Bayesian Updating: A Framework for Understanding Medical Decision Making

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Abstract

Beliefs are a fundamental component of our daily decisions, and as such, beliefs about our health have a huge impact on our health behaviors. Poor medication adherence is a well-documented problem and while it has been extensively researched, it has yet to be addressed using a Bayesian framework. This study aims to use a mixture model to understand belief updating as it affects decision making. Using an established experimental paradigm in categorical perception, we test memory and prediction in order to establish a model that can explain human belief updating. Results indicate that a mixture model provides a good explanation of participant behavior in this paradigm.

Keywords: Belief Updating; Decision making; Bayesian Models.

Introduction

Our beliefs influence the way we navigate the world at all times. If these beliefs are misaligned with reality, our behaviors will be maladaptive. Some of our most important beliefs are those about our health; indeed, medication adherence, which is a significant hurdle for the public health community, is thought to be affected by factors associated with patient beliefs (Horne & Weinman, 1999). Doctors continuously struggle with why patients do not follow their medication schedules despite the negative consequences.

According to the American Heart Association, within seven days of a heart attack, 24% of patients will not have filled their prescription. Of those who do begin their medication regimen, 34% will have stopped one prescribed medication and 12% will have stopped all three prescribed medications within a single month. Lastly, 60% of patients will be completely non-adherent to their prescribed medication after two years (Ho, Bryson, & Rumsfeld, 2009). The consequences of non-adherence in this case are life threatening; patients who do not fill any of their prescriptions have an 80% higher mortality rate within 120 days than those who do take their medication as instructed (Baroletti and Dell'Orfano, 2013).

Many investigations have proposed explanations for patients' failure to take their medication. Patients have cited forgetfulness (30%), other priorities (16%), lack of information, (9%) and emotional factors (7%) as reasons for failing to take their medication as prescribed (Ostenberg & Blaschke, 2005). The non-adherence of patients has been suggested to fit into six distinct categories: those who (1) do not understand how adherence relates to their well-being; (2) believe the costs of the medication outweigh the benefits; (3) find the regimen too complex; (4) are not vigilant; (5) hold inaccurate beliefs about medications; and (6) do not perceive the medication to be effective (Marcum, Sevick & Handler, 2013). Yet, no clear understanding of these underlying components exists, and physicians have argued that a model is necessary in order to tackle such a complex problem (Vermeire, Hearnshaw, Royen, & Denekens, 2001).

While such previous research suggests that people make medication choices irrationally, Bayesian models of cognition (a.k.a. rational models) present another way to understand medical decision making. In light of this, we propose a rational model that accounts for and makes predictions about why people would behave in this way.

The rational approach suggests that improving medication adherence is partly a problem of whether people are updating their beliefs about themselves optimally. If someone believes that they are a generally well person, they will have to update their beliefs in the face of evidence suggesting otherwise (e.g. a heart attack). After a heart attack, there may be necessary lifestyle changes (e.g. medication, exercise, and dieting), but without updating their beliefs, people may fail to make the necessary changes.

Assuming that people do Bayesian inference in their head, Bayes' rule gives a principled account of how people should update their beliefs in light of new evidence:

$$p(B|E) \propto p(E|B)p(B) \qquad \text{Eq (1)}$$

The posterior probability p(B|E) gives the probability of the belief B given the observed evidence E. This posterior probability is based on a combination of p(B), which is the prior probability (or strength) of the belief, and p(E|B), which is the likelihood of observing the current evidence given the belief. After observing the evidence E, p(B|E) becomes the new belief p(B) in the next iterative time step. This approach is useful because it characterizes the computational problem people face when trying to make sense of the world given evidence with varying degrees of uncertainty.

Bayes' rule predicts a tradeoff between prior beliefs and observed evidence. When our prior belief is strong and we encounter weak evidence, our new belief will closely reflect the prior. Conversely, in situations with a weak prior belief and strong evidence, the new belief will closely reflect the evidence. In situations where both the evidence and prior

belief are strong, the new belief will lie somewhere in between. It is, however, unclear what constitutes either a strong prior belief or strong evidence in human updating of beliefs from real-world experiences.

