Title

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Summary: The monetary character of trade, use of a common medium of exchange, is shown to be an outcome of economic general equilibrium in the presence of transaction costs and market segmentation (in trading posts with a separate budget constraint at each transaction). Commodity money arises endogenously as the most liquid (lowest transaction cost) asset. Scale economies in transaction cost account for uniqueness of the (fiat or commodity) money in equilibrium, creating a natural monopoly. Trading posts using a medium of exchange create a network externality inducing others' adoption of the same medium. Bertrand monetary equilibria (among competing trading posts) and uniqueness of 'money' are robust to threats of entry. Government-issued fiat money has a positive equilibrium value from its acceptability for tax payments and sustains its natural monopoly through the scale of government economic activity.

Keywords and Phrases: Commodity money, Fiat money, Transaction cost, Scale economy, Double coincidence of wants

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Existence and Uniqueness of 'Money' in General Equilibrium: Natural Monopoly in the Most Liquid Asset[1]

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"[An] important and difficult question...[is] not answered by the approach taken here: the integration of money in the theory of value..."
------ Gerard Debreu, *Theory of Value* (1959)

One of the oldest issues in economics is to explain the use of money, preferably in elementary terms based on the theory of value. There are contributions extending from Aristotle's *Politics* and Smith's *Wealth of Nations* to the present. The superiority of monetary trade to barter explains why monetary trade is efficient but not why monetary trade is a market equilibrium. No economic agent can individually decide to monetize; monetary exchange should be the equilibrium outcome of interaction among optimizing agents. *Money*, like *written language*, is one of the fundamental discoveries of civilization. Despite the evident superiority of monetary trade, it is puzzling; monetary trade involves one party to a transaction giving up something desirable (labor, his production, a previous acquisition) for something useless (a fiduciary token or a commonly traded commodity for which he has no immediate use) in the hope of advantageously retrading it. The foundations of monetary theory should include elementary economic conditions that allow this paradox to be sustained as an individually rational market equilibrium. Is there a (parsimonious) model of an economy where existence of a common medium of exchange is a result of the optimizing behavior

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of individual firms and households? Does the price system create money? The solution proposed in this paper focuses on transaction costs and their scale economies. The monetary character of trade, use of a common medium of exchange, is shown to be an outcome of an economic general equilibrium. Markets are assumed to be segmented[2] in trading posts, with a separate budget constraint at each transaction creating demand for a carrier of value between trading posts. Commodity money arises endogenously as the most liquid (lowest transaction cost) asset. Scale economies in transaction cost account for uniqueness of the (fiat or commodity) money in equilibrium, creating a natural monopoly. Trading posts using a medium of exchange create a network externality inducing others' adoption of the same medium. Bertrand monetary equilibria (among competing trading posts) and uniqueness of 'money' are robust to threats of entry. Government-issued fiat money has a positive equilibrium value from its acceptability for tax payments (a notion attributable to Adam Smith) and it sustains its natural monopoly due to the scale of government economic activity.

I. Money in Walrasian General Equilibrium

Consider four commonplace observations on the character of trade in virtually all economies:

(i) Trade is monetary. One side of almost all transactions is the economy's common medium of exchange.

(ii) Money is (virtually) unique. Though each economy has a 'money' and the 'money' differs among economies, almost all the transactions in most places most of the time use a single common medium of exchange.

(iii) 'Money' is government-issued fiat money, trading at a positive value though it conveys directly no utility or production.

(iv) Even transactions displaying a double coincidence of wants are transacted with money[3].

Where economic behavior displays such uniformity, a general elementary economic theory should be able to account for the universal usages. But (i), (ii), and (iii) contradict the implications of a frictionless Walrasian general equilibrium model, and (iv) contradicts the conventional view of the role of money (with regard to the double coincidence of wants). This essay presents a class of examples with a slight modification of the Arrow-Debreu general equilibrium model sufficient to derive points (i)-(iv) as outcomes. In doing so, this essay responds to a challenge expressed by Tobin (1980)

> Social institutions like money are public goods ... General equilibrium theory is not going to explain the institution of a monetary ... common means of payment.

Thus the examples below are intended to show that a general equilibrium model can explain endogenously from price theory the institution of a common monetary means of payment[4]. The price system itself designates 'money' and guides transactors to trade using

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[2] The notion of market segmentation is essential to monetization, Alchian (1977).

[3] University of California faculty whose children are enrolled at the University pay fees in money, not in kind; Ford employees buying a Ford car pay in money, not in kind; Albertson's supermarket checkout clerks acquiring groceries pay in money, not in kind. This observation suggests that the focus on the absence of double coincidence of wants --- as distinct from transaction costs --- as an explanation for the monetization of trade may miss a significant part of the underlying causal mechanism.

[4] A bibliography of the issues involved in this inquiry appears in Ostroy and Starr (1990). In

'money.' The model emphasizes complete markets and complete information. Points (ii) and (iv) involve scale economies, nonconvex transaction costs; it will typically be difficult to develop general existence of equilibrium theorems --- hence the use of examples.

It is well known that a frictionless Arrow-Debreu model cannot accomodate a role for money. The single budget constraint facing transactors in the model precludes a carrier of value between transactions. This essay is intended as a partial counterexample, demonstrating that minimal friction in trade is sufficient to induce the existence of money as a result, not an assumption. The monetary structure of the economy is derived from elementary price theory in a class of examples. Use of a common medium of exchange, a commodity money, is an outcome of the market equilibrium. Starting from a (non-monetary) Arrow-Debreu model, the monetary quality of the economic equilibrium is derived through the addition of market segmentation (with a separate budget constraint in each segment) and transaction costs. Multiplicity of budget constraints --- requiring that goods acquired be paid for by delivery of equal value at each trade separately, Ostroy (1973) --- creates a demand for media of exchange. Transaction costs imply differing bid and ask prices for each good. Liquidity is priced: its price is the bid/ask spread. The most liquid asset, the instrument that provides liquidity at lowest cost, will be chosen as the medium of exchange. Thus, the choice of a 'money' is the outcome of optimizing behavior of economic agents in a market equilibrium. Fiat money --- issued by government --- derives its positive value from acceptability in payment of taxes; it becomes the medium of exchange from its low transaction cost. Uniqueness of (fiat or commodity) money follows from scale economy in transaction costs.

Section III of the paper presents the model of segmented markets with linear transaction costs without double coincidence of wants. Commodity money arises endogenously in market equilibrium. Section IV demonstrates that the absence of double coincidence of wants is essential to monetization of trade in a linear model by considering the same problem with full double coincidence of wants. The result is a nonmonetary equilibrium. Section VI considers a (nonconvex) transaction technology with scale economies. The examples there demonstrate that uniqueness of money (uniqueness of the endogenously chosen medium of exchange) results from scale economies in transaction costs. Further, Section VI demonstrates that scale economies in transaction cost account for monetization of trade with a unique 'money' even when there is full double coincidence of wants. Section VII presents the same issues in an oligopolistic setting, as a Bertrand equilibrium. Section VIII considers government-issued fiat money whose value is supported by acceptability in payment of taxes. Scale economies in transaction

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addition, note particularly Banerjee and Maskin (1996), Hellwig (2000), Howitt (2000), Howitt and Clower (2000), Iwai (1996), Kiyotaki and Wright (1989), Marimon, McGrattan and Sargent (1990), Rajeev (1999), Rey (2001), Trejos and Wright (1995), and Young (1998). The treatment of transaction costs in this essay (as opposed to the recent focus in the literature on search and random matching equilibria) resembles the general equilibrium models with transaction cost developed in Foley (1970), Hahn (1971), Starrett (1973), and Kurz(1974). The structure of bilateral trade here however is more detailed, with a budget constraint enforced on each transaction separately, so that the Foley, Hahn, and Starrett models do not immediately translate to the present setting.

cost and government's large scale ensure that fiat money is the unique common medium of exchange.[5]

II. Formalizing Menger's 'Origin of Money'

Over a century ago, Carl Menger presented the paradox of monetary trade as a challenge to monetary theory and proposed an outline of its solution, a theory of liquidity as the basis of monetary theory, Menger (1892):

> It is obvious ... that a commodity should be given up by its owner ...for another more useful to him. But that every[one] ... should be ready to exchange his goods for little metal disks apparently useless as such...or for documents representing [them]...is...mysterious...
>
> why...is...economic man ...ready to accept a certain kind of commodity, *even if he does not need it,* ... in exchange for all the goods he has brought to market[?]
>
> [Call] goods ... *more or less saleable*, according to the ... facility with which they can be disposed of ... at current purchasing prices or with less or more diminution... Men ... exchange goods ... for other goods ... more saleable....[which] become *generally* acceptable media of exchange [emphasis in original].

Menger's proposed solution focused on the liquidity of commodities. A good is very *saleable* (liquid) in Menger's definition above, if the price at which a household can sell it (the market's prevailing bid price) is very near the price at which it can buy (the market's prevailing ask price). In this setting, price theory includes a theory of liquidity. The segmented market creates a demand for a carrier of value between transactions. Separate bid and ask prices represent transaction costs and put a price on liquidity: a

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[5] It is useful to distinguish search/random matching models of money, e.g. Kiyotaki and Wright (1989), Trejos and Wright (1995), from general equilibrium models with transaction cost e.g. Foley (1970), Hahn (1971), Starrett (1973), Ostroy and Starr (1974), Iwai (1996), Howitt(2000) and this essay. Search models emphasize very imperfect markets with limited ability of traders to locate desirable trades and with limited price flexibility. That approach is consistent with Smith (1776), v.I, book I, ch. 4. General equilibrium models typically model complete markets and a fully articulated price system. Using the general equilibrium approach allows us to pursue a parsimoniuous theory: What is a minimal set of market imperfections so that money arises endogenously?

The random matching/search formalization of the friction in trade has a very classical implication: in the rare case where two agents have a double coincidence of wants and meet to trade, they will trade their goods or services directly for one another, Kiyotaki and Wright (1991), Trejos and Wright (1993). This is a distinctive feature, distinguishing the random matching/search models from general equilibrium with transaction cost models. In the present model, direct trade between agents with reciprocal demands will take place only when that arrangement provides the lowest available transaction cost (Example IV.1). Hence, even in the rare instance of double coincidence of wants, general equilibrium models with transaction cost need not predict direct trade between parties with reciprocal demands and supplies.

In actual monetary economies, in those comparatively rare instances where double coincidence of wants occurs, it is seldom resolved by barter exchange. Trade between agents --- even with a double coincidence of wants --- usually takes a monetary form. This is typified by the examples above of a University of California professor's child's University fees, a supermarket checkout clerk's payment for groceries, and an autoworker's purchase of a car. Even in the setting most propitious for barter, those instances where double coincidence of wants occurs, monetary trade prevails. This usage contradicts the predictions of the random matching/search models. It is consistent however with Ostroy and Starr (1974) Theorem 4, and it is precisely the behavior Examples VI.2, VI.3, VI.4, VII.2, VII.3, and VIII.1 below would predict.

good's bid/ask spread is the price of using it as a medium of exchange. Hence, a good with a uniformly narrow bid/ask spread is highly liquid --- in Menger's word 'saleable' --- and constitutes a natural 'money.' Price theory implies monetary theory. Liquidity creates monetization. This is the insight that will be formalized in the examples below.

Starting from the non-monetary Arrow-Debreu model, two additional structures are sufficient to give endogenous monetization in equilibrium: multiple budget constraints (one at each transaction, not just on net trade) and transaction costs. One way of formalizing multiple budget constraints is a trading post model. Thus, if there are N goods actively traded, there are N(N-1)/2 possible trading posts. That is the starting point of the examples below. The choice of which trading posts a typical household will trade at is part of the household optimization. The equilibrium structure of exchange is the array of trading posts that actually host active trade. The determination of which trading posts are active in equilibrium is endogenous to the model and characterizes the monetary character of trade. The equilibrium is monetary with a unique money if only (N-1) trading posts are active, those trading all goods against 'money.'

The examples below derive monetary equilibrium as a market equilibrium of optimizing agents based on elementary considerations of transaction cost. Household optimization includes deciding at which trading posts the household will trade. For a given mix of goods, trade is drawn to the lowest transaction cost trading posts. The question *Why is there money?* can then be answered by presenting sufficient conditions so that an equilibrium trading array has N-1 active trading posts, those trading in a common medium of exchange versus the N-1 other goods. This is illustrated in Figures 1 and 2. Each node in the figures represents a commodity. Active trade is represented by a chord between nodes. A barter economy will have chords among a wide variety of goods --- one for each pair of goods where there is a household with a matching demand and supply (Figure 2). A monetary economy with a unique money will be a sparser array. There will be one good so that the only chords are those linking that good to all others (Figure 1). The question *why is there money?* is then reduced to asking for sufficient conditions so that the array of active trading posts in equilibrium looks like Figure 1 (spider-shaped) instead of Figure 2 (star-shaped).

<u>III. Monetization Comes from Liquidity: Monetary Competitive Equilibrium with Linear Transaction Costs</u>

The distinctive features of the model are (i) transactions exchange pairs of goods, (ii) budget constraints are enforced at each transaction separately, generating a role for a carrier of value between transactions (a medium of exchange), (iii) transaction costs are assumed to be linear in sections III and IV and nonconvex (displaying scale economies) in sections V, VI, VII and VIII. In the linear transaction cost case without double coincidence of wants, the most liquid (lowest transaction cost) good becomes the common medium of exchange. There may be multiple media of exchange when there is a tie for lowest cost.

