PARTY POLITICS VOL 11. No.3 pp. 283–298

CONSERVATION OF BALANCE IN THE SIZE OF PARTIES

Rein Taagepera

ABSTRACT

For parties of unequal seat shares (s_i), the widely used effective number of parties ($N = 1/\Sigma s_i^2$) offers an equivalent in equal-sized parties, but it needs a supplement to express the imbalance in actual shares. This is akin to supplementing the mean with the standard deviation. A suitable 'index of balance' is $b = -log\ s_1/log\ p$, where s_1 is the largest share and p is the number of seat-winning parties. It can range from 0 (utter imbalance) to 1 (perfect equality of all parties). In most individual countries, the median balance is between 0.4 and 0.6, and the worldwide median balance is close to 0.50 for any number of seat-winning parties except 2, in line with a simple logical model. Independent of electoral systems used, a rule of conservation emerges: the median product of the largest party's fractional share and the square root of the number of seat-winning parties is conserved: $s_1 p^{0.5} = 1$. The worldwide median for 603 elections is within 2 percent of 1.00.

KEY WORDS ▪ largest seat share ▪ laws of conservation ▪ logical quantitative models ▪ number of parties

When changes in electoral rules or other conditions enable more parties to win seats in a representative assembly, the seat share of the largest party tends to go down. Is there some characteristic of the party system that tends to stay constant in the process? Is something conserved?

The concept of a conserved quantity is important in many areas of science. Quantities such as energy, momentum, electric charge and (under certain conditions) mass are conserved when a closed system undergoes changes. It is worthwhile asking whether any quantities tend to be conserved in the course of political processes. Absolute in macroscopic physics, the conservation principles become probabilistic at quantum level. In social relations, a stochastic element can be expected, so that conservation could be expected to apply only to the median outcomes.

This study tests a conservation relation for party systems that connects

1354-0688[DOI: 10.1177/1354068805051778]

the seat share of the largest party to the number of parties winning seats in national assemblies. The quantity the median of which is conserved could be called the *balance* among the seat-winning parties. The index of balance thus defined could be a useful supplement to the effective number of parties,[1] roughly in the same way as the standard deviation of a normal distribution supplements the mean value.

Intuitively, we can expect an inverse relationship between the number of seat-winning parties and the largest share.[2] But we should try to be more precise than 'Number of parties up, largest share down'. Empirically, we can find the median largest share for a given number of seat-winning parties, using a large number of elections worldwide. Whatever the pattern obtained, it would lead to the next question: why this pattern and not a different one? A simple logical quantitative model[3] (Taagepera, 1999a; Taagepera and Shugart, 1993) suggests the following relationship, which applies much more broadly than just for seats in assemblies.

For any constellation of p well-defined components that add up to a well-defined total, the median fractional share of the largest component (s_1) can be expected to be the inverse of the square root of p:

$$s_1 = 1/p^{0.5}.$$

In the absence of any other information, this is the 'expectation value' in the sense that one would expect this value to be the median around which the actual values are spread. In the form shown above, this equation seems to suggest that the largest share depends on the number of components rather than vice versa, but it can be put in a more neutral form that expresses a conservation principle:

$$s_1 p^{0.5} = 1.$$

This form says that *the product of the largest component's share and the square root of the number of components is constant*. If an external factor (such as a party splitting up) should alter either s_1 or p, the other variable would be under pressure to change so as to conserve the product.

Populations as well as areas of the largest federal subunits in the US, Canada and Australia are predicted by the equation within 20 percent (Taagepera, 1999a). Party-based elections, however, offer a test with a much larger number of cases. The present study tests this conservation principle in the special case of national assembly elections. If the relationship holds, it would be of interest for the following reasons.

First, the relationship tested here is an essential link for specifying the institutional determinants of the seat share of the largest party, which itself is a crucial factor for the type of cabinet formed. The number of seat-winning parties tends to increase with increasing assembly size (S) and with increasing district magnitude (M), if all seats are allocated in districts. According to Taagepera and Shugart (1993), one should expect that

$p = (MS)^{1/4}$. If so, then confirming a specific average relationship between the number of parties and the share of the largest could enable us to estimate the institutional determinants of the largest share.

Second, this study leads to a measure of balance in party sizes, ranging from 0 for utter imbalance to 1 for perfect balance of all parties in the assembly. Consider the following two constellations where five and only five parties gain representation in a 100-seat assembly: 52–18–10–10–10 and 38–38–21–2–1. The effective number of parties is 3.00 in both cases, yet the first represents hegemony, while the second has a more balanced distribution among the top contenders. The index of balance that will be developed ($b = -\log s_1/\log p$) is 0.41 for the former and 0.60 for the latter, adding information not contained in the effective number of parties. The relationship $s_1p^{0.5} = 1$ corresponds to a balance of 0.5, halfway between 0 and 1. If this is where most constellations are, the deviation from $b = 0.5$ would be a measure of how unusual a constellation is – and we should strive to tell the unusual apart from the usual.

