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Willems, Bert

Publication Date
2005-03-01

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CSEM WP 139

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Bert Willems

March 2005

This paper is part of the Center for the Study of Energy Markets (CSEM) Working Paper Series. CSEM is a program of the University of California Energy Institute, a multi-campus research unit of the University of California located on the Berkeley campus.

2547 Channing Way
Berkeley, California 94720-5180
www.ucei.org

Cournot Competition, Financial Option Markets and Efficiency

Bert Willems[†]

www.econ.kuleuven.ac.be/ete/personal/bwillems/

ABSTRACT: Allaz and Vila (1993) show that the existence of futures markets increases the efficiency of markets in a Cournot setting. This paper looks at the efficiency effect of financial options in a similar framework. It shows that the existence of financial options also makes markets more efficient; though to a smaller extent than futures. This is particularly relevant for markets with market power and costly storage, like electricity markets.

Keywords: Futures markets, Options markets, Cournot, Market power, Electricity, Arbitrage

JEL: C72, D43, G13, L13, L50, L94

Introduction

Several countries recently decided to liberalize their electricity markets and to organize competition in electricity generation. They assumed that economies of scale and entry barriers in the generation sector were sufficiently small to make competition viable.

In practice, the generation market is not always very competitive. Generators often succeed in driving up prices significantly above competitive levels. This also happens in markets with low levels of market concentration.

The two main reasons for market power in the electricity sector are the non-storability of electrical energy and the low demand elasticity[1]. Both market characteristics make unilateral withholding of production output highly profitable for firms. Especially in periods of peak demand, generators are often producing close to the technical maximum output of their generation plants, which inevitably leads to steep supply functions. The residual demand functions faced by the generators are therefore often steep as well.

Generation market power becomes even more pronounced because technical constraints in the transmission network do not allow generators to effectively compete with each other. Furthermore, generation plants are capital intensive, and investments require a long lead time.

Prices above marginal production costs have been shown to exist in several markets. Borenstein et al. (2002) find a significant departure from competitive pricing during high demand summer months in the Californian electricity

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[†] Katholieke Universiteit Leuven, and University of California Energy Institute. The author would like to thank Severin Borenstein, Jim Bushnell, Hans Degryse, Guido Pepermans, Stef Proost, Ana Rivas, Patrick Van Cayseele, Frank Verboven, Frank Wolak and the seminar participants in Leuven and Berkeley for their valuable comments.

[1] This effect is even worse because changes in the real-time wholesales price are not passed through to consumers. Instead consumers pay a flat rate which is changed only infrequently, and does not reflect the scarcity of electricity in certain periods. Hence, consumers do not have the right incentives to reduce their demand when the market is tight.

market. Also Joskow and Kahn (2002) show that high prices in summer 2000 reflected, in part, the exercise of market power by suppliers. Wolak and Patrick (2001) argue that the two largest generators in the early England and Wales market were able to obtain prices for their output substantially above their marginal cost of generation. Wolfram (1999) shows that the British electricity prices were above the perfectly competitive prices.

Comparing different electricity markets in the US, Bushnell, et al. (2004) show that California had a relatively unconcentrated generation market[2] but that the lack of long term contracts led to high price-cost margins in the summer of 2000. With long term contracts, generators sell part of their electricity ex-ante, at a locked-in price. As a result, generators will behave more competitively in the spot market.[3]

The intuition is that of the durable goods' monopolist in Coase's conjecture.[4] See Figure 1. Graph A shows the profit maximizing price p^M for a monopolist who sells only in the spot market, has production costs $C(q)$ and faces an inverse demand function $p(q)$. The monopolist will set a price such that his marginal revenue is equal to his marginal costs. Graph B shows the same situation for a monopolist who signed long term contracts for k units of electricity. In the spot market (the second stage), k units will therefore disappear both at the demand and the supply side. The profit maximizing price is equal to $\tilde{p}^M$ and is lower than p^M. In the first stage, the contracting stage, consumers will take into account that the price in the spot market will be equal to $\tilde{p}^M$. They will only buy long run contracts for electricity at the price $\tilde{p}^M$, as they would lose money otherwise. Hence, the monopolist will receive the price $\tilde{p}^M$ in the production as well as in the contracting stage. The fact that the price should be the same in the contracting and the production stage is called the perfect arbitrage condition.

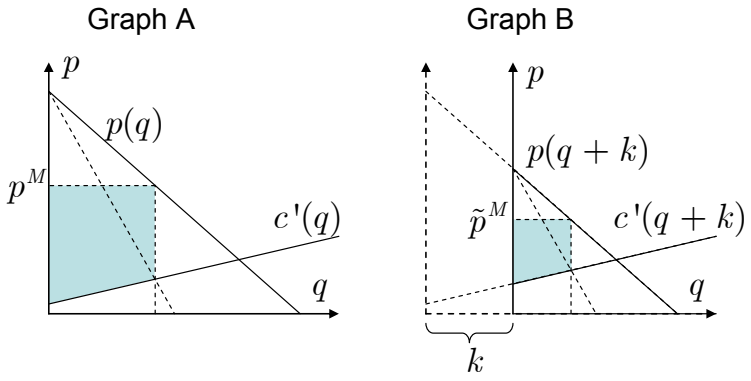

Figure 1 Effect of an ex ante contract on the behavior of a monopolist.

The study of Bushnell, et al. highlights the importance of long term contracts in electricity markets. There is however no consensus on the role of long term contracts in electricity markets.

Historically, policy makers have been opposed to long term contracting in electricity markets. They feared that long term contracts between incumbent generators and retailers might slow down entry in the generation market. They

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[2] The Herfindahl-Hirschman Index (HHI) for California, New England and PJM (Pennsylvania, New Jersey, and Maryland) were 620, 850 and 1400.

[3] The most extreme form of long term contracting between a retailer and a generation firm is of course a merger.

[4] See Coase (1972).

also assumed that long term contracting would decrease the transparency and the liquidity of the spot markets.[5] Illiquid spot markets would lead to inefficient real time production decisions, and would also make entry more difficult. A small entrant will have to rely on the spot market to balance the difference between the energy sold and the energy produced.

Currently, policy makers are changing their mind, and are becoming more favorable towards long term contracts. They hope that long term contracts will ease entry in the generation market by reducing the risk for entrants and will reduce market power in the spot market. Long term contracts will also help retailers who sell electricity at fixed regulated prices to hedge their price risks.

Nowadays there is a debate whether they should *impose* the usage of long term contracts or whether generators and retailers will sign the right amount of long term contracts on their own. See for example Creti and Fabra (2004). If regulators decide to intervene, they need to make several choices.

A first choice is on whom to impose the obligations: on the generators or on the retailers? Generators can be obliged to sell a part of their production capacity, and retailers can be obliged to contract a fraction of their estimated demand ex-ante. An obligation to retailers might help entry in generation (long run efficiency), but could place retailers in a bad bargaining position in the contract market when there is no entry in the short run.

A second choice is how the quantities of the contract should be specified. Contracts could specify a constant quantity over time, a predetermined load-shape[6], a specific fraction of system demand, or a quantity which depends on the spot price. Two standard contracts which we consider in this paper are the futures contract and the call option contract. In a futures contract the quantity which needs to be delivered is fixed. In a call option this quantity depends on the spot price: it increases with the spot price.

Options might have some advantages compared with futures.

- Options allow generators and retailers to *hedge* quantity risks, while futures can only be used to hedge price risks.[7] Given that electricity cannot be stored very easily, quantity risks are very important in the electricity market, and options play therefore an important role.
- *Market power* is most pronounced during periods of peak demand and is characterized by high spot prices. Retailers might sign option contracts to counter the market power of generators during these periods[8].

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[5] There is no theoretical evidence that forbidding long-term contracting will increase liquidity on spot markets. With liquid long term contracts, information becomes public sooner than with spot markets. This might increase liquidity. Long term contracts make spot markets also more competitive; prices will therefore be closer to competitive levels, and less prone to manipulation.

[6] A load-shape specifies a certain level of demand as a function of time. As a lot of retail consumers do not have real-time meters, these load-shapes are often used for accounting purposes.

[7] To illustrate this, consider a retailer who has the obligation to serve a number of customers at a fixed retail price. Without a long-term contract, the retailer buys his electricity on the wholesale spot market and faces two types of risks. He faces a price risk, because he does not know which price he will pay on the wholesale spot market, and a quantity risk, because he does not know how much he will need to buy, as demand depends on, for example, weather factors. Without quantity risk, the retailer would be able to hedge his entire risk by signing futures contracts for the quantity he is expected to deliver. With quantity risk, the retailer needs also to buy some call options which he will use when demand is higher than expected.

- The electricity sector is characterized by a lot of missing markets. Often options are used to correct these problems.

A third choice is whether these obligations should take the form of financial contracts, where the seller and the buyer transfer a sum of money at the end of the contract, or of physical contracts, where a specific generation plant is associated with a specific contract. Appendix II comes back to the difference between financial and physical options.

This paper

In this paper we will look at the strategic effects of financial call options in a Cournot game. We assume that there are only two markets: a financial call option market with an exogenously determined strike price and a spot market. In our set-up, firms decide themselves how many options they sell. In the model there is no uncertainty, so hedging is not an issue. The number of generators is assumed to be fixed. Hence, we do not look at the entry decision of new generation firms.

