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# Models of Scherk-Schwarz Symmetry Breaking in 5D: Classification and Calculability

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## Abstract

The form of the most general orbifold breaking of gauge, global and supersymmetries with a single extra dimension is given. In certain theories the Higgs boson mass is ultraviolet finite due to an unbroken local supersymmetry, which is explicitly exhibited. We construct: a 1 parameter  $SU(3) \times SU(2) \times U(1)$  theory with 1 bulk Higgs hypermultiplet, a 2 parameter  $SU(3) \times SU(2) \times U(1)$  theory with 2 bulk Higgs hypermultiplets, and a 2 parameter  $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$  theory with 2 bulk Higgs hypermultiplets, and demonstrate that these theories are unique. We compute the Higgs mass and compactification scale in the  $SU(3) \times SU(2) \times U(1)$  theory with 1 bulk Higgs hypermultiplet.

# 1 Introduction

The origin of symmetry breaking, one of the key questions of particle physics, is largely unknown. In four dimensions, symmetries can be spontaneously broken by scalar fields, fundamental or composite. In higher dimensional theories, a very different geometrical view is possible: symmetries can be broken by the boundary conditions on a compact space. While this idea has been known for many years [1], its early application was restricted to string motivated theories with certain six dimensional compact spaces [2].

It is remarkable that, until recently, there was no attempt to discover the simplest extensions of the standard model, or the minimal supersymmetric standard model (MSSM), in which symmetries, such as supersymmetry, Peccei-Quinn symmetry and grand unified gauge symmetry, were broken by this Scherk-Schwarz mechanism. While several such theories now exist [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14], the question of their uniqueness is unknown, and is addressed in this paper.

We consider theories based on a single compact extra dimension. We study the spacetime symmetries of this dimension in section 2.1, and construct the most general form for the breaking of supersymmetry, global symmetry and gauge symmetry in section 2.2.

In section 3 we exhibit the form of the local 5d gauge symmetry and supersymmetry which are unbroken by orbifolding. These unbroken symmetries depend on the location in the bulk – for example they are different at the two fixed points of the orbifold. These unbroken symmetries are crucial since they dictate the form of both bulk and brane interactions. There has been considerable recent debate [15] about whether the mass of the Higgs boson in certain theories of this type is finite. In section 3.5 we argue that these unbroken local 5d symmetries ensure that there are no quadratic ultraviolet divergences in the Higgs mass.

In section 4 we make a complete classification of theories with 5d local supersymmetry with one or two Higgs doublets in the bulk, with gauge group either  $SU(3) \times SU(2) \times U(1)$  or  $SU(5)$ . There are very few such theories, and we briefly describe some possible locations for the matter fields. Only a single  $SU(3) \times SU(2) \times U(1)$  theory with a single bulk Higgs has been constructed in the literature [7], and this theory is found to be an important special case of a 1 parameter family of such theories. We explore electroweak symmetry breaking in this family in section 5, with particular attention to the Higgs boson mass and the compactification scale.

The two Higgs theories, with gauge group  $SU(3) \times SU(2) \times U(1)$  and  $SU(5)$ , are shown in section 5 to each form unique two parameter families of models. The form of  $SU(5)$  breaking is unique, and the form for supersymmetry breaking involves a single free parameter, and is therefore also highly constrained. Conclusions are drawn in section 6.

## 2 The Classification

In this section we construct a classification of supersymmetric field theories in 5 dimensions, where the physical space of the fifth dimension is an orbifold of finite size.

### 2.1 5 dimensional spacetime

We begin by considering the fifth dimension to be the infinite line  $R^1$ . What are the most general spacetime transformations acting on this line, which can be used to compactify the spacetime by identifying points transforming into each other under these operations? One of them is a translation  $\mathcal{T}(2\pi R)$  which induces  $y \rightarrow y + 2\pi R$ . When we identify the points connected by this transformation, that is  $y + 2\pi R$  with  $y$ , it compactifies  $R^1$  to the circle  $S^1 = R^1/T$ . The other possibility is a parity  $\mathcal{Z}(y_0)$  which reflects the line about  $y = y_0$ . An identification using this operation, that identifies  $-(y - y_0)$  with  $y - y_0$ , produces an orbifold which is the half line,  $R^1/Z_2$ . This identification involves the choice of a special point,  $y_0$ , which is a fixed point under the transformation. This parity alone does not compactify the space. No further independent spacetime identifications can be made on the line (if there are several commensurate translations, we take  $\mathcal{T}$  to be the one of lowest  $R$ ). In this paper, we are interested in the case that both translation and parity identifications are made. In this case the physical space can be taken to be  $0 \leq (y - y_0) \leq \pi R$ , corresponding to the orbifold  $S^1/Z_2$ .

Let  $\varphi$  be a column vector representing all fields of the theory. The action of these transformations on the fields can be written as

$$\mathcal{T}(2\pi R)[\varphi(y)] = T^{-1}\varphi(y + 2\pi R), \quad (1)$$

$$\mathcal{Z}(0)[\varphi(y)] = Z\varphi(-y), \quad (2)$$

where we have chosen  $y_0 = 0$ . Acting with  $\mathcal{Z}$  twice produces the identity, so that this is a  $Z_2$  transformation, with  $Z^2 = 1$ . An identification under these operations are made by imposing the conditions  $\mathcal{T}(2\pi R)[\varphi(y)] = \varphi(y)$  and  $\mathcal{Z}(0)[\varphi(y)] = \varphi(y)$ , that is

$$\varphi(y + 2\pi R) = T\varphi(y), \quad (3)$$

$$\varphi(-y) = Z\varphi(y). \quad (4)$$

This identification makes sense only when the bulk action is invariant under the operations  $\varphi(y) \rightarrow \mathcal{T}(2\pi R)[\varphi(y)]$  and  $\varphi(y) \rightarrow \mathcal{Z}(0)[\varphi(y)]$ , since otherwise physics is not the same on all equivalent pieces of the line of length  $\pi R$ .

The simultaneous imposition of both  $\mathcal{T}(2\pi R)$  and  $\mathcal{Z}(0)$  is not automatically consistent, because the spacetime motion induced by  $\mathcal{T}(2\pi R)\mathcal{Z}(0)$  is identical to that induced by  $\mathcal{Z}(0)\mathcal{T}^{-1}(2\pi R)$ .

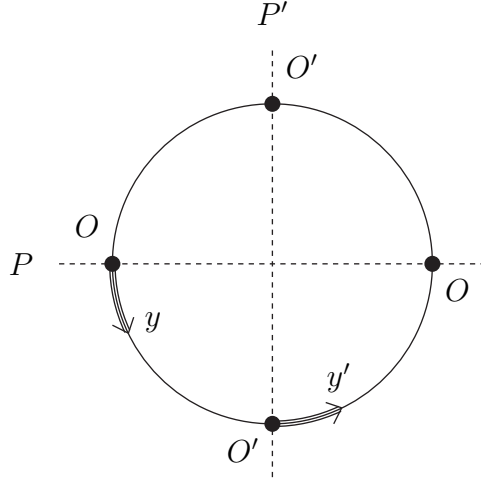


Figure 1: A diagrammatic representation of  $\mathcal{Z}(0)$  and  $\mathcal{Z}(\pi R)$  as reflections about  $y = 0$  and  $y' = 0$ , with  $y' = y - \pi R$ .

