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SURVEY PAPER

Designing and enacting instruction that enhances language for mathematics learning: a review of the state of development and research

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Abstract
After four decades of research and development on language in mathematics classrooms, there is consensus that enhancing language is crucial for promoting students' mathematics learning. After briefly sketching the theoretical contexts for work on this topic, in this paper we present six design principles for instruction that enhances language for mathematics learning. We then review the research that provides an empirical foundation for these principles, (a) concerning the design of learning environments to enhance language for mathematics learning and (b) on teaching practices (including teacher moves and classroom norms) involved in the enactment of those designed learning environments. Without claiming completeness, this review of the state of development and research shows that some aspects of design and instruction that enhance language for mathematics learning have been well researched, whereas research gaps for other aspects persist.

Keywords Design research · Enhancing language · Design principles · Teaching practices

1 Introduction

The role of language in mathematics learning has been the focus of research in mathematics education for over four decades (e.g., Austin and Howson 1979; Ellerton and Clarkson 1996; Pimm 1987). In conjunction with that research, researchers and authors of curriculum documents have called for instructional approaches that include, address, and support language in mathematics classrooms (e.g., Pimm 1987; Adler 2001; Ellerton and Clarkson 1996) as a means for enhancing mathematics learning. However, there is still a lack of empirical foundations for instructional approaches (Moschkovich 2010b; Barwell et al. 2016), for teaching practices that establish equitable classroom cultures for mathematics learning through language practices (Herbel-Eisenmann, Choppin, Wagner, and Pimm 2011), and for enhancing language for mathematics learning (Pimm 1987; Ellerton and Clarkson 1996). These are the research gaps we address in the current ZDM special issue.

In this introductory paper, we begin by outlining the theoretical contexts in which the research summary developed (Sect. 2). Next, we summarize the state of research in two areas, namely, principles for the design of learning environments (Sect. 3), and teaching practices in their enactment (Sect. 4). Rather than providing an exhaustive review of the research literature, we make explicit the criteria for selecting publications on approaches to enhancing language for mathematics learning in Sects. 3 and 4, separately. Section 5 summarizes the limitations of the current research base and recommendations for areas in which we see the field as needing to go next.

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2 Theoretical background: role of language and interaction for mathematics learning

The role of language in mathematics teaching and learning has been widely researched, as summarized in four surveys, the ICMI Study on Mathematics Education and Language Diversity (Barwell et al. 2016), the ERME survey paper, the PME survey paper on language in mathematics education research (Planas, Morgan, and Schütte 2018; Radford and Barwell 2016), and the ZDM issue on theoretical approaches for research on language in mathematics education (Planas and Schütte 2018). We reference these insightful surveys mainly to position the perspective of the current survey as building on those and using a specific focus on instruction that enhances language for mathematics learning.

Assumptions about the role of language in mathematics learning depend on the adopted theoretical perspective. This survey paper is rooted in a sociocultural perspective (Moschkovich 2015a, b), based on a thorough discussion and comparison of different theoretical approaches for describing the interplay between mathematics learning and *language as a means and resource* for mathematics learning (Lambert and Cobb 2003; Planas et al. 2018; Radford and Barwell 2016). The sociocultural perspective draws on situated views of 'mathematics learning' as a discursive activity that involves participating in a community of practice (Moschkovich 2015a, b). Learning mathematics thus involves not only mathematical knowledge, but also mathematical practices and discourse.