Let there be N+1 commodities, numbered 0,1,2,...N. They are traded in pairs --- good i for good j --- at specialized trading posts. The trading post for trade of good i versus good j (and vice versa) is designated {i,j}; trading post {i,j} is the same trading post as {j,i} . Trading post {i,j} is a business firm, the market maker in trade between goods i and j. {i,j} actively buys and (re)sells both i and j. Trade as a resource using activity is modeled by describing the post's transaction costs. The notion of transaction cost summarizes costs that in an actual economy are incurred by retailers, wholesalers, individual firms and households. The bid/ask spread summarizes these costs to the model's transactors. Thus, part of transaction cost represents the (non-marketed) time and resources used by households in arranging their transactions, summarized here imprecisely as a price spread[6].

Specify a transaction cost function for these pairwise trading posts so that all transaction costs accrue in good 0. This is obviously a restrictive convention, but it simplifies accounting for transaction costs. It is simplest to think of good 0 as the labor used in the transaction technology. Trading post {i,j} buys good 0 as an input to its transaction costs. The typical transactions of trading post {i,j} will consist of purchases $y^{\{i,j\}B}_i, y^{\{i,j\}B}_j, y^{\{i,j\}B}_0 \geq 0$, of i, j, and 0 respectively and sales $y^{\{i,j\}S}_i, y^{\{i,j\}S}_j \geq 0$ of i and j. In this section, we use the further simplifying assumption of linear transaction costs. The cost structure is generalized to non-convex costs in sections V, VI, VII, and VIII.

The transaction cost function for trading post {i,j} is

$$C^{\{i,j\}} = y^{\{i,j\}B}_0 = \delta^i y^{\{i,j\}B}_i + \delta^j y^{\{i,j\}B}_j \qquad \text{(TCL)}^7$$

where $\delta^i, \delta^j > 0$. In words, the transaction technology looks like this: Trading post {i,j} makes a market in goods i and j, buying each good in order to resell it. It incurs transaction costs in good 0. These costs vary directly (in proportions δ^i, δ^j) with volume of trade. The transaction cost structure is separable in the two principal traded goods. The trading post {i,j} buys good 0 to cover the transaction costs it incurs, paying for 0 in goods i and j. The transaction cost function $C^{\{i,j\}}$ is sufficiently flexible to distinguish transaction costs differing among commodities, including differences in durability, portability, recognizibility, divisibility.

The population of households is denoted H, consisting of a mix of subpopulations (with different tastes and endowments). A typical household $h \in H$, has an endowment $r^h \in R^N_+$; r^h_n is h's endowment of good n. For simplicity in the examples below, each household is endowed with only one commodity. This is obviously inessential. h's utility function is $u^h(x)=u^h(x_0, x_1, ..., x_N)$.

It is convenient to arrange a subpopulation H^0 to provide good 0 (transaction labor). H^0's endowment of good 0 is characterized as

$\sum_{h \in H^0} r^h_0 > \sum_{h \in H} \sum_{i=1}^{N} \delta^i r^h_i$. For typical $h \in H^0$, h's utility function is

$$u^h(x) = \sum_{i=0}^{N} x_i \ . \qquad \text{(U0)}$$

That is, a subpopulation H^0 owns all of the good 0 in sufficient quantity to cover all the transaction costs in the economy that are likely to be incurred; h's tastes, for $h \in H^0$, treat all goods as perfect substitutes with MRS equal to unity. This unrealistic assumption is designed to make accounting for transaction costs particularly easy.

6 An alternative more explicit treatment of household non-market transaction cost decisions is embodied in Kurz (1974).

7 (TCL) is intended as a mnemonic for linear transaction cost.

A typical household outside of H^0 may be denoted h=[m,n] where m and n are integers between 1 and N (inclusive). m denotes the good with which h is endowed. n denotes the good h prefers. [m,n]'s utility function can then be taken to be

$$u^{[m,n]}(x) = \sum_{i=0,\, i\neq n}^{N} x_i + 3x_n . \qquad \text{(U1)}$$

[m,n]'s endowment, $r^{[m,n]}_m$, is specified as part of the description of the subpopulation.

Households formulate their trading plans deciding how much of each good to trade at each pairwise trading post. This leads to the rather messy notation:

$b^{[m,n]\{i,j\}}_{\ell}$ = planned purchase of good ℓ by household [m,n] at trading post {i,j}

$s^{[m,n]\{i,j\}}_{\ell}$ = planned sale of good ℓ by household [m,n] at trading post {i,j}

The bid prices (the prices at which the trading post will buy from households) at {i,j} are $q^{\{i,j\}}_i$, $q^{\{i,j\}}_j$ for goods i and j respectively. The price of i is in units of j. The price of j is in units of i. The ask price (the price at which the trading post will sell to households) of j is the inverse of the bid price of i (and vice versa). That is, $(q^{\{i,j\}}_i)^{-1}$ and $(q^{\{i,j\}}_j)^{-1}$ are the ask prices of j and i at {i,j}. The trading post {i,j} covers its costs by the difference between the bid and ask prices of i and j, that is, by the spread $(q^{\{i,j\}}_j)^{-1} - q^{\{i,j\}}_i$ and the spread $(q^{\{i,j\}}_i)^{-1}-q^{\{i,j\}}_j$. Transaction costs at the trading post are incurred in good 0. Post {i,j} pays for 0 in i and j, acquired in trade through the difference in bid and ask prices. The bid price of 0 in terms of i is $q^{\{i,j\}}_{(i)0}$. The bid price of 0 in terms of j is $q^{\{i,j\}}_{(j)0}$.

Given $q^{\{i,j\}}_i$, $q^{\{i,j\}}_j$, for all {i,j}, household h then forms its buying and selling plans, in particular deciding which trading posts to use to execute his desired trades. Household $h\in H$ faces the following constraints on its transaction plans:

(T.i) $b^{h\{i,j\}}_n > 0$, only if n=i,j; $s^{h\{i,j\}}_n > 0$, only if n=i,j,0.

(T.ii) $b^{h\{i,j\}}_i \leq q^{\{i,j\}}_j \cdot s^{h\{i,j\}}_j$, $b^{h\{i,j\}}_j \leq q^{\{i,j\}}_i \cdot s^{h\{i,j\}}_i$ for each {i,j}.

There is a slightly distinct version of (T.ii), (T.ii'), applying to households in H^0.

(T.ii') For $h\in H^0$, decompose $s^{h\{i,j\}}_0$ into nonnegative elements $s^{h\{i,j\}}_{(i)0}$ and $s^{h\{i,j\}}_{(j)0}$, so that $s^{h\{i,j\}}_{(i)0}+s^{h\{i,j\}}_{(j)0}=s^{h\{i,j\}}_0$, then we have $b^{h\{i,j\}}_i\leq q^{\{i,j\}}_{(i)0} \cdot s^{h\{i,j\}}_{(i)0}$, and $b^{h\{i,j\}}_j \leq q^{\{i,j\}}_{(j)0} \cdot s^{h\{i,j\}}_{(j)0}$ for each {i,j}.

(T.iii) $x^h_n = r^h_n + \Sigma_{\{i,j\}}b^{h\{i,j\}}_n - \Sigma_{\{i,j\}}s^{h\{i,j\}}_n \geq 0,\ 0 \leq n \leq N$.

Note that condition (T.ii)[and (T.ii')] defines a budget balance requirement at the transaction level, implying the decentralized character of trade. Since the budget constraint applies to each pairwise transaction separately, there may be a demand for a carrier of value to move purchasing power between distinct transactions. h faces the array of bid prices $q^{\{i,j\}}_i$, $q^{\{i,j\}}_j$, and chooses $s^{h\{i,j\}}_n$ and $b^{h\{i,j\}}_n$, n= i, j (and n=0 for $h\in H^0$), to maximize $u^h(x^h)$ subject to (T.i), (T.ii), (T.iii). That is, h chooses which pairwise markets to transact in and a transaction plan to optimize utility, subject to a multiplicity of pairwise budget constraints.

The trading posts in sections III and IV have linear transaction technologies. A competitive equilibrium is an appropriate solution concept resulting in zero profits for the typical trading post (with the additional benefit that no account need be taken of distribution of profits). The threat of entry (by other similar trading post firms) rationalizes the competitive model, but for simplicity we take there to be a unique trading post firm making a market in goods i and j, denoted indiscriminately {i,j} = {j,i}.

A <u>competitive equilibrium</u> under (TCL) consists of $q^{o\{i,j\}}_{(i)0}$, $q^{o\{i,j\}}_{(j)0}$, $q^{o\{i,j\}}_i$, $q^{o\{i,j\}}_j$, $1\leq i,j \leq N$, so that :

• For each household $h \in H$, there is a utility optimizing plan $b^{oh\{i,j\}}_n$, $s^{oh\{i,j\}}_n$, (subject to T.i, T.ii [or T.ii' for $h\in H^0$], T.iii) so that $\Sigma_h b^{oh\{i,j\}}_n = y^{o\{i,j\}S}_n$, $\Sigma_h s^{oh\{i,j\}}_n = y^{o\{i,j\}B}_n$, for each {i,j}, each n, where

• $y^{o\{i,j\}S}_n \leq y^{o\{i,j\}B}_n$, n=i,j.

• $y^{o\{i,j\}B}_0$ can be divided into two parts, $y^{o\{i,j\}B}_{(i)0} \geq 0$, $y^{o\{i,j\}B}_{(j)0} \geq 0$, so that $y^{o\{i,j\}B}_{(i)0}+y^{o\{i,j\}B}_{(j)0}= y^{o\{i,j\}B}_0 = C^{\{i,j\}}$.

• $q^{o\{i,j\}}_{(i)0}y^{o\{i,j\}B}_{(i)0} \leq y^{o\{i,j\}B}_i - q^{o\{i,j\}}_j y^{o\{i,j\}B}_j$. $q^{o\{i,j\}}_{(j)0}y^{o\{i,j\}B}_{(j)0}\leq y^{o\{i,j\}B}_j - q^{o\{i,j\}}_i \cdot y^{o\{i,j\}B}_i$.

• $\delta^i + \delta^j q^{o\{i,j\}}_i = (q^{o\{i,j\}}_{(i)0})^{-1}(1- q^{o\{i,j\}}_i q^{o\{i,j\}}_j)$,

$\delta^j + \delta^i q^{o\{i,j\}}_j = (q^{o\{i,j\}}_{(j)0})^{-1}(1- q^{o\{i,j\}}_i q^{o\{i,j\}}_j)$

The expression in the last bullet is a marginal cost pricing condition: the transaction cost (in good 0) of buying one unit of i and enough j to pay for it (pricing the 0 in good i) is equal to the amount of i left over after completing the trade in i and j. Similarly for trade in j.

An equilibrium is said to be monetary with a unique money, μ, if --- for all households --- good μ is the only good that a household will both buy and sell. An equilibrium will be monetary with multiple moneys, μ^1, μ^2, ..., if --- for all households --- μ^1, μ^2, ... are the only goods that a household will both buy and sell.

Jevons (1875) reminds us that monetization of trade follows in part from the absence of a double coincidence of wants. In the present model, that logic is particularly powerful. Absence of coincidence of wants means that the typical traded good will be traded more than once in moving from endowment to consumption. Barter trade successfully rearranging the allocation to an equilibrium will transact an endowment first at the trading post where it is supplied and again at a distinct post where it is demanded. Hence monetary trade as an alternative (substituting retrade of money for the retrade of nonmonetary goods) can be undertaken without increasing total trading volume or transaction cost, even without scale economies. Conversely, when there is a full double coincidence of wants and linear transaction cost, equilibrium will be non-monetary even in the presence of a natural money (section IV).

We now formalize the notion of the absence of double coincidence of wants. Let N be an integer, $N\geq 3$. For m=1,2,...,N, and positive integers i, $1 \leq i \leq N-1$, let

$$m\oplus i = \begin{cases} m+i & \text{if } m+i \leq N , \\ m+i-N & \text{if } m+i > N . \end{cases}$$

That is, $m\oplus i$ denotes m+i mod N, skipping 0 (since good 0 is used primarily as an input to the transaction process). Recall that [m,n] denotes a household endowed with good m, strongly preferring good n. Using the notation above, let $H^1 = \{[m,m\oplus 1] \mid m=1,2, ..., N; r^{[m,m\oplus 1]}_m=A>0\}$. H^1 characterizes a population of N households with the same size of initial endowment, so that no pair of them have reciprocal matching endowments and preferences but so that their endowments in aggregate can be reallocated to make each one significantly better off (roughly by arranging the households clockwise in a circle ordered by endowment good and having each household $[m,m\oplus 1]$ send his endowment one place counterclockwise).

Example III.1 (Existence of monetary equilibrium with a most liquid asset, absent double coincidence of wants): Let the population of households be H = $H^0 \cup H^1$. Let $C^{\{i,j\}}$ be described by (TCL). Let $0<\delta^i<{}^1/_3$ and $0<\delta^1<\delta^i$, for i=2, 3, ... N. Transaction costs are constant and non-trivial for all goods; they are significantly lower in good 1. Then there is a unique competitive equilibrium allocation (though a range of prices may support the unique real allocation of trades and consumptions). The equilibrium is a monetary equilibrium with good 1 as the unique 'money'.

Demonstration of Example III.1: Using marginal cost pricing and market clearing, we have for each {i,j}, i≠j, 1≤i,j≤N, $q^{\{i,j\}}_{(i)0}=q^{\{i,j\}}_{(j)0}=1$, $q^{\{i,i\oplus1\}}_{i\oplus1}=1$, $q^{\{i,i\oplus1\}}_{i}=\frac{1-\delta^i}{1+\delta^{i\oplus1}}$, and for $j\neq1,i\oplus1$, $q^{\{i,j\}}_{i}=1-\delta^i$, $q^{\{i,j\}}_{j}=1-\delta^j$; $q^{\{i,1\}}_{i}=\frac{1-\delta^i}{1+\delta^1}$, $q^{\{i,1\}}_{1}=1$. $s^{[i,i\oplus1]\{i,1\}}_{i}=A$,
$b^{[i,i\oplus1]\{i,1\}}_{1}=q^{\{i,1\}}_{i}A=s^{[i,i\oplus1]\{i\oplus1,1\}}_{1}$,
$b^{[i,i\oplus1]\{i\oplus1,1\}}_{i\oplus1}=q^{\{i,1\}}_{i}q^{\{i\oplus1,1\}}_{1}A$.