Third, it will be seen that the model holds for most numbers of seat-winning parties but not at the two extremes of $p = 2$ and $p > 12$. Such deviations from the simple model may help us gain insights into the changing nature of political processes when the number of parties is very small or very large.

Fourth, political data are used here to test a broader conservation principle, which is expected to apply, on the average, whenever a well-defined total is randomly divided into a well-defined number of components.[4] Compared to sociology or economics, political science tends to be a receiver rather than a donor discipline. This is an opportunity to be a donor.

This study first tests the relationship between the largest party's share and the number of parties with worldwide data. The observed median pattern is found to fit the model, yet with systematic deviations at the extremes. Next, the derivation of the simple model is presented in a way somewhat different from Taagepera (1999a), leading to the notion of balance in party size and a refinement of the model that accounts for the product $s_1p^{0.5}$ falling below 1 at $p = 2$. The index of balance is then applied to different electoral rules and to individual country patterns.

Testing the Conservation Model

The dataset used starts out with all 753 electoral outcomes listed in Mackie and Rose (1991 and 1997), covering 25 countries.[5] Electoral coalitions sometimes introduce ambiguity about the share of the largest party. In such cases I have accepted the judgment of Mackie and Rose, so that the seat share of the largest party (s_1) can simply be read off.

The number of seat-winning parties (p) offers more difficulty. A residual 'Others' category in Mackie and Rose (1991, 1997) sometimes lumps fleeting minor parties and independents. Four seats in the 'Others' category

Table 1. The number of elections in the given range of the largest seat share (s_1) by the number of seat-winning parties (p)

	No. of seat-winning parties (p)										
Range of s_1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10–12*	*13–16*	*Sum*
Up from											
0.15[a]	–	–	–	–	1	0	0	0	1	1	3
0.20	–	–	–	–	0	1	1	0	8	5	15
0.25	–	–	–	–	4	6	6	10	14	8	48
0.30	–	2	2	3	9	5	6	6	12	3	48
0.35	–	4	10	17	13	9	4	5	8	7	77
0.40	–	7	18	17	20	8	5	3	7	1	86
0.45	–	18	14	20	9	8	3	0	4	2	78
0.50	23	24	16	8	6	1	3	0	1	2	84
0.55	28	21	10	11	2	1	2	1	1	–	77
0.60	18	9	2	5	3	1	1	–	–	–	39
0.65	13	8	2	0	0	–	–	–	–	–	23
0.70	7	3	2	0	1	–	–	–	–	–	13
0.75	1	2	1	1	–	–	–	–	–	–	5
0.80	0	0	1	1	–	–	–	–	–	–	2
0.85	0	0	1	–	–	–	–	–	–	–	1
0.90	1	0	–	–	–	–	–	–	–	–	1
0.95	2	1	–	–	–	–	–	–	–	–	3
Sum	93	99	79	83	68	40	31	25	56	29	603
Median	0.593	0.536	0.495	0.460	0.410	0.396	0.363	0.317	0.314	0.327	
$s_1 p^{0.5}$	0.84	0.93	0.99	1.03	1.00	1.05	1.03	0.95	1.04	1.25	0.985[b]
Bal.[c]	0.754	0.568	0.507	0.482	0.498	0.476	0.487	0.523	0.483	0.418	0.542[d]

[a] Ranges 0.150 to 0.199, 0.200 to 0.249, etc.
[b] 1.015 when excluding $p = 2$, 3 and 13–16.
[c] Index of balance $b = -\log s_1 / \log p$ for median s_1.
[d] 0.493 when excluding $p = 2$, 3 and 13–16.

could mean a single party winning four seats or as many as four separate parties (or independents) winning a seat. To avoid this uncertainty, the present analysis is limited to the 603 clear cases in 24 countries that have at most one seat in the 'Others' category.[6]

Table 1 lists the number of occurrences at a given number of seat-winning parties (p) and with the largest party's share (s_1) in a given range. The median s_1 for a given p is shown. Also shown is the median product $s_1p^{0.5}$, which according to the model should be 1.00. The bottom line lists the index of balance ($b = -log\, s_1/log\, p$) corresponding to the median s_1. This index should be 0.5 according to the model and will be discussed in more detail later.

