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The prevalent assumption of a universal Fermi interaction in which the coupling between two electron-neutrino pairs is identical to that between an electron-neutrino and a muon-neutrino pair[1] (except perhaps for a different neutrino) offers a possible experimental way for comparing the bare and the observed (renormalized) electron charges. The proposed measurement is based upon the observation that an effective interaction of the form

$$H = \frac{G}{\sqrt{2}} (\psi_e^+ \gamma_\lambda \psi_e)(\psi_\nu^+ \gamma_\lambda [1 + i\gamma_5] \psi_\nu) \qquad (1)$$

will involve exactly the same electromagnetic radiative corrections as the electron-photon vertex in conventional quantum electrodynamics, while the radiative corrections to μ-meson decay do not involve any charge renormalization associated with vacuum polarization.

The interaction responsible for the decay $\mu \to e + \nu + \bar{\nu}$ is

$$H' = \frac{G}{\sqrt{2}} J_\lambda^+(\mu\nu) J_\lambda(e\nu) + \text{c.c.} , \qquad (2)$$

with

$$J_\lambda(\rho\nu) = \psi_\rho^+ \gamma_\lambda [1 + i\gamma_5] \psi_\nu . \qquad (3)$$

We assume, in addition, the interaction

$$H'' = \frac{G}{\sqrt{2}} \; J_\lambda^{+}(e\nu)\, J_\lambda(e\nu) + \text{c.c.} \qquad (4)$$

An $(e\nu)(e\nu)$ coupling would follow, at least phenomenologically, if any one of the following assumptions is valid: (1) the complete universal Fermi interaction has the form $G\mathcal{J}_\lambda \mathcal{J}_\lambda^{+}$[1], with

$$\sqrt{2}\,\mathcal{J}_\lambda = J_\lambda(e\nu) + J_\lambda(\mu\nu) + \mathcal{J}_\lambda(pn) + \cdots ;$$

(2) the four-fermion interaction of Eq. (2) is due to the interaction of fermion pairs with an intermediate heavy (B) boson; (3) the vector current is conserved[1,2] even to terms of order G; (4) in any weak interaction, a $(\mu\nu)$ pair may be replaced by an $(e\nu)$ pair where the neutrinos may be different. If the $(e\nu)(e\nu)$ interaction of Eq. (4) does exist, then after a Fierz rearrangement the A-V interaction of Eqs. (3) and (4) may be re-expressed in the equivalent form as

$$H'' = \frac{G}{\sqrt{2}} \; J_\lambda(ee)\, J_\lambda(\nu\nu) + \text{c.c.} \qquad (5)$$

The constant G has been determined from the μ-meson lifetime; the perturbation theory calculation for the μ-decay rate must be decreased by a small finite electromagnetic radiative correction of 0.4%.[3] Similar corrections arise in the computation of the elastic scattering of neutrinos (or antineutrinos) by electrons from the interaction given in Eq. (5). However, there is one additional type of radiative correction to the matrix element which also follows from Eq. (5) but not Eq. (2). In Eq. (5), the vector part of the electron current is just that of Eq. (1). Its radiative corrections include all those of the electron current in quantum electrodynamics, including the unknown and possibly divergent charge renormalization. Because there is no $\mu \leftrightarrow e + \gamma$ vertex corresponding to the $e \leftrightarrow e + \gamma$ vertex of electrodynamics, no vacuum polarization correction enters into the μ-decay rate. The situation is illustrated by the Feynman diagrams of Fig. 1 for the cases where the neutrino interaction changes a μ meson to an electron (including μ decay), and for the elastic scattering of an electron by neutrinos or anti-neutrinos. The diagrams of Fig. 1(a) represent finite corrections of first order in the fine structure constant α which are common to both cases. The diagram of Fig. 1(b) is unique to the radiative corrections for the vector part of the (ee)($\nu\nu$) interaction, and has a divergent part in perturbation theory. Diagrams of the type of Fig. 1(b) also enter into the correction to the electric charge of the electron because of its net vacuum polarization.

