

# High Reynolds Number Stratified Wakes

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## Abstract

An experiment has been conducted using a large (0.46 m) towed sphere in a stratified reservoir to obtain wake data at Reynolds numbers ranging from  $5 \times 10^5$  to  $1.2 \times 10^6$ ; at least an order-of-magnitude greater than previously measured. The experiments were conducted in Triadelphia Reservoir, MD, where summer heating creates a distinct stratified profile. Mean flow and turbulence data in the nearfield stratified wake at  $x/D = 5.3$  were obtained using velocity and temperature sensors. It was found that, as can be expected, that the power spectral intensity and the turbulent dissipation rate increase with  $Re$ . Particularly noteworthy is the  $-5/3$  inertial regime spectral slope present over more than an order-of-magnitude in wavenumber.

## 1 Introduction

Although the downstream evolution of stratified wakes at high Reynolds number ( $Re > 10^6$ ) is relevant to ships and aircraft, detailed data at these Reynolds numbers are completely lacking even for canonical shapes such as a sphere. Indeed, even for unstratified wakes very limited data are available at high Reynolds numbers, with most of the available data dealing with measurements of the drag coefficient, separation, and vortex shedding frequency rather than the details of the turbulent wake velocity field.

The turbulent wake of a towed sphere has been the subject of intense study since the earliest days of fluid dynamics research with detailed studies of wake structure limited to Reynolds numbers less than  $\sim 10^5$ , e.g. Hopfinger et al. (1991). While drag and vortex shedding frequency have been measured at values up to  $Re \sim 6 \times 10^6$  (Achenbach, 1972, 1974), lift fluctuations measured up to  $Re \approx 4 \times 10^5$  (Willmarth & Enlow, 1969) and the qualitative nature of the nearfield wake observed (e.g. Taneda, 1978), wake details such as turbulent velocities and spectra have not been measured at larger values of  $Re$ . Detailed wake studies in unstratified flow have generally been at Reynolds numbers an order-of-magnitude less than  $10^6$ , i.e. in the laminar boundary layer regime at  $Re$  less than the ‘drag crisis’ at  $Re \approx 3 \times 10^5$  for a sphere (e.g. Tyagi et al., 2006). Studies of sphere wakes in stratified flows have been even more severely limited in Reynolds number, generally limited to  $Re < 10^4$  (e.g. Lin et al., 1992a, 1992b).

Studies of canonical bodies such as towed spheres provide a basis for understanding physical processes underlying realistic wakes from sources of more complex geometry. CFD codes can now readily perform detailed simulations at increasingly higher  $Re$  (e.g. Redford et al., 2015; Diamessis et al., 2011; Brucker & Sarkar, 2010; Chernykh et al., 2012), however the lack of high  $Re$  data precludes code validation. Moreover, a common approach to study wake evolution by CFD is to start the wake calculation on an initial plane downstream of the source. Data to provide appropriate initial conditions at high  $Re$  are completely lacking and may indeed be critical as there are indications that the properties of the nearfield turbulent wake of a towed body may not be universal (Xiang et al., 2015).

Stratified wake flows are generally categorized by the Reynolds and internal Froude numbers,  $Re = UD/\nu$  and  $Fr = U/ND$ , respectively, where  $D$  is the characteristic body length scale, its diameter,  $U$  the velocity,  $\nu$  the kinematic viscosity,  $N = (-g(d\rho/dz)/\rho_0)^{1/2}$ , the Brunt-Väisälä (B-V) or buoyancy frequency,  $g$  the gravitational constant,  $\rho$  the depth dependent density, and  $\rho_0$  a reference density. The nature of the early turbulent wake of a sphere in a stratified fluid is highly Reynolds and Froude number dependent as shown in the extensive experimental studies by Lin et al. (1992a, 1992b, 1993), Chomaz et al. (1993a, 1993b), Bonnier & Eiff (2002) and Brandt & Schemm (2011). Within the ranges of  $5 \leq Re \leq 10^4$  and  $0.005 \leq Fr \leq 20$  three regimes have been identified: symmetric vortex shedding at low  $Re$  and  $Fr$ , non-symmetric vortex shedding at intermediate  $Re$  and  $Fr$ , and fully turbulent (no clear coherent vortices) at high  $Re$  and  $Fr$ . Chomaz et al. (1992) have also shown the dependence of the flow separation line on Froude number. In the nearfield, the global properties of the wake, e.g. centerline velocity and length scale (power law) decay, behave similarly to those of unstratified wakes (Spedding, 1997). The detailed structure, however, is clearly affected by stratification since the wake turbulence is mixing and entraining fluid from outside the wake causing density variations on scales of the wake dimension and smaller, c.f. Brandt & Schemm (2011).