It can be seen from Table 1 that the median share of the largest party consistently decreases with an increasing number of seat-winning parties, except for p above 12. When more than 9 parties win seats, further reduction in the largest share becomes minimal, and the median s_1 seems to stabilize around 32 percent. The lowest value of the largest share observed is 18.4 percent for Belgium 1991, with 13 parties obtaining seats.[7]

The figures at the bottom of Table 1 show strong agreement with the model for 4 to 12 seat-winning parties, with median $s_1p^{0.5}$ within 5 percent of the expectation value of 1.00. The weighted mean of $s_1p^{0.5}$ in this range (1.015) is within 1.5 percent of the expectation value. Correspondingly, the median index of balance stays in the range $b = 0.50 \pm 0.02$, the weighted mean being 0.493.

The fit is borderline (within 7 percent) for three seat-winning parties. At the two extremes ($p = 2$ and $p = 13$ or larger) discrepancies increase. Even so, the overall weighted mean of $s_1p^{0.5}$ is 0.985 – again within 1.5 percent of the expectation value. However, the overall weighted mean of the index of balance ($b = 0.542$) is much higher than the expected 0.50, due to the wide discrepancy for pure two-party constellations.

Figure 1 shows the largest party's median seat share graphed against the number of seat-winning parties. Both are on logarithmic scales, so that the theoretical curve $s_1 = 1/p^{0.5}$ becomes a straight line with slope –0.5, which is shown in Figure 1. The graph highlights the previous observation that the empirical median falls short of the simple model for $p = 2$, while tending to exceed it for a very large number of parties.

To place the extent of deviation from the model into proper perspective, we have to keep in mind the range of values of s_1 which is conceptually possible. The upper limit on the largest share is 100 percent, as denoted in Figure 1 by the heavy line $s_1 = 1$. This is a limit that can be approached but not reached, since at least one seat must be left to each remaining party. The conceptual lower limit is the heavy line $s_1 = 1/p$, reflecting equal shares for all p parties. This is a limit that can be reached under some conditions, and a few actual cases with $p = 2$ and 3 are close to that line.[8] The area underneath the lower limit is labeled 'Forbidden Area'. A data point within this area would imply that the largest share is smaller than the average share – which is impossible.[9]

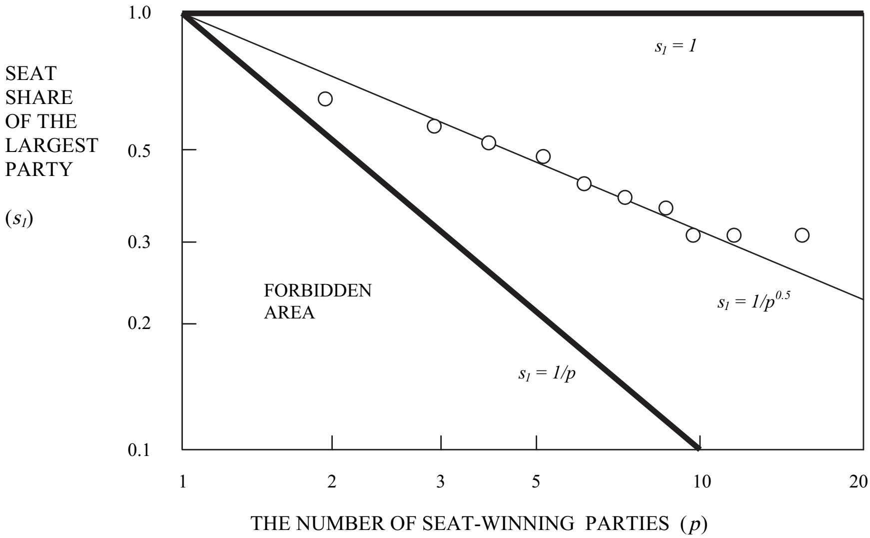

Figure 1. The median seat share of the largest party versus the number of parties in the assembly. Data from Table 1

The Basic Model and Its Extension

The simple logical quantitative model applies to a moderate number of parties but visibly needs refinement when only two parties win seats, and also when a very large number of parties do. This section first presents the basic model in a more general way, compared to Taagepera (1999a). Thereafter, some considerations of a specifically political nature will be introduced to account for the deviating result at $p = 2$.

Whenever a variable y is conceptually limited to a range between an upper limit (U) and a lower limit (L), both positive, its actual value can be expressed as $y = U^a L^b$, with both a and b positive and $a + b = 1$. Hence $y = L^b U^{(1-b)}$, with b ranging from 0 to 1. For $b = 0$, $y = U$, while for $b = 1$, $y = L$. In the absence of any further information we have no reason to expect b to be closer to one of the limits rather than the other. Thus the expectation value is $b = 0.5$, so that $y = (LU)^{0.5}$ – the geometric mean of the extremes. This means that the ratios of U to y and of y to L are equal, so that neither of the limits is favored over the other.