What's happening in Example III.1? At first household [i,i⊕1] goes to trading post {i,i⊕1} offering i in exchange for i⊕1. But no one is coming to the trading post offering i⊕1. So good i is priced at a large discount at the post, reflecting the transaction costs of both i and i⊕1. On all other markets {i,j} goods are priced to reflect their transaction costs, $q^{\{i,j\}}_{i}=1-\delta^i$. But at that pricing, since $\delta^1<\delta^i$, it is advantageous for [i,i⊕1] to trade through 1 as an intermediary. This follows since $(1-\delta^i)\cdot(1-\delta^1)>(1-\delta^i)\cdot(1-\delta^{i\oplus1})$. This pricing creates a small shortage of 1 at each trading post (since small quantities of 1 are being retained at the post to cover 1's transaction costs) so prices are readjusted so that all of the discount in bid prices at {i,1} appears in the bid price of i. This results in $q^{\{i,1\}}_{i}=\frac{1-\delta^i}{1+\delta^1}$, $q^{\{i,1\}}_{1}=1$. All trade of i for i⊕1 now goes through 1. Good 1 has become 'money,' the unique low transaction cost common medium of exchange.

In actual monetary economies we usually see a single 'money' as in Example III.1. We'll argue in sections V through VIII that the reason for uniqueness of 'money' is scale economy. Does there have to be a reason for uniqueness? Yes. US dollars, pounds sterling, and euros, all have similar low transaction costs but in their separate markets they are virtually unique in use. Economic theory should have an explanation for this uniqueness. Example III.2 below emphasizes, by counterexample, that the nonconvexity in section V is important. In Example III.2, absent the nonconvexity, when there's a tie for lowest transaction cost, there are many media of exchange in use. Is a tie realistic; isn't it a singularity? The example of dollars, sterling, and euros suggests that on the contrary, the notion of a tie for lowest transaction cost is a non-trivial event, so that uniqueness requires an explanation.

Example III.2 (Multiple 'money''s in equilibrium): Let the population of households be H = $H^0 \cup H^1$. Let $C^{\{i,j\}}$ be described by (TCL). Let $0<\delta^1 = \delta^2 = \delta^3 < \delta^i<{}^1/_3$, i=4,5,...,N. Then there is a continuum of competitive equilibrium allocations with 1,2,3 acting as 'money' in proportions from 0% to 100%. Consumptions and utilities of all households are the same as in the equilibrium of Example III.1.

Demonstration of Example III.2: The marginal cost market-clearing pricing is identical to that in Example III.1 with goods 2 and 3 priced simlarly to good 1. The exception is trade between 'money' 's where $q^{\{1,2\}}_1 = 1\text{-}\delta^1$, and similarly for 2,3, all of these bid prices being equal. The trading posts {i,1}, {i,2}, and {i,3}, i=4,5,...,N, (for trade in good i versus goods 1,2,3) are the trading posts with narrow bid/ask spreads since 1,2,3 have low transaction costs. Households can now divide their transactions among trading posts for goods 1, 2, and 3 versus all other goods in any proportion (though in equilibrium they will be the same proportions for all households). Markets clear.

The logic of Example III.2 is merely the multi-money version of III.1. Goods 1, 2, 3 are equally liquid and become media of exchange. They can be used however in any proportionate combination from 0% to 100% since absent economies of scale there is no reason further to specialize.

IV. Absence of Double Coincidence of Wants is Essential to Monetization in a Linear Model

Let $H^D = \{[m,n] \mid m,n = 1, 2, 3, ..., N, m \neq n\}$. H^D is distinctive in creating a population of households with fully complementary demands and supplies, full double coincidence of wants. We can use this population to illustrate the importance of the absence of double coincidence of wants to monetization in a linear model. Under the same conditions where monetary equilibria existed --- and indeed were the only equilibria --- in examples III.1 and III.2 in the absence of double coincidence of wants, we can show that for H^D, with full double coincidence of wants, a barter equilibrium is the unique competitive equilibrium. Hence the classical focus on the absence of double coincidence of wants is confirmed; it is essential to monetization in a linear model. Note that this result depends on the linearity (or convexity) of transaction costs; if scale economies are present, then even with full double coincidence of wants, it may be more economical to use a common medium of exchange with resulting high trading volumes.

Example IV.1 (Barter equilibrium with full double coincidence of wants): Let the population of households be $H=H^0 \cup H^D$. Let $C^{\{i,j\}}$ be described by (TCL). Let $0<\delta^i<{}^1/_3$ and $0<\delta^1<\delta^i$, for all i, i=0,2, 3, ... N-1. Transaction costs are constant and non-trivial for all goods but 1. Then there is a unique competitive equilibrium allocation. The equilibrium is non-monetary with active trade in all trading posts {i,j}, $1\leq i,j \leq N$.
Demonstration of Example IV.1: For each i,j, $1\leq i,j \leq N$, $q^{\{i,j\}}_i=(1\text{-}\delta^i)$, $q^{\{i,j\}}_j=(1\text{-}\delta^j)$. $s^{[i,j]\{i,j\}}_i=A$, $b^{[i,j]\{i,j\}}_j=q^{\{i,j\}}_iA$, $s^{[j,i]\{i,j\}}_j=A$, $b^{[j,i]\{i,j\}}_i=q^{\{i,j\}}_jA$. Markets clear. The allocation is an equilibrium.

What's happening in example IV.1? Direct barter trade works successfully in the presence of double coincidence of wants. For each household [i,j] with a supply of one good and a demand for another, there is a precise mirror image [j,i] in the population. They each go to the trading post {i,j} where their common demands and supplies are traded. They trade, each incurring the cost of trading one good. Monetary trade is not advantageous since it requires twice the transactions volume --- with corresponding cost --- of direct barter trade (similar volumes for each non-monetary good and an equal volume of trade in the medium of exchange). Monetization of trade in equilibrium in a linear model depends on absence of double coincidence of wants.

V. Uniqueness of the Medium of Exchange: Scale Economies in Transaction Cost

Monetary trade is typically characterized by a unique medium of exchange or a small number of related media (e.g. currency, credit cards, travelers' checks, all denominated in $US). How does this come about? Prof. Tobin (1980) suggests that scale economies in transaction costs are essential:

> The use of a particular language or a particular money by one individual increases its value to other actual or potential users. Increasing returns to scale ... explains the tendency for one basic language or money to monopolize the field.

When monetization takes place, households supplying good i and demanding good j are induced to trade in a monetary fashion, first trading i for 'money' and then 'money' for j, by discovering that transaction costs are lower in this indirect trade than in direct trade of i for j. But as Example III.2 points out, monetization of trade is no guarantee of uniqueness of the medium of exchange. Scale economies in transaction costs induce specialization in the medium of exchange function. High volume leads to low unit transaction costs (see also Howitt(2000), Rey (2001) and Starr and Stinchcombe (1999)). Scale economy is not a necessary condition for uniqueness of the medium of exchange in equilibrium (Example III.1), but scale economy helps to ensure uniqueness (Example VI.1, below). If there are many equally low cost candidates for the medium of exchange, then scale economy in transaction costs will allow one to be endogenously chosen as the unique medium of exchange.

The transaction cost structure of sections VI through VIII with large scale economies is unsuitable for competitive equilibrium. Competitive equilibria typically cannot exist in the unbounded scale economy environment. In section VI, instead of competitive equilibria, average cost pricing equilibria are developed. The use of average cost pricing is subject to interpretation. A literal interpretation is that the there is a natural monopoly market maker pricing at average cost to discourage new entry. An alternative is that the operation of the market is in the nature of a public good; the nonconvex technology is a summary of the interactions of many individual agents sharing an economy of scale, and hence average cost pricing reflects the common benefit from the level of activity in the market (a Marshallian externality). In section VII the scale economy allows a Bertrand equilibrium with monopoly trading posts to form. In section VIII government provides fiat money; government's large scale combined with the scale economy in transaction costs assures that government-issued fiat money becomes the common medium of exchange. Scale economy implies a cost saving resulting from uniqueness of 'money,' since only N (in the case of fiat money) or N-1 (commodity money) trading posts need to operate, incurring significantly lower costs than N(N-1)/2 (under barter). Scale economies make it cost-saving to concentrate transactions in a few firms and one intermediary instrument.

VI. Monetization Comes From Liquidity Again: Monetary General Equilibrium with Unique Money under Average Cost Pricing of Non-Convex Transaction Costs

Scale economies in the transaction cost structure induce uniqueness of the equilibrium medium of exchange. 'Money' is a natural monopoly. As Prof. Tobin(1959) tells us, "Why are some assets selected by a society as generally acceptable media of exchange while others are not? This is not an easy question, because the selection is self-justifying." Thus gold and dollar bills may have low transaction costs and be excellent candidates for medium of exchange, but if (despite high transaction cost) Yap Island stones are already the commonly chosen medium of exchange with high trading volume, then stones may have the lowest average transaction cost. The choice of Yap Island stones as the common medium of exchange is then 'self-justifying.'

The nonconvex (scale economy) cost function for trading post {i,j} is

$$C^{\{i,j\}} = y^{\{i,j\}B}_0 = \min[\delta^i y^{\{i,j\}B}_i, \gamma^i] + \min[\delta^j y^{\{i,j\}B}_j, \gamma^j] \qquad \text{(TCNC)}^8$$

where $\delta^i, \delta^j, \gamma^i, \gamma^j > 0$. In words, the transaction technology looks like this: Trading post {i,j} makes a market in goods i and j, buying each good in order to resell it. It incurs transaction costs in good 0. These costs vary directly (in proportions δ^i, δ^j) with volume of trade at low volume and then hit a ceiling after which they do not increase with trading volume. The specification in (TCNC) is an extreme case: zero marginal transaction cost beyond the ceiling. Adding additional linear terms would represent a more general case.

Since the trading posts in this economy have nonconvex transaction technologies, a competitive equilibrium is not an appropriate solution concept. The equilibrium notion used is an average cost pricing equilibrium resulting in zero profits for the typical trading post firm. The rationale for this choice of equilibrium concept may be the threat of entry (by other similar firms) if any economic rent is actually earned. The presence of potential entrants and their actions is not explicitly modeled.

An <u>average cost pricing equilibrium</u> consists of $q^{o\{i,j\}}_{(i)0}, q^{o\{i,j\}}_{(j)0}, q^{o\{i,j\}}_i, q^{o\{i,j\}}_j$, $1 \le i,j \le N$, so that :

- For each household h, there is a utility optimizing plan $b^{oh\{i,j\}}_n, s^{oh\{i,j\}}_n$, (subject to T.i, T.ii [or T.ii' for $h \in H^0$], T.iii) so that $\Sigma_h b^{oh\{i,j\}}_n = y^{o\{i,j\}S}_n$, $\Sigma_h s^{oh\{i,j\}}_n = y^{o\{i,j\}B}_n$, for each {i,j}, each n, where
- $y^{o\{i,j\}S}_n \le y^{o\{i,j\}B}_n$, n=i,j.
- $y^{o\{i,j\}B}_0$ can be divided into two parts, $y^{o\{i,j\}B}_{(i)0} \ge 0$, $y^{o\{i,j\}B}_{(j)0} \ge 0$, so that $y^{o\{i,j\}B}_{(i)0} + y^{o\{i,j\}B}_{(j)0} = y^{o\{i,j\}B}_0 = C^{\{i,j\}}$.
- $q^{o\{i,j\}}_{(i)0} y^{o\{i,j\}B}_{(i)0} = y^{o\{i,j\}B}_i - q^{o\{i,j\}}_j y^{o\{i,j\}B}_j$. $q^{o\{i,j\}}_{(j)0} y^{o\{i,j\}B}_{(j)0} = y^{o\{i,j\}B}_j - q^{o\{i,j\}}_i \cdot y^{o\{i,j\}B}_i$.

Let κ be a positive integer, $2 \le \kappa < (N/2)$. Let $H^\kappa = \{[m, m\oplus i] \mid m=1,2, ..., N; i=1,2,...,\kappa; r^{[m,m\oplus i]}_m = A>0\}$. H^κ is a set of κN households without double coincidence of wants. One way to visualize H^κ's situation is to think of the households arrayed in a circle clockwise, each one's position designated by endowment. They can arrange a Pareto improving redistribution by each taking his endowment and sending it i places counterclockwise. However, reflecting the absence of double coincidence of wants, if each of the housheholds in H^κ goes to the trading post where his endowment is traded

8 (TCNC) is intended as a mnemonic for non-convex transaction cost.

against his desired good, he finds himself alone. He's dealing on a thin market. The following Example VI.1 demonstrates that, with scale economies in transaction cost, virtually any good can become money; the designation is self-confirming.

Example VI.1 (Monetary equilibrium absent double coincidence of wants with scale economy in transaction costs): Let the population of households be $H=H^0 \cup H^\kappa$. Let $C^{\{i,j\}}$ be described by (TCNC). Let $0<\delta^i<1$ all i = 1,2, ..., N. Let $\frac{\gamma^i+\gamma^j}{\kappa A}<\frac{2}{3}$ and $(1-\frac{\gamma^i+\gamma^j}{\kappa A})>(1-\delta^j)(1-\delta^i)$ for all $i\neq j$, i,j=1,2,...N. Then for each i=1,2,...,N there is a monetary average cost pricing equilibrium with good i as the unique 'money'.