At high  $Re$  insight into the nature of the near-field wake can be obtained from the extant numerical simulations. In the study by Constantinescu & Squires (2004) simulations were performed in unstratified flow at  $Re = 1.14 \times 10^6$  and lower. These simulations confirmed the earlier qualitative observations of the nature of the near wake regime showing the decrease in the recirculation and transition regime at the higher  $Re$  as compared to values at  $Re = 10^4$ . At  $Re = 1.14 \times 10^6$ , the recirculation zone extends to  $x/D \approx 1.2$  (measured from the sphere center) and the recovery zone extends to  $x/D \approx 2.4$ . Beyond this distance the wake is unaffected by the adverse pressure gradient caused by the recirculation zone resulting from the bluff body geometry. Moreover Constantinescu & Squires clearly point out that there are no available measurements of the detailed early wake flow field at these  $Re$  values (not to mention the even greater absence of data in high  $Re$  stratified flows).

At high  $Re$  and  $Fr$  the turbulent wake is the dominant source of the internal wavefield (Lin & Pao, 1979; Gilreath & Brandt, 1985; Abdilghanie & Diamessis, 2013) rather than the body generated lee wave mechanism (Brandt & Rottier, 2015; Voisin, 1994). An extensive review of the available data and numerical simulations of body and wake generated waves is given in Brandt & Rottier (2015). As with the wake properties at high  $Re$ , virtually no data exist on the IW field in the high  $Re$ - $Fr$  regime.

In the present study detailed turbulent velocity and temperature measurements in the nearfield wake of a sphere towed in a stratified reservoir have been made in order to provide at values of  $Re \approx 10^6$ . While measurements were made at the wake centerline as well as off-center locations the present report focuses on the centerline velocity data as analysis of the full dataset is still in progress.

## 2. Experiment configuration

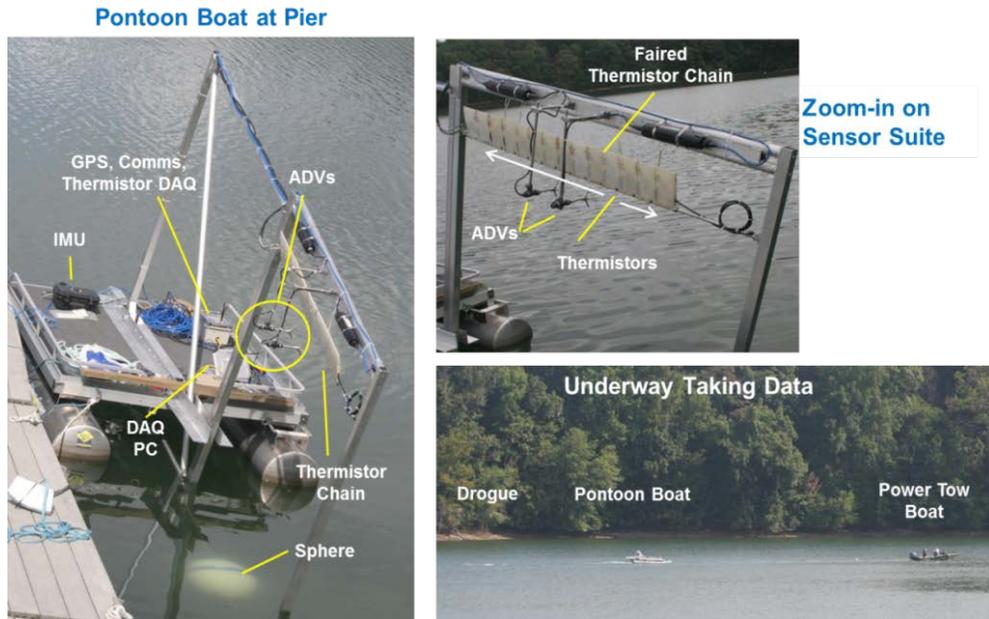
To provide high  $Re$  data an experiment has been conducted using a large (0.46 m) towed sphere in a stratified reservoir at sphere Reynolds numbers between  $5 \times 10^5$  and  $1.2 \times 10^6$ . Turbulent and mean flow field data in the nearfield stratified wake at  $x/D = 5.3$  were obtained using velocity and temperature sensors.

