e.g. b-tagging) we have also averaged across the different sources to find a typical representative size. For the systematics that impact only a single process (e.g. cross section normalization of "other" background processes), the size of that effect on the relevant process is directly reported.

sistency [118]. The length scale contribution refers to the uncertainty in determining the absolute beam separation by comparing the nominal LHC-provided beam displacements with the CMS tracker coordinate system from offline reconstructed vertices. The orbit drift contribution comes from the measurement precision of the beam position monitors at CMS. The transverse factorizability contribution is determined by using specialized scans to estimate the deviation from the assumption that the beam shapes are factorizable into independent transverse proton density functions of the transverse (x and y) coordinates. The beam-beam effects contribution comes from the electromagnetic interactions between the two beams, resulting in an optical distortion of the beams and movement of the beams away from the nominal transverse position. The remaining contributions to the overall calibration uncertainty refer to uncertainties that come from corrections that are applied to more accurately estimate the

current produced by the beams in the measurement electronics, measurements of the charge in each bunch, and consistency among measurements of beam parameters across different measurement sources.

There are four major contributions to the overall luminosity integration uncertainty: afterglow, cross-detector stability, linearity, and CMS deadtime. The afterglow contribution refers to uncertainties associated with electronic spillover or de-excitation of the surrounding material after collisions that impact the luminosity measurement. The cross-detector stability contribution comes from the instability of the luminometer efficiency due to detector-specific effects, such as irradiation or aging. The linearity contribution is derived by comparing the luminosity measurement with separate counting methods for the luminosity determination. Lastly, the CMS deadtime contribution refers to uncertainties related to downtime in the CMS electronics that impact the accurate determination of the luminosity. The total systematic uncertainty associated with the CMS luminosity measurement for Run 2 (Run 3) is 1.6(1.4)% [119–121]([122]).

Pileup: A scale factor is applied to account for differences between the simulated pileup and the pileup in data. The uncertainty on this scale factor is determined by a standard procedure that involves varying the scale factor in a way according to the CMS pileup group’s prescription [123].

L1 Prefiring: This systematic is only relevant to Run 2. This systematic is most relevant to the 2016 and 2017 years of Run 2. During these years, radiation-induced damage caused some trigger primitives to be associated with an incorrect bunch crossing, causing the L1 trigger to fire on the wrong event, which can block the subsequent event from being accepted (because of L1 trigger rules). This means that events with potentially relevant physics is not allowed to be accepted. More details are included in Appendix A of Ref. [124]. Although

2016-2017 are most impacted by pre-firing effects, the correction and associated uncertainties are applied to all years of Run 2 (as the L1 muon pre-firing remains in 2018 as well). The effects of the issue depend on the p_T and η of the objects, and the effect is not modeled in the MC. Instead, the correction is handled by event weights (and corresponding uncertainties) provided centrally by CMS in the NanoAOD v9 datasets for Run 2.

Jet energy scale (JES), resolution (JER), and p_T^{miss} : For Run 2 and Run 3, we use the JME POG-provided JES and JER corrections [125]. JES corrections are applied to both data and MC, while JER smearing is only applied to MC per the POG recommendation [126]. MET Type-1 corrections are then applied to both data and MC based on the JEC-JER corrected jets.

JEC and JER corrections are done before performing object cuts and event selections as these corrections have a direct impact on the p_T of the jets.

For Run 2, the p_T^{miss} is then corrected using the Type-1 correction. We only consider jets that do not overlap with the candidate leptons, $p_T > 15$, $|\eta| < 5.1$, and $jetEMfraction < 0.9$. In Run 3 we utilize PUPPI p_T^{miss} , and thus the Type-1 correction does not apply.

For the JEC/JER/MET uncertainties, we have a single uncorrelated uncertainty for each. Because these are treated as uncorrelated across years, this brings the total number of independent nuisance parameters to 24 (3 for each of the 8 data taking periods). For MET specifically, the uncertainty is for the unclustered energy.

Lepton identification: As described in Sec. 5.2.1, we use the “topmva” ID (and associated systematic uncertainties for electrons and muons) for Run 2. This ID was developed for CMS TOP-22-013 [93]. For electrons, we also apply the

Reco SFs (and associated uncertainties) provided by the EGM POG [127], as recommended to be applied with the the topmva SFs. For Run 3, we utilize the POG provided scale factors and associated uncertainties [128–130] for the recommended ID which is described in section 5.2.2. Since different lepton IDs are used in Run 2 and Run 3, they are not treated as correlated. We thus have 4 total nuisance parameters for lepton SFs (for electrons and for muons in Run 2 and in Run 3)

b-tag scale factors: For b-tagging corrections, this analysis uses the Method 1a approached defined by the CMS BTV POG [131]. The efficiencies are measured privately, as described in Appendix F of AN 2024-024 [132].

For UL16, an issue has been observed in centrally-provided scale factors (SFs) for light-flavored jets (visible in DY CRs e.g. as reported in AN 2021-051 [133], similarly observed by AN 2021-040 [134]). To work around this issue, we follow the same approach employed by these previous analyses, i.e., the UL16APV SFs for light flavor jets are applied for UL16.

The SFs (to account for differences between data and MC) and corresponding uncertainties are provided centrally by the BTV POG. Some of the details of the handling of the systematic variations are different between the Run 2 and Run 3, so these are discussed separately below:

- Run 2: The uncertainties include a variation specific to the heavy (b,c) and light jets; for both, there is a component which is correlated across years and a component which is uncorrelated across years. This brings the total number of systematic uncertainties to 10.
- Run 3: The uncertainties for Run 3 also include a variation specific to the heavy (b,c) and light jets. However, while there is a correlated and uncor-

related component for the heavy jets, there is currently only a correlated component for the light jets (as of Summer 2024). This brings the total number of systematic uncertainties for Run 3 to 4. Please also note that the 2023 and 2023BPix b-tag scale factors are currently not available (as of Summer 2024).

Between Run 2 and Run 3, the sources that are explicitly uncorrelated for given years remain uncorrelated. The sources that are correlated within a given run (i.e. the sources within Run 2, and sources within Run 3) are correlated between Run 2 and Run 3.

Uncertainty on nonprompt estimation: As described in Section 7.1.3, a 30% rate uncertainty is applied to the WZ contribution. Since the nonprompt contribution is related to the lepton identification, and the lepton identification is different between Run 2 and Run 3, the uncertainty is treated as uncorrelated between Run 2 and Run 3.

We note that a trigger efficiency SF is not applied (i.e. the assumption is that it is 1). The justification for this is that the analysis only studies 4-lepton final states, so the dilepton triggers used in the analysis should indeed be essentially fully efficient (and furthermore, the analysis is not a precision analysis).

9.5.2 Theoretical Uncertainties

Renormalization and factorization scale: The renormalization (μ_R) and factorization (μ_F) scales are varied up and down by a factor of 2 for the MC (theory) prediction. The weights for these variations are computed during the generation of the sample, and are provided centrally in the NanoAOD. For μ_R and μ_F , the variations are applied to the events passing the event selection, but also to the

sum of all generated events; this avoids a double counting of the cross-section uncertainty (which is usually estimated in an alternative approach, e.g. a data driven normalization, or a theory uncertainty associated with an NLO cross section to which the sample is normalized). For this analysis, the μ_R and μ_F variations are treated independently (i.e., the “envelope method” is not used, as this can lead to some shape effects being lost). This approach of treating the μ_R and μ_F independently is relatively common (e.g. used in CMS TOP-22-006 [135] and CMS HIG-17-027 [136]). The μ_R systematic is correlated across all data-taking year. The μ_F systematic is correlated across all data-taking years. Each are uncorrelated across processes.

Parton shower: The initial and final state radiation (ISR and FSR) scales are varied up and down by a factor of 2. These variations are computed during the parton shower and are provided centrally in the NanoAOD. In this analysis, these variations are applied to the events passing the event selection, but not to the sum of all generated events (as the fluctuations on the sum of all generated weights is not centrally provided in the NanoAOD). This may result in a small overestimate of the uncertainty in some cases because it will introduce a variation in the overall normalization (and the uncertainty on the normalization of most processes is already handled through a different method as explained in the μ_R/μ_F section). However, the effect should not be significantly impactful because the ISR/FSR systematic is generally not large, and furthermore the analysis is not particularly sensitive to systematics (as the uncertainty is more dominated by statistics). The ISR systematic is correlated across all data-taking year; it is uncorrelated among processes. The FSR systematic is correlated across all data-taking years; it is uncorrelated among processes.

Cross section: The normalization of ZZ , $t\bar{t}Z$, and tWZ are being extracted in a

data-driven way, so we do not apply an additional normalization uncertainty on top of this. For the “other” background processes (estimated fully from MC), we apply a 30% uncertainty to account for normalization uncertainties. For the signal processes, we apply a 15% uncertainty.

CHAPTER 10

RESULTS

"Well..... we're waiting!"

Judge Smails (Ted Knight)

Caddyshack (1980)

10.1 Overview

The SRs described in Chapter 6 and the CRs described in Chapter 7 are used for the extraction of the signal strength parameters and the evaluation of the significance, as explained in Chapter 9. The (pre-fit) yields for data and MC in the SRs and CRs are summarized in figure 10.1. The results for the Run 2 (Run 3) portion of the analysis are described in Section 10.2 (10.3). The results for the Combined (Run 2 + Run 3) analysis are presented in Section 10.4. The full set of results are summarized in Section 10.5.

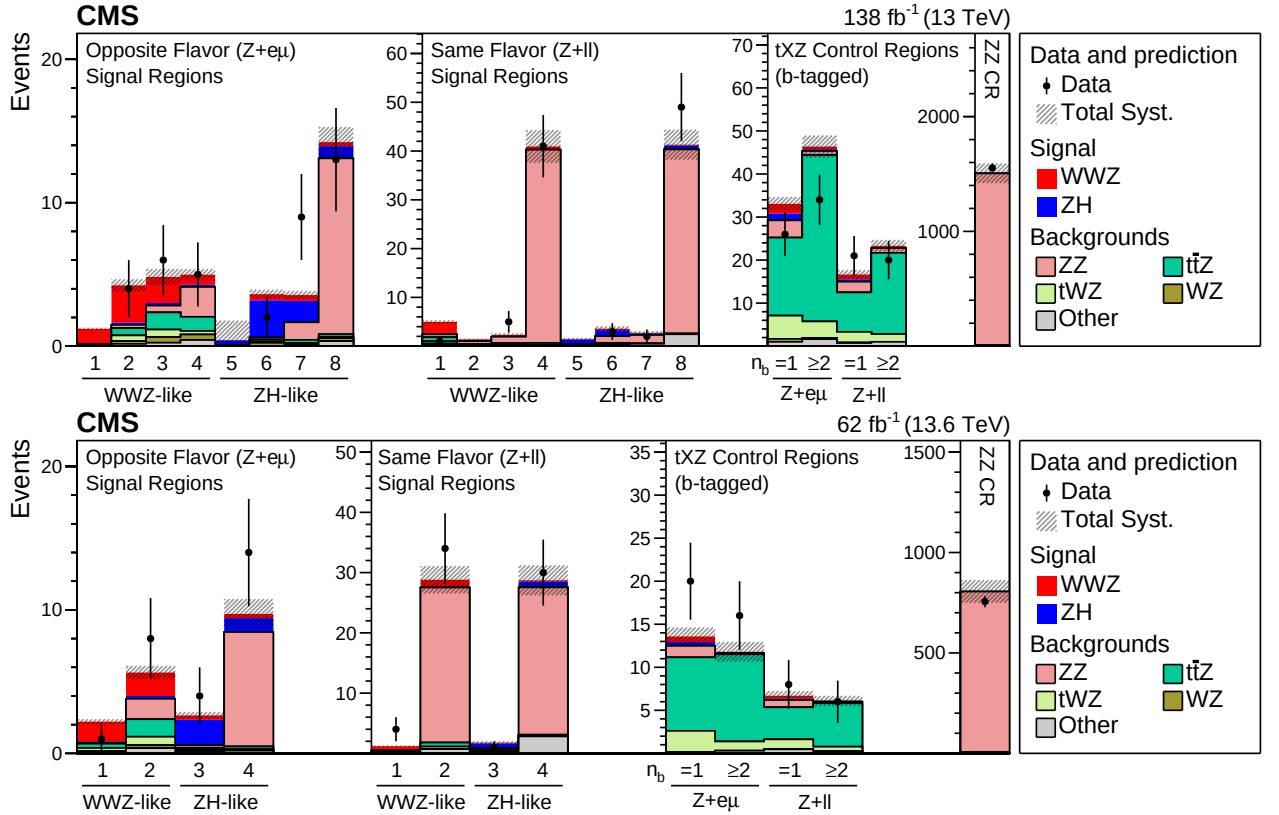


Figure 10.1: Yields for data and MC in signal and control regions for the Run 2 (top) and Run 3 (bottom) portions of the analysis.

10.2 Run 2 Results

Using the 21 SR + CR bins shown in the top portion of Figure 10.1, the signal strength and significance for the Run 2 portion of the analysis are evaluated using the methods described in Sections 9.4.1 and 9.4.2, respectively.

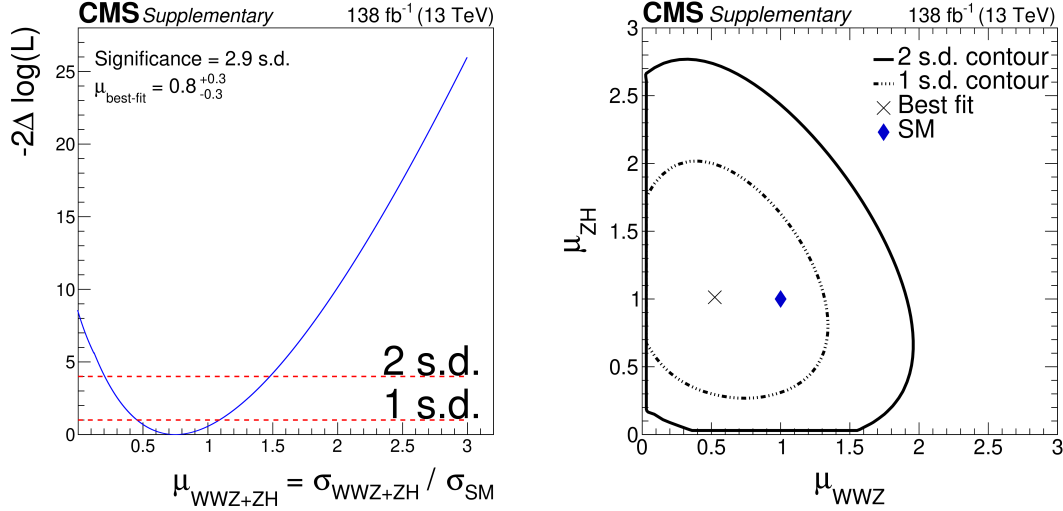


Figure 10.2: Delta-log-likelihood as a function of the inclusive signal strength parameter (left) and the individual WWZ and ZH signal strength parameters (right) for the Run 2 portion of the analysis.

Figure 10.2 shows the 1D (left) and 2D (right) likelihood scans for the Run 2 portion of the analysis; the measured signal strength for the inclusive ($WWZ + ZH$) process is:

$$\mu_{incl.}^{Run2} = 0.75^{+0.32}_{-0.28}(\text{stat.})^{+0.10}_{-0.08}(\text{sys.}) \quad (10.1)$$

corresponding to an observed (expected) significance of

$$Z_{incl.}^{Run2} = 2.9 \text{ (4.4)} \sigma \quad (10.2)$$

The theoretical cross section for the inclusive process is obtained by summing the non-resonant WWZ cross section [76] (calculated at NLO) and the ZH cross section

[88] (calculated at NNLO) times the $H \rightarrow W W$ branching fraction [137]:

$$\begin{aligned}\sigma_{\text{theo.,incl.}}^{13\text{TeV}} &= \sigma_{\text{WWZ}}^{13\text{TeV}} + (\sigma_{\text{ZH}}^{13\text{TeV}} \times \mathcal{BR}(H \rightarrow W W)) \\ &= 184 \text{ fb} + 189 \text{ fb} = 373 \text{ fb}\end{aligned}\tag{10.3}$$

which neglects contributions from the interference of these processes (which is small). Using Eq. 10.3, the cross section for the inclusive process at $\sqrt{s} = 13 \text{ TeV}$ is measured to be:

$$\sigma_{\text{obs.,incl.}}^{13\text{TeV}} = \mu_{\text{incl.}}^{\text{Run2}} \times \sigma_{\text{theo.,incl.}}^{13\text{TeV}} = 280_{-110}^{+120}(\text{stat.})_{-30}^{+40}(\text{sys.}) \text{ fb}\tag{10.4}$$

The signal strengths for the non-resonant $W W Z$ process and the $Z H$ ($H \rightarrow W W$) process are measured to be:

$$\mu_{\text{WWZ}}^{\text{Run2}} = 0.52_{-0.41}^{+0.50}(\text{stat.})_{-0.12}^{+0.13}(\text{sys.})\tag{10.5}$$

and

$$\mu_{\text{ZH}(H \rightarrow \text{WW})}^{\text{Run2}} = 1.01_{-0.51}^{+0.62}(\text{stat.})_{-0.08}^{+0.14}(\text{sys.})\tag{10.6}$$

respectively. The observed (expected) significance for the non-resonant $W W Z$ process and the $Z H$ process are found to be:

$$Z_{\text{WWZ}}^{\text{Run2}} = 1.3 (3.1) \sigma\tag{10.7}$$

$$Z_{\text{ZH}(H \rightarrow \text{WW})}^{\text{Run2}} = 2.0 (2.6) \sigma\tag{10.8}$$

Combining the measured signal strengths from Eqs. 10.5 and 10.6 with the theoretical

cross sections shown in Eq. 10.3, we obtain the following value for the cross section (cross section times $H \rightarrow WW$ branching ratio) for the non-resonant WWZ (ZH ($H \rightarrow WW$)) process:

$$\sigma_{\text{obs.,WWZ}}^{13\text{TeV}} = \mu_{\text{WWZ}}^{\text{Run2}} \times \sigma_{\text{WWZ}}^{13\text{TeV}} = 97_{-75}^{+91}(\text{stat.})_{-21}^{+24}(\text{sys.}) \text{ fb} \quad (10.9)$$

$$\begin{aligned} \sigma_{\text{obs.,ZH}}^{13\text{TeV}} \times \mathcal{BR}(H \rightarrow WW) &= \mu_{\text{ZH}(H \rightarrow WW)}^{\text{Run2}} \times \sigma_{\text{ZH}}^{13\text{TeV}} \times \mathcal{BR}(H \rightarrow WW) \\ &= 190_{-100}^{+120}(\text{stat.})_{-20}^{+30}(\text{sys.}) \text{ fb} \end{aligned} \quad (10.10)$$

10.3 Run 3 Results

Using the 13 SR + CR bins shown in the bottom portion of Figure 10.1, the signal strength and significance for the Run 3 portion of the analysis are evaluated using the methods described in Sections 9.4.1 and 9.4.2, respectively.

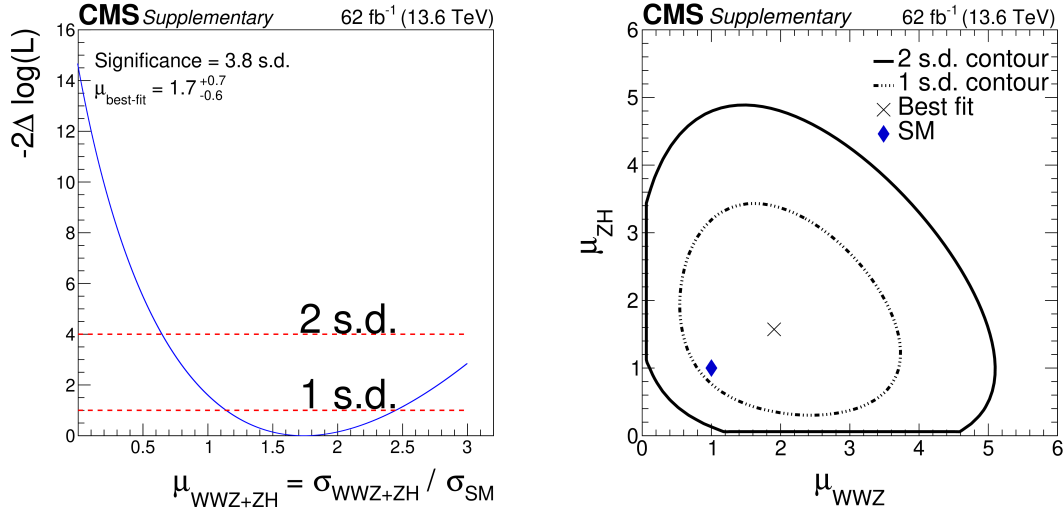


Figure 10.3: Delta-log-likelihood as a function of the inclusive signal strength parameter (left) and the individual WWZ and ZH signal strength parameters (right) for the Run 3 portion of the analysis.

Figure 10.3 shows the 1D (left) and 2D (right) likelihood scans for the Run 3

portion of the analysis; the measured signal strength for the inclusive ($WWZ + ZH$) process is:

$$\mu_{\text{incl.}}^{\text{Run3}} = 1.74^{+0.67}_{-0.58}(\text{stat.})^{+0.22}_{-0.15}(\text{sys.}) \quad (10.11)$$

corresponding to an observed (expected) significance of

$$Z_{\text{incl.}}^{\text{Run3}} = 3.8 \text{ (2.5) } \sigma \quad (10.12)$$

The theoretical cross section for the inclusive process is obtained by summing the non-resonant WWZ cross section [76] (calculated at NLO) and the ZH cross section [89] (calculated at NNLO) times the $H \rightarrow WW$ branching fraction [137]:

$$\begin{aligned} \sigma_{\text{theo.,incl.}}^{13.6\text{TeV}} &= \sigma_{\text{WWZ}}^{13.6\text{TeV}} + (\sigma_{\text{ZH}}^{13.6\text{TeV}} \times \mathcal{BR}(H \rightarrow WW)) \\ &= 200 \text{ fb} + 202 \text{ fb} = 402 \text{ fb} \end{aligned} \quad (10.13)$$

which neglects contributions from the interference of these processes (which is small). Using Eq. 10.13, the cross section for the inclusive process at $\sqrt{s} = 13.6$ TeV is measured to be:

$$\sigma_{\text{obs.,incl.}}^{13.6\text{TeV}} = \mu_{\text{incl.}}^{\text{Run3}} \times \sigma_{\text{theo.,incl.}}^{13.6\text{TeV}} = 700^{+270}_{-230}(\text{stat.})^{+90}_{-60}(\text{sys.}) \text{ fb} \quad (10.14)$$

The signal strengths for the non-resonant WWZ process and the ZH ($H \rightarrow WW$) process are measured to be:

$$\mu_{\text{WWZ}}^{\text{Run3}} = 1.91^{+1.12}_{-0.94}(\text{stat.})^{+0.25}_{-0.16}(\text{sys.}) \quad (10.15)$$

and

$$\mu_{\text{ZH}(H \rightarrow \text{WW})}^{\text{Run3}} = 1.57_{-0.89}^{+1.12}(\text{stat.})_{-0.14}^{+0.29}(\text{sys.}) \quad (10.16)$$

respectively. The observed (expected) significance for the non-resonant WWZ process and the ZH process are found to be:

$$Z_{\text{WWZ}}^{\text{Run3}} = 2.5 \text{ (1.3) } \sigma \quad (10.17)$$

$$Z_{\text{ZH}(H \rightarrow \text{WW})}^{\text{Run3}} = 2.5 \text{ (1.7) } \sigma \quad (10.18)$$

Combining the measured signal strengths from Eqs. 10.15 and 10.16 with the theoretical cross sections shown in Eq. 10.13, we obtain the following value for the cross section (cross section times $H \rightarrow WW$ branching ratio) for the non-resonant WWZ (ZH ($H \rightarrow WW$)) process:

$$\sigma_{\text{obs.,WWZ}}^{13.6\text{TeV}} = \mu_{\text{WWZ}}^{\text{Run3}} \times \sigma_{\text{WWZ}}^{13.6\text{TeV}} = 380_{-190}^{+220}(\text{stat.})_{-30}^{+50}(\text{sys.}) \text{ fb} \quad (10.19)$$

$$\begin{aligned} \sigma_{\text{obs.,ZH}}^{13.6\text{TeV}} \times \mathcal{BR}(H \rightarrow WW) &= \mu_{\text{ZH}(H \rightarrow \text{WW})}^{\text{Run3}} \times \sigma_{\text{ZH}}^{13.6\text{TeV}} \times \mathcal{BR}(H \rightarrow WW) \\ &= 320_{-180}^{+230}(\text{stat.})_{-30}^{+60}(\text{sys.}) \text{ fb} \end{aligned} \quad (10.20)$$

10.4 Combined Results

The combined (Run 2 + Run 3) results are obtained by performing a fit (see Sec. 9.4.1) and evaluating the significance (see Sec. 9.4.2) for the 34 total SR and CR bins shown in Fig. 10.1. The 1D and 2D likelihood scans as a function of the signal strength parameters are shown in the left and right panels of Figure 10.4, respectively.

The measured signal strength for the inclusive ($WWZ + ZH$) process for the

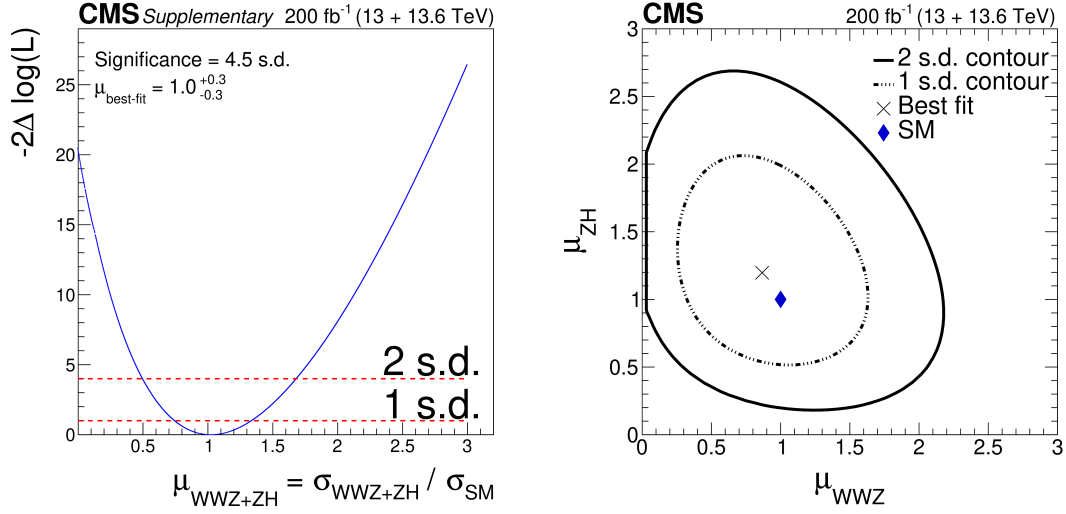


Figure 10.4: Delta-log-likelihood as a function of the inclusive signal strength parameter (left) and the individual WWZ and ZH signal strength parameters (right) for the Combined (Run 2 + Run 3) analysis.

combined (Run 2 + Run 3) analysis is:

$$\mu_{incl.}^{Comb.} = 1.03^{+0.31}_{-0.28} \quad (10.21)$$

with an observed (expected) significance of:

$$Z_{incl.}^{Comb.} = 4.5 \text{ (5.0)} \sigma \quad (10.22)$$

Breaking the contributions down by process, the observed (expected) significance of the non-resonant WWZ process combined across the two center-of-mass energies is:

$$Z_{WWZ}^{Comb.} = 2.4 \text{ (3.3)} \sigma \quad (10.23)$$

For the ZH process, the observed (expected) significance combined across both center-of-mass energies is:

$$\mu_{ZH(H \rightarrow WW)}^{\text{Comb.}} = 3.1 (3.1) \sigma \quad (10.24)$$

10.5 Summary

The cross section measurements obtained for the Run 2 and Run 3 portions of the analysis are summarized in Table 10.1. Additionally, the observed and expected significances obtained for the Run 2, Run 3, and Combined (Run 2 + Run 3) periods, broken down by process, are summarized in Table 10.2.

Era	Process	$\mu = \sigma_{\text{obs}}/\sigma_{\text{SM}}$	σ_{obs} (fb)	σ_{SM} (fb)
Run 2 ($\sqrt{s} = 13$ TeV)	WWZ only	$0.52^{+0.50}_{-0.41}(\text{stat.})^{+0.13}_{-0.12}(\text{syst.})$	$97^{+91}_{-75}(\text{stat.})^{+24}_{-21}(\text{syst.})$	184 [76]
	ZH only	$1.01^{+0.62}_{-0.51}(\text{stat.})^{+0.14}_{-0.08}(\text{syst.})$	$190^{+120}_{-100}(\text{stat.})^{+30}_{-20}(\text{syst.})$	189 [88]
	Inclusive	$0.75^{+0.32}_{-0.28}(\text{stat.})^{+0.10}_{-0.08}(\text{syst.})$	$280^{+120}_{-110}(\text{stat.})^{+40}_{-30}(\text{syst.})$	373
Run 3 ($\sqrt{s} = 13.6$ TeV)	WWZ only	$1.91^{+1.12}_{-0.94}(\text{stat.})^{+0.25}_{-0.16}(\text{syst.})$	$380^{+220}_{-190}(\text{stat.})^{+50}_{-30}(\text{syst.})$	200 [76]
	ZH only	$1.57^{+1.12}_{-0.89}(\text{stat.})^{+0.29}_{-0.14}(\text{syst.})$	$320^{+230}_{-180}(\text{stat.})^{+60}_{-30}(\text{syst.})$	202 [89]
	Inclusive	$1.74^{+0.67}_{-0.58}(\text{stat.})^{+0.22}_{-0.15}(\text{syst.})$	$700^{+270}_{-230}(\text{stat.})^{+90}_{-60}(\text{syst.})$	402

Table 10.1: Measured signal strengths and cross sections for the WWZ , ZH (with $H \rightarrow WW$) and inclusive ($WWZ + ZH$) processes at $\sqrt{s} = 13$ and 13.6 TeV. For the ZH process, the cross section times the $H \rightarrow WW$ branching fraction [137] is reported. The last column shows the SM expectation; the WWZ cross sections are calculated to NLO precision while the ZH cross sections are calculated at NNLO. In the signal strengths reported here, the uncertainty of the predicted cross section is not included.

Figure 10.5 shows the measured signal strength parameters broken down by process and run period. Good agreement is found between the measurements and the SM prediction. Due to the size of the available dataset, the statistical uncertainties are considerably larger than the systematic ones. The post-fit values of the back-

Era	Inclusive	WWZ	ZH
Run 2	2.9 (4.4)	1.3 (3.1)	2.0 (2.6)
Run 3	3.8 (2.5)	2.5 (1.3)	2.5 (1.7)
Combined	4.5 (5.0)	2.4 (3.3)	3.1 (3.1)

Table 10.2: Observed (expected) significance of the signal strength in units of standard deviations. The WWZ -only and ZH -only values are obtained by profiling the other signal process, together with all other nuisances.

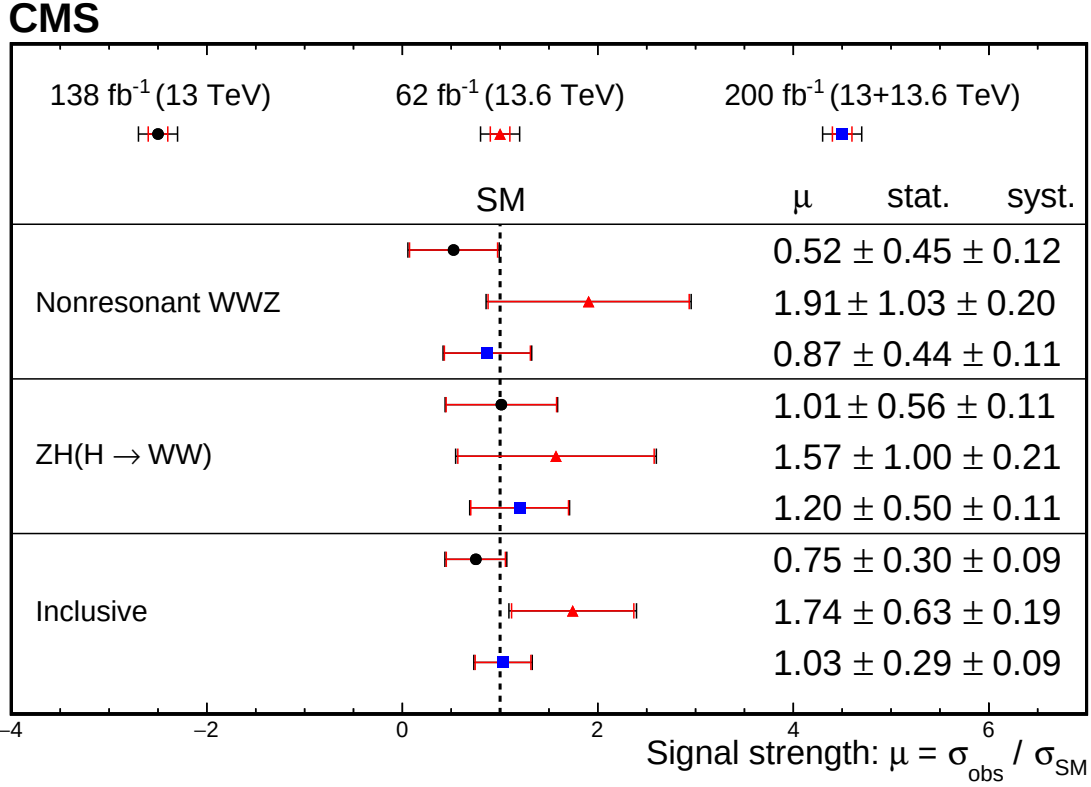


Figure 10.5: Measured values of the WWZ , ZH , and inclusive signal strength parameters. Results are shown separately by run period as well as after combining the two datasets. The red error bars on the signal strengths represent the statistical uncertainty, while the black error bars represent the total uncertainty. In the signal strengths reported here, the uncertainty of the predicted cross section is not included.

ground normalization parameters for the ZZ and tXZ processes are summarized in Table 10.3.

In summary, this thesis presents a measurement of the $pp \rightarrow WWZ$ process

in the four lepton final state using a dataset corresponding to a total integrated luminosity of $\mathcal{L} = 200 \text{ fb}^{-1}$ collected by the CMS experiment at $\sqrt{s} = 13$ and 13.6 TeV. The ratio of the cross section relative to the SM prediction is measured to be $1.03^{+0.31}_{-0.28}$, representing the most precise measurement of this process to date. With an observed significance in excess of three standard deviations, this result signifies the first evidence of a triboson production process at $\sqrt{s} = 13.6$ TeV. Additionally, this thesis marks the first simultaneous measurement of the non-resonant WWZ process and the ZH ($H \rightarrow WW$) process separately. Within the statistical uncertainties, the results are in agreement with the SM predictions.

Era	Parameter of Interest	ZZ normalization factor	tXZ normalization factor
Run 2 ($\sqrt{s} = 13$ TeV)	$\mu_{\text{incl.}}$	$1.04^{+0.06}_{-0.07}$	$0.81^{+0.12}_{-0.11}$
	μ_{WWZ}	$1.04^{+0.06}_{-0.07}$	$0.82^{+0.12}_{-0.11}$
	μ_{ZH}	$1.04^{+0.06}_{-0.07}$	$0.82^{+0.12}_{-0.11}$
Run 3 ($\sqrt{s} = 13.6$ TeV)	$\mu_{\text{incl.}}$	$0.94^{+0.07}_{-0.07}$	$1.33^{+0.26}_{-0.23}$
	μ_{WWZ}	$0.94^{+0.07}_{-0.07}$	$1.32^{+0.26}_{-0.23}$
	μ_{ZH}	$0.94^{+0.07}_{-0.07}$	$1.32^{+0.26}_{-0.23}$

Table 10.3: Post-fit values for the background normalization parameters.

CHAPTER 11
INTERMISSION

Congratulations! If you are reading this, you have successfully read the first part of my thesis, and you certainly deserve a nice break for enduring that. So get up, stretch your legs, use the restroom, or even get a snack! You'll need all the strength you can muster if you want to make it through *Part II*....

Part II

Beam Delivery System for a 10 TeV Wakefield Collider

CHAPTER 12

A BRIEF OVERVIEW OF COLLIDERS

"Whoever imagines that, because so many inventions and so many improvements in machinery have been made, there remains little else to be discovered, has but a feeble conception of the infinitude and vastness of mechanical powers, or of the unlimited reach of science. Much has been discovered, infinitely more remains unrevealed."

Senate Report Accompanying Senate

Bill No. 239, 24th Congress

(April 28, 1836)

12.1 Colliders at a glance

Colliders are the primary tools of discovery for many of the particles of the SM. Since their debut in the early 1960's, colliders and fixed target experiments have been responsible for the discovery of 13 of the 17 fundamental particles, as summarized in Table 12.1. In addition to extending the particle content of the SM, colliders have been an invaluable tool in the quest for precision measurements of the known particles, testing the predictions of the SM and informing future efforts as to where to look next. For example, LEP at CERN and SLC at SLAC were instrumental in determining the properties of the Z boson (decay channels, mass) [29] after its discovery at UA1 nearly a decade prior. As elucidated in the previous part of this thesis, despite the great advancements of the past century on determining a complete theory of the fundamental constituents and forces of nature, the SM, while a remarkably good theory, is still incomplete. While not certain, there is reason to believe that extensions of the SM would manifest themselves at future colliders.

Fundamental Particle Discoveries at Colliders and Fixed-Target Experiments		
Particle	Year Discovered	Collider
μ neutrino	1962	AGS [†] (Brookhaven)
Up/Down/Strange quarks	1969	SPEAR (SLAC)
Charm quark (J/ψ)	1974	SPEAR (SLAC) & AGS (Brookhaven)
τ lepton	1975	SPEAR (SLAC)
Bottom quark (Υ)	1977	Main Ring [†] (Fermilab)
Gluon (g)	1979	PETRA (DESY)
W and Z bosons	1983	SPS (CERN)
Top quark	1995	Tevatron (Fermilab)
τ neutrino	2000	Tevatron [†] (Fermilab)
Higgs boson	2012	LHC (CERN)

Table 12.1: List of fundamental particle discoveries at colliders and fixed-target experiments. [†] - denotes a fixed-target experiment.

There are two primary metrics by which all colliders are judged: the luminosity ($\mathcal{L}$) and the center-of-mass energy ($\sqrt{s}$). The luminosity, introduced in Section 2.2.3, describes the intensity of the collisions per unit time; a higher luminosity corresponds

to more events for a given scattering process, thus increasing the statistical power for discovering new phenomena. With regards to energy, Einstein (Eq. 2.26) tells us that matter and energy are interchangeable. Thus, the collider energy provides the total energy budget for particle production; the greater the energy, the heavier the particles that can be produced. Colliders are typically classified into one of two types based on the center-of-mass energy:

- *factories*, where $\sqrt{s}$ is chosen to be at the mass of a particular particle (e.g. SLC and proposed future Higgs factories). These are designed to study the particle in question in great detail via resonant production.
- *energy-frontier machines*, where $\sqrt{s}$ is maximized in order to search for new, heavy particles while also providing a broad physics program. The LHC falls under this category.

In the following sections, I describe the different types of colliders that have been proposed and the underlying reasons for their particular design. I also discuss the current status of the European Strategy and the recommendations of the P5 Report in motivating future collider designs.

12.2 Circular vs. Linear Colliders

It can be shown from Maxwell's equations (Eq. 1.6) that the acceleration of a charged particle results in the emission of radiation. If the acceleration is transverse to the motion of the particle, we refer to the resulting radiation as *synchrotron* radiation (SR). For circular colliders, where charged particles are bent to maintain a (pseudo) circular trajectory around the ring by dipole magnets, SR is always present. The power loss per unit length due to SR is given by the relativistic Larmor formula [138]:

$$P_\gamma = \frac{e^2 c}{6\pi\epsilon_0} \frac{E^4}{(mc^2)^4 \rho^2} \quad (12.1)$$

where E is the particle energy, m is the particle mass, and ρ is the bending radius. For fixed energy and bending radius, the ratio of SR power loss per unit length for protons versus electrons is simply the inverse ratio of their masses to the fourth power:

$$\frac{P_\gamma(p)}{P_\gamma(e)} = \left[\frac{m_e}{m_p} \right]^4 \approx 10^{-13} \quad (12.2)$$

The difference in radiated power for protons and electrons due to SR is enormous; roughly a factor of 10^{-13} . Because of this, it is highly favorable to utilize protons in circular machines as their energy loss due to SR is much lower than that for electrons. Equation 12.1 can be manipulated to give the energy loss per turn:

$$\Delta E_{\text{turn}} = \frac{4\pi}{3} \frac{r_e}{(mc^2)^3} \frac{E^4}{\rho} \quad (12.3)$$

which, for an electron, can be rewritten in the convenient form:

$$\Delta E_{\text{e,turn}}[\text{keV}] = 88.46 \frac{(E[\text{GeV}])^4}{\rho[\text{m}]} \quad (12.4)$$

At high energies in particular, the energy loss due to SR for electrons in a circular machine can approach or even exceed the energy gain from the RF system. This ultimately sets a limit on the energy and size at which a circular electron machine is no longer feasible from a power consumption standpoint, as the injected RF energy is quickly lost to SR. Table 12.2 shows the SR energy loss per turn for various electron-positron colliders, including the proposed FCC-ee and CEPC. The problem of SR for electrons at high energies can be avoided by utilizing linear accelerators, whereby electron bunches are accelerated along a straight path with minimal bending sections incorporated. Thus, SR is a central consideration for collider design and it motivates

a further classification between high-energy electron machines being linear and high-energy proton machines being circular.

Collider	SR Energy Loss		
	E_{beam} [GeV]	ρ [m]	ΔE_{turn} [GeV]
LEP (Z-pole)	45.6	3026.42	0.13
LEP (WW)	80.5	3026.42	1.23
LEP (Max. Energy)	104.5	3026.42	3.49
FCC-ee (Z-pole)	45.6	10021	0.04
FCC-ee (WW)	80	10021	0.36
FCC-ee (ZH)	120	10021	1.83
FCC-ee ($t\bar{t}$)	182.5	10021	9.79
CEPC (Z-pole)	45.6	10700	0.04
CEPC (WW)	80	10700	0.34
CEPC (ZH)	120	10700	1.71
CEPC ($t\bar{t}$)	180	10700	8.68

Table 12.2: Energy loss per turn due to synchrotron radiation for existing and proposed circular electron-positron colliders. Values for LEP obtained from ref. [139]. Values for FCC-ee obtained from ref. [140]. Values for CEPC obtained from ref. [141], assuming 50 MW SR power per beam.

Circular machines utilize radio-frequency (RF) acceleration sections that inject energy during each turn, which is highly efficient from an RF power consumption standpoint. Thus, higher energies can be achieved by more repetitions, assuming that the energy loss to SR can be compensated effectively. However, the larger beam energy increases the beam rigidity, requiring increased dipole strengths to keep the beam within a fixed bending radius (see Eq. 3.5). Thus, the beam energy for a fixed radius circular machine is primarily limited by the dipole strengths in the bending sections. Increasing the bending radius to alleviate the need for high-field dipoles introduces another problem: the overall size of the collider grows, making it considerably more expensive. Circular machines also typically utilize injection schemes that allow for bunches to occupy almost full ring, as shown in Figure 12.1 for the LHC. This allows for a very high repetition rate (the number of collisions per second) which increases the luminosity.

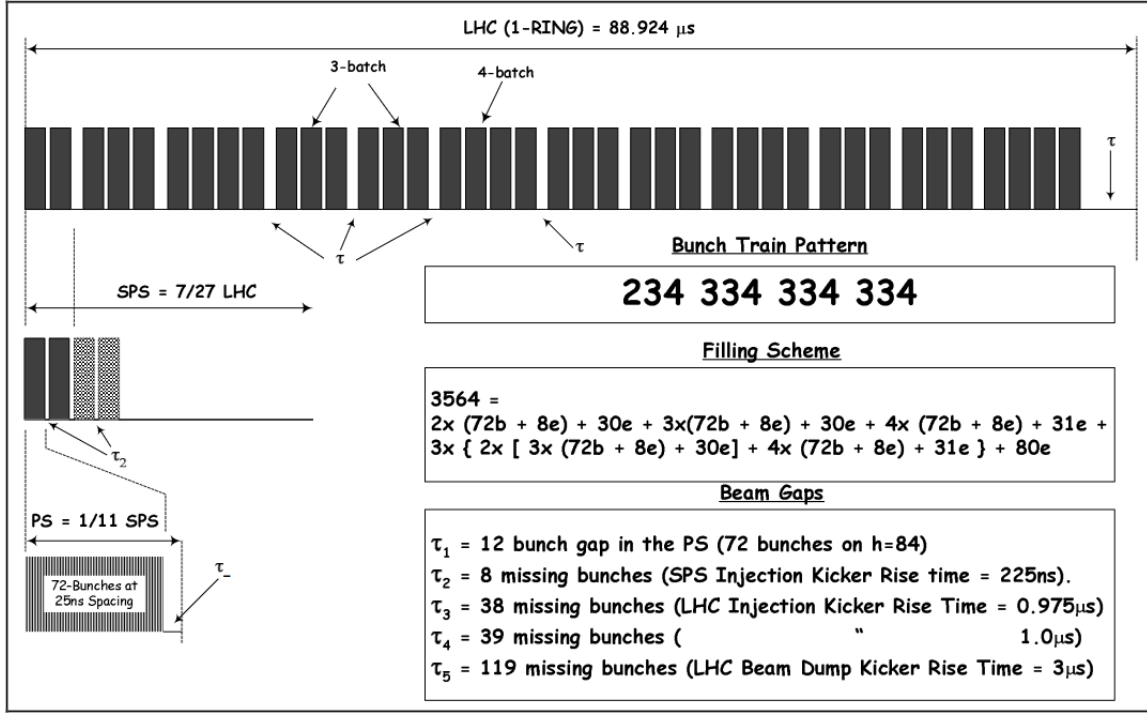


Figure 12.1: Bunch filling scheme for the LHC. Figure taken from ref. [142].

Linear colliders, in contrast to their circular counterparts, accelerate and collide bunches in a single pass. To achieve high energies, linear colliders require either long accelerating sections (which can be costly) or very high acceleration gradients. Due to the large mass of the proton relative to the electron, the acceleration of electrons is considerably easier for a fixed acceleration gradient. This, along with considerations from SR, provides a compelling rationale for utilizing electrons in linear machines. In addition to the high gradients and/or long accelerating sections required for high energy applications, linear machines typically have much smaller repetition rates than circular machines. To achieve high luminosities, linear colliders must therefore compensate by increasing the bunch charge, decreasing the beam spot size, or both (see Eq. 2.19).

12.3 Protons vs Electrons

In the previous section, I described how the relative SR power loss for protons and electrons at high energies prompts the choice for electron-positron colliders to be linear machines while protons are favorable for circular machines. Electrons and protons are chosen for (nearly) all collider designs due to the fact that they are stable particles (e.g. they do not undergo decays). One exception to this is the proposed Muon collider [143], which attempts to accelerate and collide muons prior to their decays. The basic differences between electrons and protons also manifests in different methods for manipulating the transverse phase space and achieving high luminosity, differences in the properties of the collisions, and the types of backgrounds that are produced near the interaction point.

Electrons are fundamental particles, while protons are composite particles comprised of quarks and gluons (see Section 1.2). Thus, when an interaction occurs in electron-positron collisions, the energy used for particle production is (approximately) the center of mass energy of the collisions. For proton-proton collisions, the available energy for particle production is significantly lower than the center-of-mass energy of the collider, as the quarks or gluons participating in the interaction carry some fraction of the total proton momentum (see Section 2.2.6), resulting in a broad spectrum of the luminosity with the collision energy. Consequently, electron-positron machines can have a similar energy reach (and therefore discovery potential) to proton-proton machines of considerably higher center-of-mass energy.

Near the interaction point of a collider, charged particles can interact with the electric field of the opposing beam, resulting in the emission of synchrotron radiation. This effect, known as *beamstrahlung*, becomes prominent for dense electron bunches at high energies (see Equation 13.72), while for protons this effect is negligible in comparison. The emission of SR for a given electron decreases its energy, smearing the

beam energy spectrum at the interaction point and resulting in a sizable contribution to the luminosity from collisions at lower-than-nominal energy. Figure 12.2 shows the luminosity spectrum as a function of energy for an electron-positron collider at pCM energies of 3, 10, and 30 TeV for different beam configurations; the beam-beam interaction causes the luminosity distribution to broaden in energy, and the effect is more extreme in the round-beam case, while still having a significant effect for flat-beam scenarios. The SR photons emitted from the beamstrahlung interaction can also enter the detector, or pair-produce to give additional electron-positron pairs that enter the detector, resulting in detector backgrounds.

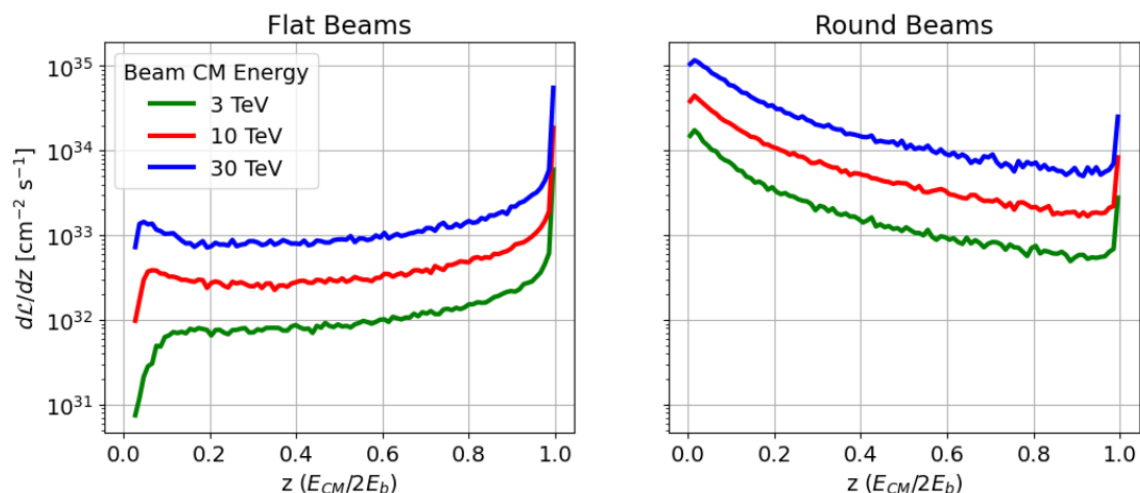


Figure 12.2: Luminosity spectra for flat-beam (left) and round-beam (right) electron-positron colliders at pCM energies of 3 TeV (green), 10 TeV (red) and 30 TeV (blue). Figure taken from ref. [144].

In high-luminosity proton-proton machines, the main source of beam-induced backgrounds results from the multiple simultaneous collisions per bunch crossing, commonly known as *pile-up*. For a single bunch crossing at the LHC during the Run-3 data-taking period, there are, on average, ≈ 60 pile-up interactions [145]. Figure 12.3 shows a visual representation of jets originating from pile-up interactions; the primary vertex is determined based on the energy of the tracks originating from the vertex while vertices with less energy are labeled as pile-up vertices and discarded.

While this contribution to the background is somewhat reducible at the detector level, difficulties can arise if pileup contributions become close to jets from the hard scattering process.

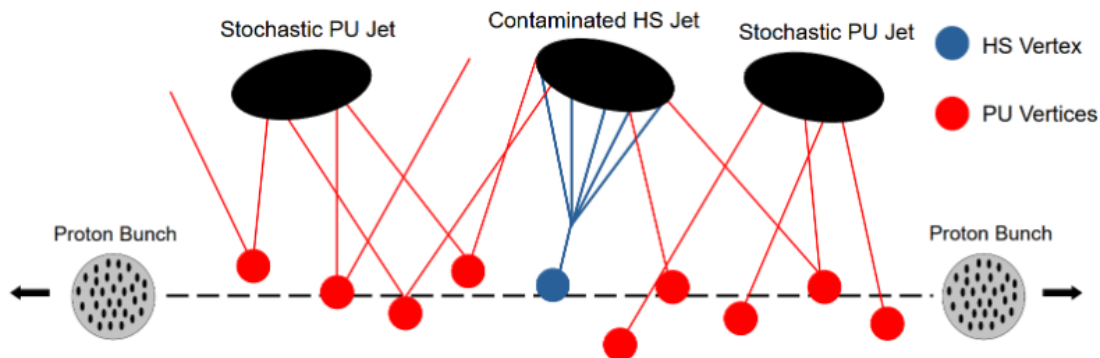


Figure 12.3: Diagram showing the interactions resulting from a proton-proton bunch collision; the hard scatter (HS) process is determined as originating from the primary vertex (PV), while additional particle production originates randomly from pileup (PU) vertices. Figure taken from ref. [146].

12.4 Plasma Wakefield Acceleration

Electron-positron colliders are an attractive option for pushing the energy frontier of colliders, as they require smaller center-of-mass energies to achieve the same sensitivity to new particles than their proton-proton counterparts. At present, the LHC has been able to probe particle masses of a few TeV (with limited statistics), and so an electron-positron collider would need to have a CM energy of $\mathcal{O}(10 \text{ TeV})$ to go beyond the LHC sensitivity. However, a considerable challenge arises when considering the lengths needed to accelerate electrons (or positrons) to TeV-scale energies using conventional acceleration methods. The current state-of-the-art method for particle acceleration is the use of superconducting radio-frequency (SRF) cavities (discussed in Sections 3.1.2 and Section 13.2.1), which can achieve acceleration gradients up to tens of megavolts per meter (MV/m). Ultimately, the maximum achievable gradi-

ents in SRF technology is limited by the dielectric breakdown voltage of the metallic structure itself. To achieve electron beams with 5 TeV energy, starting from 5 GeV electrons, the acceleration length of SRF cavities needed assuming a 10 MV/m gradient is:

$$L_{\text{accel.}} = \frac{\Delta E_{\text{beam}}}{e|\vec{E}|} = \frac{(5000 - 5)[\text{GeV}]}{(0.01[\text{GeV/m}])} \approx 5 \times 10^5 \text{ m}, \quad (12.5)$$

or approximately 310 miles. This is not feasible for a number of reasons, the most important of which are related to the economics of the collider itself. SRF cavities are expensive, and needing 310 miles of acceleration would prove, to put it mildly, incredibly costly. Furthermore, the cost of a linear collider scales roughly with overall length [147], and the length quoted above does not consider the lengths needed for the other collider components or the space needed between the SRF cavities for diagnostics and beam optics. Achieving TeV-scale beam energies utilizing SRF technology at a linear collider is simply not a realistic prospect.

An alternative method for the acceleration of electrons is *plasma wakefield acceleration*, whereby a plasma wake is driven by a laser [148] or a particle beam [149], evacuating the plasma electrons from the wake, and a witness beam of electrons is placed in the wake such that it feels a strong accelerating field. This scheme allows for acceleration gradients on the order of tens to even hundreds of gigavolts per meter (see Equation 13.29), possibly reducing the total acceleration length with respect to SRF technology by three to four orders of magnitude. A more comprehensive discussion of plasma wakefield acceleration is presented in Section 13.2.1.

12.5 The European Strategy and the P5 Report

The future of particle physics beyond the LHC is a hotly debated topic among physicists; the Standard Model is known to be incomplete, but where is the best place

to look next and what is the best machine to achieve these goals? However, future colliders require broad political support as well, as rising costs (typically in excess of a few billion CHF) mean that funding is solicited from governments and their funding agencies directly. Among the politicians and bureaucrats, the prospect of future colliders brings less scientific appeal but rather broad economic benefits and specialized workforce training. Thus, the process of choosing a future collider is an intricate dance between the physicists, the politicians, and the funding agencies, requiring broad agreement across each group for any proposal to be realized.

To answer these questions, two organizations, formed by a community of European and American physicists, provide recommendations to political and funding entities based on perceived physics merit, cost, broad applications of different technologies needed to develop such a collider, and other criteria. The American group, known as the Particle Physics Project Prioritization Panel (P5), takes input from U.S. National Laboratories, Universities, and the Future of US Particle Physics ("Snowmass") reports [150] to concoct a set of recommendations for the future of U.S. particle physics roughly every ten years. The most recent P5 report was published in 2023 [151], highlighting the desire in the U.S. for an offshore Higgs factory, and development towards a 10 TeV parton center-of-mass collider with either plasma wakefield technology (for electron-positron collisions) or based on the muon collider concept.

The European organization, referred to as the European Strategy group, undergoes a similar process whereby community input from physicists is organized into a document voicing recommendations for future collider proposals, known as the European Strategy for Particle Physics Update (ESPPU). In recent years, the ESPPU has been issued regularly to consolidate support for the Future Circular Collider (FCC), proposed by the CERN council in the wake of massive European support for a future electron-positron Higgs factory (FCC-ee) with an upgrade path towards an energy-frontier proton-proton collider (FCC-hh). As of the 2026 update to the ESPPU [152],

the FCC feasibility studies have been completed and the major recommendation appears to be in favor of the construction of the FCC at CERN to succeed the LHC.

12.6 The 10 TeV Wakefield Collider Design Study

One of the major recommendations arising from the most recent (2023) P5 report was for dedicated research and development towards "a suite of promising future projects", namely, "a 10 TeV parton center-of-momentum (pCM) collider" [151]. Similarly, the 2020 ESPP called for increased support towards advanced accelerator technologies to reach future energy goals [153]. Out of these emerged the 10 TeV Wakefield Collider Design Study [154], a proposal for a 10 TeV pCM collider based on plasma wakefield acceleration technology. Considerable work has been done on the physics potential of such a machine [155], highlighting the sensitivity to the Higgs potential at high energies and the ability to perform direct searches of BSM physics at the TeV scale with high statistics. Studies are underway to design a machine capable of achieving the machine parameter goals laid out by the 2021 Snowmass report for a wakefield collider [150, 156], however there is still more effort needed to help specify the preferred collision type (electron-positron, electron- electron, or photon-photon) and beam aspect ratio (round beams or flat beams).

We consider in this thesis the design of a flat-beam final focusing system (FFS) for the 10 TeV wakefield collider. To achieve the physics goals of such a collider, targets for key FFS parameters, outlined in ref. [156], are used as performance benchmarks. Table 12.3 summarizes the relevant parameter goals for the 10 TeV FFS, scaled down from the recommendations for the 15 TeV PWFA collider where relevant.

Target Values for 10 TeV FFS Parameters		
Parameter	Description	Value
f_{rep} [kHz]	Repetition rate	13.12
N_{bunch} [10^9]	Number of particles per bunch	5.0
$\varepsilon_x/\varepsilon_y$ [mm mrad]	Normalized horizontal/vertical emittances	0.66/0.02
σ_x^*/σ_y^* [nm]	Horizontal/vertical beam sizes at IP	18.4/0.45
$\mathcal{L}_{\text{geo}}$ [$10^{34} \text{ cm}^{-2} \text{ s}^{-1}$]	Instantaneous luminosity (geometric)	34.1

Table 12.3: Target values for the FFS parameters as specified from the 2021 Snowmass report [156], scaled down from a collision energy of 15 TeV to 10 TeV where relevant.

CHAPTER 13
BACKGROUND IN ACCELERATOR PHYSICS

13.1 Basics of Accelerator Physics

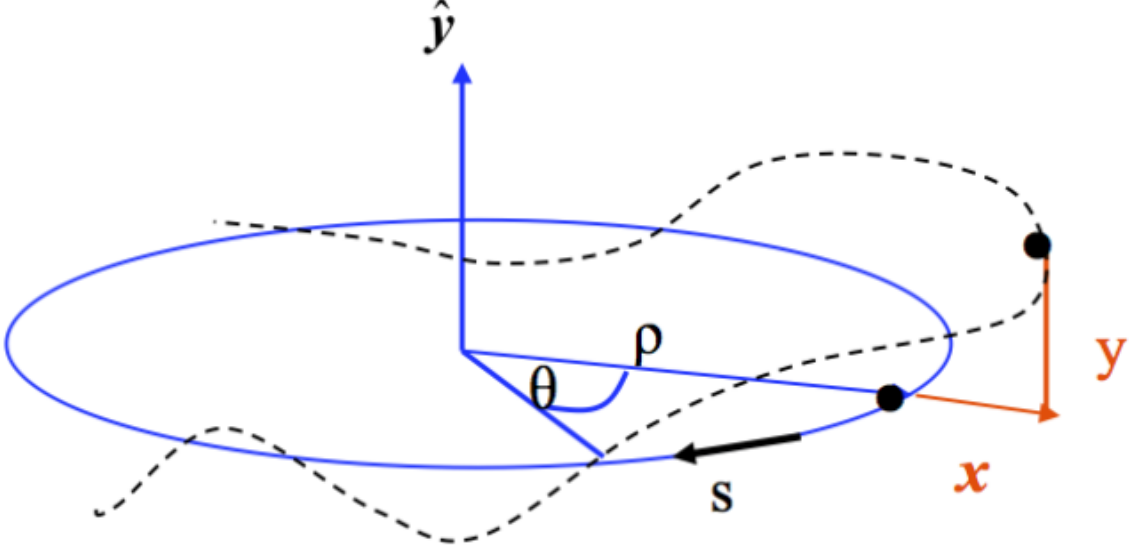


Figure 13.1: Coordinate system used in accelerator physics; s is the longitudinal coordinate along the reference trajectory and x and y are the horizontal and vertical displacements from the reference orbit (blue circle), respectively. Figure taken from ref. [157].