Demonstration of Example VI.1: Choose an arbitrary i=1,2,...,N as 'money.' For all $j\neq i$, j=1,2,...,N, let $q^{\{i,j\}}_i=1$, $q^{\{i,j\}}_j= 1-\frac{\gamma^i+\gamma^j}{\kappa A}$. For all j, and k=1,2,...,N, $j\neq k\neq i$, $q^{\{j,k\}}_j = 1-\delta^j$, $q^{\{j,k\}}_k=1-\delta^k$. For $1 \leq \ell \leq \kappa$, let $s^{[m,m\oplus\ell]\{i,m\}}_m=A$, $b^{[m,m\oplus\ell]\{i,m\}}_i =q^{\{i,m\}}_iA$, $s^{[m,m\oplus\ell]\{i,m\oplus\ell\}}_i =q^{\{i,m\}}_iA$, $b^{[m,m\oplus\ell]\{i,m\oplus\ell\}}_{m\oplus\ell} =q^{\{i,m\}}_iA$.

What's happening in Example VI.1? Virtually any good i can become money. Monetization comes from liquidity and --- with scale economies --- liquidity comes from trading volume. The economy is focusing on good i as its common medium of exchange. Since there are scale economies in transaction costs, high trading volume means low average cost with concommitant narrow bid/ask spread. The narrow bid/ask spread is the way the price system confirms and reinforces the choice of i as the medium of exchange. Trader $[m,m\oplus\ell]$ wants to trade good m for good $m\oplus\ell$. He could do so directly, but the transaction costs are heavy, reducing his return on the trade to $A(1-\delta^m)(1-\delta^{m\oplus\ell})$ units of $m\oplus\ell$ after starting with A units of good m. The alternative is to trade good m for good i and then trade i for $m\oplus\ell$. This results in $A(1-[(\gamma^i+\gamma^{m\oplus\ell})/\kappa A])$ units of $m\oplus\ell$. When κ is sufficiently large, that's a much greater return. Because of the narrow bid/ask spread on trade through i, every market with good i on one side attracts high trading volume, κ traders on each side of the market, the high trading volume needed to maintain good i's low bid/ask spreads. The scale economy means that the choice of good i as the common medium of exchange is self-confirming.

The difference between barter and monetary exchange is the contrast between a complex of many thin high transaction cost markets and an array of a smaller number of thick low transaction cost markets dealing in each good versus a unique common medium of exchange. The choice of medium of exchange is self-justifying. Any good i with sufficient scale economy in its transaction technology (with γ^i, the ceiling on its transaction costs, sufficiently low) can become the unique medium of exchange in equilibrium when trading volume κA is sufficiently high. Mint-standardized gold coins (with a low cost transaction technology) or Yap Island stones (high cost technology) may be 'money' depending on which is well established. Sufficient trading volume can confirm either choice.

Recall $H^D= \{[m,n] \mid m,n = 1, 2, 3, ..., N, m \neq n, r^{[m,n]}_m=A>0\}$. H^D is a set of N(N-1) households with full double coincidence of wants. The following Example VI.2 demonstrates that even in the presence of double coincidence of wants, sufficient scale economies in transaction costs can lead to monetization of trade, the use of a common medium of exchange.

Example VI.2 (Monetary equilibrium with full double coincidence of wants and scale economy in transaction costs): Let the population of households be $H=H^0 \cup H^D$. Let $C^{\{i,j\}}$ be described by (TCNC). Let $0<\delta^i<1$ all i = 1,2, ..., N. For some i and all j, $1\leq i,j\leq N$, $i\neq j$, let $\frac{\gamma^i+\gamma^j}{(N-1)A}<\frac{2}{3}$ and $(1-\frac{\gamma^i+\gamma^j}{(N-1)A})>(1-\delta^j)$, $(1-\frac{\gamma^i+\gamma^j}{(N-1)A})>(1-\delta^i)$. Then there is a monetary average cost pricing equilibrium with good i as the unique 'money.'

Demonstration of Example VI.2: For all $j\neq i$, j=1,2,...,N, let $q^{\{i,j\}}_i=1$, $q^{\{i,j\}}_j= 1-\frac{\gamma^i+\gamma^j}{(N-1)A}$. For all j, and k=1,2,...,N, $j\neq k\neq i$, $q^{\{j,k\}}_j = 1-\delta^j$, $q^{\{j,k\}}_k=1-\delta^k$. Let $s^{[m,n]\{i,m\}}_m=A$, $b^{[m,n]\{i,m\}}_i =q^{\{i,m\}}_iA$, $s^{[m,n]\{i,n\}}_i =q^{\{i,m\}}_iA$, $b^{[m,n]\{i,n\}}_n =q^{\{i,m\}}_iA$.

What's happening in Example VI.2? Monetization comes from liquidity and --- with scale economies --- liquidity comes from trading volume. But how can monetization of trade occur where there is double coincidence of wants? The answer is scale economies. Trader [m,n] wants to trade good m for good n. He could do so directly at post {m,n}, and he'd find a willing trading counterpart at the trading post, so he'd only have to pay for the transaction costs on one side of the trade. But the transaction costs are still substantial, reducing his return on the trade to $A(1-\delta^m)$ units of n after starting with A units of good m. The alternative is to trade good m for good i and then trade i for n. This results in $A(1-[(\gamma^i+\gamma^n)/(N-1)A])$ units of n. When N is sufficiently large, that's a much greater return. Because of the narrow bid/ask spread on trade through i, every market with good i on one side attracts high trading volume, N-1 traders on each side of the market, the high trading volume needed to maintain good i's low bid/ask spreads. The scale economy means that the choice of good i as the common medium of exchange is self-confirming.[9]

Convergence to a Unique 'Money'

Einzig (1966, p. 345), suggests "Money tends to develop automatically out of barter, through the fact that favourite means of barter are apt to arise ... object[s] ... widely accepted for direct consumption." That is, Einzig suggests those goods with high trading volumes are the most liquid (presumably reflecting scale economy in transaction cost), and evolve into common media of exchange. That medium is unique because scale economies lead to 'money' as a natural monopoly. The following example demonstrates this process.

As monetization takes place, households supplying good i and demanding good j start by trading directly. They may also consider monetary trade, first trading i for 'money' and then 'money' for j. When they discover that transaction costs are lower in this

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9 For a network externality interpretation see Hahn(1997) which notes that in the presence of market set-up costs, each transactor in the market benefits from the participation of others. "If the number who can gain from trade is ... sufficiently [large] ..., the Pareto improving trade will take place. There is thus an externality induced by set-up costs." Young (1998) assumes the externality without additional explanation. Rey (2001) denotes this interaction the "thick markets externality."

indirect trade than in direct trade of i for j, they adopt monetary trade. Starting from a barter array consisting of N(N-1)/2 active trading posts, the allocation evolves through price and quantity adjustments to a monetary array where only N-1 trading posts are active. The impetus for the concentration of the trading function in a few trading posts (those specializing in trade that includes the commodity that is endogenously designated as 'money') in the monetary equilibrium comes from pricing the scale economies in transaction technology.

Example VI.3, below, starts with an economy of diverse endowments and demands and with a double coincidence of wants. The demand structure is arranged at the outset positing some goods most "widely accepted for direct consumption." With scale economies in the transaction technology, these high volume goods will also be those with the lowest unit transaction cost. Thus they are, in Menger's view, the most saleable, and excellent candidates for "*generally* acceptable media of exchange." As they are so adopted by some households, their trading volumes increase, reducing their average transaction costs, and making them more saleable still. This process converges to an equilibrium with a unique medium of exchange, reflecting the interaction of scale economy and liquidity. As households discover that some pairwise markets (those with high trading volumes) have lower transaction costs, they rearrange their trades to take advantage of the low cost. That leads to even higher trading volumes and even lower costs at the most active trading posts. The process converges to an equilibrium where only the high volume trading posts dealing in a single intermediary good ('money') are in use. Under nonconvex transaction costs, this implies a cost saving, since only N-1 trading posts need to operate, incurring significantly lower costs than N(N-1)/2 posts. Scale economies make it cost-saving to concentrate transactions in a few trading posts and a unique 'money'.

Scale economies in the transactions technology generate a strong tendency to multiple equilibria. This creates an interest in determining which of the several equilibria the economy will actually select. One solution to this problem is to posit an adjustment process to equilibrium that makes the choice. Hence we use the following

<u>Tatonnement adjustment process for average cost pricing equilibrium</u>:

> Prices will be adjusted by an average cost pricing auctioneer. Specify the following adjustment process for prices.
>
> STEP 0: The starting point is somewhat arbitrary. In each pairwise market the bid-ask spread is set to equal average costs at low trading volume.
>
> CYCLE 1
>
> STEP 1: Households compute their desired trades at the posted prices and report them for each pairwise market.
>
> STEP 2: Average costs (and average cost prices) are computed for each pairwise market based on the outcome of STEP 1. Prices are adjusted upward for goods in excess demand at a trading post, downward for goods in excess supply, with the bid-ask spead adjusted to average cost. A market's (market making firm's) nonzero prices are specified only

for those goods where the firm has the technical capability of being active in the market; other prices are unspecified, indicating no available trade.

CYCLE 2

Repeat STEP 1 (at the new posted prices) and STEP 2.

CYCLE 3, CYCLE 4, ... repeat until the process converges.

Einzig encourages us to look for favorite means of barter as latent money; we'll define a population with some favorite means of barter. Define a household population H^F as follows: Let N be an integer, N≥ 3. Without loss of generality, designate goods 1 and 2 for distinctive roles: 1 is widely heavily traded, particularly in exchange for 2. Let $H^F = \{ [m,n] \mid 1 \le m,n \le N,\ m \ne n;\ r^{[m,n]}_m = A>0$, except $r^{[m,1]}_m = 2A = r^{[1,m]}_1$ for $m \ne 2$, $r^{[2,1]}_2 = 3A = r^{[1,2]}_1 \}$. That is, there is a distinctively high desired net trade volume in good 1, particularly in exchange for good 2 (the numerical designation is inessential).

<u>Example VI.3 (High trading volume with scale economy designates 'money')</u>: Let the population be $H^F \cup H^0$. Let transactions costs be characterized by (TCNC) with $\delta^i = {}^1/_2$, $\gamma^i = (.6)A$, all i. That is, there is full double coincidence of wants. All goods have the same transaction technology but there is higher desired net trading volume in good 1. Scale economies in transaction costs are evident at trading volumes slightly higher than the desired trade size of most traders but well within the size of traders desiring net trades in good 1, particularly in exchange for 2. Then the tatonnement process converges to a monetary equilibrium where 1 is the unique money.

<u>Demonstrating Example VI.3</u>: The economy has a full double coincidence of wants. For most pairs of goods m,n, the desired net trade is uniformly distributed; the desired trade between them is A. For pairs 1,n the desired trading volume is 2A except for the pair 1,2 where the desired volume is 3A. This structure of preferences and endowments creates a desire for relatively high trading volumes among households trading in good 1.

The scale economy in transactions costs begins to be apparent at trading volumes just slightly larger than the endowment of most households. The scale economy is manifest well within the desired trading volumes of households endowed with or desiring good 1. The progression from barter to money is then the movement from a diffuse array of many active low volume markets to the concentration on a connected family of high volume (low average cost) markets. The tatonnement proceeds as follows:

STEP 0: For all 1≤i,j≤N, i≠j, $q^{\{i,j\}}_{(i)0} = q^{\{i,j\}}_{(j)0} = 1$, $q^{\{i,j\}}_i = q^{\{i,j\}}_j = {}^1/_2$.

CYCLE 1, STEP 1:

- For $[m,n] \in H^F$, $m \ne 1 \ne n$, $b^{[m,n]\{m,n\}}_n = ({}^1/_2)A = q^{\{m,n\}}_m A$, $s^{[m,n]\{m,n\}}_m = A$; all other purchases and sales are nil.
- For $[m,1] \in H^F$, m≠2, $b^{[m,1]\{m,1\}}_1 = A = q^{\{m,1\}}_m 2A$, $s^{[m,1]\{m,1\}}_m = 2A$; all other purchases and sales are nil. For $[1,n] \in H^F$, n≠2, $b^{[1,n]\{1,n\}}_n = A = q^{\{1,n\}}_1 2A$, $s^{[1,n]\{1,n\}}_1 = 2A$; all other purchases and sales are nil.
- For the two remaining elements of H^F, [1, 2] and [2, 1], $b^{[1,2]\{2,1\}}_2 = ({}^3/_2)A = q^{\{2,1\}}_1 3A$,

$s^{[1,2]\{2,1\}}_1$=3A; $b^{[2,1]\{2,1\}}_1 = (^3/_2)A = q^{\{2,1\}}_2 3A$, $s^{[2,1]\{2,1\}}_2$ = 3A; all other purchases and sales are nil.

- For $h \in H^0$, for $i \neq 1 \neq j$, $b^{h\{i,j\}}_i = b^{h\{i,j\}}_j = A/2$, $s^{h\{i,j\}}_0 = A$; for i or j =1, $b^{h\{i,j\}}_i = b^{h\{i,j\}}_j = \gamma = (.6)A$, $s^{h\{i,j\}}_0 = 2\gamma = (1.2)A$.