If the sum of all irreducible bubbles in Fig. 1(b) times the photon propagator is written as $Q(k^2)/k^2$, where k is the photon 4-momentum and q_0 is the bare (unrenormalized) electron charge then, instead of the electron current operator $G\gamma_\lambda/2$ of Eq. (1), we have

$$\Gamma_\lambda = \frac{G}{2}\, \gamma_\lambda \left(1 - \frac{q_0^2}{k^2}\; Q(k^2)\right)^{-1} . \qquad (6)$$

We compare this with the electron-photon vertex of quantum electrodynamics[4] and write

$$\Gamma_\lambda = Z_3 \frac{G}{\sqrt{2}} \gamma_\lambda [1 + \text{finite terms of order } \alpha]. \tag{7}$$

Here Z_3 is just the ratio of the square of the measured electron charge q to the square of the bare charge q_0:[5]

$$Z_3 = \frac{q^2}{q_0^2} . \tag{8}$$

In a canonical field theory, $0 < Z_3 < 1$. For a divergent theory, $Z_3 = 0$. In perturbation theory, $Z_3 \approx 1 - (2\alpha/3\pi) \text{ Lim}_{\lambda \to \infty} (\lambda/m)$, with m as the electron mass. For the finite terms of Eq. (7) perturbation theory is expected to be adequate, especially if the ratio $|k^2/m^2|$ is not large.

In lowest-order perturbation theory, the total cross section for the scattering of a neutrino or antineutrino by an electron at rest is the sum of the separate contributions from the vector and pseudovector electron currents. When radiative corrections are included, we have for the total elastic scattering cross section, for (anti) neutrino + e → (anti) neutrino + e,

$$\sigma_{\nu e} = \frac{2G^2}{\pi} (1 + Z_3^2) (E_\nu^2) + O(\alpha) , \tag{9}$$

where $G(\text{proton mass})^2 = 1.02 \times 10^{-5}$ and E_ν is the center-of-mass energy. The second term on the right comprises the finite (after mass renormalization) radiative corrections of Fig. 1(a), and the small correction to the vector interaction from the finite part of the vacuum polarization diagrams of Fig. 1(b). Moreover, by definition of Z_3, the second term vanishes for sufficiently low-energy neutrino scattering. Neutrinos or antineutrinos may also scatter from the electrons of the polarization charge surrounding a proton. For momentum transfers less than 50 MeV/c the proton form factor is about unity, and the cross section for elastic ν or $\bar{\nu}$ scattering by a proton is

$$\sigma_{\nu p} = \frac{2G^2}{\pi}(1 - Z_3)^2 E_\nu^{\,2} + O'(\alpha) . \tag{10}$$

Thus, in principle, measurement of the ν-e or $\bar{\nu}$-e elastic cross section (at any energy), together with the assumption that the interaction of Eq. (4) is included in the universal Fermi interaction can yield Z_3, the charge renormalization constant of QED. In ν-e or $\bar{\nu}$-e, scattering the initial electron together with all of the electrons in the induced polarization cloud contributes to the scattering amplitude. This is not true in $\mu \to e + \nu + \bar{\nu}$ where we measure G for a single μ meson with only a small finite radiative correction.

The measurement of the ν-e or $\bar{\nu}$-e cross section, or even the demonstration that it is of second order[6] in G^2, is formidable. Neutrino-pair emission may play a dominant role in some phases of stellar evolution as was suggested by Pontecorvo,[7] Chiu and Morrison[8] and others, but calculations of stellar models and related observations are not yet sufficient to decide the existence of the interaction on this point. The high $\bar{\nu}$ flux of a nuclear reactor (10^{13}/cm^2/sec) may offer the most promising possibility for an eventual measurement. For a 3-MeV (lab) $\bar{\nu}$, the $\bar{\nu}$-e cross section is $4(1 + Z_3^{\,2})10^{-44}$cm^2, with $Z_3^{\,2} \simeq 1$, this is about equal to the measured $\bar{\nu} + p \to n + e^+$ reaction[9] averaged over pile antineutrinos. In the scintillator material of that experiment, electrons outnumbered protons about 5 to 1, and the production of electrons of up to a few MeV might have been the most common $\bar{\nu}$ reaction (about 3 events/min). Unfortunately, the signature of the single electron is not nearly so distinct as that of the n and e^+ produced together by $\bar{\nu}$ absorption by protons, so that at present there seems to be an apparently overwhelming background problem in the identification of possible $\bar{\nu}$-e scattering events.

FOOTNOTES AND REFERENCES

*This work was done under the auspices of the U. S. Atomic Energy Commission.

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2. S. Gershtein and J. Zeldovich, Soviet Phys.--JETP 2, 576 (1957).