Consistency therefore requires that the field transformation is the same nomatter which choice is made; thus we require  $TZ = ZT^{-1}$ , or

$$ZTZ = T^{-1}. \quad (5)$$

Thus the most general spacetime symmetries can be taken to be a reflection  $y \rightarrow -y$ , under which the fields transform as a  $Z_2$ , and a translation, under which the fields transform as Eq. (1) with any symmetry  $T$  of the action, as long as Eq. (5) is satisfied.

The compound transformation  $\mathcal{T}(2\pi R)\mathcal{Z}(0)$ , induces the spacetime motion  $y - \pi R \rightarrow -(y - \pi R)$ , which is a reflection about the point  $y = \pi R$ . Its action on the fields is  $Z' = TZ$ , and from Eq. (5) we discover that  $Z'^2 = 1$ , so that  $\mathcal{T}(2\pi R)\mathcal{Z}(0) = \mathcal{Z}(\pi R)$  is a reflection about  $\pi R$  which also induces a  $Z_2$  transformation on the fields. One can choose to describe the compactification in terms of the identifications  $\mathcal{Z}(0)$  and  $\mathcal{T}(2\pi R)$  or equivalently by  $\mathcal{Z}(0)$  and  $\mathcal{Z}(\pi R)$ ; the orbifolds  $S^1/Z_2$  and  $R^1/(Z_2, Z'_2)$  are equivalent [7]. While the physical space is the line segment  $0 < y < \pi R$ , we have found it convenient to assemble four such equivalent neighboring segments into a circle of circumference  $4\pi R$ , as shown in Figure 1. The utility of this construction is to provide a diagrammatic view of  $\mathcal{Z}(0)$  and  $\mathcal{Z}(\pi R)$  as reflections about orthogonal axes with fixed points  $O$  and  $O'$ .

In general  $Z$  and  $Z'$  do not commute. In the special case that they do,  $T^2 = 1$ , so that  $T$  is also a  $Z_2$  transformation. Acting twice with  $\mathcal{T}(2\pi R)$  induces a complete revolution of the circle of Figure 1, so that, in this commuting case, the eigenfunctions of Eqs. (3, 4) are single valued

on this circle:

$$\begin{aligned}
T &= +1 \begin{cases} (+, +) : & \cos [ny/R] \\ (-, -) : & \sin [ny/R] \end{cases} \\
T &= -1 \begin{cases} (+, -) : & \cos [(n + 1/2)y/R] \\ (-, +) : & \sin [(n + 1/2)y/R] \end{cases}
\end{aligned} \tag{6}$$

where  $(\pm, \pm)$  refer to the  $(Z, Z')$  parities.

Given a specific field content of a theory, a complete list of the possible forms for  $Z$  and  $Z'$  can be obtained. As an illustrative example, consider a theory with  $N$  complex scalars, assembled into a vector  $\phi$ . A basis can be chosen such that  $Z = P$  is a diagonal matrix. However, in this basis  $Z'$  is in general non-diagonal:

$$Z'(\alpha_i) = UP'U^\dagger, \tag{7}$$

where  $P'$  is diagonal. The  $N \times N$  unitary matrix  $U(\alpha_i)$  describes the relative orientation of the field bases which diagonalize  $Z$  and  $Z'$ , and depends on a set of continuous parameters  $\alpha_i$ .

The number of physical parameters  $\alpha_i$  is less than  $N^2$ , and depends on the numbers of positive and negative eigenvalues in  $P$  and  $P'$ . Two cases will be of particular importance to us. If either  $P$  or  $P'$  is proportional to the identity, then  $U$  can be rotated away, and there is no need to introduce any  $\alpha_i$  parameters. Next consider the case of  $N = 2$ . The only non-trivial case is when neither  $P$  or  $P'$  is proportional to the unit matrix:  $P = P' = \sigma_3$ . A general  $U$  matrix would have the form  $U = \exp(i \sum_{i=0}^3 \alpha_i \sigma_i)$ , where  $\sigma_0$  is the unit matrix and  $\sigma_{1,2,3}$  are the Pauli spin matrices. However, the parameters  $\alpha_{0,3}$  drop out of  $Z'$ , while a basis rotation which preserves  $Z = \sigma_3$  allows  $\alpha_1$  to be rotated away. Hence the only non-trivial  $2 \times 2$  case is described by a single parameter  $\alpha$ :

$$Z = \sigma_3, \quad Z'(\alpha) = e^{\pi i \alpha \sigma_2} \sigma_3 e^{-\pi i \alpha \sigma_2}. \tag{8}$$

The description in terms of  $(Z, T)$  is somewhat simpler, since  $T = e^{2\pi i \alpha \sigma_2} = R(2\pi\alpha)$ , the  $2 \times 2$  rotation matrix for angle  $2\pi\alpha$ . In this case the field  $\phi$  can be expanded in a set of Kaluza-Klein (KK) eigenfunctions of Eqs. (3, 4):

$$\phi(x, y) = R \left( \alpha \frac{y}{R} \right) \sum_{n=0}^{\infty} \begin{pmatrix} \cos [ny/R] \phi_{+n}(x) \\ \sin [ny/R] \phi_{-n}(x) \end{pmatrix} = \sum_{n=-\infty}^{\infty} \begin{pmatrix} \cos [(n + \alpha)y/R] \\ \sin [(n + \alpha)y/R] \end{pmatrix} \phi_n(x), \tag{9}$$

where  $\phi_n$  is given by

$$\phi_n(x) = \begin{cases} \frac{1}{2}(\phi_{+n}(x) + \phi_{-n}(x)) & \text{for } n > 0 \\ \phi_{+0}(x) & \text{for } n = 0 \\ \frac{1}{2}(\phi_{+n}(x) - \phi_{-n}(x)) & \text{for } n < 0. \end{cases} \tag{10}$$

The special cases  $\alpha = 0$  ( $1/2$ ) give  $T = \sigma_0$  ( $-\sigma_0$ ), so that  $T^2 = 1$ . In these cases  $Z$  and  $Z'$  commute, so that the above eigenfunctions Eq. (9) reduce to Eq. (6).

## 2.2 General form for orbifold symmetries

We consider  $N = 1$  supersymmetric gauge theories in 5d with gauge group  $G$ . The vector multiplet contains components  $\mathcal{V} = (A^M, \lambda, \lambda', \sigma)$  and the theory contains a set of hypermultiplets with components  $\mathcal{H} = (\phi, \phi^{c\dagger}, \psi, \psi^{c\dagger})$ . There may be multiple copies of a hypermultiplet of given gauge charge, and therefore, since supersymmetry allows only kinetic terms in the bulk, the bulk Lagrangian can possess some flavor symmetry  $H$ . From the 4d viewpoint the theory possesses two supersymmetries, with transformation parameters  $\Xi = (\xi_1(y), \xi_2(y))$ , which we take to be local transformations. The bulk Lagrangian possesses a global  $SU(2)_R$  symmetry under which  $\Xi = (\xi_1, \xi_2)$ ,  $\Lambda = (\lambda, \lambda')$  and  $\Phi = (\phi, \phi^{c\dagger})$  form doublets.