The paper is an extension of Allaz and Vila (1993). They showed that in a Cournot game, firms have a strategic reason to sell futures contracts, because futures contracts serve as a commitment device for the firms to obtain a larger market share in the spot market. Selling futures leads to a prisoners' dilemma type of problem. All firms sell futures, and as a result the spot price will decrease. We will use a similar framework as Allaz and Vila to analyze financial call options instead of futures contracts.

Several papers have criticized the assumptions of the Allaz and Vila paper. As we make the same assumptions, these criticisms are also applicable to the current paper.

Allaz and Vila assume Cournot competition in the spot market. The actions of the players are strategic substitutes. Mahenc and Salanie (2004) show that in a Bertrand game, players will take opposite positions in the spot market: they buy their own output instead of selling it.

The result of Allaz and Vila depends also on the assumption that the number of futures contracts a firm signs is observable by all firms. Hughes and Kao (1997) show that if the contract position is not observed by other players, then firms have no longer an incentive to sell futures contracts. [9]

A third key assumption in the model is perfect arbitrage between the contracting stage and the spot market. In practice, arbitrage in the electricity market is, however, far from perfect. The precise reason for imperfect arbitrage and how one should model its effects, remains for further investigation.[10]

Le Coq (2003) looks at whether long term contracts make tacit collusion between generators more likely. As generators interact which each other on an hourly basis, tacit collusion is certainly an issue that has to be addressed. Le Coq shows that some long term contracts might stimulate tacit collusion leading to higher prices.

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[8] Also the regulator can use options to aim its regulation more precisely at periods of high demand, minimizing its intervention in the market.

[9] They show, however, that under uncertainty and risk aversion, there are again strategic reasons to buy or sell futures contracts. The behavior of the firms will depend on the type of uncertainty that the firms face. If demand levels are uncertain, generators will sell futures contracts. If cost levels are uncertain, generators will buy futures contracts.

[10] Arbitrage might be hindered by lack of information, perceived or real regulatory restrictions on arbitrage, and entry barriers in the arbitrage market.

Finally Thille (2003) showed that the results of Allaz and Vila are weakened when storage is possible. However, storage is not possible in electricity.

Relation with Chao and Wilson

The paper is closely related to recent work of Chao and Wilson (2004). They argued that generators should be obliged to sell physical call options to retailers. They see several reasons for this. First, in the long run electricity markets are contestable and thus more competitive. Physical call options might have better strategic effects than futures contracts.[11] The regulation of market power might be easier with physical call options than with futures. And physical delivery makes sure that generation is effectively built.

Chao and Wilson discuss several interesting ideas, but make several modeling assumptions which make a straightforward comparison of futures and call options difficult.

They assume perfect regulation of the number of options that generators have to sell and free entry in the contracting stage. However, with these two assumptions a lot of contract types will give the perfect competitive outcome. A comparison of futures contracts and call options is not possible. In our paper we assume a fixed number of firms, and that generators decide themselves about the number of options they sell.

Chao and Wilson also make non-standard assumptions on the type of options and on the type of competition, which makes a comparison with the Allaz and Vila model very difficult. They assume that generators bid linear supply functions in the production stage, and sell bundles of physical options contracts in the contracting stage. Each bundle consists of one option contract of each possible strike price. Given these assumptions, the intuition of the Allaz and Vila model does not have to be valid.[12]

We assume Cournot competition in the contracting stage, and generators will sell financial call options with only one exogenously determined strike price. We think that a single option is more realistic than a linear bundle of options. It is consistent with what we see in some electricity markets where only a small number of call options is actively traded. The reason why we do not observe a larger number of option contracts is that these markets would not be liquid enough, and that transaction costs would be too high.[13]

Cournot game with futures contracts

This section explains the standard Cournot game and the Cournot game with futures contracts (i.e. the Allaz and Vila model). It presents the set up of the model, and the definition of the main variables. The next section then continues with the Cournot game with financial options.

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[11] See also Appendix II.

[12] An example: In the Allaz and Villa model generators play Cournot in the second stage. Their actions are therefore strategic substitutes. In the Chao and Wilson model they use linear supply functions. Depending on the slope of the supply functions and the demand function, their actions might be strategic substitutes or complements.

[13] With risk-averse market participants and no transaction costs, one would expect a complete set of markets. This would mean that there is a market for each possible contingency. Hence, we would see an option market for each possible strike price, i.e. a continuum of options markets. With transaction costs, market participants trade off the gains of trade and the transaction costs they would incur. If two financial contracts are relatively good substitutes, then often only one will be traded, as most gains of trade can be made in that way, while a second market would greatly increase transaction costs. See Suenaga and Williams (2004).

Our paper considers an oligopoly with two firms $i, j \in \{1,2\}$. Firm i produces q_i units at a production cost $C_i(q_i)$ with $C_i' \geq 0, C_i'' \geq 0$. Total production of both firms is equal to $q_1 + q_2$, and the spot price $P = p(q_1 + q_2)$.

Standard Cournot game

We start with the standard Cournot game without futures contracts, for which we will use the superscript 'C'.

The profit of a firm i is equal to its revenue minus production costs:

$$\pi_i^C(q_i; q_j) = P \cdot q_i - C_i(q_i) \tag{1}$$

In a Cournot game, firm *i* maximizes its profit, by setting its production quantity q_i, taking into account that the price depends on the joint production of the firms. $P = p(q_1 + q_2)$. All firms set their production level q_i simultaneously.

The Nash equilibrium of this game is the intersection of the reaction functions $q_i^C(q_j)$ of the players

$$q_i^C(q_j) = \arg\max_{q_i} \pi_i^C(q_i, q_j) \tag{2}$$

For later reference the equilibrium production quantities and spot price will be denoted by $q_1^{C,eq}, q_2^{C,eq}$ and $P^{C,eq} = p(q_i^{C,eq} + q_j^{C,eq})$.

To illustrate our paper, we will use a numerical example in which the two firms have quadratic cost functions $C_1(q_1) = .1\ {q_1}^2$ and $C_2(q_2) = .2\ {q_2}^2$, and the inverse demand function is equal to $P(q) = 1 - q$. Figure 2 presents the Cournot equilibrium of this game graphically. It shows the strategy space of the generators. The curved lines represent the iso-profit lines of each firm.

$$\pi_i(q_i, q_j) = (1 - q_1 - q_2)q_i - C_i(q_i) = cte \tag{3}$$

They are the indifference curves of each firm. The dashed lines are the contour lines for firm 2 and the solid lines for firm 1.

The reaction functions of firm *1* and 2 cross their iso-profit lines where they are horizontal and vertical respectively. The Nash equilibrium is the intersection of the two reactions functions. To simplify the notation we use vector notation: $\vec{q} = (q_1, q_2)$.

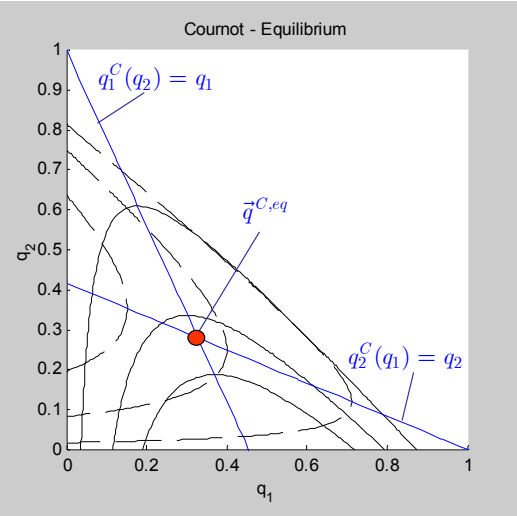

Figure 2 Standard Cournot equilibrium

Cournot game with futures contracts

If there are futures contracts, then we need to model the game with two stages: a contracting stage and a production stage. Figure 3 shows the timing of the game.

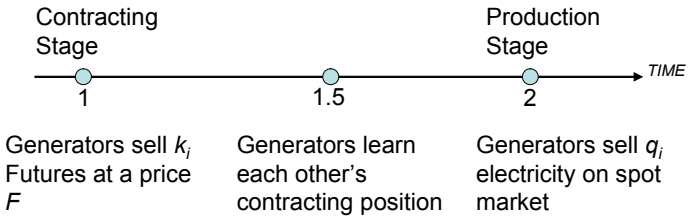

Figure 3 Timing of the Cournot game with futures

In the first stage, the *contracting stage*, generators sell futures contracts to retailers. The generators sell the contracts in a Cournot fashion: i.e. they decide simultaneously about the number of contracts k_i they sell, taking the number of contracts of the competitor as given.

Each futures contract is a two-sided insurance contract which insures the price of one unit of electricity. If the spot price P is above the futures price F, then the generator will refund the retailer the difference of the spot price P and the futures price F. If the spot price is below the futures price, then the retailer will pay the generator the difference between the futures price and the spot price. The total payment of generator i is thus $k_i(P-F)$.[14]

After the first stage and before the second stage, each firm learns the contract position of the other firms. In Figure 3 this happens at time = 1.5. There is therefore perfect information at the beginning of the second stage.

In the second stage, the *production stage*, the firms simultaneously set their production level q_i in a Cournot fashion. Each firm will take its own and its competitor's contracting position as given.

Firm i's profit is equal to revenue in the spot market, minus production costs and payments related to the futures contracts. The superscript *'F'* denotes the game with futures contracts.

$$\begin{aligned} \pi_i^F(q_i;q_j,k_i) &= P\cdot q_i - C_i(q_i) - k_i(P-F) \\ P &= P(q_i+q_j) \end{aligned} \qquad (4)$$

We will solve the game by backward induction and derive first the Nash Equilibrium in the second stage of the game as a function of the number of futures sold in the first stage $\vec{q}^{F,eq}(\vec{k})$. This is schematically presented in Figure 4. After deriving the second stage equilibrium, we will derive the reduced profit function of the first stage and solve the equilibrium of the first stage $\vec{k}^{F,eq}$.