STEP 2:

- For {m,n} where $m \neq 1 \neq n$, $1 = q^{\{m,n\}}_{(m)0} = q^{\{m,n\}}_{(n)0}$, $q^{\{m,n\}}_m = q^{\{m,n\}}_n = (^1/_2)$.
- For {m,1}, $m \neq 2$, $1 = q^{\{m,1\}}_{(1)0} = q^{\{m,1\}}_{(m)0}$, $q^{\{m,1\}}_m = q^{\{m,1\}}_1 = \frac{2A-\gamma}{2A} = .70$
- For {2,1} , $1 = q^{\{2,1\}}_{(2)0} = q^{\{2,1\}}_{(1)0}$, $q^{\{2,1\}}_2 = q^{\{2,1\}}_1 = \frac{3A-\gamma}{3A} = .80$

At this stage we can see the initial effect of the scale economy. At STEP 0 prices started essentially equivalent in all pairwise markets. But the prices announced at the end of CYCLE 1 STEP 2 show that the bid prices of goods are much higher in the high volume markets; the bid/ask spread is lower there. The high volume markets are more liquid.

On entering CYCLE 2 STEP 1 households recalculate their desired trades. Those who have been trading on {2, 1} and on {m,1} find that trade on these markets has become even more attractive since the bid-ask spreads have narrowed. Those who had been trading on {2,m} face a quandary: goods 2 and m are the goods that they want to trade, but trading indirectly through good 1 in {2, 1} and {m,1} may be a lower cost alternative. In order to make that decision the household compares $q^{\{2,m\}}_m$ to the product $q^{\{2,1\}}_1 \cdot q^{\{m,1\}}_m$. The former is the value of m in terms of 2 in direct trade, the latter through trade mediated by good 1. $q^{\{m,1\}}_m \cdot q^{\{2,1\}}_1 = .56 > .5 = q^{\{m,2\}}_m$. Household [m,2] can get more 2 for his m by trading indirectly through the markets with good 1, and household [2,m] can get more m for his 2 by trading indirectly through the markets with good 1. They decide to trade through good 1. Good 1 is beginning to take on the character of money.

The transformation of good 1 into money is not complete however. Household [m,n] for $m \neq 2 \neq n$ considers but does not adopt indirect trade through good 1. He calculates $q^{\{m,1\}}_m \cdot q^{\{n,1\}}_1 = .49 < .5 = q^{\{m,n\}}_m$. Household [m,n] still gets a better deal trading directly good m for n.

CYCLE 2, STEP 1:

- For $[m,n] \in H^F$, $m,n \neq 2$, $m,n \neq 1$, $s^{[m,n]\{m,n\}}_m = A$, $b^{[m,n]\{m,n\}}_n = Aq^{\{m,n\}}_m$; all other purchases and sales are nil.

For [m,2], $m \neq 1$, $s^{[m,2]\{m,1\}}_m = A$, $b^{[m,2]\{m,1\}}_1 = Aq^{\{m,1\}}_m$, $s^{[m,2]\{1,2\}}_1 = Aq^{\{m,1\}}_m$, $b^{[m,2]\{1,2\}}_1 = Aq^{\{m,1\}}_m q^{\{2,1\}}_1$; all other purchases and sales are nil.

For [2,n] , $n \neq 1$, $s^{[2,n]\{2,1\}}_2 = A$, $b^{[2,n]\{2,1\}}_1 = Aq^{\{2,1\}}_2$, $s^{[2,n]\{1,n\}}_1 = Aq^{\{2,1\}}_2$, $b^{[2,n]\{1,n\}}_n = Aq^{\{2,1\}}_2 q^{\{1,n\}}_1$; all other purchases and sales are nil.

For [m,1] , $m \neq 2$, $s^{[m,1]\{m,1\}}_m = 2A$, $b^{[m,1]\{m,1\}}_1 = 2Aq^{\{m,1\}}_m$; all other purchases and sales are nil. For [1,n] , $n \neq 2$, $s^{[1,n]\{1,n\}}_1 = 2A$, $b^{[1,n]\{1,n\}}_n = 2Aq^{\{n,1\}}_1$; all other purchases and sales are nil.

For [2,1], $s^{[2,1]\{2,1\}}_2 = 3A$, $b^{[2,1]\{2,1\}}_1 = 3Aq^{\{2,1\}}_2$. For [1, 2], $s^{[1,2]\{2,1\}}_1 = 3A$,

$b^{[1,2]\{2,1\}}_{1} = 3Aq^{\{2,1\}}_{1}$.

- For $h \in H^0$, for each {1,j}, $b^{h\{1,j\}}_{j}=\gamma=s^{h\{1,j\}}_{0}$; for each {i,j} so that $1\neq j\neq 2\neq i\neq 1$, $b^{h\{i,j\}}_{j}=A/2=s^{h\{i,j\}}_{0}$; all other $b^{h\{i,j\}}_{j}$ and $s^{h\{i,j\}}_{j}$ are nil. In particular $b^{h\{i,2\}}_{i}$ and $s^{h\{i,2\}}_{0}$ are nil.

STEP 2:

- For {m,n} where $m\neq 1\neq n$, $1=q^{\{m,n\}}_{(m)0}= q^{\{m,n\}}_{(n)0}$, $q^{\{m,n\}}_{m}=q^{\{m,n\}}_{n}=(^1/_2)$.
- For {m,1}, $m\neq 2$, $1=q^{\{m,1\}}_{(1)0} =q^{\{m,1\}}_{(m)0}=q^{\{m,1\}}_{1}$, $q^{\{m,1\}}_{m}=\dfrac{3A-2\gamma}{3A}= 0.60$
- For {2,1} , $1=q^{\{2,1\}}_{(2)0} =q^{\{2,1\}}_{(1)0}=q^{\{2,1\}}_{1}$, $q^{\{2,1\}}_{2}=\dfrac{(N+2)A-2\gamma}{(N+2)A}\geq 0.76$

As CYCLE 2 STEP 1 is completed, trade has become partially monetized. All trade in good 2 goes through good 1 as a medium of exchange. As STEP 2 is completed, prices reflect the higher trading volumes on markets including 1. For convenience, pricing at trading posts {1,m} dealing in good 1 is characterized by setting $q^{\{1,m\}}_{1}$ (the bid price of 1) at 1 and discounting only $q^{\{1,m\}}_{m}$ to reflect transaction cost. Going into CYCLE 3 STEP 1, typical [m,n] for $1\neq m\neq 2\neq n\neq 1$, can reconsider whether to trade in goods m and n directly or to trade through good 1 as a medium of exchange. In order to make that decision he compares $q^{\{m,n\}}_{m}$ to the product $q^{\{n,1\}}_{1}\cdot q^{\{m,1\}}_{m}$. The former is the value of m in terms of n in direct trade, the latter through trade mediated by good 1. This is the same comparison [m,n] made at CYCLE 2 STEP 1, and decided to continue to trade directly. But at the new posted prices we have $.5 =q^{\{m,n\}}_{m} < 0.60 = q^{\{n,1\}}_{1}\cdot q^{\{m,1\}}_{m}$. It is more advantageous to trade indirectly. The outcome of CYCLE 3 STEP 1 will be full monetization; all trade will go through good 1.

CYCLE 3, STEP 1:

- For $[m,n] \in H^F$, $m,n\neq 1$, $s^{[m,n]\{m,1\}}_{m}= A$, $b^{[m,n]\{m,1\}}_{1}= Aq^{\{m,1\}}_{m}$, $s^{[m,n]\{1,n\}}_{1}= Aq^{\{m,1\}}_{m}$, $b^{[m,n]\{1,n\}}_{n}=A(q^{\{m,1\}}_{m}q^{\{n,1\}}_{1})$; all other purchases and sales are nil.
- For $[m,1] \in H^F$, $m\neq 1$, $s^{[m,1]\{m,1\}}_{m}= 2A$, $b^{[m,1]\{m,1\}}_{1}= 2Aq^{\{m,1\}}_{m}$; all other purchases and sales are nil. For $[1,n] \in H^F$, $n\neq 1$, $s^{[1,n]\{1,n\}}_{1}= 2A$, $b^{[1,n]\{1,n\}}_{n}= 2Aq^{\{1,n\}}_{1}$; all other purchases and sales are nil.

For [2,1], $s^{[2,1]\{2,1\}}_{2}= 3A$, $b^{[2,1]\{2,1\}}_{1}= 3Aq^{\{2,1\}}_{2}$. For [1, 2], $s^{[1,2]\{2,1\}}_{1}= 3A$, $b^{[1,2]\{2,1\}}_{1}= 3Aq^{\{2,1\}}_{1}$.

- For $h \in H^0$, for each {i,j} with $i\neq 1\neq j$, all transactions are nil. For {1,j}, $2\leq j \leq N$, $b^{h\{1,j\}}_{j}=\gamma=s^{h\{i,j\}}_{0}$.

STEP 2:

- For {m,n} where $m\neq 1\neq n$, $1=q^{\{m,n\}}_{(m)0}= q^{\{m,n\}}_{(n)0}$, $q^{\{m,n\}}_{m}=q^{\{m,n\}}_{n}=(^1/_2)$.
- For {m,1}, $m\neq 2$, $1=q^{\{m,1\}}_{(1)0} =q^{\{m,1\}}_{(m)0}=q^{\{m,1\}}_{1}$, $q^{\{m,1\}}_{m}=\dfrac{NA-2\gamma}{NA}\geq 0.60$
- For {2,1} , $1=q^{\{2,1\}}_{(2)0} =q^{\{2,1\}}_{(1)0}=q^{\{2,1\}}_{1}$, $q^{\{2,1\}}_{2}=\dfrac{(N+2)A-2\gamma}{(N+2)A}\geq 0.76$

CYCLE 4, STEP 1: Repeat Cycle 3, Step 1

STEP 2: Repeat Cycle 3, Step 2

CONVERGENCE.

What's happening in Example VI.3? Preferences and endowments are structured so that at roughly the same prices for all goods, there is a balance between supply and demand. Some pairs of goods are more actively traded than others. Good 1 has approximately twice as much active demand (and supply) as most other goods. Good 2 has slightly more active trade than most other goods, and that active trade is concentrated in a supplier who demands good 1 and a demander endowed with good 1.

Here's how trade takes place. The starting point is a barter economy, the full array of N(N-1)/2 trading posts. For every pair of goods i-j, where $1 \leq i,j \leq N$, there is a post where that pair can be traded. The starting prices are chosen (somewhat arbitrarily) to cover average costs at low trading volume. The bid-ask spread is uniform across trading posts so trade at each post is as attractive as anywhere else. Then each household computes its demands and supplies at those prices. It figures out what it wants to buy and sell and to which trading posts it should go to implement the trades. Since all bid-ask spreads start out equal, each household just goes to the post that trades in the pair of goods that the household wants to exchange for one another; demanders of good j who are endowed with good i go to {i,j}. Because of the distribution of demands and supplies, there is twice the trading volume on posts {1,j} as on most {i,j} and three times as much on {1,2}.

Then the average cost pricing auctioneer responds to the planned transactions. He prices bid/ask spreads in all markets to cover the costs of the trade on them. Since there is a scale economy in the transactions technology, this leads to slightly narrower bid/ask spreads on the {1,j} markets and an even narrower spread on the {1, 2} market. The auctioneer announces his prices.

Households respond to the new prices. Households who want to buy or sell good 2 discover that the bid/ask spread on market {1, 2} is lower than on any other market trading 2. It makes sense to channel transactions through this low cost market, even if the household has to undertake additional transactions to do so. Ordinarily households [i,2] and [2,i] would have gone directly to the market {i,2} to do their trading. But the combined transaction costs on {i,1} and on {1,2} are lower than those on {i,2}. Households [i,2] and [2,i] find that they incur lower transaction costs by trading through good 1 as an intermediary. They exchange i for 1 and 1 for 2 (or 2 for 1 and 1 for i) rather than trade directly. The market makers on the many different {i,1} markets, $2 \leq i \leq N$, find their trading volumes increased as the [i,2] and [2,i] traders move their trades to {i,1} and {2,1}.

The average cost pricing auctioneer responds to the revised trading plans once again. Bid-ask spreads narrow on {i,1}, $2 \leq i \leq N$. Now the discounts incurred through bid-ask spreads in trading for $i \neq 1 \neq j$ indirectly --- through {i,1} and {1,j} --- are significantly smaller than those trading directly at {i,j} (particularly when N is large). The auctioneer announces his prices. Households respond to the new prices. For all households [i,j], it is now less expensive to trade through good 1 as an intermediary than to trade directly i for j or j for i. All [i,j] now trade on {i,1} and {j,1}; none trade on {i,j}, for $i \neq 1 \neq j$. Trade is fully monetized with good 1 as the 'money.'

The average cost pricing auctioneer re-prices the markets. Inactive markets, {i,j} for $i \neq 1 \neq j$, necessarily continue to post their starting prices (which reflected anticipated low trading volume). The active markets {i,1} get posted prices reflecting their high trading volumes, with narrow bid-ask spreads.

Households review the newly posted prices. The narrow bid-ask spreads on the {i,1} markets reinforce the attractiveness of their previous plans, which called for trading through good 1 as an intermediary. They leave their monetary trading plans in force. At current prices, it is much more economical to trade i for j by first trading i for 1 and then 1 for j than to trade i for j directly. High trading volumes on the {i,1} and {j,1} markets ensure low transaction costs and keep them attractive. All trade takes place at {i,1}, i=2,3,4, ..., N. Good 1 has become the unique 'money'.

Example VI.3 demonstrates price and trading adjustment to the property that scale economies in the transactions technology mean that high volume markets will be low average cost markets. The transition from barter to monetary exchange is the transition from a complex of many thin markets --- one for trade of each pair of goods for one another --- to an array of a smaller number of thick markets dealing in each good versus a unique common medium of exchange. This transition is resource saving when scale economies in transactions technology are large enough.