The symmetry  $\mathcal{Z}(0)$  induces  $y \rightarrow -y$  and the supersymmetric kinetic terms then force relative signs for the parities  $P$  of the components inside  $\mathcal{V}$  or  $\mathcal{H}$ . In particular one discovers that, from the 4d viewpoint,  $\mathcal{Z}$  necessarily breaks  $N = 2$  supersymmetry to  $N = 1$  supersymmetry. The 5d supersymmetric multiplets are then conveniently assembled into 4d supersymmetric multiplets with the  $P$  charges:  $\mathcal{V} = (V(+), \Sigma(-))$  and  $\mathcal{H} = (H(+), H^{c\dagger}(-))$ , where  $V(A^\mu, \lambda)$  is a 4d vector multiplet, whereas  $\Sigma(\sigma + iA^5, \lambda')$ ,  $H(\phi, \psi)$  and  $H^c(\phi^c, \psi^c)$  are chiral multiplets. This action of  $\mathcal{Z}$  within an  $N = 2$  multiplet we define as the set of charges  $\Sigma_3$ . However, this does not give the complete action of  $\mathcal{Z}$ . There may be different overall phase rotations for different hypermultiplets,  $P_H$ . Finally, even within a hypermultiplet,  $\mathcal{Z}$  can act differently on different components of an irreducible gauge multiplet. We label this by  $P_G$ , which we take to be an element of the gauge group. If  $P_G$  is the unit matrix there is no gauge symmetry breaking, otherwise there is. Hence we write

$$Z = \Sigma_3 \otimes P_H \otimes P_G. \quad (11)$$

The most general possibilities for  $P_H$  and  $P_G$  are given by  $P_H^2 = P_G^2 = \pm 1$  so that  $Z^2 = (P_H \otimes P_G)^2 = 1$  (not just  $P_H^2 = P_G^2 = 1$ ). An example of the case with  $P_H^2 = P_G^2 = -1$  is provided by  $G = SU(2)$  with two iso-doublet hypermultiplets  $\mathcal{H}_\pm = (H_\pm, H_\pm^{c\dagger})$ . In this case, to have a non-trivial boundary condition in the gauge space ( $P_G \neq 1$ ), we have to take  $P_G = i\sigma_3$  in the space of fundamental representation. ( $P_G = \sigma_3$  is not an element of  $SU(2)$ .) Therefore,  $P_G^2 = -1$ . Then, to have  $Z^2 = 1$ , we also have to assign  $P_H^2 = -1$  for  $\mathcal{H}_\pm$ , for instance, as  $H_\pm \rightarrow \mp i H_\pm$  and  $H_\pm^c \rightarrow \pm i H_\pm^c$ . The combined transformation,  $P_H \otimes P_G$ , is written as  $H_\pm \rightarrow \pm \sigma_3 H_\pm$  and  $H_\pm^c \rightarrow \mp \sigma_3 H_\pm^c$ , which cannot be reproduced in terms of the parity matrices  $P_H$  and  $P_G$  satisfying  $P_H^2 = P_G^2 = 1$ , since with  $P_H^2 = P_G^2 = 1$  the induced transformation is always the same for  $H_\pm$  and  $H_\pm^c$ . Indeed, in this  $SU(2)$  case, the transformations for  $H_\pm$  and  $H_\pm^c$  must be opposite under  $P_H \otimes P_G$ , in spite of the fact that  $H_\pm$  and  $H_\pm^c$  belong to the same representation,  $\mathbf{2}$ , under the  $SU(2)$ . Similar situations also occur, for instance, in the case of  $G = SO(10)$  with vector representations. However, all explicit models we discuss in this paper are described by  $P_H^2 = P_G^2 = 1$ , because the gauge breaking considered in these models are only



$SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ . Therefore, we call  $P_H$  and  $P_G$  as parity matrices, but it should be understood that the eigenvalues for them can be  $\pm i$  in general.

The argument for  $\mathcal{Z}(0)$  applies identically to the symmetry  $\mathcal{Z}(\pi R)$ , but the basis which diagonalizes  $Z'$  is in general different from that which diagonalizes  $Z$ . Choosing  $Z$  diagonal, we immediately find that  $Z'$  must take the form

$$Z' = U \Sigma_3 U^\dagger \otimes V P'_H V^\dagger \otimes W P'_G W^\dagger, \quad (12)$$

where  $P'_{H,G}$  are diagonal matrices with  $P'^2_H = P'^2_G = \pm 1$ ,  $U, V$  are unitary matrices, and  $W$  is an appropriate matrix making the action invariant under the operation  $\mathcal{Z}(\pi R)$  (for instance,  $W$  is a unitary (orthogonal) matrix for  $G = SU(n)$  ( $SO(n)$ )). Note that  $U$  is an element of  $SU(2)_R$ . Thus the action of  $\Sigma_3$  and  $U \Sigma_3 U^\dagger$  is the same on the  $SU(2)_R$  singlets:  $(\psi, \psi^c) \rightarrow (\psi, -\psi^c)$  and  $(A^\mu, \sigma + iA^5) \rightarrow (A^\mu, -(\sigma + iA^5))$ , but differs on the  $SU(2)_R$  doublets  $\Lambda$  and  $\Phi$ . From the discussion preceding Eq. (8), there is no loss of generality in taking  $\Sigma_3 = \sigma_3$  and  $U = e^{\pi i \alpha \sigma_2}$  acting on the  $SU(2)_R$  doublet space. On the other hand, the forms for  $V$  and  $W$  are highly model dependent. For example, if the flavor group  $H = U(1)^N$ , then  $V = 1$ . If  $H$  contains  $SU(2)$  factors, then in the corresponding  $2 \times 2$  blocks,  $V$  is either  $\sigma_0$  or  $e^{\pi i \beta \sigma_2}$ , depending on whether the two hypermultiplets of the same gauge charge have equal or opposite  $P$  parities.

Throughout this paper, we consider theories with local supersymmetry in the bulk. However the above classification is unchanged in the non-supersymmetric case, as long as the action of  $\Sigma_3$  is reinterpreted. It acts as  $(A^\mu, A^5) \rightarrow (A^\mu, -A^5)$  and  $(\psi, \eta) \rightarrow (\psi, -\eta)$ , where  $\psi$  and  $\eta$  are the components of any 5d Dirac fermion.

### 3 Symmetries and Symmetry Breaking

Every non-trivial entry in Eqs. (11, 12) causes symmetry breaking. One of the two supersymmetries is broken by  $\Sigma_3$ , and the other is broken by a non-trivial  $U$ . The flavor symmetry  $H$  is broken by  $P_H, P'_H, V$  and the gauge symmetry  $G$  by  $P_G, P'_G, W$ .