'Language' is also conceptualized in different ways (Planas et al. 2018; Snow and Uccelli 2009) spanning a range of perspectives, from a limited focus on the *lexical dimension* (vocabulary or word meanings), through the *syntactic dimension* (grammar of sentences and word flexions), to a perspective including also the *discursive dimension*. The discursive dimension addresses, amongst other aspects, interactional patterns and routines, and in particular discourse practices such as explaining, justifying, arguing, etc. (Schleppegrell 2007; Moschkovich 2015a; Erath, Prediger, Quasthoff and Heller 2018). Most linguists and mathematics education researchers working on language in mathematics education adopt a functional perspective of language, considering lexical and syntactical features not as ends in themselves but as means for realizing discourse practices (Schleppegrell 2007; Snow and Uccelli 2009). From the mathematics education perspective of this paper, we focus on those discourse practices, lexical and syntactic features, that are *means* to communicate, think and learn mathematical topics. This perspective includes discourse practices typical of academic activity across subjects, such as arguing, as well as topic-specific discourse practices such as describing the generality of an algebraic pattern or explaining the meaning of a specific mathematical concept (Moschkovich 2015a; Erath et al. 2018).

The increasing research focus on equity and access for all learners (Secada 1992; DIME 2007; Moschkovich 2010a; Herbel-Eisenmann et al. 2011) has shed light on the unequally distributed access to quality mathematics instruction, in particular to instruction that supports, develops, or enhances language so that all students can learn mathematics through participation in activities that involve using language. Students' familiarity with the classroom language, teachers' preparation for teaching mathematics through language, and teachers' preparation for including language learning, can all constrain students' opportunities to learn mathematics in classrooms (Schleppegrell 2007; Snow and Uccelli 2009). Thus, language is crucial for improving opportunities for students to learn mathematics. A classroom culture and socio-mathematical norms that support the participation of all students in rich classroom discourse practices (DIME 2007; Herbel-Eisenmann et al. 2011; Ingram, Andrews, and Pitt 2019), such as explaining and arguing, can positively impact all students' mathematics learning.

This survey summarizes the state of development and research on *instruction that enhances language for mathematics learning*. We use this phrase to mean instruction *through* language, but also *of* language, more precisely of those discourse practices (and lexical and syntactical means for participating in them) necessary for learning mathematics. As the next section shows, 'enhancing' includes eliciting and exploiting students' resources (across different languages, language varieties and multimodal representations: see Barwell 2018; Moschkovich 2015a; Prediger and Wessel 2013), engaging students in rich discourse practices, supporting the students' language productions, and developing their repertoires. Following Cohen, Raudenbush and Ball (2003) we see instruction as the interplay of designed learning environments (e.g., in curriculum resources and tasks) and the teaching practices by which teachers enact the instruction in the classroom interaction.

Many papers on language and learning mathematics have adopted *descriptive modes*, describing and documenting (sometimes in great detail) how students draw on language in mathematics (see Planas et al. 2018; Barwell et al. 2016 for an overview in multilingual classrooms). Some of these findings can also inform general principles for designing materials and instruction. Other studies have adopted a *prescriptive mode* and explored particular approaches for enhancing students' language for learning mathematics with respect to increasing participation, developing language or positively impacting mathematics achievement. In this paper, we summarize findings using these multiple modes, in order to provide a starting point for further work in this area.

3 Design principles for instructional approaches and their research base

Researchers have proposed, suggested, or implied design principles for instructional approaches that enhance language for mathematics learning in manifold ways, and have supported these principles with different research bases. To provide an overview of the research landscape, in Sect. 3.1 we introduce six major design principles for language-responsive mathematics materials and instruction that we have drawn from existing work. In Sect. 3.2, we then present some studies which have strengthened the empirical research base for these design principles.

For Sect. 3.1, we focused on papers that included, either implicitly or explicitly, principles or recommendations for designing instruction that enhances language and supports mathematics learning. Many of these papers are based on practical experience without empirical research. For Sect. 3.2, we restricted ourselves to the (far fewer) publications in mathematics education research that (a) provided *empirical* findings, through qualitative or quantitative data analysis, that help researchers to understand the teaching and learning processes intended in, or supported by, the instructional designs, or (b) identified conditions for successfully realizing the design principles, or (c) documented any effects of the instructional approach. In both sections, we included some papers from outside mathematics education (i.e., linguistics or language education research) when they contributed to the sociocultural perspective on language in mathematics learning, and approaches in which language and subject matter learning are assumed to support each other (Lee, Quinn, and Valdés 2013). Although those recommendations and empirical findings are also diverse (see e.g., the overview by Lee et al. 2013) and may not be specific to mathematics classrooms, this work from outside mathematics education is informed by the empirical findings on language acquisition *in general* that only such research can provide. For both sections, we selected only publications that follow two overarching orientations, as follows:

- *Amplify not simplify language* (Zwiers et al. 2017; Walqui and Bunch 2019; Schleppegrell 2007; Pimm 1987), since any approach that reduces language cannot provide the language learning opportunities required for enhancing the learning of both language and mathematics.
- *Enhance both at the same time, language and mathematics with understanding,* since any approach that addresses only one cannot enhance the language practices involved in learning and doing mathematics (Moschkovich 2010b).

3.1 Major design principles

We have extracted the following six major design principles for designing materials (tasks, lessons and units) and instruction (cf. Moschkovich 2013; Wessel and Erath 2018; Prediger and Neugebauer 2020) from the literature on enhancing language for learning mathematics. For language learning to be a catalyst for mathematics learning, materials and instruction should…:

(P1) … engage students in rich discourse practices,
(P2) … establish various mathematics language routines,
(P3) … connect language varieties and multimodal representations,
(P4) … include students' multilingual resources,
(P5) … use macro-scaffolding to sequence and combine language and mathematics learning opportunities, and,
(P6) … compare language pieces (form, function, etc.) to raise students' language awareness.

(P1) Engage students in rich discourse practices

The principle of engaging students in rich discourse practices includes a recommendation to design activities that model and provide opportunities for rich discourse practices, in multiple modes (listening, talking, reading, writing, etc.), in multiple communication settings (Herbel-Eisenmann et al. 2011), and with multiple audiences and genres useful for communicating about mathematics. These activities include any practices, language varieties, or genres that may be unfamiliar to the teacher.

Providing language learning opportunities in mathematics classrooms must not be restricted to teaching vocabulary (as in early approaches, e.g., in DfEE 2000; problematized, e.g., by Moschkovich 2002), but instead focus on enhancing rich discourse practices such as explaining meanings, constructing arguments and justifying procedures (Moschkovich 2015a). From a mathematics education perspective, this design principle resonates with one of the characteristics of teaching that supports conceptual understanding (Hiebert and Carpenter 1992). From a language learning perspective, it resonates with the principle of 'pushed input and pushed output' (Swain 1995; Meyer 2012), according to which language learning requires multiple, repeated, and extended opportunities for students to produce oral and written language, and to have access to rich language input from the teacher and/or their peers. Although the examples are from a traditional cognitivist orientation to second language acquisition, this perspective is not the only way to view second language development. There are many debates and alternative perspectives in the field, most prominently social approaches to language development (see Kibler et al. 2014; Gutiérrez et al. 2010).

However, our understanding of this principle also extends the idea of 'pushed output' by emphasizing the discursive quality of discourse practices for learning mathematics: explaining mathematical meaning or justifying procedures are richer discourse practices than reporting procedures because meaning matters for conceptual understanding (Moschkovich 2015a; Erath et al. 2018). Other researchers have emphasized that 'richness' needs to include (1) multiple modes (listening, talking, reading, writing, etc.), (2) multiple communication settings (Herbel-Eisenmann, Steele, and Cirillo 2013; Walqui 2006; Walqui and Bunch 2019; Zahner 2012), and (3) multiple audiences, language varieties, and genres (Gibbons 2002; de Araujo, Roberts, Willey, and Zahner 2018; Walqui and Bunch 2019).

On the design level, this principle can be accomplished through designing materials and instruction that support rich modes focused on mathematical concepts and are supported by materialized scaffolds, e.g., language frames (Gibbons 2002; Short 2017) and/or visuals (Driscoll, Nikula and DePiper 2016). At the teaching practices level, this principle is usually nurtured by teachers' continuous supportive moves during interactions with learners (Smit and van Eerde 2013; Ingram et al. 2019) and teacher moves for orchestrating discussions (Michaels and O'Connor 2015; Stein, Engle, Smith, and Hughes 2008), which are discussed in more detail in Sect. 4.

