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Author
Thoma, M.H.

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M.H. Thoma

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Viscosity Coefficient of the Quark-Gluon Plasma in the Weak Coupling Limit[1]

Markus H. Thoma
Nuclear Science Division
Lawrence Berkeley Laboratory
Berkeley, California 94720

Abstract

The shear viscosity coefficient of the quark-gluon plasma is calculated by considering the relaxation time approximation. Screening effects are taken into account by using an effective perturbation theory developed recently for the finite temperature QCD in the weak coupling limit. The result agrees with the one obtained from a variational approach to the Boltzmann equation, but is at variance to other results based on a Kubo-type formula.

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I. Introduction

In this note, we investigate dissipative processes in a quark-gluon plasma (QGP) supposed to be formed in ultrarelativistic heavy ion collisions. Former results for the shear viscosity of the QGP are based on two different methods. In Ref. [1-4] the kinetic theory was used. Starting from the Boltzmann equation, the shear viscosity coefficient η can be derived containing the transport cross section [5]. In the high temperature limit, corresponding to the weak coupling limit, $\eta = cT^3/[\alpha_s^2 \ln(1/\alpha_s)]$ was found. By using the relaxation time approximation for a QGP with two quark flavors, the constant c was estimated to be 0.28 [1] or 0.57 [2], whereas a variational calculation gave $c = 1.16$ [4].

For the second method the average value of the energy-momentum tensor is calculated using a nonequilibrium statistical operator [6] and compared to the energy-momentum tensor of viscous hydrodynamics (Navier-Stokes equation). This relates dissipative coefficients to equilibrium correlation functions of the energy-momentum tensor (Kubo formulas) in accordance with the dissipation-fluctuation theorem [7-11]. In [11] $\eta \gtrsim 2.6\, T^3/\alpha_s$ was inferred. Though lattice calculations based on the Kubo formula are able to find a value for η near the phase transition, they are still very crude ($0 \leq \eta \leq 9.5\, T^3$ for $T \simeq T_c$ [12]).

The physical process responsible for the viscosity to lowest order in the coupling constant g contains elastic scattering of the QGP quarks and gluons off each other via one gluon exchange. Since using a bare gluon propagator in the scattering matrix element leads to an infrared singularity, screening effects of the QGP have to be included. Up to now, these screening effects have been taken into account only phenomenologically in calculations of transport coeffients [1,2,4].

Braaten and Pisarski [13] have recently developed an effective perturbation theory in the weak coupling limit ($g << 1$), which takes screening effects into account. It can be used to generate a systematic expansion in g, which gives gauge invariant results for physical quantities. In this way leading order interaction rates can be calculated correctly [14-17]. Unfortunately, the interaction rate of particles with hard momenta ($p \equiv |\vec{p}| \gtrsim T$) still turns out to be infrared divergent, but the quadratic singularity of naive perturbation theory is reduced to a logarithmic one [15-18]. In contrast, quantities which are logarthmically divergent in naive perturbation theory (e.g., the energy loss of a charged particle in a relativistic plasma [17-19]) are finite by using the effective perturbation theory [20].

Pethick et al. [21] observed that the viscosity coefficient is finite even in the absence of static magnetic screening due to dynamical screening. We will confirm their observation by showing that the viscosity coefficient belongs to the above class of quantities which are infrared finite after applying the effective perturbation theory. We will calculate the viscosity coefficient in the relaxation time approximation. For this purpose, we have to consider the mean free path of the quarks and gluons in the QGP [5], which is the inverse of the interaction rate. It is essential to treat the transport process correctly in this interaction rate by taking into account the dominance of large angle scattering for dissipation [5]. The transport interaction rate turns out to be infrared finite, using the effective perturbation theory, in contrast to the ordinary interaction rate. We will use the method proposed by Braaten and Yuan [20] for calculating the transport interaction rate. Keeping only the leading logarithm, our final result for the viscosity coefficient confirms the dependence on the coupling constant found by the kinetic theory [1-4]. Finally, we will discuss its extrapolation to realistic values of the coupling constant.