#### 3.1 Supersymmetry breaking

If the extra dimension is not compactified, the theory is invariant under the local supersymmetry transformations  $\delta\psi(x, y) = \Xi^T(x, y) \not{\partial} \Phi, \dots$  with  $\Xi(x, y) = (\xi_1(x, y), \xi_2(x, y))$  an arbitrary function of spacetime. Compactification with the orbifold boundary conditions of Eqs. (11, 12) reduces the set of local supersymmetry transformations. The action of  $Z$  and  $Z'$  in  $SU(2)_R$  space is given by Eq. (8) so that the theory is invariant under supersymmetry transformations with

the form of Eq. (9)

$$\Xi(x, y) = \begin{pmatrix} \xi_1(x, y) \\ \xi_2(x, y) \end{pmatrix} = R \left( \alpha \frac{y}{R} \right) \sum_{n=0}^{\infty} \begin{pmatrix} \cos[ny/R] \xi_{+n}(x) \\ \sin[ny/R] \xi_{-n}(x) \end{pmatrix}. \quad (13)$$

Although this is a significant restriction, the theory still possesses local 5d supersymmetry. From the low energy viewpoint, the number of 4d supersymmetries is the number of independent modes of Eq. (13) having  $\Xi = \Xi(x)$  independent of  $y$ . There is at most a single zero-mode, since the action of both  $Z$  and  $Z'$  necessarily involves  $\sigma_3$ . In fact, an unbroken 4d supersymmetry only results if  $\alpha = 0$ , in which case it is the mode:

$$\Xi(x) = \begin{pmatrix} \xi_{+0}(x) \\ 0 \end{pmatrix}. \quad (14)$$

For  $\alpha \neq 0$ , Eq. (13) has no zero-mode and hence no 4d supersymmetry survives into the infrared. One supersymmetry is broken by  $1/R$ , the other by  $\alpha/R$ . For any  $\alpha \neq 0$ , the  $n$ th mode is proportional to  $(\cos[(n + \alpha)y/R], \sin[(n + \alpha)y/R])\xi_n(x)$ , and has an axis which rotates with  $y$ . For  $\alpha = 1/2$ , the 4d supersymmetries on the branes at  $y = 0$  and  $y = \pi R$  are orthogonal.

### 3.2 Global symmetry breaking

The global symmetry  $H$  arises from a repetition of hypermultiplets with the same gauge quantum numbers. If  $H$  is generated by  $T^\alpha$ , then the identification by  $\mathcal{Z}(0)$  breaks those generators for which  $[P_H, T^\alpha] \neq 0$ . Similarly an identification by  $\mathcal{Z}(\pi R)$  breaks those generators having  $[VP'_H V^\dagger, T^\alpha] \neq 0$ . The unbroken global group  $H'$  is generated by the set of generators which commutes with both  $P_H$  and  $VP'_H V^\dagger$ .

### 3.3 Gauge symmetry breaking

With a non-compact fifth dimension, the theory is invariant under gauge transformations of  $G$ :  $\delta A^{aM} = \partial^M \epsilon^a(y) + \dots$ , with arbitrary gauge transformation parameters  $\epsilon^a(y)$ . Compactification with non-trivial  $P_G$  implies that the gauge fields split up into two sets  $A^a = (A^{a+}, A^{a-})$ , which are  $(+, -)$  under  $y \rightarrow -y$ . The  $+$  modes have generators which commute with  $P_G$ :  $[P_G, T^{a+}] = 0$ . Hence the compactified theory possesses only a restricted gauge symmetry with gauge parameters constrained to satisfy [12]

$$\epsilon^{a\pm}(-y) = \pm \epsilon^{a\pm}(y). \quad (15)$$

On making a KK mode expansion,  $A^{a+}$  have zero-modes while  $A^{a-}$  do not, so that the low energy 4d gauge group is  $G_+$ , generated by  $T^{a+}$ . We frequently say that compactification using the parities  $P_G$  has induced the gauge symmetry breaking  $G \rightarrow G_+$ . An alternative viewpoint

is that the theory on the compact space possesses a restricted set of gauge transformations, Eq. (15), which are not broken. They do not include zero-mode transformations of  $G/G_+$ .

A precisely analogous argument applies for the gauge symmetry breaking induced by  $P'_G$ :  $G \rightarrow G'_+$ . If  $W = 1$ , so that  $P_G$  and  $P'_G$  are simultaneously diagonalizable, then the zero-mode gauge bosons correspond to the generators which commute with both  $P_G$  and  $P'_G$ , and are therefore  $(+, +)$  modes. The lightest gauge boson mode for other generators have masses of order  $1/R$ . These modes are either  $(+, -)$ ,  $(-, +)$  or  $(-, -)$ . For  $W = 1$ , the mode eigenfunctions are given by Eq. (6).

In the case that  $W(\gamma)$  has a non-zero Euler angle,  $\gamma$ , further gauge symmetry generators are broken  $[WP'_G W^\dagger, T^a] \neq 0$ , with some previously massless gauge bosons acquiring mass  $\gamma/R$ . In this case the KK modes of the local gauge transformations have forms which depend on the continuous parameter  $\gamma$ . Thus in general the total structure of gauge symmetry breaking,  $G \rightarrow G'$ , is very rich.

### 3.4 The brane action

The action has both bulk and brane contributions

$$S = \int d^4x dy [\mathcal{L}_5 + \delta(y)\mathcal{L}_4 + \delta(y - \pi R)\mathcal{L}'_4]. \quad (16)$$

The form of the bulk action is very tightly constrained by the unbroken local 5d gauge and supersymmetry transformations discussed above. What interactions are allowed on the branes at  $y = 0, \pi R$ ?

The constraints imposed by the local symmetries are found in the following way: the brane actions  $\mathcal{L}_4, \mathcal{L}'_4$  are the most general allowed by the gauge and supersymmetry transformations that act at  $y = 0, \pi R$ . At  $y = 0$  these transformations are:

$$\epsilon^{a+}(x)T^{a+}, \quad \Xi(x, 0) = \begin{pmatrix} \xi(x) \\ 0 \end{pmatrix}, \quad (17)$$

while, at  $y = \pi R$ , for the case  $W = 1$ , the transformations are

$$\epsilon^{a'+}(x)T^{a'+}, \quad \Xi(x, \pi R) = R(\pi\alpha) \begin{pmatrix} \xi'(x) \\ 0 \end{pmatrix}. \quad (18)$$

For  $W \neq 1$ , the form of the gauge transformations at  $y = \pi R$  may be more complicated. For  $\alpha \neq 0$  the supersymmetries on the two branes are different — for  $\alpha = 1/2$  they are orthogonal.

What restrictions are imposed on the brane actions  $\mathcal{L}_4, \mathcal{L}'_4$  by the global symmetries  $SU(2)_R$  and  $H$ ? These symmetries may be accidental symmetries of the bulk — a consequence of 5d local supersymmetry — so that the brane actions need not respect them. However, if orbifolding

leaves some part of the global symmetry unbroken, we may choose to impose this on both the bulk and brane actions.

The orbifold transformations  $\mathcal{Z}(0, \pi R)$  may contain non-trivial contributions from the global symmetries  $H$  and  $SU(2)_R$ . However, this does not restrict the form of  $\mathcal{L}_4, \mathcal{L}'_4$ . It only says that  $H$  and/or  $SU(2)_R$  must be symmetries of the bulk action  $\mathcal{L}_5$ . Of course the brane action at any point can only involve fields that are even about that point (or derivatives of odd fields). However, a brane interaction at  $y = 0$  can transform non-trivially under  $Z'$  or, equivalently,  $T$ . This transformation simply serves to fix the brane action at  $y = 2\pi nR$  in terms of that at  $y = 0$  — it does not constrain the action at  $y = 0$ .