[14] This discussion explains the futures contract as a financial insurance contract. An alternative explanation considers the futures contract as a physical contract where the generator sells k_i units in the first stage at a price F, $q_i - k_i$ units in the second stage at a price P, and produces q_i. Generator i's profit becomes $F\cdot k_i + P\cdot(q_i - k_i) - C(q_i)$, which is equivalent with equation (4). It can be shown that financial and physical futures are equivalent, but that this is not the case for financial and physical options.

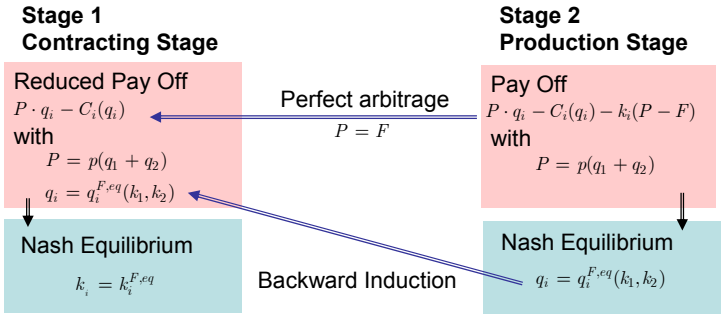

Figure 4 Cournot game with futures contracts and backward induction

Second Stage

Firm i maximizes its profit by setting its production q_i taking into account the number of futures contracts which itself and its competitor own (??). The best response functions of the second stage are

$$q_i^F(q_j, k_i) = \arg\max_{q_i} \pi_i^F(q_i, q_j, k_i) \tag{5}$$

Note that firm i's second stage reaction function does *not* depend on the number of futures that firm j owns, as also its profit function (4) does not depend on it.

The fact that firm i owns futures contracts, changes its incentives to produce in the second stage. Firm i needs to refund buyers of the futures contract for high strike prices. It has therefore less interest in high spot prices, and produces more in the second stage of the game.

$$\frac{\partial q_i^F(q_j, k_i)}{\partial k_i} \geq 0 \tag{6}$$

Hence, owning futures contracts makes a firm more aggressive in the second stage, i.e. it produces more, and its reaction function moves outwards. This effect is based on exactly the same intuition as Coase's conjecture as explained in Figure 1.

Figure 5 shows this effect in the numerical example. The solid and the dashed curved lines are the iso-profit lines of firm 1 when k_1 units were sold in the contracting stage. ($\pi_1^F(q_1, q_2, k_1) =$ constant) and when no futures were sold ($\pi_1^C(q_1, q_2) =$ constant) respectively. The best reaction function of firm 1 $q_1 = q_1^F(q_2, k_1)$ crosses its iso-profit lines where they are horizontal. It shifts to the right when more futures contracts are sold.

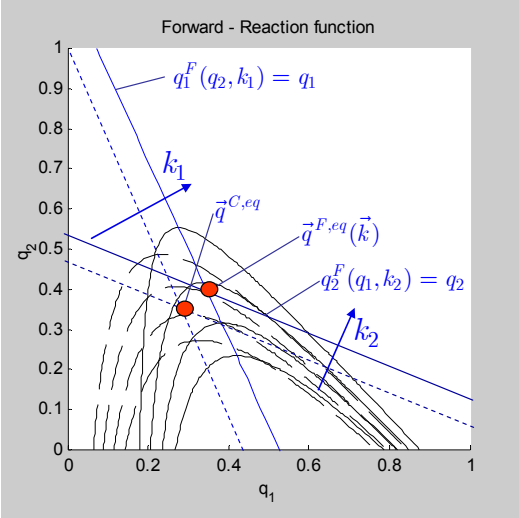

Figure 5 Second stage equilibrium in the Cournot game with futures contracts

The second stage Nash equilibrium is defined by the intersection of the best response functions. Given that k_i, k_j futures contracts are sold in the first stage, the equilibrium quantities are $q_i^{F,eq}(k_i,k_j)$, and the equilibrium price is $p^{F,eq}(k_i,k_j)$.

Figure 5 shows further how the reactions functions of *both* firms shift out with larger numbers of forwards. The intersection of the reaction functions is the second Nash equilibrium. Note that the *equilibrium quantity* that is produced by firm i depends on the number of forwards sold by *both* firms.

First Stage

In the first stage the firms maximize their profit (4)

$$P \cdot q_i - C_i(q_i) - k_i(P - F) \tag{7}$$

taking into account that q_i is determined by the second stage behavior of the firms, $q_i = q_i^{F,eq}(k_i,k_j)$ and that the price is determined by the demand function $P = p(q_1 + q_2)$.

We now need to define how the futures price $F(\vec{k})$ depends on the number of futures sold by the generators, i.e. the inverse demand function for futures in the contracting stage.

Allaz and Vila, assume perfect arbitrage between the contracting and the production stage. This means that there is no profit to be made by arbitraging between the spot market and the futures market, i.e.

$$F(k_1,k_2) = p^{F,eq}(k_1,k_2) \tag{8}$$

Arbitrageurs are not modeled in the Allaz and Vila paper, and they will not be modeled in this paper either. Future work could look at alternative assumptions for the demand function for futures. Note that (8) implies that arbitrageurs correctly anticipate the strategic effects of the futures contracts on the spot price P.

Define the first stage reduced pay-off function of generator i as $\Pi_i^F(k_i,k_j)$. This is the profit firm i will obtain when the players sell $\vec{k}$ futures contracts in the first stage, and play Cournot in the second stage:

$$\Pi_i^F(k_i,k_j) = P \cdot q_i - C_i(q_i) - k_i(P - F) \tag{9}$$

where P, $\vec{q}$ and F are determined by the following equations:

$$
\begin{aligned}
q_i &= q_i^{F,eq}(k_i, k_j) \\
P &= p(q_i, q_j) \\
F &= P
\end{aligned}
\tag{10}
$$

The first equation of (10) is the second stage equilibrium condition, the second equation is the inverse demand function for electricity in the spot market, and the third equation is the arbitrage condition. Note that given perfect arbitrage, a firm has no pecuniary reason to buy or sell futures contracts. There is only a strategic reason to sell futures. This can easily be shown by substituting the perfect arbitrage condition in the objective function of the firm. The objective function of the firm depends only indirectly on the number of futures sold.

By selling more futures in the first stage, a generator can change the second stage equilibrium. Figure 6 shows this for firm 1. In Figure 6A, firm 1 does not sell futures; while in Figure 6B it sells k_1 futures. By selling futures, total production in the second stage is increased, leading to a lower price. This influences firm 1's profit negatively. However, selling futures increases the market share of firm 1, which increases profit.

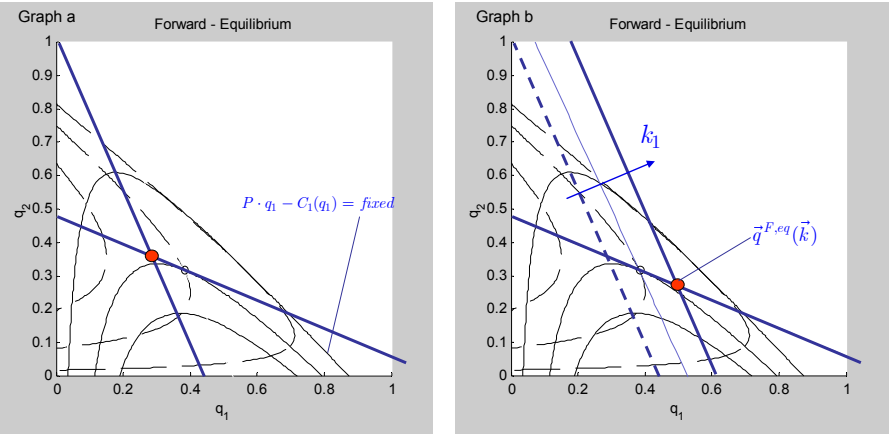

Figure 6 Impact of selling more futures by firm 1, on the second stage equilibrium.

At the optimal number of futures $k_1^F(k_2)$ both effects are balanced. This trade-off defines the first stage reaction functions of both firms (Figure 7).

$$
k_i^F(k_j) = \arg\max_{k_i} \Pi_i^F(k_i, k_j) \tag{11}
$$

The equilibrium in the first stage is determined by the intersection of the first stage reaction functions. The equilibrium number of futures contracts are $k_i^{F,eq}$ and $k_j^{F,eq}$.