II. Calculation of the Viscosity Coefficient

We calculate the viscosity coefficient η by using the elementary kinetic theory [5] for a QGP of massless quarks and gluons. In this approximation, the viscosity coefficient of the QGP is given by the sum of a quark and a gluon contribution ($\eta = \eta_q + \eta_g$) [2]:

$$\eta_i \simeq \frac{4}{15} n_i \langle p_i \rangle \lambda_i, \tag{1}$$

where n_i is the density of particles of type i in the QGP, $\langle p_i \rangle$ the average momentum of the particle and $\lambda_i = 1/\Gamma_i$ its mean free path. The interaction rate Γ_i can be calculated to lowest order in g from the imaginary part of the quark or gluon self energy. Let us consider first the quark self energy shown in Figure 1, where we have included screening effects by using the effective gluon propagator defined in the high temperature approximation [22]. According to the rules of the effective perturbation theory [13], it is sufficient to use this gluon propagator because the momentum of the quark is hard ($\langle p_q \rangle \sim T$).

The calculation of the self energy at finite temperature using the imaginary time formalism is straightforward [13]. In Ref. [18] it was shown that the main contribution to the interaction rate comes from the soft momentum transfer region i.e., $\omega, q \sim gT << p \simeq p' \sim T$ in the weak coupling limit, where ω is the energy and $q = |\vec{q}|$ the magnitude of the momentum of the exchanged gluon. Neglecting the Pauli-blocking factor $1 - n(p')$ for the outgoing particle of momentum p', which is justified because of $\langle p' \rangle \simeq 3T$ in the QGP, we obtain [17,18]

$$\Gamma_q = \frac{C_F g^2 T}{2\pi} \int_0^\infty dq\, q^2 \int_{-1}^1 d\mu \int_{-\infty}^\infty \frac{d\omega}{\omega} \delta(\omega - \hat{p}\cdot\vec{q}) \left(\rho_l(\omega, q) + (1 - \frac{\omega^2}{q^2})\rho_t(\omega, q)\right) , \quad (2)$$

where $C_F = 4/3$ is the Casimir invariant, $\mu \equiv \hat{p}\cdot\hat{q}$ and $\rho_{l,t}$ are the discontinous parts of the spectral densities corresponding to the longitudinal and transverse parts of the effective gluon propagator. Inserting the expressions for $\rho_{l,t}$ given in Ref. [23] into (2), we find that the contribution coming from the exchange of a longitudinal gluon is given by $\Gamma_q^l = 0.732\alpha_s T$, while the transverse part of the interaction rate is logarithmically infrared divergent [17]. Using a magnetic gluon mass $m_{mag}^2 \simeq 26\alpha_s^2 T^2$ [2], as infrared cutoff [15], we can fit the transverse part of the interaction rate by $\Gamma_q^t \simeq 0.13\alpha_s T\ [\log(2.44/\alpha_s)]^{1.63}$. Thus for $\alpha_s < 1$, we find $\Gamma_q = \Gamma_q^l + \Gamma_q^t > 0.84\alpha_s T$. The gluon interaction rate is obtained from the quark interaction rate by replacing C_F by $C_A = 3$ in accordance with the result found by Braaten [16]. Substituting these results for the interaction rates in (1) and using the energy densities of a non-interacting gas of massless quarks of two flavors and gluons, $n_q\langle p_q\rangle = 6.9T^4$ and $n_g\langle p_g\rangle = 5.3T^4$, we find $\eta < 3.0T^3/\alpha_s$. This agrees with the result of the Kubo-type calculations [11].

It is well known [5], however, that the use of the interaction rate (2) is not a good approximation in the case of the viscosity, because large angle scattering is the most efficient mechanism for the dissipative momentum transfer. Therefore, the interaction rate should be multiplied by a factor $\sin^2\theta$ under the integral in (2), where θ is the scattering angle in the center of mass system: $\sin^2\theta = 1 - (\hat{p}\cdot\hat{p}')^2$. For small momentum transfer $q << p$ the scattering angle is small. Thus the transport interaction rate Γ_{trans} is much smaller than the regular rate Γ. Therefore, η estimated by using Γ_{trans}, is much larger than by using Γ. As a matter of fact, η is enlarged by an additional factor of $1/\alpha_s$ as we will show in the following. For small momentum transfers we may use the approximation $\sin^2\theta \simeq (q^2/p^2)\ [1-(\hat{p}\cdot\hat{q})^2]$. Substituting this expression under the integral of (2), we see that the infrared behavior is improved due to the q^2-factor in $\sin^2\theta$, but that hard momentum transfers also contribute now. Using the method of Ref. [20], we have to distinguish between soft and hard contributions to Γ_{trans} by introducing a separation scale $gT << q^* << T$. Then the soft quark contribution reads

$$\Gamma_{trans,q}^{soft} = \frac{C_F g^2 T}{2\pi p^2} \int_0^{q^*} dq\, q^3 \int_{-q}^{+q} \frac{d\omega}{\omega}(1-\frac{\omega^2}{q^2}) \left(\rho_\ell(\omega,q) + (1-\frac{\omega^2}{q^2})\rho_t(\omega,q)\right) . \quad (3)$$