We conclude that the global symmetries  $H$  and  $SU(2)_R$  do not necessarily place any restrictions on  $\mathcal{L}_4, \mathcal{L}'_4$ , which may be taken to be the most general set of interactions invariant under the gauge and supersymmetry transformations at  $y = (0, \pi R)$ .

### 3.5 Calculability

The short distance divergence structure of the theory must reflect the unbroken local gauge symmetry and supersymmetry. Since the action was taken as the most general respecting the local symmetries, all short distance radiative corrections must take the form of local operators which are already present in Eq. (16). Thus any quantity which is forced by the local symmetries to vanish at tree level will have finite UV radiative corrections. Such quantities need not vanish; they may be generated by IR physics.

As an example, consider the case of  $G = SU(3) \times SU(2) \times U(1)$  with a single Higgs doublet hypermultiplet in the bulk [7]. The zero-mode structure is precisely that of the one Higgs doublet standard model. There has been considerable debate recently about the radiative structure of the zero-mode Higgs boson mass [15], with some arguing that it is finite and some that it is quadratically divergent. The exact unbroken local supersymmetry, given by Eq. (13) with  $\alpha = 1/2$ , is sufficient to guarantee that the mass of the zero-mode Higgs boson is radiatively UV finite to all orders in perturbation theory. Those who claim divergent behavior have apparently not realized that there is an unbroken 5d local supersymmetry in the theory. The Higgs mass is non-zero because at distances larger than  $1/R$  there are non-local IR contributions. In the low energy effective theory (in this case the standard model) these contributions appear to be quadratically divergent (the usual top loop contribution to the Higgs mass). However, at shorter distances the locality of the fifth dimension becomes operative and removes the divergence. In calculating explicit loop diagrams, this is seen most transparently by going to position space for the extra dimension, as in Ref. [8]. When using momentum space, the internal propagators are expanded in KK modes, and the sum of these KK modes must be done in a way which preserves the local supersymmetry. A simple way of doing this is to include the contributions from all

modes.

Whether a quantity is finite and calculable simply depends on whether a local operator can be written which contributes to it. In any theory with the local supersymmetry Eq. (13), all magnetic dipole moment operators vanish at tree level and will therefore be radiatively finite. Recently a finite one-loop contribution to  $b \rightarrow s\gamma$  has been computed [16] in the theory of Ref. [7]. On the other hand the electroweak  $\rho$  parameter arises at tree level from the supersymmetric operator  $\delta(y) \int d^4\theta (H^\dagger e^{gV} H)^2$ , and hence is subject to quadratically divergent radiative corrections.

In calculating radiative corrections to such quantities as the Higgs boson mass and  $b \rightarrow s\gamma$  one may wonder whether contributions from gravitino exchange are important. Since we have local supersymmetry in the bulk, such interactions are certainly present. They cannot change the above arguments about finiteness, and now we argue that they contribute only very small amounts to the finite quantities. The local symmetry structure of the theory becomes apparent at distances smaller than  $1/R$ , and hence all contributions from shorter distances are cut off. However, the gravitino interactions are weak at scale  $1/R$  and do not make a substantial contribution. At some higher energy scale the gravitational interaction becomes strong, but these local interactions cannot contribute significantly to the finite quantities.

## 4 Simple Models with Bulk Higgs

In this section we consider simple supersymmetric models with  $G = SU(5)$  or  $G = SU(3) \times SU(2) \times U(1)$ , with the Higgs doublet(s) in the bulk. The breaking of electroweak symmetry is then linked to the physics of the bulk. In  $SU(5)$  theories, a crucial role of the bulk is to accomplish doublet-triplet splitting. In the non-unified case, the Higgs is also a near zero-mode. In all cases we consider the role the bulk plays in breaking supersymmetry.

In general we are interested in non-trivial  $U, V, W$  so that  $Z$  and  $Z'$  are not simultaneously diagonalized. However, some of the simple theories do have  $[Z, Z'] = 0$ , while in other theories the lack of commutativity is small, so that it is convenient to think first about the commuting case.

After global and gauge symmetry breaking, there are a collection of  $H' \times G'$  irreducible hypermultiplets  $(\phi, \psi; \phi^c, \psi^c)$ . Given the gauge and global parities  $P_G, P'_G, P_H$  and  $P'_H$  of Eqs. (11, 12), the fermion  $\psi$  has four possibilities for its  $(P, P')$  parities:  $\psi(p, p')$  with  $p, p' = \pm 1$ . The  $(P, P')$  parities of all other components of the hypermultiplet are now fixed in terms of  $P_R \equiv \exp(2\pi i \alpha \sigma_2) = (+1, -1)$  for  $\alpha = (0, 1/2)$ :

$$[\phi(p, P_R p'), \psi(p, p'); \phi^c(-p, -P_R p'), \psi^c(-p, -p')]. \quad (19)$$

There are four different types of hypermultiplet according to whether  $p = \pm$  and  $t \equiv pp' = \pm$ . If  $P_R = +1$  supersymmetry is unbroken, and there is a zero-mode chiral multiplet only for parities

of equal signs ( $t = +1$ ). If  $P_R = -1$  supersymmetry is broken, and for parities of equal signs ( $t = +1$ ) the zero-mode is a fermion, while for parities of opposite signs ( $t = -1$ ) the zero-mode is a scalar.

Similarly, after gauge symmetry breaking, a gauge boson may have any combination of parities,  $A^\mu(p, p')$ , but the other components of the 5d vector multiplet are then given:

$$\left[ A^\mu(p, p'), \lambda(p, P_R p'); (\sigma + iA^5)(-p, -p'), \lambda'(-p, -P_R p') \right]. \quad (20)$$

The KK mode expansion for any of these fields is given by its  $(P, P')$  quantum numbers according to Eq. (6). If  $t = +1$  ( $-1$ ) the KK modes have mass  $m_n = n/R$  ( $(n + 1/2)/R$ ). The fermion masses of the tower are Dirac type.

It is remarkable that in 5d the most general possible supersymmetry breaking is described by just a single parameter  $\alpha$ . For arbitrary  $\alpha$ , but keeping  $\beta = \gamma = 0$ , the eigenfunctions of the  $SU(2)_R$  doublets  $(\phi, \phi^{c\dagger})$  and  $(\lambda, \lambda')$  pass from Eq. (6) to Eq. (9) with the eigenvalues shifted by  $\alpha/R$

$$m_n \rightarrow \begin{cases} m_n \pm \alpha/R & \text{non zero-mode} \\ \alpha/R & \text{zero-mode.} \end{cases} \quad (21)$$

The gauginos become Majorana and are shifted in mass relative to their gauge boson partners. Similarly the hypermultiplet scalars are shifted in mass relative to their fermionic partners. In both cases the mass of the zero-mode is lifted by  $\alpha/R$ , while the excited members of the  $SU(2)_R$  doublets get split in mass by  $\pm\alpha/R$  relative to the corresponding  $SU(2)_R$  singlet states.

#### 4.1 Models with $G = SU(3) \times SU(2) \times U(1)$

We choose the orbifold symmetries to preserve the gauge group, so that  $P_G$  and  $P'_G$  are trivial. All the vector multiplets therefore have  $p = p' = 1$  in Eq. (20).