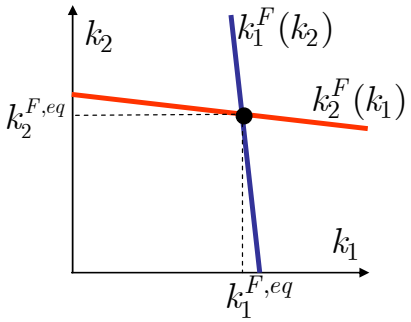

Figure 7 Equilibrium in the first stage of the Cournot game with futures contracts

Figure 8 shows the second stage equilibrium when the generators sell the equilibrium futures quantities $\vec{k}^{F,eq} = (k_1^{F,eq}, k_2^{F,eq})$. In the equilibrium point both reaction functions are tangent to the iso-profit lines of their competitor. For further reference we define the Allaz and Vila spot price as:

$$P^{AV} = p^{F,eq}(k_i^{F,eq}, k_j^{F,eq}) \tag{12}$$

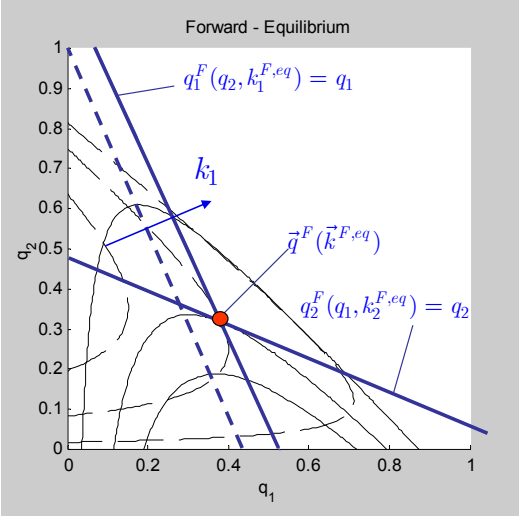

Figure 8 Allaz and Vila equilibrium

Cournot game with options

This section discusses the Cournot game with financial options. We will use the superscript 'O' to denote this game. As already mentioned above, we assume that only financial call options with an exogenously fixed strike price S are traded. The call option is a one-sided insurance contract which insures retailers against price increases above the strike price S. If the spot price is above the strike price, then the generator will refund the retailer the difference between the spot price and the strike price: $P - S$. When the spot price is below strike price, then there is no payment. In short, the generator pays the retailer the amount $V(P)$ with

$$V(P) = \max\{P - S, 0\} \tag{13}$$

The Cournot game with financial options will be modeled similarly to the Cournot game with Futures contracts. In the first stage, the contracting stage, firms sell financial call options in a Cournot fashion: Firm *i* sells k_i options at a price F. After the first stage each firm learns about the contract position of the other firms. In the second stage, the production stage, the firms simultaneously decide about their production level in the spot market.

Second Stage : reaction functions

Firm i's profit is equal to the sum of its profit in the physical market (revenue in the spot market minus production costs) and its profit in the financial market:

$$\pi_i^O(q_i; q_j, k_i) = P \cdot q_i - C_i(q_i) + k_i \cdot (F - V(P))$$
$$\text{with} \tag{14}$$
$$P = P(q_i + q_j)$$

The best response function of firm i maximizes profit (14).

$$q_i^O(q_j; k_i) = \arg\max_{q_i} \pi_i^O(q_i, q_j, k_i) \tag{15}$$

Figure 9 shows the reaction functions of the standard Cournot game (graph A), the second stage of the Cournot game with futures (graph B), and the second stage of the Cournot game with options contracts (graph C). The -45 degrees downward sloping line in graph C is the set of production quantities at which the spot price is equal to the strike price ($p(q_1 + q_2) = S$). For production quantities above this line, the spot price is below the strike price $p(q_1 + q_2) < S$. For production quantities below this line, the spot price is above the strike price $p(q_1 + q_2) > S$. We will now describe the best response functions of the generators.

If the spot price is above the strike price, in which case the option is said to be "in-the-money", the option behaves as a standard Allaz and Vila futures contract. The best response function of the firm is the best response function of a firm who sold k_i futures contracts $q_i^F(q_j;k_i)$. This is the South-West corner in Figure 9C ($p(q_1 + q_2) > S$).

If the spot price is below the strike price, in which case the option is said to be "out-the-money", the option has no strategic effect. The decision of firm i is not influenced by the number of options that it sold. Its best response function is equal to $q_i^C(q_j)$, the standard Cournot reaction function. This is the north-east corner in Figure 9C ($p(q_1 + q_2) < S$).

If the spot price is equal to the strike price, a generator will sell a quantity between the optimal quantity with futures and the standard Cournot quantity.

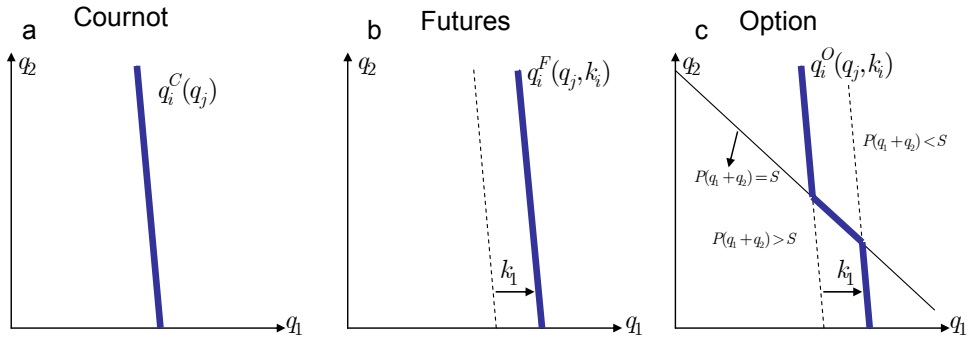

Figure 9 Reaction function of the standard Cournot game, the Cournot game with futures, and the Cournot game with option contracts.

Formally, the best response function for a firm that sold k_i option contracts is described by

$$q_i^O(q_j,k_i) = \begin{cases} q_i^C(q_j) & \text{IF} \quad q_i^C(q_j) + q_j > q^S \\ q_i^F(q_j,k_i) & \text{IF} \quad q_i^F(q_j,k_i) + q_j < q^S \\ q^S - q_j & \text{otherwise} \end{cases} \tag{16}$$

where $q^S = p^{-1}(S)$ is the demand level at which the spot price is equal to the strike price. Equation (16) describes the three parts of the reaction function in Figure 9C.

Second stage: Nash Equilibrium

The Nash equilibrium in the second stage of the game is defined by the intersection of the reaction functions.

If the strike price is higher than the Cournot price $p^{C,eq} < S$, then, independently of the number of options sold in the first stage, the second stage equilibrium is the standard Cournot equilibrium.

$$\vec{q}^{O,eq}(k_i,k_j) = \vec{q}^{C,eq} \tag{17}$$

The options are out–the–money and have no strategic effect. This is shown graphically in Figure 10, which shows that the number of options sold by firm 1 has no effect on the second stage equilibrium.

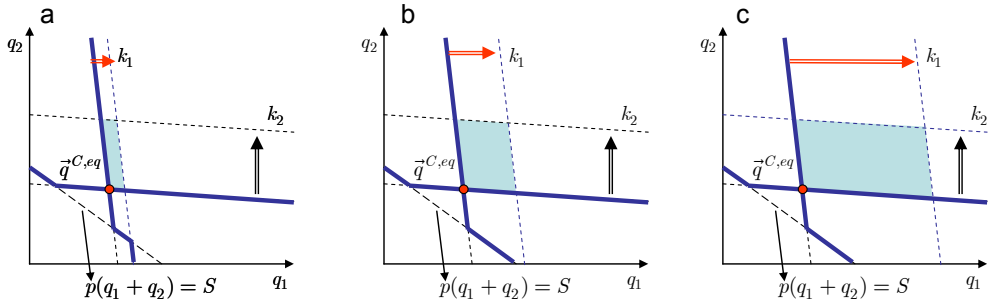

Figure 10 Impact of options sold by firm 1 on the second stage equilibrium, High Strike price.

If the strike price is below the Cournot price $p^{C,eq} > S$, then the second stage equilibrium $\vec{q}^{O,eq}(\vec{k})$ depends on the number of options sold in the first stage. Figure 11 shows the impact of the number of options sold by generator 1 on the second stage equilibrium.

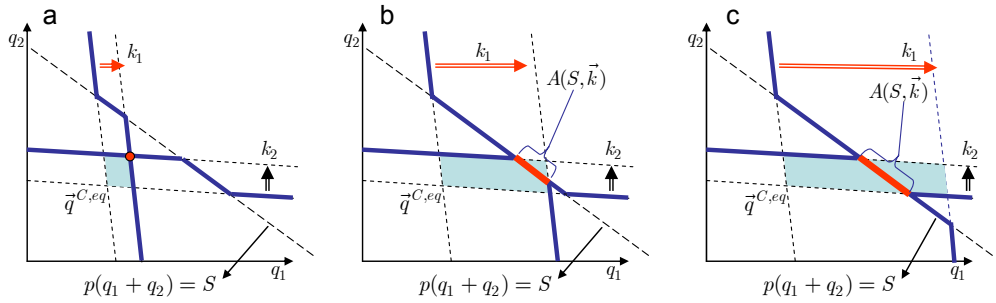

Figure 11 Impact of options sold by firm 2 on the second stage equilibrium, Low Strike price.

If a *small number of options* is sold ($P^{F,eq}(k_i,k_j) > S$), then the options will be in-the-money in the second stage, and have the same strategic effect as futures contracts. See Figure 11A. The second stage equilibrium with options is the same as with futures contracts:

$$\vec{q}^{O,eq}(\vec{k}) = \vec{q}^{F,eq}(\vec{k}) \tag{18}$$

If a *large number of options* is sold in the first stage ($P^{F,eq}(k_i,k_j) < S$), then generators will increase their output in the spot market, until the price drops to the strike price of the option. None of the firms will increase their output further, as the option is out-the-money. There is a set of equilibriums $A(S,\vec{k})$ in the second stage of the game

$$\vec{q}^{O,eq}(\vec{k}) \in A(S,\vec{k}) \tag{19}$$

each with a spot price $P = S$, and where the output of both firms is in between the reaction function of the standard Cournot game and the Cournot game with futures. Formally, $A(S,\vec{k})$ is described as follows:

$$A(S,\vec{k}) = \left\{ \vec{q} \;\middle|\; \begin{array}{l} P(q_1+q_2) = S \\ q_1^C(q_2) \le q_1 \le q_1^F(q_2,k_1) \\ q_2^C(q_1) \le q_2 \le q_2^F(q_1,k_2) \end{array} \right\} \tag{20}$$

If a lot of options are sold, we will say that the option market is ‘flooded’. See Figure 11B and Figure 11C.