The gluon contribution is obtained again by replacing C_F by C_A. Using the expression for the spectral functions given in Ref. [23], we obtain

$$\Gamma_{trans,q}^{soft} = \frac{3C_F g^2 T}{4\pi p^2} m_g^2 \left[\log\left(\frac{q^*}{m_g}\right)^2 + A_{soft}\right], \quad (4)$$

where $m_g = 2gT/3$ is the thermal gluon mass in the case of two flavors and $A_{soft} = -1.379$ was found from a numerical integration.

The calculation of the hard contribution to the transport interaction rate is much more difficult. For this purpose, we have to consider the interaction rate caused by all tree level diagrams which contribute to the $qq \to qq$, $qg \to qg$ and $gg \to gg$ processes [24]. In the case of a heavy quark this calculation can be performed [18,19] assuming the energy of the massive quark to be much higher than the energy of the thermal quarks and gluons. For a massless quark we did not succeed in calculating the hard contribution. But from general considerations [20] we know that it has to be of the form $\Gamma_{trans,q}^{hard} = B\log(T/q^*)^2 + A_{hard}$, where $B = (3C_F g^2 T m_g^2/4\pi p^2)$ is the factor in front of the logarithm in (4), and the constant A_{hard} contains contributions from the scattering amplitudes beyond the leading logarithm and from the fact that the small momentum approximation for $\sin^2\theta$ does not hold for $q \simeq T$. Keeping only the logarithmic term, we obtain

$$\Gamma_{trans,q} = \frac{16\pi C_F T^3}{3p^2}\, \alpha_s^2 \, \log\left(\frac{1}{\alpha_s}\right). \quad (5)$$

Note that the transport interaction rate depends on the momentum in contrast to the ordinary one. Replacing the momentum p of the incident quark or gluon by its average value in the QGP yields for quarks ($\langle p_q \rangle = 3.2T$)

$$\Gamma_{trans,q} = 2.3\, T\, \alpha_s^2 \, \log\left(\frac{1}{\alpha_s}\right) \quad (6)$$

and for gluons ($\langle p_g \rangle = 2.7T$)

$$\Gamma_{trans,g} = 6.9\, T\, \alpha_s^2\, \log\left(\frac{1}{\alpha_s}\right). \tag{7}$$

Combining (6) and (7) with (1), the quark and gluon contributions to the viscosity coefficient are given by

$$\eta_q = 0.82\, \frac{T^3}{\alpha_s^2 \log(1/\alpha_s)}, \tag{8}$$

$$\eta_g = 0.20\, \frac{T^3}{\alpha_s^2 \log(1/\alpha_s)}. \tag{9}$$

Therefore we end up with

$$\eta = 1.02\, \frac{T^3}{\alpha_s^2 \log(1/\alpha_s)}. \tag{10}$$

The coefficient $c = 1.02$ is close to the one found by Baym et al. [4] ($c = 1.16$), where the effective gluon propagator was introduced adhoc in the scattering matrix element (Figure 2) in order to prevent infrared singularities. This procedure was justified in the case of the energy loss of a heavy fermion by comparing its result to the one obtained by using the effective perturbation theory for the imaginary part of the self energy of Figure 1 [18], but there is no proof for its validity in general.

If we try to extrapolate our result to realistic values of the coupling constant e.g., $\alpha_s \simeq 0.2$ corresponding to $g \simeq 1.6$ [25], we encounter serious problems. First of all, the leading logarithm approximation is no longer justified, because the constants behind the $\log(1/\alpha_s)$-term may be of the same order as the logarithmic term. These corrections can be obtained in principle by calculating A_{soft} and A_{hard} as described above.