Accelerator physics is broadly concerned with the acceleration and transport of particle beams for various applications, including colliders, medical research, and free electron lasers (among others). Typically, the beams are comprised of a large number of particles (usually electrons, protons, or ions) and thus it becomes convenient to treat it as a single object with a set of parameters describing the collective dynamics. However, to construct these collective parameters, we must first start with a single particle and work our way up. In three dimensions, a free particle can be described by a set of six coordinates $\vec{x}(t)$; a position and a velocity for each dimension. In Cartesian coordinates:

$$\vec{x}(t) = \left(x(t), v_x(t), y(t), v_y(t), z(t), v_z(t) \right) \quad (13.1)$$

In an accelerator context, where the velocity along the accelerating direction is highly

relativistic, it is convenient to use a set of co-moving, curvilinear coordinates with the beam. In general, a particle in an accelerator will be bent transversely to its nominal trajectory; this is particularly true in circular accelerators and also applies to linear accelerators as well. We can define a reference trajectory along an accelerator, representing the ideal path that a particle would take. The longitudinal position along the accelerator for the reference particle, in curvilinear coordinates, is denoted by the coordinate s , as shown in Figure 13.1, and its energy is denoted by E_{ref} . Defining the reference particle coordinates as:

$$\vec{x}_{\text{ref}}(t) = \left(x_0 = 0, \quad x'_0 = 0, \quad y_0 = 0, \quad y'_0 = 0, \quad s, \quad E_{\text{ref}} \right) \quad (13.2)$$

where x and y are the coordinates transverse to the longitudinal coordinate s , and the primed coordinates are the angular coordinates defined as:

$$x' = \frac{dx(s)}{ds}, \quad y' = \frac{dy(s)}{ds} \quad (13.3)$$

We have also made the coordinate choice such that the reference particle is centered at the origin in the transverse coordinates and has no transverse momenta. The coordinates of an arbitrary particle can be expressed with respect to the reference particle coordinates:

$$\begin{aligned} \vec{r}_p &= \left(x_p = (x - x_0), \quad x'_p = (x' - x'_0), \quad y_p = (y - y_0), \quad y'_p = (y' - y'_0), \quad l = (s - s_0), \quad \delta = \frac{E - E_{\text{ref}}}{E_{\text{ref}}} \right) \\ &= \left(x, \quad x', \quad y, \quad y', \quad l, \quad \delta \right) \end{aligned} \quad (13.4)$$

In each dimension, an arbitrary particle occupies some point in phase space, which is specified by the corresponding coordinate and momentum. Consider a large number

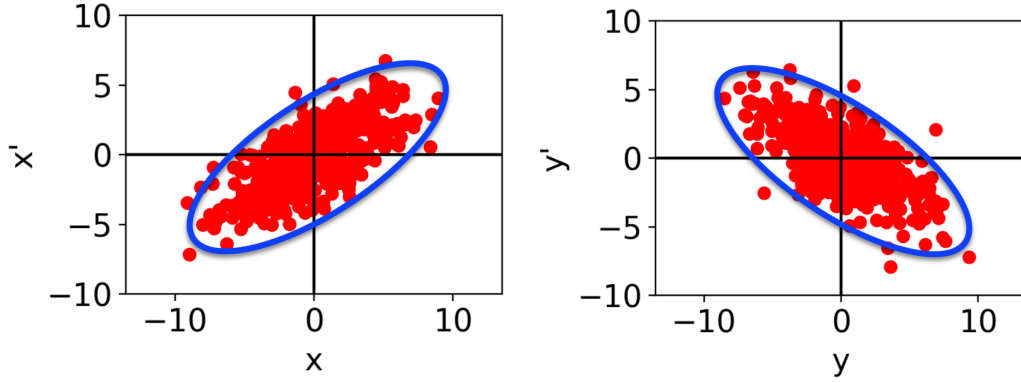


Figure 13.2: Representative transverse phase space occupancy of a particle beam in the horizontal (left) and vertical (right) directions. Figure taken from ref. [158].

of particles constituting, say, a particle bunch in an accelerator. At any given point along the accelerator, the particles in the bunch will occupy a certain volume of phase space, as shown in Figure 13.2. To describe the collective behavior of the beam, the phase space volume occupied by the beam is inscribed within an ellipse with the equation [55]:

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon \quad (13.5)$$

where α , β , γ are the Twiss parameters and ϵ is referred to as the *geometric emittance*. The area of phase space A occupied by the beam is thus related to the geometric emittance by the formula:

$$A = \int_{\text{ellipse}} dx dx' = \pi \epsilon \quad (13.6)$$

The Twiss parameters define the properties of the phase space ellipse, as shown in Figure 13.3, and can be interpreted to have the following physical meanings:

- β : the (square root of) the amplitude of transverse oscillations for a particle. This is known as the *beta function*, and it describes the width of the beam ellipse.

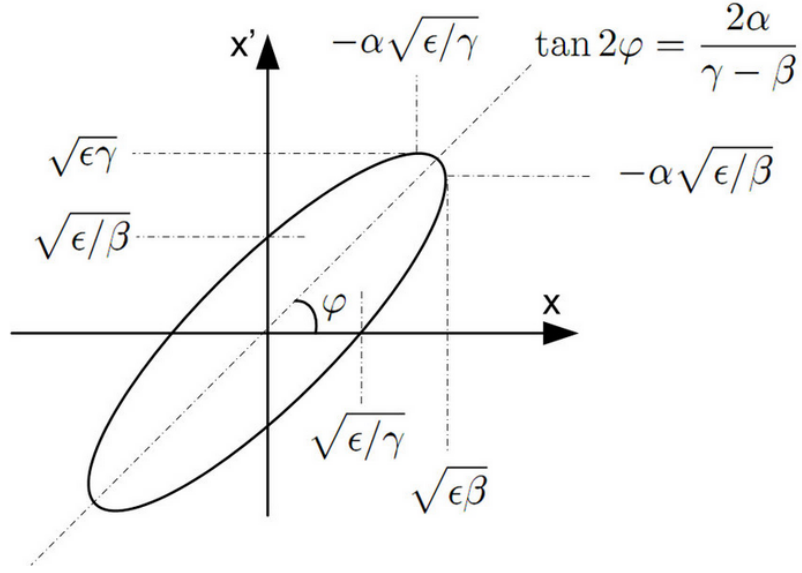


Figure 13.3: The phase space ellipse, defined in terms of the Twiss parameters. Figure taken from ref. [159].

- α : (half) the (negative) change in the beta function with respect to the longitudinal coordinate s . Defined by $\alpha \equiv -\frac{1}{2}d\beta/ds$, this parameter specifies the orientation of the beam ellipse.
- γ : specifies the divergence of the beam ellipse at a given point. Defined as $\gamma \equiv \frac{1+\alpha^2}{\beta}$.

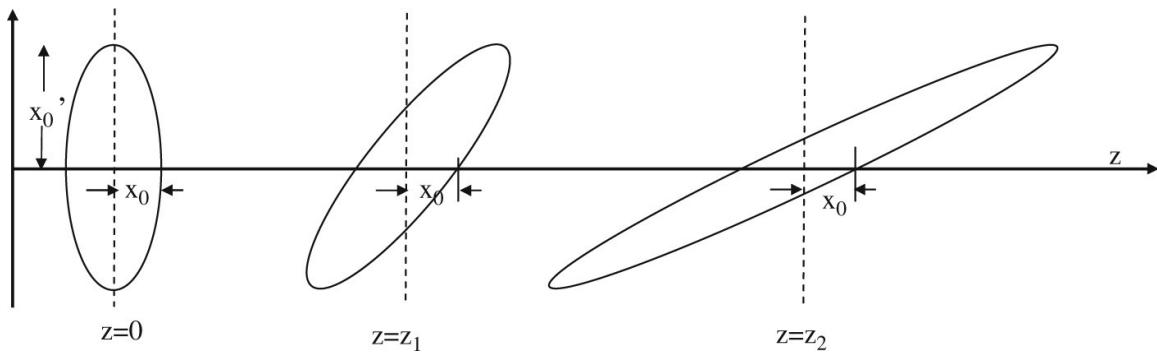


Figure 13.4: Rotation of phase space ellipse traversing through a drift section. Figure taken from ref. [55].

Liouville's theorem states that the phase space density of particles subject to

conservative forces remains constant, and so the ellipse volume remains constant for a fixed number of particles. As the bunch traverses along the beamline through drift spaces, dipoles, and quadrupoles, each individual particle rotates in phase space and traces out its own ellipse. This results in a rotation of the phase space ellipse for the whole bunch, as shown in Figure 13.4. Using this along with Liouville's theorem, it is then clear that particles within the phase space ellipse stay within the ellipse while the rotations occur. During acceleration, the geometric emittance is reduced via a process known as *adiabatic damping*, where the increase in the momentum of the beam requires a corresponding decrease in the geometric emittance of the beam to keep the phase space volume constant. This phase-space volume, referred to as the *normalized emittance* ε_n , is defined in terms of the geometric emittance as:

$$\varepsilon_n = \beta\gamma\varepsilon \quad (13.7)$$

where $\beta = v/c$ is the particle velocity normalized to the speed of light and γ is the Lorentz factor. This definition, along with adiabatic damping, removes essentially any dependence of the normalized emittance on the particle momentum, making it more aligned with Liouville's Theorem.

Of great importance in accelerator physics is the knowledge of the particle trajectories in the transverse coordinates, as this determines the beam spot size along the lattice. The evolution of a transverse coordinate u for a single particle subject to some focusing potential along the beamline $k(s)$ is governed by Hill's equation [160]:

$$u'' + k(s)u = 0. \quad (13.8)$$

The general solution to this equation is an oscillating solution with a position-dependent amplitude:

$$u(s) = \sqrt{\varepsilon\beta(s)} \cos(\psi(s) - \psi_0), \quad (13.9)$$

where $\psi(s)$ is the *betatron phase*, specifying the period of betatron oscillations. The solution for u in Equation 13.9 also shows another useful interpretation of the β -function and geometric emittance; their product specifies the amplitude of transverse oscillations of a single particle. The s dependence of the β -function highlights the relationship between the β -function and the placement of the focusing elements (quadrupoles) along the accelerator lattice. In this case, the β -function now represents that of a single particle, which can be representative of the entire beam itself by noting that particles within the phase space ellipse stay within the phase space ellipse. Thus, by choosing a particle on the edge of the ellipse, we can sufficiently describe the beam dynamics by the dynamics of a single particle. This then allows the definition of the beam envelope function $E_u(s)$ containing all particles within the beam:

$$E_u(s) = \sqrt{\varepsilon_u\beta_u(s)} \quad (13.10)$$

The beam envelope can also be interpreted to be the total transverse beam size $\sigma_u(s)$, assuming the particle distribution in phase space is uniform. If the distribution is non-uniform, as is the case with Gaussian bunches, it is typical to utilize a different definition, based on the standard deviation σ_u of the Gaussian. In the absence of dispersion, the beam size for Gaussian beams is

$$\sigma_u(s) = \sqrt{\varepsilon_u\beta_u(s)} \quad (13.11)$$

Other contributions to the beam spot size, such as those arising from non-zero dispersion and chromaticity will be discussed later.

A useful analogy in accelerator physics is the connection of particles traversing magnets to that of light passing through focusing and defocusing lenses in optics. To describe the evolution of transverse coordinates in accelerator physics, it is useful to adopt a formalism, akin to the ray transfer matrix formalism for light optics, whereby the transverse particle coordinates are represented by a state vector $\vec{\theta}$:

$$\vec{\theta} = \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}. \quad (13.12)$$

The transverse coordinates at a final position $\vec{\theta}_f$ are related to the initial coordinates $\vec{\theta}_0$ by matrix multiplication with the transfer matrix M :

$$\vec{\theta}_f = M \cdot \vec{\theta}_0. \quad (13.13)$$

The transfer matrix itself is a left product of the transfer matrices M_i for the individual elements comprising the beamline between the initial point s_0 and the final point s_f :

$$M = \prod_{i=1}^n M_i = M_n \cdot \dots \cdot M_2 \cdot M_1, \quad (13.14)$$

where $i = 1$ denotes the element starting at s_0 and $i = n$ denotes the element ending at s_f . The general form of the transfer matrix for a single beamline element with uncoupled x and y motion is a 2x2 block matrix:

$$M_i = \begin{pmatrix} c(x; x_0) & c(x; x'_0) & 0 & 0 \\ c(x'; x_0) & c(x'; x'_0) & 0 & 0 \\ 0 & 0 & c(y; y_0) & c(y; y'_0) \\ 0 & 0 & c(y'; y_0) & c(y'; y'_0) \end{pmatrix}, \quad (13.15)$$

where $c(u, u_0)$ is a function mapping coordinate u_0 at some s to coordinate u at a different position s . As we will see in Section 13.2, this form is sufficient to describe upright dipole and quadrupole magnets and drift spaces; the basic building blocks used for beam steering and focusing. Sextupole magnets will introduce additional terms coupling the x and y motion. The transfer matrix formalism is valid such that the magnet strength parameters are constant throughout the magnet, and is useful in that it can be applied to analyze the lower-order dynamics of any arbitrary beam matrix.

For treatment of higher-order contributions to the beam evolution, a generalization of the transfer matrix formalism is required. The transfer map X relates the transverse particle coordinates z_f at a point along the beamline to the initial set of coordinates $(x_i, p_{x,i}, y_i, p_{y,i}, \delta_i)$ [161]:

$$z_f = \sum_{jklmn} X_{z,jklmn} x_i^j p_{x,i}^k y_i^l p_{y,i}^m \delta_i^n, \quad (13.16)$$

where $X_{z,jklmn}$ are the map coefficients for the particular final coordinate. The order of the map, M , is the largest sum of the powers among the considered terms:

$$M = j + k + l + m + n. \quad (13.17)$$

Above $M = 2$, calculation of map coefficients is analytically challenging, however many modern particle tracking codes (such as MAD-X [162] with the Polymorphic Tracking Code [163]) can numerically calculate the coefficients for arbitrary M . The

beam spot size at the IP is a complicated function of various terms at different M (see Eq. 14.20), and calculation is usually carried out with programs such as MAPCLASS2 [164] interfaced with MAD-X and the Polymorphic Tracking Code.

Particle beams are not fully monochromatic, meaning that there is always some finite energy spread δ :

$$\delta = \frac{p - p_0}{p_0} \quad (13.18)$$

where p_0 is the reference beam momentum. For a fixed magnetic field strength, particles of different energies will be bent differently. This is particularly relevant when considering beam focusing, which is done primarily via quadrupole magnets. The quadrupole focusing strength is given by:

$$k = 0.2998 \frac{g \text{ [T/m]}}{\beta E \text{ [GeV]}} \quad (13.19)$$

differentiation with respect to E and setting $\beta = 1$ (ultrarelativistic limit) gives:

$$\frac{dk}{dE} = -0.2998 \frac{g}{E^2} \quad (13.20)$$

For a particle with energy difference dE , the new focusing strength k_{new} is thus given by:

$$k_{\text{new}} = k + \frac{dk}{dE} dE = k - \frac{(0.2998)g \cdot dE}{E^2} = k - \left(k \cdot \frac{dE}{E}\right) = k(1 - \delta). \quad (13.21)$$

This energy-dependent focusing is illustrated in Figure 13.18, whereby particles with different energy offsets passing through a quadrupole are focused to different longitudinal points, resulting in what is known as a *chromatic* spot size growth. A further discussion of this effect is presented in Section 13.4.1.

Another relevant phenomenon related to the energy dependence of particle bending in magnetic fields is that of *dispersion*, which occurs in dipole magnets. Particles entering a dipole with different energies are bent differently, resulting in a transverse kick Δx that depends on the energy offset δ of the particle and the *dispersion function* D_x :

$$\Delta x = D_x \delta \quad (13.22)$$

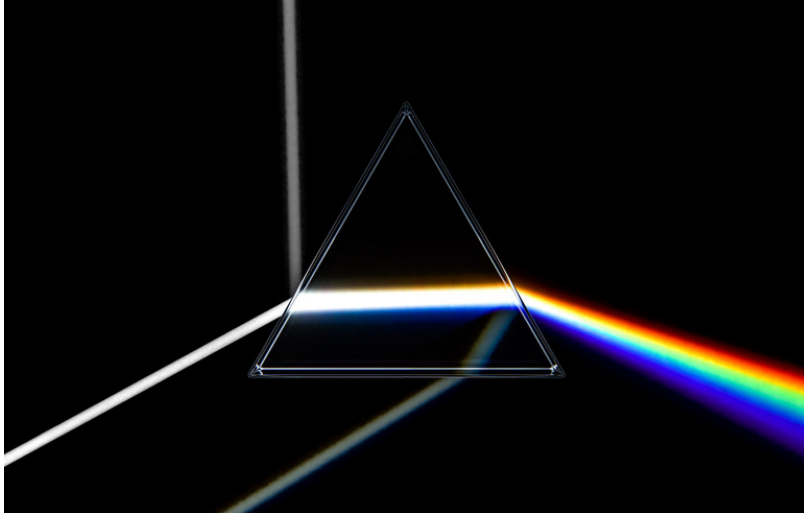


Figure 13.5: Separation of white light in a prism.

A similar phenomena is present in light optics; the spatial separation of monochromatic light passing through a prism, as shown in Figure 13.5. The overall effect of dispersion on the transverse particle coordinates can be found by incorporating momentum-dependence into Hill's Equation (Eq. 13.8). To lowest order, this gives

$$u'' + k(s)u = \frac{\delta}{\rho(s)} \quad (13.23)$$

where ρ is the bending radius of the dipole field. The homogeneous solution to Eq. 13.23 is the betatron oscillation solution given by Eq. 13.9. The particular solution to this equation is defined to be the transverse perturbation given by Eq. 13.22. Thus,

the general solution for the transverse position $u(s)$ and velocity $u'(s)$ is given by:

$$\begin{aligned} u(s) &= c(u; u_0)u_0 + c(u; u'_0)u'_0 + D_u(s)\delta \\ u'(s) &= c(u'; u_0)u_0 + c(u'; u'_0)u'_0 + D'_u(s)\delta \end{aligned} \quad (13.24)$$

where the c are the same ones present in the transfer matrix (Eq. 13.15) and $D'_u(s) = dD_u/ds$ is the second-order dispersion.

The net effect of dispersion on the beam spot size at a given longitudinal position is also of importance. For a Gaussian beam, the beam size (to lowest order) with dispersion is given by [55]:

$$\sigma_u(s) = \sqrt{\varepsilon_u \beta_u(s) + D_u^2(s) \sigma_\delta^2} \quad (13.25)$$

where σ_δ is the standard deviation of the momentum offset. Suppression of dispersion is thus an important consideration for accelerator design.

13.2 Accelerator Building Blocks

13.2.1 Methods of Acceleration

Charged particles can be accelerated along their direction of motion by the application of a strong electric field via the Lorentz force (see Equation 3.1). The energy gain of a particle in an electric field is obtained by integrating the force experienced along the length of the electric field:

$$\Delta E_{\text{beam}} = e \int_0^L E(z) dz. \quad (13.26)$$

As discussed in Section 3.1.2, high electric fields can be obtained within regular conducting or superconducting metallic structures driven by RF electromagnetic fields.

Modern superconducting RF technology allows for resonant driving of cavities at 1-3 GHz, resulting in peak gradients of up to 20 MV/m. Cavities designed for the Compact Linear Collider (CLIC) allow this limit to be pushed further to frequencies of up to 12 GHz with maximum achievable fields closer to 100 MV/m. The limiting factor to the maximum gradient in RF cavities is *dielectric breakdown*, whereby the fields become so strong that they damage the cavity materials.

The problem of dielectric breakdown can be fully avoided by simply utilizing something that is already broken down: a plasma! A plasma is a system of mobile charges, comprised of ions and electrons with number densities n_i and n_e , respectively, with a well-defined temperature T . Plasmas are differentiated from ionized gases by a set of criteria:

- **Quasi-neutrality:** the density of electrons and ions are approximately equal: $n_i = n_e = n_0$, where n_0 is referred to as the *plasma density*.
- **Spatial Extent:** the spatial extent of the plasma in any dimension L should be much larger than the Debye length $\lambda_D = \sqrt{\epsilon_0 k_B T / n_0 e^2}$, which is the characteristic length scale of shielding for a potential within the plasma.
- **Large plasma parameter:** in a sphere with radius λ_D , the number of particles N_D in the unperturbed part of the plasma for a given test charge is large (e.g. $N_D \gg 1$).
- **Few collisions:** the time between collisions τ_{coll} should be long compared to the plasma period $\omega_p = \sqrt{n_0 e^2 / m_e \epsilon_0}$, e.g. $\tau_{\text{coll}} \omega_p \gg 1$.

Plasma acceleration starts with a driver; either an intense laser pulse or an electron beam is sent into a plasma, whereby the plasma electrons are fully expelled from a region (the so-called *nonlinear blowout regime*) leaving only the (semi-) stationary ions. For now, I consider only the case where the driver is an electron bunch, referred

to as "beam-driven" plasma wakefield acceleration or PWFA for short. The laser-driven case, referred to as laser-wakefield acceleration, or LWFA, is summarized in refs. [165, 166]. The blowout regime is achieved by utilizing a drive bunch that is more dense than the plasma itself:

$$n_b > n_0, \quad (\text{blowout regime}) \quad (13.27)$$

A witness electron beam, placed inside the blowout bubble at the correct longitudinal position, will experience a strong forward electric field from the ejected electrons occupying the "sheath" around the blowout bubble. The longitudinal electric field in the bubble E_z in the ultrarelativistic limit ($v \rightarrow c$) is related to the radius of the blowout $r_b(\xi)$ via the equation [167]:

$$E_z(\xi) = \frac{en_0}{\varepsilon_0} \frac{r_b(\xi)}{2} \frac{d}{d\xi}(r_b(\xi)) \quad (13.28)$$

where $\xi = ct - z$ is the longitudinal position in co-moving coordinates. Equation 13.28, highlights the dependence of the longitudinal electric field on the geometry of the bubble; the field is linear in the blowout radius and also depends on the derivative of the bubble radius with respect to the longitudinal coordinate. This means that the electric field is negative (accelerating for electrons) in the back half of the bubble, while it is positive (decelerating for electrons) in the front half of the bubble, as illustrated in Figure 13.6. The magnitude of the electric field within the wakefield is given approximately by the expression [168, 169]:

$$E_z [\text{V/m}] \approx 96 \sqrt{n_0 [\text{cm}^{-3}]}. \quad (13.29)$$

This means that for a plasma density of $n_0 = 10^{18} \text{ cm}^{-3}$, the corresponding electric field would be $\approx 100 \text{ GV/m}$; three orders of magnitude larger than the CLIC

RF cavities. Thus, PWFA offers high-gradient acceleration without the challenge of dielectric breakdown, allowing linear colliders to achieve higher energies over smaller distances at a fraction of the cost of RF-based accelerators.

The nonlinear blowout wake also induces strong transverse wakefields, providing linear axisymmetric focusing of the witness electron bunch due to the electric field of the ions in the bubble. The radial electric field within the bubble, denoted by $\vec{W}_\perp$ is given by [170]:

$$\vec{W}_\perp(r) = \frac{r}{2} \frac{en_0}{\varepsilon_0} \hat{r} \quad (13.30)$$

This focusing capability of the blowout wake can be leveraged to create plasma-based focusing elements, referred to as *plasma lenses*, which are discussed in more detail in Section 13.2.7.

Despite the great benefits with respect to conventional RF technology, there are a number of challenges associated with PWFA that make its implementation difficult for collider applications. The first of which is the charge asymmetry of PWFA, whereby plasmas have a different response to electrons versus positrons. This fact has made positron acceleration in plasma extremely difficult to achieve experimentally, and is still an open topic of research at this time. A review of the challenges and methods of positron acceleration in plasma is presented in ref. [172]. For plasma acceleration to be a viable technology for collider applications ($\sqrt{s} > \mathcal{O}(100 \text{ GeV})$), the accelerating length of the plasma must be long enough to attain these high beam energies. This then requires either long plasma stages, which is limited by head erosion, dephasing between the witness and drive bunches, and plasma instabilities, or many plasma stages, which require precise matching of the beam with interstage optics, many beams to drive the wake of each stage, and careful removal of the spent drive beams after each stage. Motion of the plasma ions also impacts the stability and efficiency of

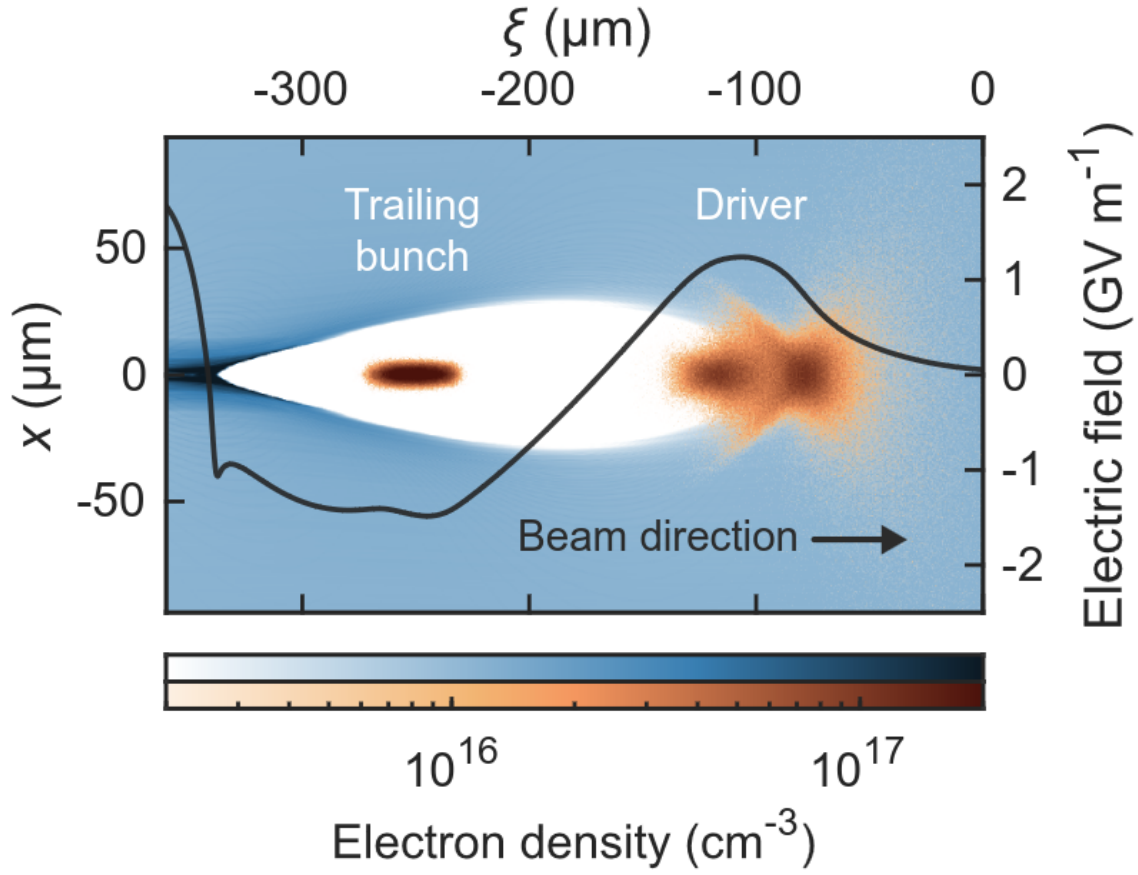


Figure 13.6: Principle of beam-driven PWFA; a driver excites a nonlinear wake (white bubble), and a trailing bunch is placed within the bubble such that it experiences a strong electric field in the beam direction. Figure taken from ref. [171].

the plasma acceleration process, and plasma heating from successive bunches passing through the plasma limit the overall repetition rate that a plasma-based collider can achieve.

Beam-driven plasma wakefield acceleration is a rather deep subject and still a very active area of research. More details on the physics of PWFA, experimental progress, and challenges are presented in ref. [173].

13.2.2 Magnets

Magnets are indispensable elements for the bending and focusing of particle beams. The transverse properties of a magnet can be derived from the complex magnetic potential $P(z)$, which, in general, is represented by a sum over all multipole components [55]:

$$P(z) = A(z) + iV(z) = \sum_{n \geq 0} C_n z^n = \sum_{n \geq 0} C_n (x + iy)^n \quad (13.31)$$

where the C_n are the coefficients for the order n multipoles and A (V) is the real (imaginary) magnetic potential component. The potential is related to the real and imaginary components of the magnetic field by

$$\vec{B} = \vec{\nabla} \times P(z) = \vec{\nabla} \times (A(z) + iV(z)) \quad (13.32)$$

Using this, along with Maxwell's equations yields the following expressions for the magnetic field terms:

$$\begin{aligned} B_x &= -\frac{\partial V}{\partial x} = \frac{\partial A}{\partial y}, \\ B_y &= -\frac{\partial V}{\partial y} = -\frac{\partial A}{\partial x} \end{aligned} \quad (13.33)$$

The general solution for the coefficients C_n are expressed in terms of an upright multipole component λ_n and a skew multipole component μ_n :

$$C_n = \lambda_n + i\mu_n \quad (13.34)$$

where the upright and skew components at order n are obtained by taking derivatives of the magnetic field components:

$$\begin{aligned} \lambda_n &= -\frac{1}{n!} \frac{\partial^{n-1} B_y}{\partial x^{n-1}}, \\ \mu_n &= \frac{1}{n!} \frac{\partial^{n-1} B_x}{\partial x^{n-1}} \end{aligned} \quad (13.35)$$

In the following sections, I will give examples of the magnetic field terms and properties of the most commonly used magnetic elements in accelerator design: the $n = 1$ (dipole), $n = 2$ (quadrupole), and $n = 3$ (sextupole) magnets.

13.2.3 Dipoles

Dipole magnets are comprised of two poles, and correspond to the first term in the multipole expansion (Equation 13.31). There are two primary types of geometries for dipole magnets; sector (or wedge) and rectangular dipole magnets, as shown in Figure 13.8. We consider here rectangular geometries, as they are used predominantly in the cases we are concerned with. In general, the transverse field of a dipole magnet can be written as:

$$\vec{B}_\perp = B_x \hat{x} + B_y \hat{y} \quad (13.36)$$

In most accelerator applications, it is customary to only consider bending in the

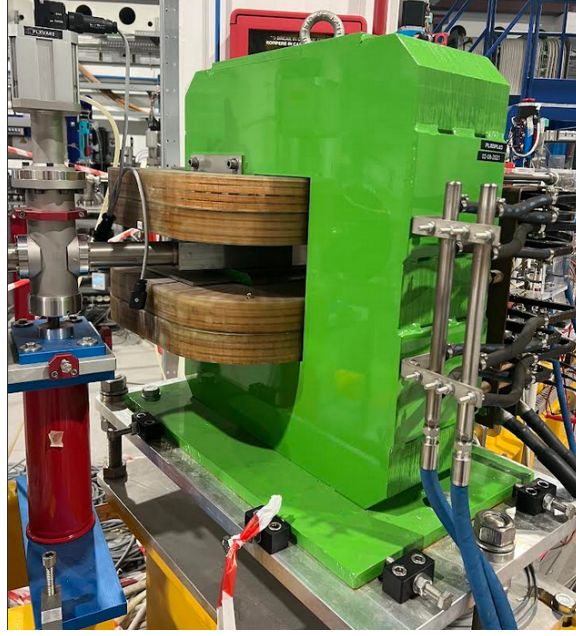


Figure 13.7: Dipole magnet at the INFN SPARC-Lab.

horizontal plane ($B_x = 0$). Furthermore, the bending of a charged particle in a magnetic field depends on the momentum of the charged particle, and thus it is convenient to normalize the magnetic field strength according to the *beam rigidity* e/p_0 , where p_0 is the particle momentum along the z direction. Thus, we can re-write the magnetic field of a dipole magnet in the following form:

$$\frac{e}{p_0} \vec{B}_\perp = \frac{e}{p_0} B_y \hat{y} = \frac{1}{\rho_0} \hat{y} \quad (13.37)$$

where ρ_0 is the dipole bending radius of the particle (see Equation 3.5). The field lines from the dipole potential are visualized in Figure 13.9; a particle traveling into the page would experience a force to the right (left) if it has positive (negative) charge.

Within the linear optics formalism, dipoles are represented (in the bending plane) by the following transfer matrix:

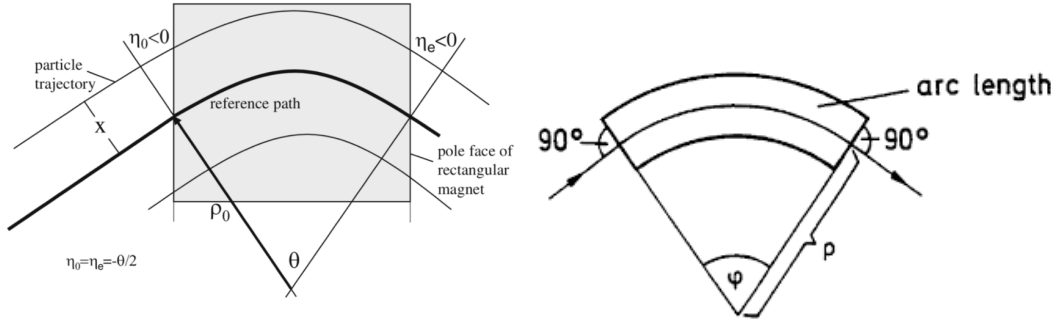


Figure 13.8: Common dipole magnet geometries: rectangular (left) and sector (right). Figures taken from refs. [55, 174].

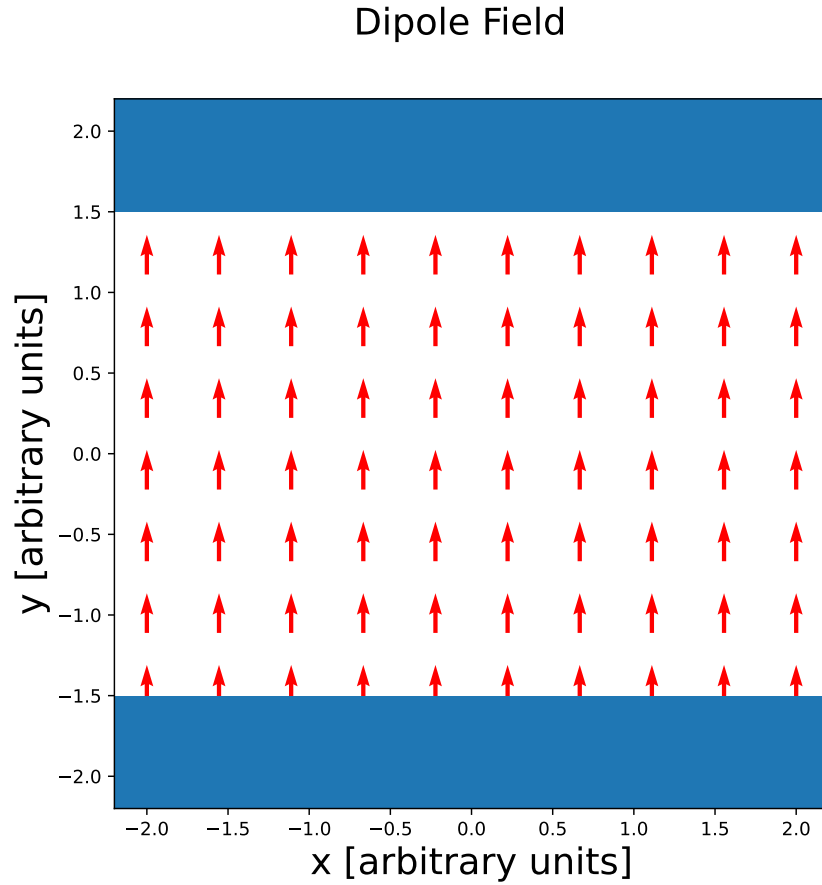


Figure 13.9: Magnetic field lines (red arrows) in the transverse plane for a dipole magnet, represented by the blue rectangles.

$$M_{\text{dipole}} = \begin{pmatrix} \cos(\theta) & \rho_0 \sin(\theta) \\ -\frac{\sin(\theta)}{\rho_0} & \cos(\theta) \end{pmatrix} \quad (13.38)$$

where $\theta = \ell/\rho_0$ is the bending angle of the dipole (ℓ is the dipole length). In the non-bending plane, the dipole can be represented as a drift space of length ℓ . Dipoles are also a source of dispersion, which changes the trajectory of off-momentum particles with respect to the reference trajectory. For small bending angles ($\theta \ll 1$), the first- and second-order dispersion generated by a rectangular dipole in the horizontal direction is given by:

$$\begin{aligned} D_x &= \rho_0(1 - \cos \theta), \\ D'_x &= \sin \theta \end{aligned} \quad (13.39)$$

In a linear accelerator, dipoles are utilized for a multitude of applications, including in chicanes (for bunch compression), damping rings, and also in the final focusing system to generate dispersion for correcting chromatic effects (along with sextupoles). Rather importantly, bending sections also generate synchrotron radiation, which can increase the beam energy spread δ .

13.2.4 Quadrupoles

A quadrupole magnet is comprised of four poles with alternating polarity, as shown in Figure 13.10. For an upright quadrupole, the transverse magnetic field, normalized to the magnetic rigidity, can be written as:

$$\frac{e}{p_0} \vec{B}_\perp = \frac{e}{p_0} (gy\hat{x} + gx\hat{y}) = ky\hat{x} + kx\hat{y} \quad (13.40)$$

where g is the quadrupole field gradient obtained from the λ_n expansion:

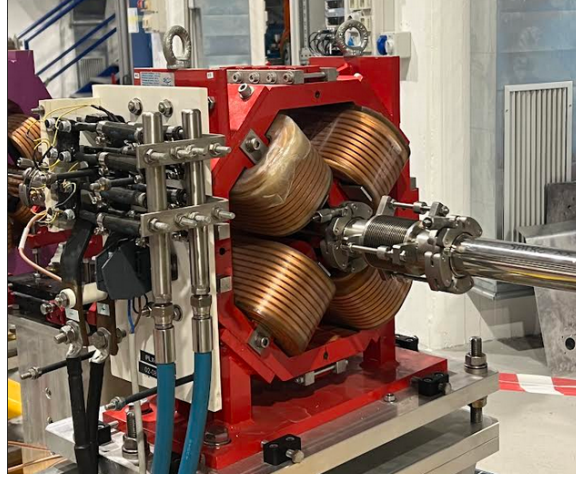


Figure 13.10: Quadrupole magnet at the INFN SPARC-Lab.

$$g = -\frac{\partial B_y}{\partial x} = -\frac{\partial B_x}{\partial y} \quad (13.41)$$

and $k = eg/p_0$ is the normalized field strength. A particle traversing a quadrupole magnet with some transverse offset thus experiences a focusing force in one plane and a defocusing force in the other plane, as shown in Figure 13.11. To achieve focusing in both planes, pairs of quadrupoles (or "doublets") can be placed together in particular arrangements that obey certain stability criteria.

The focusing properties of a quadrupole magnet draw parallels to the focusing of light from lenses in optics. In the context of linear optics, one can define a focal length f for a quadrupole in analogy to that of a thin lens. In the case where the length of the quadrupole L is much shorter than the focal length of the lens (weak focusing), the focal length can be written as:

$$f = (kL)^{-1} \quad (13.42)$$

Using this definition, it is easy to then write the transfer matrix for a thin quadrupole (focusing in x) in the following form:

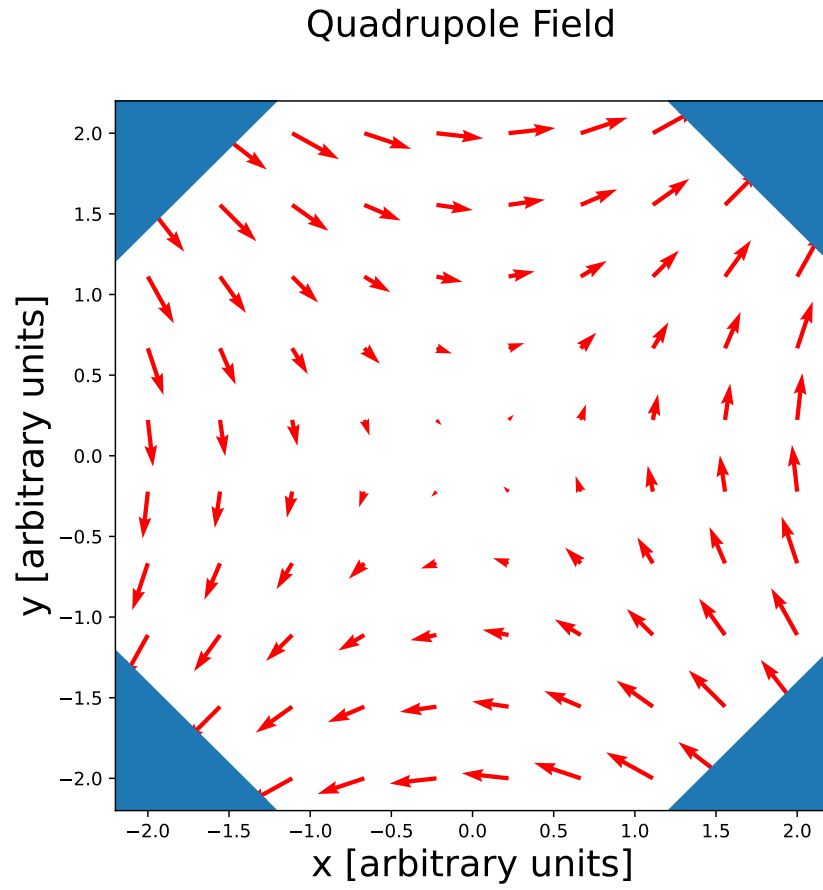


Figure 13.11: Magnetic field lines (red arrows) in the transverse plane for a quadrupole magnet, represented by the blue rectangles.

$$M_{\text{QFX}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{f} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f} & 1 \end{pmatrix} \quad (13.43)$$

For quadrupoles where the focusing is strong and the phase advance $\varphi = \sqrt{kL}$ is considerable within the quadrupole, the transfer matrix can be written in the more general form (assuming focusing in x and defocusing in y):

$$M_{\text{QFX}} = \begin{pmatrix} \cos \varphi & \frac{\sin \varphi}{\sqrt{k}} & 0 & 0 \\ -\sqrt{k} \sin \varphi & \cos \varphi & 0 & 0 \\ 0 & 0 & \cosh \varphi & \frac{\sinh \varphi}{\sqrt{|k|}} \\ 0 & 0 & \sqrt{|k|} \sinh \varphi & \cosh \varphi \end{pmatrix} \quad (13.44)$$

Quadrupole magnets, along with dipoles, are the main magnets utilized in accelerator design due to their focusing capabilities. In the absence of focusing magnets, particle beams diverge in free-space, and thus quadrupoles are necessary for beam transport to different points along the accelerator. Ideal quadrupoles do not generate additional dispersion, although dispersion created upstream by dipoles can be influenced by quadrupoles.

13.2.5 Sextupoles

A magnet with six poles is referred to as a *sextupole* magnet, with a perpendicular magnetic field for an upright sextupole given by:

$$\frac{e}{p} \vec{B}_{\perp} = mxy\hat{x} + \frac{1}{2}m(x^2 + y^2)\hat{y} \quad (13.45)$$

where m , the normalized field strength, is related to the sextupole gradient $g' =$

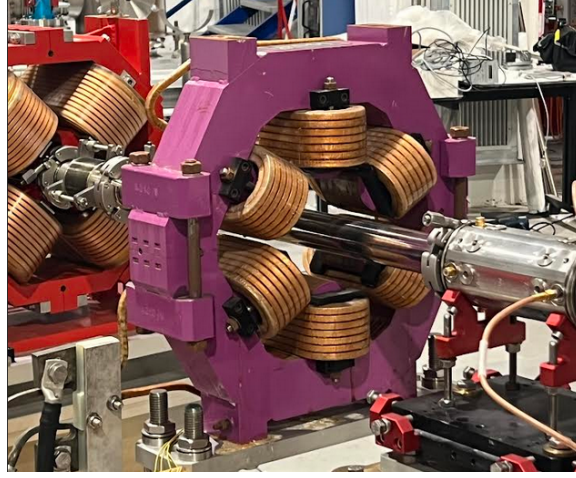


Figure 13.12: Sextupole magnet at the INFN SPARC-Lab.

$\partial^2 B_y / \partial x^2$ via:

$$m = \frac{e}{p} g' \quad (13.46)$$

The magnetic field for a sextupole is visualized in Figure 13.13. One can then consider the force that such a sextupole would exert along the horizontal plane (e.g. $y = 0$). The perpendicular field reduces to

$$\frac{e}{p} \vec{B}_\perp = \frac{1}{2} m x^2 \hat{y} \quad (13.47)$$

and we take the velocity along the z direction $\vec{v} = v \hat{z}$. The force is then obtained by Lorenz's Law:

$$\vec{F} = e \vec{v} \times \vec{B}_\perp = ev(\hat{z} \times \frac{p}{2e} m x^2 \cdot \hat{y}) = -\frac{vp}{2} (m \delta^2 D_x^2) \hat{x} = -\frac{ev}{2} g' \delta^2 D_x^2 \hat{x} \quad (13.48)$$

where I have made the substitution $x = \delta D_x$, implying a source of upstream dispersion to separate the particles of different energy along the x direction. Equation 13.48 highlights the quadratic dependence of the force on the momentum offset of the

particle, implying that particles of higher energy $\delta > 0$ are focused by a sextupole in x while particles of lower energy $\delta < 0$ are defocused in x , as illustrated in Figure 13.14. This means that, when placed after quadrupoles in regions of finite dispersion, sextupoles can be used for correcting chromatic effects, which arise from the different focusing of off-momentum particles by quadrupoles. A more detailed discussion of chromatic correction schemes for use in the final focusing system of a linear collider is presented in Section 13.4.1.

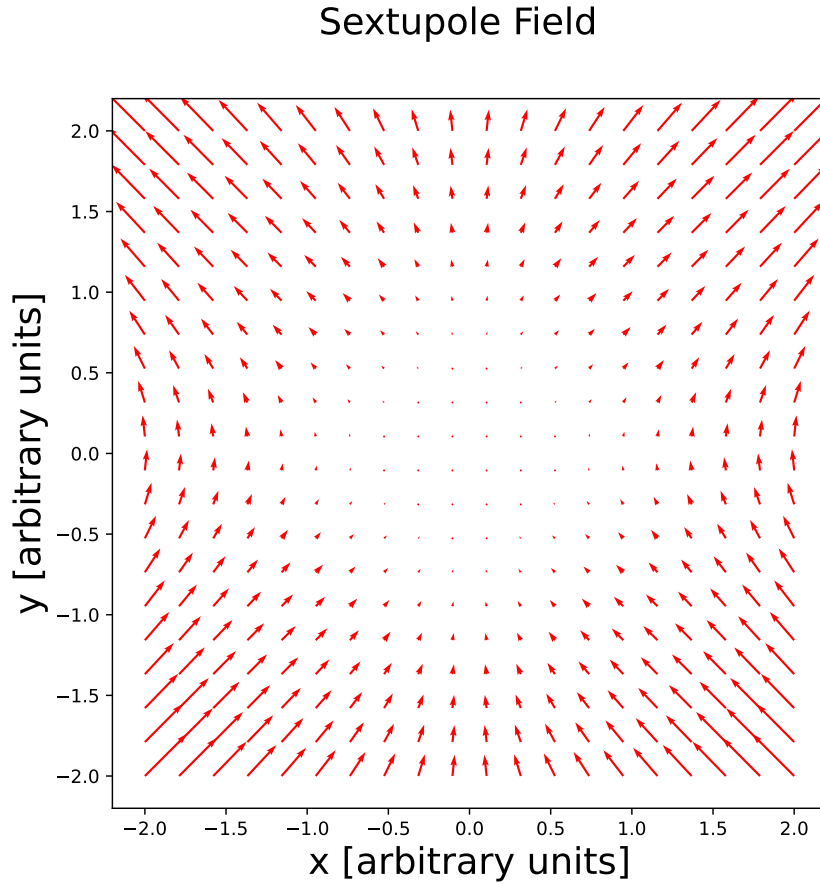


Figure 13.13: Magnetic field lines (red arrows) in the transverse plane for a sextupole magnet.

In the limit where the sextupole lengths are thin, we can consider the sextupoles as giving kicks to the particle momenta in the transverse directions by the amount:

$$\begin{aligned}\Delta x' &= -\frac{1}{2}m\ell_s(x^2 - y^2) \\ \Delta y' &= -m\ell_s xy\end{aligned}\tag{13.49}$$

This allows us to write the thin-lens transfer matrix for a sextupole in the following form:

$$M_{\text{sextupole}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2}m\ell_s x & 1 & \frac{1}{2}m\ell_s x & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & m\ell_s x & 1 \end{pmatrix}\tag{13.50}$$

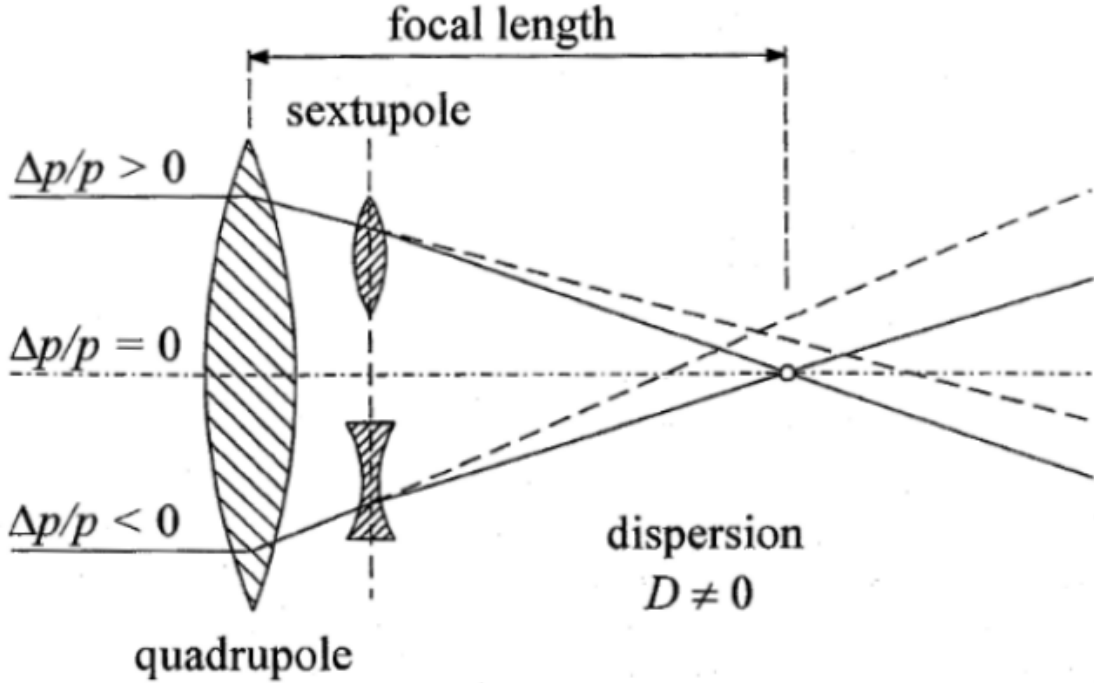


Figure 13.14: Principle of chromatic correction of a quadrupole with a sextupole magnet. Figure taken from ref. [175].

13.2.6 Higher-Order Multipoles

Magnets with $n > 3$ (eight or more poles) are less common than dipoles, quadrupoles, and sextupoles in accelerator lattices, however more modern designs are increasingly using octupoles ($n = 4$) and decapoles ($n = 5$) for correction of higher-order aberrations. The normalized transverse magnetic field for an upright octupole is:

$$\frac{e}{p}\vec{B}_{\perp,\text{oct}} = \frac{1}{6}r(3x^2y - y^3)\hat{x} + \frac{1}{6}r(x^3 - 3xy^2)\hat{y} \quad (13.51)$$

where $r = (e/p)s_4$ is the normalized octupole gradient and $s_4 = \partial^3 B_y / \partial x^3$ is the octupole gradient. The field lines for an octupole are shown in Figure 13.15. As discussed in Section 13.4.2, octupoles can be useful for mitigating the impact of SR in the final focusing system of a collider.

13.2.7 Plasma Lenses

Plasma-based focusing elements have been proposed as alternatives to conventional quadrupole magnets, utilizing the axisymmetric, strong transverse focusing fields produced by a nonlinear blowout wake (see Eq. 13.30). This provides multiple advantages with respect to quadrupoles:

- Stronger field gradients by many orders of magnitude, allowing for better focusing over shorter distance scales. Table 13.1 shows a comparison of gradients and focal lengths for a plasma lens ($n_0 = 10^{17} \text{ cm}^{-3}$) and typical quadrupole magnets for a 5 TeV electron beam.
- Axisymmetric focusing, whereby a beam is focused in both transverse planes simultaneously, while a quadrupole can only focus in one dimension.

Additionally, plasma lenses can be shaped to include components from different orders

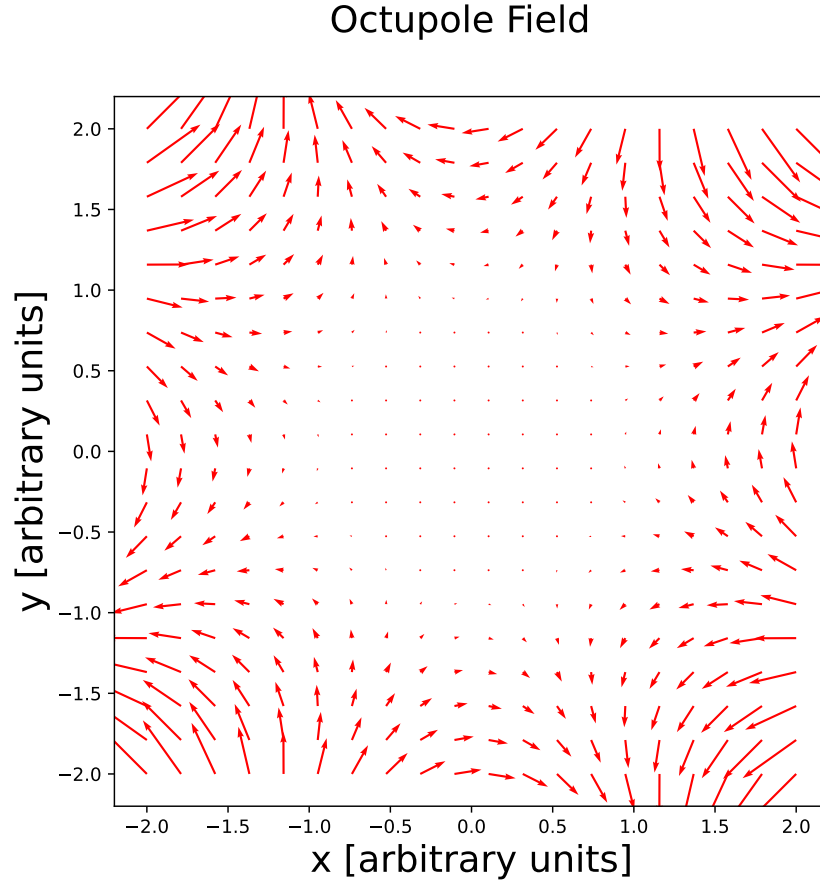


Figure 13.15: Magnetic field lines (red arrows) in the transverse plane for an octupole magnet.

Focusing Element	K [m^{-2}]	L [mm]	f [cm]
Superconducting Quadrupole: $G = 100$ T/m	0.06	410	4100
Permanent Magnetic Quadrupole: $G = 500$ T/m	0.3	180	1800
Underdense Plasma Lens: $n_0 = 10^{17} \text{ cm}^{-3}$	180	7.4	74

Table 13.1: Comparison of different focusing elements for a 5 TeV electron beam assuming a uniform phase advance $\Delta\psi = \sqrt{K}L = 0.1$ across the element.

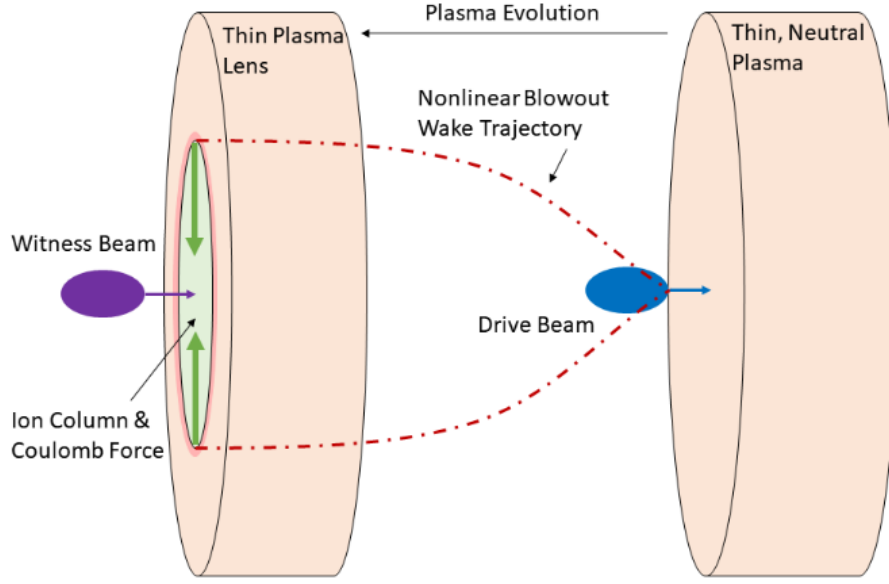


Figure 13.16: Operating principle of a thin, passive plasma lens. Figure taken from ref. [56].

in the multipole expansion, enabling the correction of higher-order effects without the need for additional magnets.

Plasma lenses are typically thin, such that the phase advance within the lens is minimal to avoid betatron radiation emission, and also to limit the amount of acceleration felt by the beam in the longitudinal direction. In this arrangement, we refer to the plasma lens as a *thin underdense passive plasma lens* (TUPPL), where "underdense" refers to the fact that the plasma density of the lens is less than that of the drive beam (thus creating the nonlinear blowout wake), and "passive" refers to the fact that the lens is formed by the driving of a nonlinear wake from a beam, rather than from applying a current to a capillary to create the lens (a so-called *active plasma lens*). Assuming radial symmetry, the focal length of a TUPPL is related to the integral of its longitudinal density profile [176]:

$$f = \frac{1}{2\pi r_e} \frac{\gamma_b}{\int n_0(z) dz} \quad (13.52)$$

where γ_b is the Lorentz factor of the beam. For a plasma lens to be effective in focusing the witness beam, it must be spatially large enough (longitudinally and radially) to contain the entire witness beam. The β -function at the beam waist resulting from the focusing of a thin plasma lens is given by [176]:

$$\beta^* = \frac{1}{K^2 L^2 \beta_0 + 2KL\alpha_0 + \gamma_0} \quad (13.53)$$

where $K = 1/fL$ is the focusing strength and β_0, α_0 , and γ_0 are the Twiss parameters at the start of the plasma lens. Typical L values for plasma lenses are determined by noting that the phase advance $\Delta\psi = \sqrt{K}L$ should be much less than one.

The main application for plasma lenses in a collider application is for the purpose of matching electron beams into the plasma acceleration stages [177], which is done to prevent unwanted emittance growth in the linac. A particularly useful application of the plasma lens in the final focusing system of a TeV-scale linear collider would be the replacement of quadrupole pairs (such as the final doublet) in order to shrink the overall length needed for the final focusing system [178]. Although plasma lenses offer high focusing gradients, the hard SR emission (Oide effect, see Section 13.4.2) can be avoided by tapering the density profile of the lens such that it acts as an adiabatic focuser [179]. This provides a great advantage over conventional quadrupole focusing near the IP as it allows for smaller spot sizes. Furthermore, in the adiabatic regime, the focusing provided by plasma lenses is apochromatic; particles of different energies are focused to the same point. Consequently, this property could alleviate the need for chromatic correction in the final focusing system (see Section 13.4.1) and reduce the overall length by eliminating the need for long bending sections and sextupoles.

Despite the numerous advantages over conventional focusing elements, plasma lenses do pose significant challenges to implement in future colliders. Precise shaping of the plasma lens to achieve adiabatic focusing is difficult to achieve in practice,

and this remains an active field of research. For colliders with high repetition rates (kHz or above), plasma lenses may create concerns as to their robustness, as the plasma recovery time may result in density profile variations on a shot-to-shot basis, leading to different focusing properties. It has also been demonstrated that the driving of plasmas using flat beams, which are typically used in linear colliders to reduce beamstrahlung, can lead to asymmetric focusing fields experienced by the witness bunch and variation in the focusing fields along the longitudinal coordinate [180], necessitating precise experimental control and matching to prevent chromatic beam size growth and proper focusing along both axes. Additionally, the desire for adiabatic focusing would increase the plasma density closer to the IP, increasing the rate of beam-plasma scattering that would result in emittance growth [179] and background events in the detector [178]. Finally, despite successful experimental demonstrations of plasma lensing of positron beams [181], it remains an open question as to whether or not positrons can be focused to small spot sizes using a plasma lens without geometric aberrations [182].

Experimental demonstration of plasma lens focusing in the nonlinear blowout regime is still a work in progress. Future studies by the E308 experiment at SLAC National Accelerator Laboratory hope to provide evidence for such a process in the near future.

13.2.8 Drift Spaces

Not all space in an accelerator is dedicated to magnets or accelerating structures. In fact, most of the length of an accelerator is comprised of drift spaces; spaces in which there are no magnets present. In the absence of focusing forces, a beam traversing a drift space will eventually begin to diverge due to intra-beam forces and uncorrected transverse velocities of the beam particles. For a drift of length L_{drift} , the linear

transfer matrix is given by:

$$M_{\text{drift}} = \begin{pmatrix} 1 & L_{\text{drift}} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_{\text{drift}} \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (13.54)$$

The use of drift spaces in an accelerator is pragmatic; magnets are expensive and careful design of the lattice (e.g. magnet placement) can typically achieve desired focusing capabilities while maintaining reasonable cost.

13.3 Anatomy of a Linear Collider

All linear colliders are comprised of a series of complex systems, each performing an essential task in the creation of a beam, managing the beam quality, accelerating the beam up to the terminal energy, and delivering the beam to the collision point (or interaction point). The following components are common among most electron-positron linear collider designs:

- **Sources:** electrons are gathered by means of a *photo-injector*, whereby a laser is shot at a metallic target (typically *GaAs*) and electrons are produced via the photoelectric effect. Positrons are created by sending an electron beam into a high-density, high- Z target (such as Tungsten), resulting in photon emission and electron-positron pair creation (recall Section 2.2.4). The positrons are separated from the electrons and photons using magnets.
- **Damping Rings:** electron and positron bunches are accelerated to a few GeV and injected into damping rings, designed to cause SR emission. The emission of SR reduces the emittance of the bunches, which is necessary to achieve the small beam spot sizes needed to reach high luminosity.