Figure 12 shows the second stage equilibrium as a function of the number of options sold by both firms and of the strike price. In Figure 12A the strike price is high, and the generators play the standard Cournot equilibrium. In Figure 12B, the strike price is low, and the equilibrium type depends on the number of options sold in the first stage.

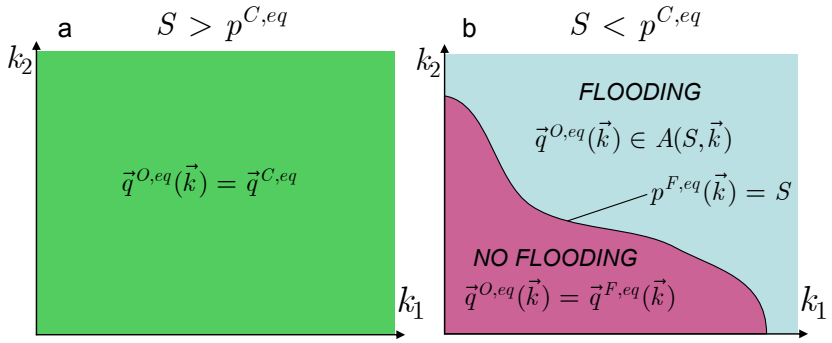

Figure 12 Equilibriums in the second stage as a function of the number of options sold in the first stage $\vec{k}$ and the spot price S.

First Stage : Profit

As before we assume that there is perfect arbitrage between the contracting stage and the production stage; the price of the option F is therefore equal to the pay-out of the option:

$$F = V(P) \tag{21}$$

Given arbitrage, the reduced first-stage profit function of the firm simplifies to

$$\Pi_i^O(k_i, k_j) = P \cdot q_i - C_i(q_i)$$
$$\text{with} \tag{22}$$
$$P = p(q_i + q_j)$$

where the quantities are determined by the second stage Nash equilibrium. The profit of a firm depends only indirectly on the number of options it sells, through the dependencies of q_i and q_j on the number of option contracts sold.

First Stage: Equilibrium

The fact that there are several Nash equilibriums in the second stage if a lot of options are sold and the strike price is low (See Figure 12B), complicates the solution of the first stage. As there is no obvious focal point in the second stage, it is hard to predict how the second stage behavior of the players depends on their first stage commitments. In particular, firms can use one second stage equilibrium as a 'punishment equilibrium', for 'misbehavior' of a firm in the first stage, and play another equilibrium when both firms behave. 'Punishment' strategies are credible as they are Nash equilibriums in the second stage of the game.[15]

In order to narrow down the number of equilibriums, we assume that generators co-ordinate on the equilibrium with the highest price. We think this is a valid assumption because of the following reasons:

First, the low price equilibriums rely on the fact that both generators extensively flood the market in the first stage of the game, i.e. the number of options they sell by far exceeds their actual production in the second stage. This type of behavior is very risky. It is however a Nash Equilibrium because the generators perfectly co-ordinate their production in the second stage on one of the infinite number of equilibriums and make sure that the spot price is

[15] Appendix III shows the large set of equilibriums of the game.

exactly equal to the strike price. If the generators do not succeed in co-ordinating on a Nash Equilibrium in the second stage, they might end up losing a lot of money. Furthermore, if we add some uncertainty to the model, this extensive flooding becomes even more risky.

Second, as generators play the game regularly, it is likely that generators will learn how to play the equilibrium with the highest price and a lower risk.

Before we derive and prove the equilibrium of the game, we first discuss the results. Figure 13 shows the equilibrium spot price as function of the strike price of the financial option.

- If the strike price is above the Cournot price, then options are never in-the-money, and they have no strategic effect. We obtain the Cournot outcome. (Segment A in Figure 13)
- If the strike price is below the price of the equilibrium that would prevail with futures (cfr. Allaz and Vila), then the generators co-ordinate on the Allaz and Vila equilibrium. The options are in-the-money, and have the same strategic effect as futures. (Segment B in Figure 13)
- If the strike price is between the Allaz and Vila and the standard Cournot price, then the firms will make sure that the spot price is equal to the strike price. (Segment C in Figure 13)

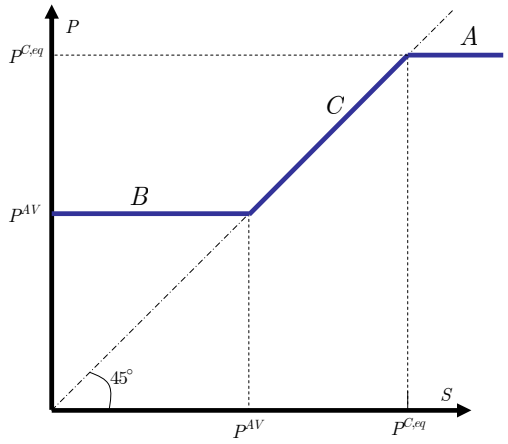

Figure 13 Price in the equilibrium as a function of the strike price.

Figure 13 clearly shows that the price with futures is (weakly) lower than with options. Options have the same effect as futures when their strike price is low.

In the appendix we will prove that Figure 13 represents indeed the equilibriums of the game. The proof consists of the following four theorems:

Theorem 1: If the strike price is above the standard Cournot price ($S > p^{C,eq}$), then the Cournot equilibrium is the equilibrium of the game.

Theorem 2: If there exists an equilibrium with a spot price above the strike price, then it has to be the Allaz and Vila equilibrium.

Theorem 3: If the strike price is below the Allaz and Vila price P^{AV}, then the Allaz and Vila equilibrium is an equilibrium.

Theorem 4: If the strike price is between the Allaz and Vila price and the standard Cournot price, then flooding is an equilibrium. The spot price becomes equal to the strike price.

Theorem 1 defines the equilibrium price when the strike price is above the Cournot price.

Theorem 2 and 3 can be used to show that for a strike price below the Allaz and Vila price, the Allaz and Vila equilibrium is an equilibrium (Theorem 3) and that there are no other equilibriums which would give a higher spot price. (Theorem 2)

Theorem 2 and 4 can be used to show that for intermediate prices, the generators will flood the market, and set the spot price equal to the strike price (Theorem 4) and that there are no other equilibriums which would give a higher spot price. (Theorem 2)

Conclusion

This paper discusses the efficiency effects of options in a Cournot oligopoly, extending the work of Allaz and Vila. Instead of looking at futures contracts, it looks at financial call options. It is assumed that only options with one specific strike price are traded. This strike price is exogenous in the model.

Options might be better than futures because they allow the players to hedge quantity risk, they allow more precise regulation of market power, and they solve some of the problems which are associated with the missing markets in the electricity market. This paper does not look at these effects and concentrates on the strategic effects of options when generators decide endogenously on the number of options.

We show that options make markets more competitive but to a smaller extent than futures. The precise effects depend on the strike price of the options. If the strike price is high, financial options have no effect on the efficiency of markets because they are out–the–money. If the strike price is intermediate, firms will sell a lot of options in the first stage, and flood the market. The equilibrium price is equal to the strike price. For low strike prices, options have the same strategic effect as futures, firms will sell the same number of futures as in the Allaz and Vila model.

In order to restrict the number of equilibria in the game, we assumed that generators will co-ordinate on the equilibriums which lead to the highest price in the first stage of the game. In theory they could also co-ordinate on other equilibriums, but we think these equilibriums are less likely to occur in practice.

References

Allaz, B. and Vila, J. L. (1993). "Cournot Competition, Forward Markets and Efficiency." Journal of Economic Theory, 59, 1-16.

Borenstein, S., Bushnell, J. B., and Wolak, F. A. (2002). "Measuring Market Inefficiencies in California's Restructured Wholesale Electricity Market." American Economic Review, 92(5), 1376-1405.

Bushnell, J., Mansur, E., and Saravia, C. (2004). "Market Structure and Competition: A Cross-Market Analysis of U.S. Electricity Deregulation." Rep. No. CSEM WP-126, University of California Energy Institute.

Chao, H.-P. and Wilson, R. (2004). "Resource Adequacy and Market Power Mitigation via Option Contracts." University of California Energy Institute Power Conference.

Coase, R. H. (1972). "Durability and Monopoly." Journal of Law & Economics, 15(1), 143-149.

Creti, A. and Fabra, N. (2004). "Capacity Markets for Electricity." Rep. No. CSEM WP-124, University of California Energy Institute.

Hughes, J. S. and Kao, J. L. (1997). "Strategic Forward Contracting and Observability." International Journal of Industrial Organization, 16(1), 121-133.

Joskow, P. L. and Kahn, E. (2002). "A Quantitative Analysis of Pricing Behavior in California's Wholesale Electricity Market During Summer 2000." Energy Journal, 23(4), 1-35.

Le Coq, C. (2003). "Long-Term Supply Contracts and Collusion in the Electricity Markets." Rep. No. 552, Stockholm School of Economics.

Mahenc, P. and Salanie, F. (2004). "Softening Competition Through Forward Trading." Journal of Economic Theory, 116(2), 282-293.