Example VI.3 shows that the transition progresses through individually rational decisions when prices reflect the scale economy and the initial condition includes a commodity (the latent 'money') with a relatively high transaction volume (hence low average transaction cost). Then, as Einzig notes, "favourite means of barter are apt to arise" and a barter economy thus converges incrementally to a monetary economy. Menger (1892) describes this transition:

> when any one has brought goods not highly saleable to market, the idea uppermost in his mind is to exchange them, not only for such as he happens to be in need of, but...for other goods...more saleable than his own...By...a mediate exchange, he gains the prospect of accomplishing his purpose more surely and economically than if he had confined himself to direct exchange...Men have been led...without convention, without legal compulsion,...to exchange...their wares...for other goods...more saleable...which ...have ...become generally acceptable media of exchange.

Thus, Menger argues that starting from a relatively primitive market setting, some goods will be more liquid than others. As they are adopted as media of exchange, markets for trade in them versus other goods become increasingly liquid. Eventually they become the common media of exchange in equilibrium. Example VI.3 formalizes this argument emphasizing that the increasing liquidity develops endogenously as a result of scale economy in the transaction process.

A Large Pure Trade Economy with Average Cost Pricing Monetary Equilibrium

Since scale economies enter into this argument in an essential way, we'd now like to consider a large economy. This class of examples starts with the same structure as in Example VI.2, but we allow the economy to be large in the sense that there are G (positive integer) households of each type [m,n]. Let $H^{D\times G}$ denote the G-fold replication of H^D with typical element [m,n,g] where m and n are integers between 1 and N (inclusive), m≠n, and g is an integer between 1 and G. m denotes the good with which h is endowed. n denotes the good he prefers. g is a serial number for the agent of type [m,n].

<u>Example VI.4 (Average Cost Pricing Monetary Equilibrium in a Large Economy)</u>: Let $H=H^{D\times G}\cup H^0$. Let transaction technology be characterized by (TCNC). For all $1\leq i,j\leq N$, let $\delta^i>0$, $A-[(\gamma^i+\gamma^j)/G(N-1)]>A(1-\delta^i)$ and $A-[(\gamma^i+\gamma^j)/G(N-1)]>(^1/_3)A$. Without loss of generality, distinguish any single good, 1,...,N, as μ . Then the economy has a monetary average cost pricing equilibrium with good μ as 'money'.

<u>Demonstration of Example VI.4</u>:

For $j \neq \mu$, $q^{\{\mu,j\}}_\mu = 1 = q^{[\mu,j]}_0$.

$q^{\{\mu,j\}}_j=1-[(\gamma^i+\gamma^j)/GA(N-1)]$.

For all other i, j, combinations, $q^{\{i,j\}}_i=(1-\delta^i)$, $q^{\{i,j\}}_j=(1-\delta^j)$.

For h = [m,n,g] (where $m,n \neq \mu$) we have

$b^{[m,n,g]\{\mu,n\}}_n = Aq^{\{\mu,m\}}_m$

$s^{[m,n,g]\{\mu,n\}}_\mu = Aq^{\{\mu,m\}}_m$

$b^{[m,n,g]\{\mu,m\}}_\mu = Aq^{\{\mu,m\}}_m$

$s^{[m,n,g]\{\mu,m,\}}_m = A$.

For h = [m,n,g] (where $m = \mu$) we have

$b^{[m,n,g]\{n,\mu\}}_n = A$.

$s^{[m,n,g]\{i,\mu\}}_\mu = A$.

For h = [m,n,g] (where $n = \mu$) we have

$b^{[m,n,g]\{\mu,m\}}_\mu = A\ q^{\{\mu,m\}}_m$.

$s^{[m,n,g]\{\mu,m\}}_m = A$.

For some elements $h'' \in H^0$, $\sum_{h''} b^{h''\{\mu,m\}}_m = \gamma^\mu+\gamma^m$, $\sum_{h''} s^{h''\{\mu,m,\}}_0 = \gamma^\mu+\gamma^m$.

The examples of section VI demonstrate Tobin's (1959) argument: the choice of the medium of exchange is self-justifying. There is a significant resource saving --- and a competitive pricing advantage to the market-maker --- in moving from a barter to a monetary equilibrium, but the choice of what is 'money' is (under these assumptions) essentially arbitrary[10]. Once the choice is made, the equilibrium, including the designation of 'money,' is stable against small perturbations and entry by alternative media of exchange. These characteristics of the monetary equilibrium reflect the underlying transactions technology: the complementarity among pairwise goods markets implicit in the structure of the problem and the scale economies in transaction costs encourage concentration of trading activity in a few market-makers and a single medium of exchange[11]. Conversely, Theorems III.1 and III.2 suggest that scale economies are essential to monetization of the economy. Without assuming properties peculiar to the designated 'money' as in Example III.1 (that 'money' is the single good so that trades that

[10] This arbitrariness is in contrast to the example of Banerjee and Maskin (1996) where, without explicit transaction costs, in a convex model, the choice of 'money' is fully determined by the parameters of the model as the unique good whose quality is most easily recognized.

[11] The notion of scale economy is consistent with the models of Iwai (1995) and Kiyotaki and Wright (1989) where concentrating trading activity on a single transaction medium reduces waiting times for the completion of trades.

include it are achieved at the lowest possible transaction cost) there seems to be no impetus in a convex model driving the equilibrium toward a single distinguished medium of exchange. Unique monetization results from scale economies in the transaction technology.

VII. Barter and Monetary Bertrand Equilibria with Scale Economies

Under nonconvex transaction costs, competitive equilibria are unlikely to exist. Hence the focus on average cost pricing equilibria in sections V and VI. Imperfectly competitive equilibria, Bertrand equilibria among competing trading posts, may exist and be monetary. A single good will be distinguished in equilibrium as the medium of exchange common to virtually all transactions. Monetary Bertrand equilibria with nonconvex transaction costs share the strong stability property of the average cost pricing equilibria investigated in section VI. They are stable against entry by a new post offering an alternative medium of exchange. This will typically be true even if the alternative medium is superior in the sense that total transaction costs would be lower if it were generally adopted. A superior (lower cost) medium will typically not be adopted in preference to the prevailing medium, precisely because it is not prevailing. Markets using the alternative medium are thin, displaying high average transaction costs. In order to be attractive to individual buyers and sellers the markets must become thicker; there must be additional posts available where the alternative medium is in use.

The transaction cost structure with scale economies is a very suitable setting for Bertrand equilibrium. Each potential market-making firm operating a trading post can survey prevailing prices and demand functions and decide to enter by determining prices to post. This section develops a class of examples of barter and monetary Bertrand equilibrium in a pure trade economy with pairwise goods markets and nonconvex technology. A Bertrand equilibrium is a Nash equilibrium of best responses in price based on imputed demand functions from the households.

In order to model oligopoly, expand the set of market making firms in the following way. For each pair of goods i,j, let there be several firms capable of making the market in i and j. Denote the firms {i,j;1}, {i,j;2},...,{i,j,ℓ},...,{i,j;Λ}, where $\Lambda \geq 2$ is the number of potential entrants into making the market in i and j, indexed by ℓ. Most of the firms {i,j,ℓ} will remain inactive, but their potential to enter the market affects equilibrium prices. We'll focus on the population structure $H=H^D \cup H^0$ as in example IV.1. The reason for focusing on this setting with full double coincidence of wants is not that double coincidence is essential, but that it gives us a fair sized economy, with enough symmetry that the algebra is relatively simple. The economy is large enough that if traders concentrate their transactions on a few trading posts, there may be scope for scale economies. It is small enough that if trading activity is dispersed, then markets are thin and no scale economy is experienced (at the cost of greater complexity we could alternatively use $H^\kappa \cup H^0$, $1 \leq \kappa \leq N-1$). We'll take the market makers' cost functions to display scale economies following the specification (TCNC). The scale economies become active only at relatively high trading volumes.

We'll consider a range of cases. First, example VII.1, we'll suppose that market makers price for low trading volume and that turns out to be an equilibrium: a barter economy with thin markets is a Bertrand equilibrium. Then we'll consider the opposite tack, example VII.2. The interesting case, reflecting a network externality, shows up where most active trading post market makers have adopted a common medium of exchange but others are deciding whether to enter in active market-making. We'll show that once most active market makers have adopted the plan of making a market using a common medium of exchange, the remaining market makers will find that demand facing them makes using the common medium of exchange compelling as well. Once a common medium has been widely adopted, the pressure to adopt universally is decisive.

Example VII.1 (A Bertrand barter equilibrium; When you're not [hot], you're not): Let the population of households be $H=H^D \cup H^0$. Let $C^{\{i,j\}}$ be described by (TCNC). Let $0<\delta^i< {}^1/_3$ for all i = 1,2, ..., N. Let $(\delta^i+\delta^j)A< \gamma^i + \gamma^j$ for all $i\neq j$, i,j=1,2,...N. Let $\Lambda \geq 2$. Note that this setting implies that scale economies in transaction costs are available, but that they are inactive when each trading post deals in quantities of each good comparable to the endowment of each household. Then there is a Bertrand barter equilibrium. For all i-j commodity pairs there is an active market where that pair is transacted.
Demonstration of Example VII.1: There are no active scale economies in this example, but it is convenient to concentrate on a single trading post for each commodity pair. For i,j=1,2,...,N, $i\neq j$, let $1 = q^{\{i,j,\ell\}}_0$, $q^{\{i,j,\ell\}}_i = (1-\delta^i)$, $q^{\{i,j,\ell\}}_j= (1-\delta^j)$. For i=m or n and j=n or m respectively and (without loss of generality) for and $\ell = 1$, let $b^{[m,n]\{i,j,\ell\}}_n = q^{\{i,j,\ell\}}_m A$, $s^{[m,n]\{i,j,\ell\}}_m = A$. Suppose all other trading posts $\{i,j,\ell\}$, $\ell > 1$, are inactive. Then markets clear. Each trading post covers its transaction costs. The pricing and allocation is a Bertrand equilibrium.

The equilibrium in Example VII.1 is essentially Walras's (1874) trading post model. For each pair of distinct goods in active trade, i,j=1, 2, 3, ..., N, there is a market maker dealing in the pair. The volume of trade at each trading post is modest; A units (a single endowment) of each of two goods is traded at each post. For each good there is a bid price and a higher ask price, so that the market maker retains a surplus (in the proportions δ^i and δ^j) from each transaction. The market maker incurs transaction costs (in the proportion, δ^i and δ^j) in good 0; to provide for the transaction costs the market maker buys 0 from agents in H^0 in exchange for the surplus i and j left over from the direct transactions. A zero profit condition is fulfilled, enforced by the threat of entry of other identical market makers $\ell > 1$.

The starting point of the following Example VII.2 is the same as the previous Example VII.1, but the result is a monetary, not a barter equilibrium. This demonstrates the notion that, reflecting the scale economies at the level of individual trading posts, use of a single common medium of exchange in trade of one good encourages trade in that medium for all goods. There are multiple equilibria; any good can become 'money.' The same initial conditions can result in a barter equilibrium or a monetary equilibrium. Reflecting the scale economy, a common usage of monetary trade encourages monetary trade in all goods. Common usage of barter trade discourages monetary trade in all goods.

Example VII.2 (When you're hot, you're hot; A Bertrand Monetary Equilibrium): Let the population of households be $H=H^D \cup H^0$. Let $C^{\{i,j\}}$ be described by (TCNC). Let $0<\delta^i< {}^1/_2$

for i = 1,2, ..., N. Let $(\delta^i+\delta^j)A< \gamma^i + \gamma^j$, $(N-1)A-(\gamma^i + \gamma^j) > (N-1)A(1-\delta^i)$, for all $i\neq j$, i,j=1,2,...N. Let $\Lambda\geq 2$. Note that this setting implies that scale economies in transaction costs are available, and active when each active trading post deals in quantities of each good comparable to the total endowment of that good. Without loss of generality, distinguish any single good, 1,...,N, as μ . Then there is a Bertrand monetary equilibrium with good μ acting as 'money'. For all i-μ commodity pairs there is an active market where that pair is transacted and these are the only active markets in equilibrium.

Demonstration of Example VII.2: The following prices and allocations constitute a Bertrand equilibrium.

For $\ell = 1$, all $j \neq \mu$: $q^{\{\mu,j,1\}}_0 = 1$

$q^{\{\mu,j,1\}}_j = [1-(\gamma^j + \gamma^\mu)/(N-1)A]$, $q^{\{\mu,j,1\}}_\mu = 1$.

For h = [m,n] (where $m,n \neq \mu$) and $\ell = 1$ we have

$b^{[m,n]\{\mu,n,1\}}_n = Aq^{\{\mu,m,1\}}_m$

$s^{[m,n]\{\mu,n,1\}}_\mu = Aq^{\{\mu,m,1\}}_m$

$b^{[m,n]\{\mu,m,1\}}_\mu = Aq^{\{\mu,m,1\}}_m$

$s^{[m,n][\mu,m,1]}_m = A$

For all m,n so that neither $m,n=\mu$, and for all $\ell \neq 1$, $q^{\{m,n,\ell\}}_m=(1-\delta^m)$ and $b^{[m,n]\{m,n,\ell\}}_n = 0$, $s^{[m,n]\{m,n,\ell\}}_m = 0$, $b^{[m,n]\{m,n,\ell\}}_m = 0$, $s^{[m,n]\{m,n,\ell\}}_n = 0$.