(P2) Establish various mathematics language routines

The principle of using various mathematics language routines suggests a recommendation that tasks, lessons, and units include mathematics language routines that provide support for language learning far beyond sentence frames or starters. Zwiers et al. (2017) define a mathematics language routine as "a structured but adaptable format for amplifying, assessing, and developing students' language" (p. 9). These routines can support self-, peer-, and teacher assessment, enable teacher feedback and provide opportunities for students to revise and refine their language production. In this way, mathematics language routines can help students to refine "not only the way they organize and communicate their own ideas, but also ask questions to clarify their understandings of others' ideas" (p. 9). Examples of these routines include "Convince yourself, a friend, a skeptic" (Mason, Burton, and Stacey 2010; Schoenfeld 1985), Number Talks (Humphreys and Parker 2015), and "Always, sometimes, never" (Swan 2006). An example spanning over several phases is a cycle of "pre-write, think, structured pair shares, listen in pairs, post-write" (Zwiers et al. 2017). Some of these routines originated in mathematics education, others in second language education, and some may resonate with the Commognition framework focus on routines (Sfard 2012). Zwiers et al. (2017) emphasize that language routines should always be applied in "meaningful and purposeful [ways], not inauthentic or simply answer-based" (p. 9).

(P3) Connect language varieties and multimodal representations

The principle of connecting language varieties (e.g., everyday, academic, technical language) and multimodal representations (symbolic, graphical, diagrammatic…) suggests designing activities that flexibly and deliberately relate these components (see Fig. 1).

This principle extends the classical principle of using multiple multimodal representations for enhancing conceptual understanding, as the symbolic representations require the graphical and concrete representations to be understood (Lesh 1979; Duval 2006). It is based on the general observation that students' understanding requires *making connections* to aspects they have already understood (Hiebert and Carpenter 1992). The same idea can be extended to different language varieties, as also formal technical language, and academic language needs to be connected to the language resources students bring into classrooms (Schleppegrell 2007).

Whereas early articulations of the principles started from sequencing once from concrete, through graphical, towards symbolic representations (Bruner 1967) or from everyday, through academic, towards technical language (Gibbons 2002), the actual principle emphasizes the need for relating registers and representations forwards and backwards, rather than sequencing through them once (see Fig. 1, adapted from Prediger and Wessel 2013; similar in Moschkovich 2013).

Additionally, the emphasis is not on *changing between*, but on *connecting* the language varieties and representations.

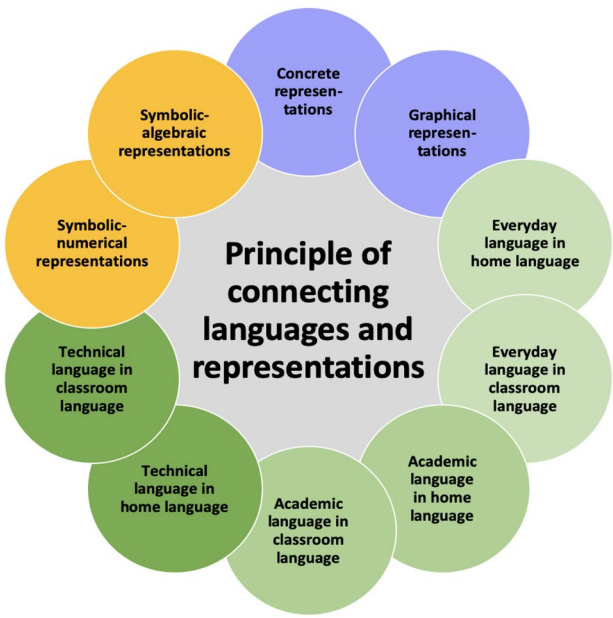

Fig. 1 Design principle of relating language varieties and representations as combination of P3 and P4 (adapted from Prediger and Wessel 2013)

This means that tasks (and teacher moves, see Sect. 4) require that students reason and articulate how different linguistic descriptions and representations are specifically related (Prediger and Wessel 2013). For example, students should not only translate the multiplication 5×3 into an array model with five rows of 3 points each, but also explain how to see the unitizing structure in the rows ('five threes' or 'five sets of three') in order to verbalize the meaning of multiplication as unitizing (Götze 2019).

