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DYNAMICAL BIRTH AND THERMAL DEATH OF ANGULAR MOMENTUM IN HEAVY ION REACTIONS

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where

$$\xi_i(u_1,u_2,\beta) = \int_{u_1}^{u_2} x^{-(i+3/2)} e^{-\beta x} dx \quad . \tag{5b}$$

The integrals ξ_i satisfy a recurrence relation

$$\xi_i = \frac{1}{i+\frac{1}{2}} \left[\frac{e^{-\beta u_1}}{u_1^{i+\frac{1}{2}}} - \frac{e^{-\beta u_2}}{u_2^{i+\frac{1}{2}}} - \beta\xi_{i-1} \right] \quad . \tag{6}$$

In order to evaluate the desired integrals, it is sufficient to evaluate one of them separately. Application of the recurrence relation now allows evaluation of all desired ξ_i's, and thus all necessary μ_m's.

Consider a system in which essentially all impact parameters contribute to the deep inelastic process. In order to leave out events in which the kinetic energy in incompletely damped, it is assumed that $\eta_1 = 0$ and $\eta_2 = 0.9$, thus allowing 20% of the cross section for incompletely damped (QE) events. Three very striking features, shown in Fig. 1, emerge: 1) The mean angular momentum $\langle \eta \rangle$ decreases as a function of β. 2) The ratio $\rho = \sigma/\langle \eta \rangle$ exceeds the $2\ell+1$ value for nearly all asymmetries. 3) The skewness γ changes sign as a function of asymmetry.

In order to compare with experiment, it is necessary to correlate β with Z. From previous work in fitting data from 620 MeV $^{86}Kr + ^{197}Au$, the following parameters are used: $\mu_2 = 6.6 \times 10^{21}\ e^2\ sec^{-1}$, $\tau_0 = 4.0 \times 10^{-21}$ sec. To calculate the γ-ray multiplicity, $M_\gamma(Z)$, we assume rigid rotation. Then one has $M_\gamma(Z)/\frac{1}{2}\ \ell_{max} = f(Z)\ \langle \eta(\beta) \rangle$, where f(Z) is the fraction of the angular momentum tied up in the fragment spins. The curve of asterisks in Fig. 1 is a plot of this quantity. As can be seen, the multiplicities are approximately constant as a function of Z, in agreement with the data. Large values of σ are predicted, in excess of those expected from a $2\ell+1$ distribution.

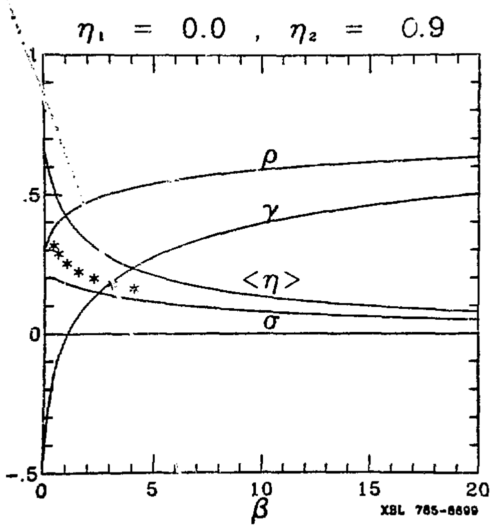

Fig. 1

It is interesting to note that angular momentum fractionation is expected even when statistical equilibrium is attained along the mass asymmetry coordinate, either directly as the end-product of diffusion, or through population from compound nucleus. The reason for this can easily be seen. The potential along the mass asymmetry coordinate has a minimum at symmetry with the second derivative increasing with angular momentum. At equilibrium, the high angular momentum mass distributions are more sharply peaked about symmetry than the low angular momentum mass distributions. It follows that, after summation over all partial ℓ-waves, the angular momentum decreases with increasing asymmetry.

For two touching liquid drop spheres arising from compound nucleus fission, the first moment of the angular momentum is

$$\bar{\ell}(y)/\bar{\ell}_D = \tfrac{3}{2}(\exp \mathcal{R}E_R^{mx}/T-1)^{-1}\left[1-\sqrt{T/\mathcal{R}E_R^{mx}}\; F\left(\sqrt{\mathcal{R}E_R^{mx}/T}\right)\right] \exp \mathcal{R}E_R^{mx}/T$$

where T is the temperature; $\mathcal{R} = -0.547 + 1.296\, y^2$; $y = \frac{1}{2}(A_2 - A_1)/A$ is the asymmetry parameter; E_R^{mx} is the rotational of the equivalent sphere at the maximum angular momentum; $F(x)$ is the Dawson's integral, and $\bar{\ell}_D$ is the average of the entrance channel angular momentum distribution. The second moment of the angular momentum is

$$\overline{\ell^2}(y)/\overline{\ell^2} = 2(\exp \mathcal{R}E_R^{mx}/T-1)^{-1}\left[(1-T/\mathcal{R}E_R^{mx})\exp \mathcal{R}E_R^{mx}/T + T/\mathcal{R}E_R^{mx}\right] .$$