The experiments were conducted in Triadelphia Reservoir, MD, where summer heating creates a distinct stratified profile. A modular frame was used to support the sphere and instrumentation beneath the deck of a pontoon boat that was towed at several constant speeds. The sphere and instrumentation were rigidly mounted beneath the pontoon boat in order to position the instrumentation at a known fixed position with respect to the sphere and suspended within the stratified layer, nominally 3 m below the surface, a depth where the ambient stratification was significant, nominally 20 - 30 cph.

The experimental rig, shown in fig. 1 consists of a pontoon boat towed by a power boat to provide stability. A modular frame onto which a sphere and instrumentation are mounted is rigidly suspended beneath the deck of the pontoon boat in order to position the instrumentation at a known fixed location with respect to the sphere wake source. When not deployed for testing, the frame can be rotated up 90 degrees such that the entire frame and instrument suite are at or above the water line. This enables launch and recovery from shallow water and access to the instrumentation for installation and adjustments.

The primary operational challenge was to maintain approximately zero pitch and yaw of the frame so that the sphere wake would be located in line with the sensor locations as the pitch and yaw imparted to the pontoon platform varied with towing speed. This issue was ultimately resolved through a combination of appropriate towing line/bridle assemblies, weight redistribution, installation of a rudder assembly and, most significantly, a mechanism for on-the-fly adjustment of the pitch of the frame relative to the pontoon platform to compensate for any platform pitch and achieve zero pitch of the experiment assembly relative to the sphere free stream velocity. Real time read-outs of the frame pitch and comparison of the heading and course from a GPS compass enabled maintenance of the desired orientation and for post-test confirmation.

Instrumentation included two Nortek Vectrino Acoustic Doppler Velocimeters (ADV) to simultaneously measure three components of the instantaneous turbulent wake velocities at two locations – one at the wake centerline and one near the wake edge. An array of nine fast response (FP07, 7 msec response time) thermistors mounted in faired housings was placed along the vertical centerline of the wake cross-section to measure the fluctuating temperature field (and thus density fluctuations). To enable capture of the higher frequency fluctuations pre-emphasis was employed in the thermistor circuits. One thermistor was located immediately adjacent to the each of the two ADV sampling volumes in order to provide simultaneous velocity and temperature/density measurements and a measure of the velocity-temperature (density) correlation,  $\overline{\rho'u'}$ . Velocity and temperature data were sampled and recorded at 200 Hz. Background stratification profiles were measured and recorded periodically with a CastAway CTD profiler.



**Figure 1. Photographs of experimental rig, instrumentation and operation.**

### 3. Results

A large set of data was collected from multiple deployments at Lake Triadelphia during the summers of 2014 and 2015. The sphere was towed at a range of speeds between 2 and 4.5 kts resulting in  $5 \times 10^5 \leq Re \leq 1.2 \times 10^6$ . Approximately 6 hours of data in run segments of ~2 to 8 min duration were collected. The Froude numbers obtained ranged from 20 to nearly unstratified. Preliminary results from a subset of the centerline velocity data are presented below.

#### 3.1 Mean velocity and turbulence intensity

Centerline mean velocity ( $U_{cl}$ ) and turbulence intensity ( $u'$ ) at four tow speeds are shown in fig. 2, black circles. The mean velocities are essentially constant over the  $Re$  range considered. The turbulent intensity levels, however, decrease somewhat with increasing  $Re$ ; this is likely due to the highly variable nature of the intense turbulent flow at this small distance downstream (Constantinescu & Squires, 2004). Also shown in fig. 2 are the data of Tyagi et al. (2006) which is the only other data set at high  $Re$ , albeit at an order-of-magnitude less than the present study and under unstratified (wind tunnel) conditions. Notably the present values of  $u'/U_{cl}$  at  $x/D = 5.3$  fall between the values of Tyagi et al. at  $x/D = 3.75$  and  $7.5$  despite being beyond the drag crisis.