The way evidence from a negative health event is integrated into our belief system is also important. Information has been shown to decay at different rates based on the source (Yang, Mohan, Mehrotra & Varshney, 2002). Therefore, information from the body is treated differently than information communicated from a doctor. If a person is informed of a problem but cannot feel it, they may integrate this information differently or the information might decay more rapidly. Some information will not decay until new evidence is present, and some will decay at an exponential rate after it is presented. This decay function modifies a traditionally static belief-updating model ($P(B|E)$) to a time-dependent conditional probability such that $P(B{:}t_B|E{:}t_E)$ (Yang et al., 2002).

In a standard belief-updating model, the prior distribution for a given belief should be combined with the data from the environment to create a posterior distribution. Afterwards, the posterior distribution should become the new belief. However, human behavior suggests that beliefs are not updated as efficiently as this model would suggest; for example, after a negative health event, people appear to maintain the belief that they are healthy (i.e. there is a lag in updating of beliefs). Lag in updating occurs when evidence accumulates before an update is prompted. In order for the belief system to update, there must be a sufficient amount of evidence, which may take several trials to accumulate (Sanborn, Griffiths, & Navarro, 2010). Even after the belief is updated, people appear to slowly return to the belief that they are healthy, which can be considered a regression back to a baseline belief. This occurs when beliefs are initially updated after evidence, but slowly return to the initial prior.

These factors might be explained by the inclusion of multiple prior distributions. Rather than just one belief that updates to include new evidence, there might be two or more: one that remains relatively static and others that update with new evidence. This suggests that people make choices based on a mixture of these two distributions.

In the case of medical decision making, people may have two prior distributions: one baseline prior and one updated prior that integrates negative health information (e.g. a heart attack). This baseline prior likely integrates information about our wellness throughout our lives—both when we were sick and when we were not—which may leave us with a general impression that illnesses tend to come and go. When deciding whether to take medication, people are making a prediction about their future wellness, and to do this they must choose from which prior—their baseline prior or their updated prior—to sample. This choice may be based on the relative strength of the priors; while the new prior may be relevant at the moment, the baseline prior may be used for making predictions about their health. When choosing a distribution for prediction, they will likely choose the one with more accumulated evidence, which is often the baseline prior. If people sample from the baseline prior, they might be less likely to take their medication because they believe that this illness, like previous ailments, is likely to come and go.

The existence of multiple distributions is further supported by evidence in the animal literature on spontaneous recovery. Spontaneous recovery occurs in rats when they are trained on a reward schedule for a long period of time, then trained on a new reward schedule, and after a break, regress to the initial schedule (Gleitman, 1971). If they are reminded of the new schedule after this recovery, they will switch to it much more rapidly than during initial training. Memory theorists have argued that rats are storing two separate distributions: one for reinforcement and one for non-reinforcement, and that these two distributions compete (e.g. Gleitman, 1971).

In the current study, we investigate how the strength and duration of evidence influences belief updating, and explore possible factors affecting optimal belief updating and future prediction, including lag and regression to baseline beliefs. We model this data within a simple rational model that assumes that memory and prediction are integrated with beliefs about the underlying environment. To this end, we have constructed an experimental paradigm to simulate real world belief updating. The paradigm is based on previous research on categorical perception (Huttenlocher, Hedges, and Duncan, 1991) in which the authors found a regression to the mean effect in recall, as predicted by Bayes rule. This effect suggests a trade-off between memory and environmental evidence.

Experiment

It is not ideal to manipulate people's beliefs about their health in the real world; therefore, in this experiment we used an artificial paradigm that manipulates the same components. The experiment tested the model's predictions of how variation in prior beliefs should influence stimulus estimation. In this task, participants were asked to record the location of a dot in a circle and reconstruct that location from memory. Dots were presented inside a circle one at a time in clusters at different locations around the circle, and participants were asked to remember where they saw each dot. The task was chosen because it has been demonstrated that participants are able to learn the underlying distribution of dots, thereby making it an effective tool for studying belief updating (Huttenlocher et al., 1991). Our task extends the Huttenlocher paradigm by including prediction in addition to recall. These prediction trials were meant to assess participants' beliefs about the future. We predicted a regression to the mean effect similar to that of Huttenlocher et al. (1991) and a prediction bias such that predictions further in the future will be biased further away from the current cluster mean and toward the overall distribution.