- **Linac:** bunches are accelerated using primarily metallic radio-frequency (RF) cavities up to the design energy of the collider while preserving the small bunch emittances.
- **Beam Delivery System:** the accelerated beam is then collimated to protect the magnets and inner detector systems (e.g. tracker) and then de-magnified in the transverse dimensions at the collision point to maximize the luminosity.

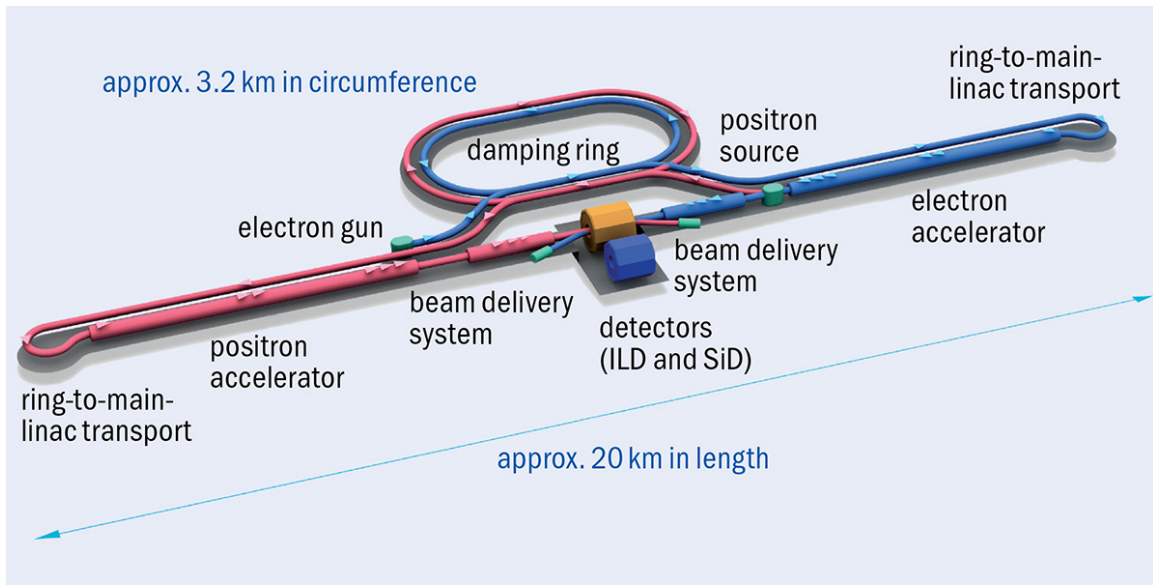


Figure 13.17: Layout of the proposed International Linear Collider (ILC). Figure taken from ref. [183].

Figure 13.17 illustrates these components for the International Linear Collider (ILC) design. In this thesis, we are primarily concerned with the final ingredient: the beam delivery system. In the following section I will describe this in more detail.

13.3.1 The Beam Delivery System

In a linear collider, the beam delivery system (BDS) is typically comprised of two to three subsystems: the final focusing system (FFS), the collimation section, and (sometimes) a beam switch yard.

The role of the FFS in a linear collider is to demagnify the beams in order to minimize the transverse beam spot sizes (σ_x, σ_y) and thus maximize the luminosity $\mathcal{L}$:

$$\mathcal{L} \propto \frac{1}{\sigma_x \sigma_y} \quad (13.55)$$

This can be achieved by arranging pairs of quadrupole magnets into a telescope, which provides parallel-to-parallel ($R_{12} = 0$) and point-to-point ($R_{21} = 0$) imaging [184]. In the thin-lens approximation, the one-dimensional transfer matrix $R_{\text{telescope}}$ for a telescope can be written in the following form:

$$R_{\text{telescope}} = \begin{pmatrix} -M & 0 \\ 0 & -\frac{1}{M} \end{pmatrix} \quad (13.56)$$

where M is the magnification, or the ratio of the final transverse displacement to the initial transverse displacement. Complications arise when considering beams with a finite energy spread δ , as particles of different energies are focused to different longitudinal positions along the beamline. Figure 13.18 shows the different focusing of particles with momentum deviations for a quadrupole focusing element. These chromatic aberrations result in an increase in the beam spot size at the nominal focal point (or interaction point), and thus can degrade the luminosity if left untreated. Additional elements are required in order to correct these chromatic effects, as discussed in Section 13.4.1. For the beams to be focused to the small spot sizes required at the interaction point, the quadrupoles closest to the interaction point will typically have the largest field strengths. These strong quadrupole fields induce the emission of incoherent SR, which increases the beam energy spread and sets a limit on the minimum achievable spot size. This effect, known as the Oide effect [185], is discussed further in Section 13.4.2. Furthermore, the desire for high-intensity electron

beams at future plasma wakefield colliders also poses a problem for the achievable luminosity. As mentioned in Section 12.3, electron beams can also radiate energy away in the field of the opposing beam (beamstrahlung), further increasing the beam energy spread and resulting in collisions at lower CM energy, thereby broadening the luminosity spectrum as a function of energy [186].

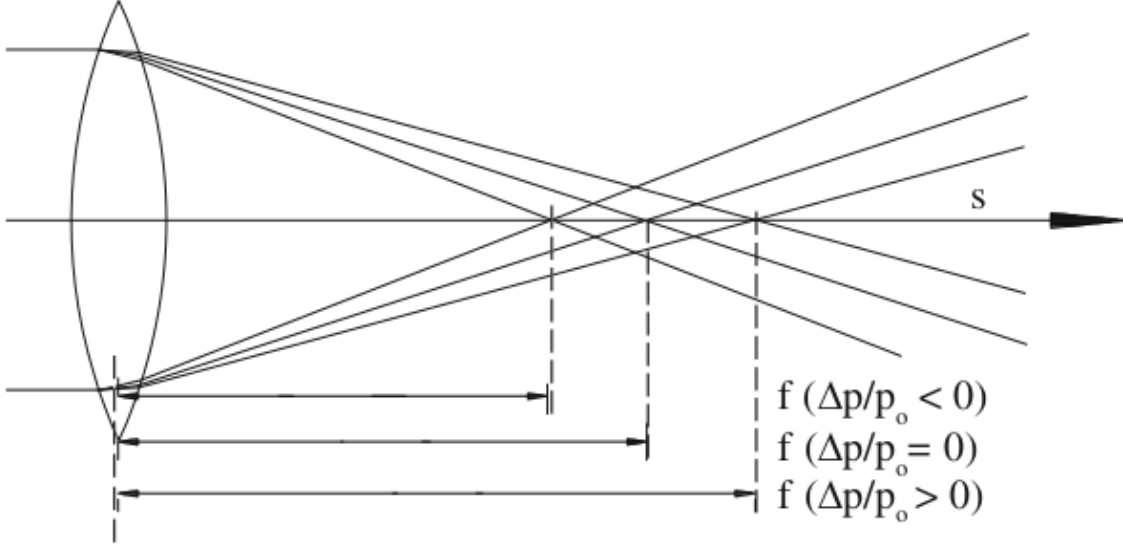


Figure 13.18: Diagram illustrating the focal points for particles with momentum deviations with respect to the nominal reference energy p_0 . Figure adapted from Ref. [55].

The collimation section sits upstream of the FFS and is responsible for the removal of particles with large amplitudes within the bunch, in both transverse and longitudinal phase space, commonly referred to as *beam halo*. To achieve this, the collimation section is comprised of two portions: an energy collimation section, to remove particles with large δ , and a transverse collimation section, to remove particles with large transverse amplitudes x or y . This is done for the protection of the magnets close to the IP (namely the final doublet) as well as the inner parts of the detector (e.g. silicon vertex detectors), as photons from SR emission and electrons or positrons hitting these elements would cause severe damage (see Figure 13.19). Collimation is

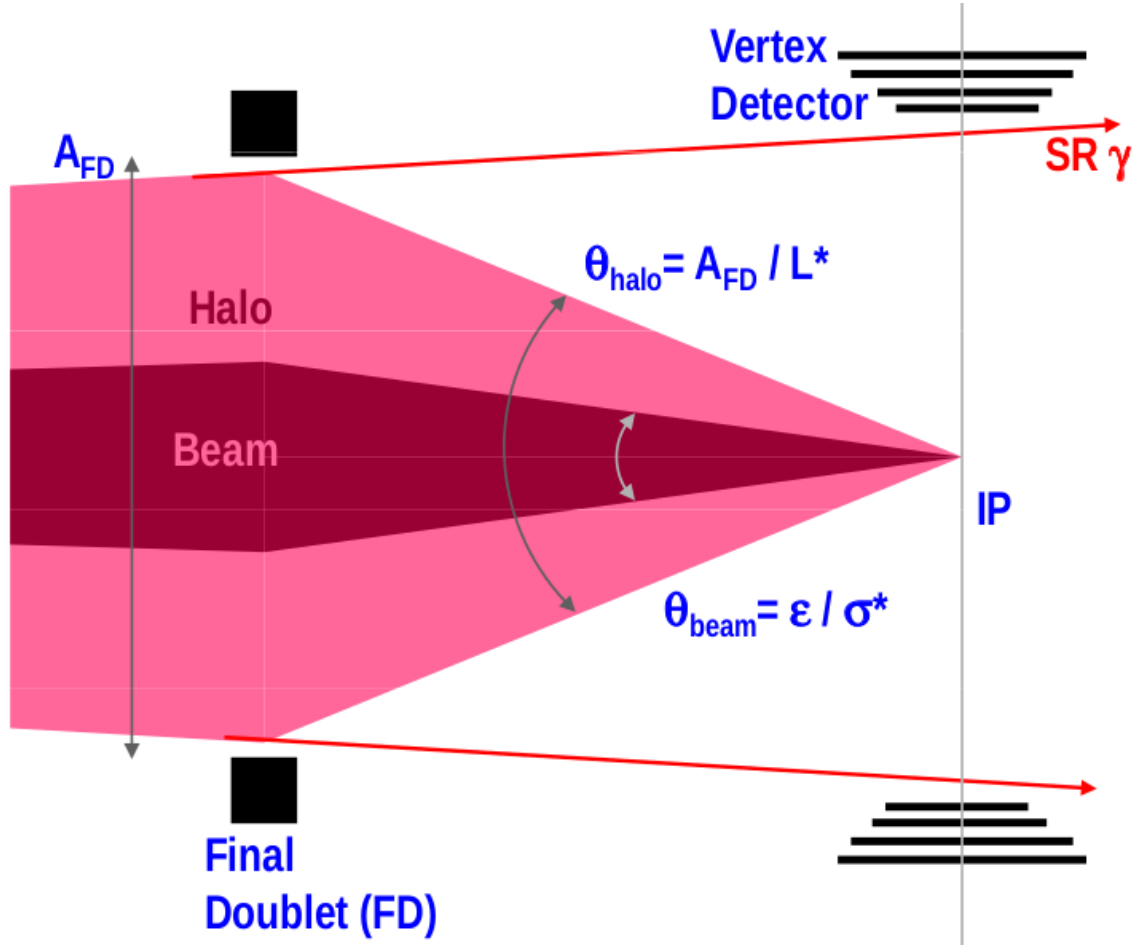


Figure 13.19: Schematic showing the impact of beam halo at the final doublet and interaction point. Figure taken from ref. [187].

typically performed using a spoiler-absorber scheme (see Figure 13.20), whereby a thin piece of metal (a spoiler) is used to intercept beam halo and increase the phase space occupancy of the beam, which is then sent into a thick absorber designed to capture the halo or remove errant beams. The overall length L_{coll} of the collimation system in this scheme is mostly dependent on the length of focusing quadrupoles, resulting in an approximate scaling with the beam energy: $L_{\text{coll}} \propto E_{\text{beam}}$ [188]. A more detailed discussion on the details of collimation systems and their requirements is presented in ref. [189].

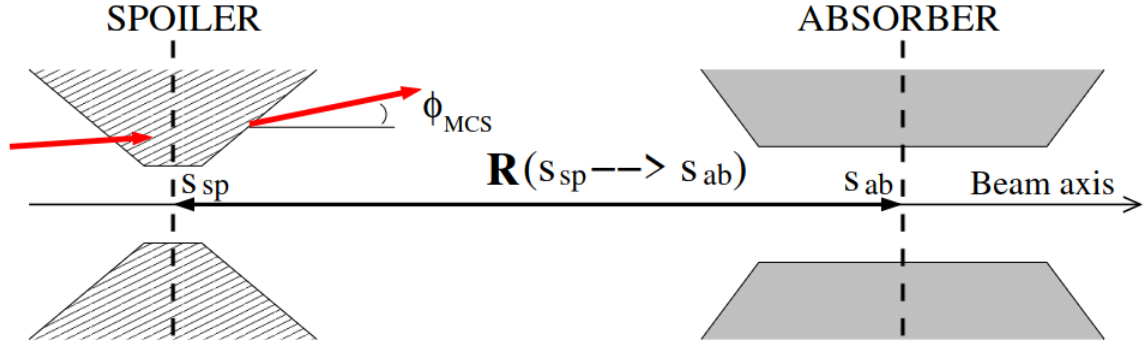


Figure 13.20: Diagram of the spoiler-absorber scheme; halo particles are kicked transversely from multiple Coulomb scattering in the spoiler in order to be removed by the absorber. Figure adapted from ref. [189].

13.4 Challenges in Final Focusing

In the following sections, I discuss in detail the effects that limit the ability to focus an electron beam down to arbitrarily small spot sizes.

13.4.1 Chromatic Effects

In practice, all particle beams have some finite RMS energy spread δ_p , and the focusing of particles of different energies is different in a quadrupole field (see Eq. 13.21). Since the strongest fields are present in the quadrupoles closest to the IP, the largest

contribution to the chromatic spot size growth at the IP is from these quadrupoles. The transverse spot size at the IP σ_i^* , as a function of δ_p , can be written (to lowest order) as [190]:

$$\sigma_i^* = \sqrt{\epsilon_i \beta_i^* (1 + \xi_i^2 \delta_p^2) + D_i^2 \delta_p^2} \quad (13.57)$$

where ϵ_i is the geometric emittance for coordinate i , β_i^* is the IP beta function for coordinate i , D_i is the dispersion for the coordinate i and ξ_i is the *chromaticity*. The chromaticity, a second-order chromatic effect which is manifested as changes to the twiss parameters due to energy offsets, is defined in terms of the change in phase advance $\Delta\psi_i$:

$$\xi_i \equiv \frac{1}{\delta_p} \frac{\Delta\psi_i}{\psi_i}. \quad (13.58)$$

To lowest order, the chromaticity at a given longitudinal position along an accelerator can be written in terms of the corresponding Twiss parameters via the chromatic amplitude W :

$$\xi_i = W = \sqrt{\left(\frac{\partial\alpha_i}{\partial\delta} - \frac{\alpha_i}{\beta_i} \frac{\partial\beta_i}{\partial\delta}\right)^2 + \left(\frac{1}{\beta_i} \frac{\partial\beta_i}{\partial\delta}\right)^2} \quad (13.59)$$

In a typical FFS, the design such that the IP dispersion D_i and chromaticity are as close to zero as possible to avoid spot-size blowup.

Chromaticity correction can be achieved by the introduction of sextupoles into regions of finite dispersion, which is generated by dipoles and perturbations to quadrupole fields. To lowest order, and using the thin-lens approximation, the x and y chromaticities are given by [55]:

$$\begin{aligned}\xi_x &= \xi_{x,0} + \frac{1}{4\pi} \sum_{i=1}^n m_i D_{x,i} \beta_{x,i} \ell_{s,i} \\ \xi_y &= \xi_{y,0} + \frac{1}{4\pi} \sum_{i=1}^n m_i D_{x,i} \beta_{y,i} \ell_{s,i}\end{aligned}\tag{13.60}$$

where the sum is over the number of sextupoles n . The subscripts in Eq. 13.60 denote the values at the sextupole locations (i.e. $\beta_{x,1}$ is the horizontal beta function at the location of the first sextupole), and we have assumed $D_y = 0$ as typically bending only occurs in the horizontal direction. The values $\xi_{x,0}$ and $\xi_{y,0}$ are referred to as the *natural chromaticities*, which are also the chromaticities in the absence of sextupoles:

$$\begin{aligned}\xi_{x,0} &= -\frac{1}{4\pi} \sum_{j=1}^q \beta_{x,j} k_j \\ \xi_{y,0} &= \frac{1}{4\pi} \sum_{j=1}^q \beta_{y,j} k_j\end{aligned}\tag{13.61}$$

where now the sum is over the number of quadrupoles in the lattice q , with k being the quadrupole strength. Cancellation of the chromaticity is thus achieved (at lowest order) by having the following criteria satisfied:

$$\begin{aligned}-\sum_{j=1}^q \beta_{x,j} k_j &= \sum_{i=1}^n m_i D_{x,i} \beta_{x,i} \ell_{s,i} \\ \sum_{j=1}^q \beta_{y,j} k_j &= \sum_{i=1}^n m_i D_{x,i} \beta_{y,i} \ell_{s,i}\end{aligned}\tag{13.62}$$

Thus, the minimum number of sextupoles needed to cancel the chromaticity at lowest order is two: one for each transverse direction. For a given lattice comprised of

focusing elements, one can then solve Equation 13.62 to determine the sextupole strengths, lengths, and dispersion needed for chromaticity correction.

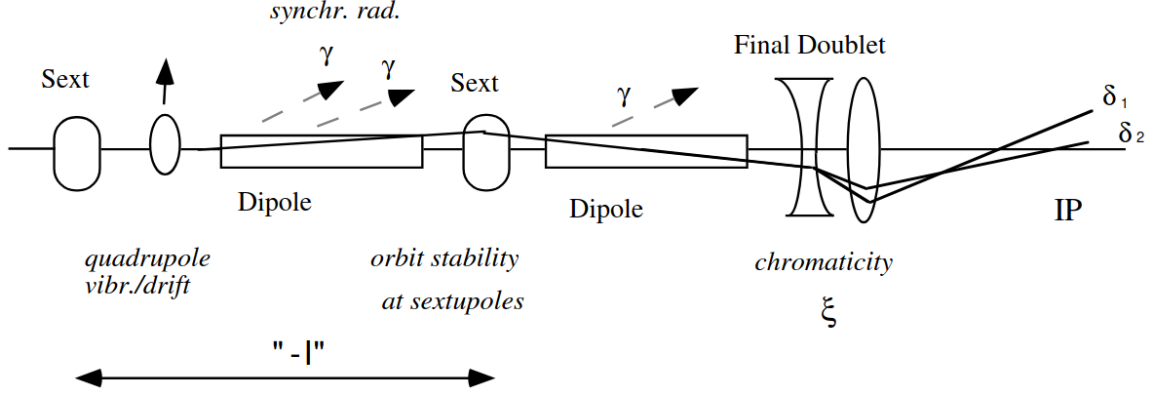


Figure 13.21: Diagram showing the design for the final focusing scheme with dedicated chromatic correction sections. Figure adapted from Ref. [191].

There are two primary methods employed at linear colliders for correcting the chromaticity, differing primarily in the arrangement of the bending sections and the sextupoles. The first scheme developed [192, 193] utilizes dedicated sections for cancelling the vertical and horizontal chromaticities, as shown in Figure 13.21. Each section is comprised of a bending section with a pair of sextupoles, and the sections themselves are placed in regions of high dispersion and large β . The sextupoles generate geometric aberrations, which are corrected by separating the sextupoles by a minus identity transformation:

$$R = -\mathcal{I} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{between sextupoles} \quad (13.63)$$

A different scheme [194] relies on the principle of local chromaticity correction by placing sextupoles just upstream and between the FD quadrupoles with a long bending section upstream to generate dispersion, as shown in Figure 13.22. These sextupoles generate horizontal second-order dispersion, which is corrected by a careful choice in the optics design; the FFS is designed such that half of the total horizontal chro-

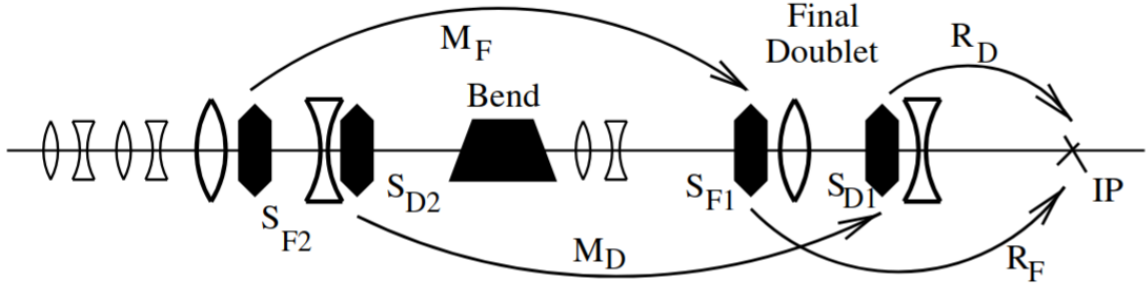


Figure 13.22: Diagram showing the optics arrangement for the local chromatic correction scheme; S_D (S_F) denote the defocusing (focusing) sextupoles in the x plane. Figure adapted from Ref. [194].

maticity is generated upstream of the FD. Another pair of sextupoles is introduced prior to the bending section with the purpose of cancelling the geometric aberrations, provided that they have the opposite phase of the FD sextupoles. The constraints for cancellation of second-order aberrations is such that the x and y sextupole pairs must be separated by transfer matrices M_x and M_y [194] of the form:

$$M_x = \begin{pmatrix} A & 0 & 0 & 0 \\ A_{21} & \frac{1}{A} & 0 & 0 \\ 0 & 0 & A & 0 \\ 0 & 0 & A_{43} & \frac{1}{A} \end{pmatrix}, \quad (13.64)$$

$$M_y = \begin{pmatrix} B & 0 & 0 & 0 \\ B_{21} & \frac{1}{B} & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & B_{43} & \frac{1}{B} \end{pmatrix},$$

where all A and B are arbitrary. Additionally, the chromatic effects at second order are also canceled if the following criteria are met:

$$\begin{aligned}
 K_{SF2} &= -A^3 K_{SF1}, \\
 K_{SD2} &= -B^3 K_{SD1}, \\
 K_{SF1} &= \frac{\xi_{x1} + \xi_{x2}}{M_{F12}^3 D'_x}, \\
 K_{SD1} &= \frac{\xi_y}{M_{D34}^3 D'_x}
 \end{aligned} \tag{13.65}$$

where the K are the integrated sextupole strengths of the sextupoles, with subscripts denoting the names as defined in Figure 13.22. Additionally, the chromaticities ξ_{x1} and ξ_{x2} are those generated upstream and downstream of the bending section, and should be equal to each other. D'_x is the angular dispersion at the IP, and the M_{F12} and M_{D34} are the elements of the transfer matrix between the focusing and defocusing pairs of quadrupoles, respectively.

13.4.2 The Oide Effect

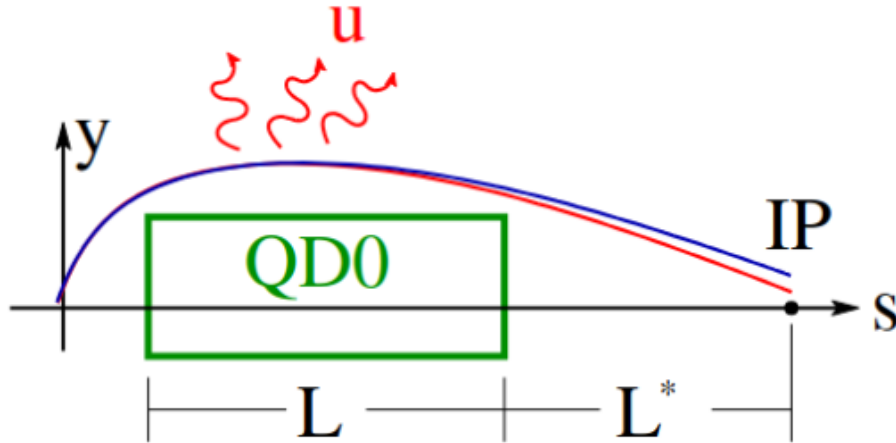


Figure 13.23: Differences in particle trajectory for design orbit (blue) and trajectory due to synchrotron radiation emission in the final quadrupole (red). Figure taken from Ref. [195].

The strong focusing fields of the FD quadrupoles acting on the electron beam

result in the emission of hard SR near the IP, as shown in the diagram in Figure 13.23. This process, known as the Oide effect, causes emittance growth in the beam, which increases the spot size at the IP. Furthermore, the Oide effect sets a bound on the minimum achievable spot size, known as the Oide limit. In linear colliders, where flat beams are typically preferred, the spot size growth is much more pronounced in the vertical direction due to the fact that the final quadrupole is usually focusing in the vertical direction.

With SR emission, the vertical spot size at the IP for a linear collider is given by:

$$\sigma_y^{*2} = \beta_y^* \epsilon_y + \frac{110}{3\sqrt{6}\pi} r_e \lambda_e \gamma^5 F(\sqrt{K}L, \sqrt{K}\ell^*) \left(\frac{\epsilon_y}{\beta_y^*} \right)^{5/2} \quad (13.66)$$

where $r_e = 2.818 \times 10^{-15}$ m is the classical electron radius, $\lambda_e = 2.426 \times 10^{-12}$ m is the electron's Compton wavelength, γ is the relativistic Lorentz factor, and F is a dimensionless function given by:

$$F(\sqrt{K}L, \sqrt{K}\ell^*) = \int_0^{\sqrt{K}L} |\sin \phi + \sqrt{K}\ell^* \cos \phi|^3 \times \left[\int_0^\phi (\sin \phi' + \sqrt{K}\ell^* \cos \phi') d\phi' \right]^2 d\phi \quad (13.67)$$

where K is the focusing gradient of the quadrupole, L is the thickness of the quadrupole, and ℓ^* is the distance between the front of the quadrupole and the IP. The function in Equation 13.66 has a minimum when β_y^* is

$$\beta_y^* = \left[\frac{275}{3\sqrt{6}\pi} r_e \lambda_e F(\sqrt{K}L, \sqrt{K}\ell^*) \right]^{2/7} \gamma(\epsilon_{Ny})^{3/7} \quad (13.68)$$

which results in the following expression for the minimum vertical spot size at the IP, known as the Oide limit:

$$\sigma_y^* = \left(\frac{7}{5}\right)^{1/2} \left[\frac{275}{3\sqrt{6\pi}} r_e \lambda_e F(\sqrt{K}L, \sqrt{K}\ell^*) \right]^{1/7} (\epsilon_{Ny})^{5/7} \quad (13.69)$$

Despite the Oide limit not being measured directly at collider experiments, it is a fundamental ingredient in the design of future proposed linear colliders.

A few solutions exist for the mitigation or even circumvention of the Oide effect at linear colliders. The most obvious solution arises from increasing the length L or decreasing the gradient K of the final quadrupole [196], although this leads to diminishing returns beyond $L = 10$ meters and increases the FFS length. Another solution is the use of octupole magnets placed between and after the final quadrupoles to compensate the average vertical deflection from SR energy loss [197], as shown in Figure 13.24. Plasma lenses with a density ramp (the so-called *adiabatic plasma lens*) have also been shown to mitigate the Oide effect from a theoretical standpoint [179], however practical implementation of such a lens is challenging and experimental validation has yet to be demonstrated.

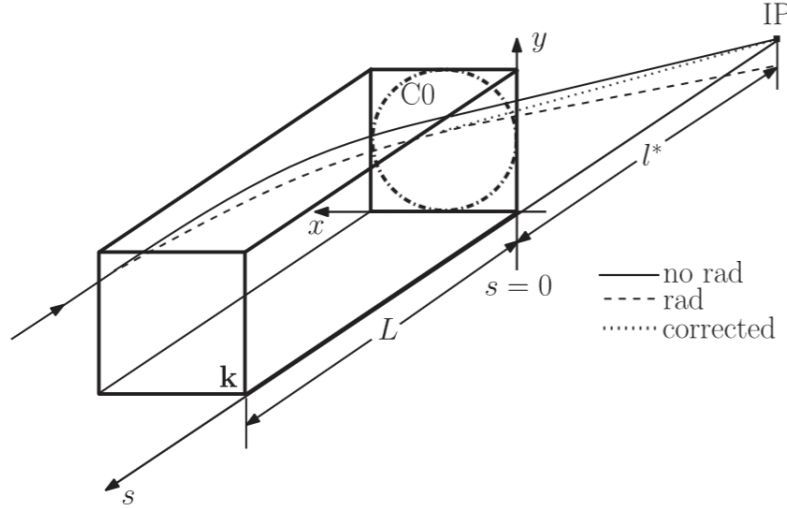


Figure 13.24: Trajectories of particles through final quadrupole to IP considering no SR (solid line), with SR included (dashed line), and with octupole correction (dotted line). Figure taken from Ref. [198].

There is also a dependence of the Oide limit on the horizontal emittance, but to

a lesser degree. A generalization considering both transverse dimensions is derived in ref. [198], exhibiting a scaling of the vertical beam size strongly with the vertical emittance while scaling quadratically with the horizontal emittance.

13.4.3 Beam-Beam Effects

The presence of a high density electron beam creates a strong electric field, which can interact with the particles of the opposing beam once they are sufficiently close together. Near the IP, this interaction can have a deleterious effect on the beams themselves, causing the emission of SR photons which reduces the energy of the electrons. The characteristic electric field strength at which these beam-beam interactions becomes significant is the Schwinger critical field $\mathcal{E}_{\text{cr}}$ [199]:

$$\mathcal{E}_{\text{cr}} = \frac{m_e^2 c^3}{e \hbar} = 1.323 \times 10^{18} \text{ V m}^{-1} \quad (13.70)$$

The Schwinger critical field is, notably, the field strength at which quantum electrodynamics becomes nonlinear, which enables the production of electron-positron pairs from the vacuum via coherent (beamstrahlung pair creation and trident cascade) processes along with the contributions from incoherent (Breit-Wheeler, Bethe-Heitler, and Landau-Lifshitz) processes [200]. The products of these interactions can provide additional backgrounds for detector experiments, which can pose serious challenges for event reconstruction. Additionally, the production of electron-positron pairs creates further distortion of the luminosity spectrum as the additional low-momentum particles contribute to collisions at smaller values of the CM energy, as shown in Figure 13.25.

The ratio of the electric field of an electron beam $\mathcal{E}$ to the Schwinger field, characterizing the level of quantum beamstrahlung effects, is known as the *beamstrahlung parameter* Υ [200]:

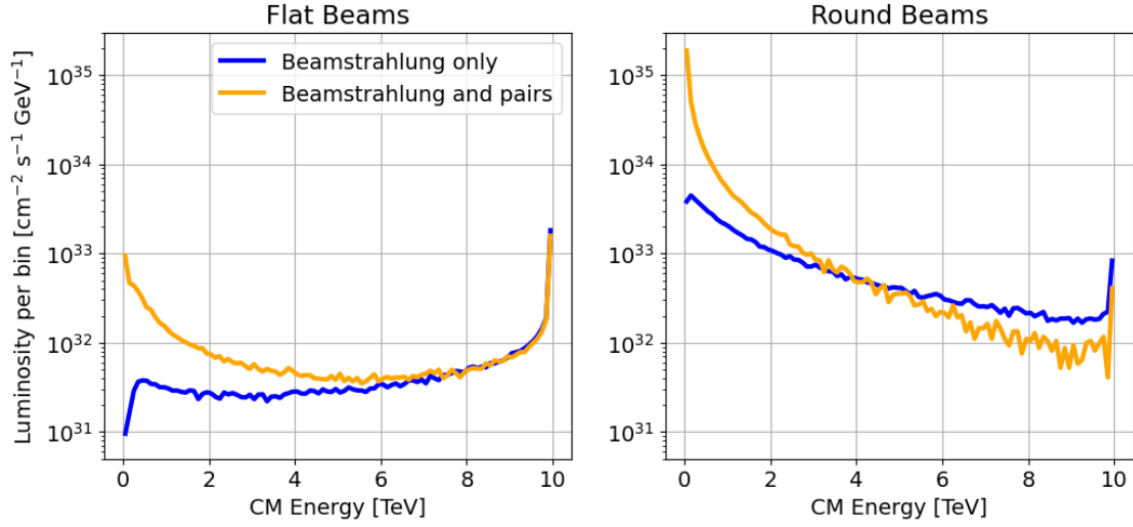


Figure 13.25: Luminosity spectrum for flat-beam (left) and round-beam (right) collisions at 10 TeV CM energy with (orange curve) and without (blue curve) pair production processes included. Figure taken from Ref. [144].

$$\Upsilon = \frac{\mathcal{E}}{\mathcal{E}_{\text{cr}}} = \frac{2}{3} \frac{\hbar \omega_c}{E_b} \quad (13.71)$$

where $\omega_c = 3\gamma^3 c/2\rho$ is the critical frequency for SR [201] and E_b is the electron beam energy. For $\Upsilon > 1$, this is the so-called "quantum beamstrahlung regime", where the SR is in the quantum regime, and the coherent production of electrons and positrons is significant. For lower values of Υ ($\Upsilon \lesssim 0.3$), the incoherent processes become dominant, producing a large number of low-energy particles that can cause unwanted interactions. As the bunches interact, the field experienced by the opposing beam changes, resulting in fluctuations of the Υ parameter. It was shown by Chen [202] that the net effect due to beamstrahlung for Gaussian beams is well-estimated by the mean beamstrahlung parameter over the effective collision time:

$$\Upsilon_{\text{mean}} \approx \frac{5}{6} \frac{r_e^2 \gamma N}{\alpha \sigma_z (\sigma_x + \sigma_y)} \quad (13.72)$$

where N is the bunch population (number of electrons), and α is the fine structure con-

stant. Equation 13.72 is a useful starting point to estimate, for a given collider design, the impact that beamstrahlung will have. Table 13.2 shows the collider parameters for a number of proposed future electron-positron machines and the corresponding Υ_{mean} values.

Beamstrahlung at Proposed Colliders					
Collider	γ	N	σ_x^*/σ_y^* [nm],	σ_z [μm]	Υ_{mean}
CLIC 380	3.52×10^5	5.2×10^9	149.0/2.0	70	0.16
CLIC 1.5 TeV	1.47×10^6	3.7×10^9	60/1.5	44	1.8
CLIC 3 TeV	2.94×10^6	3.7×10^9	50/1.5	44	4.4
CLIC 7 TeV	6.85×10^6	3.7×10^9	39.4/1.8	44	13
ILC 250	2.44×10^5	2×10^{10}	515/7.66	300	0.03
ERLC 500	4.89×10^5	8.1×10^8	900/7.4	890	4.4×10^{-4}
PWFA 1 TeV	9.78×10^5	5×10^9	58.07/1.43	5	15
PWFA 3 TeV	2.94×10^6	5×10^9	33.53/0.83	5	78
PWFA 15 TeV	1.47×10^7	5×10^9	15/0.4	5	867

Table 13.2: Mean beamstrahlung parameters for proposed future collider designs, calculated using Equation 13.72. Values for IP beam parameters obtained from refs. [156, 203, 204]. Values for PWFA colliders are based on projected physics goals, and do not reflect a mature design concept.

The effects of beamstrahlung can be mitigated by the use of flat beams rather than round beams for fixed beam area at the IP, as the flat beams provide fewer collisions at lower CM energy for the same total luminosity. For collision energies $\sqrt{s} > 1$ TeV, many of the relevant collider designs operate in the regime where $\Upsilon \gg 1$, primarily due to the high energies and short bunch lengths. In this regime, for fixed $\mathcal{L}$, beam energy, and beam sizes at the IP, the number of photons emitted per lepton n_γ and induced energy spread from beamstrahlung scale according to [156]:

$$\begin{aligned}
 n_\gamma &\propto \sigma_z^{1/3} \Upsilon^{2/3}, \\
 \delta_\gamma &\propto \sigma_z^{1/3} \Upsilon^{2/3}.
 \end{aligned}
 \tag{13.73}$$

These scalings suggest that shorter bunch lengths can help to reduce adversarial effects from beamstrahlung. Recent studies, operating under the assumption that beamstrahlung is an inevitability for collisions at CM energies of 10 TeV and above, have explored the physics potential of machines possessing broad luminosity spectra [144, 155, 156].

CHAPTER 14

A FINAL FOCUSING SYSTEM AT 10 TEV

As part of the 10 TeV Wakefield Collider Design Study, I present here the first design of a flat-beam final focusing system (FFS) for a 10 TeV collider. The design is based off the FFS for the Compact Linear Collider (CLIC) at a design energy of 7 TeV [204, 205]. Figure 14.1 shows the lattice along with the β -function and dispersion evolution while table 14.1 summarizes the parameters for the CLIC 7 TeV FFS design. The instantaneous luminosity, with SR effects included, is calculated to be [205]:

$$\mathcal{L}_{\text{CLIC},7\text{TeV}} = 12.5 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1} \quad (14.1)$$

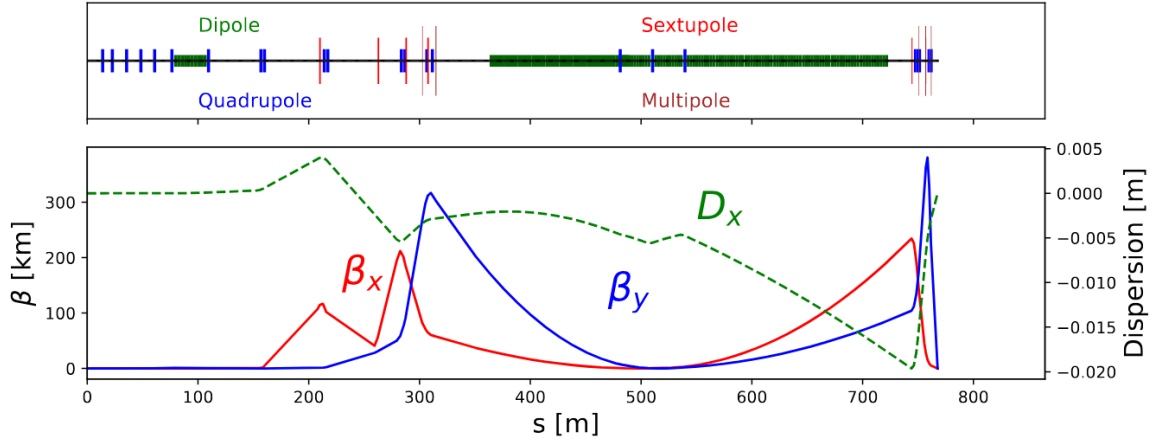


Figure 14.1: Horizontal and vertical β functions and dispersion for the 7 TeV CLIC FFS with survey shown above. Dipoles are shown in green, quadrupoles in blue, sextupoles in red, and higher-order multipoles in brown.

14.1 Methodology

The change in the magnetic rigidity going from a beam energy of 3.5 TeV for CLIC to the 5 TeV beam energy needed for the 10 TeV wakefield collider necessitates an energy-dependent scaling of the CLIC 7 TeV lattice. While this can be achieved by a proportional scaling of the magnetic field strength with the increased energy, this is not a feasible option as the integrated field strengths required would quickly

7 TeV FFS Parameters		
Parameter	Description	Value
Length [m]	Total length	767.671
L^* [m]	Distance between final quad and IP	6
β_x^*/β_y^* [mm]	Vertical/horizontal beta functions at IP	7.00/0.16
ϵ_x/ϵ_y [nm]	Vertical/horizontal emittances at entrance to FFS	660/20
σ_x^*/σ_y^* [nm]	Vertical/horizontal spot sizes at IP (no SR)	34.44/1.55
$\sigma_{x,SR}^*/\sigma_{y,SR}^*$ [nm]	Vertical/horizontal spot sizes at IP (with SR)	40.67/3.02
$\delta_{p,rms}$ [%]	RMS bunch energy spread	0.3
f_{rep} [Hz]	Repetition rate (collisions per second)	60
n_b	Number of bunches per pulse	312
N	Number of particles per bunch	3.72×10^9

Table 14.1: Parameters for the 7 TeV CLIC FFS design.

exceed the capabilities of even the highest-field superconducting technologies. A more sustainable approach, detailed in ref. [206], scales the lengths of the magnets according to a factor κ , related to the original energy E_0 and the final energy E_f via the equation:

$$\kappa = \left(\frac{E_f}{E_0} \right)^p \quad (14.2)$$

where $p \in \mathbb{R}$ is determined by constraints for the scaling. The increase in the magnet length at fixed gradient increases the integrated quadrupole strength. In order to maintain a fixed integrated quadrupole strength, the focal length must also be scaled by the same factor. This results in the two conditions:

$$\begin{aligned} \tilde{L}_{\text{quadrupole}} &= \kappa L_{\text{quadrupole}} \\ \tilde{f} &= \kappa f \end{aligned} \quad (14.3)$$

where the quantities with tildes denote the scaled parameters. With these two conditions specified, we can determine the necessary scaling for the normalized quadrupole

strength k , which is given by (in the thin-lens approximation):

$$k = \frac{1}{L_{\text{quadrupole}} \times f} \quad (14.4)$$

which then implies that the scaled normalized quadrupole gradient is given by:

$$\tilde{k} = \frac{1}{\tilde{L}_{\text{quadrupole}} \times \tilde{f}} = \frac{1}{(\kappa^2)(L_{\text{quadrupole}} \times f)} = \frac{1}{\kappa^2} k \quad (14.5)$$

The expected scaling of the β function can be derived utilizing Equations 14.3 and 14.5 with Hill's equation:

$$\frac{d^2}{ds^2} x(s) - kx(s) = 0 \quad (14.6)$$

where $x(s)$ is the transverse offset of a particle at position s . Equation 14.6 has a general solution of the form:

$$x(s) = \sqrt{\beta(s)} \epsilon \cos(\psi(s) + \psi_0) \quad (14.7)$$

where $\psi(s)$ is the phase at position s and ψ_0 is the initial phase. Substituting this solution back into Eq. 14.6, one obtains:

$$\frac{d^2}{ds^2} \left(\sqrt{\beta(s)} \right) - \frac{1}{(\sqrt{\beta(s)})^3} - k \left(\sqrt{\beta(s)} \right) = 0 \quad (14.8)$$

After scaling, we have $\beta \rightarrow \tilde{\beta}$, $s \rightarrow \kappa s$, and $k \rightarrow k/\kappa^2$, and Equation 14.8 can be re-written as:

$$\frac{1}{\kappa^2} \frac{d^2}{ds^2} \left(\sqrt{\tilde{\beta}(s)} \right) - \frac{1}{\left(\sqrt{\tilde{\beta}(s)} \right)^3} - \frac{1}{\kappa^2} k \left(\sqrt{\tilde{\beta}(s)} \right) = 0 \quad (14.9)$$

The solution for $\tilde{\beta}(s)$ is chosen such that the factors of κ in each term of Equation 14.9

are equal to each other, which occurs for

$$\tilde{\beta}(s) = \kappa\beta(s) \quad (14.10)$$

With this solution, the factors of κ can be divided by on both sides of Equation 14.9, reducing it back to Equation 14.8.

The scaling procedure can be further extended to different species of magnets, under the assumption of constant integrated field strengths. For a magnet with N poles, the integrated field strength K_N is related to the magnet length L and the magnetic field via:

$$K_N = L \times \frac{\partial^{(N/2-1)} B_y}{\partial x^{(N/2-1)}} = L \times k_{(N/2-1)} \quad (14.11)$$

where $k_{(N/2-1)}$ the normalized field strength for a magnet with N poles and has units of $[\text{m}^{-(N/2)}]$. After scaling, we have $L \rightarrow \kappa L$ and $\partial x \rightarrow \kappa \partial x$, which gives the relation:

$$K_N = L \times k_{(N/2-1)} = \kappa L \times \frac{\tilde{k}_{(N/2-1)}}{\kappa^{(N/2-1)}} = \frac{L \times \tilde{k}_{(N/2-1)}}{\kappa^{N/2}} \quad (14.12)$$

which means that the normalized field strength for a magnet with N poles scales like:

$$\tilde{k}_{(N/2-1)} = \kappa^{-(N/2)} k_{(N/2-1)} \quad (14.13)$$

Table 14.2 summarizes the scalings of the relevant parameters under this scheme.

The choice of the parameter p in Equation 14.2 depends on additional constraints imposed by the particular FFS design. One design consideration is the case in which the beam "stay-clear" N_{sc} is preserved:

$$N_{\text{sc}} = r_{\text{coil}}/\sigma_x \quad (14.14)$$

Scalings of relevant parameters			
Quantity	Units	Pre-scaling	Post-scaling
Beta Function	[m]	β	$\kappa\beta$
Magnet and Drift Lengths	[m]	ℓ	$\kappa\ell$
Norm. Dipole strength	[m ⁻¹]	k_0	$\kappa^{-1}k_0$
Norm. Quadrupole strength	[m ⁻²]	k_1	$\kappa^{-2}k_1$
Norm. Sextupole strength	[m ⁻³]	k_2	$\kappa^{-3}k_2$

Table 14.2: Scaling of parameters for increased FFS design energy. The value of the parameter κ is given by Eq. 14.16.

where r_{coil} is the coil aperture radius of a quadrupole and σ_x is the horizontal spot size. From this definition, the stay clear represents the distance between the beam and the magnet coil, normalized to the horizontal spot size. Preservation of the stay clear thus ensures that a scaling of the magnet strengths and length scales keeps the machine protected from the beam as well as beam jitter. This constraint also ensures the shortest possible interaction region length [206]. Assuming fixed normalized emittances when moving to a higher beam energy, the resulting value of p for fixed beam stay clear is:

$$p = \frac{1}{3}, \quad (\text{constant } N_{\text{sc}}) \quad (14.15)$$

Starting with a beam energy of 3.5 TeV for the CLIC design and scaling up to 5 TeV for the 10 TeV wakefield design, the value of the parameter κ for constant beam stay clear is:

$$\kappa = \left(\frac{5 \text{ TeV}}{3.5 \text{ TeV}} \right)^{1/3} \approx 1.126 \quad (14.16)$$

Another scheme, which preserves the magnet apertures and gradients, stipulates:

$$p = \frac{1}{2}, \quad (\text{constant } r_{\text{coil}}, g) \quad (14.17)$$

As a starting point, we choose κ under the assumption of constant beam stay clear and also test different values of p in order to maximize the luminosity.

A number of different software tools are utilized for the design of the lattice and evaluation of the lattice parameter evolution, the tracking of particles, the calculation of beam spot size contributions at different orders in the transfer map, and the simulation of the beam-beam effects to determine the luminosity. The lattice is implemented in the MAD-X software [162], which allows for the evolution of the Twiss and other beam parameters along the lattice. Particle tracking is implemented in two different codes, each with their own use cases. PLACET [207] is used for the evaluation of the beam spot size, and the resulting particle distribution is fed into the particle-in-cell (PIC) code GUINEA-PIG [208] to simulate the beam-beam effects and calculate the luminosity. The MAD-X polymorphic tracking code (PTC) [163] is utilized separately for the computation of the map coefficients (see Eq. 13.16) up to arbitrary order, which is interfaced with MAPCLASS2 [164] to compute the beam spot size at different map orders.

14.2 Results

Under the scaling scheme summarized in Table 14.2 (where κ is given by Eq. 14.16), the length of the FFS increases from 767.671 m at 7 TeV to 864.588 m at 10 TeV. Figure 14.2 shows the evolution of β -function and dispersion along with the lattice survey for the 10 TeV FFS design. The key parameters for the 10 TeV FFS are summarized in Table 14.3; many of the values, including the vertical and horizontal normalized emittances, bunch energy spread, repetition rate, bunches per pulse, and particles per bunch, are borrowed from the CLIC 7 TeV design. These parameters are also compatible with the goals from the Snowmass 2021 report [156]. Values for the IP β functions are obtained from MAD-X, while the reported beam spot sizes

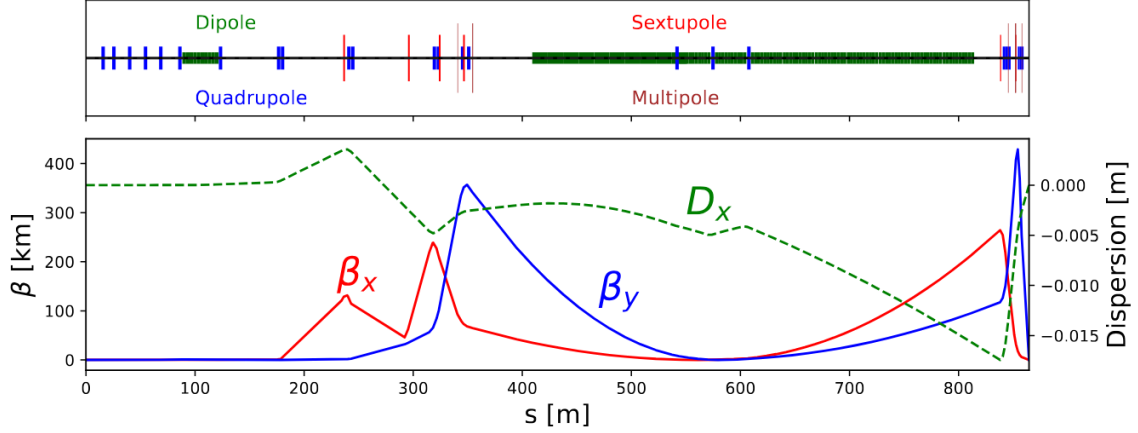


Figure 14.2: Horizontal and vertical β functions and dispersion for the 10 TeV FFS with survey shown above. Dipoles are shown in green, quadrupoles in blue, sextupoles in red, and higher-order multipoles in brown.

at the IP are from particle tracking in PLACET. The luminosity is calculated by performing particle tracking using PLACET and using the IP particle distribution with GUINEA-PIG to simulate the beam-beam effects and calculate the luminosity. The resulting luminosity from GUINEA-PIG for the 10 TeV FFS is calculated to be:

$$\mathcal{L}_{10\text{TeV}} = 10.8 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1} \quad (14.18)$$

and the parameters for the GUINEA-PIG simulation are summarized in Table G.1.

The impact of SR on the beam spot size and luminosity are considered as well. Table 14.4 shows the beam spot sizes calculated with PLACET with and without SR effects included. The resulting luminosity, calculated from GUINEA-PIG, shows a factor of ≈ 2 reduction due to SR. Additionally, the spot sizes and luminosity were calculated turning off SR in different magnets as shown in Table G.2, showing the individual contributions from dipoles, quadrupoles, and multipoles (sextupoles and higher). From this, we find that the majority of the vertical spot size growth comes from SR emission in the quadrupoles, while in the horizontal direction the spot size growth is shared between radiation from dipoles and quadrupoles. As discussed in

10 TeV FFS Parameters		
Parameter	Description	Value
Length [m]	Total length	864.588
L^* [m]	Distance between final quad and IP	6.757
β_x^*/β_y^* [mm]	Vertical/horizontal beta functions at IP	7.88/0.18
ϵ_x/ϵ_y [nm]	Vertical/horizontal emittances at entrance to FFS	660/20
σ_x^*/σ_y^* [nm]	Vertical/horizontal spot sizes at IP (no SR)	32.40/2.31
$\sigma_{x,SR}^*/\sigma_{y,SR}^*$ [nm]	Vertical/horizontal spot sizes at IP (with SR)	43.02/4.21
$\delta_{p,rms}$ [%]	RMS bunch energy spread	0.3
f_{rep} [Hz]	Repetition rate (collisions per second)	60
n_b	Number of bunches per pulse	312
N	Number of particles per bunch	3.72×10^9

Table 14.3: Parameters for the 10 TeV FFS design.

	σ_x^* [nm]	σ_y^* [nm]	$\mathcal{L}$ [$10^{34} \text{ cm}^{-2} \text{ s}^{-1}$]
Without SR	32.40	2.31	19.2
With SR	43.02	4.21	10.8
Target Values	18.4	0.45	34.1

Table 14.4: IP Spot Sizes with and without SR effects included, calculated from PLACET, and luminosity calculated using GUINEA-PIG. Target values extrapolated from 15 TeV values for a flat-beam PWFA collider provided in the 2021 Snowmass report [156].

Section 13.4.1, cancellation of the chromaticity requires certain conditions relating the dispersion generated by dipoles to the sextupole strengths (see Equation 13.62). The dipoles generate SR, and the overall impact to the spot size and luminosity was investigated by introducing a bending factor b_f to the dipole strength k_0 and the sextupole strength k_2 such that:

$$\begin{aligned} k_0 &\rightarrow b_f k_0, \\ k_2 &\rightarrow k_2/b_f, \end{aligned} \tag{14.19}$$

which preserves the relationship between the dipole and sextupole strengths needed for the chromatic correction. Figure 14.3 shows the spot sizes (top) and luminosity (bottom) as a function of this bending factor; the maximum luminosity and minimum IP spot size is obtained for the nominal values of the dipole and sextupole strengths ($b_f = 1$). This suggests, in this particular configuration of the FFS, that the dipole and sextupole strengths are optimal.

Significant contributions to the beam spot size can arise from higher-order aberrations introduced by the lattice itself. As described in Section 13.1, the particle coordinates at a single point along the lattice are related to the initial 6D-phase-space coordinates using the transfer map formalism. The transverse coordinates for each particle at the IP can then be computed from knowledge of the map coefficients, resulting in a distribution from which the RMS beam size can be extracted. The full expression for the (squared) standard deviation of a coordinate at the IP z_f is a sum over many terms [161]:

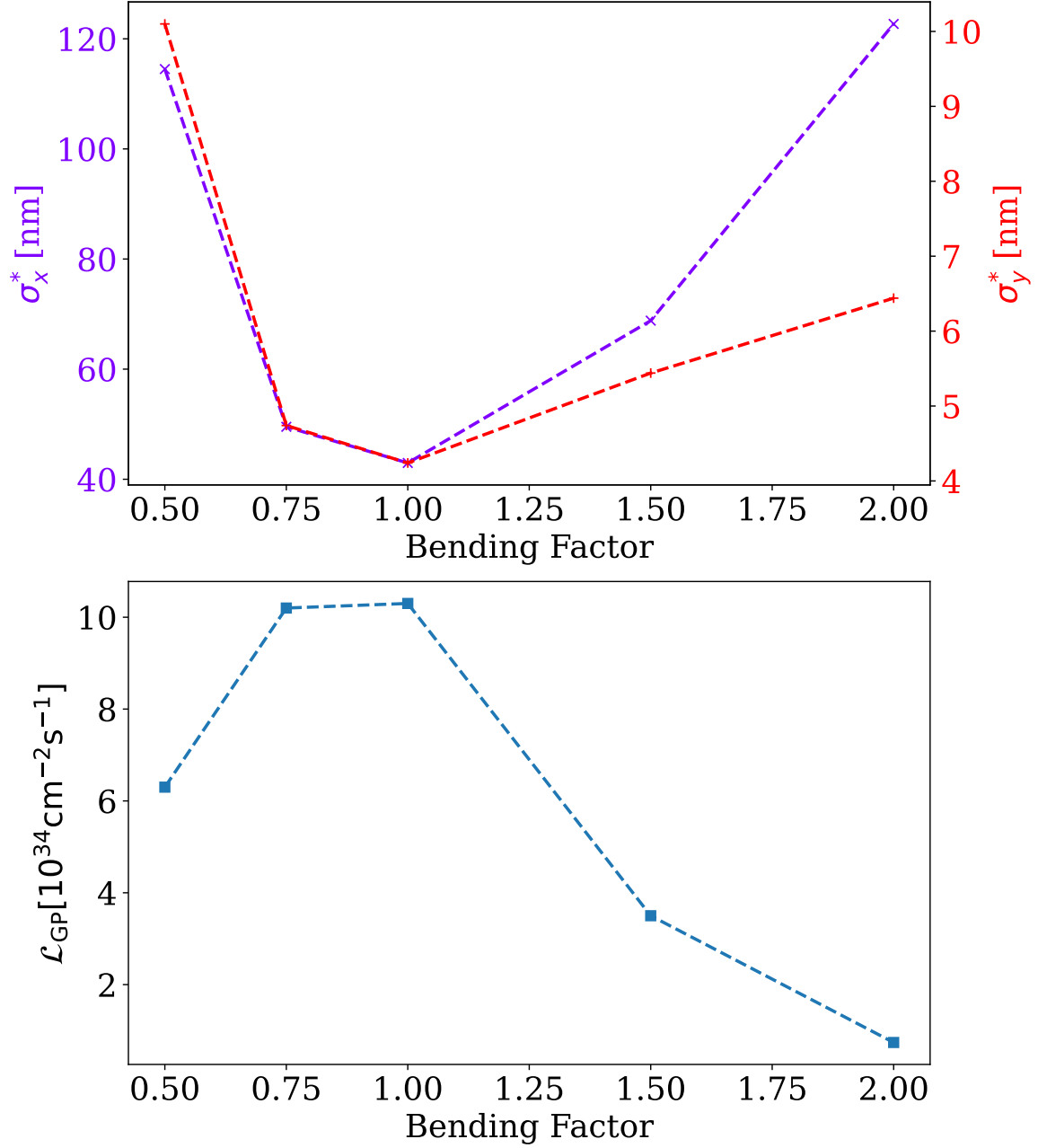


Figure 14.3: Beam sizes (top) and luminosity (bottom) for different values of the bending factor. Values for the spot sizes and luminosity were obtained via PLACET simulations and GUINEA-PIG, respectively.

$$\begin{aligned}
 \langle z_f^2 \rangle = & \sum_{jklmn} X_{jklmn}^2 \Gamma\left(\frac{1+2j}{2}\right) \Gamma\left(\frac{1+2k}{2}\right) \Gamma\left(\frac{1+2l}{2}\right) \Gamma\left(\frac{1+2m}{2}\right) \\
 & \times \frac{2^{j+k+l+m-2n}}{(2n+1)\pi^2} \sigma_x^{2j} \sigma_{p_x}^{2k} \sigma_y^{2l} \sigma_{p_y}^{2m} \sigma_\delta^{2n} \\
 & + \sum_{jklmn > j'k'l'm'n'} 2X_{jklmn} X_{j'k'l'm'n'} \\
 & \times \Gamma\left(\frac{1+j+j'}{2}\right) \Gamma\left(\frac{1+k+k'}{2}\right) \Gamma\left(\frac{1+l+l'}{2}\right) \Gamma\left(\frac{1+m+m'}{2}\right) \\
 & \times \frac{2^{([j+k+l+m+j'+k'+l'+m']/2)-n-n'}}{(n+n'+1)\pi^2} \sigma_x^{j+j'} \sigma_{p_x}^{k+k'} \sigma_y^{l+l'} \sigma_{p_y}^{m+m'} \sigma_\delta^{n+n'},
 \end{aligned} \tag{14.20}$$

where Γ is the gamma function. Written in a more compact way, the standard deviation of the transverse coordinates $q_f = (x_f, y_f)$, corresponding to the RMS beam sizes, is an infinite sum over terms at different orders in the map expansion:

$$\sigma_q^2 = \sum_M^\infty \langle q_f^2 \rangle_M = \langle q_f^2 \rangle_1 + \langle q_f^2 \rangle_2 + \langle q_f^2 \rangle_3 + \dots \tag{14.21}$$

where the subscripts denote the map order M of the contribution to the spot size. The advantage of this expression is in its interpretation; the $M = 1$ term accounts for the spot size resulting from the linear Twiss parameters (see Eq. 13.11), the $M = 2$ term accounts for chromatic effects and lowest-order non-linear effects from sextupole fields, and the $M = 3$ term contains the octupole contributions, and so on. Rather than computing the full infinite sum, it is more efficient to calculate the spot size up to some maximum order p such that the contribution at order $p + 1$ is small in comparison, allowing for a reasonable approximation without large computational costs. Figure 14.4 shows the order-by-order contribution to the IP spot size for the 10 TeV FFS, calculated using the MAPCLASS2 program without SR effects included up to $M = 5$ with values provided in Table 14.5. From this, the largest contributions

Beam sizes for 10 TeV FFS		
Map Order	σ_x^* [nm]	σ_y^* [nm]
1	25.6	0.497
2	28.5	0.532
3	28.8	1.15
4	31.1	2.22
5	31.9	2.34

Table 14.5: Horizontal and vertical spot sizes calculated from MAPCLASS2 for the 10 TeV CLIC FFS lattice up to $M = 5$ in the map.

to the spot size come from $M = 2$ and $M = 4$ for the horizontal spot size, while in the vertical direction the largest contributions arise at $M = 3$ and $M = 4$.

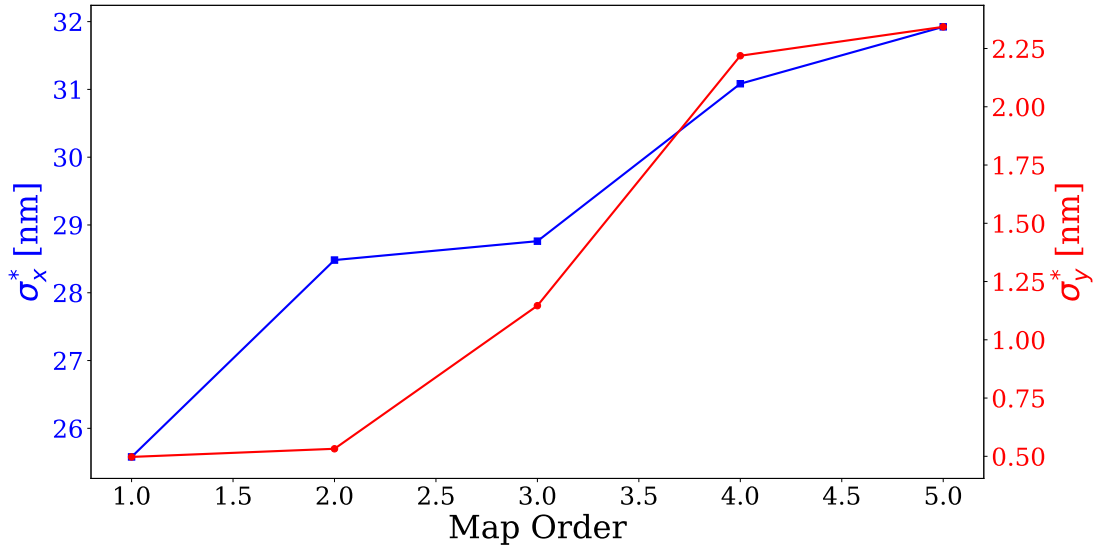


Figure 14.4: Horizontal (blue) and vertical (red) beam sizes at 10 TeV calculated to different orders in the map. Values calculated using MAPCLASS2, and do not include contributions from synchrotron radiation.