#### 3.2 Turbulence Spectra

In figure 3(a) are shown the streamwise velocity spectra at the four tow speeds corresponding to  $Re = \{0.5, 0.8, 1.0, 1.2\} \times 10^6$ . As would be expected the spectral power increases with speed. At these high  $Re$  values a clear  $-5/3$  slope inertial region is present for more than an order-of-magnitude in wavenumber. The spectra appear free of noise contamination (possibly related to vibration) below 0.1 cy/cm. The spikes in the vicinity of 0.1 cy/cm are believed to be due to a vibration of the frame.

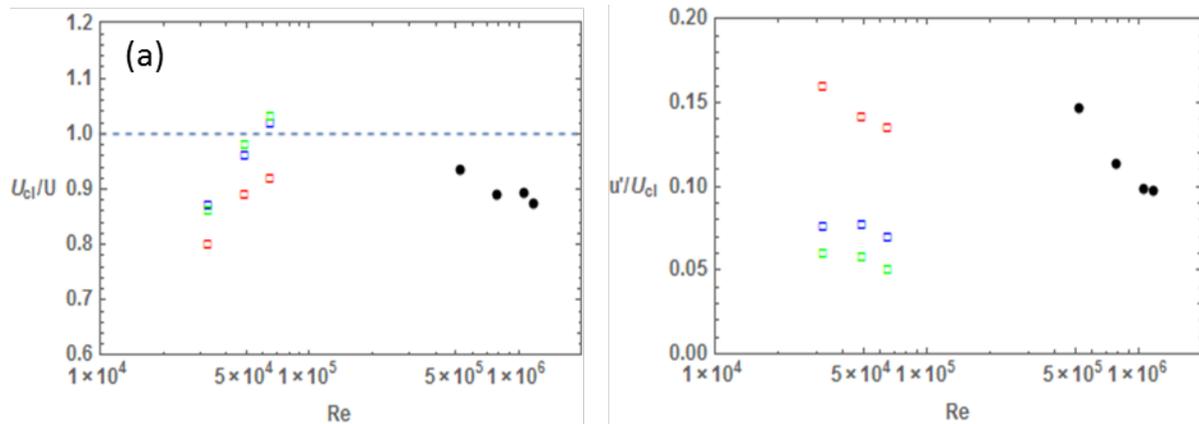


Figure 2. Centerline velocity (a) and turbulence intensity (b) for the present experiments,  $x/D = 5.3$  – black points. Data from Tyagi et al. (2006),  $x/D = \{3.75, 7.5, 11.25\}$  – red, blue and green square points respectively.

In order to ensure the measurements were due to the turbulent wake of the sphere and not due to the frame structure or ambient background turbulence, a number of tests were performed with the sphere removed leaving only the 0.75 in diameter rod on which the sphere is mounted in place. A sample comparison of the results of the  $u$  velocity spectra with and without the sphere is shown in fig. 3(b). The spectrum without the sphere drops rapidly to a noise floor that is more than an order-of-magnitude lower than when the sphere is present. Future testing will include accelerometers to mitigate vibration at wavenumbers  $> 0.1$  cy/cm.