Suenaga, H. and Williams, J. (2004). "The Natural Number of Forward Markets for Electricity." University of California Energy Institute Power Conference.

Thille, H. (2003). "Forward Trading and Storage in a Cournot Duopoly." Journal of Economic Dynamics & Control, 27(4), 651-665.

Wolak, F. A. and Patrick, R. (2001). "The impact of Market Rules and Market Structure on the Price Determination Process in England and Wales Electricity Market." Rep. No. Working paper 8248, NBER.

Wolfram, C. D. (1999). " Measuring Duopoly Power in British Electricity Spot Market." American Economic Review, 89(4), 805-826.

Appendix I : Proofs

Theorem 1

If the strike price is above the standard Cournot price ($S > p^{C,eq}$), then the Cournot equilibrium is the equilibrium of the game.

The second stage equilibrium is equal to the standard Cournot equilibrium independent of the number of options sold,

$$\vec{q}^{O,eq}(k_i, k_j) = \vec{q}^{C,eq} \tag{23}$$

This is therefore also the equilibrium of the two-stage game. This result is trivial.

Theorem 2

If there exists an equilibrium with a spot price above the strike price, then it has to be the Allaz and Vila equilibrium

Any equilibrium of the two-stage game has to be an equilibrium in the second stage of the game. The spot price can therefore only be above the strike price if the players do not flood the market, and sell a small number of options. See Figure 12.

Assume there exists an equilibrium of the first stage of the game $\vec{\hat{k}}$, such that the players do not flood the market in the second stage($P^{F,eq}(k_1, k_2) > S$). We have to prove that this equilibrium is the Allaz and Vila equilibrium $\vec{\hat{k}} = \vec{k}^{F,eq}$.

Given that $\vec{\hat{k}}$ is an equilibrium of the game, $\hat{k}_i$ is locally optimal for firm *i*, with other words $\frac{\partial \Pi_i^{O,eq}(\vec{\hat{k}})}{\partial k_i} = 0$. As only a small number of options are sold, the profit of firm *i* is the same as when the firm would sell futures instead of options:

$$\Pi_i^{O,eq}(\vec{\hat{k}}) = \Pi_i^{F,eq}(\vec{\hat{k}})$$

Hence, $\hat{k}_i$ is also the local optimum of $\Pi_i^{F,eq}(\vec{\hat{k}})$:

$$\frac{\partial \Pi_i^{F,eq}(\vec{\hat{k}})}{\partial k_i} = 0 \tag{24}$$

If the function $\Pi_i^{F,eq}(k_i, k_j)$ is concave in k_i, it follows that $\hat{k}_i$ is also a global optimum of $\Pi_i^{F,eq}(\vec{k})$. Hence, $\vec{\hat{k}}$ is the Nash Equilibrium of the Cournot game with futures contracts: $\vec{\hat{k}} = \vec{k}^{F,eq}$.

Theorem 3

If the strike price is below the Allaz and Vila price $S < P^{AV}$, then the Allaz and Vila equilibrium is an equilibrium.

We will prove that playing $\vec{k} = \vec{k}^{F,eq}$ is an equilibrium of the first stage of the game, when the players play a punishment equilibrium in the second stage of the game. The punishment equilibrium ensures that unilateral

flooding of the market is not optimal. We will now explain the punishment equilibrium, and then we show that it is an equilibrium in the first stage.

Figure 14 shows the strategy space of second stage of the game. In the first graph, both firms sell the equilibrium quantities $k_i^{F,eq}$. In the second graph, firm 1 deviates from the first stage equilibrium and floods the market. If the market is flooded, there are an infinite number of equilibriums. In the punishment equilibrium, the firms co-ordinate on that equilibrium which gives the deviator the smallest market share. Note that once firm 1 has flooded the market, selling even more options has no effect on the second stage equilibrium.

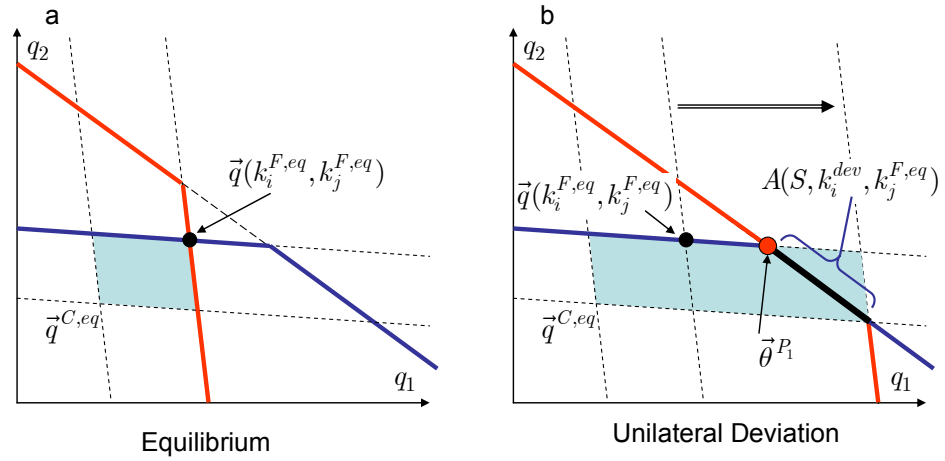

Figure 14 Deviation of $\vec{k}^{F,eq}$ by firm 1

Formally, if firm i unilaterally floods the market, the firms will co-ordinate on the punishment equilibrium $\vec{q}^{P_i}(\vec{k})$ in the second stage, where $\vec{q}^{P_i}(\vec{k})$ is defined by the following two equations

$$\begin{aligned} p(q_i^{P_i} + q_j^{P_i}) &= S \\ q_j^{P_i} &= q_j^F(q_i^{P_i}, k_j^{F,eq}) \end{aligned} \tag{25}$$

We will now verify that $\vec{k}^{F,eq}$ is an equilibrium in the first stage, given the punishment equilibrium $\vec{q}^{P_i}(\vec{k})$ in the second stage. Figure 15 shows the first stage strategy space. We need to prove that when player 2 plays $k_2^{F,eq}$, it is optimal for player 1 to play $k_1^{F,eq}$.

If firm 1 does not flood the market, i.e. it sells fewer options than in the point B, then its profit $\Pi^O(k_1, k_2^{F,eq})$ is the same as with future contracts.

$$\Pi^O(k_1, k_2^{F,eq}) = \Pi^F(k_1, k_2^{F,eq}) \tag{26}$$

If firm 1 floods the market by selling more options than in point B, then it will obtain the same profit as in point B; its profit $\Pi^O(k_1, k_2^{F,eq})$ becomes:

$$\Pi^O(k_1, k_2^{F,eq}) = \Pi^F(k_1^B, k_2^{F,eq}) \tag{27}$$

This is a direct consequence of the fact that once the market is flooded, selling more options has no effect on the second stage equilibrium, and hence the profit level of firm 1.

As $k_1^{F,eq}$ maximizes $\Pi^F(k_1, k_2^{F,eq})$, first stage deviation is not profitable for firm 1.

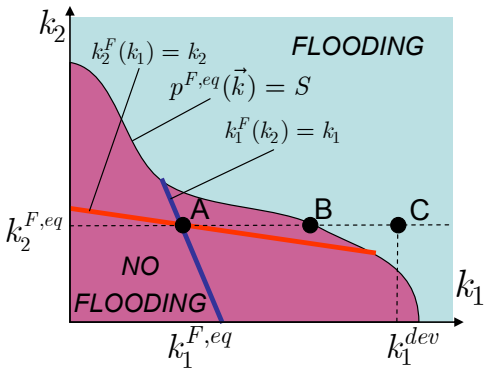

Figure 15 Deviation from first type of equilibrium

Theorem 4

If the strike price is between the Allaz and Vila price and the standard Cournot price $P^{C,eq} < S < P^{AV}$, than flooding is an equilibrium.

We will prove that any $\vec{q}^* \in \overline{A}(S) \cap B$ is a second stage equilibrium outcome, where $\overline{A}(S)$ and B are defined as

$$\overline{A}(S) = \left\{ \vec{q} \middle| \begin{array}{l} P(q_1 + q_2) = S \\ q_1^C(q_2) \leq q_1 \\ q_2^C(q_1) \leq q_2 \end{array} \right\} \tag{28}$$

and

$$B = \left\{ \vec{q} \middle| \begin{array}{l} C_1'(q_1) \leq P(q_1 + q_2) \\ C_2'(q_2) \leq P(q_1 + q_2) \end{array} \right\} \tag{29}$$

$\overline{A}(S)$ is the set of production quantities where each player produces more than his Cournot best response function, and where the spot price is equal to the strike price. B is the set of production quantities where for each generator the spot price is higher than his marginal cost. For all production quantities in B each generator would like to obtain a larger market share given constant prices.

We will assume for now that both firms sell a very large number of option contracts $k_i \to \infty$, in such way that when one firm unilaterally decides to sell fewer contracts, the remaining number of contracts still floods the market.[16] If a lot of options are sold in the first stage, then the set of equilibriums in the second stage $A(S, \vec{k})$ becomes equal to $\overline{A}(S)$

$$\lim_{k_i \to \infty} A(S, \vec{k}) = \overline{A}(S) \tag{30}$$

Any element $\vec{q}^* \in \overline{A}(S)$ is thus a second stage equilibrium of the game.