The simplest possibility is that there is a single Higgs hypermultiplet in the bulk. From Eq. (19) we see that if  $t = pp' = 1$  for this hypermultiplet, there is a single zero-mode Higgsino, so that this case is forbidden by anomalies. For  $t = -1$  and supersymmetry unbroken,  $R_P = 1$ , there is no zero-mode Higgs boson. Such a situation is hard to reconcile with observation: supersymmetry is unbroken and the Higgs mass squared has a large positive value comparable to the masses of the KK excitations of the standard model gauge particles. The unique theory with 1 Higgs hypermultiplet has  $t = -1$  and  $\alpha \neq 0$ . The case of  $\alpha = 1/2$  was studied in Ref. [7]. In this theory the Higgs potential depends on only 1 unknown parameter,  $1/R$ , and since the Higgs mass is finite it can be predicted:  $m_h = 127 \pm 8$  GeV. A deformation of this theory is possible by allowing  $\alpha$  to deviate from  $1/2$ , so that  $[Z, Z'] \neq 0$ . We study this deformation in section 5. The unique 1 Higgs hypermultiplet theory may therefore be described by  $Z, T$  in the

supersymmetry and Higgs flavor spaces as

$$Z = \Sigma_3 \otimes 1, \quad (22)$$

$$T = e^{2\pi i(1/2+\theta)\sigma_2} \otimes -1. \quad (23)$$

In many theories it is useful to consider the  $Z, T$  basis, since the symmetry breaking is transparently summarized by  $T$ . The simplest assignment of matter is for quark and lepton superfields to all be in the bulk with positive  $T$  parity so that they all contain a single zero-mode fermion. Thus the orbifold quantum numbers in the matter flavor space are  $(Z_M, T_M) = (+1, +1)$ . Indeed, the requirement that all charged fermions have Yukawa coupling to the zero-mode Higgs and that the KK modes would not yet have been discovered makes this all but unique. The only other possibility known to us has  $u_R$  and  $d_R$  superfields located on the branes at  $y = 0$  and  $y = \pi R$ , respectively, and the rest of the matter in the bulk.

The most general theory with two Higgs hypermultiplets is conveniently described in the supersymmetry and Higgs flavor spaces by

$$Z = \Sigma_3 \otimes \sigma_3, \quad (24)$$

$$T = e^{2\pi i\alpha\sigma_2} \otimes e^{2\pi i\beta\sigma_2}, \quad (25)$$

and involves two free parameters:  $\alpha, \beta$ .<sup>1</sup> This theory was written down by Pomarol and Quiros [5], who took the view that  $\alpha$  and  $\beta$  were of order unity. At the compactification scale,  $1/R$ , supersymmetry is broken, so that the theory below  $1/R$  is non-supersymmetric and must contain a Higgs zero-mode. This happens only for the case  $\alpha = \beta$ , which was the focus of their work [5, 6]. Such a light Higgs requires a relation between the breaking of supersymmetry,  $\alpha$ , and the breaking of Peccei-Quinn symmetry,  $\beta$ . We have recently advocated an alternative view [14] where  $\alpha$  and  $\beta$  are taken to be extremely small. In this case the effective theory below  $1/R$  is the MSSM. The parameters  $\alpha$  and  $\beta$  force a non-trivial  $y$  dependence for the zero-mode Higgs,  $h_{u,d}$ , and gauginos,  $\lambda$ , so that on compactification they lead to the mass terms

$$\begin{aligned} \mathcal{L} = & -\frac{\alpha}{2R}(\lambda\lambda + \text{h.c.}) - \frac{\alpha^2}{R^2}(h_u^\dagger h_u + h_d^\dagger h_d) \\ & -\frac{\beta}{R}(\tilde{h}_u \tilde{h}_d + \text{h.c.}) - \frac{\beta^2}{R^2}(h_u^\dagger h_u + h_d^\dagger h_d) \\ & +\frac{2\alpha\beta}{R^2}(h_u h_d + \text{h.c.}). \end{aligned} \quad (26)$$

The first line gives the supersymmetry breaking soft masses determined by  $\alpha$  alone, while the second line gives the Peccei-Quinn breaking terms induced by  $\beta$  alone. The third term is proportional to both supersymmetry and Peccei-Quinn symmetry breaking. It is remarkable that these

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<sup>1</sup> For  $Z = \Sigma_3 \otimes \sigma_0$  the two lightest Higgs modes have the same hypercharge.

mass terms correspond precisely to those of the MSSM. The common scalar and gaugino mass is  $\alpha/R$ , the  $\mu$  parameter is  $\beta/R$  and the soft parameter  $B$  is predicted to be  $2\alpha/R$ . The signs of the two Peccei-Quinn breaking terms are correlated such that the conventional  $\mu$  parameter is negative. It is remarkable that the most general 2 Higgs hypermultiplet theory in 5d, Eqs. (24, 25), leads to the MSSM soft operators, with a unified origin for both supersymmetry breaking and the  $\mu$  parameter. The smallness of supersymmetry breaking and the  $\mu$  parameter are both due to the smallness of the commutator  $[Z, T]$ . Quarks and leptons can be on either brane, or, if they are in the bulk, they have orbifold quantum numbers in the matter flavor space of  $(Z_M, T_M) = (+1, +1)$ .

## 4.2 Models with $G = SU(5)$

The weakest aspects of conventional 4d grand unified theories are the breaking of the unified gauge symmetry and arranging the mass splitting of Higgs triplets from Higgs doublets. Assigning a non-trivial action for the orbifold symmetries in the gauge space, and taking  $1/R$  to be the scale of gauge coupling unification, opens up new, higher dimensional possibilities for grand unification, with orbifold breaking of the gauge group and orbifold doublet-triplet splitting. In the case of 5d, there are two parities  $Z$  and  $Z'$  available for gauge symmetry breaking. If the gauge group is  $SO(10)$ , there is no choice for these parities which gives a set of zero-modes for the 5d vector multiplet corresponding to a successful weak mixing angle prediction. We therefore confine our attention to the case that the gauge group is  $SU(5)$ .

To obtain a theory below  $1/R$  with (approximate) 4d supersymmetry and two Higgs doublets, one should start with two Higgs hypermultiplets in the  $\mathbf{5}$  of  $SU(5)$ . The most general orbifold symmetry which breaks  $SU(5)$  to  $SU(3) \times SU(2) \times U(1)$ , and does not give unwanted zero-modes from the 5d vector multiplet, or from the two Higgs hypermultiplets, is [14]

$$Z = \Sigma_3 \otimes \sigma_3 \otimes I_5, \quad (27)$$

$$T = e^{2\pi i \alpha \sigma_2} \otimes -e^{2\pi i \beta \sigma_2} \otimes \begin{pmatrix} I_3 & 0 \\ 0 & -I_2 \end{pmatrix}, \quad (28)$$

where  $I_n$  is the  $n \times n$  unit matrix. This theory has unbroken, local 5d supersymmetry transformations of the form Eq. (13). It also has unbroken, local 5d gauge transformations. Those corresponding to the generators of the standard model gauge group,  $\epsilon^{3-2-1}(y)$ , have  $(Z, Z') = (+, +)$ , while the remaining transformation parameters,  $\epsilon^X(y)$ , have  $(Z, Z') = (+, -)$ . These transformation parameters therefore have the appropriate KK mode expansions of Eq. (6). Notice that the full  $SU(5)$  gauge transformations are operative at the brane at  $y = 0$ , while only those of  $SU(3) \times SU(2) \times U(1)$  act at the brane at  $y = \pi R$ .