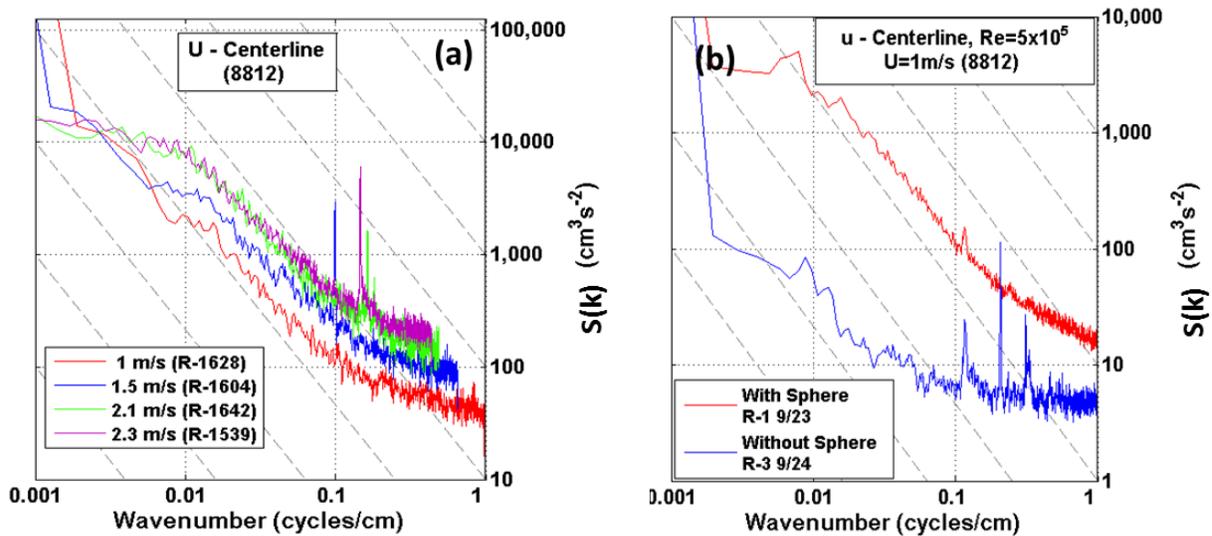


Figure 3. Spectra of measured centerline streamwise velocity profiles. (a)  $Re = \{0.5, 0.8, 1.0, 1.2\} \times 10^6$ . Dashed lines show a  $-5/3$  slope. (b)  $Re = 0.5 \times 10^6$  experimental rig with and without the towed sphere.

Figure 4 presents representative turbulent velocity spectra of the three velocity components ( $u, v, w$ ; streamwise, vertical and span-wise, respectively) at the wake centerline ( $x/D = 5/3$ ). The

spectral distributions are similar, all showing the  $-5/3$  inertial range, indicating the isotropic nature of the wake as would be expected in this early-time wake (Spedding, 1997).

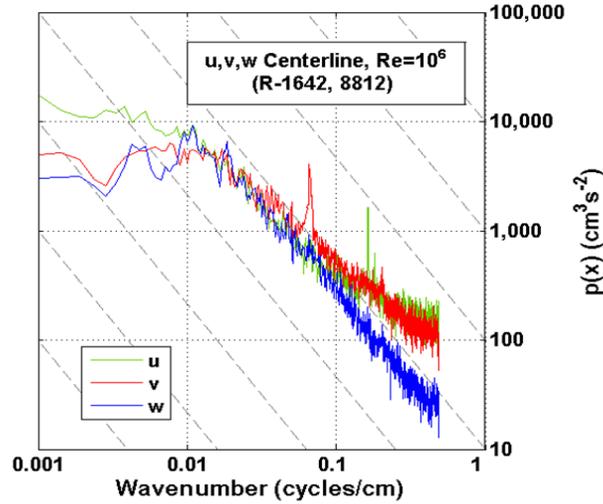


Figure 4. Comparison of  $u, v, w$  spectra at  $Re = 5 \times 10^5$ .

### 3.2 Turbulent Dissipation

As there is a well-defined spectral inertial range the turbulent dissipation,  $\varepsilon$ , can be estimated by fitting the power spectra with the form (Tennekes & Lumley, 1972)

$$S(k) = a_0 \varepsilon^{2/3} k^{-5/3},$$

where  $k$  is the wavenumber and  $a_0$  is the Kolmogorov constant with the accepted value of  $a_0 = 0.68$ . For the streamwise centerline spectra shown in fig. 3(a) values of  $\varepsilon = \{0.04, 0.09, 0.16, 0.19\} \text{ m}^2/\text{s}^3$  were obtained, showing increasing values with increasing  $Re$  for the four  $Re$  conditions considered. These values are orders-of-magnitude greater than typical ambient ocean values as can be expected due to the intense turbulent source. Estimates of dissipation values at  $x/D = 5.3$  based on the results of Tyagi et al. (2006) at lower values of  $Re$  range are significantly higher ranging from  $5 - 15 \text{ m}^2/\text{s}^3$ ; although these values were computed by computing velocity derivatives directly from the time series data, which may not be a comparable approach.

### Discussion

A methodology for obtaining turbulent wake data at high  $Re$  with “laboratory” like control and precision has been developed, by towing a large sphere in a quiescent stratified reservoir. Results provide a first look at the near field turbulent wake at  $Re$  up to  $1.2 \times 10^6$ , above the ‘drag crisis’ at  $Re \approx 3 \times 10^5$ . Velocity spectra show a distinct  $-5/3$  inertial regime spectral slope over more than an order-of-magnitude in wavenumber. High levels turbulent dissipation levels have been found resulting from the intense turbulent source. Continued analysis of the present dataset will investigate the off-center turbulent velocity and the temperature fluctuation intensity across the wake. Future studies will focus on obtaining comparable data at further downstream distances.

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