Method

Participants were recruited from Rutgers University, New Brunswick. There were eight participants in this study and they were compensated with $10 for their participation, which lasted approximately 30 minutes.

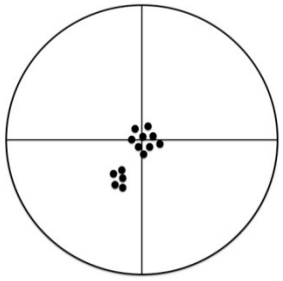

Figure 1: This circle illustrates the stimuli presented to participants. The dots represent two clusters, with the centermost cluster illustrating baseline training. Axes were not visible during the study.

Procedure In this experiment, participants were asked to record the location of a dot presented in a circle (see Figure 1) and reconstruct that location from memory. Participants were given a cover story in order to keep the task engaging; they were told that the circle was a garden and the dots were moles. In order to save their garden, they had to "catch" the moles by clicking on the locations where they saw them.

Baseline was established over a training period by presenting sequences of dots near the center of the circle. Belief updating was measured by presenting a single dot or sequence of dots either close to or far from baseline. Three manipulations on dot location and distribution were used to assess updating in this task: variance, location (consisting of a distance and angle measure), and number of trials. The distances and angle measures were informed by Huttenlocher et al. (1991). There were 24 angle measures including the axes, and the measures consisted of the same relative angles in each quadrant. Four different distances measuring out from the center of the circle to the circumference were chosen, and represented in each quadrant. Dots were presented in blocks (3, 6, 9, or 12 presentations in a cluster), sampled from a multinomial normal distribution with a mean of a given radius and one of three variances (0.01, 0.04, and 0.06 in a unit circle) chosen respectively to represent weak, average, and strong evidences. All presentations at baseline were at the smallest variance in order to strengthen baseline evidence. Each of the relative angles had a different distance, variance, and number of trials. The trial order was randomized, starting with 20 dots presented at baseline.

Each dot was viewed for one second followed by a combined visual mask and distractor task designed to remove the dot from participants' visual field and introduce uncertainty in the memory process. This mask consisted of a grid of black and white squares; after this mask was removed, an "X" appeared on the screen and participants were asked to report the color of the square (black or white) previously in that location. Data from the distractor task was recorded but not analyzed. After the completion of the distractor task, participants were asked to reconstruct the location of the dot from memory by clicking a spot in the circle.

After every three trials, participants were asked to make a prediction about a future dot location. Prediction trials alternated between prediction for the next trial and prediction for five trials from now. Each block (defined as a cluster of trials at one mean) was followed by a prediction for the expected dot location 10 trials from the current trial. This resulted in a total of 280 trials: 80 prediction trials and 200 recall trials.

Results

Recall

Figure 2, panel a, shows the mean radial bias (recalled minus studied dot location) as a function of the five radius locations. The dashed black line shows the regression of radial bias on study radius averaged across time steps in a cluster. As expected, participant responses regressed towards the mean radius such that dots studied at smaller distances from the circle center were overestimated while dots studied at the larger distances from the circle center were underestimated. This replicates the findings of Huttenlocher et al. (1991).

Figure 2, panel a, also shows the mean bias for each of four time steps within a cluster—at trial 1, 3, 6, and 9. This suggests that bias is reduced as the strength of evidence in a cluster increases; that is, after only one trial in a given cluster there was a greater regression to the overall mean, whereas after nine trials in a cluster there was significantly less regression to the overall mean location and accuracy was closer to the true mean of the current cluster. This suggests that participants were learning the underlying distribution of the dot cluster. Initial regression towards the overall mean also appeared to be less for cluster locations below the overall mean. This might be a result of the initial baseline training. In addition, regression lines were fit to recall performance at each of the time steps (see Table 1).