In the present case, s_1 can range from $L = 1/p$ to $U = 1$, so that the general formula $y = L^b U^{(1-b)}$ simplifies into $s_1 = (1/p)^b$, and the expectation value $y = (LU)^{0.5}$ becomes $s_1 = 1/p^{0.5}$. Of course, not all actual values are expected to be exactly at $s_1 = 1/p^{0.5}$ – only their median is.[10]

When proposing logical quantitative models based on nothing but conceptually possible upper and lower limits, the following objection frequently arises, even when the model fits the data: this is a nice parsimonious explanation, but it fails to involve any political factors such as, in the present case, party organization or electoral systems. But why should we ban Occam's razor and add superfluous factors? Or why should we favor specific factors over more general ones? Indeed, the model based on limits applies not only to parties but also to sizes of federal subunits (Taagepera, 1999a). This being so, why should we look for separate reasons in the two cases?

When the distribution of a political indicator follows a normal pattern, we do not look for specifically political reasons to explain the bell shape. Only deviations from normal distribution may call for political explanations (and, of course, political reasons may underlie the values of the mean and the standard deviation). The same is true here. The given upper and lower limits on y as a function of x predict a median pattern of $y = 1/x^{0.5}$ regardless of what x and y stand for. If the actual data fit (as is the case here, from $p = 3$ to $p = 12$), this means that the political factors (be they general or country-specific) must cancel themselves out in the worldwide picture, so as to yield precisely the pattern expected on the basis of pure probabilistic considerations.

Political explanations are called for only when facing deviations from the simple model. On the worldwide level, this is most markedly the case when only two parties win seats. On the country level, deviations do occur that make us wonder about the effect of electoral rules and types of party organization, to be discussed later.

The approach followed leads to a way to measure balance in the size of parties. Recall that any observed value of s_1 can be expressed as $s_1 = (1/p)^b$. The corresponding value of b can be calculated as $b = -\log s_1/\log p$. Recall that the allowed range of b is from 0 to 1, and the expected median is at the center of this range. The index of balance b will be discussed shortly in more detail, but for the moment it will be used to explain the discrepancy at $p = 2$.

Why is the simple model visibly off the mark when only two parties gain representation? Could the model involve hidden assumptions that fail at $p = 2$? When using the mean of the logical boundaries (0 and 1) as the expectation value for the index of balance, the model implicitly assumes that the distribution of $\log s_1$ between the conceptual limits is symmetrical. If we do not know the direction of possible lopsidedness, symmetry is of course our best assumption, as a first approximation. But for a second approximation, let us now introduce a minimal amount of information of a political nature. For simplicity, let us visualize an assembly with 100 seats.

When only two parties achieve representation, political competition may push for a balance between them. Hence constellations close to the conceptual lower limit (50–50) are politically quite plausible. Constellations approaching the upper limit (99–1), however, are unlikely in democracies, because the minor party could survive and the major party could avoid a split only under very peculiar circumstances.[11] The simplest modification to take this asymmetry into account would be to assume a linear decrease in probability of values of *log* s_1, starting with a positive value at $s_1 = 0.5$ and reaching zero at $s_1 = 1$. Instead of $s_1 = 70.7$ percent of the simple model, such a second approximation places the median expectation for $p = 2$ at $s_1 = 61.2$ percent, close to the observed median (59.3 percent – cf. Table 1).[12]

Pure two-party parliaments occur mainly when single-member districts are used, often (but not only) with plurality rule.[13] Compared to disparity in votes among the parties, the plurality allocation rule tends to magnify the disparity in seats, enhancing the largest party's share (Duverger mechanical effect). Even so, this share falls short of what the simple model predicts, but it is well in line with the second approximation.

When a third party also gains representation, it may reduce the largest party's share or, to the contrary, split the opposition and thus help the largest party. Thus the distribution of *log* s_1 is likely to become more symmetrical. When more than three parties win seats, heavy hegemony continues to be unstable in democratic politics. The opposite extreme, however, near-equality of all parties, is also rare. With probabilities reduced at both extremes, a more symmetrical probability profile results, leading back toward the simple model.

New considerations may enter when a large number of tiny parties win a few seats each. It may whittle down the largest party share – or, to the contrary, it might actually reinforce the largest party by atomizing the remaining field. The addition of still more parties may hardly affect the share of the largest party. With s_1 steady and p up, $s_1 p^{0.5}$ would increase beyond 1.00. No simple way has been found to model a second approximation for a large number of parties.