In the case that  $\alpha = \beta = 0$  the orbifold does not break 4d supersymmetry or the Peccei-Quinn symmetry. This is the case introduced by Kawamura [10] and extended to include matter and



the unification of gauge couplings [12]. It is important to stress that the orbifold symmetries in Higgs flavor space must take the form

$$(Z_H, T_H) = (\sigma_3, -\sigma_0). \quad (29)$$

Other assignments do not lead to zero-mode Higgs doublets. For example, the  $\sigma_3$  ensures that the light doublets are vector-like with respect to the unbroken gauge group.

Three generations of grand unified matter  $(T, \bar{F})$  can be placed on the brane at  $y = 0$ . The Yukawa couplings to the bulk Higgs fields are also located at this point, and should therefore be  $SU(5)$  invariant.<sup>2</sup> These Yukawa interactions do not induce  $d = 5$  proton decay, because the form of the masses for the Higgs triplets, generated by the orbifolding, possesses a symmetry which sets the amplitude to zero [12]. The Yukawa couplings lead to the successful  $b/\tau$  mass relation for the third generation. Similar relations for the lighter generations can be avoided by mixing with heavy bulk matter [12].

Alternatively, matter may be placed in the bulk [12, 13]. A single generation requires two sets of  $\mathbf{10} + \bar{\mathbf{5}}$  hypermultiplets:  $T, T', \bar{F}, \bar{F}'$ . On the two dimensional space,  $(T, T')$  and  $(\bar{F}, \bar{F}')$ , the orbifold symmetry acts as

$$(Z_M, T_M) = (\sigma_0, \sigma_3), \quad (30)$$

for each generation. The  $\sigma_0$  ensures that the light matter is chiral under the unbroken gauge group, while the  $\sigma_3$  ensures that an entire generation,  $q, u, d, l, e$ , is massless. Strictly speaking, the unification of quarks and leptons is largely lost:  $T(u, e), T'(q), \bar{F}(d), \bar{F}'(l)$ . However, the  $SU(5)$  understanding of the quantum numbers of a generation is preserved. This combination of bulk matter is the smallest which leads to anomaly-free, chiral zero-modes, and it automatically gives the quantum numbers of a generation. There is no proton decay from  $SU(5)$  gauge exchange, and there are no  $SU(5)$  fermion mass relations [12]. Whether matter is placed on the brane or in the bulk, the theory beneath  $1/R$  is the MSSM without supersymmetry breaking or Peccei-Quinn symmetry breaking.

The only remaining freedom in the structure of the orbifold symmetry Eqs. (27, 28) is a non-zero value for  $\alpha$  and  $\beta$  [14]. Since these parameters break 4d supersymmetry and the Peccei-Quinn symmetry, they must be extremely small. As in the case of the standard model gauge group, they lead to the zero-mode mass terms of Eq. (26).

The color triplet Higgsino mass matrix from orbifolding turns off dimension 5 proton decay, and bulk matter turns off proton decay from the  $SU(5)$  gauge interactions and from scalar Higgs triplet exchange. Hence  $1/R$  could be reduced to the TeV scale.<sup>3</sup> While the precise weak mixing angle prediction is lost, KK modes of standard model particles, as well as those of  $X$  and  $Y$  gauge

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<sup>2</sup> Those of Ref. [11] are not invariant under the  $SU(5)$  gauge transformations discussed above.

<sup>3</sup> Grand unified theories at the TeV scale were considered in Ref. [17] with a different mechanism of suppressing proton decay from the  $SU(5)$  gauge interactions.

bosons and fermions, could be produced at high energy colliders. In such grand unified theories at the TeV scale, supersymmetry could be broken by the orbifold via a large  $\alpha$  parameter, and there could be one or two Higgs quasi zero-modes. Above the compactification scale the running of the gauge couplings is dominated by  $SU(5)$  symmetric power law running [12, 18]. Thus, for this scheme to be viable, there must be some large new exotic contributions to the gauge couplings either at or beneath the compactification scale.

## 5 $SU(3) \times SU(2) \times U(1)$ Model with One Higgs Doublet

In this section, we investigate radiative electroweak symmetry breaking in one Higgs doublet theories with  $G = SU(3) \times SU(2) \times U(1)$ . We consider the case where all three generations of matter and a single Higgs propagate in the 5d bulk. The most general orbifold boundary conditions are given by Eqs. (22, 23), so that we have a 1 parameter family of theories parameterized by a real number  $\theta$  ( $0 \leq \theta < 1/2$ ). For any member of this family, the Higgs effective potential depends on only one free parameter  $1/R$ , so that the physical Higgs boson mass  $m_h$  and the compactification scale  $1/R$  are calculable. The  $\theta = 0$  case corresponds to the model in Ref. [7].

The KK mass spectrum for the  $\theta = 0$  case is given by

$$\begin{aligned} m, h, A^\mu & : n/R \\ \tilde{m}, \tilde{m}^c, \tilde{h}, \tilde{h}^c, \lambda, \lambda' & : (n + 1/2)/R \\ m^c, h^c, \sigma & : (n + 1)/R, \end{aligned} \tag{31}$$

where  $n = 0, 1, 2 \dots$  and  $m$  represents  $q, u, d, l, e$ . A non-zero value for  $\theta$  modifies the above mass spectrum such that the scalar and gaugino masses are shifted by  $\theta/R$ . In particular, the Higgs boson  $h$  obtains a tree-level mass  $\theta/R$ , and the two linear combinations of  $\tilde{m}$  and  $\tilde{m}^c$  have split masses of  $(n + 1/2 \pm \theta)/R$ . As a result, the Higgs effective potential depends on  $\theta$  and the values for  $m_h$  and  $1/R$  also depend on  $\theta$ .

Radiative electroweak symmetry breaking occurs only when  $\theta$  is small. Since the tree-level Higgs mass squared is given by  $\theta^2/R^2$  and one-loop negative contribution through the top Yukawa coupling is  $\sim -(1/\pi^4)(y_t^2/R^2)$ ,  $\theta \lesssim 1/\pi^2$  is required to break electroweak symmetry. This small  $\theta$  perturbs the field dependent masses of the top and stop KK towers as

$$m_{F_n} = \left\{ n + \frac{1}{\pi} \arctan(\pi y_t H R) \right\} \frac{1}{R}, \tag{32}$$

$$m_{B_n^\pm} = \left\{ n \pm \theta + \frac{1}{\pi} \operatorname{arccot}(\pi y_t H R) \right\} \frac{1}{R}, \tag{33}$$

where  $n = -\infty, \dots, +\infty$ ,  $H \equiv |h|$ , and there are one Dirac fermion ( $F_n$ ) and two complex scalars ( $B_n^\pm$ ) at each level  $n$ . With these masses, we can calculate the one-loop Higgs effective potential

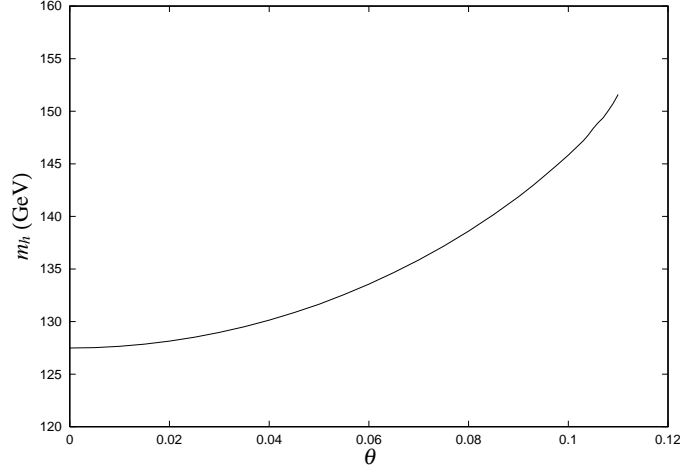


Figure 2: The physical Higgs boson mass  $m_h$  as a function of  $\theta$ .