Of particular concern for the 10 TeV FFS design is the impact of the Oide effect, arising from the strong focusing fields of the final quadrupole (QD0). The sensitivity of the chosen parameters to the Oide limit was studied by scanning the β^* and emittance values. At fixed β_y^* , the vertical emittance was scanned for different values of the horizontal emittance, as shown in Figure 14.5. The values for the 10 TeV

FFS, denoted by the black dot, is in a region where the Oide effect starts to become significant, as the slope of the red curve at this point falls between the spot size scaling predicted by Equation 13.11 (purple line) and Equation 13.69 (blue line). This behavior is also apparent when varying β_y^* for fixed emittances. Figure 14.6 shows the evolution of the vertical beam size at the IP (purple curve) and luminosity (red curve) for different β_y^* values. As indicated by the teal line, the β_y^* chosen for the 10 TeV FFS maximizes the luminosity, while the minimum σ_y^* is achieved for a β_y^* that is ≈ 4 times larger. This increase in the beam size at peak luminosity is due to the fact that the beam size is sensitive to tails in the beam that are generated by higher-order aberrations and SR from quadrupoles (e.g. Oide effect), while the luminosity is largely dependent on the core of the beam distribution.

The transport of particles along the FFS also causes an increase in the emittance through various processes. Fixing the vertical and horizontal β functions to their nominal values, the vertical and horizontal emittances were scanned. Figure 14.7 shows the IP vertical emittance $\varepsilon_{y,f}$ as a function of the vertical emittance at the start of the FFS $\varepsilon_{y,0}$ for different values of the horizontal emittance, which are denoted by the different color lines (with units [10^{-7} m]). For the nominal values used for the 10 TeV FFS ($\varepsilon_{x,0} = 660$ nm, $\varepsilon_{y,0} = 20$ nm), the vertical emittance growth along the FFS is approximately by a factor of 5.

Beamstrahlung effects were studied for the 10 TeV FFS using GUINEA-PIG. Figure 14.8 shows the luminosity spectrum as a function of collision energy ($\sqrt{s}$) for the 10 TeV FFS design without pair production processes enabled; the red line denotes the luminosity for collisions occurring within 20% of the nominal design energy. This luminosity, denoted by $\mathcal{L}_{20\%}$ is taken as a figure of merit for energy-frontier machines, as the precise calibration of the luminosity with CM energy is not necessary in searches for new particles [155]. The resulting $\mathcal{L}_{20\%}$ from GUINEA-PIG is

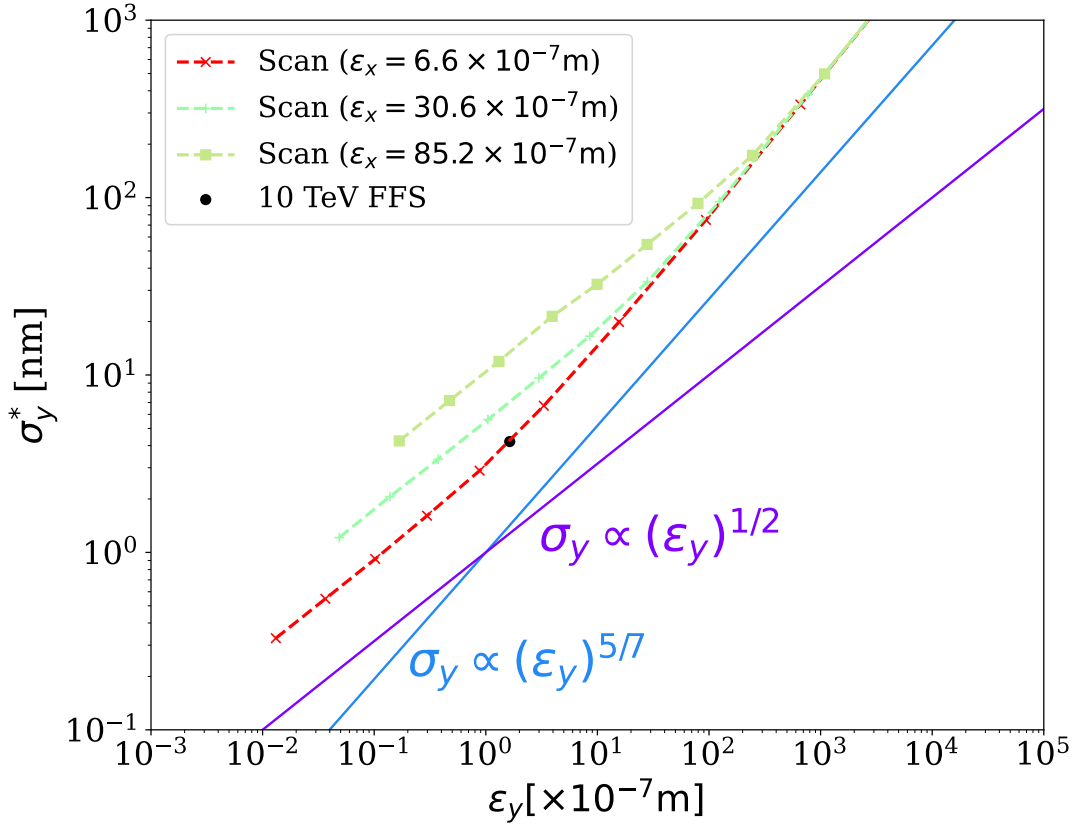


Figure 14.5: Vertical beam size at the IP as a function of the normalized vertical emittance. Green and red curves show beam size for different values of the normalized horizontal emittance, while the blue and purple lines are shown for reference, highlighting the expected beam size scaling with (Eq. 13.69) and without (Eq. 13.11) the Oide effect, respectively. Beam sizes are calculated using PLACET.

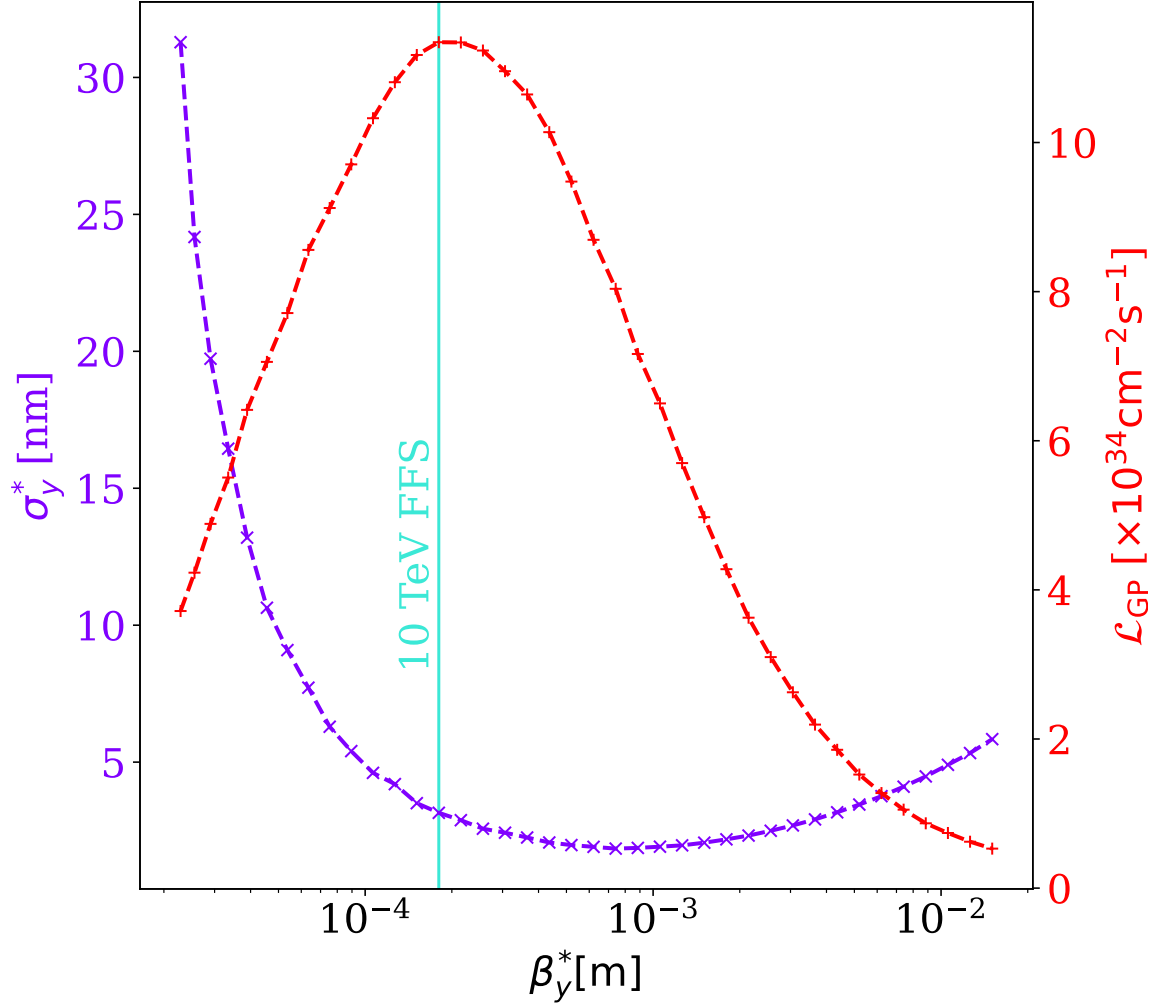


Figure 14.6: Vertical beam size (purple) and luminosity (red) at the IP as a function of the vertical beta function at the IP. The vertical line shows the nominal beam size for the 10 TeV FFS design. Values for β_y^* obtained from MAD-X, while spot sizes and luminosity obtained via PLACET simulations and GuineaPIG, respectively.

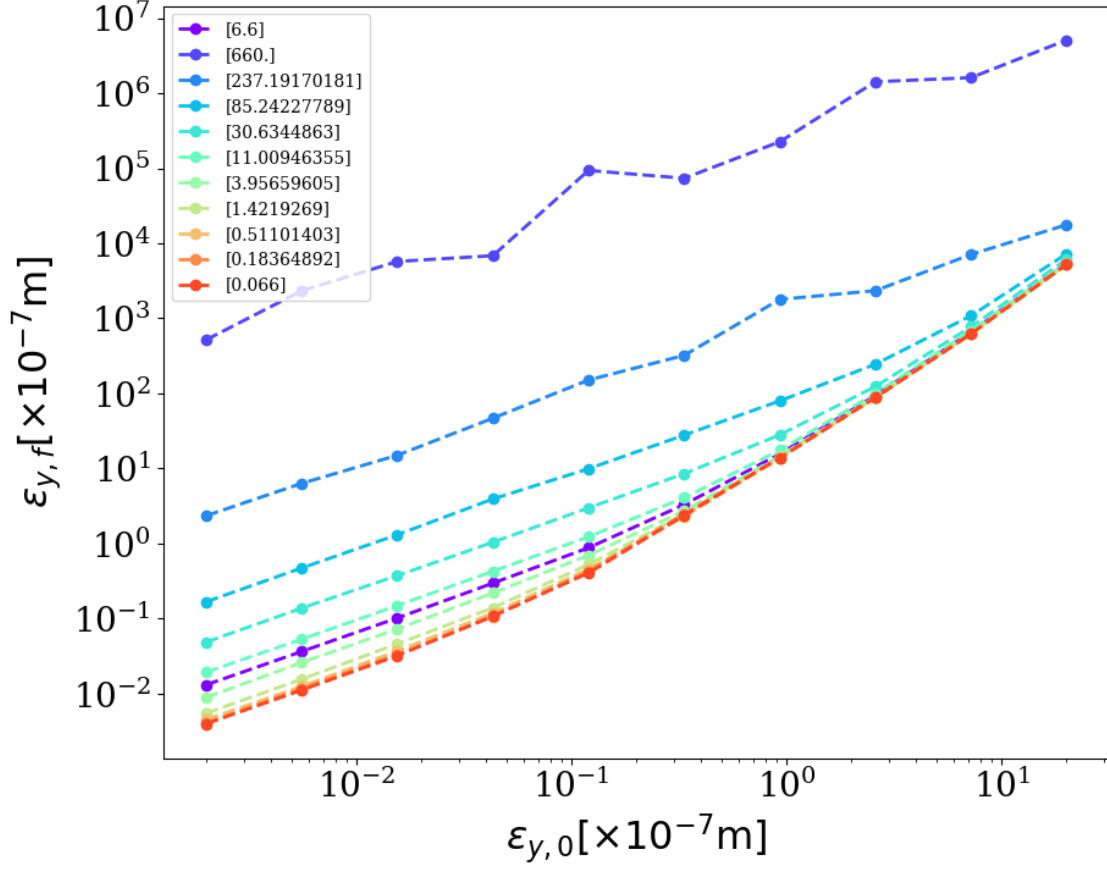


Figure 14.7: IP vertical emittance (vertical axis) as a function of initial vertical emittance (horizontal axis) for the 10 TeV FFS. Colored lines correspond to different values of the initial horizontal emittance, and are in units of $[10^{-7} \text{ m}]$.

$$\mathcal{L}_{20\%} = 3.07 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}. \quad (14.22)$$

The ratio of the top-20% luminosity to the total luminosity for the 10 TeV FFS is thus:

$$\mathcal{L}_{20\%}/\mathcal{L}_{\text{tot}} = 3.07/10.8 = 0.284 \quad (14.23)$$

meaning that approximately 30% of the total luminosity at 10 TeV is coming from collisions within 20% of the design energy.

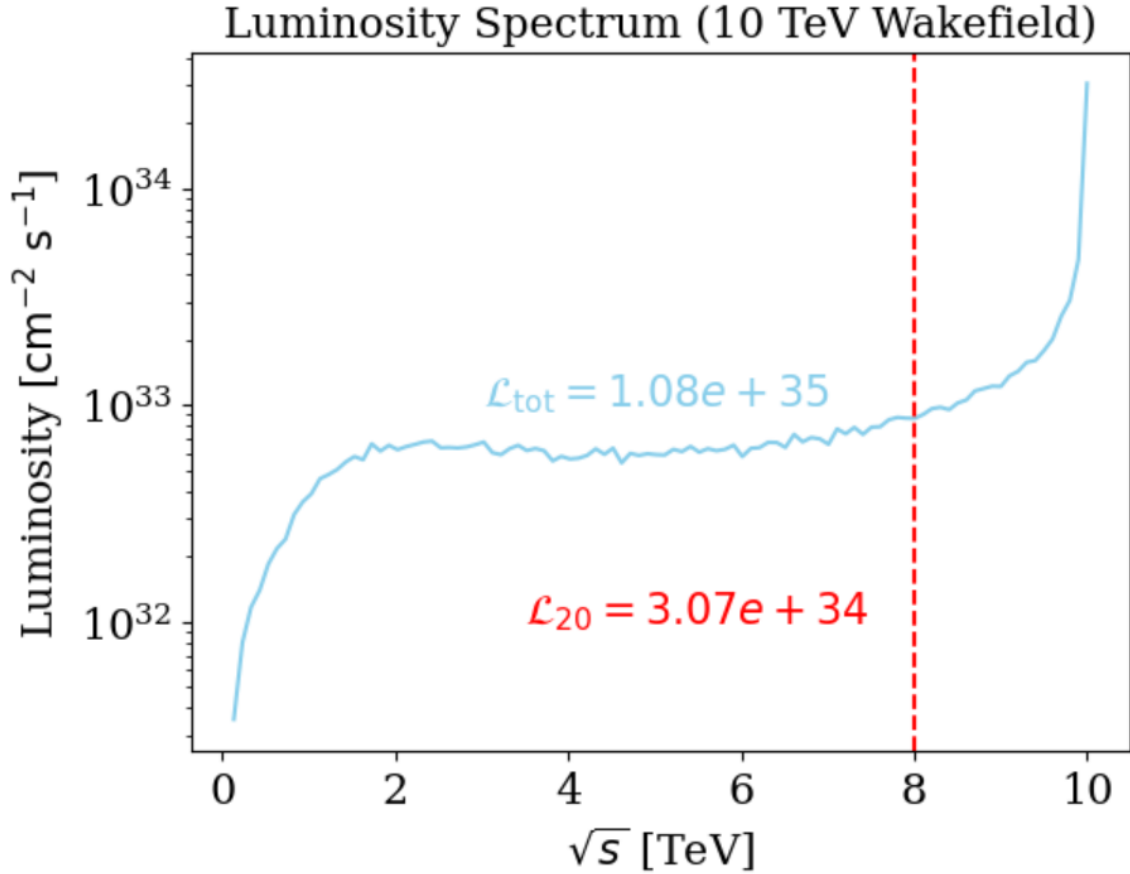


Figure 14.8: Luminosity spectrum as a function of the CM collision energy $\sqrt{s}$. The red line denotes the collision energy within 20% of the design energy, and the value for $\mathcal{L}_{20}$ is obtained by taking the integral of the blue line to the right of the red line.

The average beamstrahlung parameter, given by Equation 13.72, is calculated from

the maximum beamstrahlung parameter (Υ_{max}) returned by GUINEA-PIG [186]:

$$\Upsilon_{\text{mean}} \approx \frac{\Upsilon_{\text{max}}}{2.4} \approx 28.7. \quad (14.24)$$

It should be noted that this value is approximate, as it assumes Gaussian bunches at the IP. The value of Υ_{mean} suggests that this collider is firmly in the quantum beamstrahlung regime.

CHAPTER 15
CONCLUSIONS AND FUTURE STUDIES

15.1 Compatibility with PWFA Experiments

Plasma wakefield acceleration experiments have demonstrated some, but not all, of the parameters assumed in the design of this FFS. Experiments at FLASH-Forward at DESY [171] have demonstrated plasma acceleration of electron bunches resulting in energy spreads as low as 0.12% for the witness beam, compatible with the 0.3% RMS energy spread considered in this study. Furthermore, the witness bunch lengths obtained at FLASH-Forward and FACET-II at SLAC National Accelerator Laboratory [209] range from 10-20 μm , similar to the nominal bunch length of 44 μm for the 10 TeV FFS. However, both experiments to date have only been able to achieve witness bunch emittances on the order of 1 μm , requiring a reduction of 1-2 orders of magnitude to be compatible with the 0.66 μm horizontal and 0.02 μm vertical emittances that I assume here for the 10 TeV design. Furthermore, the witness bunch charges in both experiments are smaller by an order of magnitude than the values considered for the 10 TeV FFS. Table 15.1 summarizes the status of the experimental progress and its compatibility with the values assumed for the 10 TeV FFS design.

Metric	Achieved Values	This Study	Compatible?
Bunch energy spread	0.12% [171]	0.3%	Yes
Bunch length	10-20 μm [209]	44 μm	Yes
Bunch charge	$\mathcal{O}(10 \text{ pC})$ [171, 209]	596 pC	No
Emittance	$\mathcal{O}(1 \mu\text{m})^*$ [171, 209]	0.66 μm (x) / 0.02 μm (y)	No

Table 15.1: Comparison of achieved parameter values from PWFA experiments with the values assumed for the 10 TeV FFS design. * Emittances are (approximately) equal in both horizontal and vertical directions.

Recent simulation efforts [180, 210] have shown challenges with flat-beam PWFA, and limited experimental results are available in this regime. It may be difficult to preserve a large asymmetry between the horizontal and vertical emittances in the plasma acceleration process because of non-linear effects that lead to emittance mixing. If flat beams cannot be effectively accelerated in plasma, then an additional

matching section is needed to convert the round beams from the PWFA stages to the flat beams required at the entrance of the FFS.

15.2 Summary and Conclusions

The 10 TeV FFS lattice presented here is the first conceptual design of a FFS for a linear collider operating at a parton center of mass energy of 10 TeV. However, there are various improvements that can be made to maximize its performance. The increase in the vertical and horizontal beam sizes due to SR can be reduced by increasing the lengths and decreasing the gradients of the final focusing quadrupoles. Contributions from higher-order aberrations can also be mitigated by introducing octupoles, decapoles, and higher-order multipole magnets into the design.

In the context of the 10 TeV wakefield collider design study [154], this first 10 TeV FFS design falls short of the luminosity target of $\mathcal{L} = 34.1 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ for a flat-beam PWFA collider outlined by the Snowmass 2021 report [156]. Adjusted to 10 TeV from the initial 15 TeV proposed design energy, the target beam sizes in the vertical and horizontal directions are 0.45 nm and 18.4 nm, respectively; a factor of ≈ 10 smaller in y and ≈ 2.5 smaller in x. It should be noted that these reported target values were computed without any consideration for realistic beam dynamics and thus represent an idealized scenario. Even with the proposed improvements stated above, to achieve this decrease in the beam size and increase in luminosity, a thorough redesign of the lattice is required.

15.3 Outlook and Future Studies

Possible future studies include the design of a round-beam FFS at 10 TeV, leveraging the fact that PWFA is easier to achieve with round beams. For this, two designs could be considered: one consisting of conventional focusing elements while the other would

incorporate plasma lenses into the FFS in an attempt to reduce the overall length and focusing capabilities. A discussion on the properties and plasma lenses is presented in Section 13.2.7. To establish compatibility between PWFA and the current flat-beam 10 TeV FFS design, a matching section to convert the round beams to flat beams should be added as well. Other studies could include the characterization of the FFS performance due to magnet misalignment and jitter, similar to the method described in ref. [196].

For a 1-10 TeV center-of-mass plasma-based collider to remain competitive, the design assumptions for the collimation section must be fundamentally reconsidered due to the roughly linear scaling of the collimation system with energy. A collimation system optimized for plasma colliders could involve a distributed concept that is incorporated between plasma acceleration stages, or new methods for final doublet and inner detector protection. In summary, the design of the collimation system will be an important aspect of future studies for an optimized BDS.

APPENDIX A

VARIABLE DEFINITIONS

A.1 Variable Definitions

A.1.1 Lepton Identification

The following variables are used in the identification of leptons:

d_{xy} : the transverse distance from the primary vertex at which the lepton track originates. Commonly referred to as the *transverse impact parameter*.

d_z : the longitudinal distance from the primary vertex at which the lepton track originates. Commonly referred to as the *longitudinal impact parameter*.

$\frac{d}{\sigma_d}$: the 3D impact parameter (d) divided by the significance of the 3D impact parameter with respect to the primary vertex.

I_{rel} : the *relative isolation*, defined as the scalar p_T sum of PF objects in a cone of radius $\Delta R = 0.3$ around the lepton, divided by the lepton p_T :

$$I_{\text{rel}} = \frac{1}{p_T^\ell} \left[\sum_{\text{hadrons, charged}} p_T^{\text{PV}} + \max \left[0, \sum_{\text{hadrons, neutral}} p_T + \sum_{\text{photons}} p_T - \rho A_{\text{eff}} \right] \right] \quad (\text{A.1})$$

where the first sum is over charged hadrons originating from the primary vertex, the second (third) sums are over the neutral hadrons (photons). The factor ρA_{eff} is a term used to account for the contribution from pileup, as described in ref. [211].

Conversion Rejection: boolean stating whether the electron is not consistent with an internal conversion.

Charge consistency: agreement between the three different electron charge computation algorithms.

Missing hits: number of missing hits within the inner layer of the tracker detector.

p_T^{ratio} : relative muon p_T with respect to the closest jet p_T . If no jet is present, then $1/(1 + I_{rel})$ is used instead.

pflsId: ID stating whether the electron is sufficiently isolated from other particle-flow candidates.

A.1.2 Jet Identification

The following variables are used in the identification of jets:

$\Delta R_{j,\ell}$: angular separation (see Eq. 2.8) between candidate jets (see Section 5.3) and candidate leptons (see Section 5.2).

Neutral Hadron Fraction: Fraction of jet energy attributed to neutral energy deposits in the HCAL.

Neutral EM Fraction: Fraction of jet energy attributed to neutral energy deposits in the ECAL.

Number of Constituents: the number of particles within the jet.

Muon Fraction: Fraction of jet energy from muons.

Charged Hadron Fraction: Fraction of jet energy from charged hadrons.

Charged Multiplicity: Number of charged particles within the jet.

Charged EM Fraction: Fraction of jet energy attributed to charged energy deposits in the ECAL.

APPENDIX B

THE STANDARD MODEL LAGRANGIAN

"The horror..... the horror"

Colonel Walter E. Kurtz

Apocalypse Now (1979)

The Standard Model Lagrangian is given by [212]:

$$\begin{aligned}
 \mathcal{L}_{\text{SM}} = & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e + \\
 & \frac{1}{2}ig_s^2(\bar{q}_i^\sigma \gamma^\mu q_j^\sigma)g_\mu^a + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g_\mu^c - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \\
 & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2}M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \\
 & \frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2}M\phi^0 \phi^0 - \beta_h[\frac{2M^2}{g^2} + \\
 & \frac{2M}{g}H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z_\mu^0(W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - Z_\nu^0(W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + Z_\mu^0(W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)] - \\
 & ig s_w[\partial_\nu A_\mu(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu(W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + \\
 & A_\mu(W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)] - \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- + \\
 & g^2 c_w^2(Z_\mu^0 W_\mu^+ Z_\nu^0 W_\nu^- - Z_\mu^0 Z_\mu^0 W_\nu^+ W_\nu^-) + g^2 s_w^2(A_\mu W_\mu^+ A_\nu W_\nu^- - \\
 & A_\mu A_\mu W_\nu^+ W_\nu^-) + g^2 s_w c_w[A_\mu Z_\nu^0(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-] - \\
 & g\alpha[H^3 + H\phi^0 \phi^0 + 2H\phi^+ \phi^-] - \frac{1}{8}g^2 \alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + \\
 & 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2] - gMW_\mu^+ W_\mu^- H - \frac{1}{2}g\frac{M}{c_w^2}Z_\mu^0 Z_\mu^0 H - \\
 & \frac{1}{2}ig[W_\mu^+(\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^-(\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \\
 & \frac{1}{2}g[W_\mu^+(H \partial_\mu \phi^- - \phi^- \partial_\mu H) - W_\mu^-(H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z_\mu^0(H \partial_\mu \phi^0 - \\
 & \phi^0 \partial_\mu H) - ig\frac{s_w^2}{c_w}MZ_\mu^0(W_\mu^+ \phi^- - W_\mu^- \phi^+) + ig s_w MA_\mu(W_\mu^+ \phi^- - W_\mu^- \phi^+) - \\
 & ig\frac{1-2c_w^2}{2c_w}Z_\mu^0(\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + ig s_w A_\mu(\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \\
 & \frac{1}{4}g^2 W_\mu^+ W_\mu^- [H^2 + (\phi^0)^2 + 2\phi^+ \phi^-] - \frac{1}{4}g^2 \frac{1}{c_w^2}Z_\mu^0 Z_\mu^0 [H^2 + (\phi^0)^2 + 2(2s_w^2 - \\
 & 1)^2 \phi^+ \phi^-] - \frac{1}{2}g^2 \frac{s_w^2}{c_w}Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{s_w^2}{c_w}Z_\mu^0 H (W_\mu^+ \phi^- - \\
 & W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - \\
 & g^2 \frac{s_w}{c_w}(2c_w^2 - 1)Z_\mu^0 A_\mu \phi^+ \phi^- - g^1 s_w^2 A_\mu A_\mu \phi^+ \phi^- - \bar{e}^\lambda (\gamma \partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda \gamma \partial \nu^\lambda -
 \end{aligned}$$

$$\begin{aligned}
 & \bar{u}_j^\lambda (\gamma \partial + m_u^\lambda) u_j^\lambda - \bar{d}_j^\lambda (\gamma \partial + m_d^\lambda) d_j^\lambda + i g s_w A_\mu [-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \\
 & \quad \frac{1}{3}(\bar{d}_j^\lambda \gamma^\mu d_j^\lambda)] + \frac{i g}{4 c_w} Z_\mu^0 [(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4 s_w^2 - 1 - \gamma^5) e^\lambda) + \\
 & \quad (\bar{u}_j^\lambda \gamma^\mu (\frac{4}{3} s_w^2 - 1 - \gamma^5) u_j^\lambda) + (\bar{d}_j^\lambda \gamma^\mu (1 - \frac{8}{3} s_w^2 - \gamma^5) d_j^\lambda)] + \frac{i g}{2 \sqrt{2}} W_\mu^+ [(\bar{\nu}^\lambda \gamma^\mu (1 + \\
 & \quad \gamma^5) e^\lambda) + (\bar{u}_j^\lambda \gamma^\mu (1 + \gamma^5) C_{\lambda \kappa} d_j^\kappa)] + \frac{i g}{2 \sqrt{2}} W_\mu^- [(\bar{e}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^\kappa C_{\lambda \kappa}^\dagger \gamma^\mu (1 + \\
 & \quad \gamma^5) u_j^\lambda)] + \frac{i g}{2 \sqrt{2}} \frac{m_e^\lambda}{M} [-\phi^+ (\bar{\nu}^\lambda (1 - \gamma^5) e^\lambda) + \phi^- (\bar{e}^\lambda (1 + \gamma^5) \nu^\lambda)] - \frac{g}{2} \frac{m_e^\lambda}{M} [H (\bar{e}^\lambda e^\lambda) + \\
 & \quad i \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda)] + \frac{i g}{2 M \sqrt{2}} \phi^+ [-m_d^\kappa (\bar{u}_j^\lambda C_{\lambda \kappa} (1 - \gamma^5) d_j^\kappa) + m_u^\lambda (\bar{u}_j^\lambda C_{\lambda \kappa} (1 + \gamma^5) d_j^\kappa) + \\
 & \quad \frac{i g}{2 M \sqrt{2}} \phi^- [m_d^\lambda (\bar{d}_j^\lambda C_{\lambda \kappa}^\dagger (1 + \gamma^5) u_j^\kappa) - m_u^\kappa (\bar{d}_j^\lambda C_{\lambda \kappa}^\dagger (1 - \gamma^5) u_j^\kappa) - \frac{g}{2} \frac{m_u^\lambda}{M} H (\bar{u}_j^\lambda u_j^\lambda) - \\
 & \quad \frac{g}{2} \frac{m_d^\lambda}{M} H (\bar{d}_j^\lambda d_j^\lambda) + \frac{i g}{2} \frac{m_u^\lambda}{M} \phi^0 (\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \frac{i g}{2} \frac{m_d^\lambda}{M} \phi^0 (\bar{d}_j^\lambda \gamma^5 d_j^\lambda) + \bar{X}^+ (\partial^2 - M^2) X^+ + \\
 & \quad \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \frac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + i g c_w W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \\
 & \quad \partial_\mu \bar{X}^+ X^0) + i g s_w W_\mu^+ (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + i g c_w W_\mu^- (\partial_\mu \bar{X}^- X^0 - \\
 & \quad \partial_\mu \bar{X}^0 X^+) + i g s_w W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + i g c_w Z_\mu^0 (\partial_\mu \bar{X}^+ X^+ - \\
 & \quad \partial_\mu \bar{X}^- X^-) + i g s_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) - \frac{1}{2} g M [\bar{X}^+ X^+ H + \bar{X}^- X^- H + \\
 & \quad \frac{1}{c_w^2} \bar{X}^0 X^0 H] + \frac{1 - 2 c_w^2}{2 c_w} i g M [\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \frac{1}{2 c_w} i g M [\bar{X}^0 X^- \phi^+ - \\
 & \quad \bar{X}^0 X^+ \phi^-] + i g M s_w [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \frac{1}{2} i g M [\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]
 \end{aligned}$$

APPENDIX C
LEPTON ID VALIDATION

C.1 Lepton ID Validation

C.1.1 Run 2

In the Run 2 portion of the analysis, electrons and muons are selected according to the TOP-UL-MVA ID [93], a BDT-based lepton ID developed to distinguish leptons from top decays from non-prompt leptons. This appendix contains plots validating the performance of this Lepton ID for muon candidates and electron candidates.

The agreement between data and MC can be checked for the Run 2 lepton ID using the background control regions. To get adequate statistics, the validation is performed in the ZZ CR which is defined in Section 7.1.1. For validation of the muon ID, we restrict ourselves to the 4μ final state in the ZZ CR. The muon p_T and η distributions for data and MC are shown in Figure C.1, exhibiting good agreement.

A similar approach is used to check the data and MC agreement for electrons using the Run 2 lepton ID. The validation is also performed in the ZZ CR, and we restrict ourselves to the $4e$ final state. The electron p_T and η are shown in Figure C.2, and good agreement is observed.

C.1.2 Run 3

For the Run 3 portion of the analysis, electrons and muons are selected according to the ID requirements recommended by the CMS EGamma

and Muon Physics Object Groups, respectively. Details of these IDs are discussed in Section 5.2.2.

Agreement between simulation and data is checked for the Run 3 muon ID by looking at dilepton events around the Z boson mass peak. Plots of muon kinematic variables with the Run 3 ID are shown in Figure C.3. Good agreement between data and simulation is observed.

For the Run 3 electron ID, we also check the agreement between simulation and data by looking at dilepton events around the Z boson mass peak. Plots of the electron kinematic variables with the Run 3 ID are shown in Figure C.4, and good agreement is seen between the data and the MC simulation.

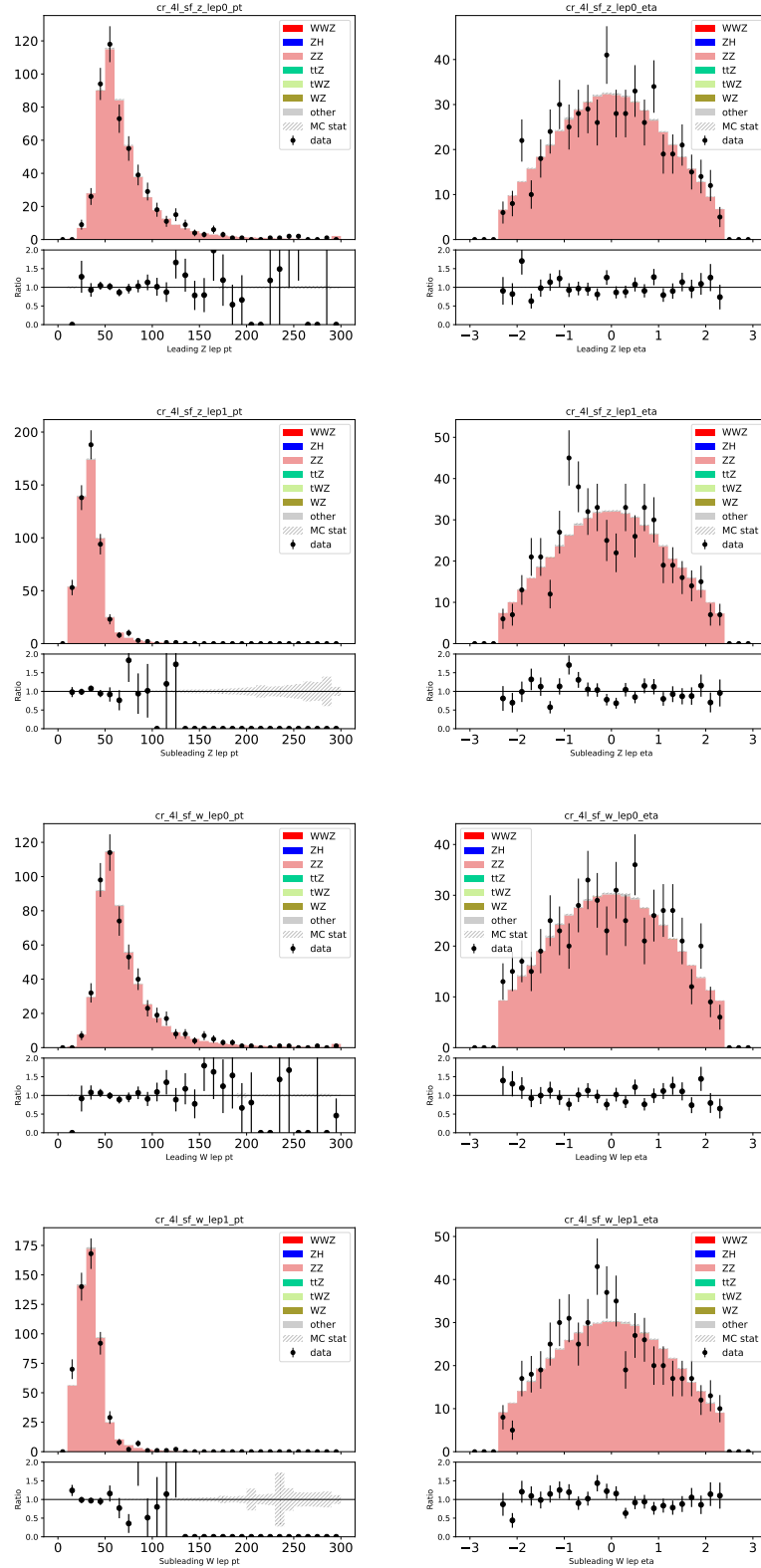


Figure C.1: Muon p_T and η distributions in the ZZ CR for data and MC. All plots are made in the 4μ final state.

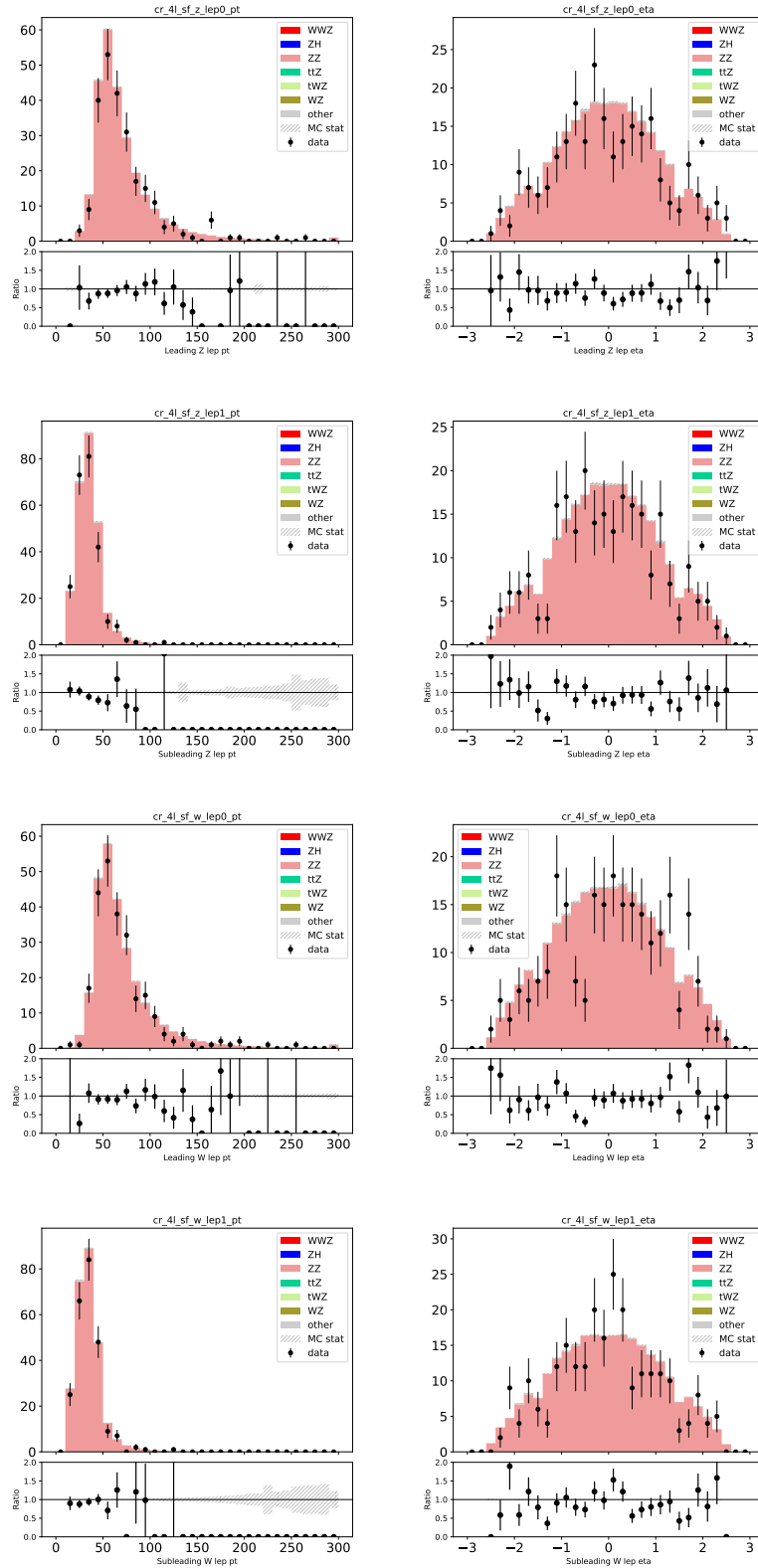


Figure C.2: Electron p_T and η distributions in the ZZ CR for data and MC. All plots are made in the $4e$ final state.

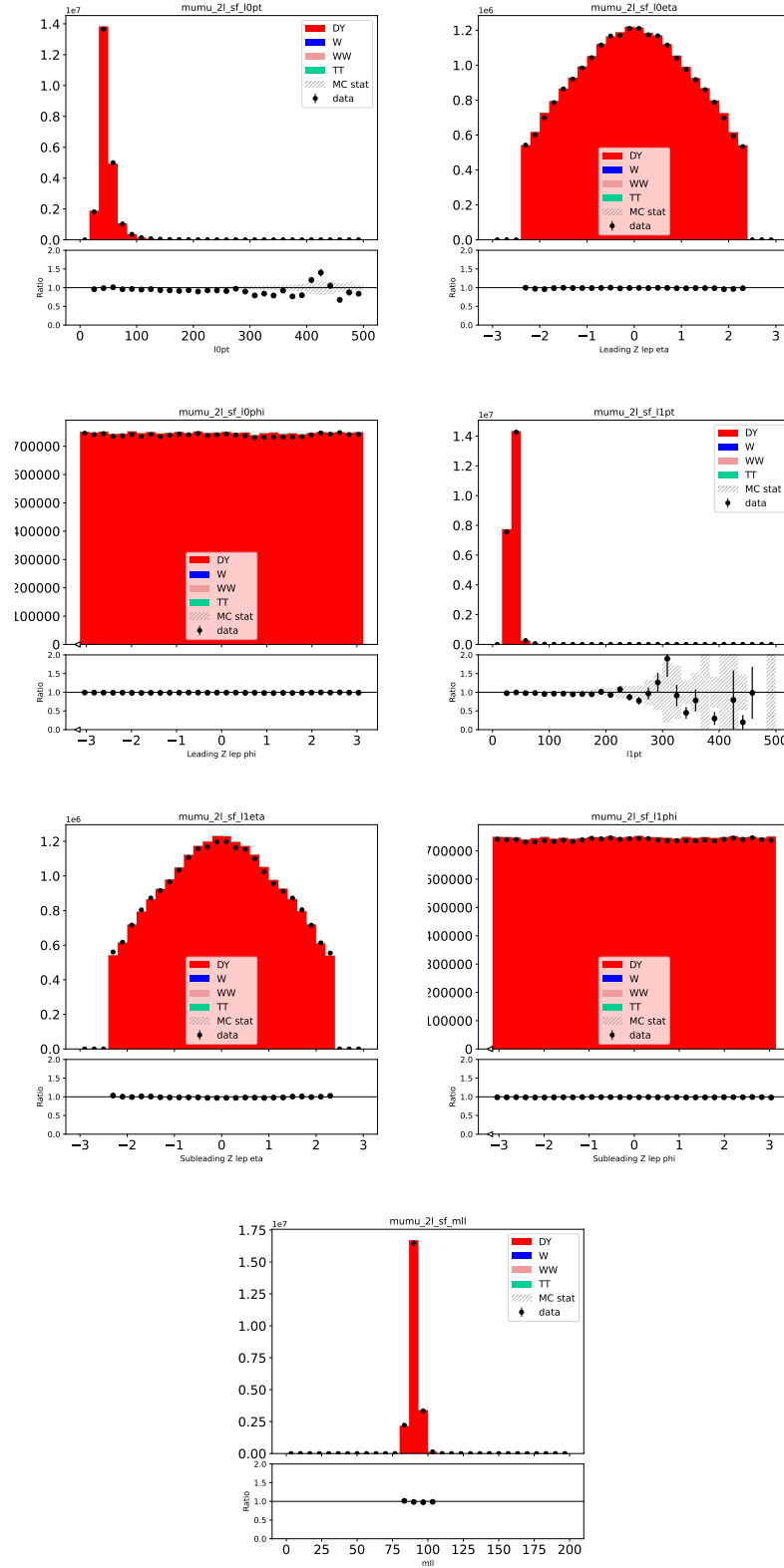


Figure C.3: Muon kinematic distributions in same-flavor dilepton events with Run 3 Muon ID (2022).

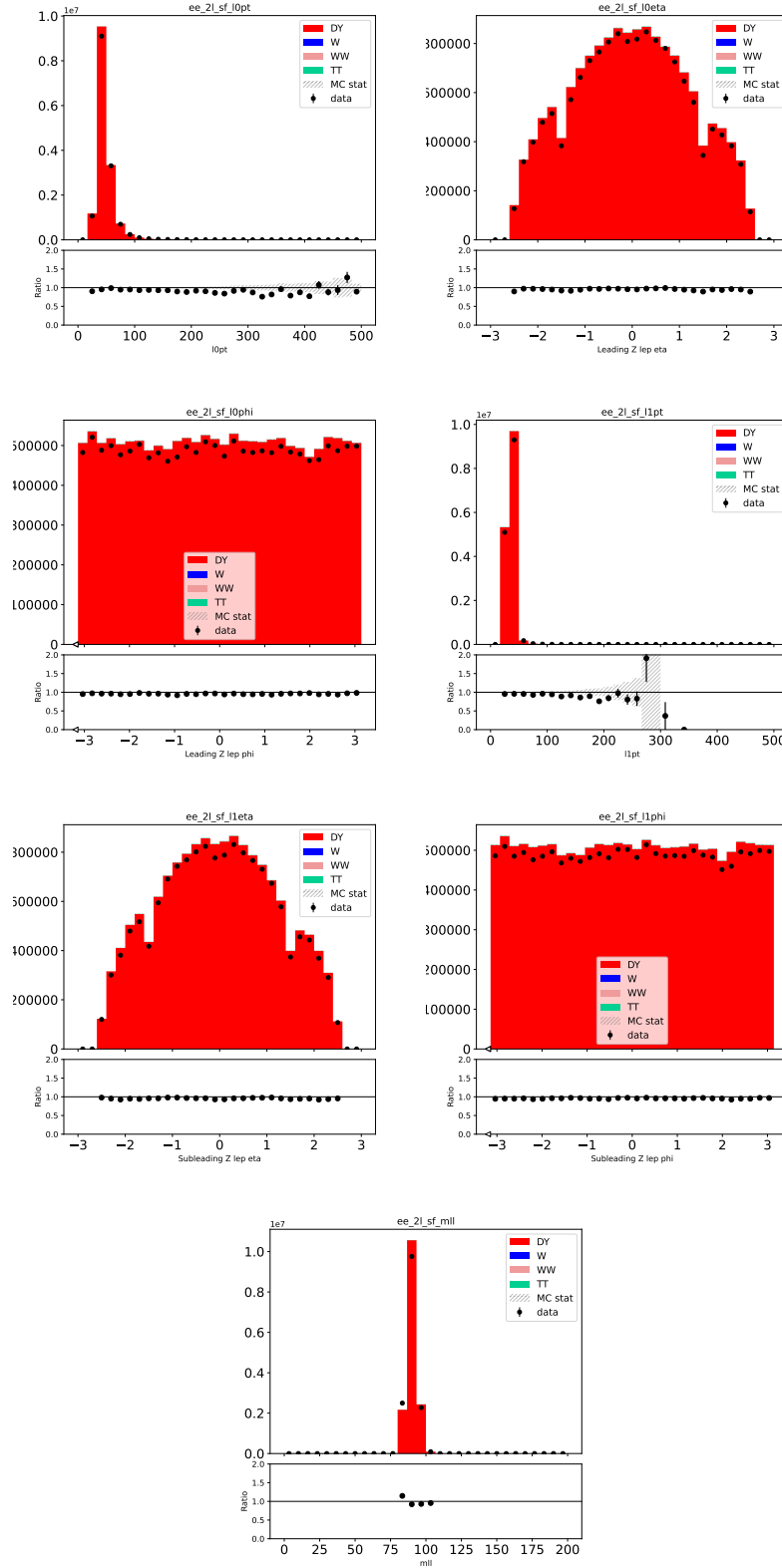


Figure C.4: Electron kinematic distributions in same-flavor dilepton events with Run 3 Electron ID (2022).

APPENDIX D

BDT VALIDATION

D.1 BDT Validation

D.1.1 BDT Training Regions

D.1.2 BDT Validation in Control Regions

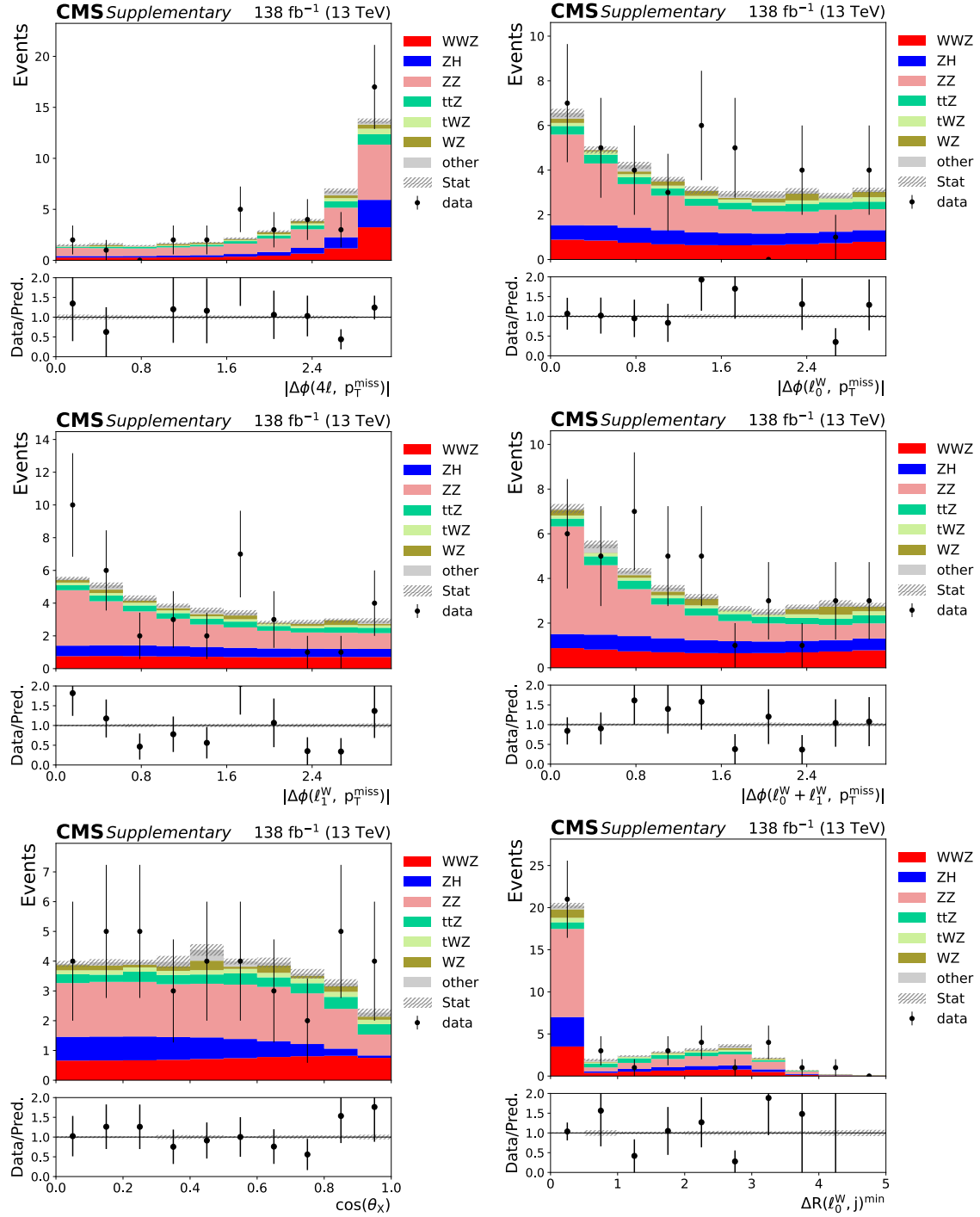


Figure D.1: BDT input variables shown in the OF training region (Run 2).

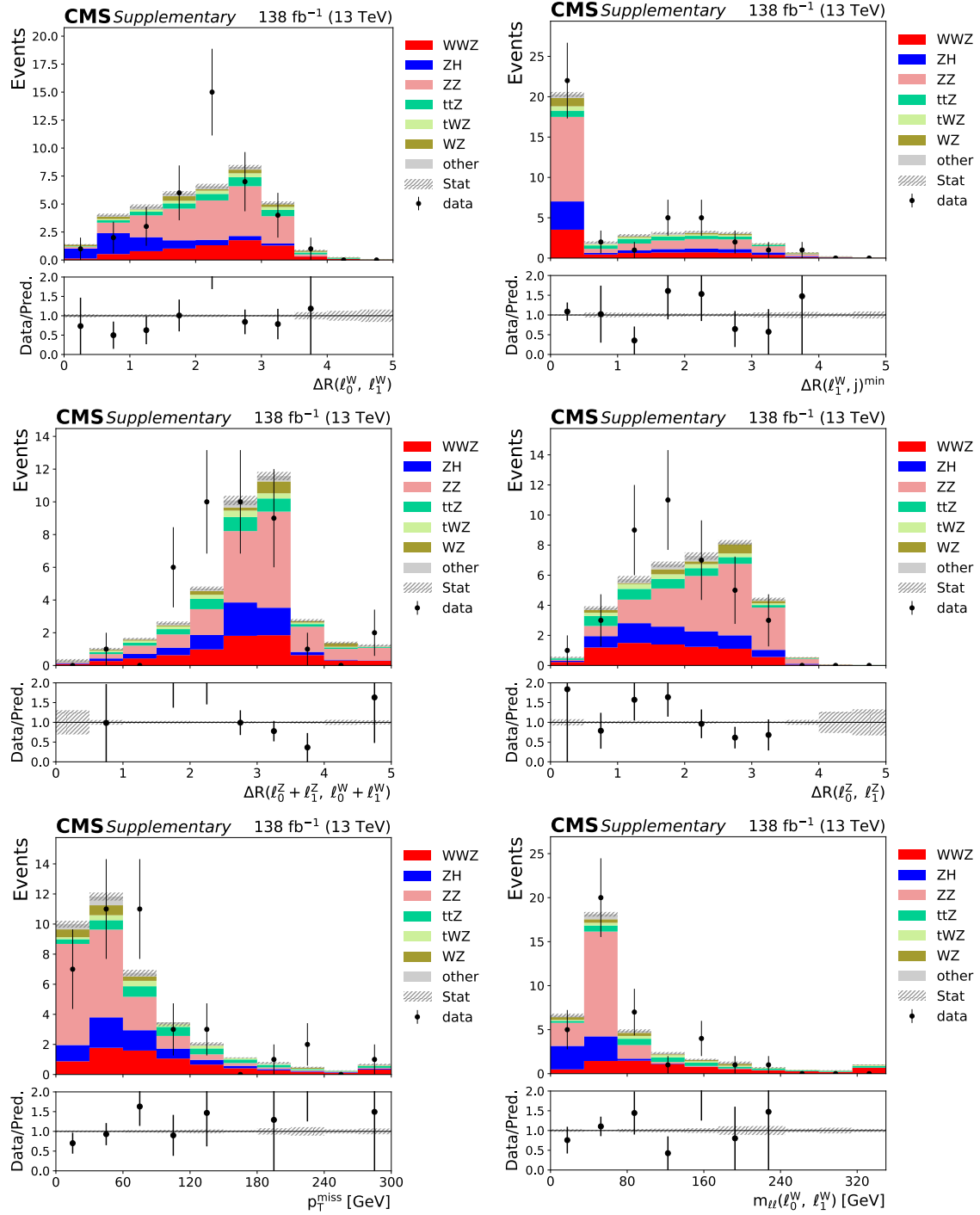


Figure D.2: BDT input variables shown in the OF training region (Run 2).

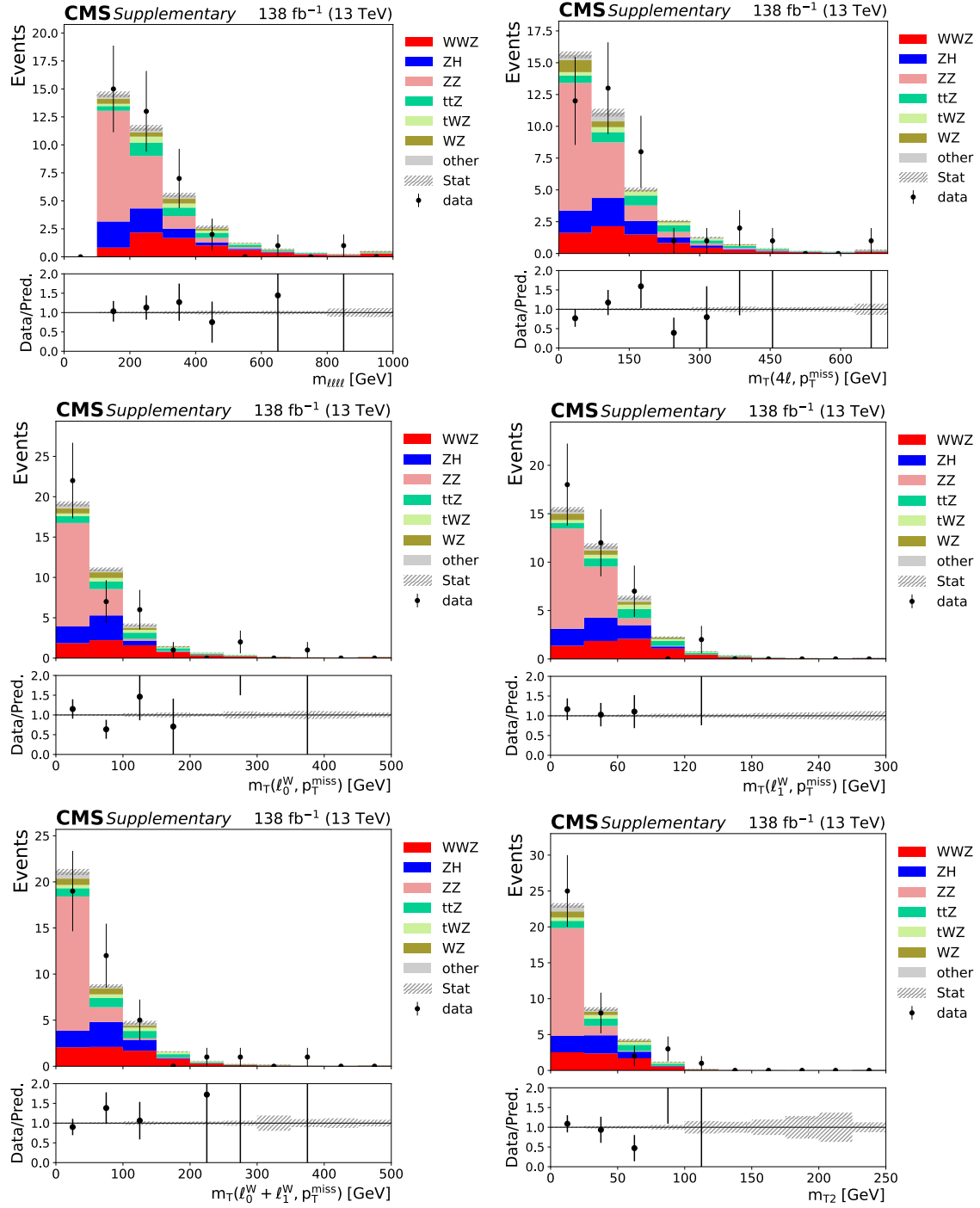


Figure D.3: BDT input variables shown in the OF training region (Run 2).