Index of Balance

The widely used effective number of parties ($N = 1/\Sigma s_i^2$) yields the number of equal-sized parties to which the actual set of parties is in some ways equivalent. This is useful, but N does not indicate how unbalanced the actual distribution of party sizes is. In a 100-seat assembly, $N = 3.00$ could represent a very balanced 34–33–33 or a very unbalanced 57–7, plus 36 parties or independents at 1 seat each.[14] This is where the index of balance ($b = -log\ s_1/log\ p$) offers a supplement (not a substitute!). For 34–33–33, $b = 0.98$, reflecting near-perfect balance, while for 57–7–1– . . . –1, $b = 0.15$, reflecting strong imbalance. Reporting both the effective number and balance describes the party constellation more thoroughly than does the effective number alone. This is analogous to reporting not only the mean but also the standard deviation of a normal distribution. The mean and the effective number measure the central tendency, while standard deviation and balance measure the spread around the central tendency.[15]

A different way to supplement the effective number has been suggested (Taagepera, 1999b). In a more general family of measures of the number of parties (N_n) that involves a parameter (n), N_0 corresponds to the total number of non-zero components (our p), while N_∞ is the inverse of the largest party share ($1/s_1$). The commonly used effective number of parties ($N = 1/\Sigma s_i^2$) corresponds to N_2, and it can be shown that $N_0 \geq N_2 \geq N_\infty$ for all constellations. Taagepera (1999b) proposes supplementing N_2 with N_∞ (the inverse of the largest share), while recognizing the problem of considerable collinearity between the two. Indeed, N_2 can only have values in the range $N_\infty^2 > N_2 > N_\infty$ and, conversely, $N_2 > N_\infty > N_2^{0.5}$. This collinearity is well in evidence in the country averages of N_2 ('*ENPP*') and N_∞ reported by Siaroff (2003).

In contrast, for any given value of N_2, the index of balance can in principle take any values between 0 and 1, provided the total number of seats is sufficiently large.[16] In this notation, the index of balance assumes the elegant form $b = \log N_0/\log N_\infty$, while the conservation statement $s_1 p^{0.5} = 1$ corresponds to $N_0 = N_\infty^2$.

Given that the median value of b is fairly constant around 0.50 when more than 2 parties win seats, we may consider the distribution of b jointly for all such cases. If the extremes ($b = 0$ and $b = 1$) are rather depopulated, the distribution might be close to the normal.[17] This would not be expected for $p = 2$.

Table 2 shows the distribution of the index of balance separately for two groups: only 2 and more than 2 parties winning seats. For more than 2 parties, the mean ($b = 0.493$) is close to 0.500. The distribution is quite symmetrical, but fails the Kolmogorov–Smirnov test for normality[18] at

Table 2. Distribution of index of balance b for 2 and more than 2 seat-winning parties

	No. of seat-winning parties	
Range of b	*Only 2*	*More than 2*
0 to 0.05	1	0
0.05 to 0.15	1	2
0.15 to 0.25	1	12
0.25 to 0.35	0	47
0.35 to 0.45	3	111
0.45 to 0.55	7	157
0.55 to 0.65	12	109
0.65 to 0.75	19	59
0.75 to 0.85	26	6
0.85 to 0.95	16	5
0.95 to 1	7	2
Sum	93	510

probability <0.01. In particular, the central peak is more pointed. No logical or political reason has been found for why elections worldwide should lead to party constellations so close to half-balance.

For elections where only two parties won seats, the distribution in Table 2 is skewed, as expected. The median index of balance is 0.754, corresponding to a largest party share of 59.1 percent, which is close to the 61.2 percent given by the second-approximation model.[19]

Country Patterns and Electoral Rules

Do some countries or electoral rules consistently feature a balance different from 0.5? Going beyond qualitative description, the index of balance introduces a quantitative measure, making comparisons possible among elections and among countries.

Table 3 gives the median balance for 33 periods with essentially the same electoral system in 23 countries. Of these, two-thirds are within 0.1 of the expected overall median of 0.5. Among the recent systems, Spain, Italy, and Israel have unusually low balance, combining a large major party with a sprinkling of small ones. At the opposite extreme, the median balance is highest for Malta, New Zealand, and Iceland, among recent systems, reflecting unusually equal shares among a small number of seat-winning parties.

There seems to be no significant correlation between electoral rules and balance. In particular, in the 9 countries that have used both proportional representation (PR) and majority or plurality, PR shows a lower balance in 5 cases and a higher one in 4. If a country switches from single-member districts with plurality allocation rule (SMP) to PR, the number of seat-winning parties may well increase – but also the largest party's share is likely to decrease, so that the balance need not be affected.[20] On the other hand, the index of balance can shift even in the absence of significant change in electoral rules. Finland offers the most glaring example. It had a median balance of 0.49 for 1907–1939 and a markedly higher 0.62 for 1945–1995.