(P4) Include students' multilingual resources

Principle of including students' multilingual resources extends principle P3 for multilingual students to the home language and classroom language (already included in Fig. 1). As many multilingual students attend schools in countries where the classroom language is not their home language, including students' home language allows instruction to exploit and extend student resources for mathematics learning (Adler 2001; Moschkovich 2002; Setati 2005; Barwell et al. 2016).

For example, several language practices such as code-switching or translanguaging (Moschkovich 2019; Schüler-Meyer, Prediger, Kuzu, Wessel, and Redder 2019), using cognates (such as rectangle in English and rectángulo in Spanish), and repeating an explanation in one's other language for politeness or deeper understanding, can support students in making meaning (Barwell 2018; Moschkovich 2019; Schüler-Meyer et al. 2019). Recent research hypothesized that indeed, these multilingual resources can be turned into sources for meaning making, under the condition that principle P4 is combined with P1, P5, and P6 (Barwell 2018; Schüler-Meyer et al. 2019).

(P5) Use macro-scaffolding to sequence and combine language and mathematics learning opportunities

In general, many different approaches are called scaffolding when they support learners to leverage their resources in succeeding zones of proximal development (Walqui 2006; Gibbons 2002). Whereas micro-scaffolding refers to in-the-moment support in the interaction, Gibbons (2002) suggested the term macro-scaffolding to refer to supports in lesson design and planning. The principle of macro-scaffolding suggests sequencing, connecting, and coordinating sequenced opportunities to focus on language and mathematics learning deliberately in teaching units, while tightly combining them (Gibbons 2002; Pöhler and Prediger 2015).

The principle is particularly important for conceptual understanding in mathematics and leading to instruction that starts from everyday experiences and leverages those towards more formal concepts. The approach of hypothetical learning trajectories sequences learning opportunities along hypothetical steps; with multiple supports for student progress, students reach specified learning goals along the intermediate steps (Simon 1995; Gravemeijer 1998). For example, in the Realistic Mathematics Education approach (Gravemeijer 1998), a mathematical trajectory starts from everyday problems and students construct meanings through guided emergent modelling. In parallel to these early ideas of sequencing mathematics learning opportunities, Gibbons (2002) suggests sequencing *language learning* starting from students' everyday language resources and successively developing student's language use towards academic and technical language (similarly, Pimm 1987; Adler and Ronda 2015). Walqui (2006) also emphasized the need to coordinate this sequencing with the subject matter learning.

Pöhler and Prediger (2015) suggested combining content and language learning trajectories with respect to the function of language on each conceptual level (Gravemeijer 1998): In the discursive dimension, explaining structures and meanings is the discourse practice required for the first levels, whereas the formal level requires reporting procedures. In the lexical dimension, the first levels start from students' everyday resources, but explaining meanings often requires collective explanations and therefore shared meaning-related vocabulary. This is later completed by technical vocabulary used for describing structures and reporting procedures. For example, if a language routine summarized as a cycle of 'pre-write, think, structured pair shares, listen in pairs, post write' (Zwiers et al. 2017) includes teacher and peer feedback, then it can support students shifting from using informal to more formal language.

(P6) Compare language pieces (form, function, etc.) to raise students' language awareness

The principle of comparing for raising awareness suggests that comparing or contrasting language pieces can enhance students' awareness of how these pieces vary across language varieties and function in various language practices for communicating mathematically.