Table 1. *Recall bias as a function of study radius and trial number within a dot cluster*

	One Trial	Three Trials	Six Trials	Nine Trials
Slope	-.15	-.11	-.076	-.054
Intercept	.07	.06	.04	.03

Prediction

Figure 2, panel b, shows prediction bias relative to the mean of the current cluster as a function of cluster length and prediction type. For predictions of the dot location on the next trial, participants showed little bias and predicted the future dot location to be close to the mean of the current cluster with some initial underestimation. Predictions for locations five trials in the future also fell close to the current cluster mean, but with some initial overestimation. Predictions for locations ten trials in the future are the most interesting because they give some indication that subjects might hold multiple beliefs about the stimulus environment—one for the current cluster and one for the overall dot distribution. After studying a twelve-dot cluster, participants appeared to make predictions away from the mean of the current cluster; specifically, they predicted the future dot location to be closer to the circle center, with a systematic 10% underestimation. This suggests that predictions for the next dot location were drawn from the belief about the current state of the environment as quantified by the mean location of the current cluster. For predictions of dot locations further in the future, participants no longer appeared to use their belief about the current dot environment; rather, they appeared to be using a different belief based on the overall stimulus distribution and biased to the circle center. This suggests that the baseline manipulation influenced the direction of bias in the overall distribution.

Figure 2, panel c, shows prediction bias relative to the mean of the current cluster as a function of study radius and prediction type. Again, prediction for the next trial showed less overall bias, prediction for 5 trials from now showed incrementally more bias, and prediction for 10 trials from now showed a strong regression to the mean of the overall stimulus distribution. Predictions while studying dot locations at a .7 and .9 radius resulted in a greater regression effect, with predictions at .9 showing a systematic 25% underestimation relative to the current cluster mean.

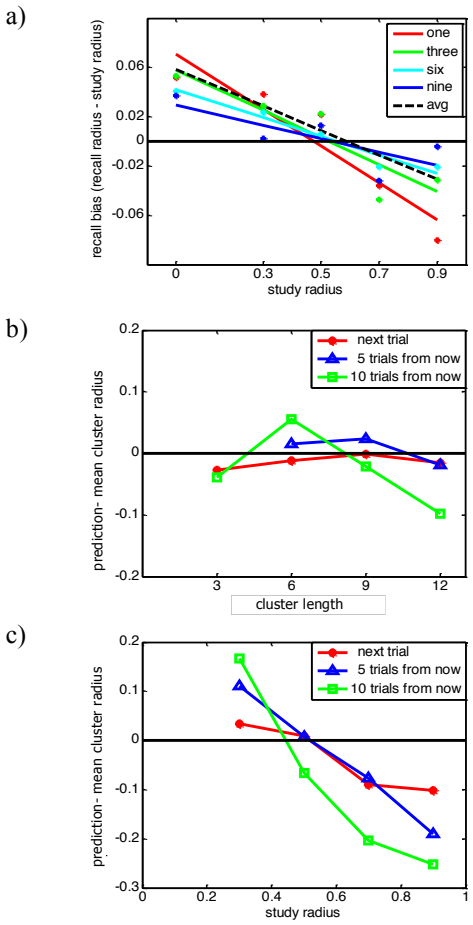

Figure 2: Panel a shows recall bias as a function of study radius and trial number within a dot cluster (either trial 1, 3, 6, or 9). Panel b shows prediction bias relative to the mean of the current dot cluster as a function of cluster length and prediction trial. Panel c shows prediction bias relative to the mean of the current cluster as a function of study radius and prediction type.

Discussion

The results suggest that people incrementally update their beliefs and that this incremental updating may be based on multiple prior distributions—one about the overall distribution, and one about the local cluster. As time progresses, it seems participants assign progressively more weight to the evidence from the local cluster. This evidence motivates our use of a Bayesian framework, which should give an accurate representation of the data.

In terms of health beliefs, these findings suggest that when deciding whether or not to take medication, people might be sampling from one of two distributions: a distribution with the recent evidence (e.g. a negative health event), and the overall distribution (e.g. their wellness over their lifetime). For predictions in the near future, they may believe they will still be sick, but for long-term predictions they may not believe that they will still have the ailment.

This reliance on the overall distribution for future predictions may be based on the strength of their beliefs. If their baseline prior is stronger (i.e. has accumulated more evidence), it makes sense that people would sample from it when trying to predict their future health. If the prior based on the more recent evidence were to become stronger than the baseline prior, it is possible that the former would replace the latter and be used for future predictions. Based on these findings, we model the relationship between the environment, recall, and prediction using a rational model. Furthermore, because it appears that participants are using multiple prior distributions, we utilize a mixture model to simulate this relationship.