The lowest median balance (Italy 1895–1921, $b = 0.18$) and the highest (Belgium 1847–1898, $b = 0.72$) both occur with 2-rounds majority rule. Balance tends to be the lowest in majority systems (overall median 0.46), followed by PR (0.51) and SMP (0.54), but the extensive overlap suggests that electoral rules have little effect on the balance of party sizes.[21]

Since pure two-party assemblies with their high balance arise most frequently with SMP, this allocation rule would indirectly lead to a relatively high balance. However, SMP countries do not stand out as a group. Some SMP countries have few independents and regional parties, which enhances balance ($b = 0.66$ for New Zealand), while some others have many of them, reducing balance (0.41 for the UK). Most SMP countries have a mean balance in the central range, 0.45 to 0.55, like other countries.

Table 3. Median values of index of balance for periods with same electoral rule

Period and no. of elections[a]	*Electoral rule*	*Median balance*[b]
Italy 1895–1921, 7	Majority	0.177
Spain 1977–1996, 7	PR[b]	0.285
Switzerland 1885–1908, 5	Majority	0.318
Italy 1946–1992, 11	PR	0.376
Israel 1949–1990, 14	PR	0.396
Greece 1926–1993, 15	Varied	0.405
UK 1885–1992, 16	SMP[b]	0.410
Norway 1921–1993, 19	PR	0.441
France 1958–1993, 10	Majority	0.454
Sweden 1911–1994, 27	PR	0.461
Norway 1882–1918, 12	(Majority)	0.464
Denmark 1901–1913, 6	SMP	0.471
Belgium 1900–1996, 31	PR	0.485
Netherlands 1918–1994, 21	PR	0.502
Portugal 1975–1995, 9	PR	0.507
Denmark 1918–1994, 32	PR	0.508
Germany 1919–1933, 7	PR	0.515
Germany 1871–1912, 7	Majority	0.524
Canada 1878–1993, 20	SMP	0.528
Luxembourg 1919–1994, 17	PR	0.548
Switzerland 1919–1995, 19	PR	0.559
USA 1828–1994, 77	SMP	0.561
Netherlands 1886–1913, 8	Majority	0.568
Finland 1907–1995, 31	PR	0.579
Germany 1949–1994, 12	MMP	0.580
Australia 1901–1996, 3	Ordinal	0.596
Austria 1919–1995, 21	PR	0.597
Iceland 1916–1995, 28	PR	0.633
New Zealand 1890–1993, 19	SMP	0.662
France 1910–1956, 7	(PR)	0.663
Malta 1921–1992	Ordinal	0.689
Sweden 1887–1908, 9	SMP	0.701
Belgium 1847–1898, 29	Majority	0.717

[a] Elections with several ‘Others’ seats are not included.
[b] PR = Proportional representation; SMP = single-member districts and plurality rule.

One electoral rule that might increase balance among seat-winning parties is PR for sufficiently large parties, combined with a high legal threshold that weeds out small parties – the post-World War II German formula. The balance for Germany 1957–1994 is on the high side indeed (0.58), but we would need more cases.[22]

The dispersion of balance for different elections in the same country varies appreciably, tending to be wider for SMP and narrower for PR.[23] The widest dispersion is observed for Belgium 1847–1914, where the balance ranges all the way from 0.04 to 0.95. Up to 1892, only two parties won seats, and their seats ratio ranged from nearly equal to highly unequal. The range is also wide for New Zealand (*b* ranging from 0.21 to 0.96) over its long SMP period (1890–1993) and for the US, especially in its early stage of 1828–1882 (from 0.23 to nearly 0.99), where the balance of major parties varied and third parties entered erratically. The range of variation is the narrowest in Israel 1949–1996 (from 0.31 to 0.53) and Portugal 1975–1995 (from 0.27 to 0.57).

In sum, electoral rules have little demonstrable impact on the balance of party sizes. There is no other visible factor either that would be common to countries with unusually low or high balance in Table 3. In particular, types of party organization do not seem to offer any explanation. Marked deviation from the world median of 0.5 seems to be highly country-specific.

Conclusions

The concept of a conserved quantity is important in many areas of science. This study has used extensive worldwide data on the number and size of parties in assemblies to test a general rule of conservation of balance. This rule should apply whenever a well-defined total is randomly divided into a well-defined number of components. Median agreement is good when assemblies contain 3 to 12 parties, while deviations are marked in two-party assemblies and in an extremely splintered field. One may wonder whether a similar pattern arises with components different from parties, such as populations and areas of federal subunits.