But even after including these terms, problems arise when these calculations are extrapolated to realistic values of α_s. For example, in Ref. [19] an unphysical negative result for the energy loss was found if g exceeds a critical value of 1.1. The reason for this is the assumption of a separation scale $gT << q^* << T$. Alternative methods avoiding this scale introduce a gauge dependent subset of diagrams and lead to results which are not consistent in the order of g [19].

However, it should be noted that there is a trend to larger values of the viscosity coefficient [1,2,4], which indicates that a hydrodynamical calculation, neglecting dissipative effects, of the expansion phase of the QGP formed in an ultrarelativistic heavy ion collision is questionable [2].

On the other hand, close to the phase transition the viscosity may be small [2] because according to lattice calculations the mean free path λ_i may be reduced due to an increase of the screening length near the critical temperature [25].

III. Conclusions

Taking into account the transport process (dominance of the large angle scattering) in the relaxation time approximation and including screening effects by using Braaten and Pisarski's effective perturbation theory of high temperature QCD (weak coupling limit) [13], we obtained an infrared finite result for the shear viscosity coefficient of the QGP, even in the absence of static magnetic screening. Keeping only the contribution of the leading logarithm, we found $\eta \sim T^3/[\alpha_s^2 \log(1/\alpha_s)]$ in contrast to results obtained by considering the Kubo formula [11]. On the other hand, the dependence on the coupling constant and temperature of our result agrees with estimates from kinetic theory [1,2,4]. Furthermore, there is a quantitatively good agreement with the result of the variational approach to the Boltzmann equation, where an effective gluon propagator was introduced in the scattering amplitude [4]. Therefore, our result, which contains the sum of all contributions to the lowest order in g in a gauge invariant way [13], can be regarded as a justification of the screening procedure used in Ref. [4]. In addition, we conclude that the relaxation time ansatz is a reliable approximation for the transport coefficients of the QGP in the weak coupling limit. A naive extrapolation of these results to realistic values of the coupling constant is not possible, but there are indications that dissipative effects of the QGP are not negligible.

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Figure Captions

1. The quark self energy containing the effective gluon propagator.

2. Elastic scattering of a quark in the QGP via the exchange of an effective gluon.

Fig.1

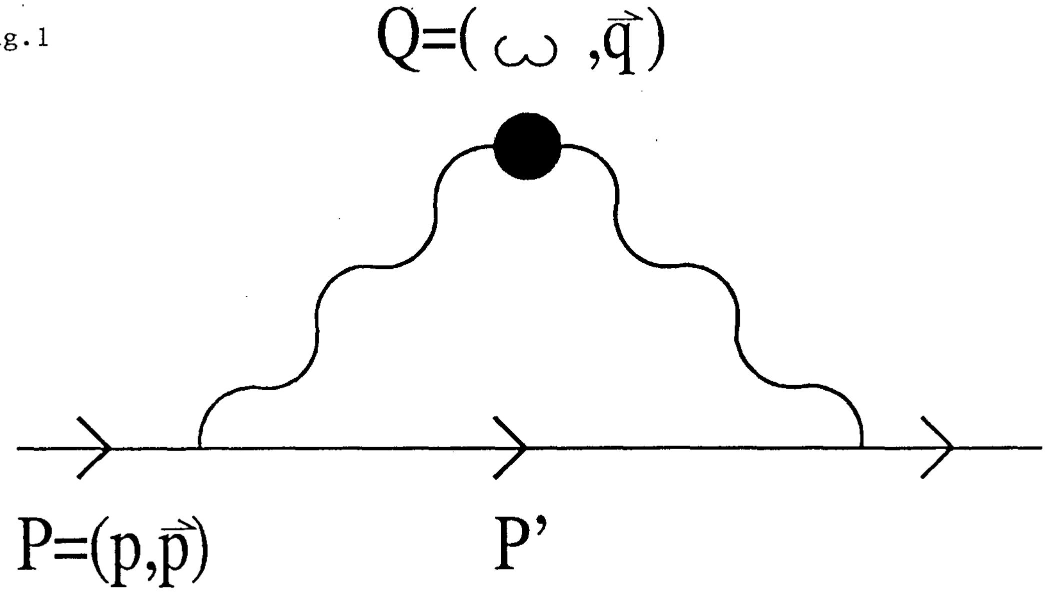

Fig.2