We will now prove that extensively flooding of the market ($k_i \to \infty$) is a first stage equilibrium. Therefore we need to make an assumption on the equilibrium the generators will co-ordinate on in the second stage of the game; also off-the-equilibrium path. We assume that the players co-ordinate on $\vec{q}^*$, as long as it is a second stage

[16] Analytically this means that $P^{F,eq}(0, k_j) < S$ and $P^{F,eq}(k_i, 0) < S$.

equilibrium, and co-ordinate on any other second stage equilibrium when $\vec{q}^*$ is not an equilibrium in the second stage.

Figure 16A shows the strategy space of the second stage when the players both extensively flood the market. We will now prove that it is not profitable for firm 1 to deviate from this equilibrium. For small deviations in the first stage, the firms will still play the equilibrium $\vec{q}^* = (q_1^*, q_2^*)$ in the second stage, as shown in Figure 16B. Firm 1 will thus not increase its profit. For larger deviations, the set of second stage equilibriums becomes smaller $A(S,\vec{k}) \subset \overline{A}(S)$. As a result the equilibrium $\vec{q}^*$ is no longer possible. In any of the second stage equilibriums, firm 1's market share decreases while firm 2's market share increases. This has a negative impact on firm 1's profit as we assumed that $\vec{q}^* \in B$. Also, large deviations are thus not profitable. See Figure 16C.

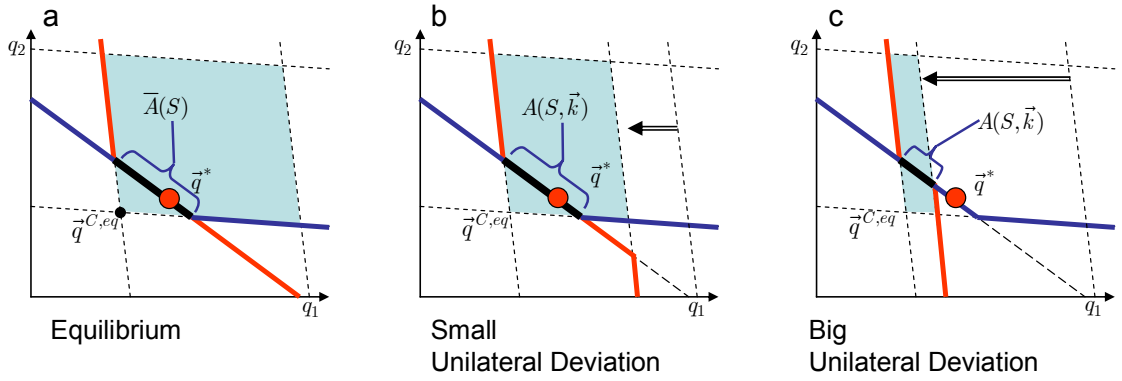

Figure 16 Unilateral deviation by firm 1.

To conclude the proof, we need to show that the set $\overline{A}(S) \cap B$ is not empty for $P^{AV} < S < P^{C,eq}$. Take production quantities $\vec{q}^* = (q_1^*, q_2^*)$ such that each firm produces more than the Cournot quantity and less than Allaz and Vila quantity

$$q_i^{C,eq} \leq q_i^* \leq q_i^{F,eq}(\vec{k}) \tag{31}$$

and such that the spot price equal is equal to the strike price

$$q_1^* + q_2^* = q_S \tag{32}$$

with $P(q_S) = S$.

Given that the strike price is between the Allaz and Vila price and the Cournot price ($P^{AV} < S < P^{C,eq}$) it follows that

$$\begin{aligned} q^S &\geq q_1^{C,eq} + q_2^{C,eq} \\ q^S &\leq q_1^{F,eq}(\vec{k}^{F,eq}) + q_2^{F,eq}(\vec{k}^{F,eq}) \end{aligned} \tag{33}$$

Hence a $\vec{q}^*$ which satisfies (31) and (32) certainly exists. We now will prove that $\vec{q}^* \in \overline{A}(S) \cap B$

$\vec{q}^*$ is an element of $\overline{A}(S)$ when $q_i^C(q_j^*) \leq q_j^*$. This is the case given the following derivation:

$$q_i^C(q_j^*) \underset{(1)}{\leq} q_i^C(q_j^{C,eq}) \underset{(2)}{=} q_i^{C,eq} \leq q_j^* \tag{34}$$

The first step is valid because the reaction function is downward sloping and the second step is valid because $\vec{q}^{C,eq}$ is the Cournot equilibrium.

$\vec{q}^{*}$ is an element of B when the spot price is larger than the marginal cost. This is the case given the following derivation:

$$C_i^{'}(q_i^{*}) \underset{(1)}{\leq} C_i^{'}(q_i^{F,eq}) \underset{(2)}{\leq} P^{AV} \leq S \qquad (35)$$

The first step is valid because the marginal cost function is upward sloping and the second step is valid because generators will choose in the Cournot game with futures an allocation where the price is above the marginal costs. This proves the existence of the equilibriums.

Appendix II : Difference between Financial and Physical Call Options

As mentioned before we will now discuss the difference between financial and physical contracts on market power. Most authors implicitly assume that both contracts are equivalent. This is true for futures contracts, but it is not the case for option contracts as we will show now.

Above we discussed financial option contracts; we will now discuss physical option contracts and show the difference with financial options.

The main difference is that a physical option is associated with a specific generation plant. If a retailer buys a physical option in the contracting stage, he takes an option on one MW of production capacity of a specific plant, i.e. he reserves the generation capacity. In the production stage, the retailer can decide to run 'his' plant and produce electricity. For this electricity he will pay the strike price of the option S. The generators can only use the generation plants which are not reserved by retailers to sell electricity in the spot market. They are thus not allowed to use the generation plants that were already committed in the physical call options.

As a result of selling physical options, a generator does no longer decide on the production level of all his plants. He sold the "right to set the production level" of the reserved generation plants to the retailers. Retailers will turn on these generation plants when the spot price P increases above the strike price S.

The advantage of physical options is that because specific production plants are assigned to the contracts, the probability that electricity is physically delivered increases.

Physical option contracts can be modeled in a two-stage game as we did with financial options. In the first stage, generators sell physical call options and assign some production capacity to their commitment. In the second stage retailers decide about the production level of the plants they reserved, and generators decide about the production level of the plants which were not reserved. Market clearing will determine the spot price for electricity.

Because retailers turn on plants when the spot price is above the strike price, the production level of a generator increases with the spot price. Hence, a generator will produce more, when the sales of the other generators in the market decrease.

When generators decide about the production level of the unreserved plants in the spot market they will take into account that the total production level of the other generators is not constant. The residual demand function that a generator faces, becomes flatter when other generators sell physical call options.[17] This leads to a more competitive behavior by the players in the second stage. The best response function of a generator i is a function of the options sold by the firm itself and by competitors. Note that this is different with financial options. There, the best response function depends only on the number of options sold by the firm itself.

[17] See also Chao and Wilson (2004).

The price of physical options is determined by arbitrage. The value of a reservation of one MW of production capacity is equal to the spot price minus the strike price when the option is in–the–market, and zero when it is out-the-market.[18]

$$F = \max\{P - S, 0\}$$

When there is perfect arbitrage, then the direct (i.e. the monetary) value of a financial option is zero for a firm. With physical options, this is no longer the case. By selling an option a generator receives the amount F in the contracting stage. In the production stage he receives the strike price S and incurs a cost $C_i^{'}(q_i)$ if the option is in-the-money. If the option is out-the-money, he does not receive a payment, but also does not incur production costs. The monetary value of a physical option for generator i is

$$F - 1_{\{P-S\}} \cdot (S - C_i^{'}(q_i))$$

with $1_{\{x\}}$ the indicator function. When x is positive, then $1_{\{x\}} = 1$, otherwise it is zero. Given perfect arbitrage, we can rewrite the monetary value of the option as the price-cost margin if the option is in-the-money and zero otherwise

$$1_{\{P-S\}}\{P - C_i^{'}(q_i)\}$$

We did not succeed in finding an equilibrium of the Cournot game with physical options and a fixed strike price. The second stage profit functions of the players are more intertwined than with financial options, and there is the extra monetary reason to sell options in the first stage.

In a working paper[19], I compare bundles of physical and financial call options of the Chao and Wilson type. Generators sell bundles of options, where each bundle consists of one option of each possible strike price. I show that with this type of contracts, financial options are better than futures and physical options are worse.

Appendix III : Numerical example

This section uses the numerical example to illustrate the wide range of equilibriums in the game when no assumption is made on the co-ordination of the generators.

Figure 17 presents the equilibriums of the game graphically. It shows the strategy space of the second stage of the game. The curved lines represent the iso-profit lines of each firm.

$$(1 - q_1 - q_2)q_i - C_i(q_i) = cte \qquad (36)$$

They are the indifference curves of each firm. The dashed lines are the contour lines for firm 2 and the solid lines for firm 1.

Under *perfect competition*, the firms maximize their profit, assuming that the price level is fixed. The firms assume that if they change their output they will have no effect on the price. In the strategy space, a line with a fixed price level is a straight line with a downward slope of -45 degrees. At the optimal production level for firm *i* the price is equal to its marginal cost.

$$p(q_1 + q_2) = C_i^{'}(q_i) \qquad (37)$$

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[18] Because physical call options can only be offered when they are backed by generation capacity, arbitrageurs cannot sell physical call options in the contracting stage, as they do not own production capacity. Retailers themselves will therefore need to arbitrage between the two markets.

[19] Unpublished, contact the author for a copy.