Within the limits of validity thus established, the following principle of conservation of balance can now be asserted: the median product of the largest party's fractional share and the square root of the number of seat-winning parties is conserved: $s_1 p^{0.5} = 1$. The worldwide median is within 2 percent of 1.00.

A partly novel approach to developing the underlying logical quantitative model has pinned down an underlying assumption that does not hold under pure two-party conditions. A resulting second approximation leads to agreement with actual median size distribution in two-party assemblies.

The new approach to the model suggests an index of balance that can range from 0 for extreme imbalance to 1 for perfect equality. While the widely used effective number measures the central tendency of party constellations, the index of balance adds a measure of spread around the central tendency. Two-thirds of individual elections produce a balance ranging from 0.35 to 0.65, and two-thirds of country medians range from 0.40 to 0.60.

Which factors tend to produce low or high balance? Only now that balance has been operationalized can this question be asked. Electoral rules immediately come to mind, but their impact cannot be demonstrated, except for an indirect impact of single-member plurality rule through relatively frequent purely two-party assemblies. Unusually high or low balance seems due to country-specific factors or previously unsuspected general ones. This is the field of study opened up by this article.

Notes

1 The effective number of parties ($N = 1/\Sigma s_i^2$, where s_i is the fractional share of the *i*-th party) has become the most widely used measure of the number of parties (Lijphart, 1994: 70).

2 At the electoral level, the inverse relationship is confirmed by Anckar (2000), worldwide and in Finnish and British local elections. With a higher number of parties running, the vote share of the largest party tends to be lower, and this relationship holds separately for plurality and proportional systems. Instead of the electoral level, where a huge number of parties can conceivably run, the present study focuses on the assembly level, where the number of parties represented is more severely limited by the sheer number of seats available.

3 A logical quantitative model is defined here as a model that can be constructed without data input, on logical grounds, and then can be quantitatively tested.

4 The number of parties represented in an assembly is well defined: a party either has at least one seat, or it does not. In contrast, the number of electoral parties is less well defined in principle (a party can run and not get a single vote) and even more by availability of data, given that all too many minor parties may be grouped under the 'Others' category. Hence I focus on parties in the assembly.

5 Using a more extensive and up-to-date dataset is not expected to change the findings, given the universal nature of the rule of conservation tested. All that is needed is an extensive dataset selected by someone else than the present author. The collections by Mackie and Rose (1991, 1997) satisfy these conditions.

6 Using an index different from balance, a preliminary study (Roopalu, 2002) did include the elections with several 'Others' seats. The latter presented difficulties in operationalization of the number of parties, without altering the broad picture. In the following countries, more than 25 percent of all elections involved several 'Others' seats: Japan (all 33 elections), Ireland (24 out of 25), New Zealand (15 out of 34), UK (13 out of 29), Canada (12 out of 32), Greece (7 out of 22), and Germany (9 out of 35).

7 The median for all 603 elections is 5 parties winning at least 1 seat, with the largest party's share around 46 percent. In 70 percent of the cases, the number of seat-winning parties ranges from 2 to 6. Beyond 6, the drop-off is marked. The record number of seat-winning parties (in the absence of several 'Others' seats) is 16 (Israel 1984 and 1988, Italy 1992, Switzerland 1991). The prevalent range for the largest party's share is 35 to 59 percent. In only 19 percent of the cases does the largest party's share fall below 35 percent, and in only 15 percent of the cases does it exceed 59 percent.

8 US 1836 has $s_1 = 0.504$, the conceptual limit for $p = 2$ being 0.500. Iceland 1922 and 1926 are technically right at the limit of 0.333 for $p = 3$, but these were *Landskjör* elections with only 3 seats involved.

9 In the presence of two variables s_1 and p, the gut reaction of many political scientists is to apply simple OLS, and they may ask why this approach is not followed here. Application of OLS to logarithms of s_1 and p would yield a straight line in

Figure 1, roughly $s_1 = 0.8p^{-0.4}$, that heads into the conceptually forbidden area at low p. At $p = 1$, it would predict a value of s_1 around 0.8, which is absurd – when only one party wins seats, its seat share can only be 1 (100 percent). Such a mechanical application of OLS would make us overlook the essential observation that the parsimonious logical model $s_1 = 1/p^{0.5}$ fits well from $p = 3$ to $p = 12$ (as well as at $p = 1$), leaving only the deviations at $p = 2$ and $p > 12$ to be explained. OLS is not always the best course to follow.

10 This derivation glances over the fact that in this case L is a reachable limit and U an unreachable one. It may have implications when assemblies are very small.

11 A non-democratic dominant party could allow symbolic representation for a minor party as window-dressing (as the Communists did in Poland). Among stable democracies as defined by Lijphart (1999), only Botswana has one very large and one very small party, maintained by the ethnic nature of their support.