The only trading firms active in equilibrium in Example VII.2 are those trading good μ for other goods m = 1, 2, ..., μ–1, μ+1, ..., N-1. Only one trading post in each pair μ-n is active, reflecting the scale economy. Without loss of generality, that trading post is designated number 1. Other potential entering trading posts in μ-n are inactive, but their threat of entry keeps the active post's pricing at average cost. Household $h \in H^D$, h = [m,n], goes to the trading post {m, μ, 1} dealing in good m, his endowment, and sells his endowment at the bid price in exchange for μ. Household h then takes the proceeds of the sale to {n, μ, 1} the trading post dealing in n, his desired good, and buys n for μ at the ask price. Buyers and sellers are evenly matched so all demands are fulfilled. Since ask prices of good m exceed bid prices, post {m, μ, 1} accumulates net stocks of m. Post {m, μ, 1} incurs transaction costs in good 0. Households $h \in H^0$ supply good 0, the needed input to the transaction process, receive payment in μ, and spend the μ on good m, absorbing post {m, μ, 1}'s net accumulation of m (prices for these transactions are unity). The allocation and market structure constitute an equilibrium since no firm finds it profitable, taking other firms' announced prices as given, to change its prices.

Example VII.2 is the strategic counterpart of VI.2 (a price-taking average cost pricing equilibrium). Note that the preferences and endowments in this example fullfil 'double coincidence of wants.' Nevertheless, the structure of transaction costs keeps agents from trading directly with those whose endowments and preferences are reciprocal to their own (as they do in Example IV.1), but encourages them to use monetary trade. Firms trading in i and j where $i \neq \mu \neq j$, find that entry is unprofitable because their markets are thin. Since most trade goes through μ, transaction volumes in markets μ-j are much higher than in i-j. The i-j firms cannot successfully (at positive trading volumes) charge wide enough margins between bid and ask prices to cover costs. It is in this sense that the choice of μ as 'money' is self-justifying.

Example VII.2 emphasizes as well that there are multiple equilibria. Virtually any of the N goods can become μ , the common medium of exchange. The only ones ruled out are those with insufficient scale economies (excessively high γ^i). There is no assurance that any single equilibrium designation of μ will be the best choice. On the contrary, there are monetary equilibria with a choice of 'money' that is dominated by possible alternatives. A best choice would be one with the lowest γ^i, but there is no mechanism posited in Example VII.2 to seek out the lowest cost medium. On the contrary, once a common medium of exchange has been selected (by chance, history, an invisible hand), its costs become <u>locally</u> lowest by scale economy. Hence the choice is sustained despite the availability of superior alternatives. Their superiority is clear to us as observing economists globally, but it is not locally evident, because (with nonconvex transaction costs) only global changes, not incremental local changes, result in a cost saving. Once the equilibrium with μ as 'money' is established, the markets for the firms {i,j,ℓ} for i≠μ≠j are thin and unprofitable, even if they have a lower set-up cost.

There is a network externality associated with the choice of a common medium of exchange. As additional markets and traders use a particular medium of exchange, that medium becomes more attractive for others, reflecting complementarities among markets. The strategic situation facing a market-making firm as the economy approaches the Bertrand equilibrium in Example VII.2 reflects the power of the network externality in the common medium of exchange. As the economy approaches a Bertrand equilibrium with good μ as common medium of exchange, consider the situation of a trading post firm deciding to enter the market. Suppose that almost all goods except one, good n*, are already traded for μ. That is, there are active trading posts {μ, n, 1} for all goods n=1,2, ..., n*-1, n*+1, ..., N, but the market for n* is still unsettled. Trade in all goods but n* is already monetized. What is the demand situation confronting trading posts entering the market in n*? All the households [m, n*] who want to trade good m for n* face low transaction costs in trading their endowments, m, for the prevailing medium of exchange μ. All the traders [n*, m] who want to trade their endowment n* for a variety of other goods m face low transaction costs in buying m for the prevailing medium of exchange μ. Potential entrants to trading post activity in n* and μ, {μ,n*,ℓ} see this immense latent demand. There are N-1 buyers and N-1 sellers who will find it advantageous (other things being equal) --- based on the low costs at complementary trading posts {μ,n,1}, n≠n* --- to trade at {μ,n*,ℓ} if the price is right. The demand facing {μ,n*,ℓ} promises a thick market at the trading post. Conversely, the (barter) trading posts {m,n*,ℓ}, m≠μ , face the prospect of trading in a thin market. At break-even prices, these trading posts face low volume. The only traders interested in their markets are those with precisely matching demands [m, n*] and [n*, m]. The message to potential entrant trading posts is clear. There is immense demand for trading post {μ,n*,ℓ} providing transactions in the prevailing medium of exchange, μ. There is little demand for trading post {m,n*,ℓ} providing transactions to a thin barter market.

The (network) externality here follows these lines: High trading volume at active trading posts using the prevailing medium of exchange, μ, leads to low average transaction costs at those posts, implying low Bertrand pricing of transaction services. That leads to high demand for potential entrant trading posts (in goods not currently served) trading in the prevailing medium of exchange μ. When those posts enter and find their average costs are low, their Bertrand prices are also low. Use of μ as a common medium of exchange builds on itself. Each trading post making a market in μ adds to the demand for other trading posts making a market in μ and other goods. Each additional active market $\{\mu,n,\ell\}$ increases trading volumes and reduces average costs for the complementary markets.

Bertrand Monetary Equilibrium in a Large Economy[12]

As in Example VI.4, we now consider a large economy.

Example VII.3 (Bertrand Monetary Equilibrium in a Large Economy): Let $\Lambda \geq 2$. Let $H=H^{D\times G}\cup H^0$. Let transaction technology be characterized by (TCNC). For all $1\leq i,j\leq N$, let $A-[(\gamma^i+\gamma^j)/G(N-1)] > (1-\delta^i)A$, and $A-[(\gamma^i+\gamma^j)/G(N-1)] > (^1/_3)A$. Without loss of generality, distinguish any single good, 1,...,N, as μ. Then the economy has a Bertrand monetary equilibrium with good μ as 'money'.

Demonstration of Example VII.3:

The following prices and quantities constitute a Bertrand equilibrium. Without loss of generality designate the active trading post firm in each μ-n market as $\ell = 1$.

For $j \neq \mu$, $\ell = 1$, $q^{\{\mu,j,1\}}_{\mu} = 1 = q^{\{\mu,j,1\}}_0$.

$q^{\{\mu,j,1\}}_j=1-[(\gamma^i+\gamma^j)/GA(N-1)]$.

For all other i, j, ℓ, combinations, $q^{\{i,j,\ell\}}_i=(1-\delta^i)$, $q^{\{i,j,\ell\}}_j=(1-\delta^j)$.

For h = [m,n,g] (where $m,n \neq \mu$) we have

$b^{[m,n,g]\{\mu,n,1\}}_n = Aq^{\{\mu,m\}}_m$

$s^{[m,n,g]\{\mu,n,1\}}_{\mu} = Aq^{\{\mu,m\}}_m$

$b^{[m,n,g]\{\mu,m,1\}}_{\mu} = Aq^{\{\mu,m\}}_m$

$s^{[m,n,g]\{\mu,m,,1\}}_m = A$.

For h = [m,n,g] (where $m = \mu$) we have

$b^{[m,n,g]\{n,\mu,1\}}_n = A$.

$s^{[m,n,g]\{i,\mu,1\}}_{\mu} = A$.

For h = [m,n,g] (where $n = \mu$) we have

$b^{[m,n,g]\{\mu,m,1\}}_{\mu} = A\, q^{\{\mu,m\}}_m$.

$s^{[m,n,g]\{\mu,m,1\}}_m = A$.

For some elements $h'' \in H^0$, $\sum_{h''} b^{h''\{\mu,m,1\}}_m = \gamma^{\mu}+\gamma^m$, $\sum_{h''} s^{h''\{\mu,m,1\}}_0 = \gamma^{\mu}+\gamma^m$.

As in example VII.3, the firms $\ell > 1$ are potential market entrants and their threat of entry affects price determination. The firms with positive levels of actual transactions are $\ell = 1$. There are zero profits. The allocation and market structure constitute an equilibrium since no firm finds it profitable, taking other firms' announced prices as given, to change its price offers. Again as in example VII.2, firms $\{i,j,\ell\}$ where $i \neq \mu \neq j$, find that entry is

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[12] A version of this example appeared in Starr and Stinchcombe (1998). See also Howitt (2000), for a large economy with a Bertrand monetary equilibrium in a trading post model.

unprofitable because their markets are thin. They post prices $q^{\{i,j,\ell\}}_i=(1-\delta^i)$, $q^{\{i,j,\ell\}}_j=(1-\delta^j)$ that cover their operating costs at low volume, but these prices are unattractive to active traders, dominated by the posted prices of firms trading through μ. It is in this sense that the choice of μ as 'money' is self-justifying. Since most trade goes through μ, transaction volumes in markets $\{\mu,j,1\}$ are much higher than in $\{i,j,1\}$ for $i \neq \mu \neq j$. Hence $\{\mu,j,1\}$ firms can successfully operate at narrower bid/ask spreads than $\{i,j,1\}$ firms can.

The distinction between examples VII.2 and VII.3 is that the large numbers of traders in VII.3 allow the economy to overcome larger set-up costs on the transactions technology than would otherwise be possible. This shows up as the assumption that $A-[(\gamma^i+\gamma^j)/G(N-1)] > (1-\delta^i)A$ in example VII.3 versus $(N-1)A-(\gamma^i + \gamma^j) > (N-1)A(1-\delta^i)$ in example VII.2.

The examples of section VII demonstrate Tobin's (1959) argument: the choice of the medium of exchange is 'self-justifying.' There is a significant resource saving --- and a competitive pricing advantage to the market-maker --- in moving from a barter to a monetary equilibrium, but the choice of what is 'money' is (under these assumptions) essentially arbitrary. Once the choice is made, the equilibrium, including the designation of 'money,' is stable against small perturbations and entry by competing market-makers or by alternative media of exchange. These characteristics of the monetary equilibrium reflect the underlying transactions technology: the complementarity among pairwise goods markets implicit in the structure of the problem and the scale economies in transaction costs encourage concentration of trading activity in a few market-makers and a single medium of exchange.[13] Conversely, example III.2 shows that scale economies are essential to unique monetization of the economy. Without assuming properties peculiar to the designated 'money' (e.g. that 'money' is the single good so that trades that include it are achieved at the lowest possible transaction cost, as in example III.1) there seems to be no impetus in a convex model driving the equilibrium toward a single distinguished medium of exchange. Unique monetization of trade results from scale economies in the transaction technology.

VIII. Government-Issued Fiat Money

In order to study fiat money we introduce a government with the unique power to issue fiat money. Fiat money is intrinsically worthless; it enters no one's utility function. But government is uniquely capable of declaring it acceptable in payment of taxes. Adam Smith (1776) notes “A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money…” (v. I, book II, ch. 2). Abba Lerner (1947) comments, "The modern state can make anything it chooses generally acceptable as money...if the state is willing to accept the proposed money in payment of taxes." Taxation --- and fiat money's guaranteed value in payment of taxes --- explains the positive equilibrium value of fiat money[14]. Scale economies explain its uniqueness as the medium of exchange.

[13] The notion of scale economy is consistent with the models of Iwai (1995) and Kiyotaki and Wright (1989) where concentrating trading activity on a single transaction medium reduces waiting times for the completion of trades.

[14] See also Li and Wright (1998) and Starr (1974).

As an economic agent, government is denoted G. Government sells tax receipts, the N+1[st] good. It also sells good N+2, an intrinsically worthless instrument, (latent) fiat money, that government undertakes to accept in payment of taxes, that is, in exchange for N+1. The typical household [m,n] in H^1 or H^K desires to purchase tax receipts to the extent it prefers not to have a quarrel with the government's tax authorities. Government sets a target tax receipt purchase by the taxayer of $\tau^{[m,n]}$. Then we rewrite [m,n]'s utility function as

$$u^{[m,n]}(x) = \sum_{i=0,\ i\neq n}^{N} x_i + 3x_n - 10[\max[(\tau^{[m,n]} - x^{[m,n]}_{N+1}), 0]] \qquad \text{(UT).}$$

That is, household [m,n] values paying his taxes with a positive marginal utility up to his tax bill $\tau^{[m,n]}$ and with zero marginal utility for tax payments thereafter. Government uses its revenue to purchase a variety of goods n=1,...,N, in the amount x^G_n .

Good N+2 good represents latent fiat money. Government, G, sells N+1 (tax receipts) for N+2 at a fixed ratio of one-for-one. The trading post {N+1, N+2} where tax receipts are traded for N+2 operates with zero transaction cost. Acceptability in payment of taxes ensures N+2's positive value. If, in addition, N+2 is assumed to have sufficiently low transaction cost, then it becomes the common medium of exchange. Thus if we assume a low linear transaction cost, the existence of a fiat money equilibrium is merely an application of Example III.1 and need not be repeated here.

Government-issued fiat money is typically the unique common medium of exchange: in the US virtually all transactions are denominated in US dollars; in the UK virtually all (nonfinancial) transactions are denominated in pounds sterling. The virtual uniqueness of the monetary instrument is not merely a possibility; it seems to be a general fact. Dollars, euros, pounds sterling, and other government-issued fiat money's all seem to have similar low transaction costs. But in any single market economy precisely one of these instruments is likely to be the unique common medium of exchange. Example VIII.1 harnesses scale economy to explain why fiat money is (almost universally) the unique common medium of exchange.

Particularly in the case of scale economies in the transactions technology, there is a strong tendency to multiple equilibria (recall Example VI.1). This creates an interest in determining which of the several equilibria the economy will actually select. Hence we posit the same tatonnement adjustment process for average cost pricing equilibrium as in section VI. That plausible adjustment process explains why government-issued fiat money becomes the unique common medium of exchange ---- and would do so even in the absence of legal tender rules. Government has two distinctive characteristics: it has the power to support the value of fiat money by making it acceptable in payment of taxes; it is a large economic presence undertaking a high volume of transactions in the economy. Hence, government can make its fiat money the common medium of exchange merely by using it as such. The scale economies implied will make fiat money the low transaction cost instrument and hence the most suitable medium of exchange, not just for government but for all transactors.