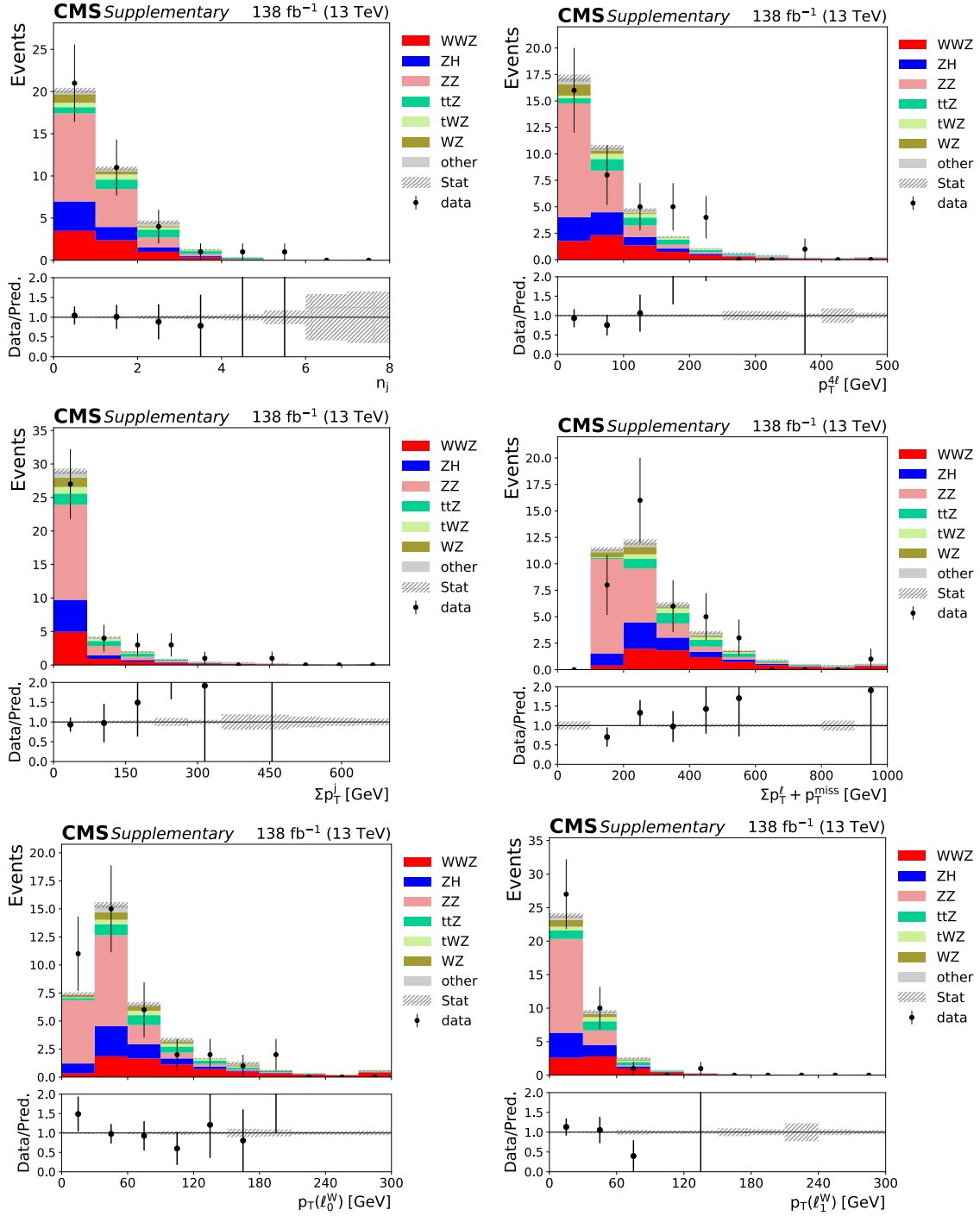


Figure D.4: BDT input variables shown in the OF training region (Run 2).

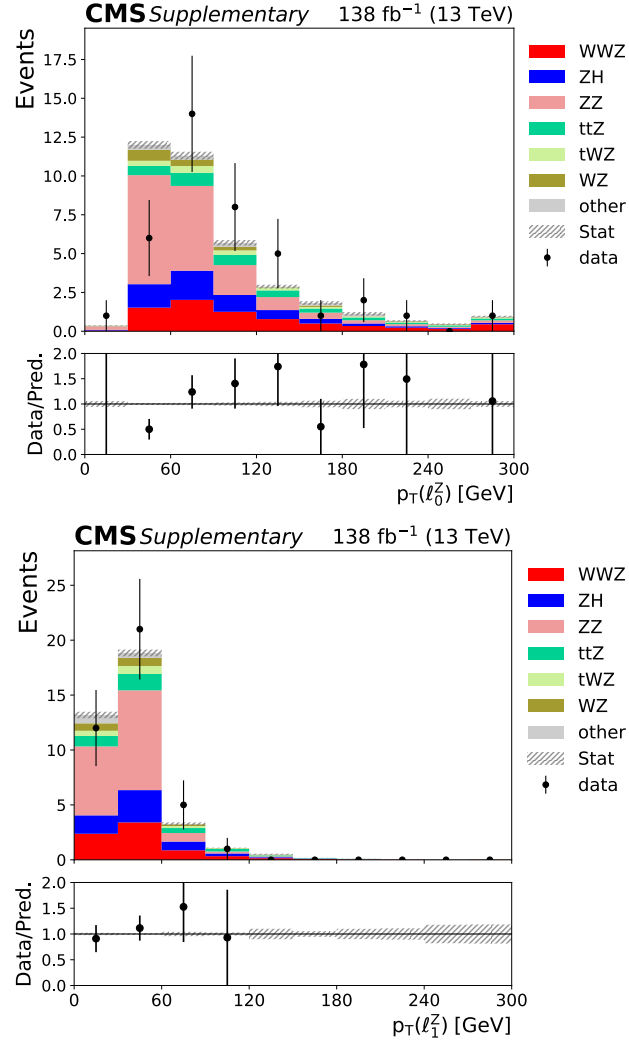


Figure D.5: BDT input variables shown in the OF training region (Run 2).

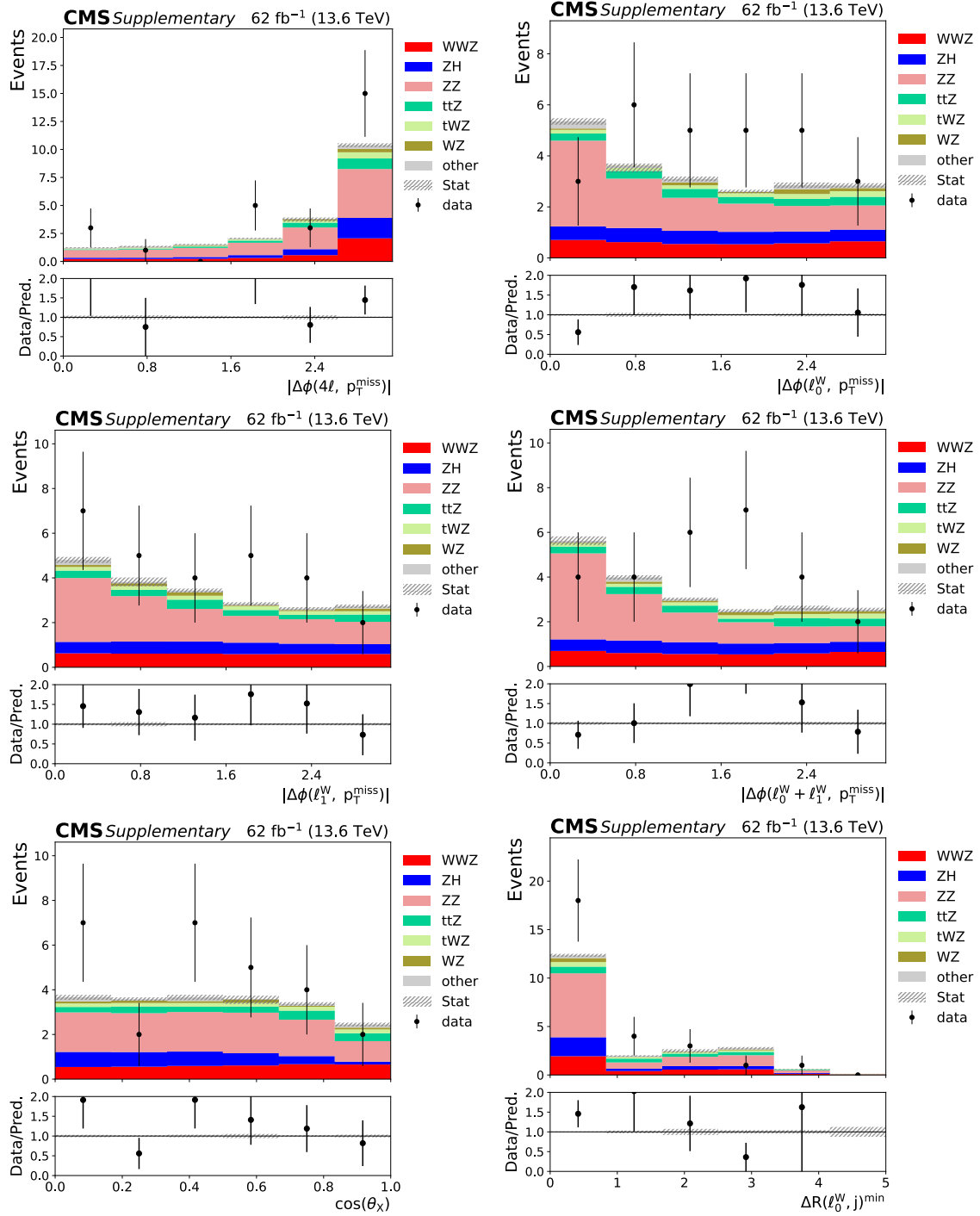


Figure D.6: BDT input variables shown in the OF training region (Run 3).

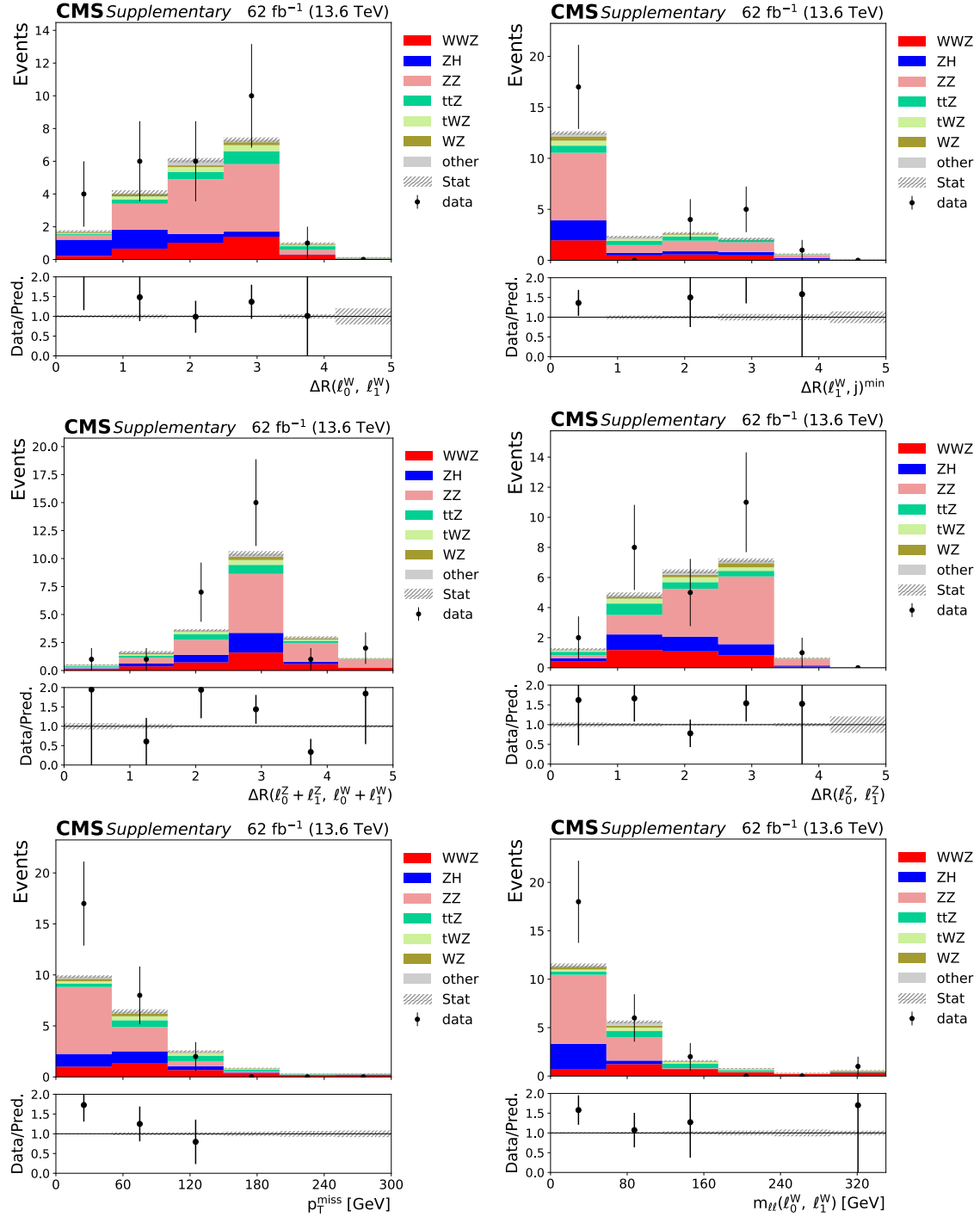


Figure D.7: BDT input variables shown in the OF training region (Run 3).

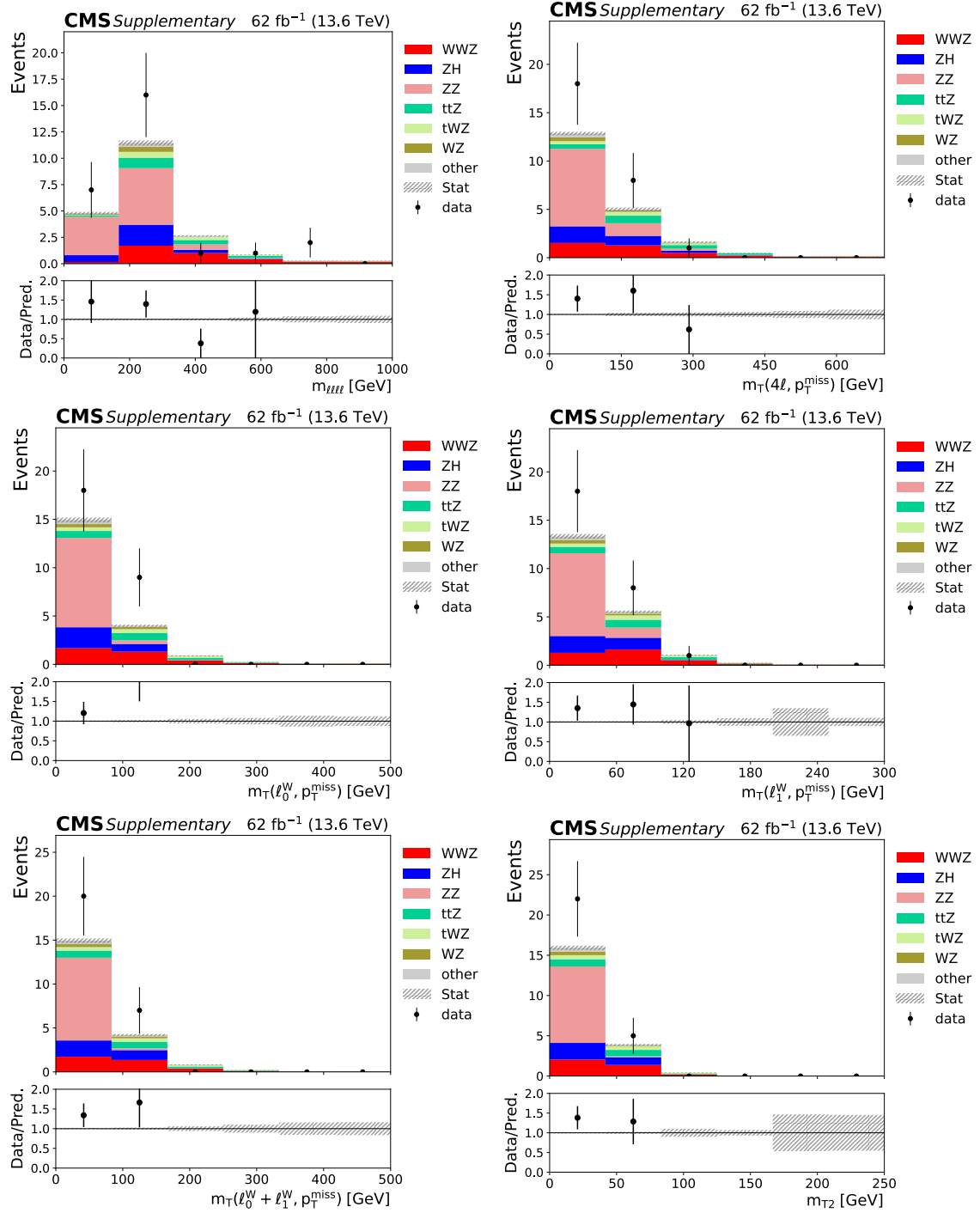


Figure D.8: BDT input variables shown in the OF training region (Run 3).

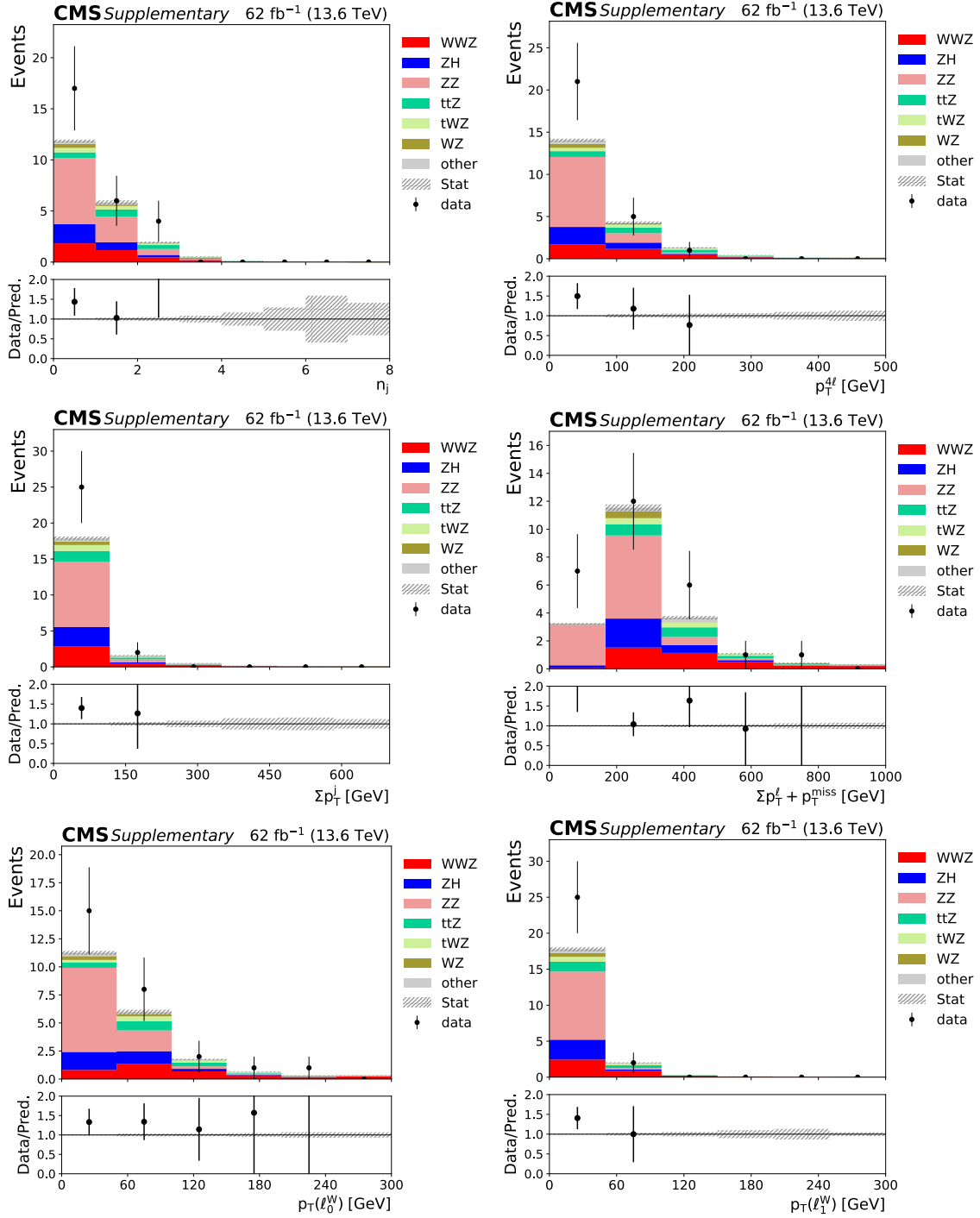


Figure D.9: BDT input variables shown in the OF training region (Run 3).

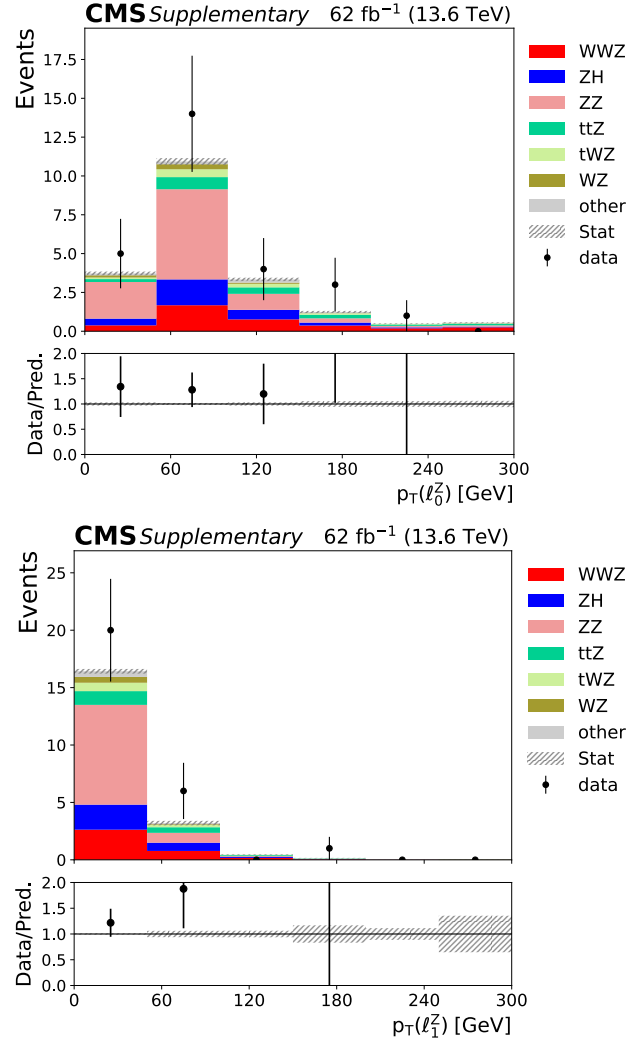


Figure D.10: BDT input variables shown in the OF training region (Run 3).

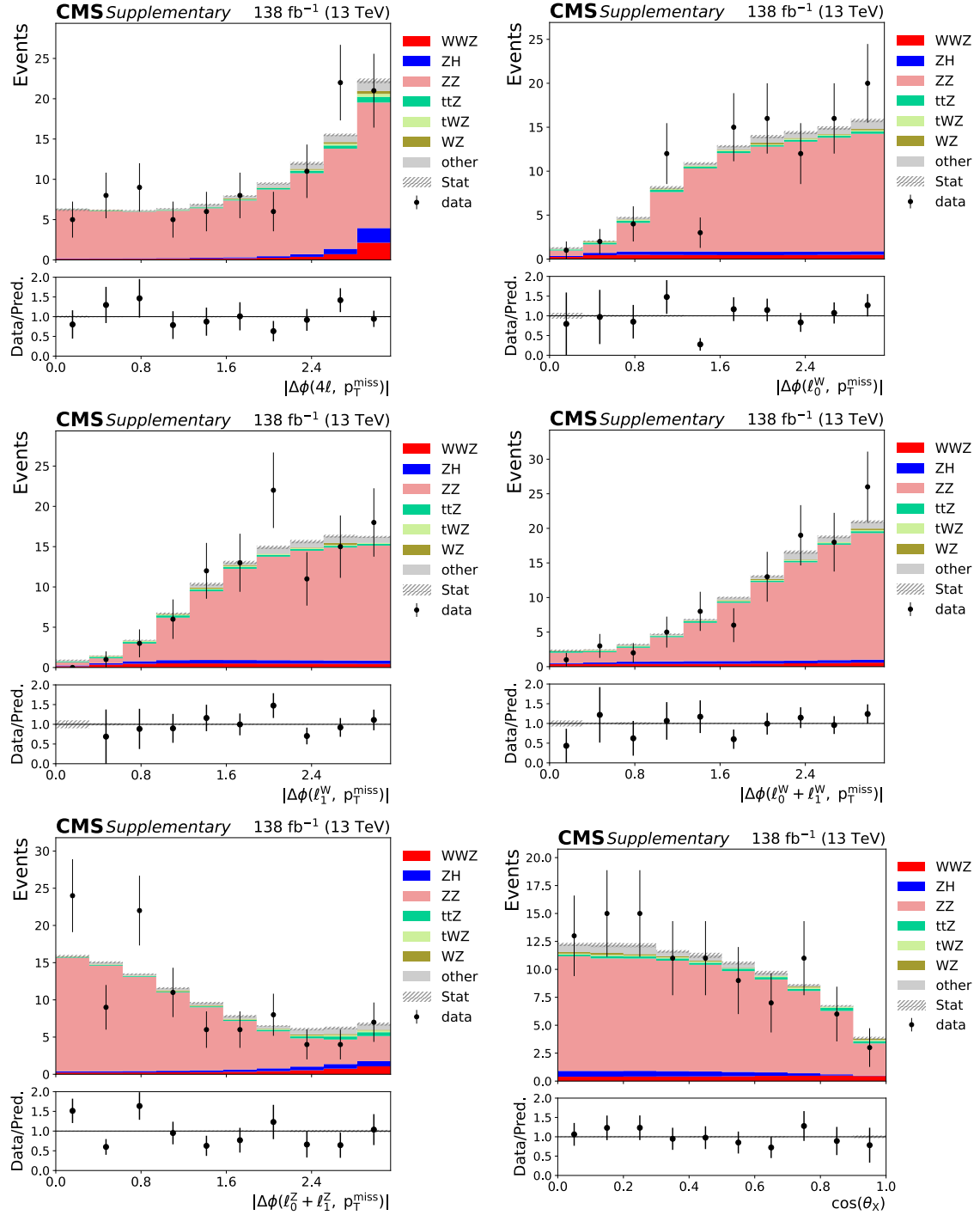


Figure D.11: BDT input variables shown in the SF training region (Run 2).

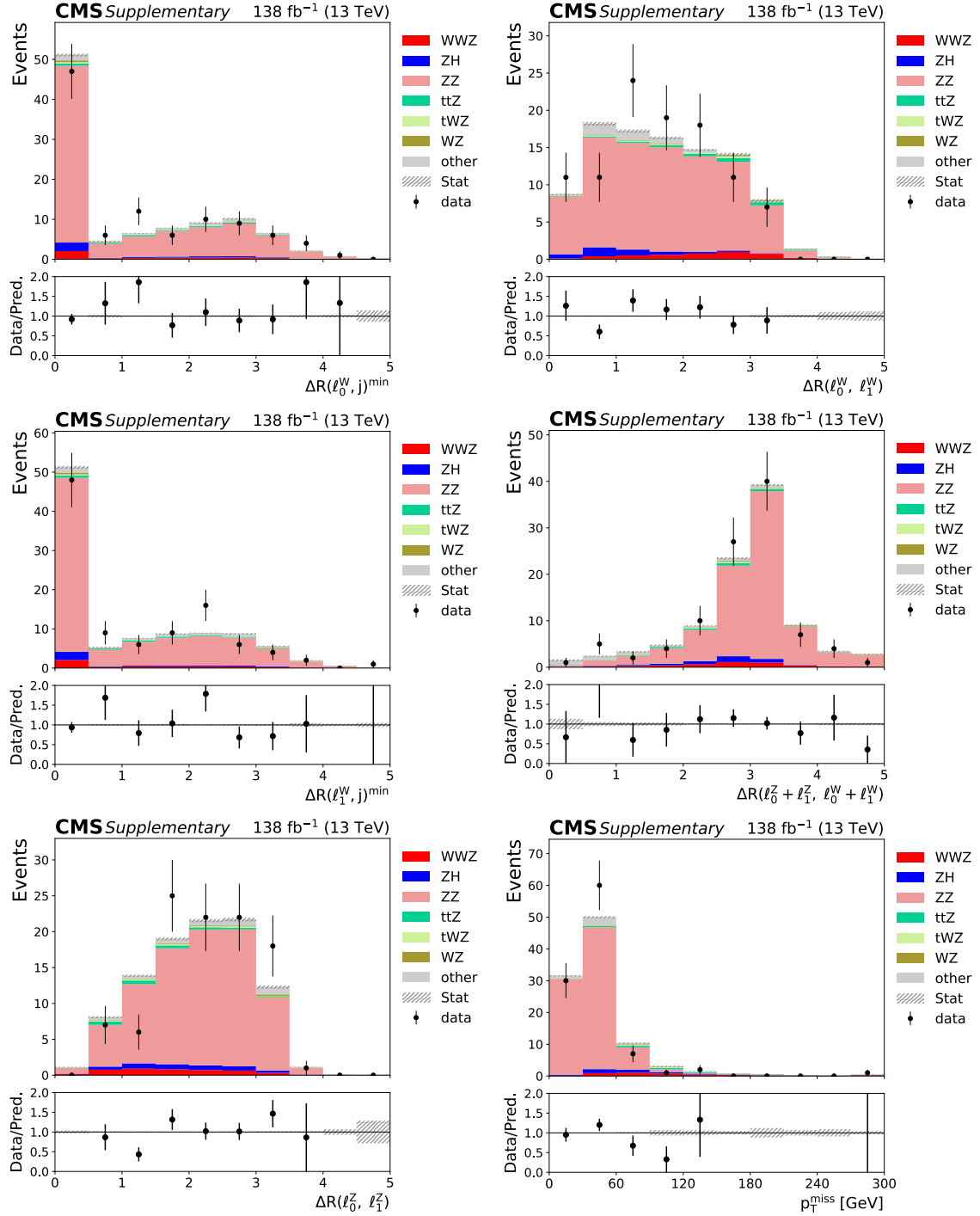


Figure D.12: BDT input variables shown in the SF training region (Run 2).

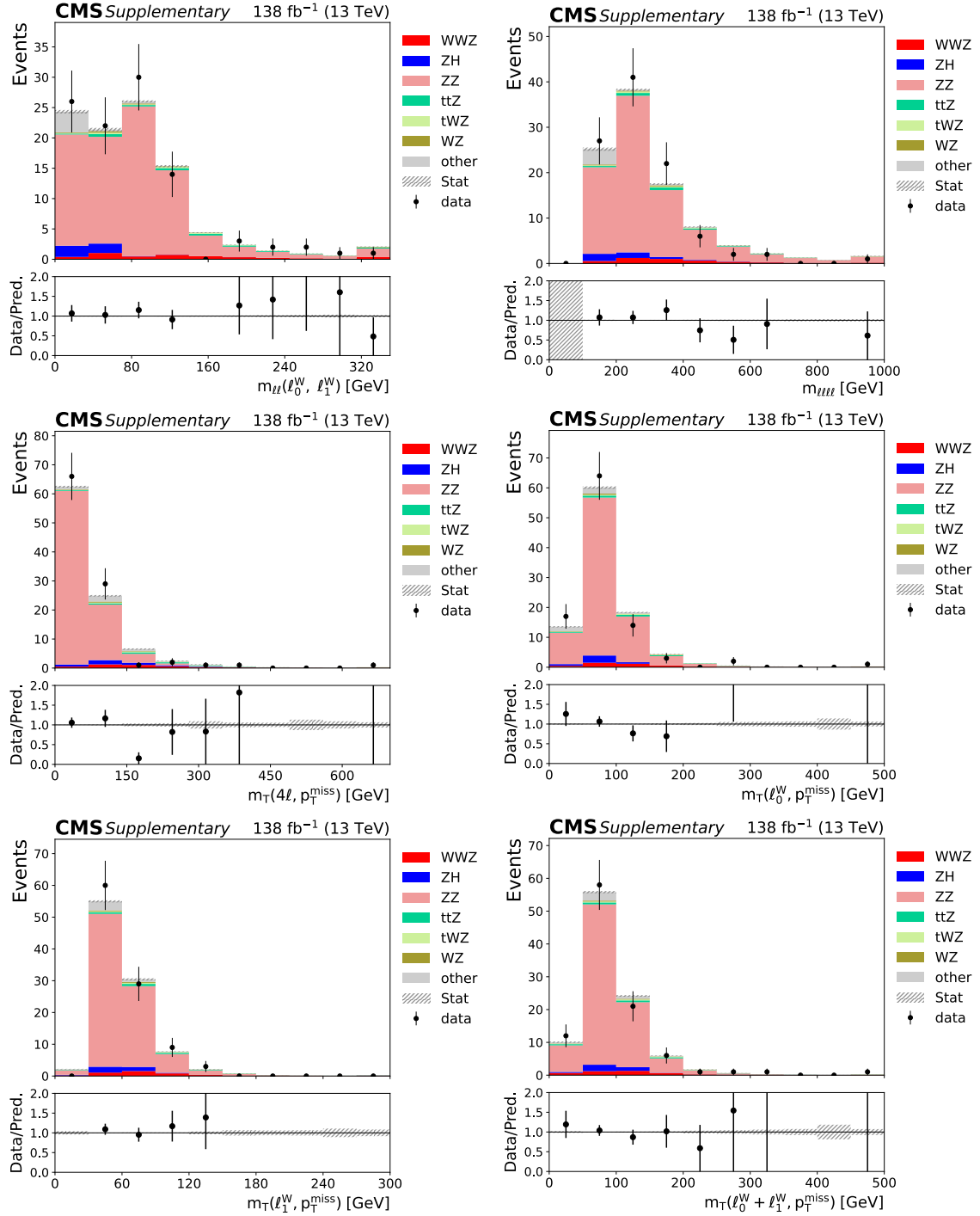


Figure D.13: BDT input variables shown in the SF training region (Run 2).

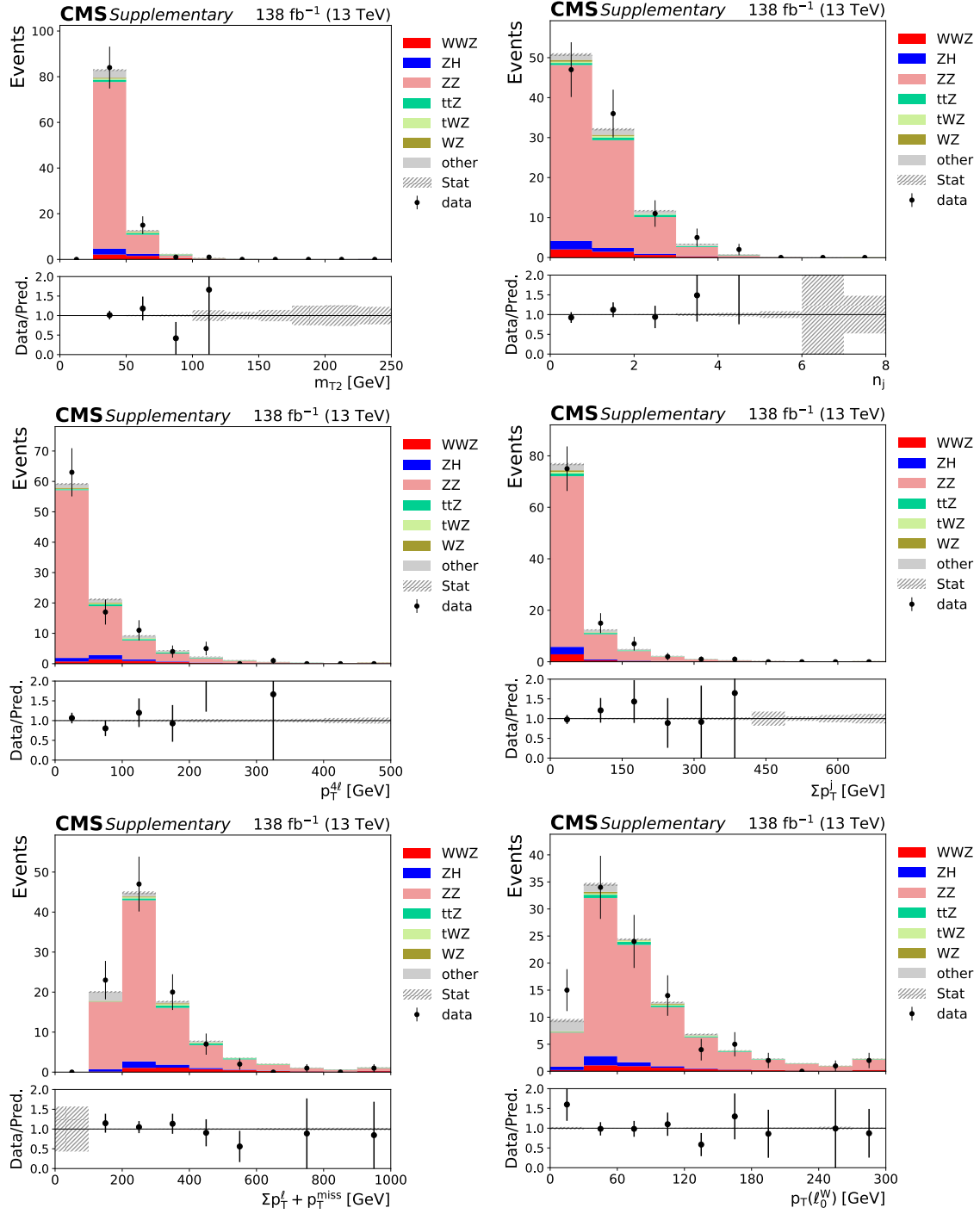


Figure D.14: BDT input variables shown in the SF training region (Run 2).

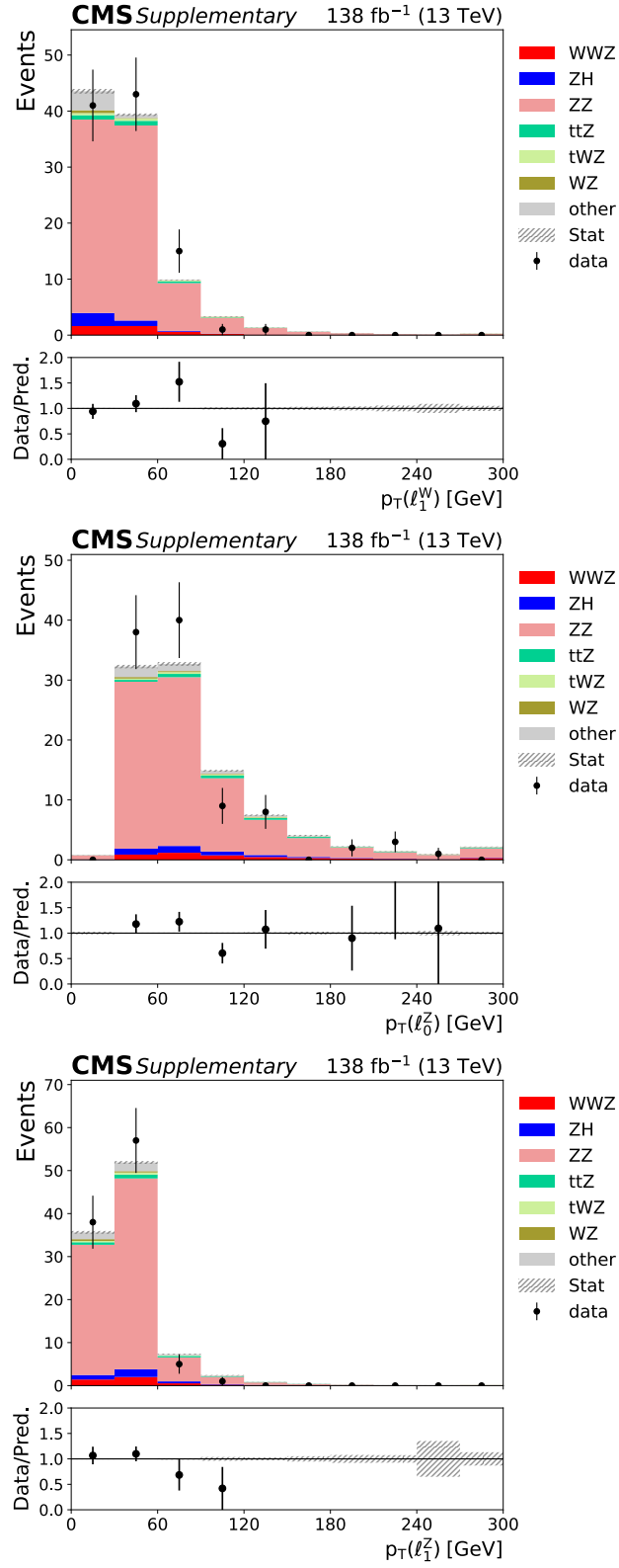


Figure D.15: BDT input variables shown in the SF training region (Run 2).

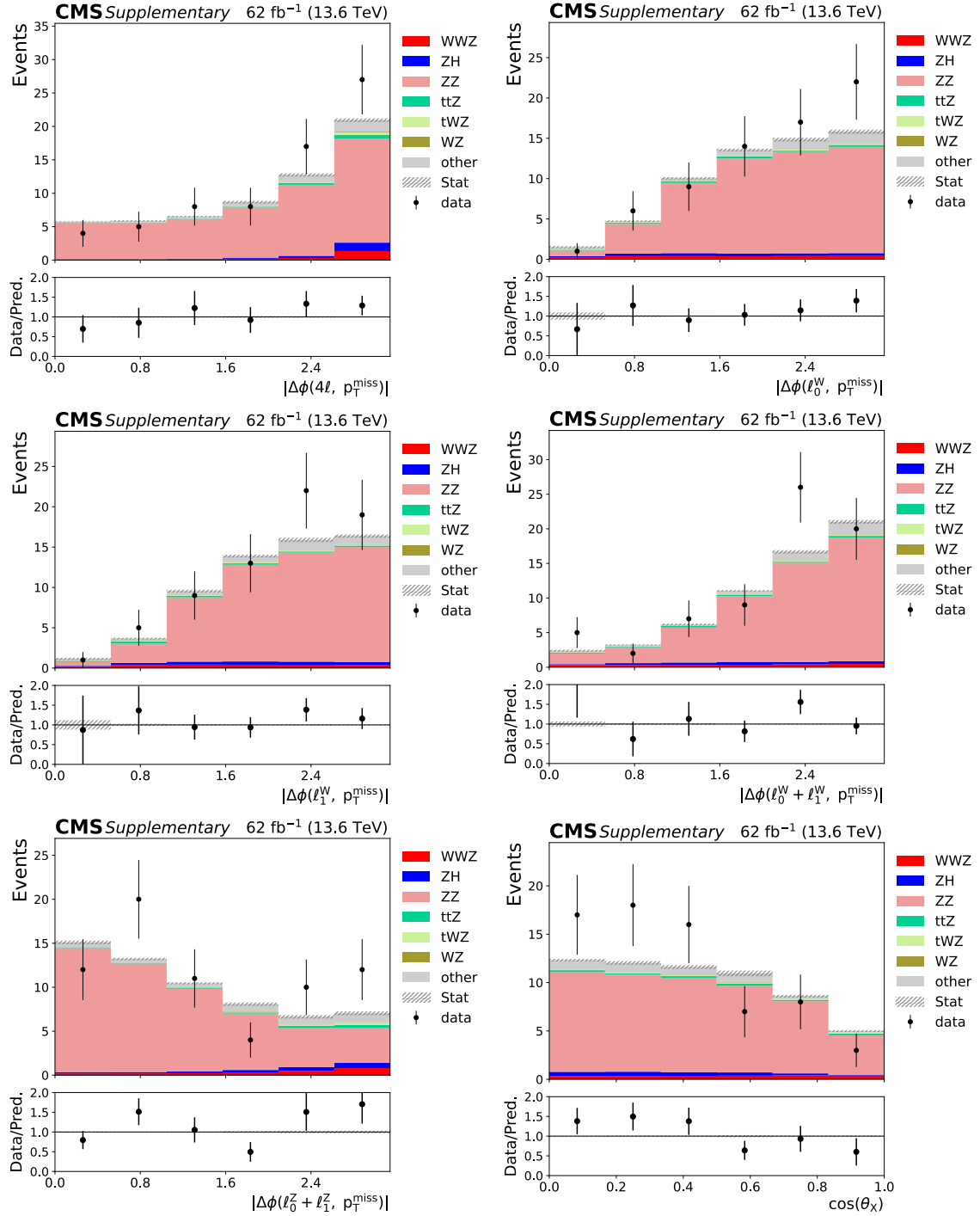


Figure D.16: BDT input variables shown in the SF training region (Run 3).

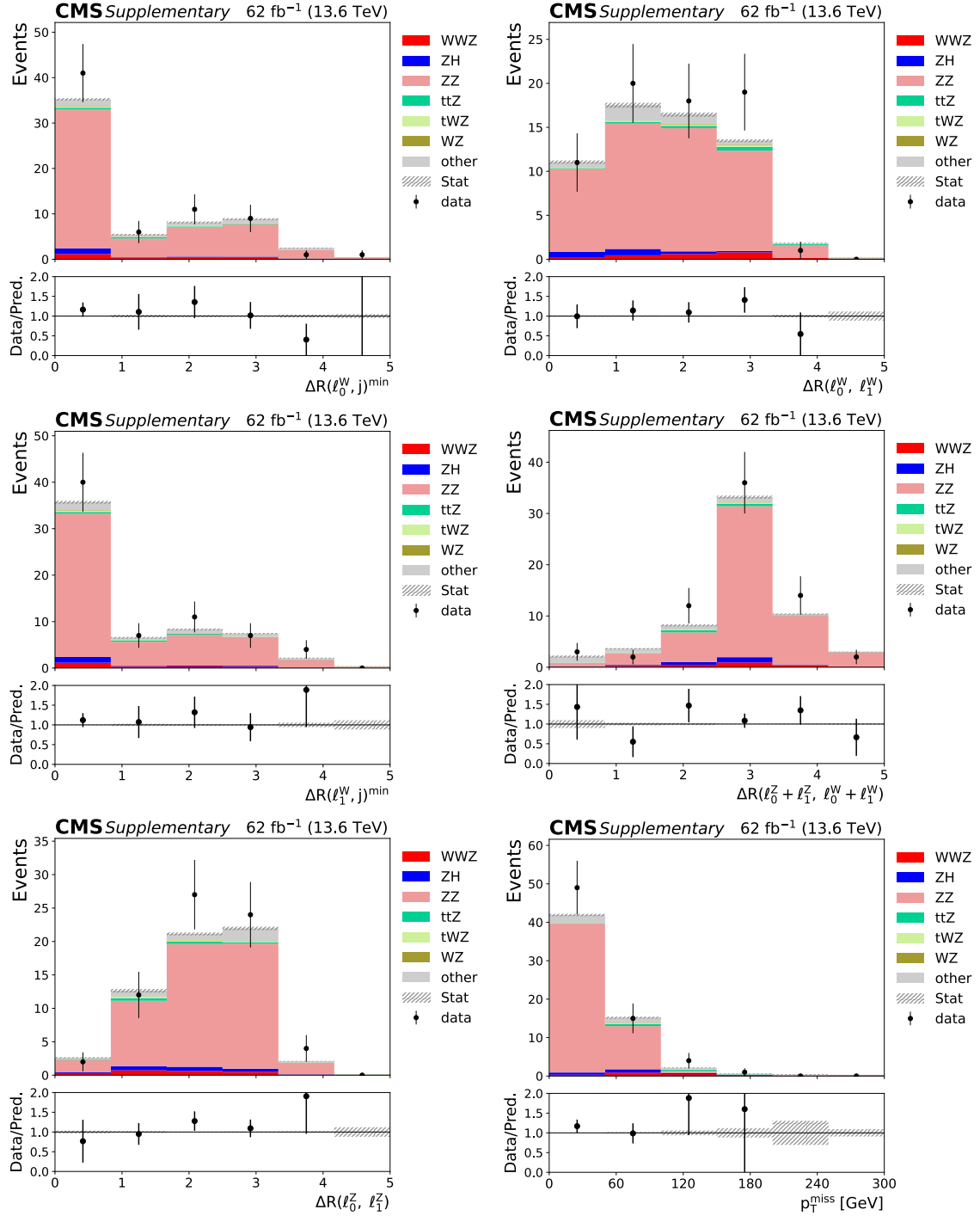


Figure D.17: BDT input variables shown in the SF training region (Run 3).

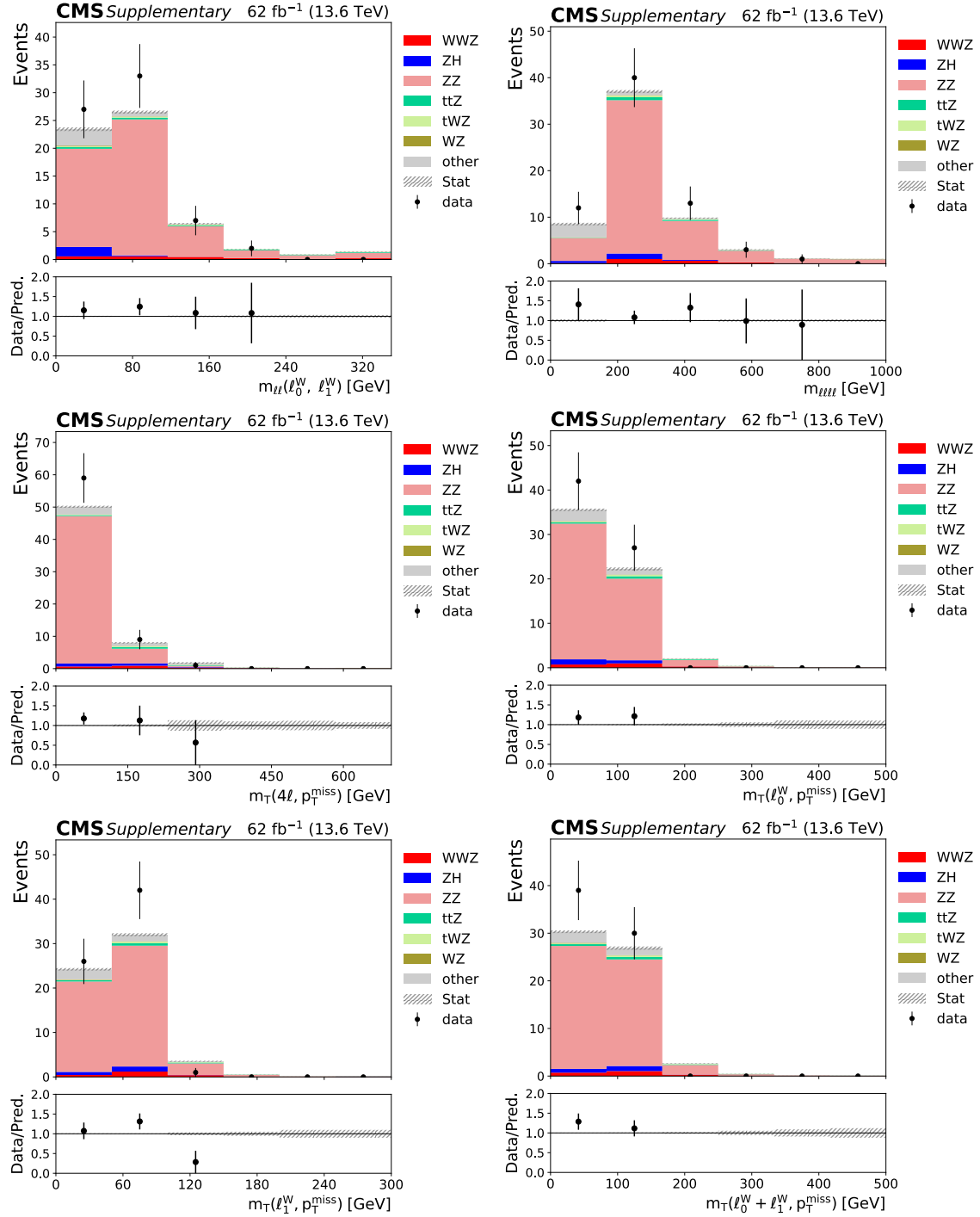


Figure D.18: BDT input variables shown in the SF training region (Run 3).

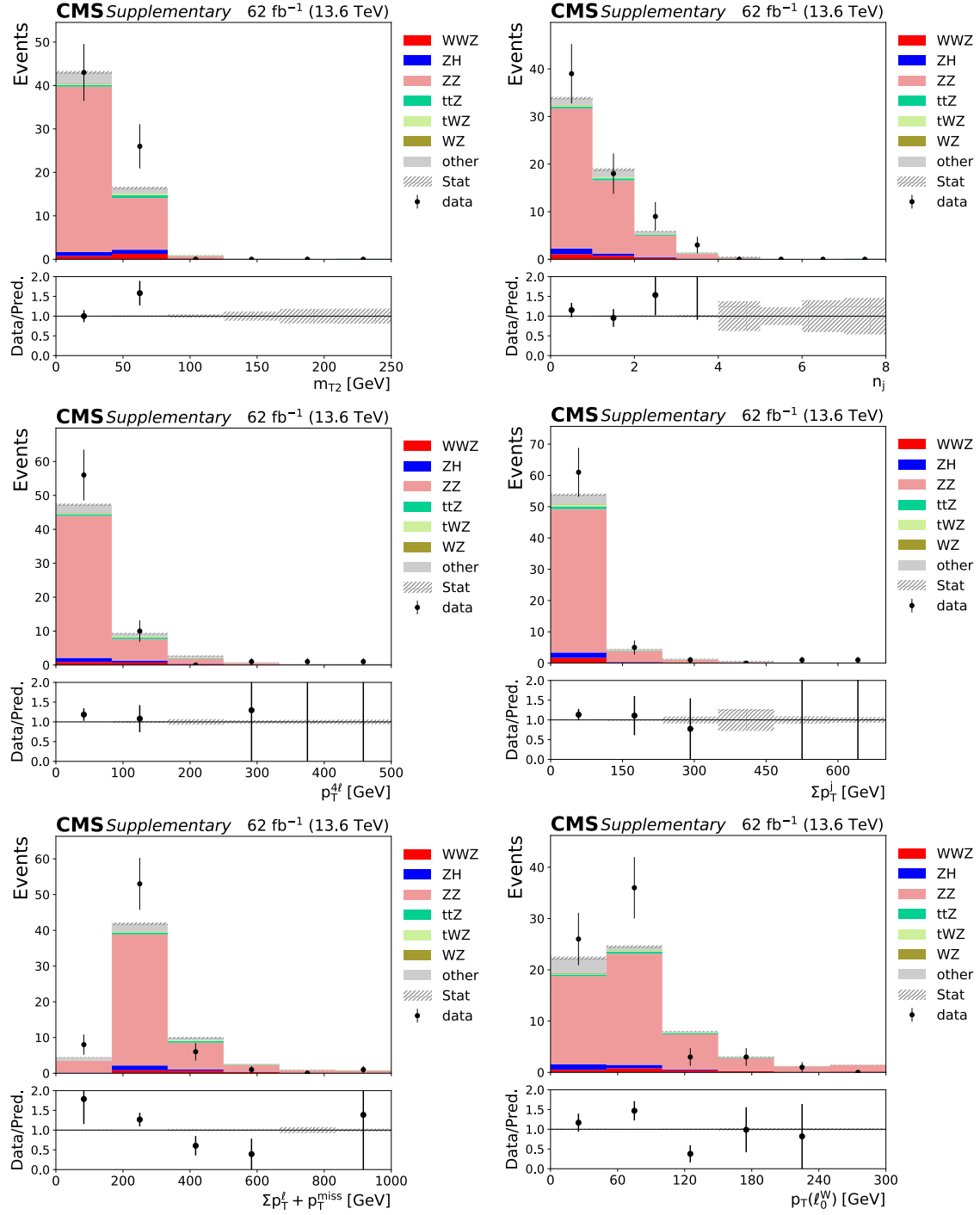


Figure D.19: BDT input variables shown in the SF training region (Run 3).