12 More precisely, linear decrease of probability of $\log s_1$ between log 0.5 and log 1 places the median of occurrences at $s_1 = x$ such that $\log x = -\log 2/2^{0.5}$. Hence $x = 0.6125$. Empirically, frequency increases slightly, from the low to the high 50s, before starting to decrease (cf. Table 1), but the simple linear approximation (on log scale) is still fairly close. While the simple model posits an index of balance 0.500, the second approximation brings it up to 0.707, still somewhat short of the observed median (0.754).

13 Pure two-party parliaments occurred most frequently in the nineteenth century. Out of the 93 cases, the US contributes 33 (throughout 1828–1988), Belgium 26 (1847–1892), New Zealand 11 (1946–1987), Malta 8 (1945–1992), Canada 7 (1878–1917), and four other countries together 8. The shift away from $b = 0.5$ (and $s_1 = 71$ percent) occurs not only with single-member districts (US: median $b = 0.759$, $s_1 = 59.1$ percent; Belgium: $b = 0.724$, $s_1 = 60.5$ percent; New Zealand: $b = 0.798$, $s_1 = 57.5$ percent), but also with multi-seat quasi-PR (Malta: $b = 0.935$, $s_1 = 52.3$ percent).

14 Independents cannot just be ignored when calculating the effective number of parties, without running into inconsistencies.

15 It may be asked whether the standard deviation could be used to supplement the effective number. The answer is 'no', because the standard deviation can supplement only a mean, and even then only if the distribution is normal.

16 In the example 57–7–1– . . . –1, b is still relatively high (0.15), because the total number of seats is only 100. With 1000 seats, distributed as 577–1– . . . –1, the effective number of parties remains 3.00, while b drops to 0.09.

17 The normal distribution extends from minus to plus infinity. Here the conceptual range is limited (0 to 1), truncating the normal distribution. If the standard deviation is much less than one-half of the conceptual range, the truncation matters little.

18 The Kolmogorov–Smirnov estimation procedure starts out with the cumulative sample function $S(x)$. It can be used to test whether the cumulative probability function $F(x)$ has a specified form $F0(x)$ – see Neter et al. (1992). If, despite the lack of normality, the standard deviation is calculated, it comes out as 0.136, meaning about one-quarter of the conceptually possible maximum. I thank Kaili Roopalu for carrying out this test.

19 Remarkably, almost the entire possible range is populated, from $b = 0.989$ (US 1836, $s_1 = 0.504$) down to $b = 0.042$ (Belgium 1884 partial elections, $s_1 = 0.971$).

Thus, extreme balance and extreme imbalance can occur in a two-party system, but only under rare conditions.

20 Thus, in Denmark, an average of 4.5 parties won seats in the 6 elections (1901–1907) preceding the gradual transition from SMP to PR, and the average largest share was 0.51. In the 6 elections following the transition (1924–1939), it was 6.8 and 0.41. The product $s_1p^{0.5}$ stays 1.07.

21 Both countries using an ordinal ballot (Australia in single-member and Malta in multi-member districts) show high balance. Ireland, however, uses the same system as Malta, yet tends to have a low balance, to the extent it can be estimated in the pervasive presence of the 'Others' category.

22 Among the relatively stable electoral systems, Turkey may have the highest nationwide legal threshold (along with various other restrictions): since 1983, it has been 10 percent, except for independents. The mean balance for the 4 elections in 1983–1995, where the independents received no seats (Nohlen et al., 2001), is 0.58. This means that the party seat shares are not appreciably more balanced than in the general case.

23 More variation in balance could be expected for SMP than for PR because SMP is relatively hospitable to local parties and independents, whose number may vary from election to election without much affecting the largest party's seat share. While SMP has a high district-level 'threshold of exclusion', it is also true, somewhat counter-intuitively, that the *nationwide* 'threshold of inclusion' is actually lower for SMP than for PR (Grofman, 1999: 322).

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REIN TAAGEPERA is Research Professor at the University of California, Irvine, where he has taught since 1970, and also at Tartu University, Estonia, since 1992. His PhD was in physics, and his main research interest is in electoral systems. He is senior author of *Seats and Votes* (Yale University Press, 1989) and was the founding chair (2001–2002) of a new political party, Res Publica, that won the Estonian parliamentary elections of March 2003 and formed a coalition cabinet.

ADDRESS: (October–March) School of Social Sciences, University of California, Irvine, CA 92697, USA. [email: rtaagepe@uci.edu]. (April–August): Department of Political Science, Tartu University, 50090 Tartu, Estonia.

Paper submitted 2 April 2004; accepted for publication 24 May 2004.