Example VIII.1: Let the population of households be $H=H^0 \cup H^K$. Let $u^{[m,n]}$ be described by (UT). Let $\tau^o > 0$ be a constant. Let $0 < \tau^{[m,n]} = \tau^o < A(1-\delta^{N+2})(1-\delta^m)$, all $[m,n] \in H^K$. Let

$x^G_n=\kappa\tau^o q^{\{N+2,n\}}_{N+2}$ all n=1,2,...,N. Let $C^{\{i,j\}}$ be described by (TCNC). Let $(\gamma^{N+2}/\kappa\tau^o)<\delta^i<1/3$ all i = 1,2, ..., N. Then there exists a monetary average cost pricing equilibrium with taxation with good N+2 as the unique 'money.' That monetary equilibrium is the unique limit point of the tatonnement adjustment.

Demonstration of Example VIII.1:

Step 0: For $n\neq m$, set $q^{\{m,n\}}_n=(1-\delta^n)$.

Cycle 1, Step 1:

For i=1,2,...,κ, let $s^{[n,n\oplus i]\{n,n\oplus i\}}_n=A-(\tau^o/q^{\{N+2,n\}}_n)$, $b^{[n,n\oplus i]\{n,n\oplus i\}}_{n\oplus i}=(A-(\tau^o/q^{\{N+2,n\}}_n))q^{\{n,n\oplus i\}}_n$, $s^{[n,n\oplus i]\{N+2,N+1\}}_{N+2}=\tau^o=b^{[n,n\oplus i]\{N+2,N+1\}}_{N+1}$; $b^{[n,n\oplus i]\{N+2,n\}}_{N+2}=\tau^o$, $s^{[n,n\oplus i]\{N+2,n\}}_n=\tau^o/q^{\{N+2,n\}}_n$. For n=1,2,...,N, let $s^{G\{N+2,n\}}_{N+2}=\kappa\tau^o$, $b^{G\{N+2,n\}}_n=\kappa\tau^o q^{\{N+2,n\}}_{N+2}$.

Cycle 1, Step 2: For $n,m\neq N+2$, $n\neq m$, set $q^{\{m,n\}}_n=(1-\delta^n)$.

$q^{\{N+2,n\}}_n=(1-\min[\delta^n,\gamma^n/\kappa\tau^o])(1-\gamma^{N+2}/\kappa\tau^o)$, $q^{\{N+2,n\}}_{N+2}=1$.

Cycle 2, Step 1: For n=1,2,...,N, let $s^{G\{N+2,n\}}_{N+2}=\kappa\tau^o$, $b^{G\{N+2,n\}}_n=\kappa\tau^o q^{\{N+2,n\}}_{N+2}$; $s^{G\{N+1,N+2\}}_{N+1}=N\kappa\tau^o$, $b^{G\{N+1,N+2\}}_{N+2}=N\kappa\tau^o$; $b^{[n,n\oplus i]\{N+2,N+1\}}_{N+1}=\tau^o$, $s^{[n,n\oplus i]\{N+2,N+1\}}_{N+2}=\tau^o$; $s^{[n,n\oplus i]\{N+2,n\}}_n=A$, $b^{[n,n\oplus i]\{n,N+2\}}_{N+2}=Aq^{\{N+2,n\}}_n$; $s^{[n,n\oplus i]\{n\oplus i,N+2\}}_{N+2}=Aq^{\{N+2,n\}}_n-\tau^o$, $b^{[n,n\oplus i]\{n\oplus i,N+2\}}_{n\oplus i}=(Aq^{\{N+2,n\}}_n-\tau^o)q^{\{n\oplus i,N+2\}}_{N+2}$.

Cycle 2, Step 2: For $n,m\neq N+2$, $n\neq m$, set $q^{\{m,n\}}_n=(1-\delta^n)$.

$q^{\{N+2,n\}}_n=(1-\min[\delta^n,\gamma^n/\kappa A])(1-\gamma^{N+2}/\kappa A)$, $q^{\{N+2,n\}}_{N+2}=1$.

Cycle 3, Step 1: Repeat Cycle 2, Step 1.

Cycle 3, Step 2: Repeat Cycle 2, Step 2.

Convergence.

What's happening in Example VIII.1? Scale economies are taking their course! Government expenditures in all goods markets in exchange for N+2 (and large household demand to acquire N+2 to finance tax payments) result in a large trading volume on the trading posts for good N+2 versus n=1,...,N. Volume is large enough that scale economies kick in. The average cost pricing auctioneer adjusts prices, the bid/ask spread, to reflect the scale economies. The bid/ask spreads incurred on trading m for m⊕i by way of good N+2 become considerably narrower than on trading m for m⊕i directly. The price system then directs each household to the market {m,N+2} where its endowment is traded against good N+2. The household sells all its endowment there for N+2 and trades N+2 subsequently for tax payments and desired consumption. Scale economy has turned N+2 from a mere tax payment coupon into 'money,' the unique universally used common medium of exchange.

IX. Conclusion

The monetary structure of trade in general equilibrium, the uniqueness of money, and the existence of a fiat money equilibrium can be demonstrated as the outcome of a market general equilibrium with transaction costs. The monetary character of trade, the existence of a common medium of exchange in economic equilibrium, is logically derived from price theory. Starting from a (non-monetary) Arrow-Debreu Walrasian model the addition of two constructs is sufficient: segmented markets with multiple budget constraints (one at each transaction) and transaction costs. The multiplicity of budget constraints creates a demand for a carrier of value (medium of exchange) between transactions. Money (the common medium of exchange) arises endogenously as the most liquid (lowest transaction cost) asset. Government-issued fiat money derives its value from acceptability in payment of taxes. Uniqueness of the monetary instrument (fiat or commodity money) in equilibrium comes from scale economies in transaction cost.

The taxonomy of cases developed is depicted in the table.

Equilibrium Monetary Structure

	Returns to Scale in Transaction Technology	
Demand Structure	*Linear Transaction Technology*	*Increasing Returns Transaction Technology*
Absence of Double Coincidence of Wants	Monetary Equilibrium where the low transaction cost instrument becomes 'money' (Example III.1); Possibly multiple 'moneys' (Example III.2)	Monetary Average Cost Pricing Equilibrium with Unique 'Money' (Example VI.1)
Absence of Double Coincidence of Wants with Fiat Money	Fiat Money Equilibrium if fiat money is the low transaction cost instrument (Apply Example III.1)	Fiat Money Equilibrium (‘money’ is unique) when tax payments and government purchases are sufficiently large (Example VIII.1)
Full Double Coincidence of Wants	Nonmonetary equilibrium (Example IV.1)	Monetary Equilibria (Average Cost Pricing and Bertrand) with Unique 'Money' (Examples VI.2, VII.2)

Absent double coincidence of wants, with linear transaction costs, a low transaction cost instrument is endogenously chosen as a medium of exchange. In the case of linear transaction costs, absence of double coincidence of wants is essential to monetary equilibrium. Alternatively scale economies in transaction cost (nonconvex transaction costs) lead to a corner solution, uniqueness of the common medium of exchange. Fiat money derives its positive equilibrium value from acceptability in payment of taxes. Fiat money becomes the unique common medium of exchange when government taxation and purchases are sufficiently large that scale economies in transaction costs make it the low (average) transaction cost instrument.

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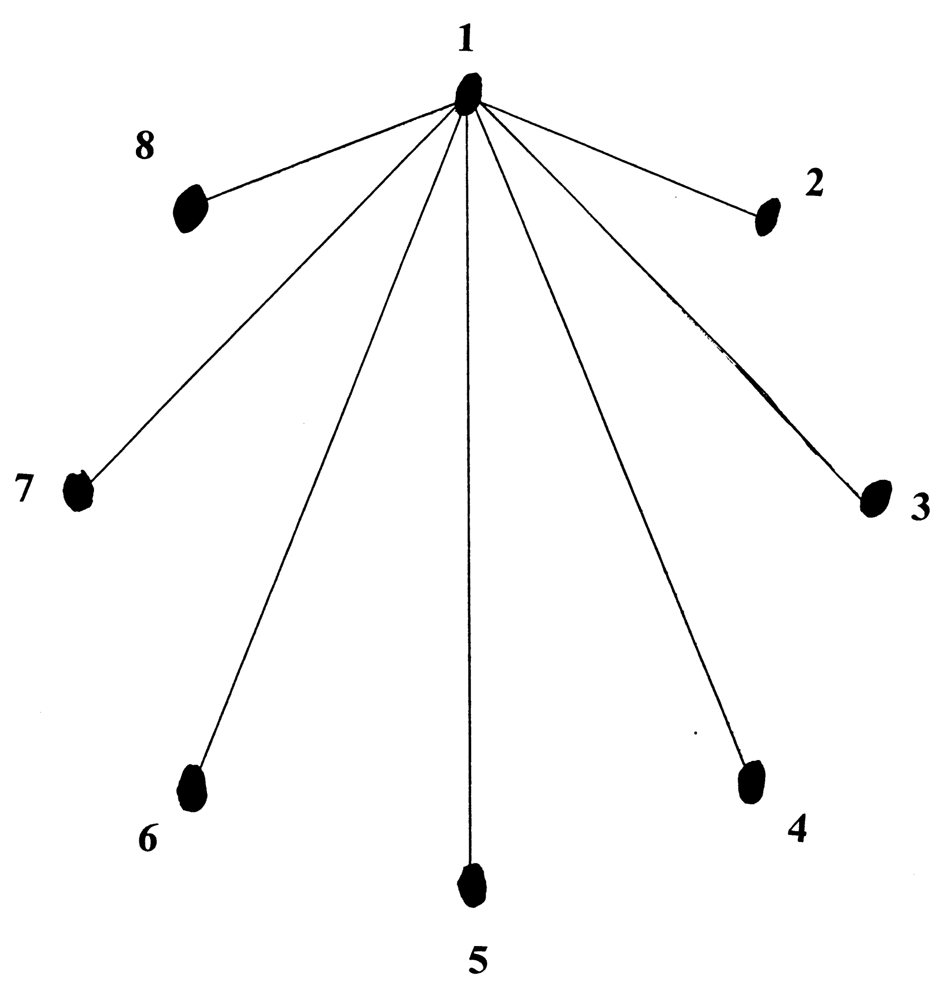

Figure 1: Monetary Equilibrium with Unique Money

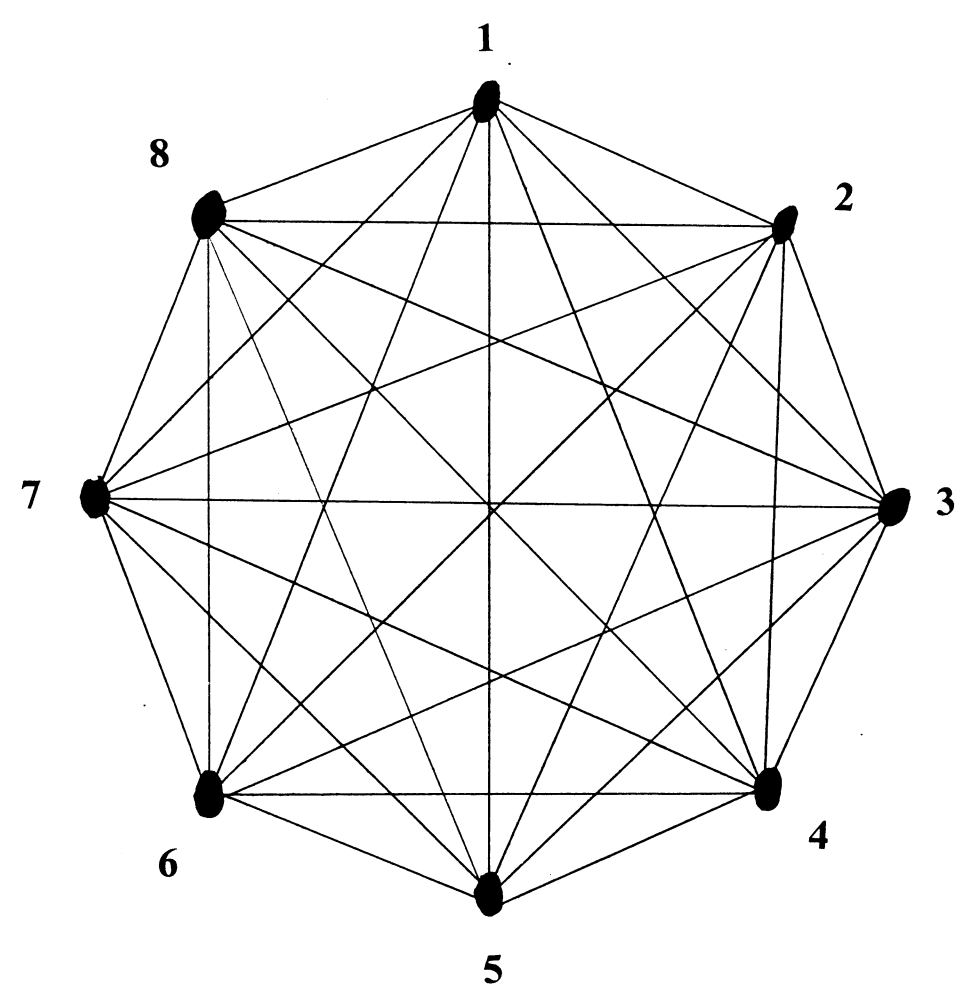

Figure 2 : Barter Equilibrium for H^D