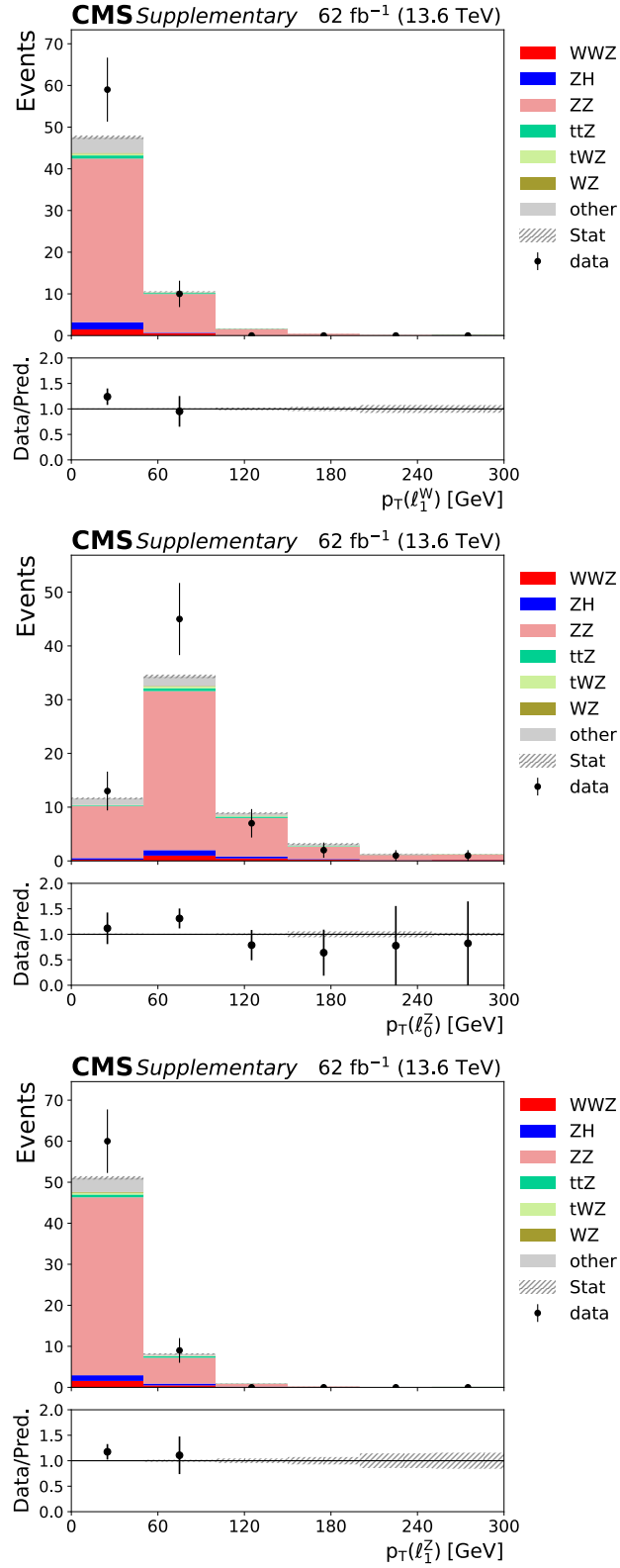
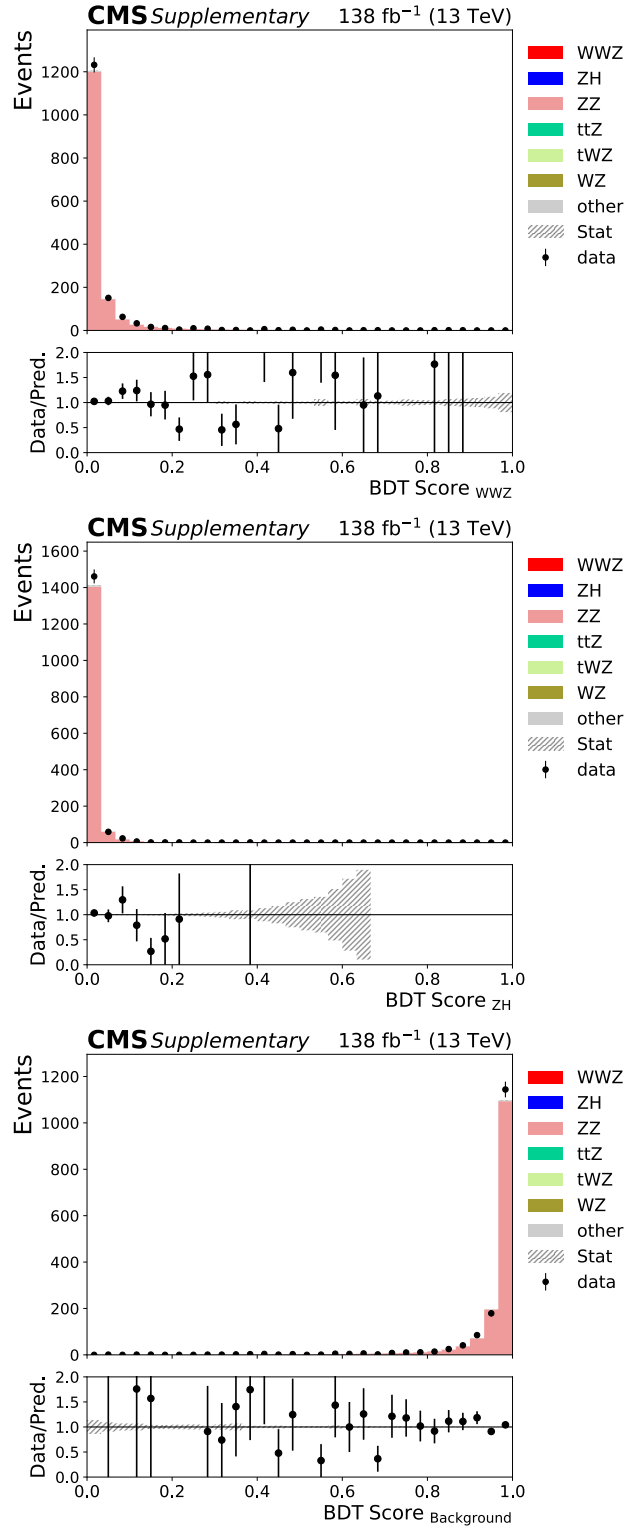
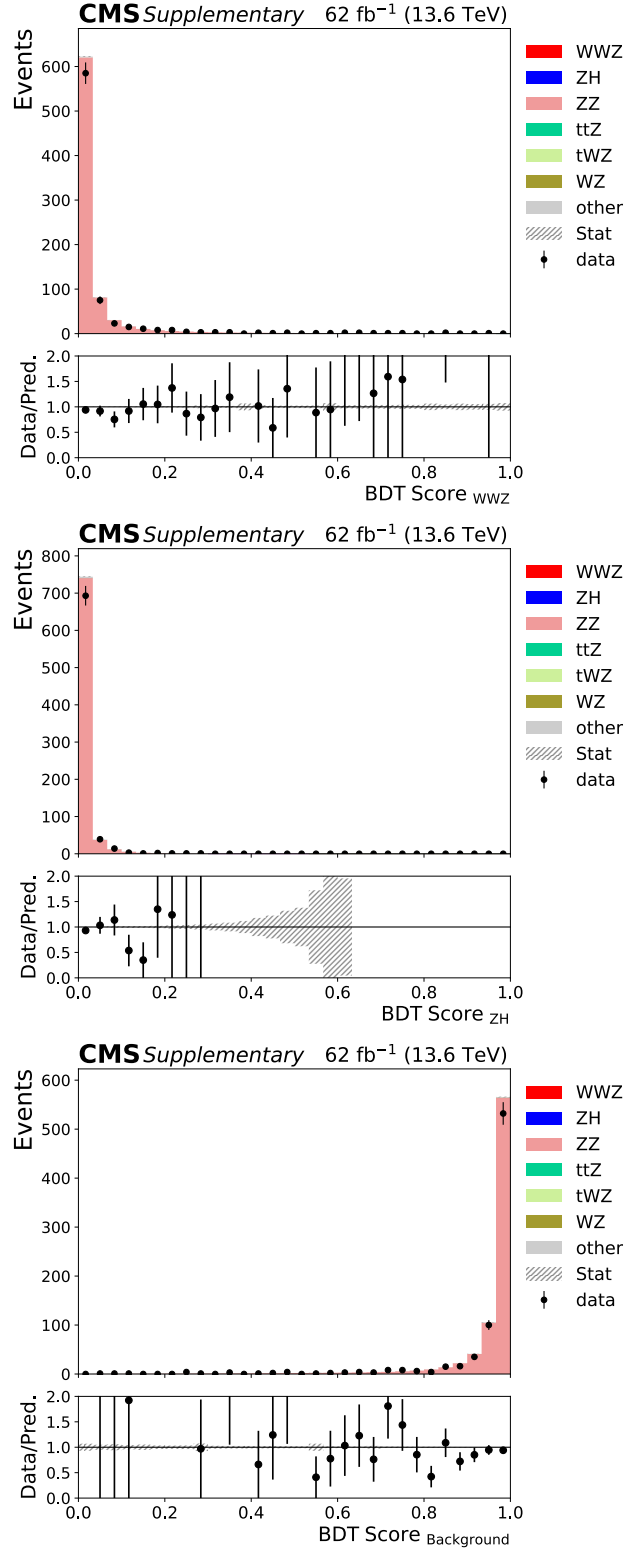
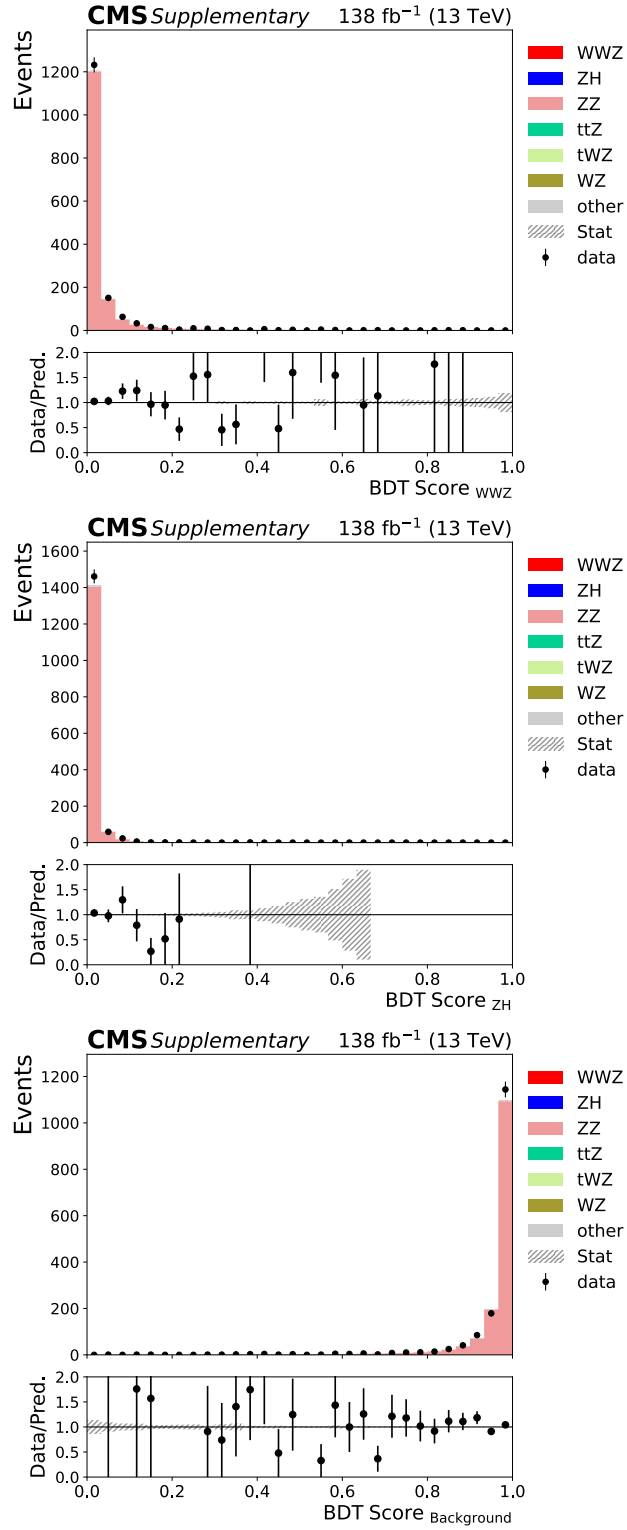
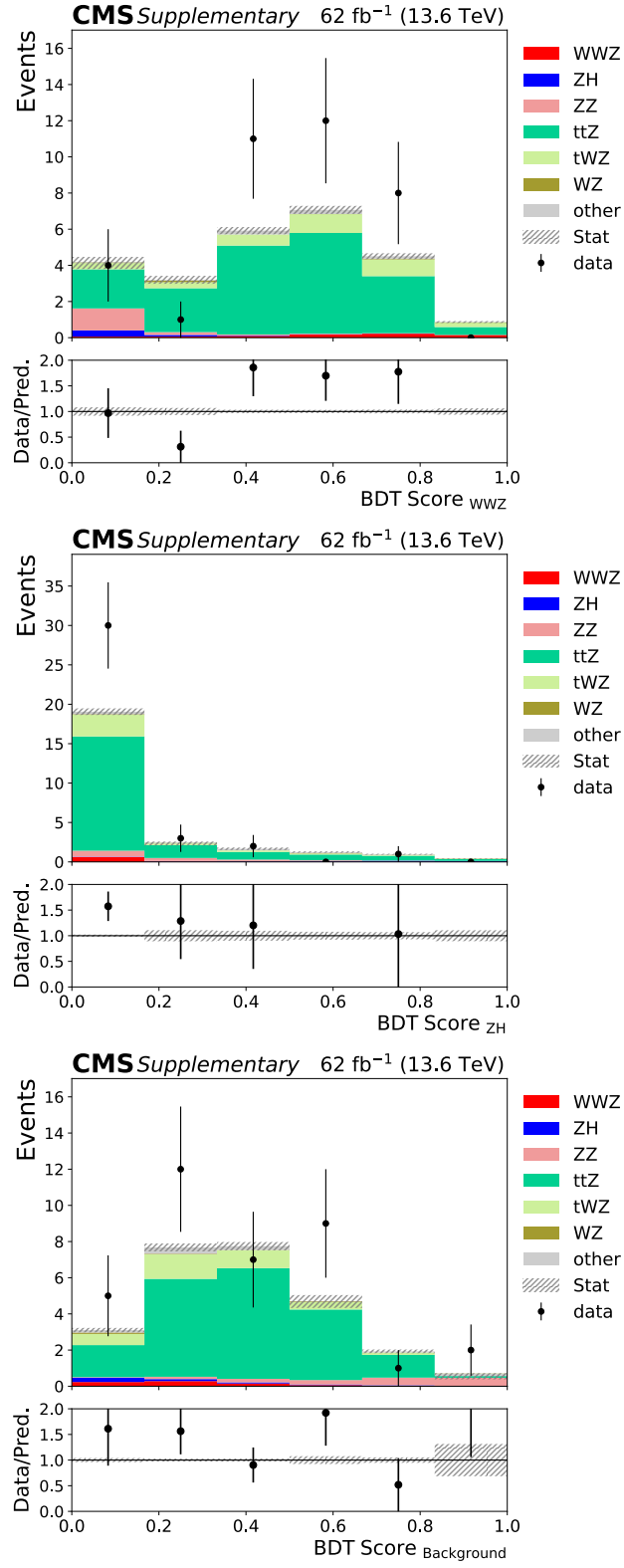


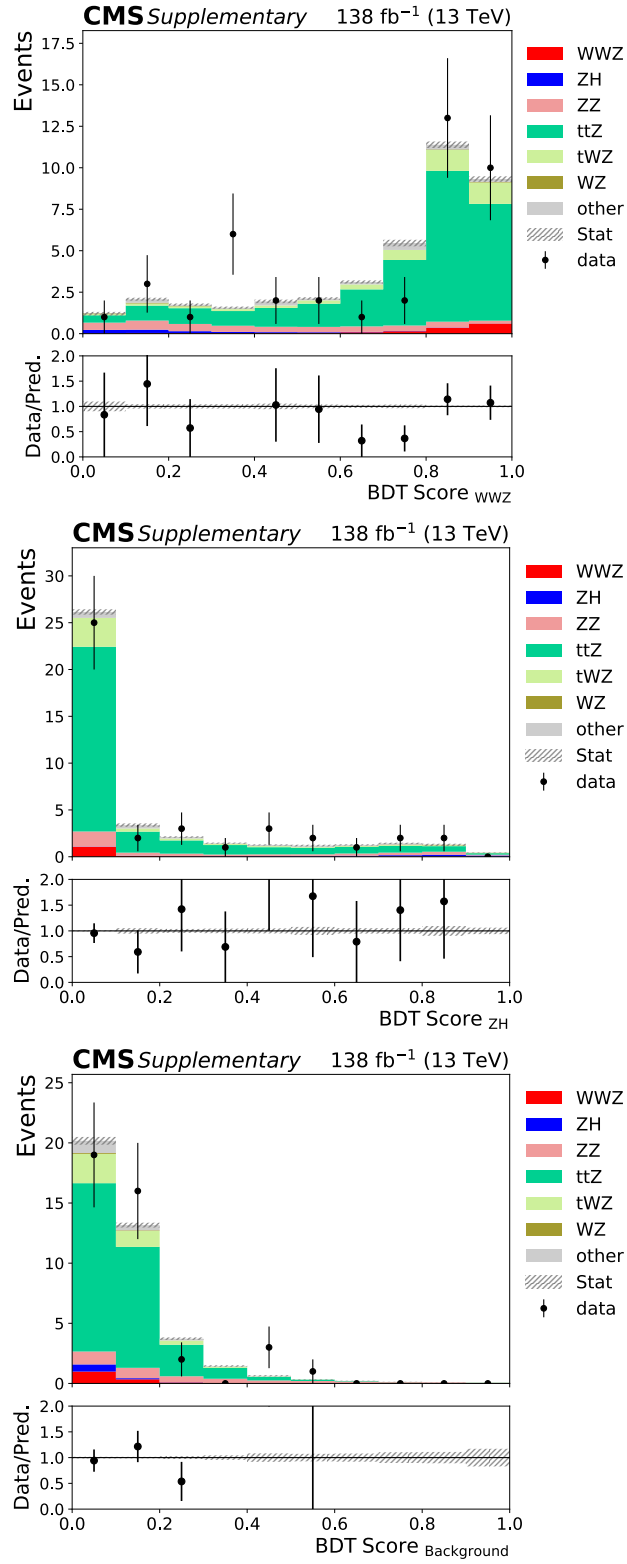
Figure D.20: BDT input variables shown in the SF training region (Run 3).

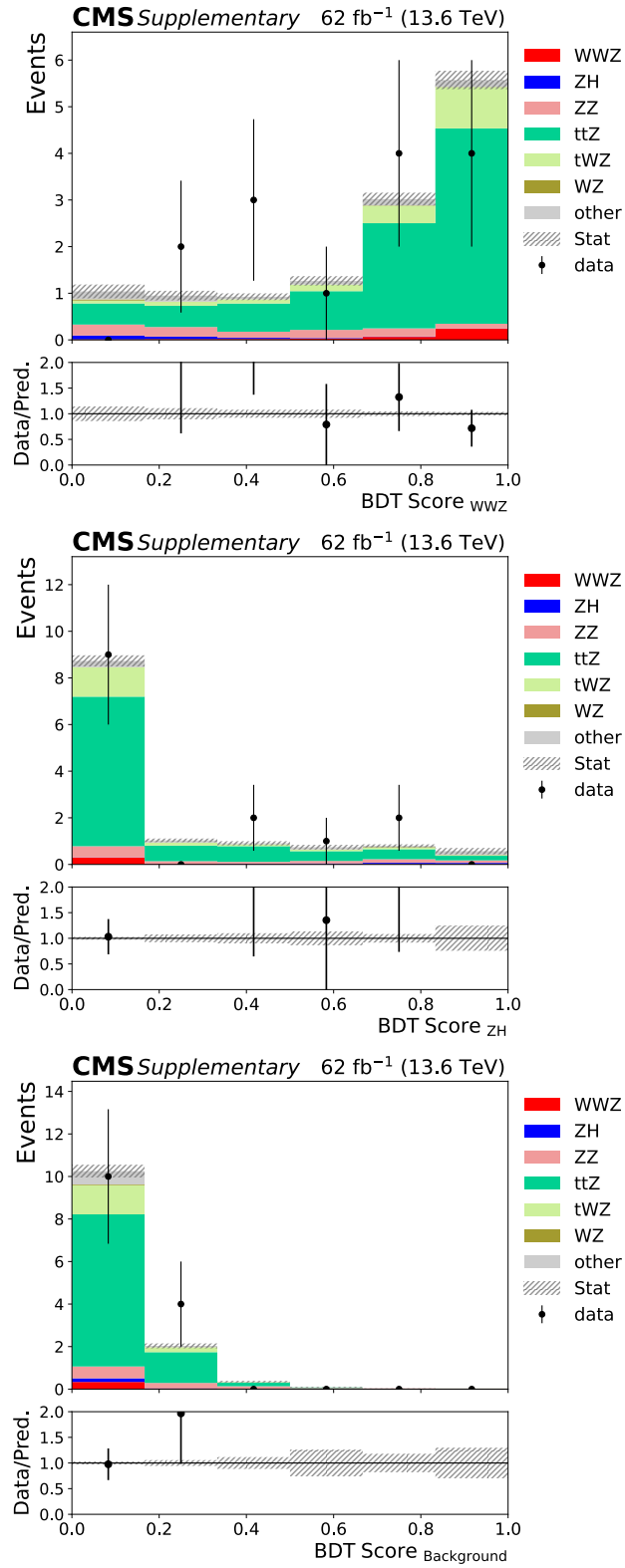

 Figure D.21: BDT scores shown in the ZZ control region (Run 2).


 Figure D.22: BDT scores shown in the ZZ control region (Run 3).

Figure D.23: BDT scores shown in the tXZ OF control region (Run 2).

Figure D.24: BDT scores shown in the tXZ OF control region (Run 3).

Figure D.25: BDT scores shown in the tXZ SF control region (Run 2).

Figure D.26: BDT scores shown in the tXZ SF control region (Run 3).

APPENDIX E

CONTROL REGION VALIDATION

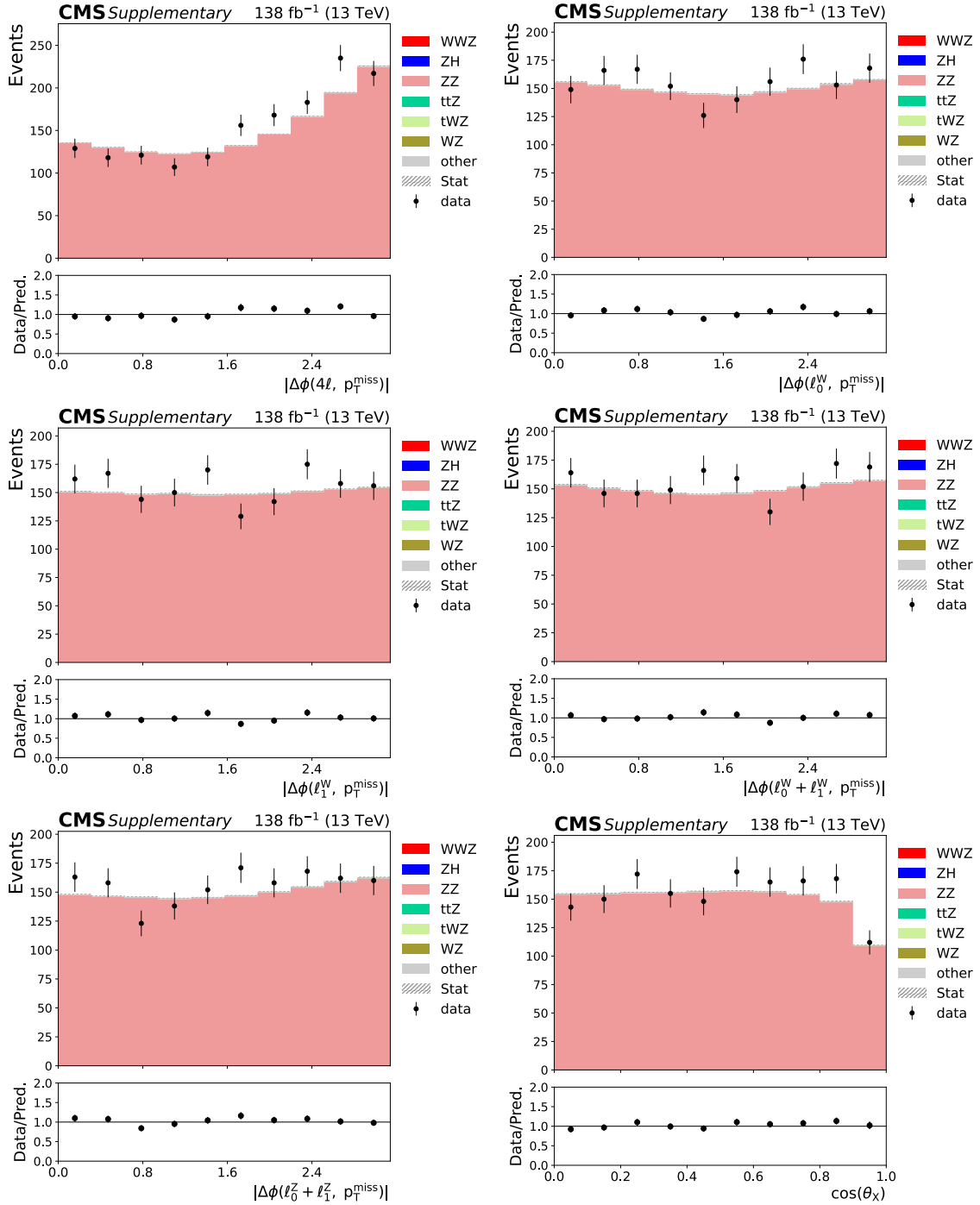
E.1 Control Regions

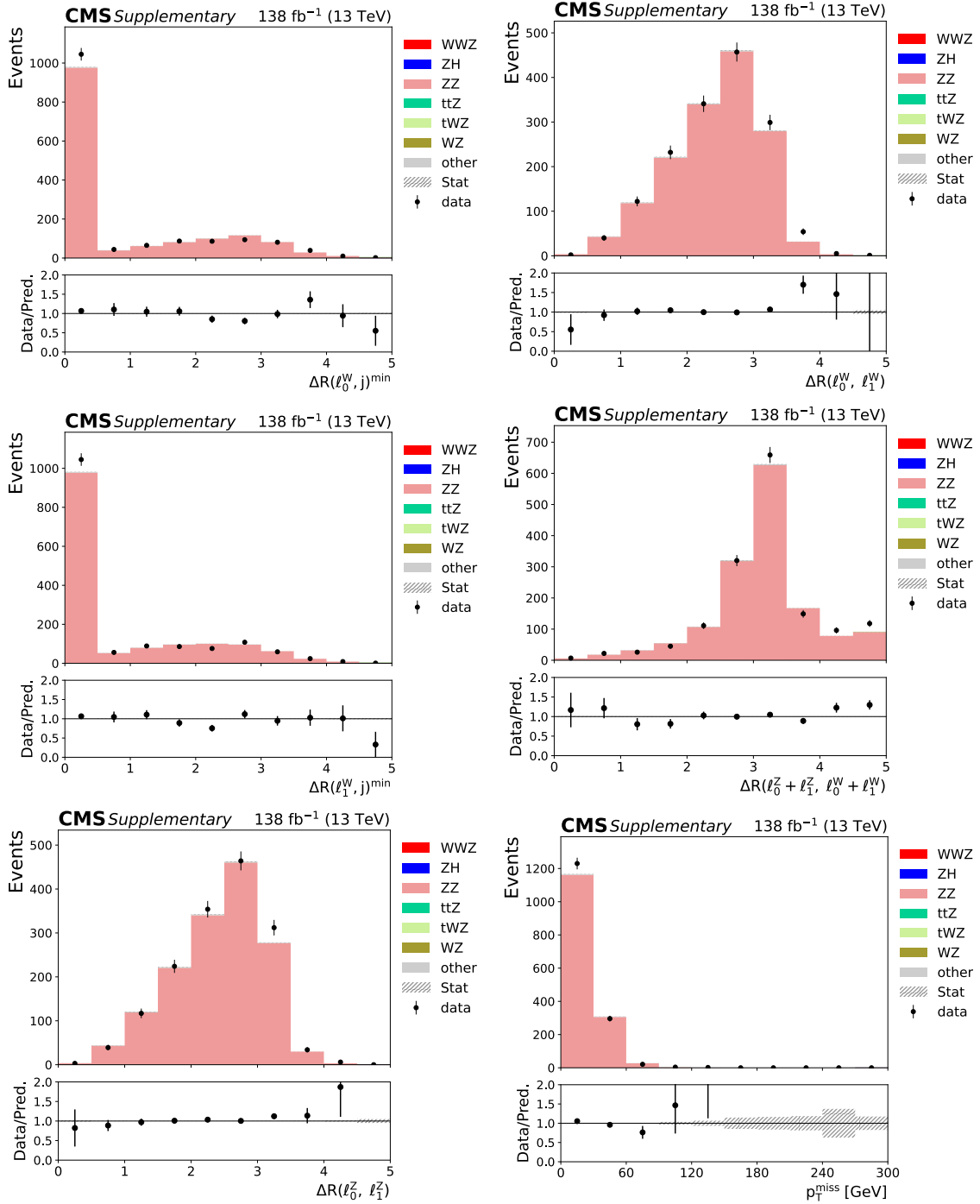
Additional plots for the control regions are shown in ref. [132].

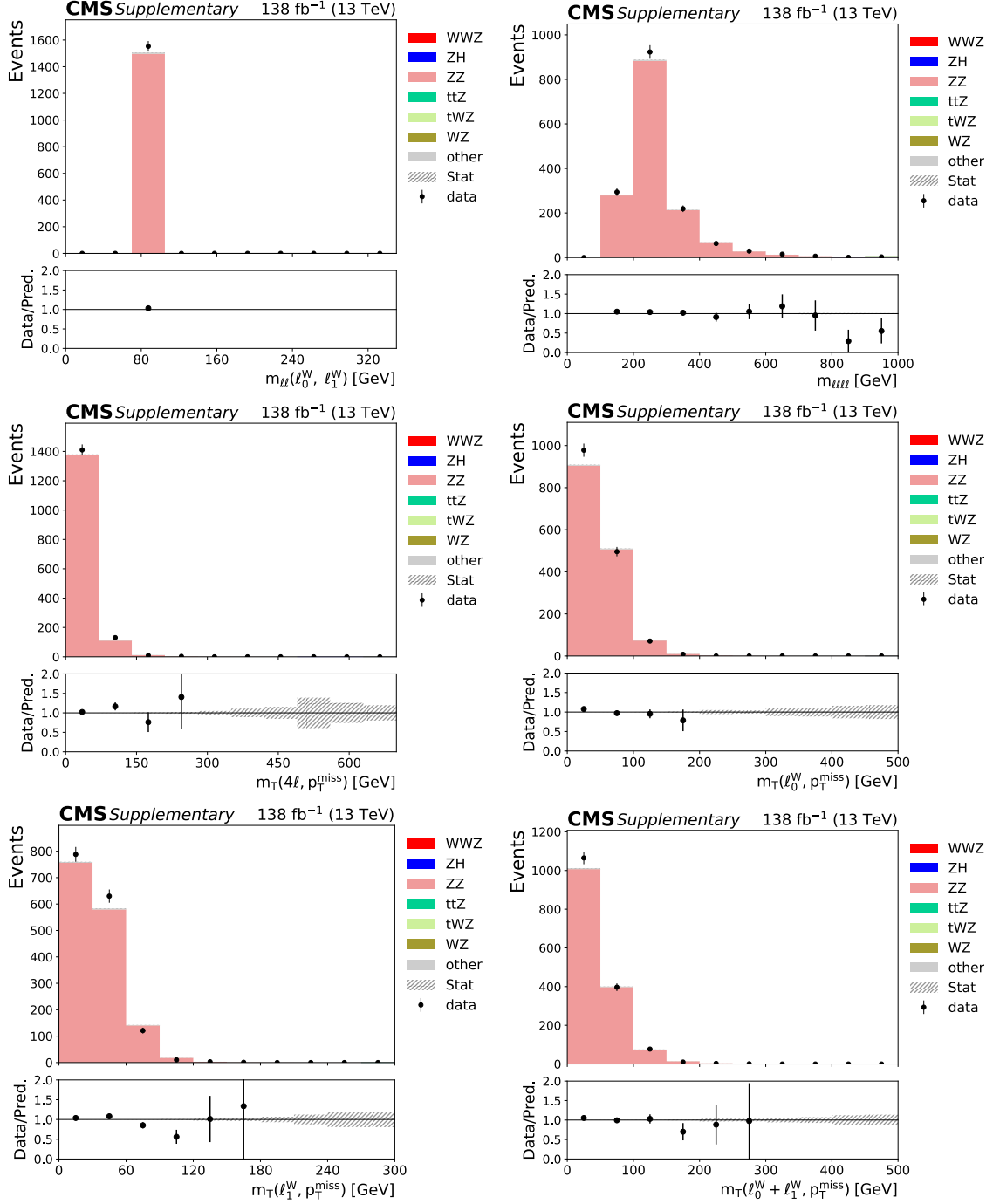
E.1.1 ZZ CR

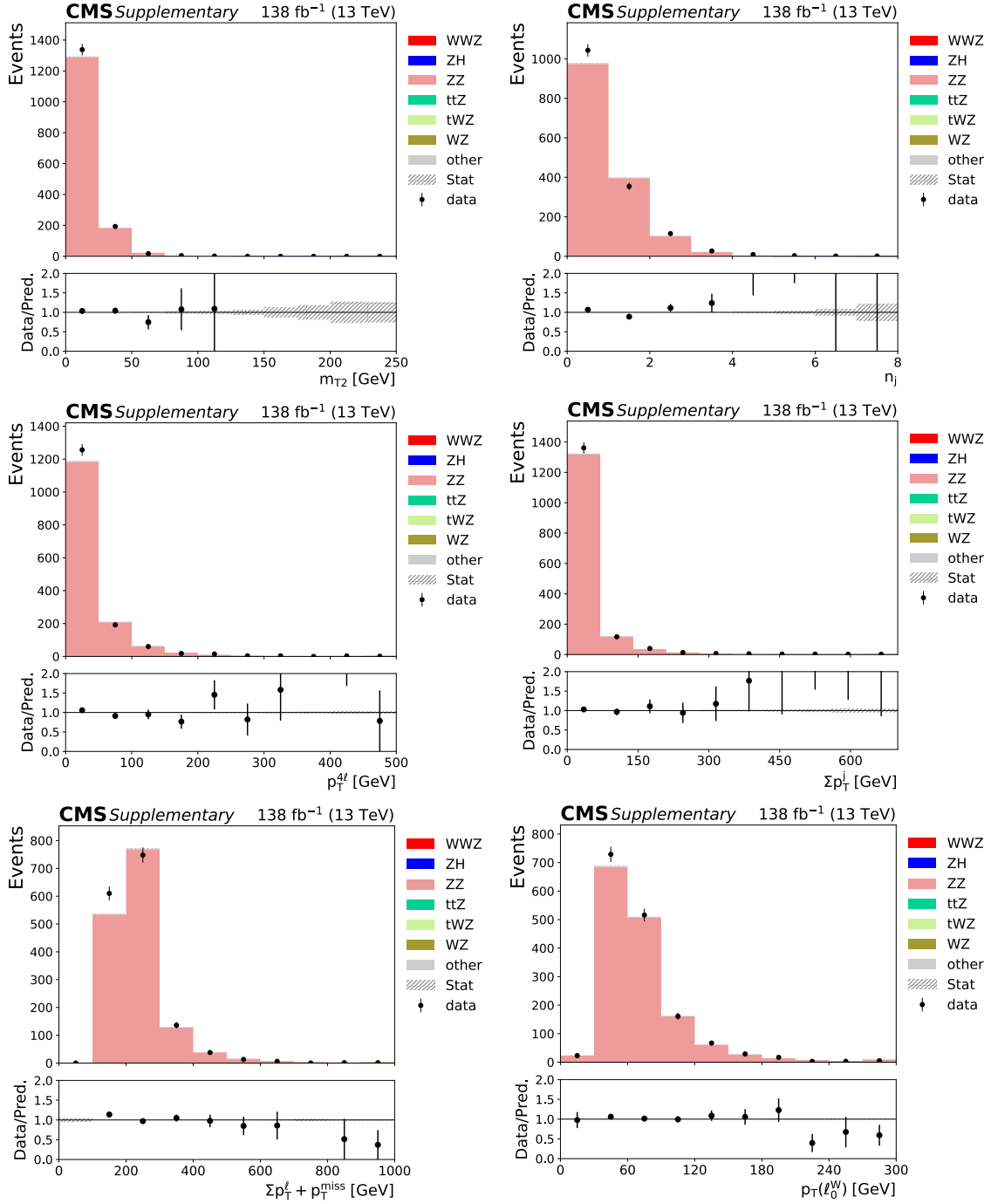
E.1.2 tXZ OF CR

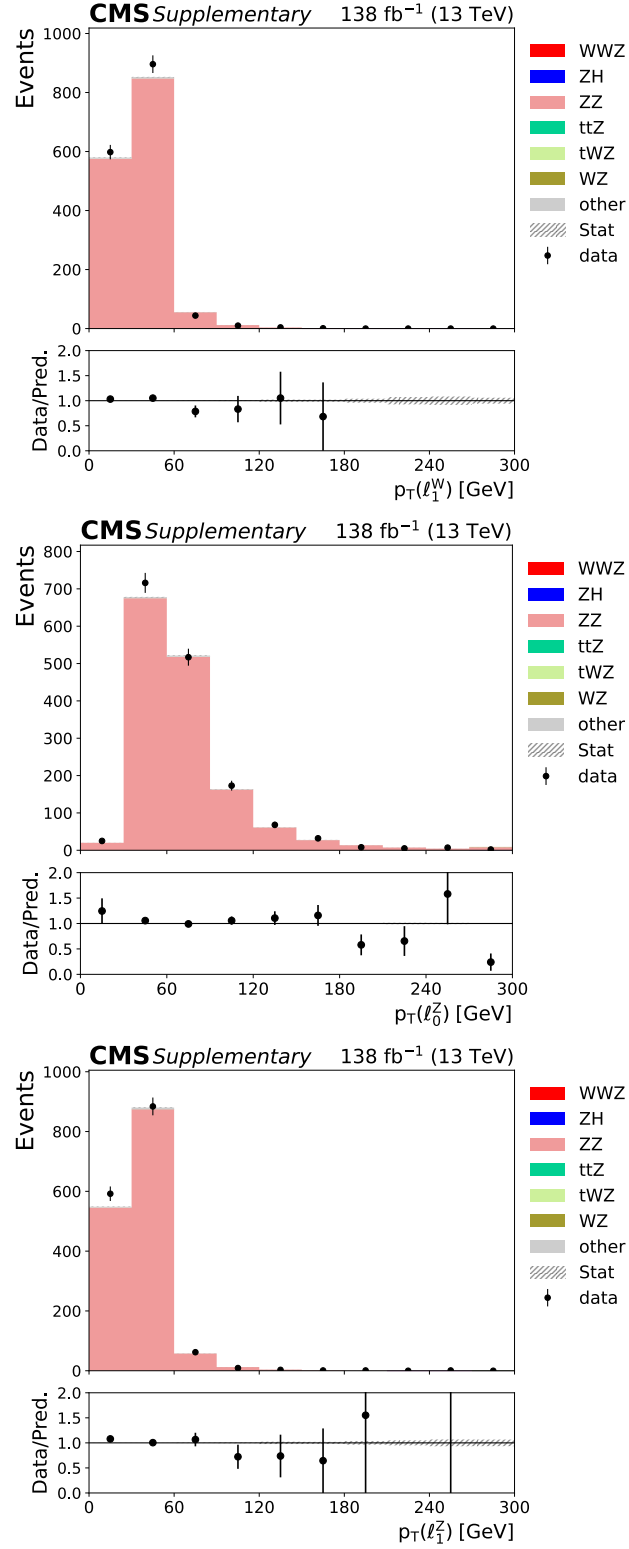
E.1.3 tXZ SF CR


 Figure E.1: BDT input variables shown in the ZZ control region (Run 2).


 Figure E.2: BDT input variables shown in the ZZ control region (Run 2).


 Figure E.3: BDT input variables shown in the ZZ control region (Run 2).


 Figure E.4: BDT input variables shown in the ZZ control region (Run 2).


 Figure E.5: BDT input variables shown in the ZZ control region (Run 2).

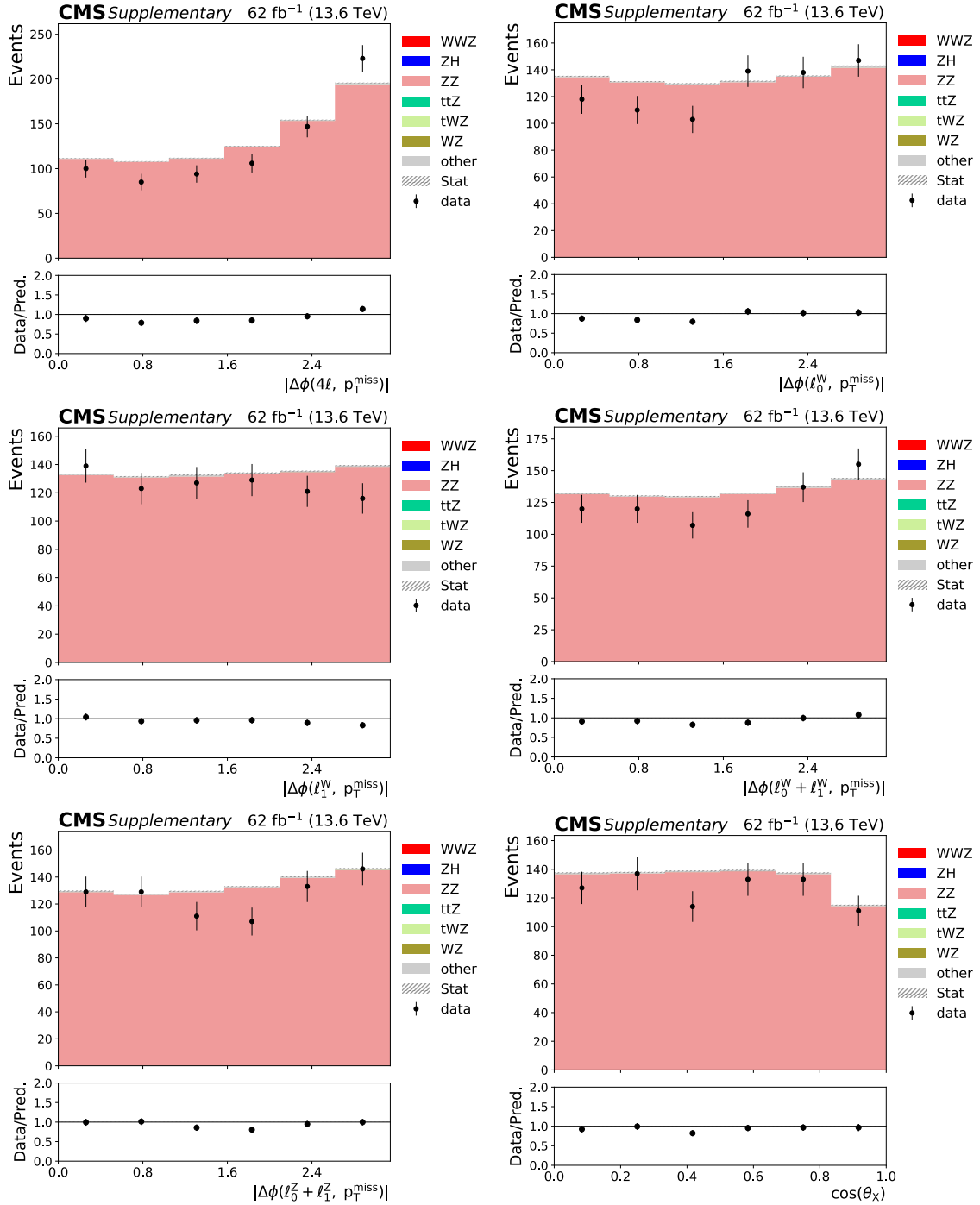
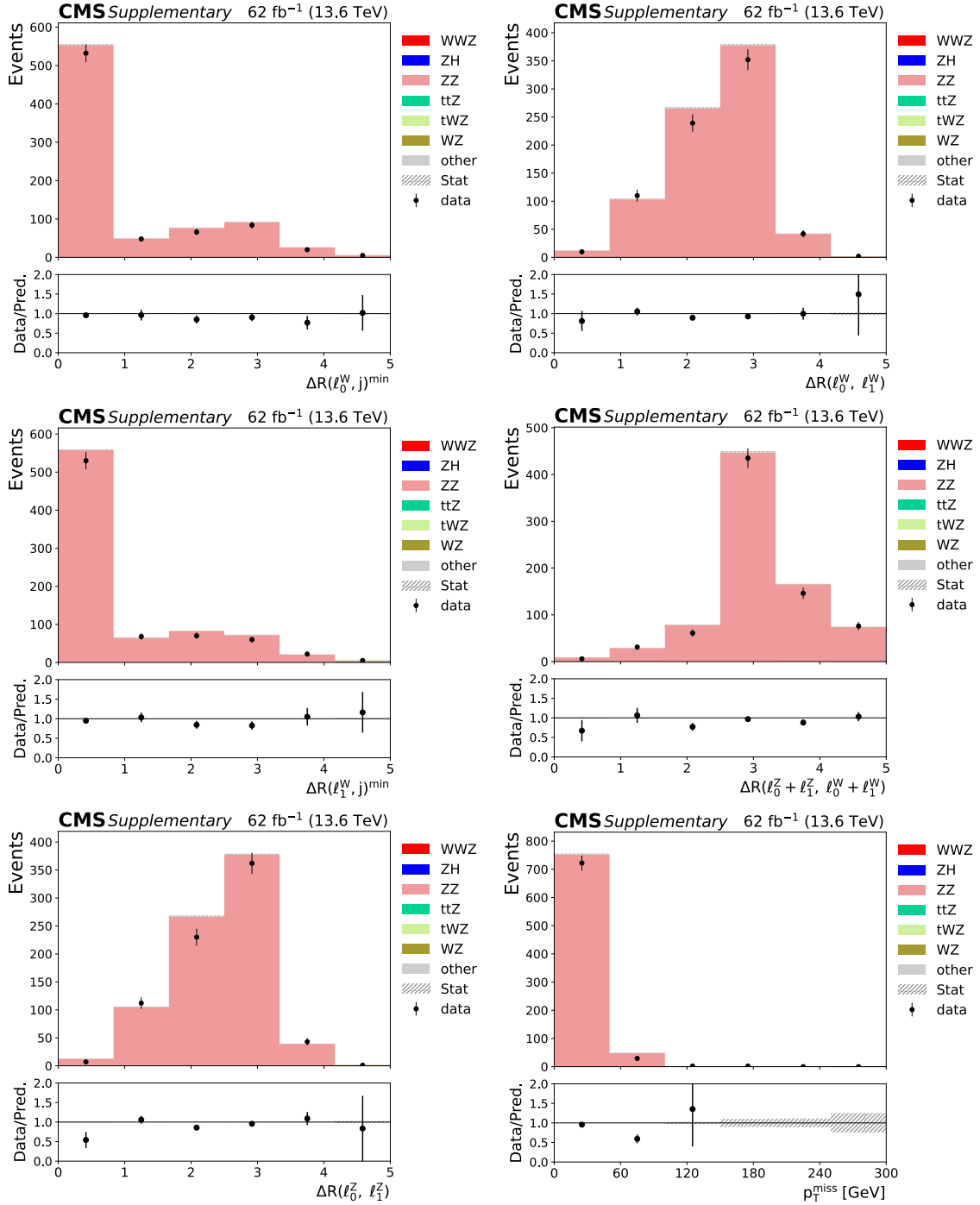
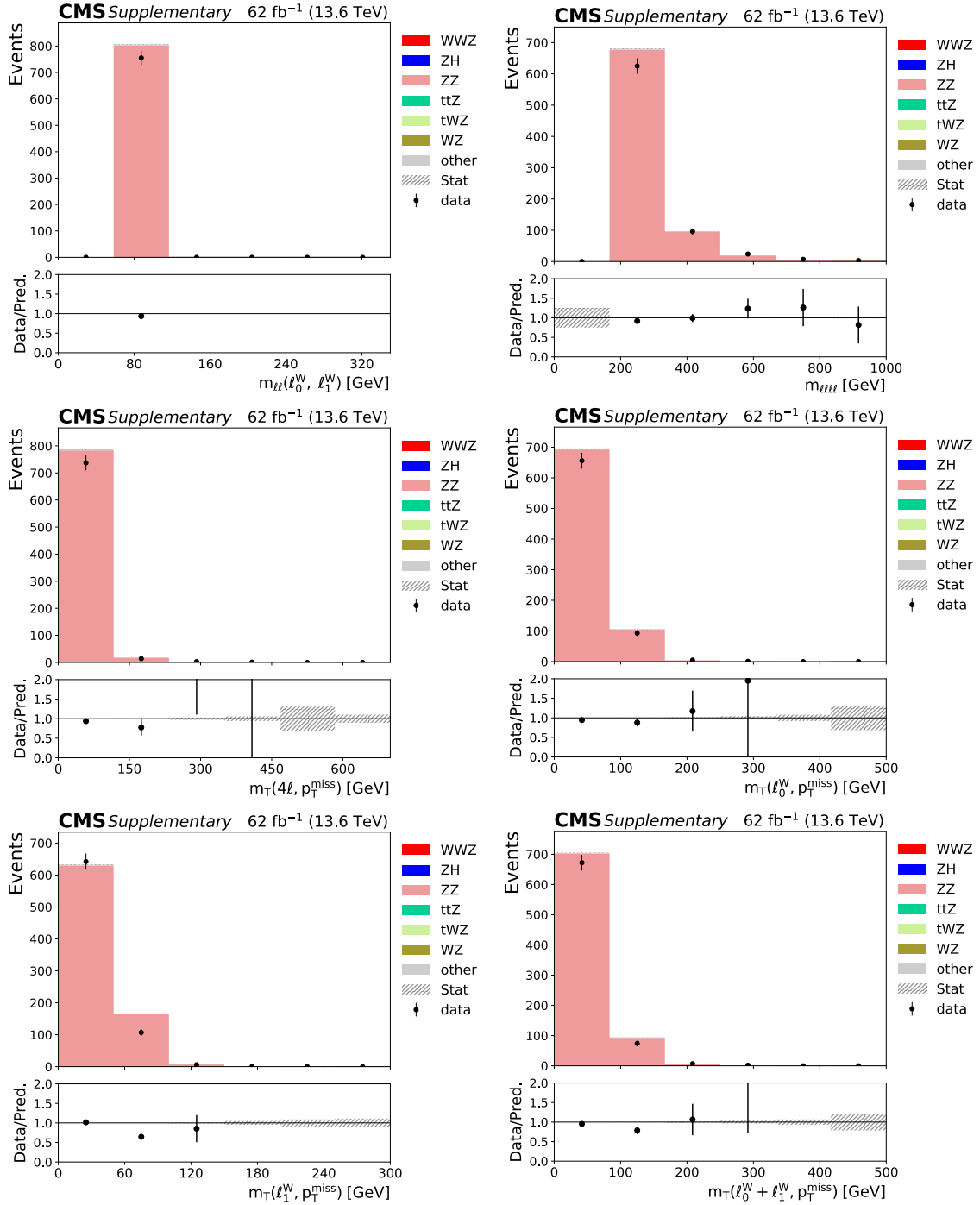
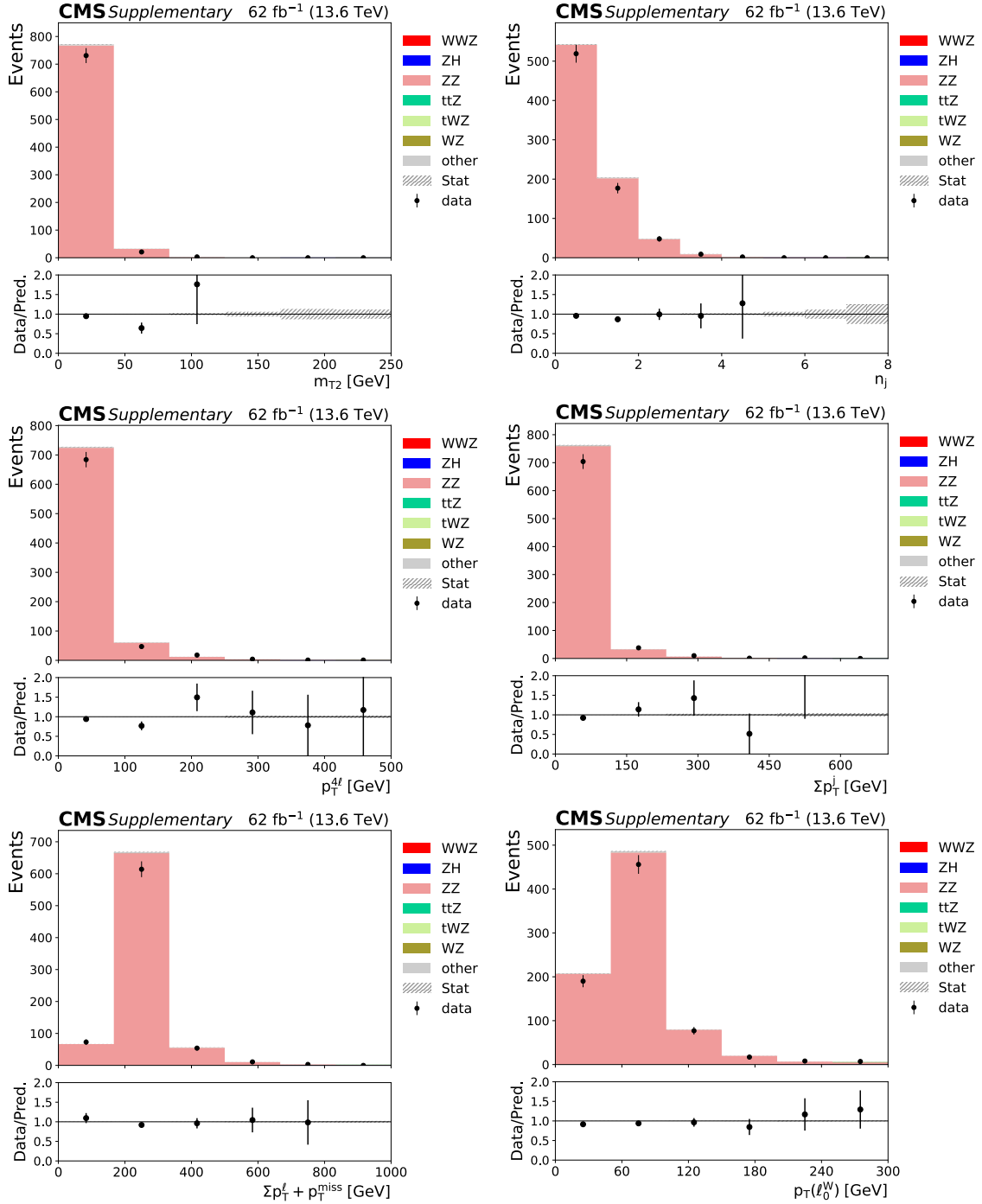
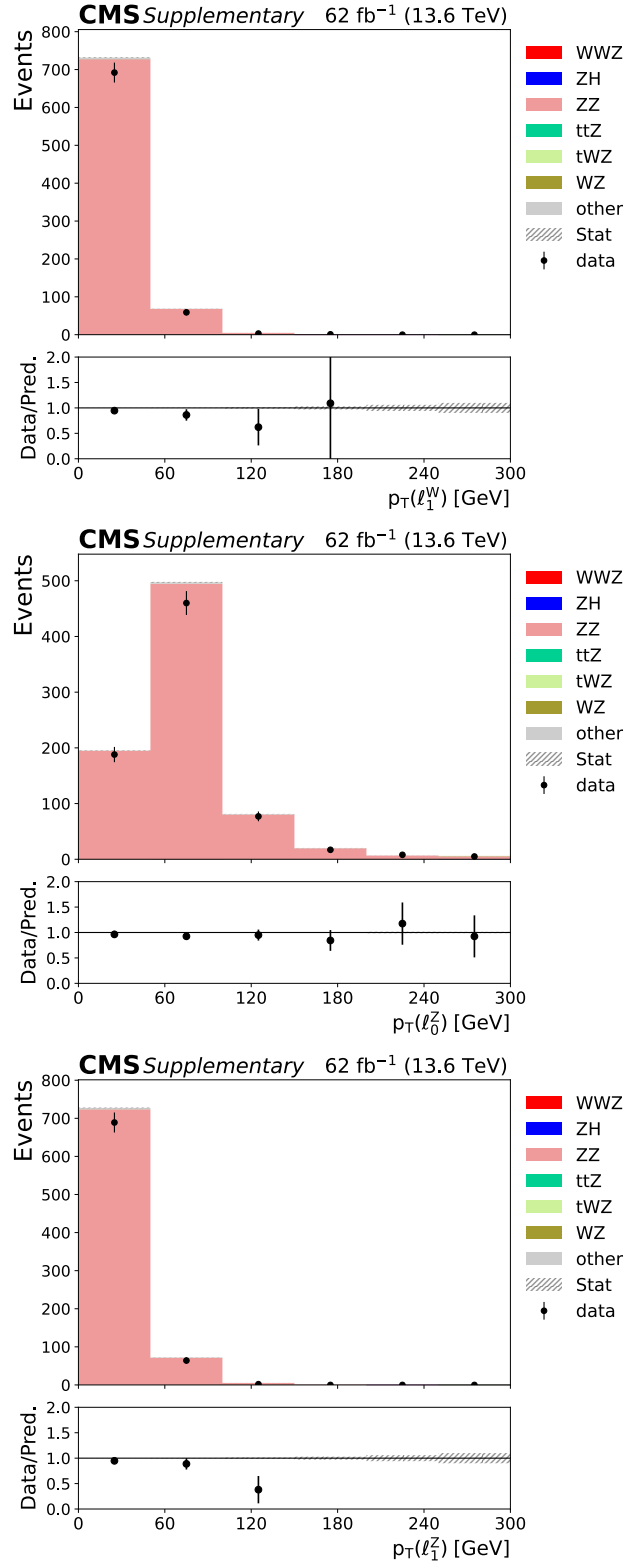


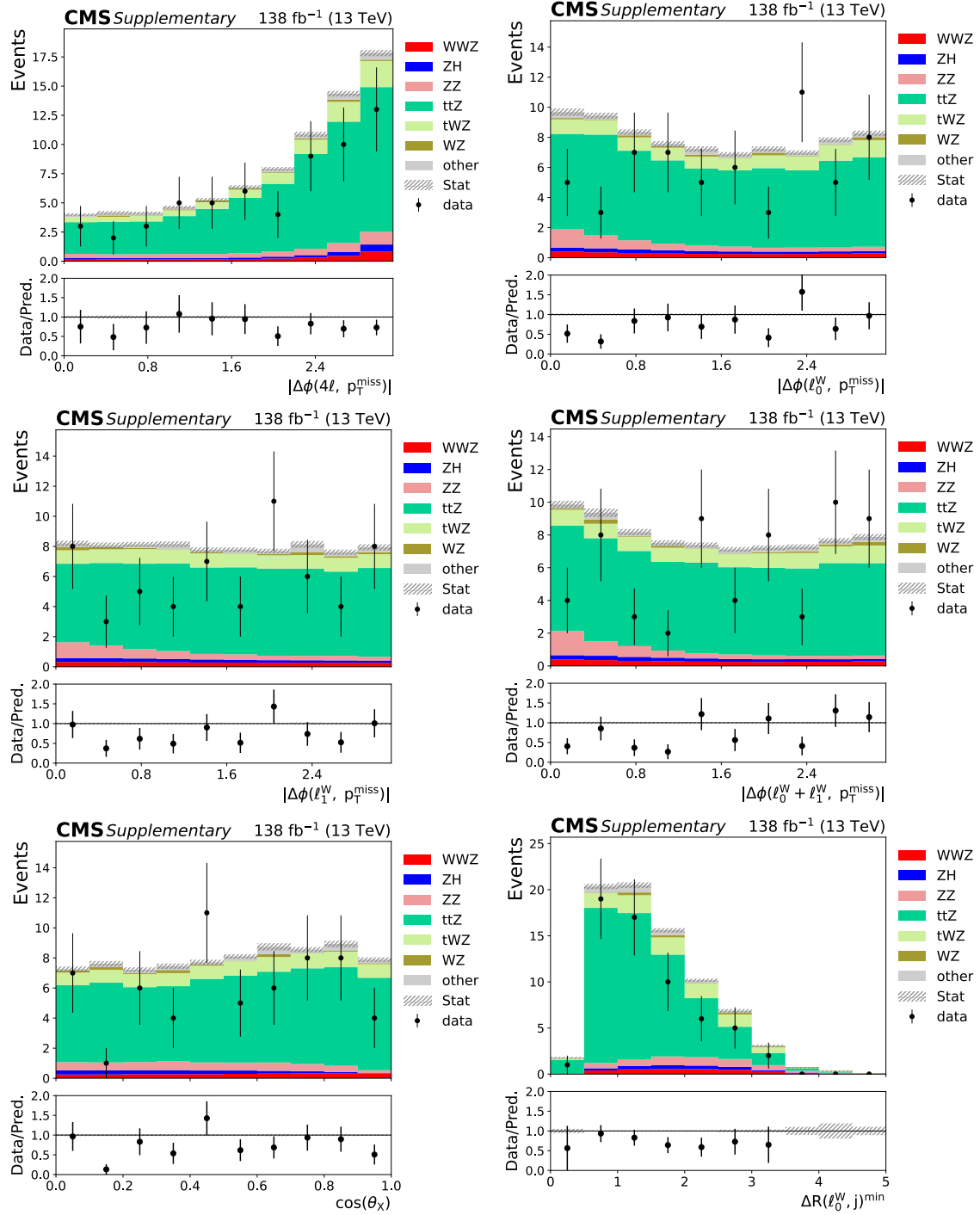
Figure E.6: BDT input variables shown in the ZZ control region (Run 3).

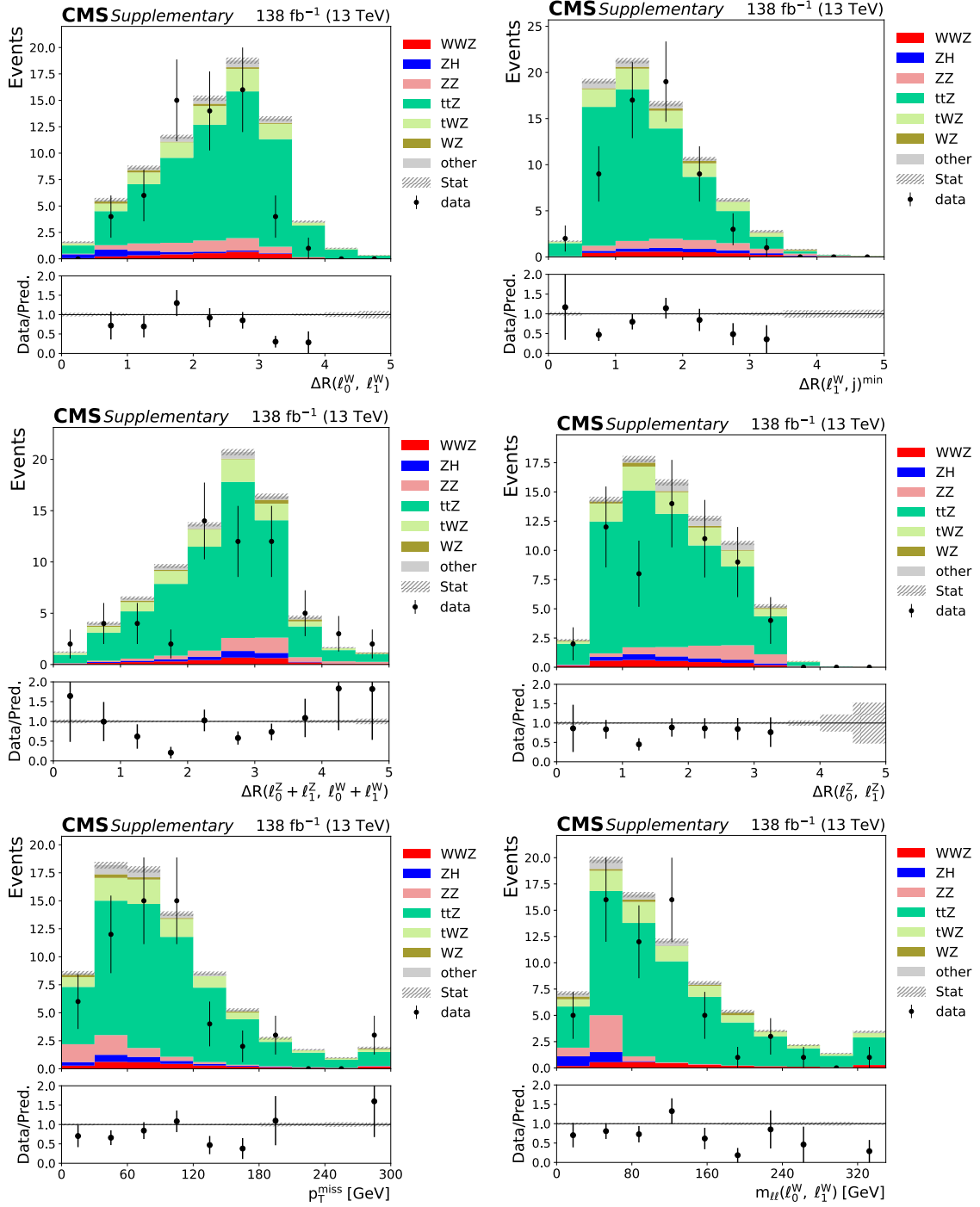

 Figure E.7: BDT input variables shown in the ZZ control region (Run 3).

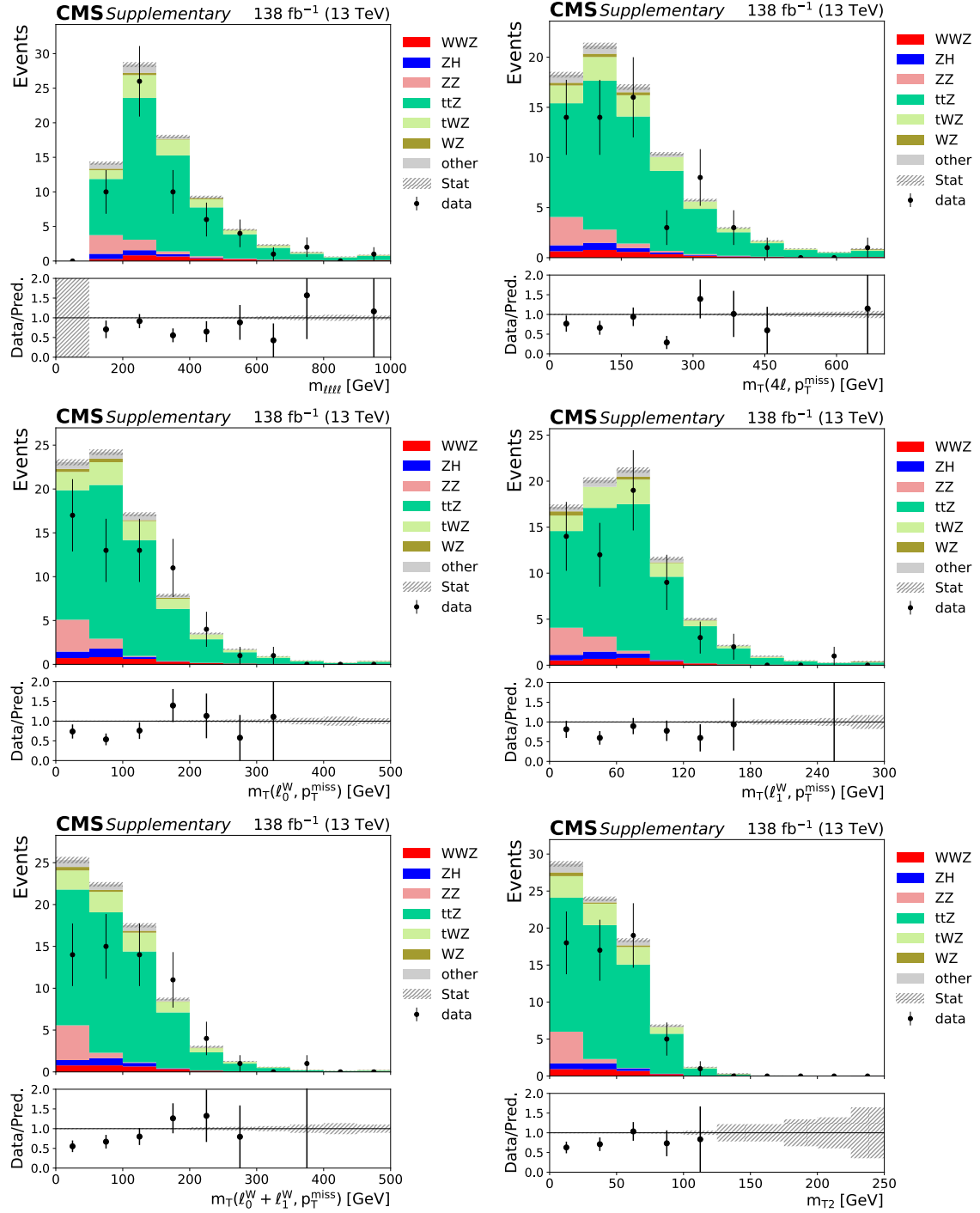

 Figure E.8: BDT input variables shown in the ZZ control region (Run 3).