From the above equations one sees that a slight angular momentum fractionation, concentrating high angular momenta close to symmetry is displayed at low temperatures, while at high temperatures the fractionation disappears as expected.

In the above discussion fission was competing with neutron emission. Such a competition sharply limits fission to the highest ℓ waves and this prevents a strong angular momentum fractionation. If, as in deep inelastic processes, the equilibrium is immediately attained along the mass asymmetry coordinate without competition from neutron decay, the angular momentum fractionation is dramatically enhanced. Experimental study of the angular momentum fractionation in symmetric mass distributions may help to decide whether or not compound nucleus formation is involved.

Statistical Coupling Between Orbital and Intrinsic Angular Momenta

In the spirit of simplicity let us assume that we can assimilate the exit channel configuration to that of two touching, equal, rigid spheres with all the associated rotational degrees of freedom. First, let us consider the equilibrium between intrinsic rotation of the fragments and their orbital rotation, assuming that the relevant angular momenta are all parallel to each other. If the total angular momentum is I and the orbital momentum is ℓ, the energy, for an arbitrary partition between orbital and intrinsic angular momentum is:

$$E(\ell) = \ell^2/2\mu r^2 + 2(I-\ell)^2/(4\times 2\mathcal{J}) = \ell^2/2\mu r^2 + \tfrac{1}{4}(I-\ell)^2/\mathcal{J} .$$

The first term is the orbital and the second the intrinsic rotational energy, $\mathcal{J}$ being the moment of inertia of one of the two equal spheres. The average orbital angular momentum $\bar{\ell}$ is given by: $\bar{\ell} = I\mu r^2/(\mu r^2 + 2\mathcal{J}) = \frac{5}{7} I$. The second moment $\overline{\ell^2}$ is given by: $\overline{\ell^2} = T/(1/\mu r^2 + 1/2\mathcal{J}) + I^2/[4\mathcal{J}^2(1/\mu r^2 + 1/2\mathcal{J})^2]$.

The standard deviation is $\sigma_\ell^2 = 2\mathcal{J}\mu r^2 T/(\mu r^2 + 2\mathcal{J}) = \frac{10}{7}\mathcal{J}T$.

Thermal Fluctuation of the Angular Momentum Projection on the Disintegration Axis

Above, we have assumed that the two touching fragments are aligned with their common axis perpendicular to the total angular momentum. Because of the thermal fluctuations, this condition can be relaxed. Assuming now that the two fragments are rigidly attached one to the other, the energy is given by $E = (I^2-K^2)/2\mathcal{J}_\perp + K^2/2\mathcal{J}_\parallel = I^2/2\mathcal{J}_\perp + K^2/2\mathcal{J}_{eff}$, where $\mathcal{J}_\perp = 2\mathcal{J} + \mu r^2$; $\mathcal{J}_\parallel = 2\mathcal{J}$; $\mathcal{J}_{eff}^{-1} = \mathcal{J}_\parallel^{-1} + \mathcal{J}_\perp^{-1}$; K is the projection of the angular momentum I on the two fragment axes. We obtain $\bar{K}^2 = \mathcal{J}_{eff} T = \frac{14}{5}\mathcal{J}T$. The total fragment spin is given by $I_{sp}^2 = K^2 + \frac{4}{49}(I^2-K^2)$. The average spin, on the other hand, is to order $\bar{K}^2/I^2$, $\bar{I}_{sp}^2 = \frac{2}{7} I + \frac{9}{2}\mathcal{J}T/I$. Furthermore, the fluctuation $\sigma_{sp}^2 = \bar{I}_{sp}^2 - \bar{I}_{sp}^2 \cong 0$ up to order $\bar{K}^2/I^2$ can be neglected in most cases.

