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Quantifying heavy quark transport coefficients with an improved transport model

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Abstract

The heavy-flavor transport coefficients contain important information on the strong interaction at finite temperatures. The extraction of these numbers from experimental data requires dynamical modeling of heavy-flavor transport that is coupled to realistic medium evolution. Furthermore, meaningful extractions necessitate both a faithful implementation of the physical inputs to be tested and the quantification of model uncertainty. For these purposes, we have developed a partonic transport model LIDO [1, 2]. It has an improved treatment of in-medium parton bremsstrahlung, which has been calibrated to theoretical calculations in a simple medium to reduce modeling uncertainty. Regarding the interaction between heavy quark and the medium, few-body perturbative scatterings are applied to large-momentum transfer (q) processes, while a diffusion equation models the dynamics of small-q processes. Such a separation restricts the explicit use of medium quasi-particles to large-q processes only. Another advantage is that deviations from the leading-order probe-medium coupling can be parametrized as an additional contribution to the diffusion constant.

The heavy quark transport coefficients are then extracted with uncertainty estimation from a Bayesian analysis including both the RHIC and the LHC data. The results are found to be consistent with earlier extraction of the light-quark transport coefficients at high momentum and be comparable with lattice calculations of the heavy-flavor diffusion constant in the static limit at low momentum.

Keywords: Heavy quark, transport coefficients, Bayesian analysis

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1. Introduction

Heavy quarks are flavor-tagged particles that are created almost exclusively from initial hard processes in relativistic heavy-ion collisions, and they interact with the rest of the system through strong interaction. They are ideal probes of the "strong degrees-of-freedom" for the entire medium evolution, in particular, the quark-gluon plasma phase (QGP). A vital step in this endeavor is to quantify the strength of the interaction between heavy quarks and the QGP. The strongly-coupled nature of QGP makes it hard to answer such questions directly from the first principle approach. So we take another path to extract the interaction strength, the so-called heavy quark transport parameter $\hat{q}$, as a function of both momentum and temperature via a comparison between transport model calculations and experimental data including both the RHIC and the LHC measurements. Note that $\hat{q}$ at high-momentum ($p \gg M$) is also known as the jet transport parameter, while $\hat{q}(p \ll M) \to 2\kappa(T)$ is related to the momentum diffusion of heavy quarks in the QGP thermal bath. The results quantify heavy-flavor coupling to the QGP in a broad momentum range. The model is reviewed in section 2, and the results of the heavy quark in-medium interaction properties are discussed in section 3. A summary is given in section 4.

2. The model

2.1. Heavy-flavor dynamics and implementations in an improved transport model

Heavy quark dynamics is greatly simplified at small momentum $p \lesssim M$. Its gluon radiation process is suppressed by the mass effect [3]. Recoils in collisions with QGP constituents are small compared to the mass $\sqrt{MT} < M$. Therefore, the heavy quark dynamics can be described by a diffusion equation, with the input of the momentum diffusion coefficient κ. At higher momentum $p > M$, radiative energy loss becomes comparable to the elastic energy loss; and the phase-space opens up for rare but hard kicks from the medium that significantly change the heavy quark momentum. These processes are not well approximated by the diffusion dynamics. Instead, they are included in a cross-section based linearized Boltzmann equation. Finally, at very high momentum, heavy quarks momentum change is completely dominated by gluon radiations. Such energetic radiations inside a dense QGP are further complicated by the Landau-Pomeranchuk-Migdal (LPM) effect due to a very long gluon formation time, and one needs to modify the traditional Boltzmann equation to mimic such effect. To capture the feature of heavy-flavor dynamics in a broad range of momentum, we carefully designed a partonic transport model `LIDO`. It interpolates the diffusion dynamics for soft interaction to a linearized Boltzmann equation for hard collisions, with an approximate implementation of the LPM effect for gluon radiation. The ingredients of the model can be formally summarized as,

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f = \mathcal{D}[f] + C_{1\leftrightarrow 2}[f] + C_{2\leftrightarrow 2}[f] + C_{2\leftrightarrow 3}[f]. \tag{1}$$

The time evolution of the heavy-quark distribution function is governed by a drifting term and interactions with the medium. Processes that involve a small change in heavy-quark momentum $|q| < Q_{\rm cut} \propto m_D$ is treated by a diffusion term $\mathcal{D}$; while the collision term $C_{2\leftrightarrow 2}$ contains processes with a large momentum change $|q| > Q_{\rm cut}$. The induced radiations due to diffusion and large-q elastic collisions are treated as effective $n \to n+1$ body collisions in $C_{1\leftrightarrow 2}$ and $C_{2\leftrightarrow 3}$.

This transport model is coupled to an event-by-event simulation package [4] of the medium based on 2+1D viscous hydrodynamics. At the transition temperature, the heavy quarks are converted into heavy-mesons using a fragmentation plus recombination hadronization model [5]. Finally, charmed mesons undergo hadronic rescatterings with pions and ρ-mesons [5, 6].