$V_t(H)$  from the top-stop loop, using calculational techniques in Ref. [19]:

$$V_t(H) = \frac{9}{16\pi^6 R^4} \sum_{k=1}^{\infty} \left\{ \frac{\cos[2k \arctan(\pi y_t R H)]}{k^5} - \cos[2\pi k \theta] \frac{\cos[2k \operatorname{arccot}(\pi y_t R H)]}{k^5} \right\}. \quad (34)$$

Then, together with the tree-level Higgs potential

$$V_{H,0}(h) = \frac{\theta^2}{R^2} H^2 + \frac{g^2 + g'^2}{8} H^4, \quad (35)$$

we can derive the values for  $m_h$  and  $1/R$  by requiring that  $\langle H \rangle = 175$  GeV.

In Figures 2 and 3, we have plotted the predicted values for  $m_h$  and  $1/R$  as a function of  $\theta$ , including one-loop gauge contributions to the quadratic term in the potential. They are both monotonically increasing functions with respect to  $\theta$ . The  $\theta = 0$  case reproduces the values obtained in Ref. [7]:  $m_h = 127$  GeV and  $1/R = 731$  GeV. Note that the definition of  $R$  here is different from that in Ref. [7] by a factor of 2, so that it corresponds to  $1/R = 366$  GeV in the notation of Ref. [7]. (A slight increase of  $1/R$  compared with the previous value comes from an improved treatment of higher order effects. This also changes the central value for the estimate of the lightest stop mass to  $m_{\tilde{t}_-} = 211$  GeV.) As  $\theta$  is increased to  $\theta \gtrsim 0.1$ ,  $1/R$  approaches infinity meaning that electroweak symmetry breaking does not occur beyond that value of  $\theta$ . It is interesting that we can obtain larger values for  $1/R$  by perturbing the model of Ref. [7] with small non-zero  $\theta$ . It reduces the amount of tuning required to obtain phenomenologically acceptable value of the  $\rho$  parameter, since the contribution from KK towers to the  $\rho$  parameter scales as  $(1/R)^{-2}$ .

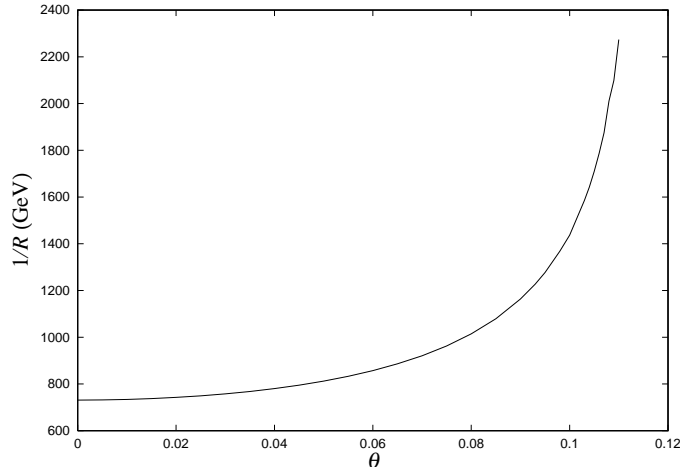


Figure 3: The compactification scale  $1/R$  as a function of  $\theta$ .

## 6 Conclusions

In this paper we have given the most general form for the orbifold breaking of symmetries from a single extra dimension. While the structure of gauge and flavor symmetry breaking is very rich, the breaking of supersymmetry is described by a single free parameter. The supersymmetry breaking from all 5d theories is therefore guaranteed to have a simple form. We have explicitly exhibited the form of the local supersymmetry transformations which are left unbroken by the orbifolding. All ultraviolet divergences of the theory must correspond to local operators which respect this unbroken local supersymmetry. It is this symmetry that results in the ultraviolet finiteness of the Higgs mass in certain theories.

We have explicitly constructed the most general orbifold symmetries for  $N = 1$  supersymmetric, 5d models with gauge group  $SU(3) \times SU(2) \times U(1)$  and  $SU(5)$ , having either one or two Higgs hypermultiplets in the bulk. There are very few such theories. There is a unique one parameter family of  $SU(3) \times SU(2) \times U(1)$  theories with a single Higgs hypermultiplet. We have studied radiative electroweak symmetry breaking in this family of theories, and the Higgs boson mass and the compactification scale are shown as a function of this parameter,  $\theta$ , in Figures 2 and 3, respectively. The special case  $\theta = 0$  gives a central value for the Higgs mass of 127 GeV [7].

There is a unique two parameter family of  $SU(3) \times SU(2) \times U(1)$  theories with two Higgs hypermultiplets. One parameter breaks supersymmetry and the other breaks Peccei-Quinn symmetry. This family was first constructed by Pomarol and Quiros [5] where the compactification scale was taken to be in the TeV region. To obtain a zero-mode Higgs boson, the two orbifold parameters were taken equal, giving a one dimensional parameter space. After radiative elec-

trouweak symmetry breaking, the Higgs boson was found to be lighter than 110 GeV throughout this one dimensional parameter space, almost excluding the theory. Theories with a heavier Higgs boson might result when the two parameters are allowed to differ by a small perturbation. Another possibility is that the compactification scale is taken much larger than the TeV scale, and both parameters are taken very small [14]. In this case the theory below the compactification scale is the MSSM with a constrained form for the soft supersymmetry breaking operators as shown in Eq. (26). Radiative electroweak symmetry breaking requires that the compactification scale be in the interval  $10^6 - 10^9$  GeV.

There is a unique two parameter family of  $SU(5)$  theories with two Higgs hypermultiplets, where the orbifolding breaks  $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ . In the special case that the two free parameters vanish, the orbifolding corresponds to that introduced by Kawamura [10] and developed in Ref. [12]. The theory including the two free orbifold parameters gives a unified origin for  $SU(5)$ , supersymmetry and Peccei-Quinn breaking [14].

These three families of theories are the only ones in 5d with the stated gauge groups and bulk Higgs modes. While each family has variants depending on the location of the quarks and leptons, we have stressed the tightly constrained form of orbifold symmetry breaking. In *all* cases, the group theoretic structure of the symmetry breaking is given by Eqs. (27, 28), where  $Z$  is the orbifold parity and  $T$  the translation symmetry. If there is no  $SU(5)$  unification the last space is removed in these equations with appropriate sign changes in the second space. If there is a single Higgs doublet, then in the second space  $Z$  is +1 and  $T$  is -1.

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