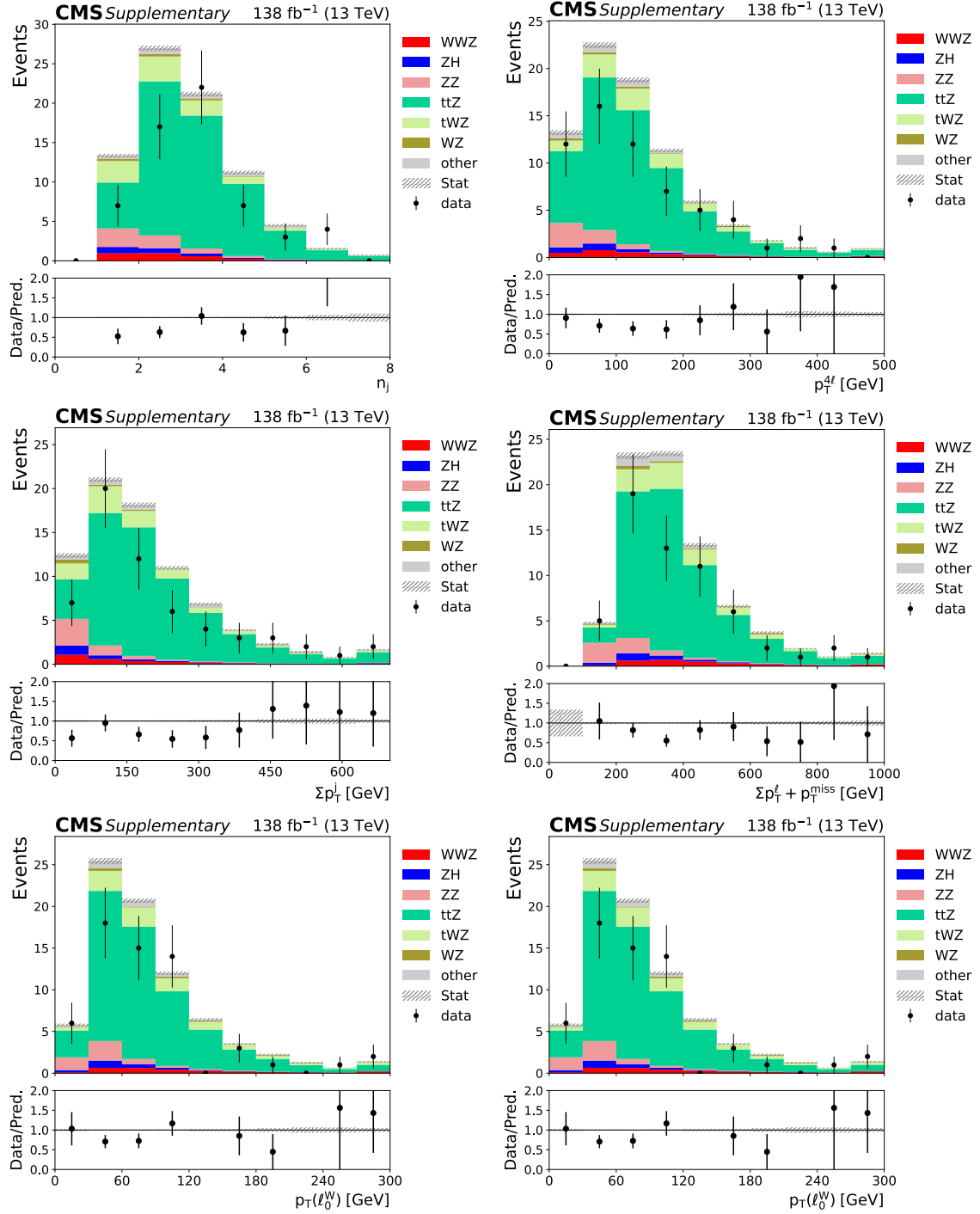

 Figure E.9: BDT input variables shown in the ZZ control region (Run 3).


 Figure E.10: BDT input variables shown in the ZZ control region (Run 3).


 Figure E.11: BDT input variables shown in the tXZ OF control region (Run 2).


 Figure E.12: BDT input variables shown in the tXZ OF control region (Run 2).


 Figure E.13: BDT input variables shown in the tXZ OF control region (Run 2).


 Figure E.14: BDT input variables shown in the tXZ OF control region (Run 2).

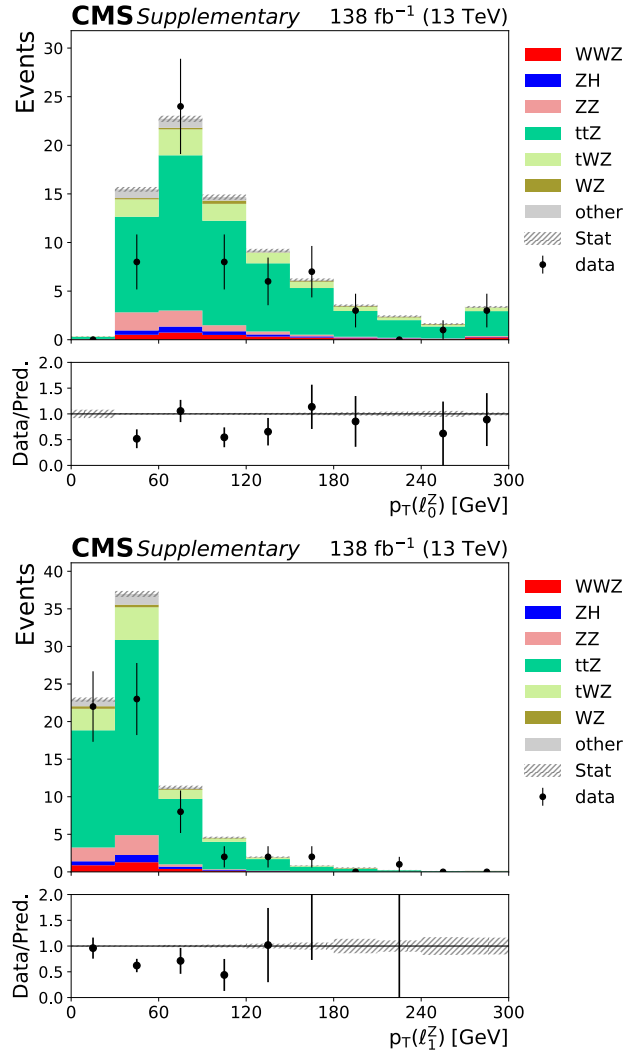
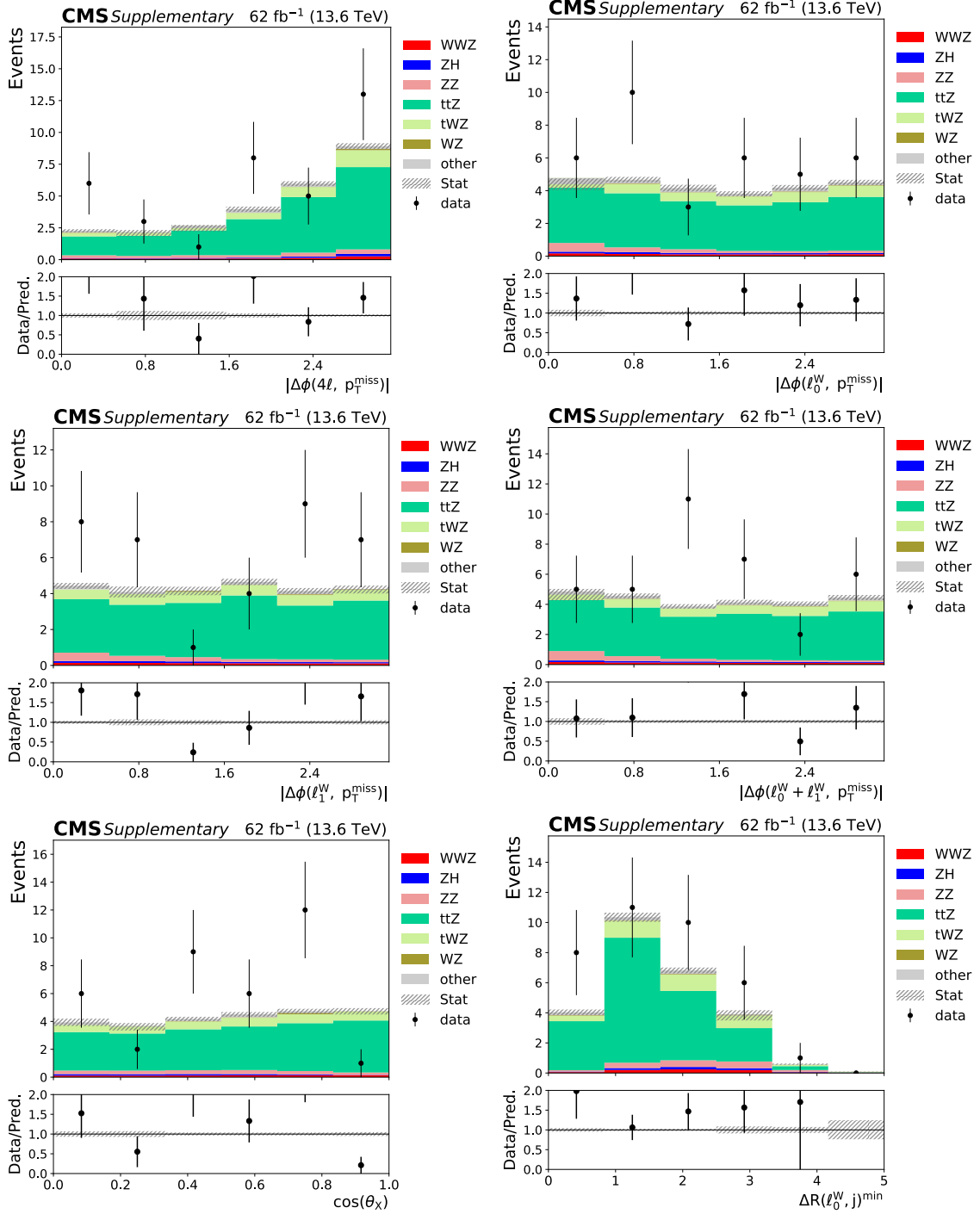
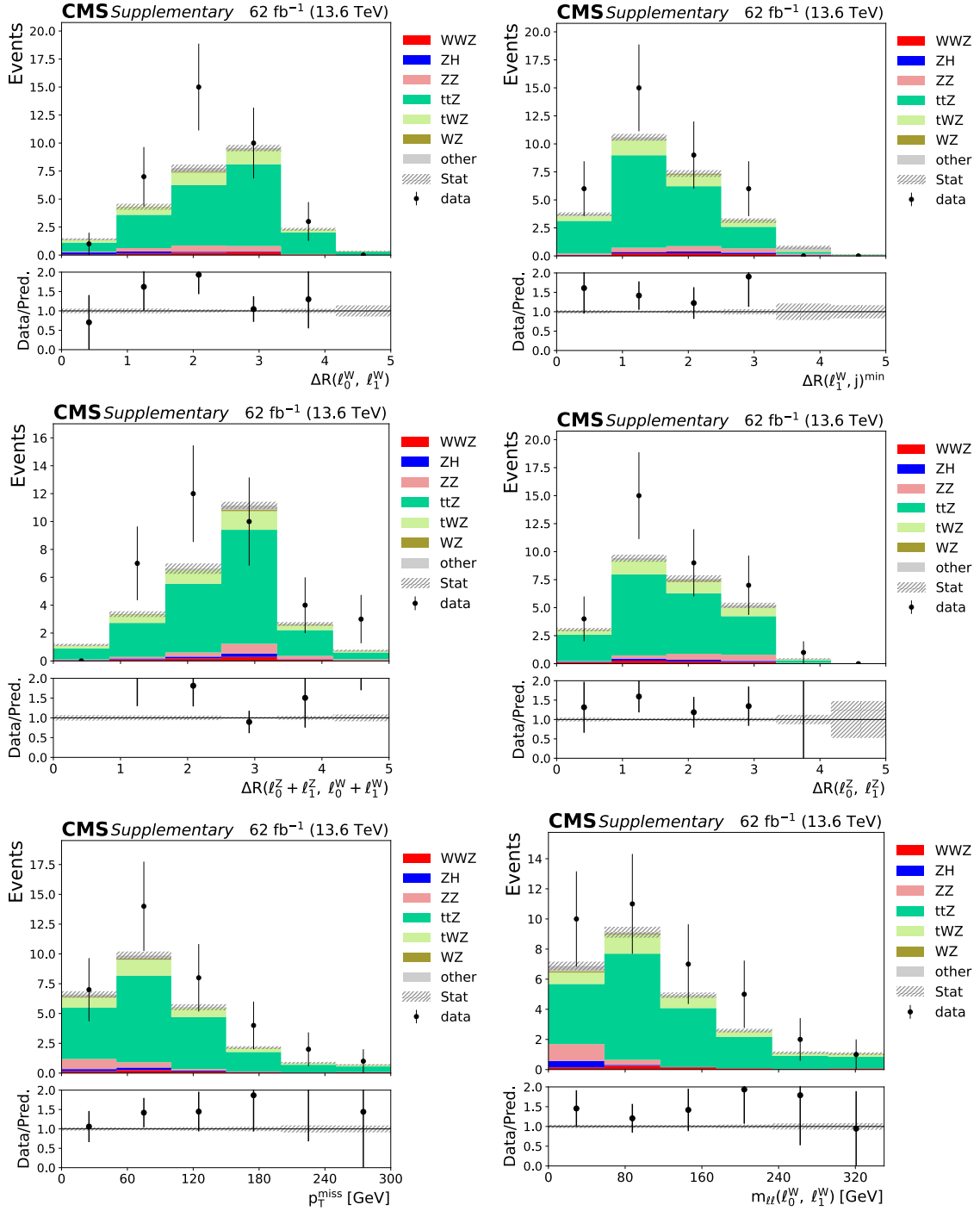
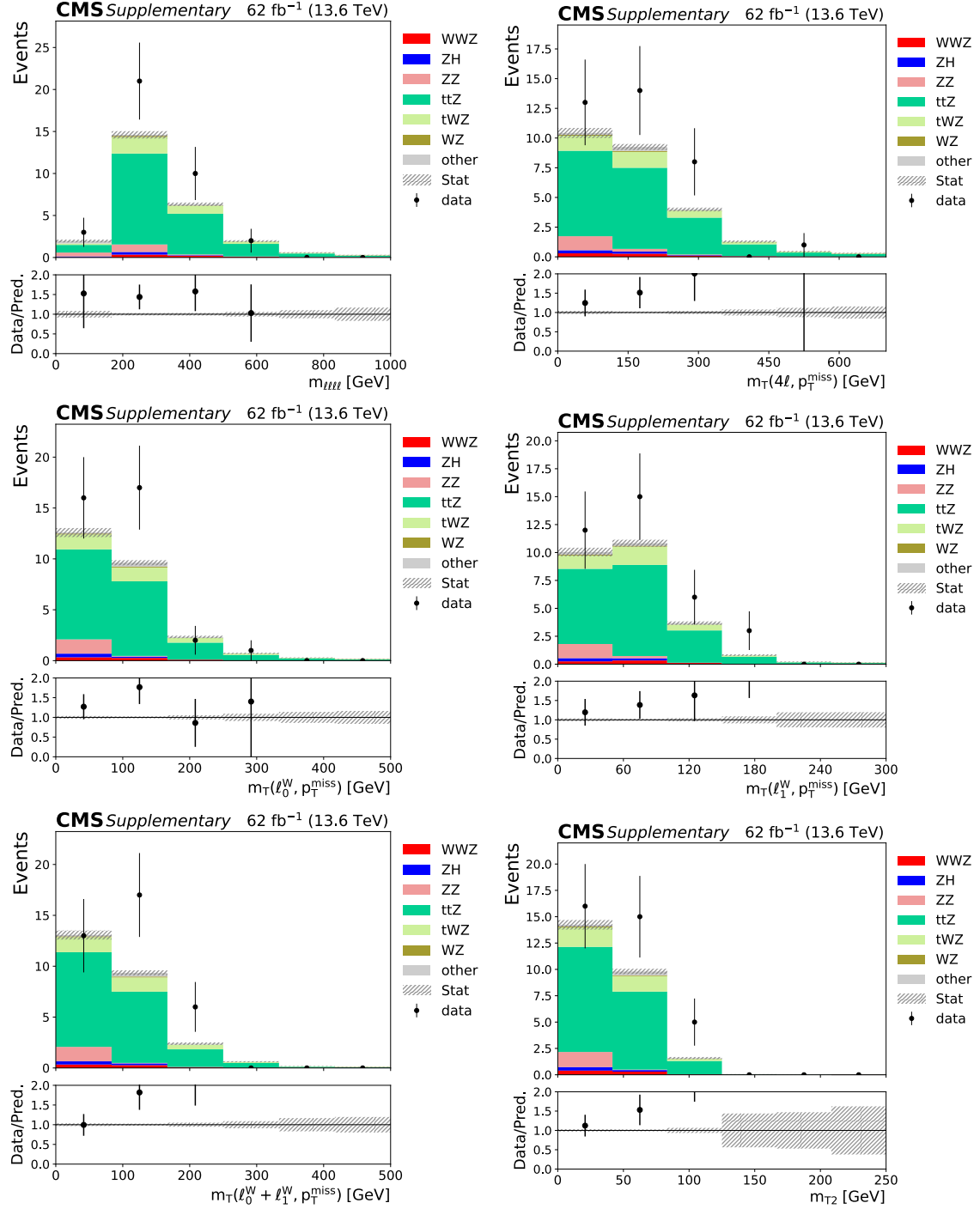
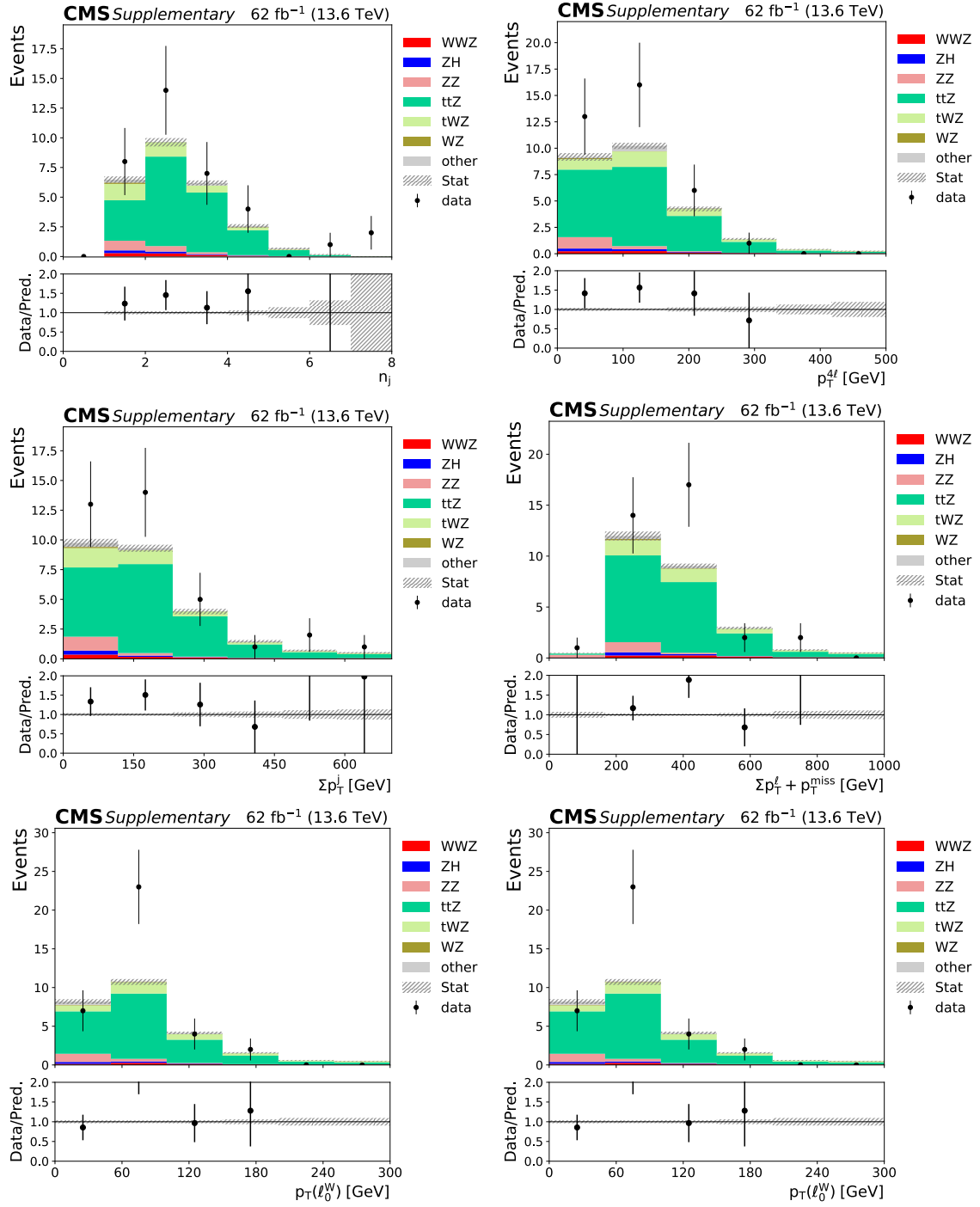


Figure E.15: BDT input variables shown in the tXZ OF control region (Run 2).


 Figure E.16: BDT input variables shown in the tXZ OF control region (Run 3).


 Figure E.17: BDT input variables shown in the tXZ OF control region (Run 3).


 Figure E.18: BDT input variables shown in the tXZ OF control region (Run 3).


 Figure E.19: BDT input variables shown in the tXZ OF control region (Run 3).

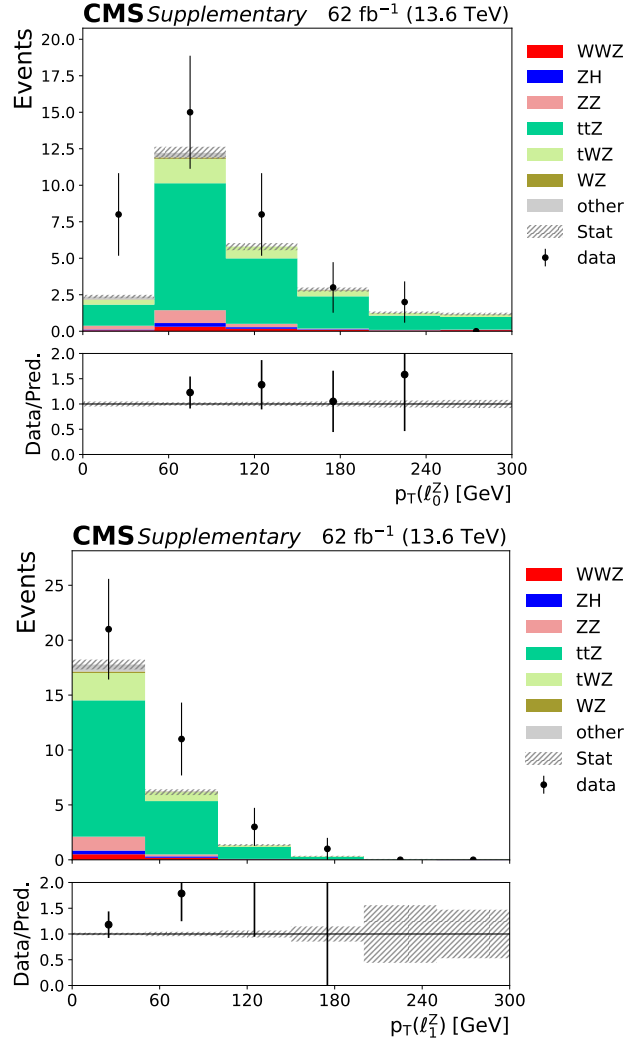
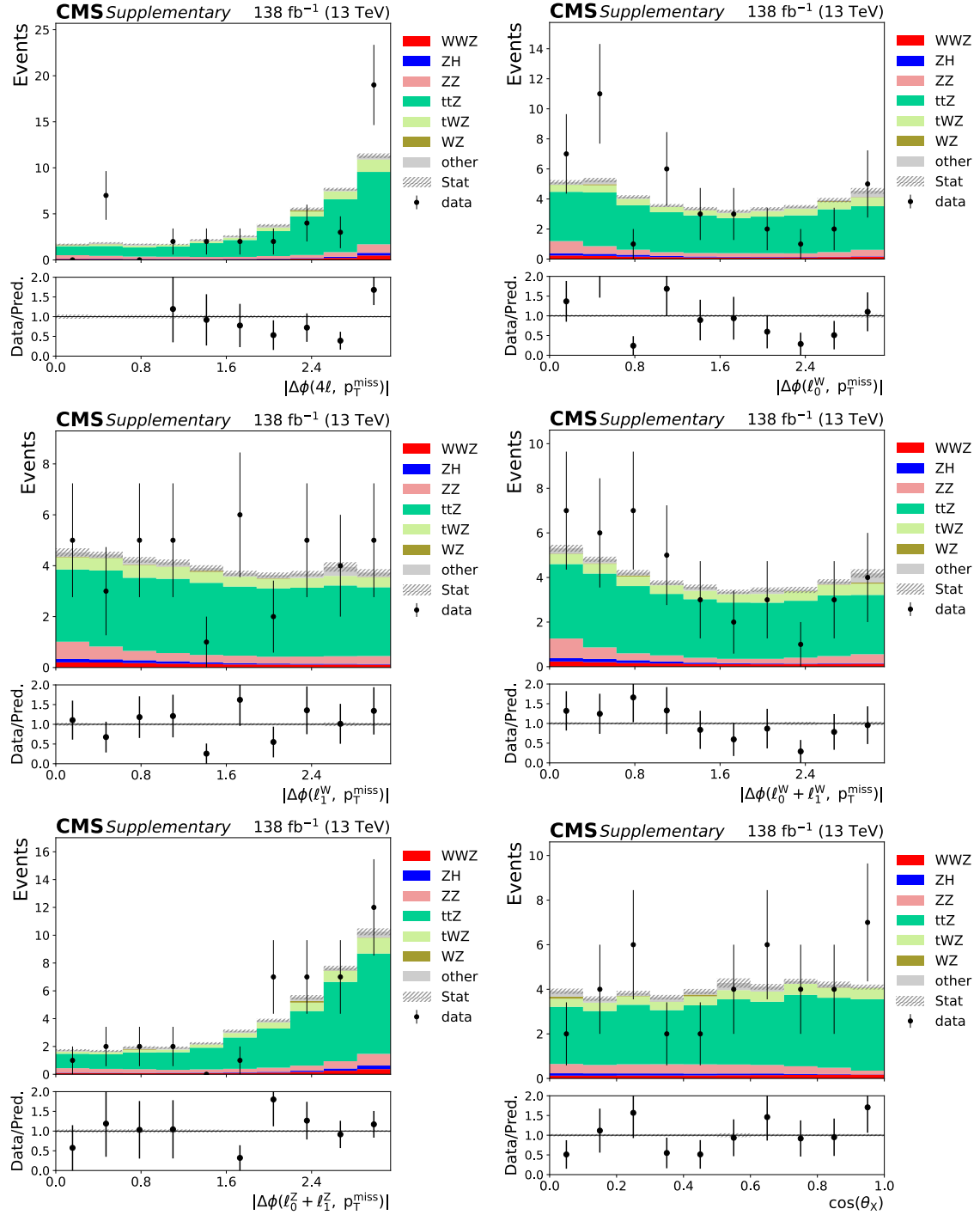
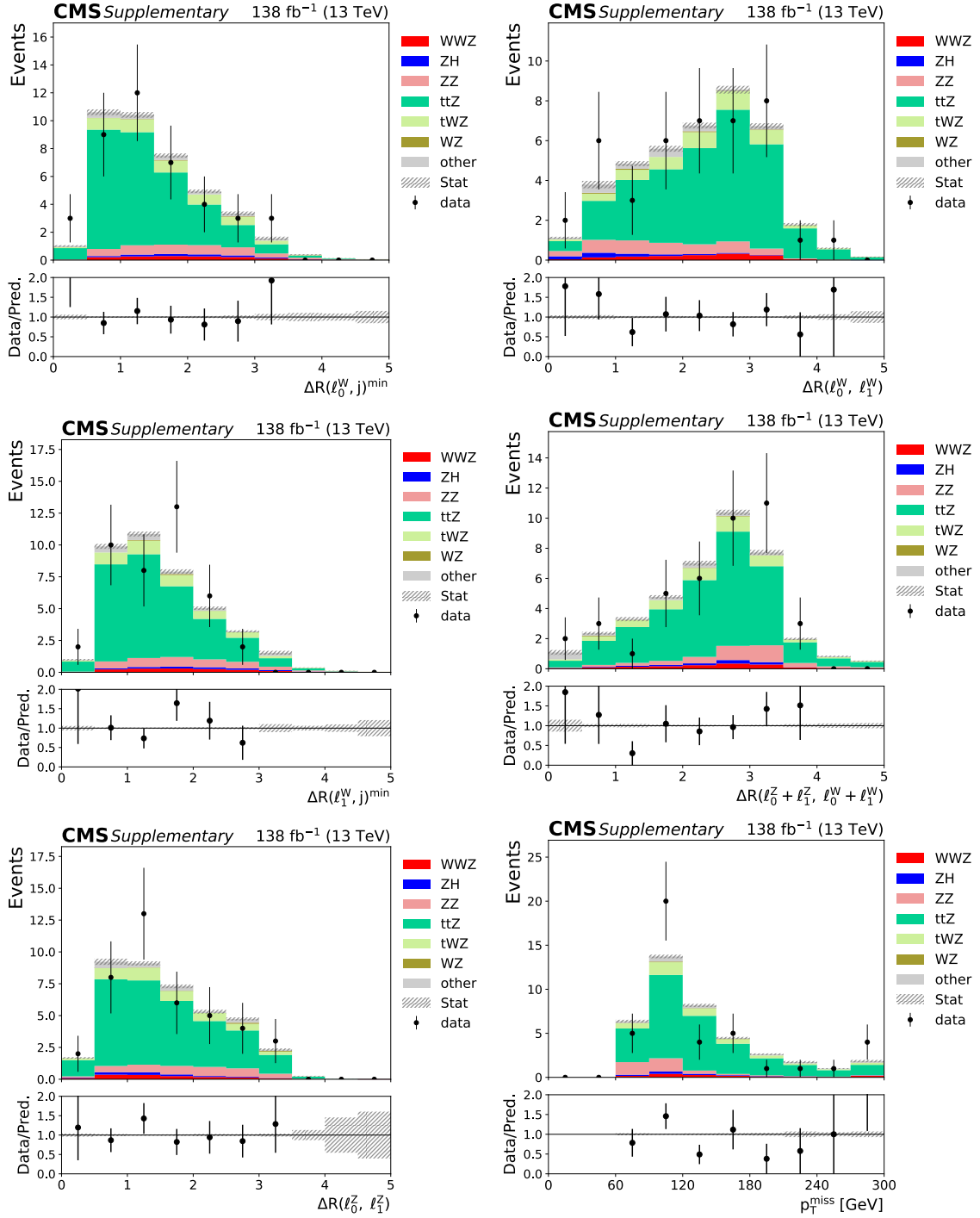
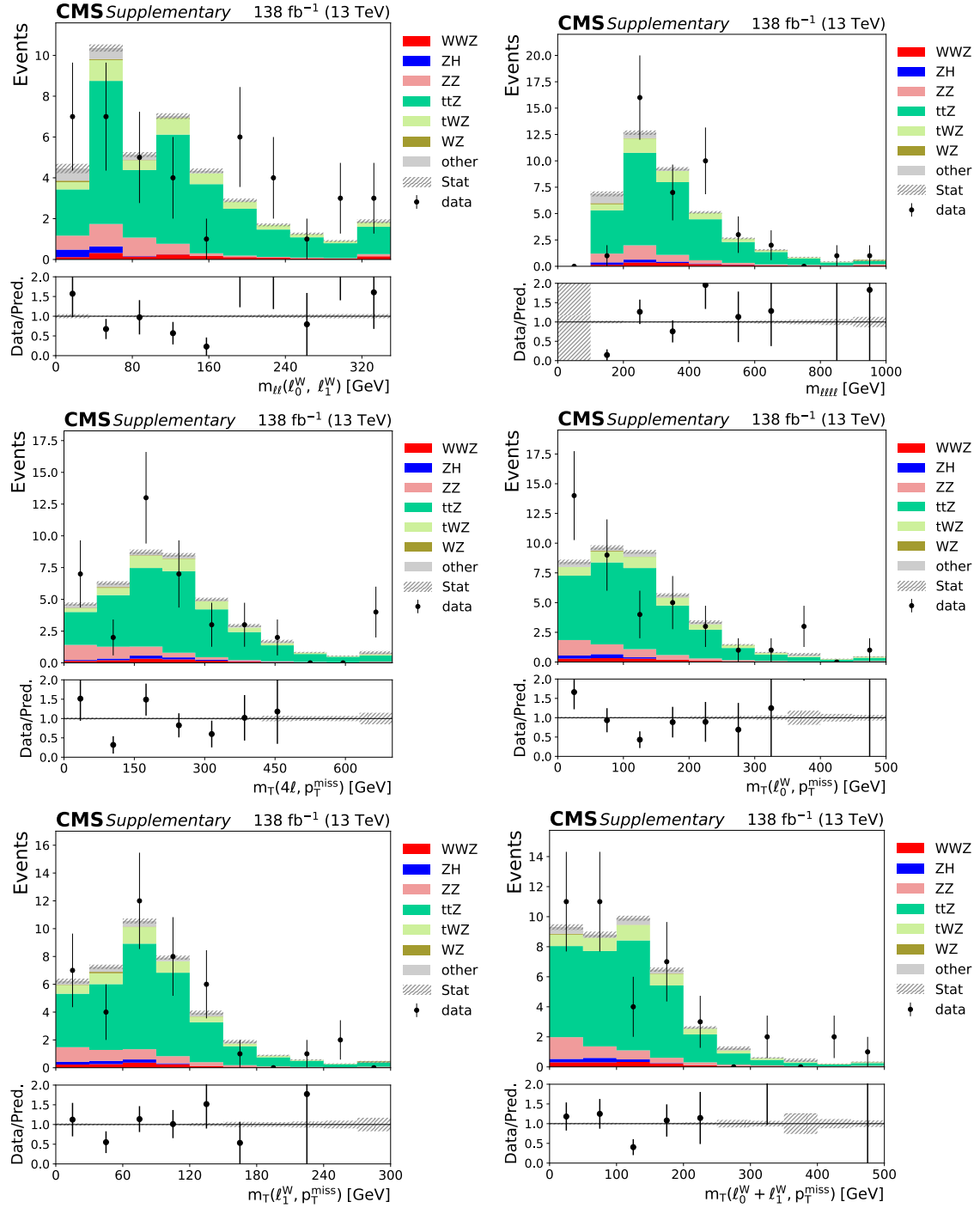
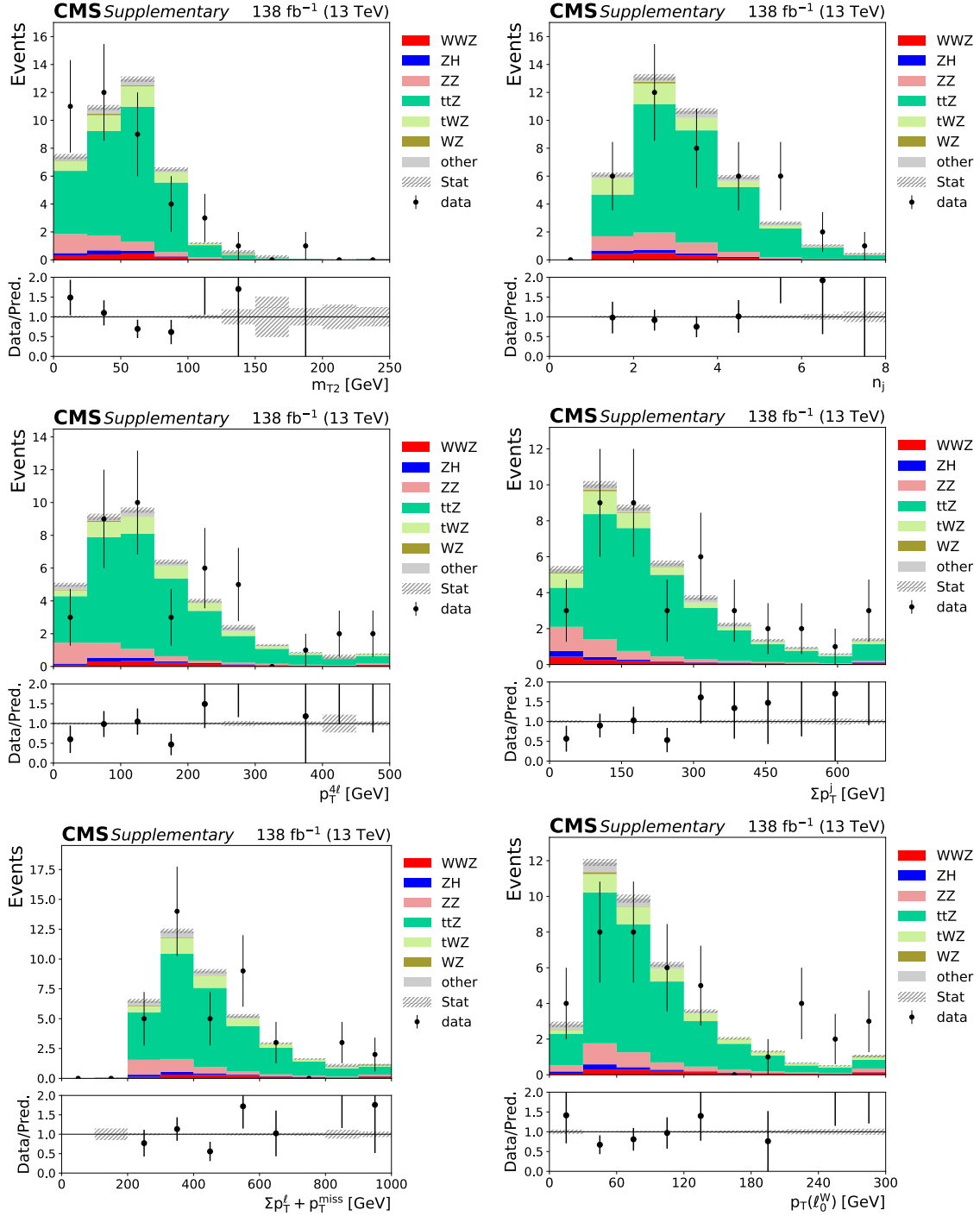


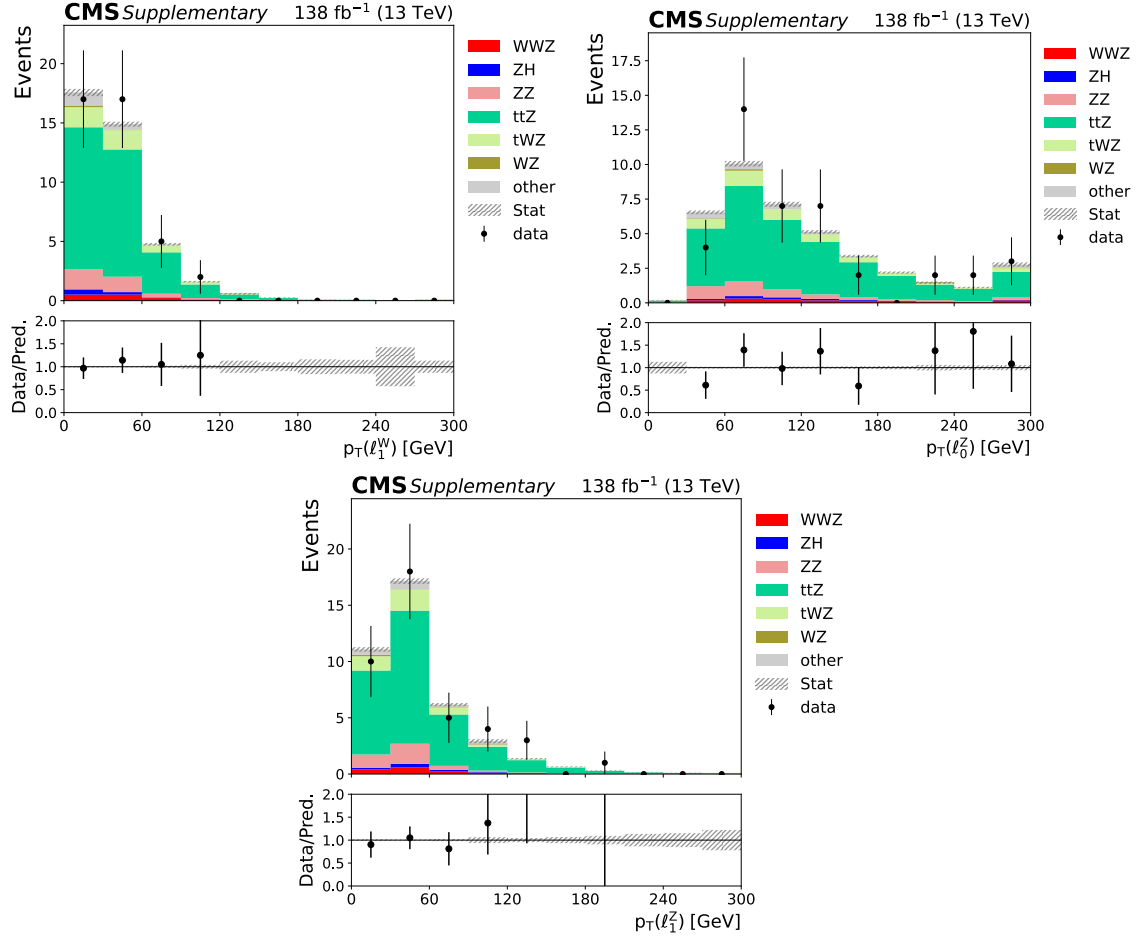
Figure E.20: BDT input variables shown in the tXZ OF control region (Run 3).

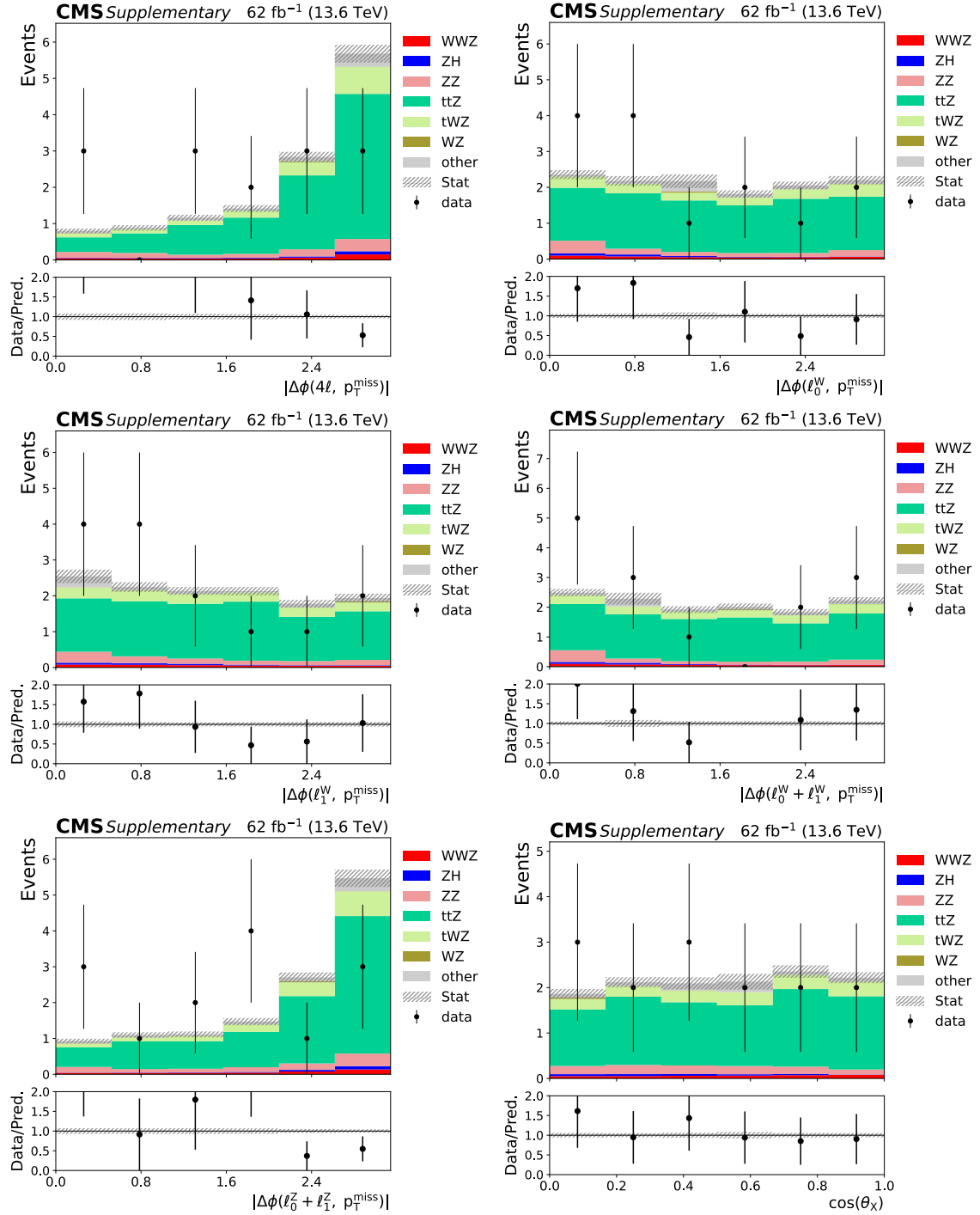

 Figure E.21: BDT input variables shown in the tXZ SF control region (Run 2).

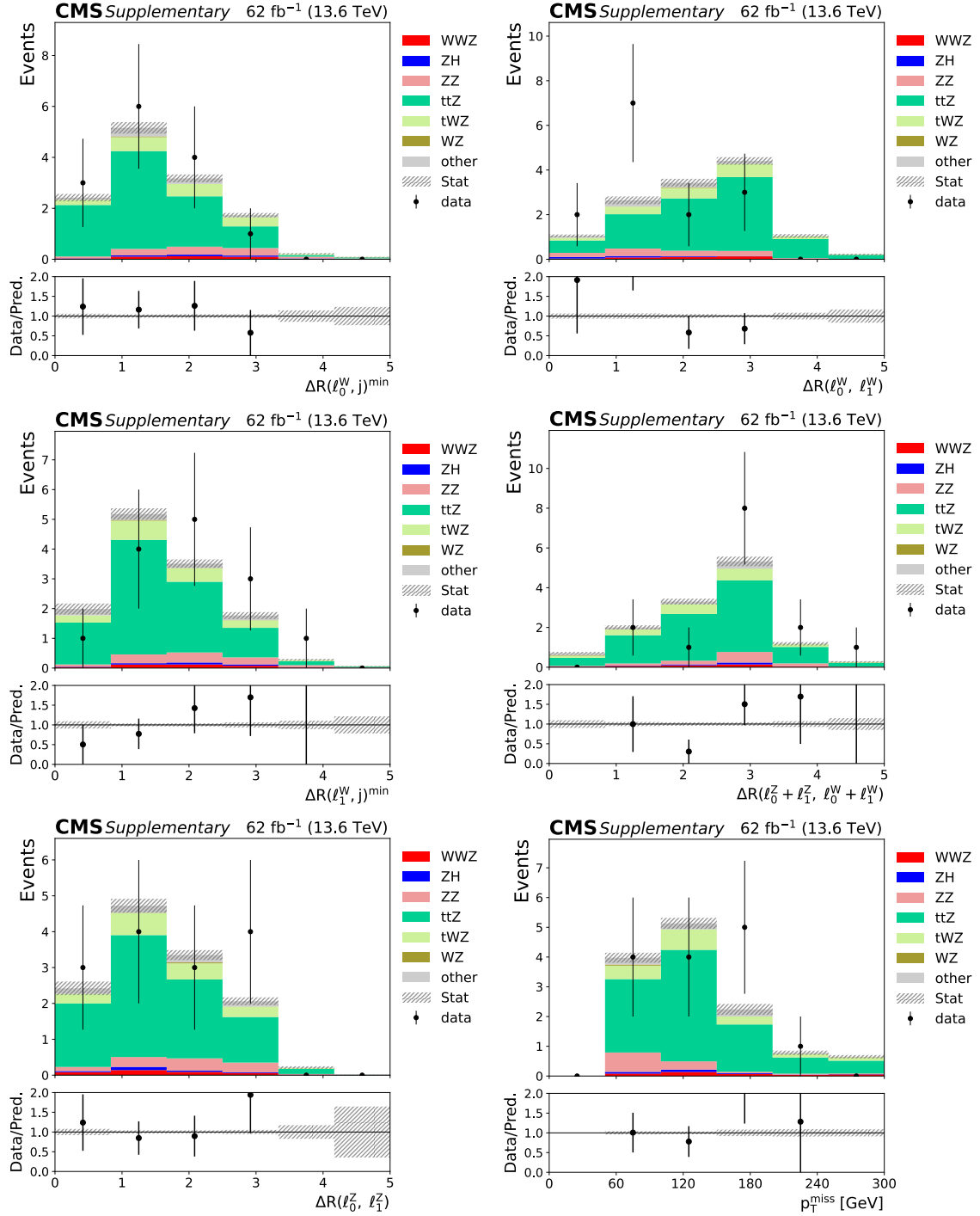

 Figure E.22: BDT input variables shown in the tXZ SF control region (Run 2).

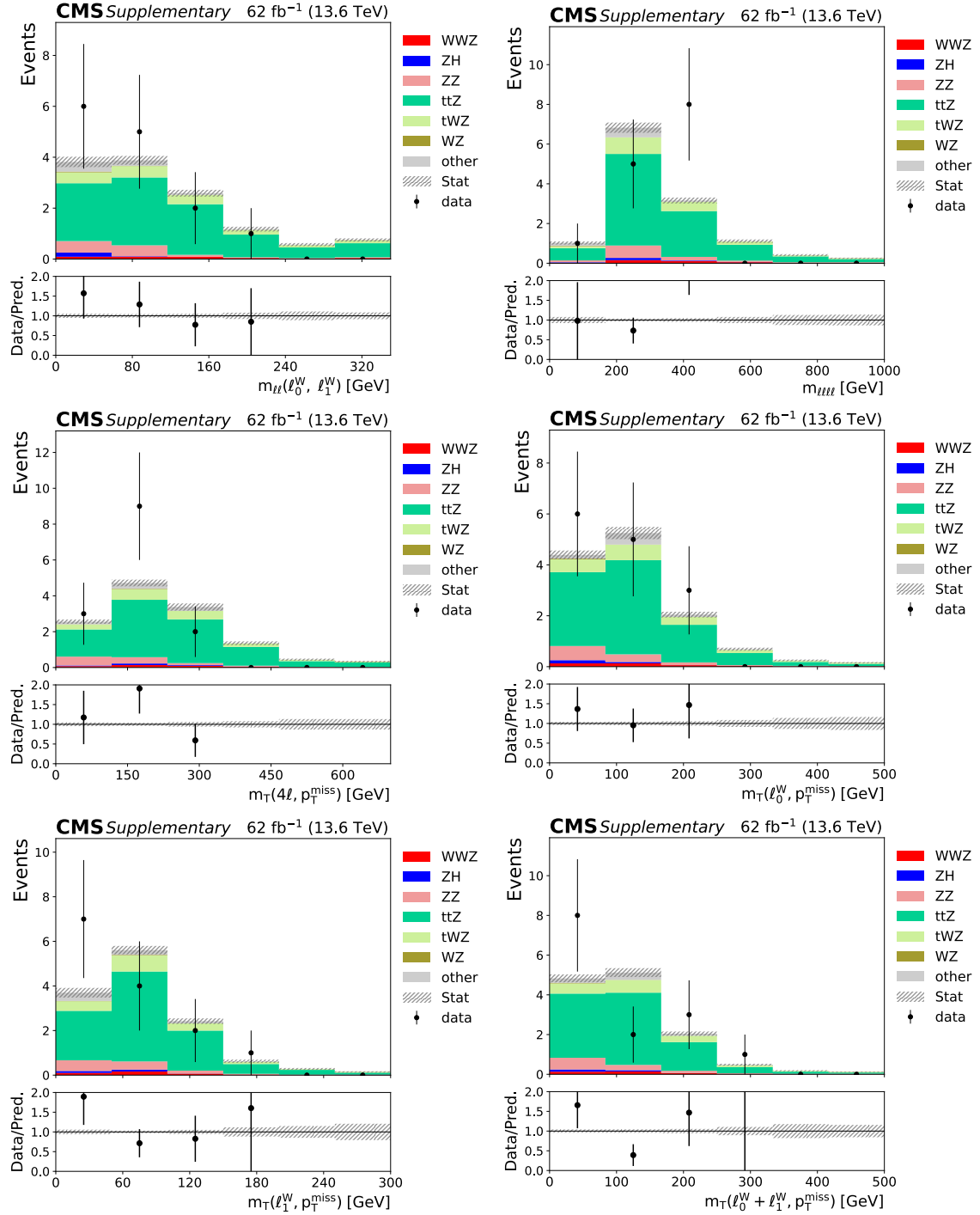

 Figure E.23: BDT input variables shown in the tXZ SF control region (Run 2).

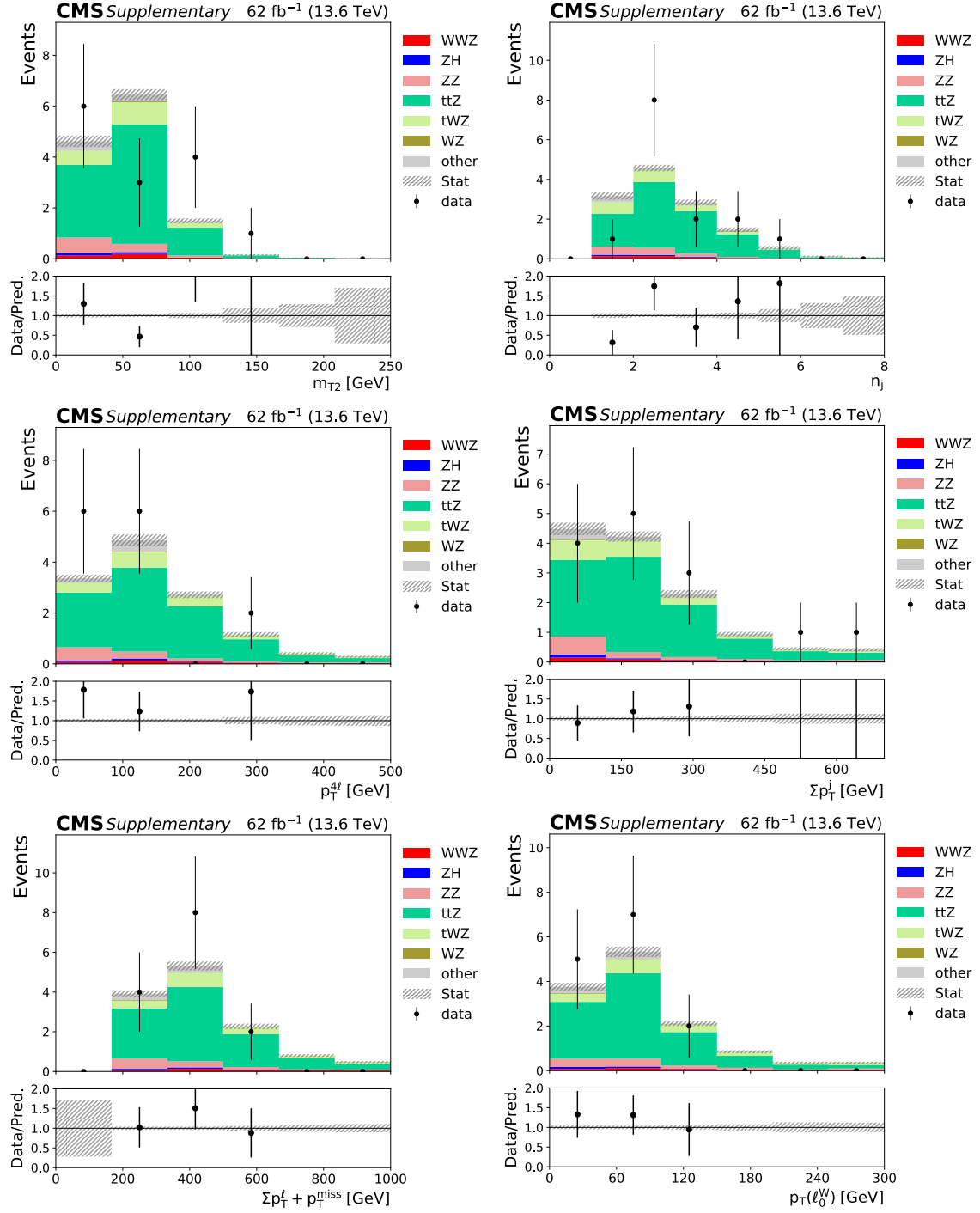
Figure E.24: BDT input variables shown in the tXZ SF control region (Run 2).

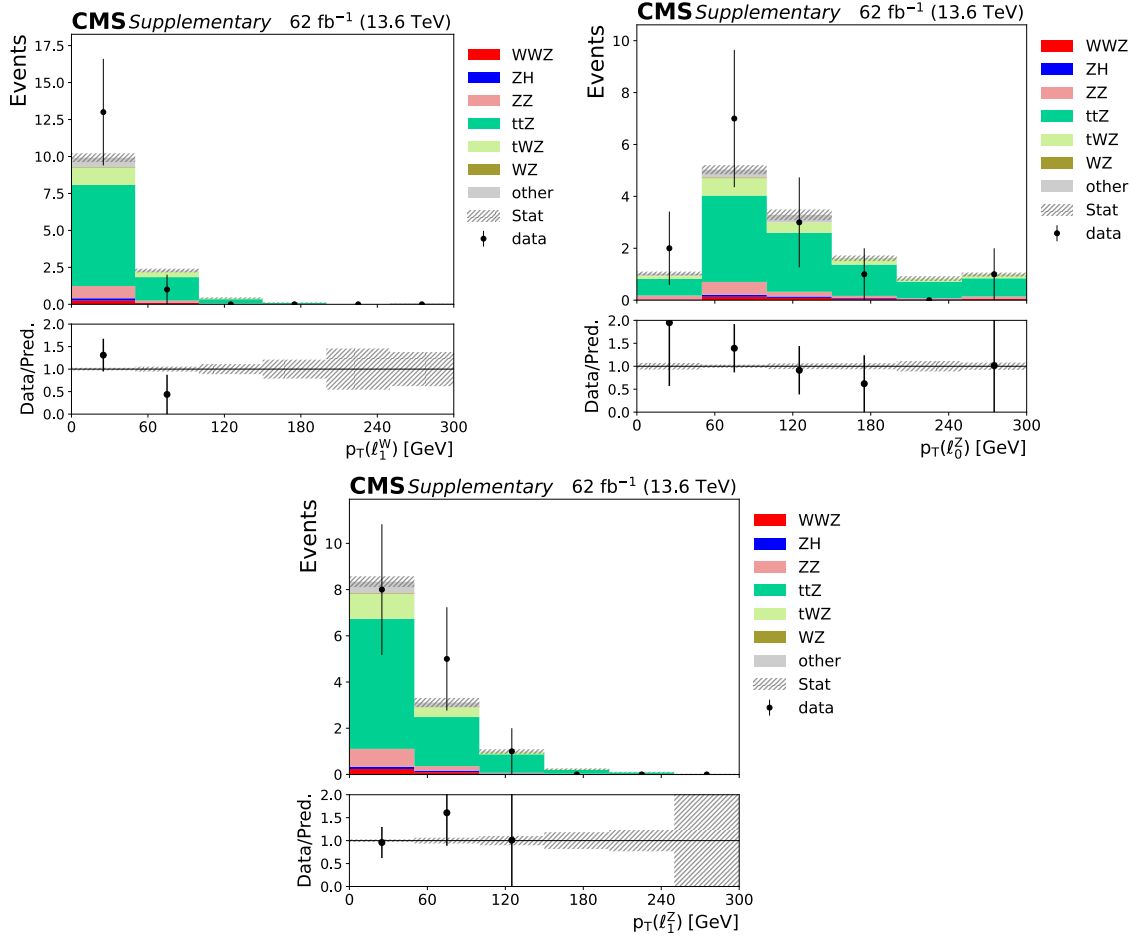

 Figure E.25: BDT input variables shown in the tXZ SF control region (Run 2).


 Figure E.26: BDT input variables shown in the tXZ SF control region (Run 3).

Figure E.27: BDT input variables shown in the tXZ SF control region (Run 3).


 Figure E.28: BDT input variables shown in the tXZ SF control region (Run 3).