Modeling

The strength of rational models is that they can be used to characterize the computational problems people face when trying to make sense of the world given the sparse and noisy input from the senses. For example, an observer in our experimental task is faced with recalling features θ of a stimulus presented at study (i.e. locations of dots). Based on our experimental design we will assume that these features are Gaussian distributed, $\theta \sim N(\mu, \tau)$, where μ and τ are the prior mean and precision of the dot locations. When a specific dot location θ is studied, we assume this leads to noisy representations y in episodic memory, where $y \sim N(\theta, \psi)$. That is, the memory representation is centered on the original studied dot location and is stored with some noise ψ, where ψ expresses the resemblance of the stored representation to the studied location. The goal of the observer on a test trial is to recall the studied dot location θ as best as possible using noisy samples y retrieved from memory. Extending Equation 1 to the memory task, the inference problem for the observer is $p(\theta | y, \psi, \mu, \tau)$. The posterior probability $p(\theta | y, \psi, \mu, \tau)$ describes how likely dot locations θ are given the noisy memory contents y and prior beliefs about the dot locations. We assume that the observer has a prior belief that corresponds to the observed stimulus distribution in the experiment (i.e., the environmental statistics). Furthermore, experimental results suggest that observers hold multiple beliefs about the environment: one for the local cluster, and one for the overall stimulus distribution. This can be modeled with a mixture model where the mean and precision (μ, τ) of beliefs are a combination of overall and cluster specific beliefs, $\mu = z\mu_c + (1-z)\mu_o$, and $\tau = z\tau_c + (1-z)\tau_o$, where (μ_c, τ_c) represents the belief associated with cluster i and (μ_o, τ_o) represents the overall belief about the stimulus distribution (Hemmer & Steyvers, 2009). The variable z weights the contribution of the cluster belief relative to the overall belief. This weighting is determined by $z \sim Bernouli(X_i)$, where χ_i is a constant that represents the familiarity of a cluster. In this way, familiar clusters lead to a belief that is more dependent on the cluster rather than the overall distribution. This implements the intuitive notion that for unfamiliar clusters it is unlikely that the cluster belief is reliable and inference instead reverts to a higher-level belief based on the overall stimulus distribution.

Standard Bayesian techniques (Gelman, Carlin, Stern & Rubin, 2003) can be used to calculate the mean of the posterior distribution:

$$\hat{\theta} = w\mu + (1-w)\bar{y} \qquad \text{Eq (2)}$$

where $w = (1/\sigma_0^2)/[(1/\sigma_0^2)+(n/\sigma_m^2)]$ and n is the number of samples taken from episodic memory. In this way, recall can be modeled as a weighted linear combination of beliefs and memory content, where the strength of the prior belief and episodic memory trades off as described in the introduction.

Here, the rational analysis is applied to the experiment without directly estimating any parameters. Instead, it is assumed that the observer has a belief that corresponds to a mixture of the local and overall environmental statistics. On the first time-step, the observer is assumed to have a belief about the mean stimulus location that is biased towards the overall distribution, but by the 9th time step to have a belief that is identical to the local cluster's environmental distribution (only time steps 1, 3, 6 and 9 are simulated here). Therefore, χ (i.e., familiarity with the local cluster) was set to 0.6, 0.7, 0.8, and 1 for each of the four successive times in a cluster modeled here to simulate an increasing level of familiarity with the cluster environment. The mean of the overall distribution μ_o was set to 0.5 for all radius locations, except radius 0, where μ_o was set to 0.35 to reflect the baseline training. The precision for the overall distribution τ_o was set to the exact precision in the total set of observed data, except for radius 0 which was always observed at the smallest variance manipulation. The means