Language awareness (García 2017; James and Garrett 1992) refers to "explicit knowledge about language, and conscious perception and sensitivity in language learning, language teaching and language use" (García 2017, p. 264); it is sometimes also called meta-linguistic awareness (Malakoff and Hakuta 1991; Bialystok and Barac 2012) and includes word consciousness.

Word consciousness can be enhanced (e.g., by looking at families of words such as equal, equation, equate, etc.) and discussing what kinds of words these are (Scott and Nagy 2004). Also, in mathematics classrooms, when pieces of language (e.g., two near concepts or two only slightly varied sentence structures) are compared and contrasted, students can become sensitized to the subtleties of language. In this idea, the principle resonates with the variation principle in mathematics (Pang et al. 2017; Marton and Pang 2006), in which students are required to compare varied pieces (e.g., tasks) in order to discern the essential features.

Dröse and Prediger (2020) have transferred that principle to comparing mathematical word problems for raising

students' *syntactic* awareness, e.g., 'He has 5 €' versus 'He has 5 € more' or 'she gives him 5 €' versus 'she is given 5 €', which lead to other choices of operations. By systematically comparing the syntactic forms, students can be sensitized to syntactic features.

Language awareness is not only about lexicon and syntax, it can also include discourse practices such as informal explanations or arguments in small groups that become more formal explanations. These use not only formal words and syntax, but also a more formal discourse structure. Again, when students compare different explanations or argumentations, their awareness for specificities of language in mathematics classrooms can be enhanced (García 2017).

3.2 Overview on empirical research base for six design principles

Although many authors state or use the design principles from Sect. 3.1, there is only limited research that provides an empirical base grounded in the topic-specific realization of the principles and empirical insights into typical effects of those learning environments. Without claiming completeness, we have selected 32 articles published in English medium refereed journals with explicit and methodologically rigorous research. More classroom observation studies that address P1 (engaging students in rich discourse practices) with a focus on teaching practices required for enacting the principle are considered separately in Sect. 4. Here, we included only studies relevant to the design of tasks, lessons, or units. Table 1 lists 24 reviewed articles on qualitative research that focus on language *for mathematics* learning, and Table 2 lists 10 articles on quantitative research (of which two have quantitative and qualitative parts). Both tables summarize the format of the research and the nature of the empirical evidence. While the qualitative studies usually focus on identifying *conditions of success* for realizing one or two design principles, the quantitative studies evaluate the *effectiveness* of different combinations of design principles.

Table 1 shows multiple research studies providing a qualitative research base for the six principles, 11 *non-interventionist* observation studies and 13 with *interventionist* formats such as design research, action research or learning process studies. Both research formats can reveal important insights: non-interventionist studies can identify necessary (typically absent) conditions, and interventionist studies can go more deeply into specifying conditions for the success of an instructional approach.

Even if there is not the space to discuss all of the findings listed in Table 1 separately, we see a common tendency in the identified conditions, namely that enacting one principle often depends on other principles. For example, engaging students in rich discourse practices requires providing a shared meaning-related language (P5 as a condition for P1). Another example is that including students' multilingual resources was mainly successful when the instruction systematically connected language varieties (P3 as a condition for P4). Although the studies in Table 1 are presented here as examples of single principles, their qualitative findings repeatedly point to the need to combine the principles when designing or enacting instruction. Additionally, nearly every study revealed that the instructional design can be successfully enacted only when the classroom norms and discourse resonate with the design principles. This led us to take a deeper look into research studies on teacher moves, classroom norms, and other aspects of teaching addressed in Sect. 4.

Besides these general insights from the studies in Table 1, the topic-specific research on various mathematical topics is crucial for specifying topic-specific language demands required to design macro-scaffolding (Moschkovich 2010b; Pöhler and Prediger 2015). Indeed, 19 out of 24 studies tackle a specific topic but elaborate the topic-specificities to different degrees. This is less crucial for P1, P2, P4, P6, but very important for P3 and P4 for the design of representations and meaning-related language supports for the mutual topic.