Twisting and Bending Modes Excited in a Zero Angular Momentum System

These three degrees of freedom are degenerate in our two-equal sphere model. A splitting of the degeneracy could easily occur in the case of fragment deformation. If we call R the angular momentum of each fragment, we obtain $R^2 = \frac{3}{2}\mathcal{J}T$ and $\sigma_R^2 = 0.227\mathcal{J}T$. Furthermore, for each fragment the resulting angular momentum is randomly oriented. It is worth pointing out again that this angular momentum can exist even when the total angular momentum is zero because of the pairwise cancellation mentioned above.

Coupling of Twisting and Bending Modes to Rigid Rotation

We want to generalize the previous calculation to the case of non-zero total angular momentum. Let us assume that each fragment has an aligned angular momentum component, I_R, arising from rigid rotation, and a random component, R, due to the bending and twisting modes. The overall rotational energy is

$$E = \left\{(I_R^2 + R^2 + RI_R\cos\theta) + (I_R^2 + R^2 - 2RI_R\cos\theta)\right\}/2\mathcal{J} = (I_R^2 + R^2)/\mathcal{J} .$$

The average total angular momentum of the fragments is:

$$\bar{I} = 2I_R + \tfrac{2}{3}\,\overline{R^2}/I_R = 2I_R + \mathcal{J}T/I_R = \tfrac{2}{7} I + 7\mathcal{J}T/I = \tfrac{2}{7} I(1 + \tfrac{49}{2}\mathcal{J}T/I).$$

Similarly the average square angular momentum to order $\overline{R^2}/I_R^2$, yields $\overline{I^2} = 4I_R^2 + \frac{11}{3}\overline{R^2}$. Again, to order R^2/I_R^2 we have $\sigma_I^2 = R^2 = \frac{3}{2}\mathcal{J}T$. Of greatest importance is the fact that a sizeable "tilt" of the angular momentum of each fragment about the direction of the total angular momentum is introduced: $\tan\theta = \sqrt{\overline{R^2}} / I_R$. This depolarization is of great importance for the proper interpretation of the out-of-plane angular distribution of gamma rays emitted by the fragments, and for the out-of-plane angular distribution of the sequential fission fragments.[3)]

A Simple Application to a Typical Heavy-Ion Reaction

The reaction we want to consider is 600 MeV $^{86}Kr + ^{197}Au$. If we allow the system to evolve to the configuration of two touching spheres ($r_o = 1.22$), we have $\mathcal{J}T = 110\ \hbar$ or $\sqrt{\mathcal{J}T} \cong 10\text{-}11\ \hbar$.

Let us now consider the statistical fluctuations between orbital and intrinsic angular momentum. The total fragment spin for the average total angular momentum is $I_{sp} = 54.0\ \hbar$, $\sigma_{sp} = (^{10}\!/_7\, \mathcal{J}T)^{1/2} = 12.5\ \hbar$. The fluctuation of the separation axis about the normal to the angular momentum yields the following results: $\overline{K^2} = {}^{14}\!/_5\, \mathcal{J}T = 308\ \hbar$, $\sqrt{\overline{K^2}} = 17.55\ \hbar$, $I_{sp} = 54.0 + 2.62\ \hbar$. The out-of-plane angle is $\theta = 5.3°$. For the twisting and bending modes, one obtains $\overline{R}^2 = {}^3\!/_2\, \mathcal{J}T = 165\ \hbar$, $\sqrt{\overline{R}^2} = 12.85\ \hbar$. When the twisting and bending modes are coupled to rigid rotation, we obtain (average) $I_{sp} = {}^2\!/_7\, I + 7\mathcal{J}T/I = 54.0 + 4.07$, $\sigma_{sp} = 12.85\ \hbar$. This produces an angular momentum depolarization of $\theta \approx 25.5°$, where we see that this effect greatly dominates over the fluctuation of the separation axis.

Summarizing, the spin fluctuations amount to $\sigma^2_{total} = 165 + 157 = 322$, or $\sigma_{total} = 18\ \hbar$. If we assume a triangular distribution for the angular moment, we obtain for the spin of both fragments an entrance channel $\sigma^2_{e.c.} = 357\ \hbar^2$. By combining all the fluctuations we have $\sigma^2 = 690\ \hbar^2$ or $\sigma = 26.3\ \hbar$. In conclusion, without applying for angular momentum fractionation, we obtain for the overall fragment spin, $I_{spin} \cong 60 \pm 26\ \hbar$. It appears that the combination of these fluctuations with that shown in Fig. 1 gives rise to an overall fluctuation larger than 50%, comfortably larger than that arising from the $2\ell+1$ angular momentum distribution and in satisfactory agreement with experiment.[1)]

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