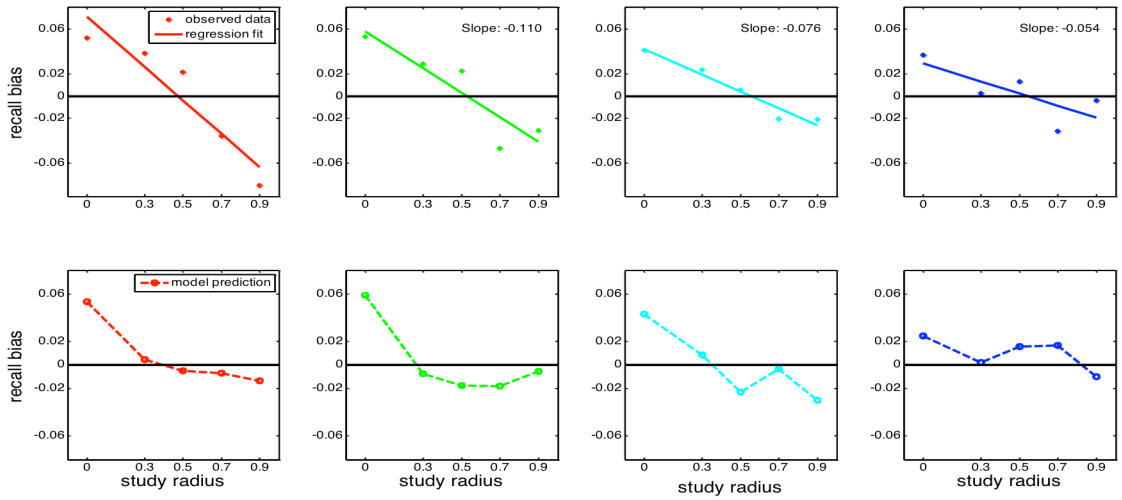

Figure 3: The graphs above show recall bias as a function of study radius at one, three, six, and nine time-steps, with the top row showing participant performance and the bottom row showing model predictions.

for the local clusters μ_c was set to the true radius locations, and the precision τ_c was set to the mean precision for clusters at each radius. Finally, the memory noise ψ was set to 0.11.

The goal of this analysis is to compare the predictions of the rational model and the empirical data at a qualitative level. As seen in Figure 3, the model produces results that are qualitatively consistent with the responses given by human observers. It appears as though participants incrementally change their beliefs with an increasing number of time-steps within a cluster. Results show that memory estimation errors can be explained by the use of beliefs about the environment, since smaller radius distances from the circle center were later recalled to be further away and larger radius distances were later recalled to be closer. Furthermore, beliefs change as a function of increasing familiarity with the underlying local environment.

The observer in our task was also asked to make predictions about future stimulus locations. The posterior predictive distribution of future dot locations $p(\theta_{future}|\ \theta)$ is determined by averaging the predictive probability across all possible values of beliefs weighted by the strength of the belief. The mean of the posterior predictive distribution can then be shown to be equal to the prior mean, with the variance drawn from both the variance of the observed stimulus and the uncertainty in the current belief. Therefore, prediction for a future stimulus is centered on the mean of the current belief. Yet, as demonstrated in the experimental results, this only holds for short-term predictions; for long-term predictions, people appear to use a mixture of the current belief about the cluster and the overall belief, similar to that of recall. It is now trivial to extend the rational model to assume long-term predictions to be a mixture of belief, but that will be outside of the scope of the current paper.

General Discussion

A rational model that makes predictions about belief updating based on different types of evidence has important applications for public policy, and could help tailor treatment strategies for patients encountering new illnesses like cardiovascular disease. By understanding which methods of updating beliefs are the most effective, doctors might learn how to help patients integrate their illnesses into their belief system. The results indicate that people hold multiple beliefs that simultaneously effect decision making: one about their current environment and one about the overall environment.

The results above provide preliminary support for the existence of multiple distributions in belief updating. It was found that bias decreased as time steps in a cluster increased, and that a mixture model provided a good explanation of this pattern. The inclusion of multiple prior distributions was further supported by results from the prediction trials. For predictions for one trial in the future, participants appeared to sample from the current cluster mean, while for predictions for ten trials in the future they appeared to sample from the overall distribution. This provides support for the initial hypothesis that people sample from multiple prior distributions when making future predictions. While this experimental paradigm was successful in simulating a real world belief-updating scenario, in order to further our understanding of how people make choices about their health this investigation should be expanded into more realistic environments.

Acknowledgements

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