Table 2 provides an overview of eleven quantitative studies we could identify in mathematics education journals or collections that provide empirical evidence for the efficacy or effectiveness of a particular language-related instructional approach. Even if effect sizes must always be interpreted with caution and within the specific research context (Bakker, Cai, English, Kaiser, Mesa, and van Dooren 2019), the studies provide existence proofs that there *can be* considerable effect sizes for language-related instructional approaches. Studies situated in small group settings do not yet reach ecological validity for whole class teaching, as teachers' micro-scaffolding is much easier in these cases; thus the transferability of these findings should be studied in future research.

The first seven studies relate to the listed design principles enacted within topic-specific language-related mathematical interventions, and all report high effect sizes for the intervention group. Four other studies adopted another approach. While one of these, the subject-specific approach, was also effective for enhancing students' mathematics learning (Shilo and Kramarski 2019), two subject-independent approaches (Hagena, Leiß, and Schwippert 2017; Short 2017) did not reach significant effects for the treatment group. The last study (Ing et al. 2015) showed that student participation in rich discourse practices predicted student achievement, with teacher support for these practices predicting student participation, but not directly predicting student achievement.

Table 1 Overview of design research and qualitative studies on design principles

Design principle	Qualitative study with research format	Mathematical topic and age group	Identified conditions for realizing a design principle successfully
(P1) Engage in rich discourse practices	Moschkovich (1999): CO	Geometry (age 8–9)	• Discourse should focus on students' meaning-making with rich tasks
	Moschkovich (2002): CO	Functions and patterns (age 14–15)	• Some students may need discursive support as they participate (i.e. teacher revoicing, requests to clarify, etc.)
	Prediger & Wessel (2013): DR	Fractions (age 12–13)	• Mathematically *rich* activities. e.g., systematic variation or relating representations provide rich opportunities for explaining • Discourse practice of explaining meanings requires focused language support by teacher and material
	Michaels & O'Connor (2015): CO and PD	Multiple topics, (age 6–12)	• Support and orchestrate mathematical discussions focused on central mathematical concepts
(P2) Establish mathematics language routines	Moschkovich (2015b): CO	No specific topic, no specific grade	• Use cycles of activity structures with teacher feedback
	Mercer & Sams (2006): LP	Numeracy (age 9–10)	• For establishing productive language routines (not called as such by the authors), teachers' facilitation is crucial
	Smit et al. (2016): DR	Diagrams (age 10–11)	• Model writing of rich discourse practices by Genre Pedagogy Cycle • Include routinized feedback on writing
(P3) Connect languages and multimodal representations	Zahner et al. (2012): LP	Linear functions, (age 12–13)	• Develop meaning for technical language after use of informal language. Make strong connections between representations, not only switches (in multiple representation activities) • Allocate enough time for students to engage in thinking about representations and discuss problems using different language varieties
	Prediger & Wessel (2013): DR Götze (2019): DR	Fractions (age 12–13) Multiplication (age 7–9)	• Both studies show: • Instruction needs to connect rather than only switch between language varieties and representations • Language support is required for developing a collective meaning-related language
	Zolkower, Shreyar & Pérez (2015): CO	Algebra (age 11–12)	• Student understanding of multi-semiotic text can be enhanced by repetition, conjunction, demonstrative pronouns and pointing gestures
	Pöhler & Prediger (2015): DR	Percentage (age 12–13)	• Focused language support is required for establishing a collective meaning-related language
	Rønning & Strømskag (2017): DR	Polygons (age 7–8)	
	Prediger & Şahin-Gür (2020): DR	Qualitative calculus (age 16–17)	• Tasks and activity structures are required that initiate the collective striving for precision and conciseness, in order to unfold condensed mathematical language
(P4) Include students' multilingual resources	Adler (2001): CO Barwell (2018): CO	Various	• Consider not only named languages, but also informal & formal language varieties • Embed the use of named languages into use of all language varieties and representations and other sources of meaning making **(P3)**
	Setati (2005): CO	Multiplication (age 9–10)	• Focus on conceptual talk; combine with **P3** • Establish norms for equitably including non-dominant languages
	Planas & Setati-Phakeng (2014): CO	Various	See above
	Moschkovich (2019): CO	Geometry (age 8) Patterns (age 14)	• Encouraging and allowing students to code switch and translanguage can support mathematical reasoning
	Schüler-Meyer et al. (2019): DR	Fractions (age 12–13)	• Focus on conceptual talk; combine with **P3** • Bilingual connective mode (which goes beyond mutual compensation) is more beneficial for mathematics learning than only switching between languages