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5. If the interaction of Eq. (4) is an approximation valid only for low-momentum transfer, as in the case of an intermediate boson, then Eq. (7) is no longer necessarily precise. The value of an irreducible bubble of Fig. 1(b) then is Q only if it ends with a photon vertex, and $\bar{Q}$ if it ends with a neutrino pair vertex. In this case instead of $G \rightarrow Z_3 G$ for the vector interaction we have, for small k^2, $G \rightarrow Z_3 G + q^2 G \lim_{k^2 \rightarrow 0} (\bar{Q} - Q)/k^2$.

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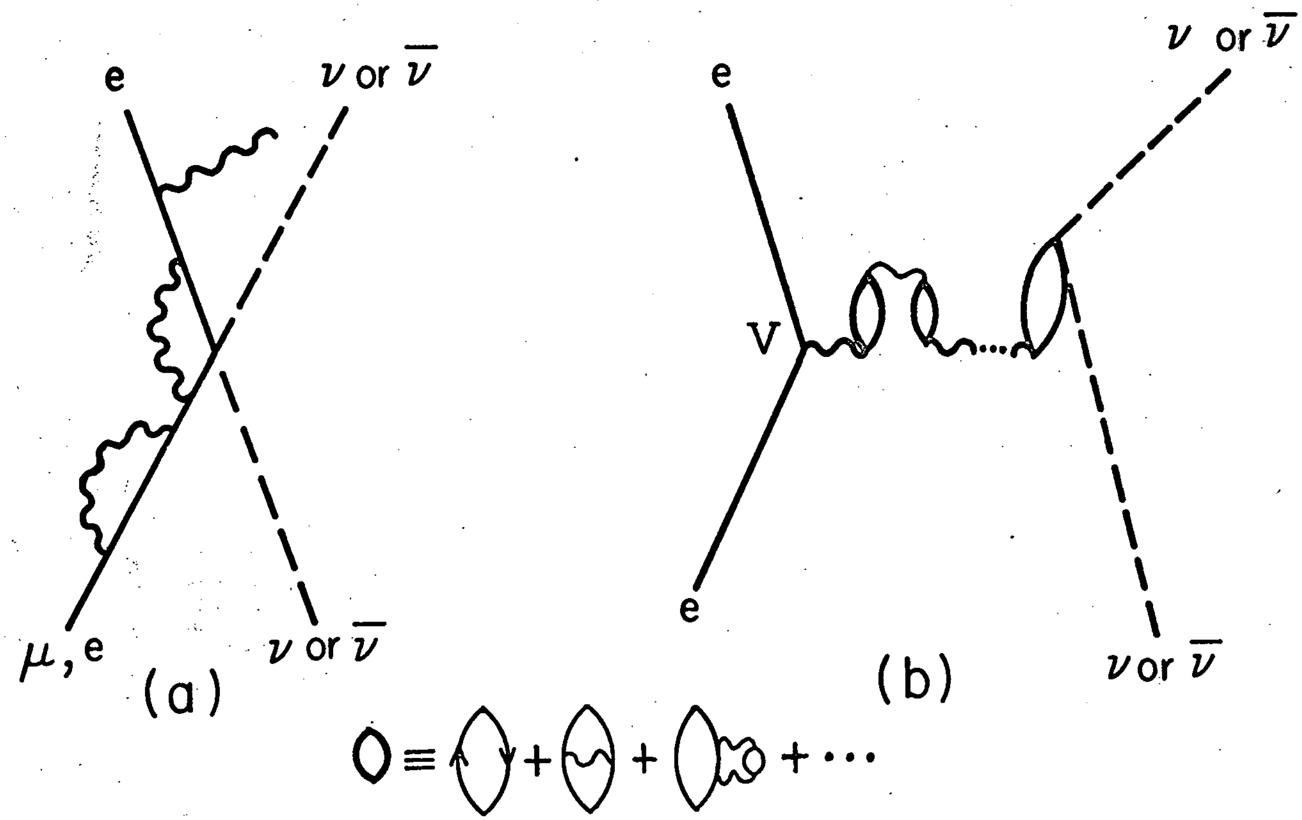
FIGURE CAPTION

Fig. 1. Feynman diagrams for electromagnetic radiative corrections to $(\mu\nu)(e\nu)$ and $(e\nu)(e\nu)$ interactions. The bubbles of (b) are electron positron loops; the wavy lines are photons.



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