2.2. A flexible parametrization of the heavy-flavor transport parameter

The interaction between the heavy-quark and the medium is encoded in $\mathcal{D}$ and $C_{2\leftrightarrow 2}$. At leading order in the weakly coupled limit, the contribution to the transport parameter at high-momentum due to small-q processes ($\hat{q}_{\mathcal{D}}$) and large-q collisions ($\hat{q}_C$) are

$$\hat{q}_{\mathcal{D}} = C_F m_D^2 T \int_0^{Q^2_{\rm cut}} dq^2 \frac{\alpha_s(q^2)}{q^2 + m_D^2}, \tag{2}$$

$$\hat{q}_C = \frac{1}{E_1} \sum \int \frac{dp_2^3}{(2\pi)^3 E_2} f_0(p_2)(s - M^2) \int_{Q^2_{\rm cut}}^{s} \frac{d\sigma_{2\to 2}}{dq^2} q^2 dq^2, \tag{3}$$

Here, $\hat{q}_C$ is simply the transverse momentum broadening obtained by integrating the q^2-weighted large-q two-body elastic collision, averaging over the medium distribution functions, and then sum over all channels and degeneracies. The running coupling constant in a medium is assumed to be,

$$\alpha_s(q^2) = \frac{4\pi}{9} \frac{1}{\ln\left(\max\{|q|^2, (\mu\pi T)^2\}/\Lambda^2\right)}. \tag{4}$$

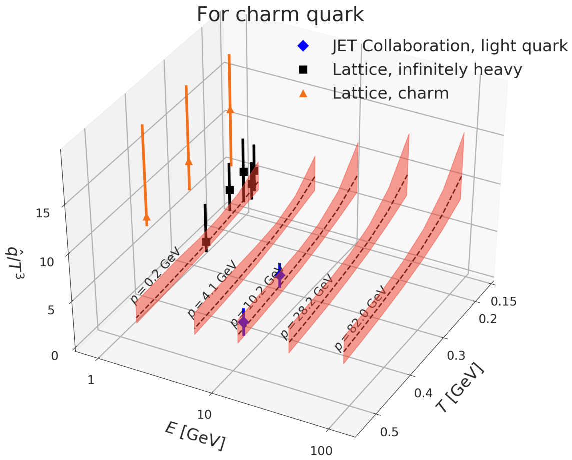

Figure 1: The T^3 scaled heavy quark transport parameter $\hat{q}/T^3$ extracted from global model-to-data comparison as a function of temperature at various heavy quark momentum. The red bands denote 90% credible limits of the extraction. They are compared to JET Collaboration extraction for light quark [14] and two lattice calculations from [16] (orange) and [15] (black).

For a hard enough scale $q > \mu\pi T$, this is the same as the usual running α_s for $N_f = 3$. For smaller q, α_s takes the value at a medium scale $\mu\pi T$, where μ is the tunable parameter. To parametrize possible deviations from the leading-order weakly-coupled formula and to include mass corrections at low momentum, we include an extra momentum diffusion strength $\Delta\hat{q}$ [1],

$$\Delta\hat{q} = \frac{KT^3}{[1 + (aT/T_c)^p]\,[1 + (bE/T)^q]}, \tag{5}$$

with five additional parameters (K, a, b, p, q). One can explore a class of $\Delta\hat{q}$ as a function of heavy quark energy and temperature. As a remark, at low momentum, the combination of E/T in this ansatz goes to M/T and the correction becomes mass-dependent for different heavy flavors. Including all the contributions, the final heavy quark transport parameter in the model is

$$\hat{q} = \hat{q}_{\mathcal{D}} + \hat{q}_{C} + \Delta\hat{q}. \tag{6}$$

3. Results

We include open heavy-flavor data taken by CMS and ALICE Collaborations at the LHC [7, 8, 9, 10, 11] and by STAR Collaboration at the RHIC [12, 13]. In figure 1, we show the extracted probability density (posterior distribution) of $\hat{q}(E, T)$ for the charm quark. We find that $\hat{q}$ increases slowly with heavy quark energy and has an increase when the temperature is decreasing toward the pseudo-critical temperature. At $E = 10$ GeV, where the charm mass effect is small for elastic processes, the current results agree with the JET Collaboration extraction of the light quark transport coefficient [14] within uncertainty. At $p = 0$, we compare the extracted charm quark momentum diffusion coefficient ($\kappa = \hat{q}/2$) to various lattice QCD calculations. One should be careful when comparing the phenomenological extraction to the lattice calculations. First, different lattice approaches give quite different results. The extracted κ is comparable to the lattice calculation for infinite heavy quark limit [15], but is significantly below the other lattice evaluation from the charmonia spectral function [16]. Second, we make a distinction between elastic interaction and inelastic interaction in the transport model, and only elastic processes contribute to $\hat{q}$ in equation 6.

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[1] Such deviations can be a result of higher-order contributions or non-perturbative physics, but we will not make a physical interpretation of $\Delta\hat{q}$ for the result from this phenomenological extraction

This artificial separation of different types of interaction certainty introduces ambiguity when comparing to results from the first-principle approach. Future improvement is to “measure” the effective spatial / momentum diffusion directly from transport model simulations with both elastic and inelastic interactions, instead of employing equation 6.

4. Summary

We extracted the momentum and temperature dependent heavy quark transport parameter $\hat{q}$ using both the RHIC and the LHC open heavy flavor data. For this purpose, we model the heavy-flavor in-medium dynamics in a wide momentum range using the LIDO transport model. It combines heavy-flavor diffusion dynamics, perturbative large-angle scatterings processes, and medium-induced gluon radiations. The resultant $\hat{q}$ is comparable to the previous estimation of light quark transport parameters at finite momentum. We also comment on the possible ambiguity when comparing this result directly to first principle lattice calculations at zero momentum.

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