 Figure E.29: BDT input variables shown in the tXZ SF control region (Run 3).


 Figure E.30: BDT input variables shown in the tXZ SF control region (Run 3).

APPENDIX F
NUISANCE PARAMETER IMPACTS

F.1 Nuisance Parameter Impacts

The impacts of the nuisance parameters described in Section 9.5 on the measured signal strengths are presented in this section. We first evaluate the impacts of the nuisances on the "Asimov" dataset, where the observation is fixed to the MC prediction in the SRs and taken from real data in the CRs; this is done to validate the fitting procedure. These "Asimov" impacts for the systematics are shown in Section F.1.1. The nuisance parameter impacts on the signal strengths are then determined for the observed dataset collected during the Run 2 and Run 3 periods of CMS data-taking. These impacts are presented in Section F.1.2.

F.1.1 Asimov Impacts

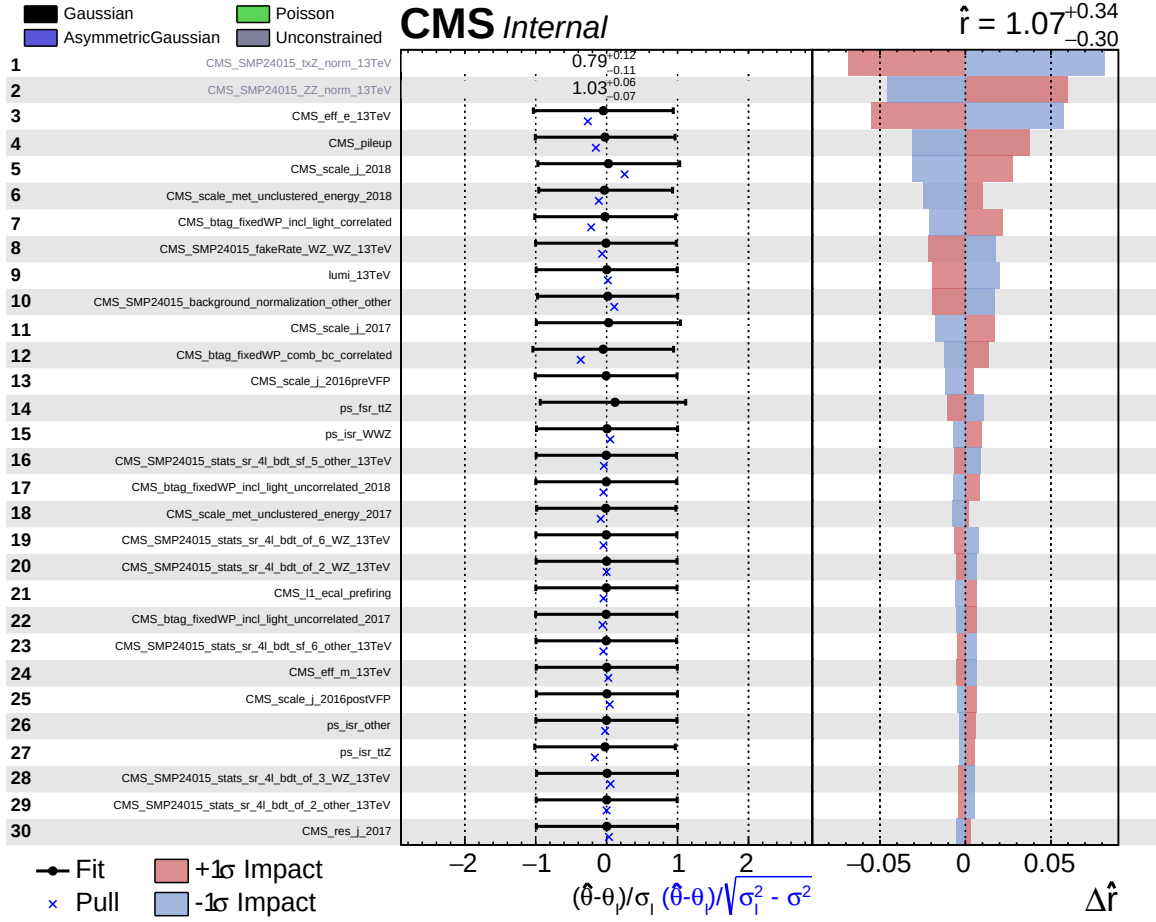


Figure F.1: Nuisance parameter impacts for the inclusive ($WWZ + ZH$) signal strength (Run 2)

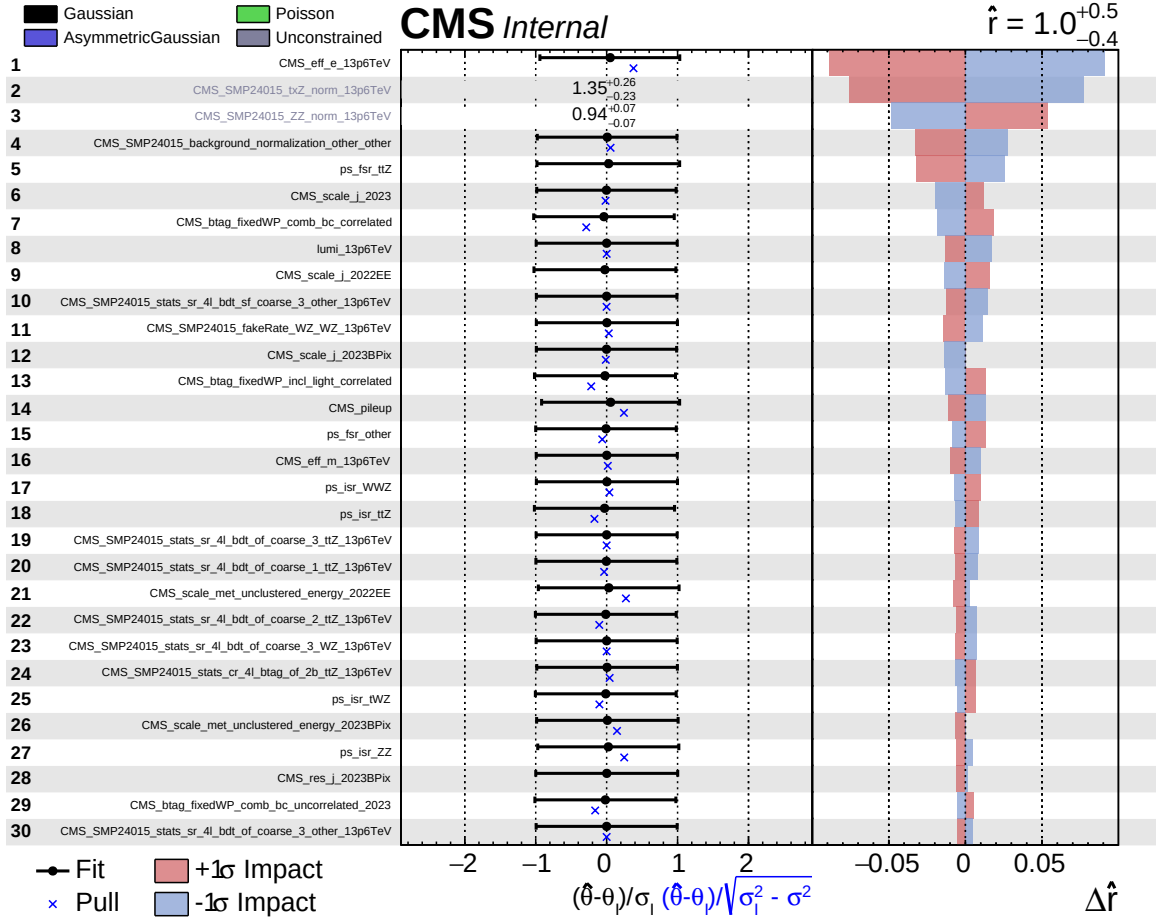


Figure F.2: Nuisance parameter impacts for the inclusive ($WWZ + ZH$) signal strength (Run 3)

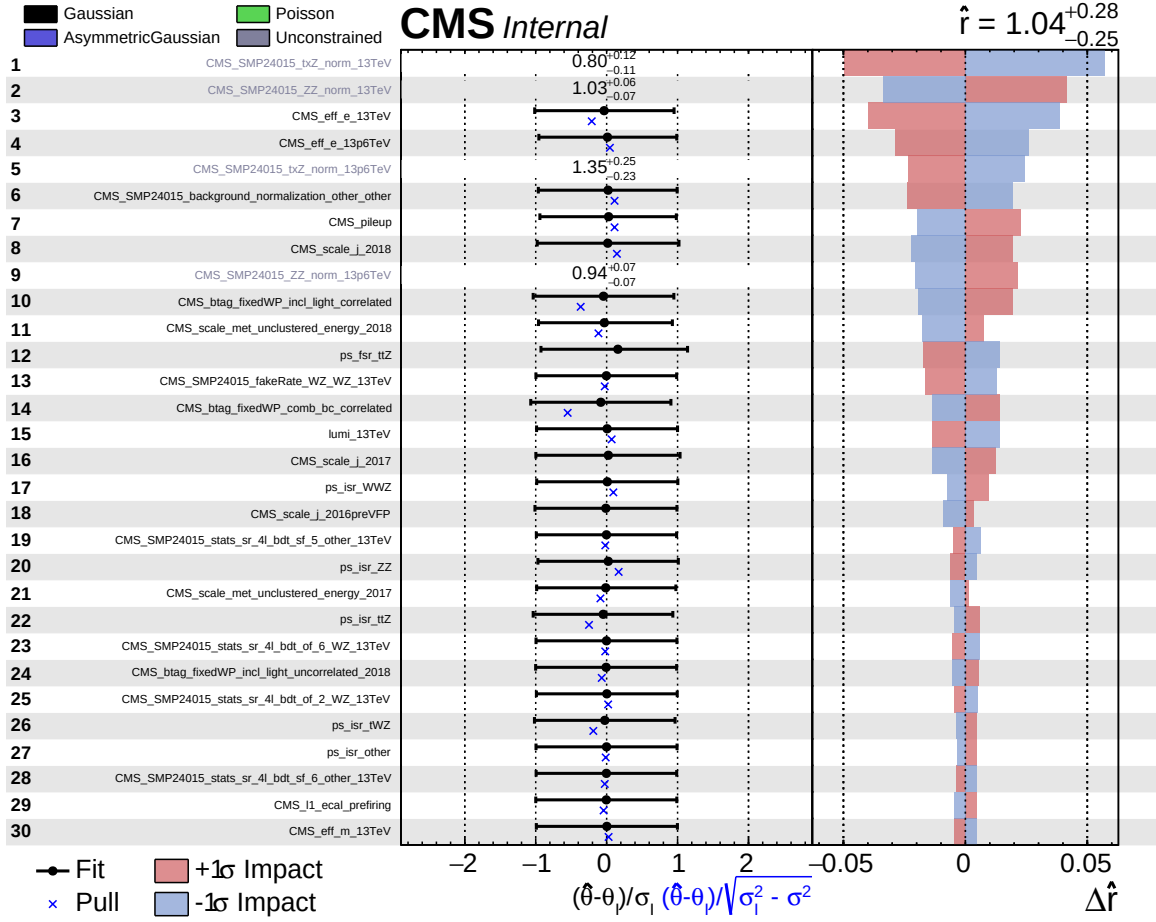


Figure F.3: Nuisance parameter impacts for the inclusive ($WWZ + ZH$) signal strength (Combined)

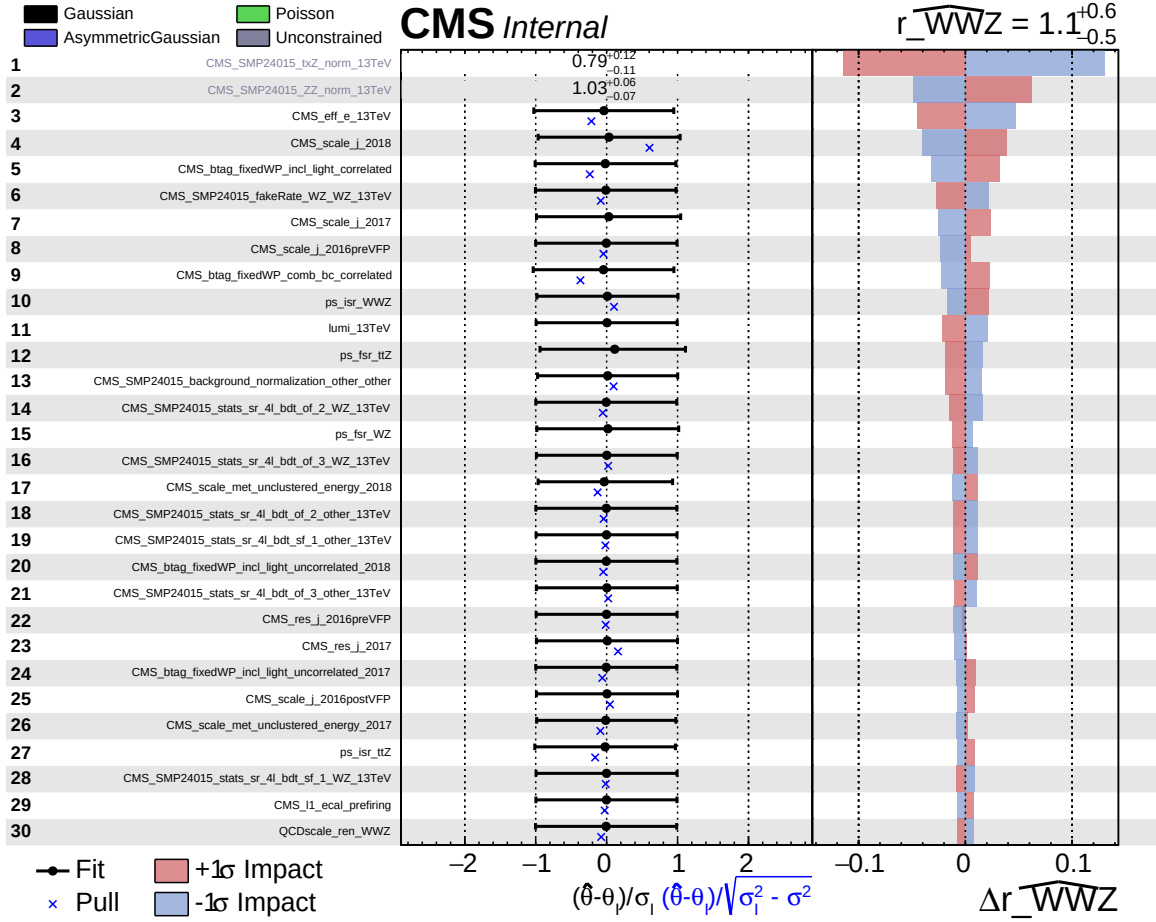


Figure F.4: Nuisance parameter impacts for the non-resonant WWZ signal strength (Run 2)

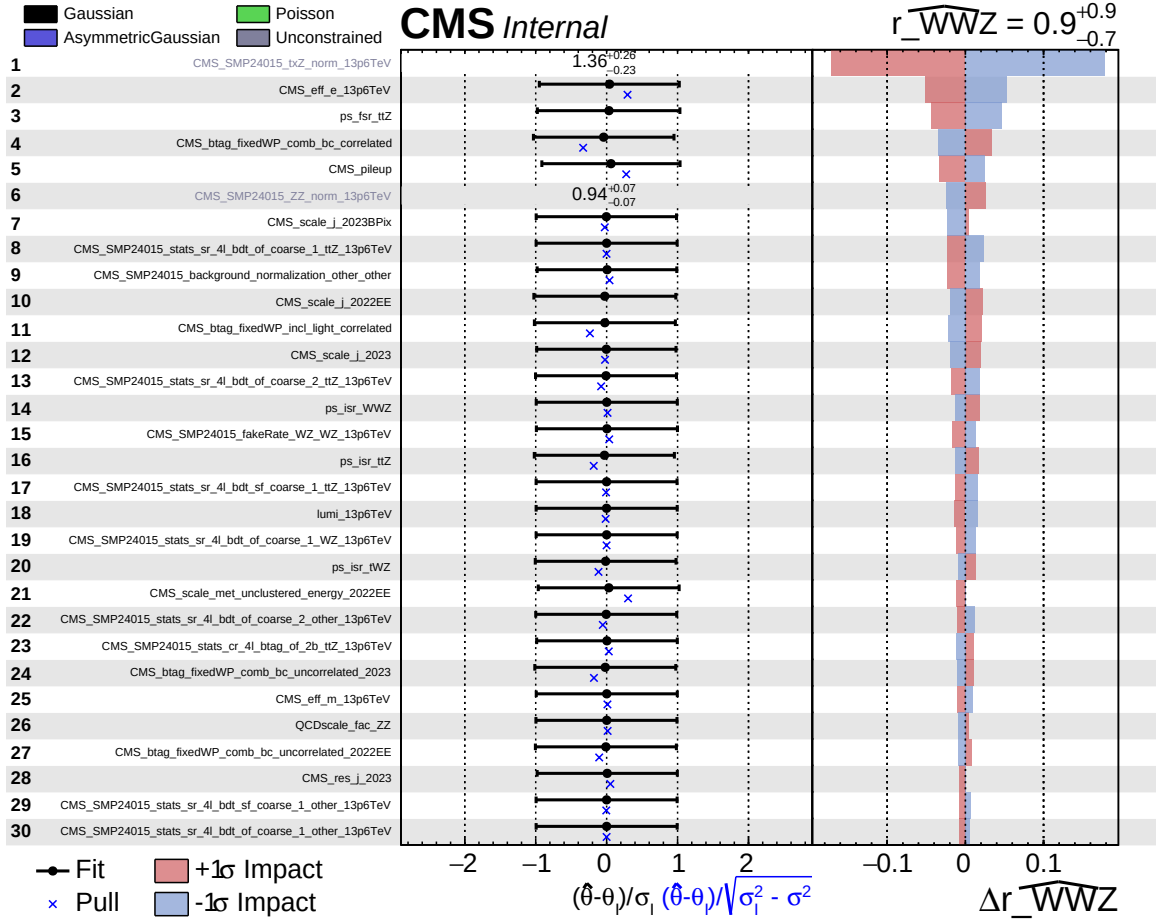


Figure F.5: Nuisance parameter impacts for the non-resonant WWZ signal strength (Run 3)

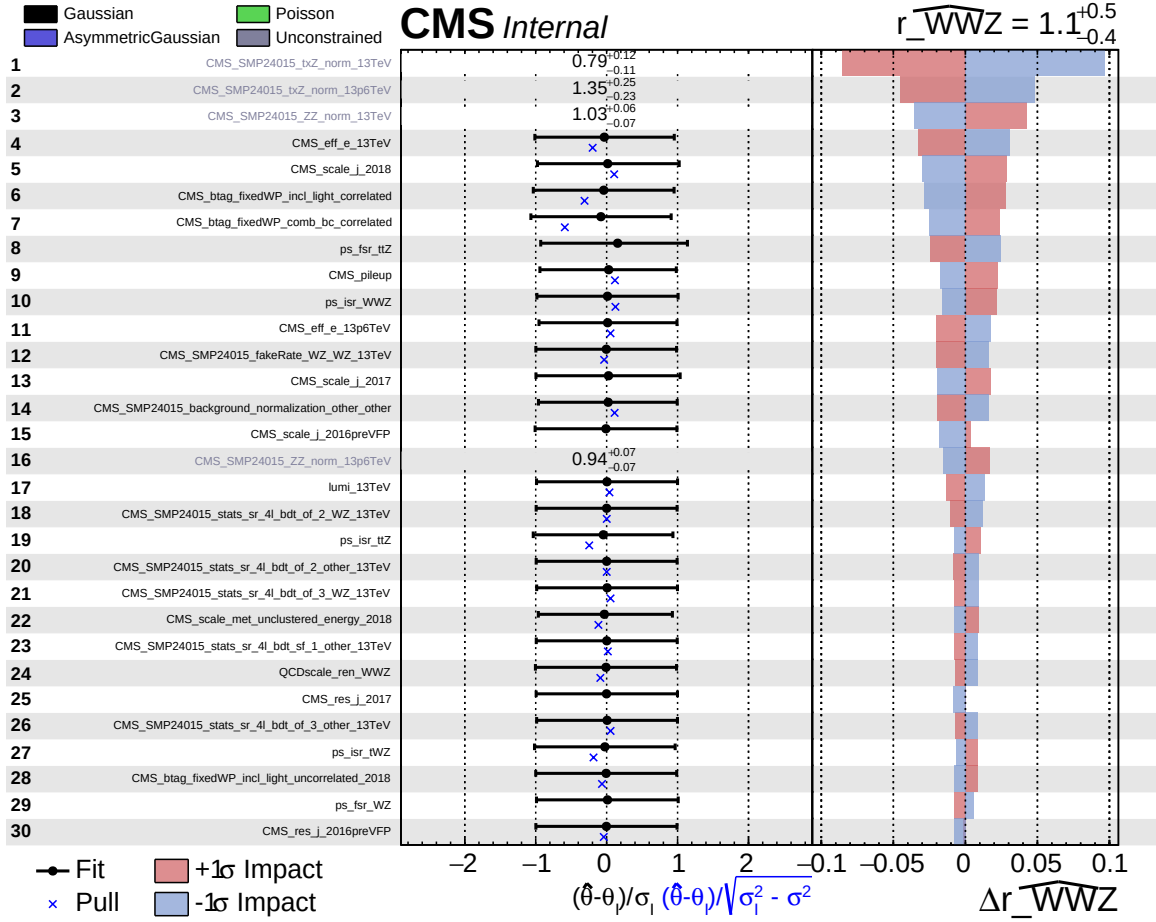
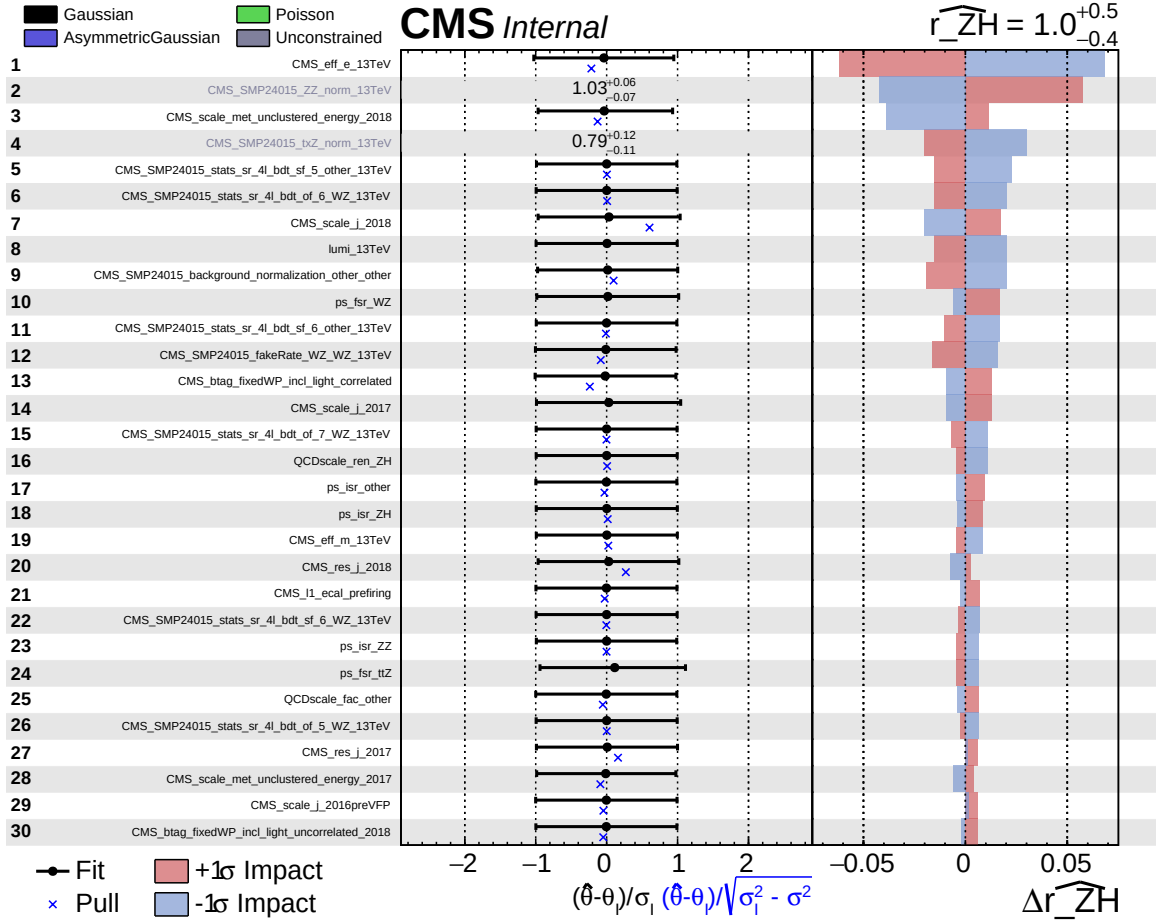
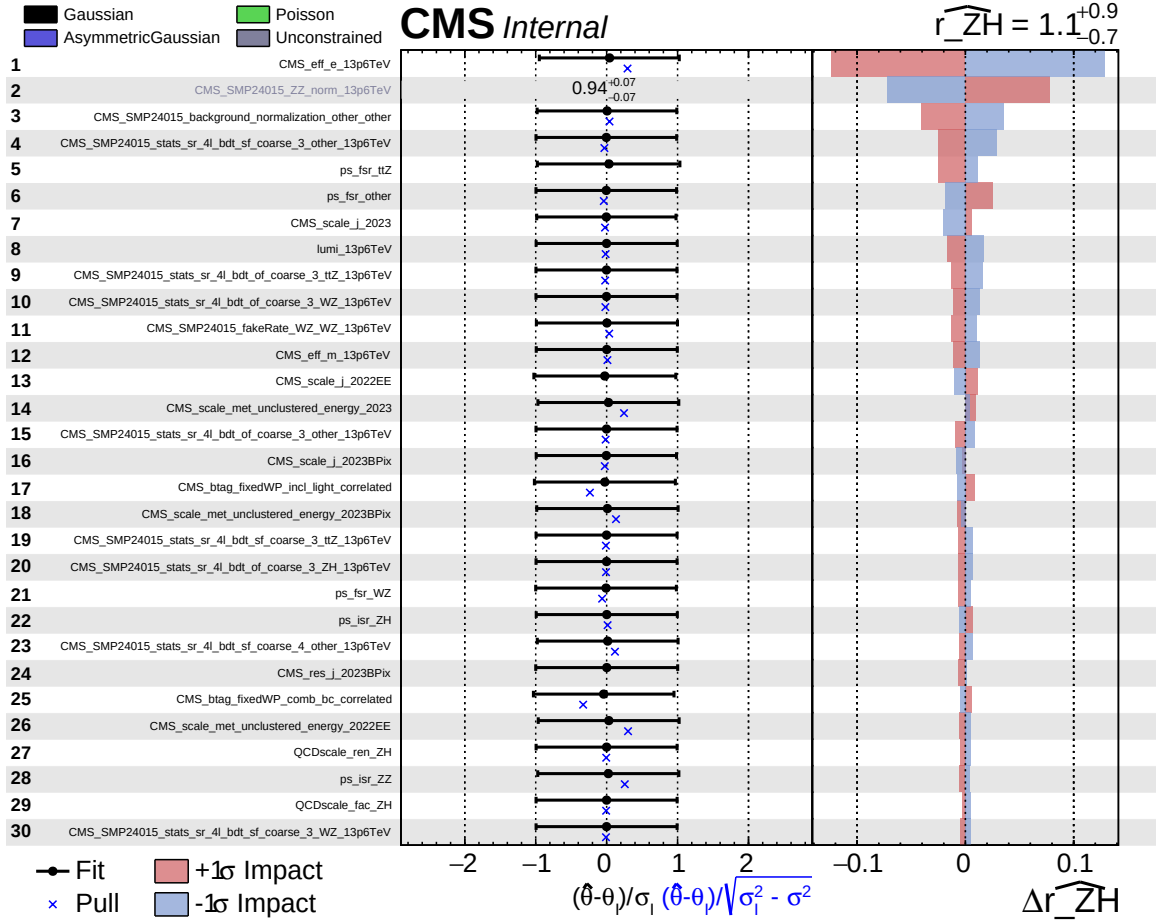


Figure F.6: Nuisance parameter impacts for the non-resonant WWZ signal strength (Combined)


 Figure F.7: Nuisance parameter impacts for the ZH signal strength (Run 2)


 Figure F.8: Nuisance parameter impacts for the ZH signal strength (Run 3)

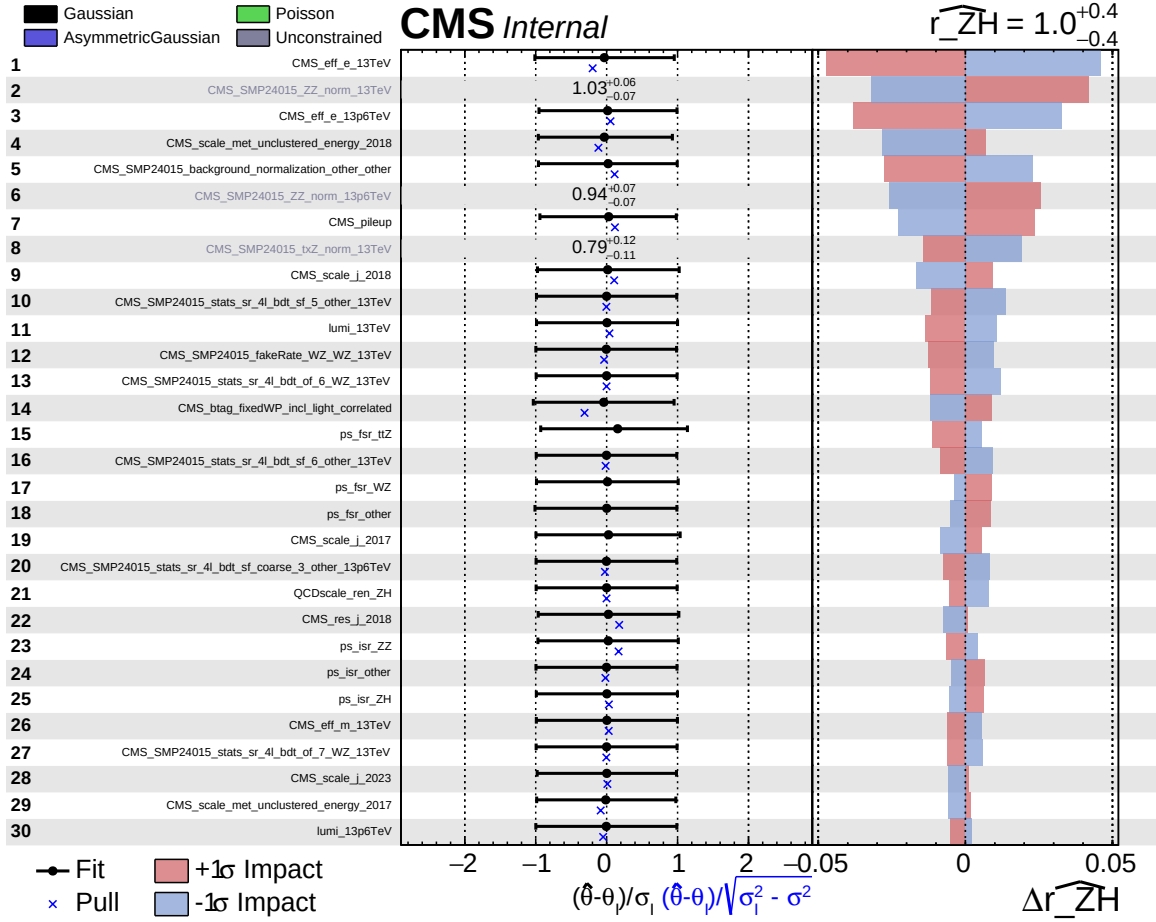


Figure F.9: Nuisance parameter impacts for the non-resonant ZH signal strength (Combined)

F.1.2 Unblinded Impacts

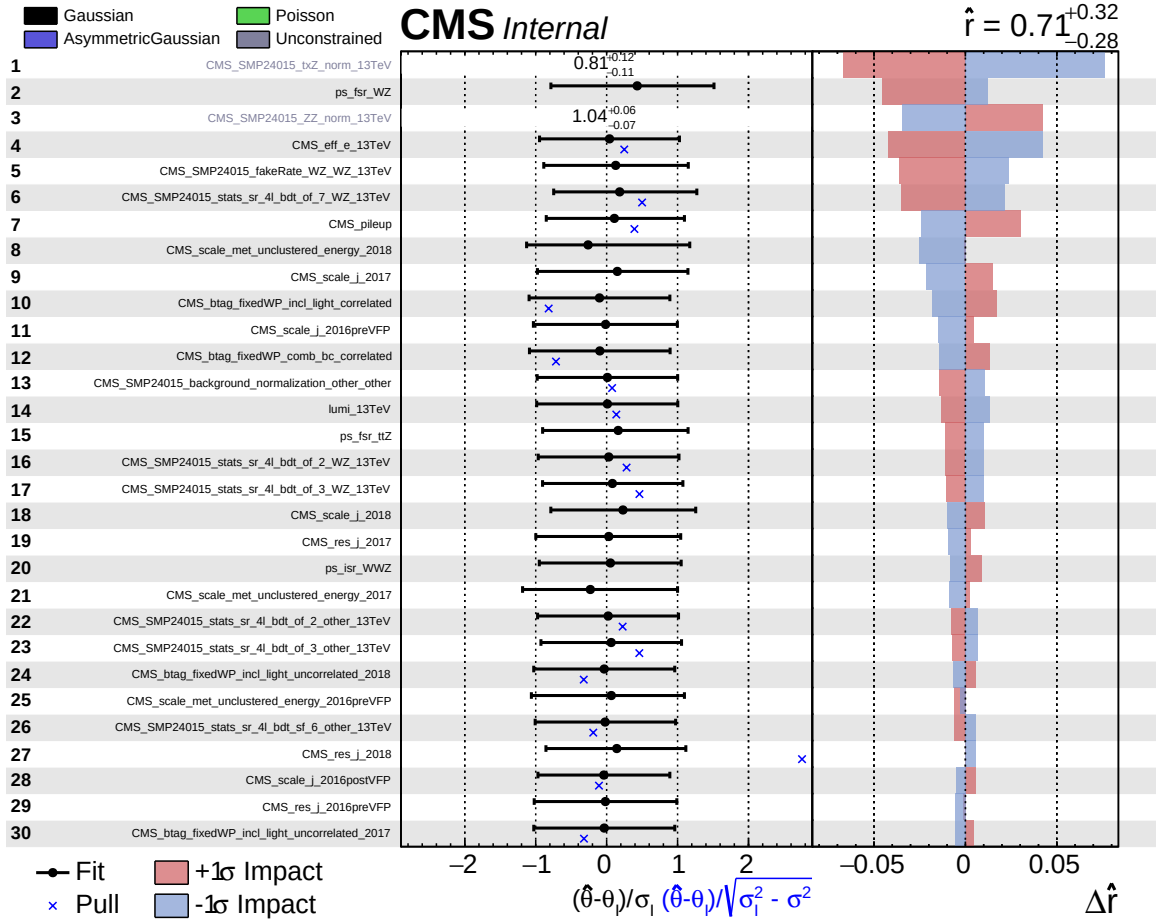


Figure F.10: Unblinded nuisance parameter impacts for the inclusive ($WWZ + ZH$) signal strength (Run 2)

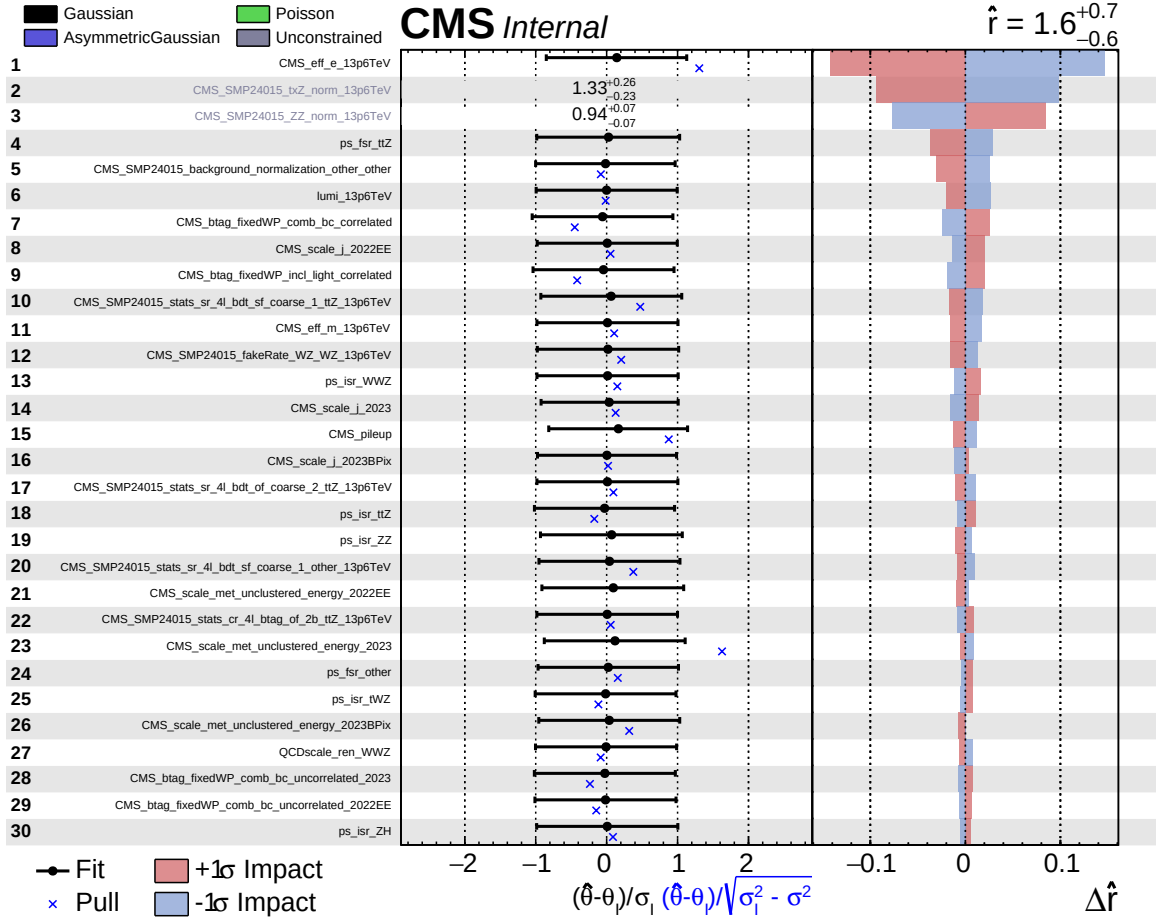


Figure F.11: Unblinded nuisance parameter impacts for the inclusive ($WWZ + ZH$) signal strength (Run 3)

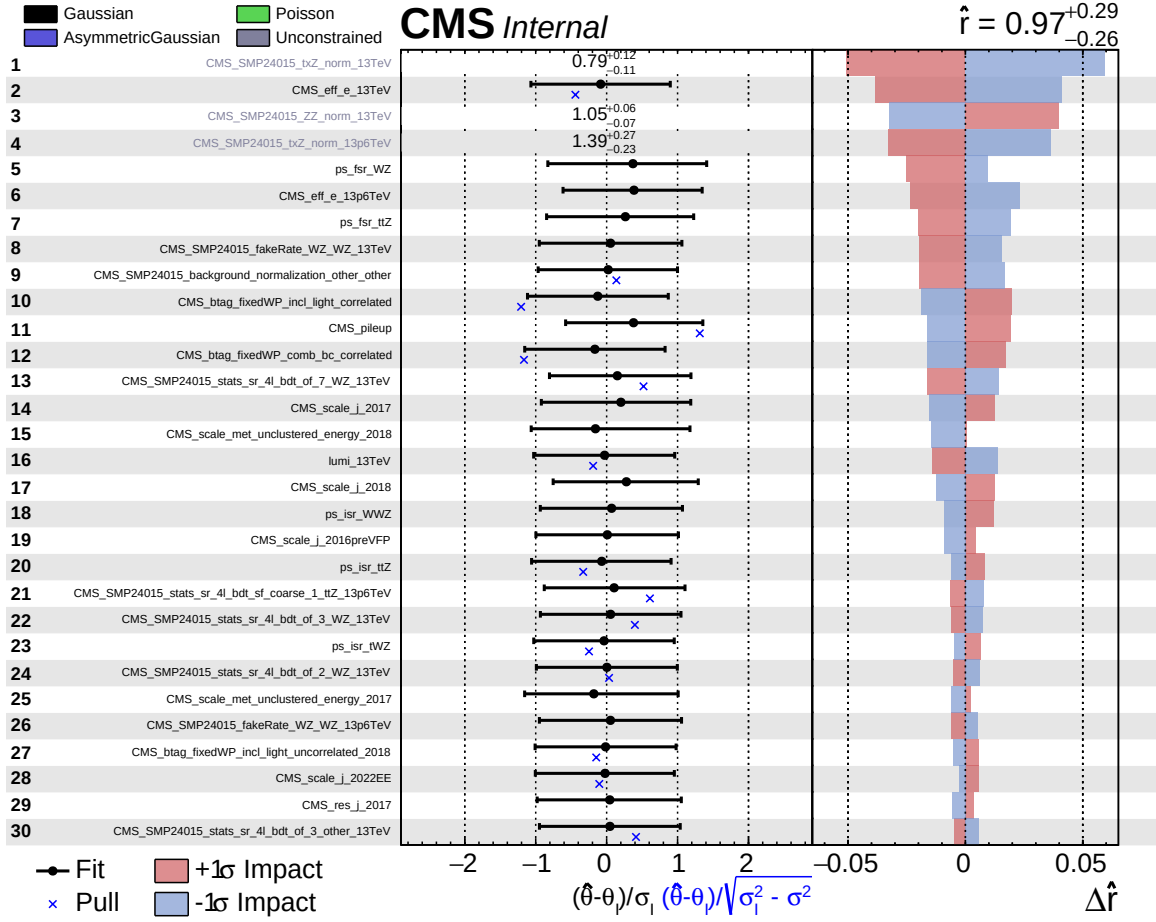


Figure F.12: Unblinded nuisance parameter impacts for the inclusive ($WWZ + ZH$) signal strength (Combined)

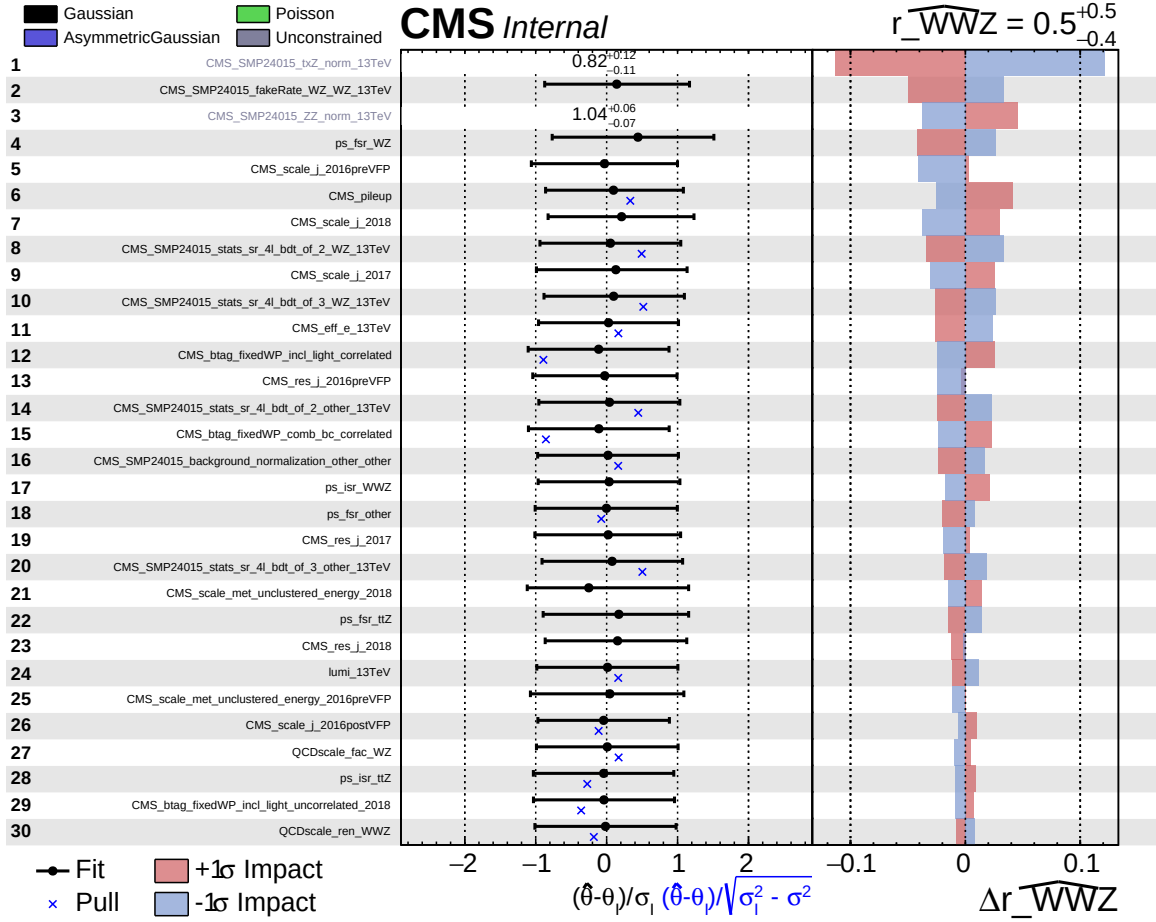


Figure F.13: Unblinded nuisance parameter impacts for the non-resonant WWZ signal strength (Run 2)

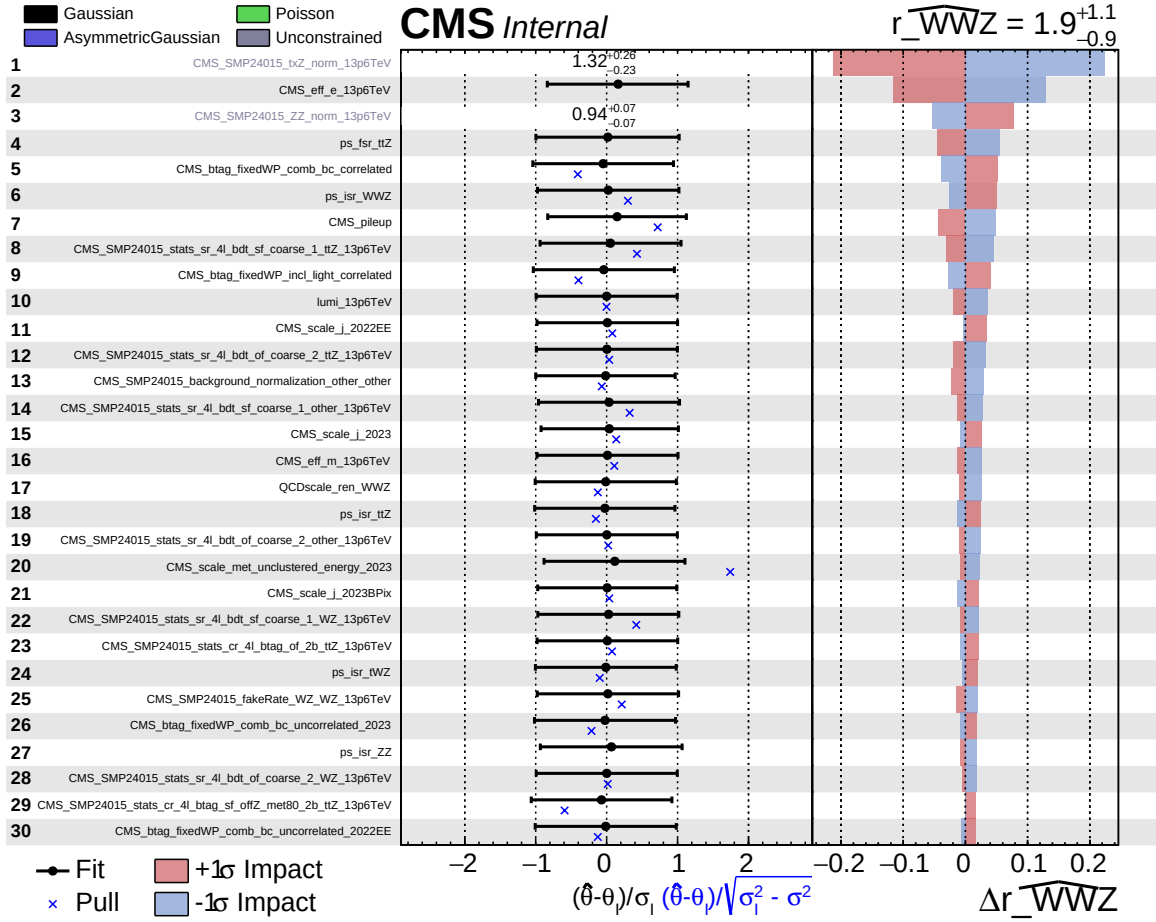


Figure F.14: Unblinded nuisance parameter impacts for the non-resonant WWZ signal strength (Run 3)

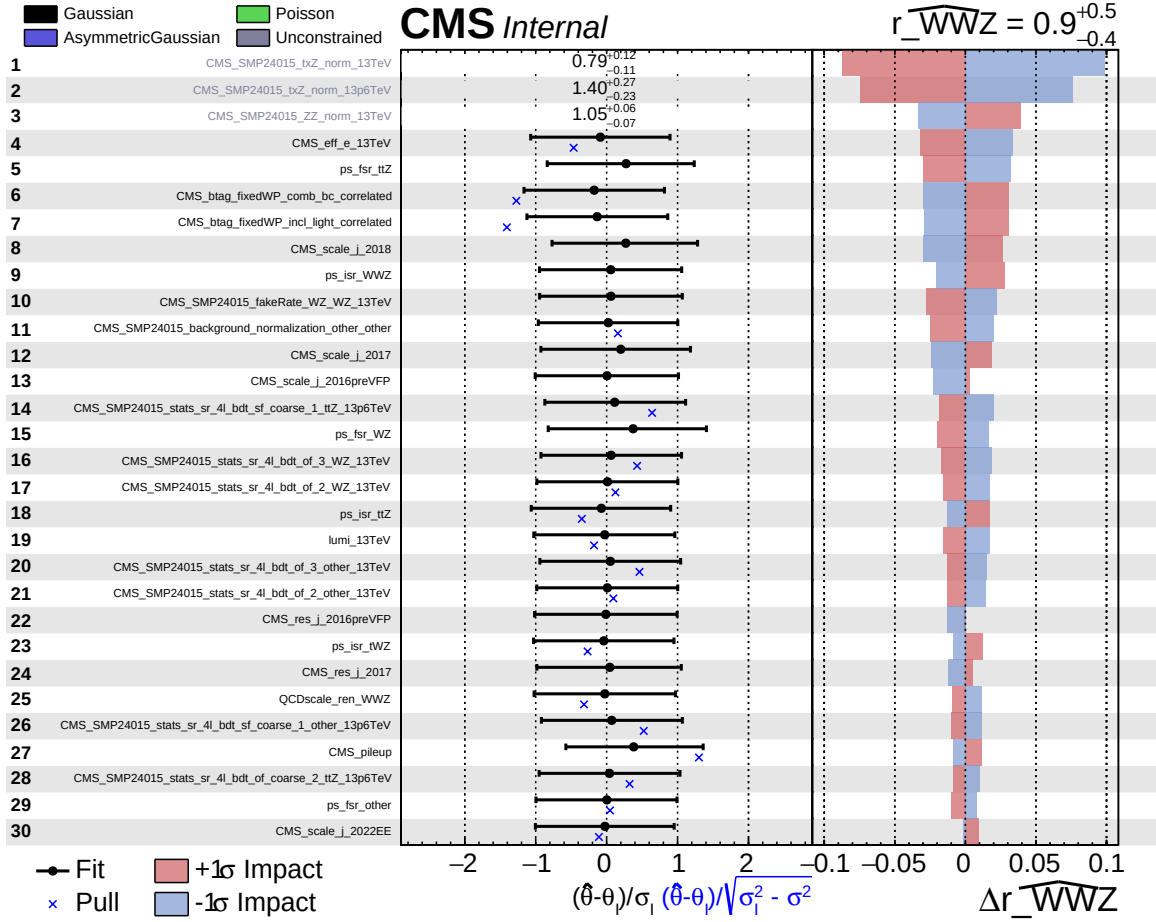


Figure F.15: Unblinded nuisance parameter impacts for the non-resonant $W W Z$ signal strength (Combined)

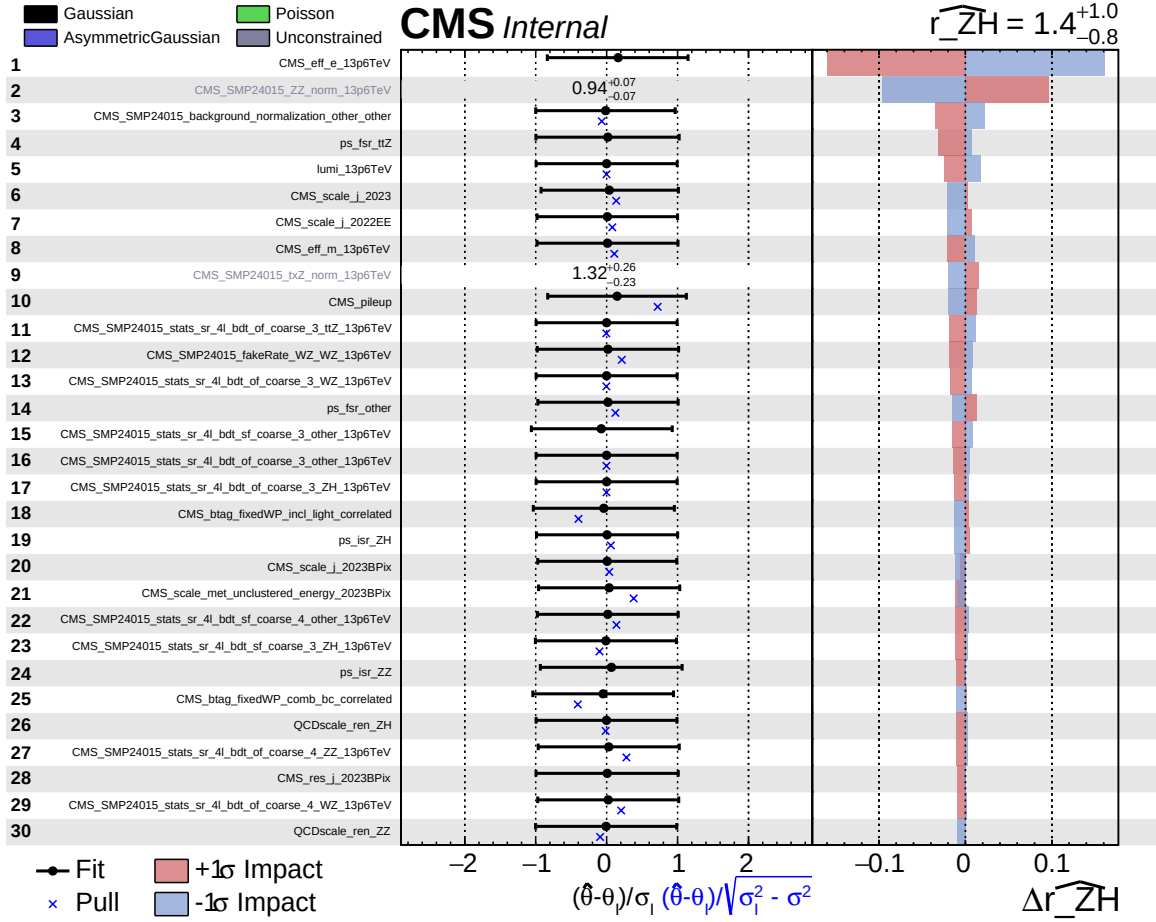


Figure F.17: Unblinded nuisance parameter impacts for the ZH signal strength (Run 3)

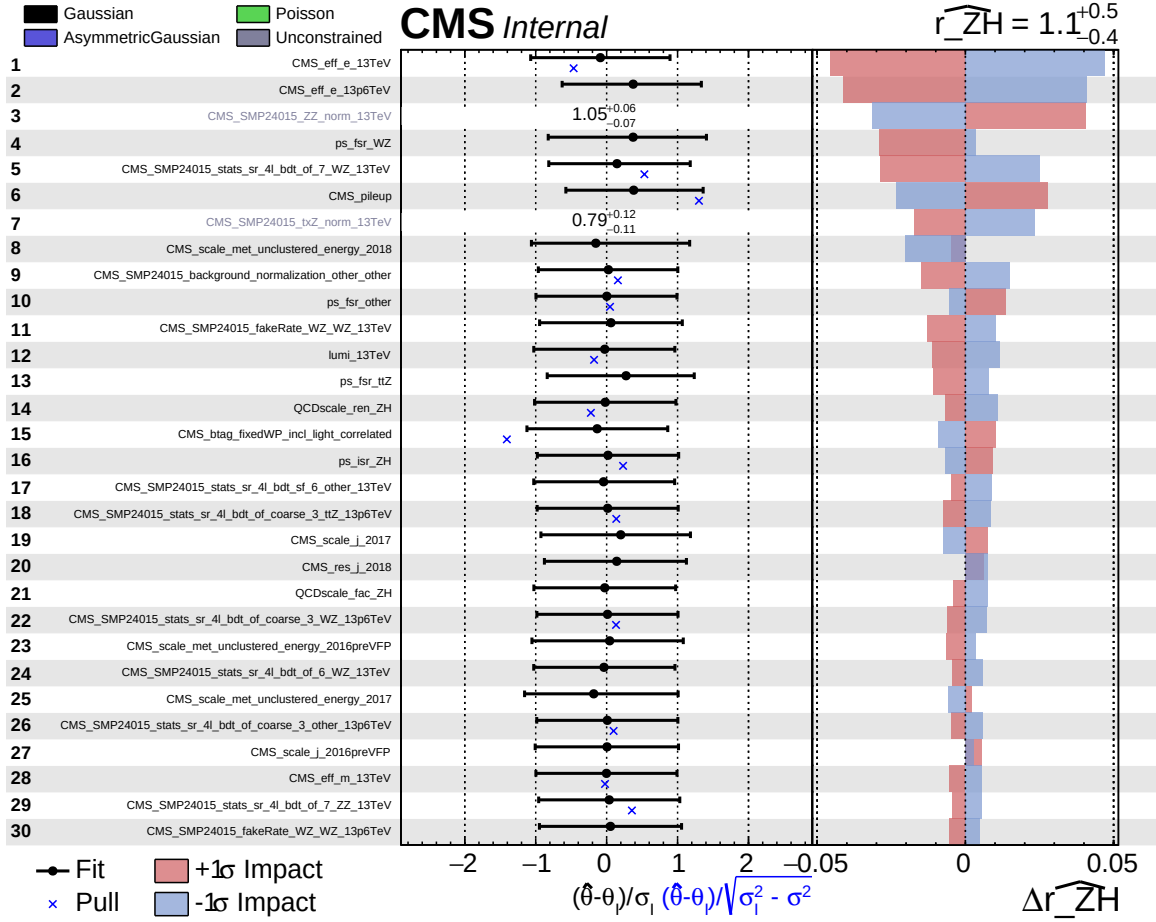


Figure F.18: Unblinded nuisance parameter impacts for the non-resonant ZH signal strength (Combined)

APPENDIX G

ADDITIONAL INFORMATION FOR 10 TEV BDS

G.1 Parameters for GUINEA-PIG

Variable	Definition	Value
n_x	# of cells (horizontal)	110
n_y	# of cells (vertical)	100
n_z	# of cells (longitudinal)	50
n_t	# of timesteps to move bunch slices	1
n_m	# of macroparticles (per beam)	100000
cut_x	size of grid in x [nm]	$3\sigma_x^*$
cut_y	size of grid in y [nm]	$3\sigma_y^*$
cut_z	size of grid in z [μm]	$3\sigma_z$
electron_ratio	fraction of electrons used in lumi spectrum calculation	0.2
photon_ratio	fraction of photons used in background calculations	0.2

Table G.1: GUINEA-PIG parameters used in the luminosity calculation. Parameters not shown take default values.

G.2 Impact of SR from different sources

	σ_x^* [nm]	σ_y^* [nm]	$\mathcal{L}$ [$10^{34} \text{ cm}^{-2} \text{ s}^{-1}$]
Without SR	32.40	2.31	19.2
Only Dipole SR	36.06	2.40	12.9
Only Quadrupole SR	40.67	4.33	14.8
Only Multipole SR	32.40	2.31	19.1
Dipole & Quadrupole SR	42.60	4.23	10.8
Dipole & Multipole SR	36.06	2.40	13.0
Quadrupole & Multipole SR	40.10	4.16	14.6
All SR	43.02	4.21	10.8

Table G.2: IP Spot Sizes with and without SR effects included in different magnet classes, calculated from PLACET, and luminosity calculated using GUINEA-PIG.

APPENDIX H

GLOSSARY

- **Asimov data:** Named after the famous science fiction writer Isaac Asimov, the Asimov dataset is a dataset where the observable quantities are fixed to their expected values.
- **Center of Mass Energy ($\sqrt{s}$):** A Lorentz-invariant quantity dictating the amount of energy available for producing new particles in a collision
- **Factorization Scale (μ_f):** Defines the minimum energy scale (Infrared Cutoff) that is considered in the QCD calculation.
- **Hermetic:** covering the full solid angle (4π radians) around the interaction point.
- **Moliere Radius:** the size (transverse to the incident particle) of a region in a given material that contains 90% of the electromagnetic shower from the incident particle.
- **Nuclear Interaction Length (λ_I):** the mean distance over which a hadron travels prior to interacting with a nucleus via inelastic scattering.
- **Radiation Length (X_0):** the mean length, for a given material, that an electron travels to radiate away $1/e$ of its total energy.

- **Renormalization Scale** (μ_r): Energy scale at which the calculation of the strong coupling constant is performed.

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