Solving equation (37) for q_i gives the prefect competitive supply function as $q_i^{PC}(q_j)$, it specifies the production level of the competitive firm *i*, when firm *j* produces q_j units. The competitive supply function crosses the indifference lines of firm *i* when they have a -45 degree slope. The intersection of both competitive supply functions is the perfect competitive equilibrium. (the point PC in the graph) In the equilibrium, the profit contours of both firms are tangent and have a slope of -45 degrees.

In the *standard Cournot* game, each firm will maximize its profit assuming the output of the other player fixed. The best response function of firm 1, $q_1^C(q_2)$, crosses its profit indifference curve in the region where it is horizontal. The best response function of firm 2, $q_2^C(q_1)$, crosses its profit indifference curve in the region where it is vertical. The intersection of both best response functions is the Cournot equilibrium, which is represented by the letter *C* in the model.

In the *Cournot game with futures contracts,* a firm can shift the equilibrium in the second stage, by selling futures in the futures market. If firm 1 sells future contracts in the first stage, then its best response function shifts outwards. The best response function of firm 2 will *not* change. By selling futures contracts, firm 1 will shift down the equilibrium over the reaction function of firm 2. At the optimal level of futures contracts, the best response function of firm 2 is tangent to the profit indifference lines of firm 1. The intersection of both reaction functions gives the Allaz and Vila equilibrium *F*.

In the *Cournot game with financial call options,* the equilibrium depends upon the strike price.

If the strike price is above the Cournot price, then the only possible equilibrium is the Cournot equilibrium (point *C*). If the strike price is below the Allaz and Vila price, then one of the Nash equilibriums is the Allaz and Vila equilibrium. (point *F* in the figure)

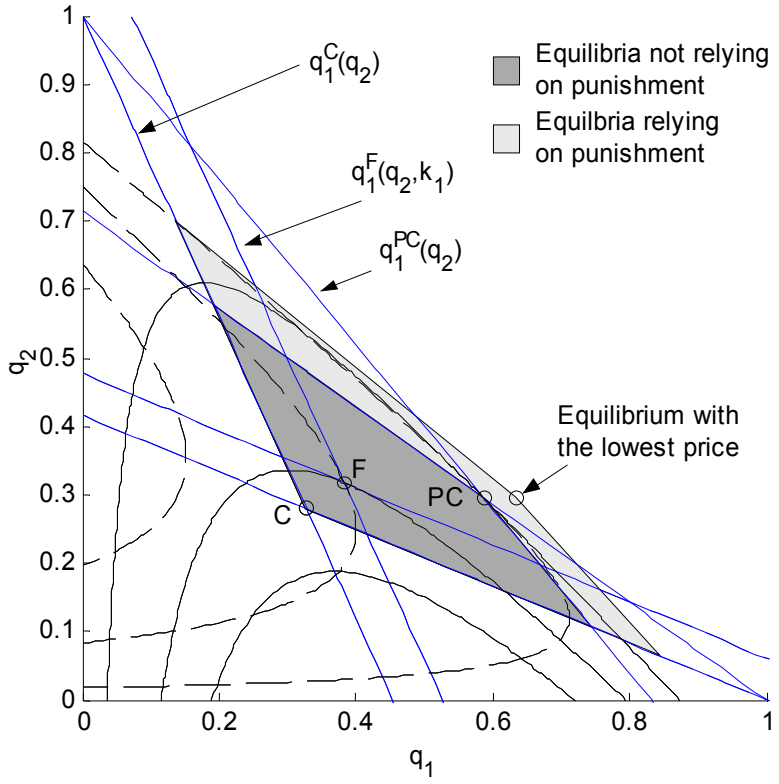

Figure 17 Second stage strategy space

We will now look for the equilibriums where generators flood the market.

In theorem 4 we proved that flooding is an equilibrium in the set $\overline{A}(S) \cap B$. This is the dark shaded area in the figure. The equilibriums in this set rely on extensive flooding in the first stage of the game but do not require a punishment strategy in the second stage. Note that any price between the perfect competitive price and the Cournot price can be obtained as an equilibrium.

Other flooding equilibriums than the dark shaded area exist when we allow firms to use punishment strategies in the second stage of the game. We will now look at some of these equilibriums. As before, we assume that the players sell a large number of options in the first stage of the game. In the second stage they use a punishment strategy θ^P. More in particular, we assume that the firms will co-ordinate on the equilibrium that gives the smallest market share to the deviator. Figure 18 shows the strategy space in the second stage of the game when firm 1 deviates in the first stage. We will check when it is profitable for firm 1 to deviate and sell fewer options in the first stage of the game.

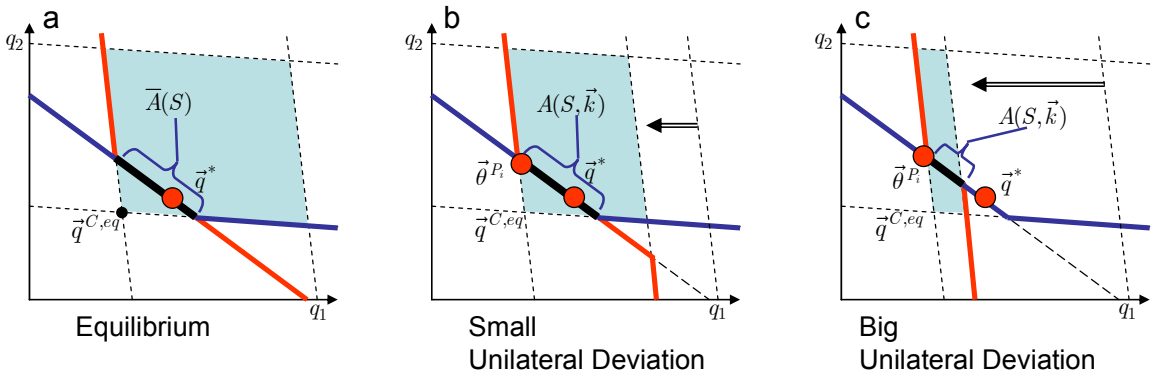

Figure 18 Second stage punishment strategy $\vec{\theta}^{P_1}$, if firm 1 deviates in the first stage of the game.

Selling a large number of options is optimal for firm 1, if its profit in equilibrium $\vec{q}^*$ is higher than in the punishment strategy $\vec{\theta}^{P_1}$. In the equilibrium $\vec{q}^*$ as well as in the punishment equilibrium $\vec{\theta}^{P_1}$ the spot price is equal to the strike price S. Production quantities are, however, different.

Figure 19 shows graphically the effect of a deviation on firm 1's profit. In equilibrium $\vec{q}^*$, firm 1's profit is equal to area abc minus area B. In the punishment equilibrium, firm 1's market share is smaller, and its profit is equal to the area aedc. Firm 1 prefers equilibrium $\vec{q}^*$ to $\vec{\theta}^{P_1}$, when area A is larger than area B. Note that even if the strike price S is below marginal costs in equilibrium $\vec{q}^*$, it still might be an equilibrium, when infra-marginal profits are sufficiently large (area A).

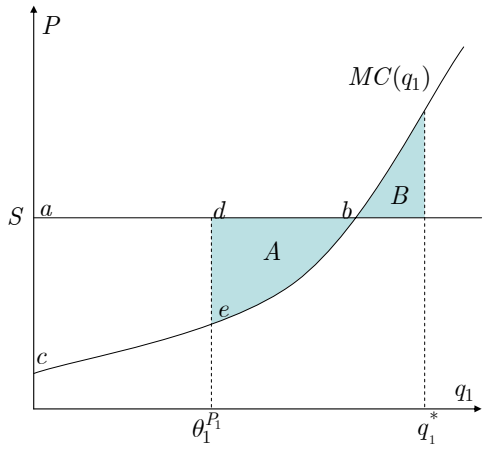

Figure 19 Firm 1 prefers the equilibrium $\vec{q}^*$, as the punishment equilibrium $\vec{\theta}^{P_1}$ gives it a lower profit.

Flooding the market in stage 1 and playing $\vec{q}^*$ in stage 2 is an equilibrium when

$$\pi_1(\vec{q}^*) > \pi_1(\vec{\theta}^{P_1}) \tag{38}$$

where $\vec{\theta}^{P_i}$ is defined by the following two equations:

$$\begin{aligned} p(\theta_i^{P_i} + \theta_j^{P_i}) &= S \\ \theta_i^{P_i} &= q_i^C(\theta_j^{P_i}) \end{aligned} \tag{39}$$

Equations (38) and (39) define a set of equilibriums of the two-stage game, which allows for punishment strategies in the second stage:

$$C(S) = \left\{ \vec{q} \left| \begin{matrix} \pi_2(\vec{q}) \geq \pi_2(\vec{\theta}^{P_2}) \\ \pi_1(\vec{q}) \geq \pi_1(\vec{\theta}^{P_1}) \end{matrix} \right., \begin{matrix} q_1^C(q_2) \leq q_1 \\ q_2^C(q_1) \leq q_2 \end{matrix}, \; P(q_1 + q_2) = S \right\} \tag{40}$$

In Figure 17 this set is the union of the light shaded area and the dark shaded area.

Overview

Table 1 shows the equilibrium quantities and the equilibrium price in the Competitive equilibrium, the Cournot equilibrium, the Cournot equilibrium with futures contracts, and the two-stage equilibrium with the lowest price. The price is low under competitive equilibrium, and high in the standard Cournot game.

	q_1	q_2	P
Cournot	.327	.280	.393
Futures	.383	.317	.300
Competitive	.588	.294	.118
Lowest possible price	.634	.297	.069

Table 1 Solution of the game