Table 1 (continued)

Design principle	Qualitative study with research format	Mathematical topic and age group	Identified conditions for realizing a design principle successfully
(P5) Use macro-scaffolding	Smit & van Eerde (2013; Smit et al. 2013): DR	Diagrams (age 10–11)	• Language routines must be combined with macro-scaffolding as planned sequencing, since meaning-making for more elaborate language builds upon elicited informal language resources
	Pöhler & Prediger (2015): DR	Percentage (age 12–13)	• Focused language support is necessary for establishing the meaning-related language, the language support should be sequenced in line with the content trajectory
(P6) Compare language pieces for raising language awareness	Adler & Ronda (2015): CO	Algebra (age 15–16)	• Careful selection of examples to be compared can support student discourse on differences and similarities of mathematical and • language-related structures
	Dröse & Prediger (2020): DR	Subtraction word problems (age 10–11)	• Teacher initiation of reflections on text variation can be used to raise student language awareness • Structural scaffold can support the comparison of syntactic structures
	Scott & Nagy (2004): AR	Generic outside math (age 10–11)	• Comparing concepts in the same semantic field can support word consciousness which then contributes to vocabulary acquisition (finding is not specific to mathematics but generalizable)

DR = design research, i.e., deliberate iterative instructional design with qualitative study of design experiments and theorizing; AR = action research with some formative assessment; LP = interventionist learning process study with small groups and a given instruction; CO = non-interventionist whole class observation study without influence on design

Although the quantitative studies are too few to derive any stable and reliable pattern, we observe that *topic-specific* interventions tended to be more effective for affecting mathematics learning than *topic-independent* interventions. We see a need for more quantitative research studies and more topic-specific interventions in order to validate that hypothesized pattern.

4 Productive teaching practices and their research base

While in Sect. 3 we presented principles for designing materials and instruction, in this section we consider examples of productive, enacted teaching practices for enhancing language for learning mathematics through interaction, with a particular focus on teacher moves and practices (Sect. 4.1) (summarized as teacher moves) and classroom norms (Sect. 4.2). Although Sects. 3 and 4 are related, the different kinds of research bases require different structures in subsections as they cover two different research areas. The first goes beyond the designs arising from the principles outlined in Sect. 3 to consider the enactment of these principles in classroom interactions. The second offers insight into the contexts in which the design principles are sustained over time and embedded within teaching practices. For these two sections, we selected articles mainly in mathematics education journals that considered teacher moves, and document how they support the enactment of language-responsive instruction. Articles were included only when they provide *empirical* findings, through qualitative or quantitative data analysis, that give us further insight into the roles that teacher moves play in enhancing students' language in mathematics classrooms. Some articles overlap with those in Sect. 3, if the authors investigated the design as well as its enactment.

There are many *descriptive* studies that identify, exemplify and contrast different teaching practices, but so far there is limited empirical data supporting a *predictive mode*, i.e., qualitative or quantitative results on the effectiveness of these practices, especially across a range of classrooms, or how changes in these teaching practices can impact students' learning of mathematics.

4.1 Teacher moves aiming at enhancing language

In total, we selected 24 reviewed articles in mathematics education journals that present an empirical base for showing that specific teacher moves can have the potential situatively to support and enhance language for learning mathematics (see Table 3). Most of the articles adopted qualitative research formats, two of them quantitative formats (Howe,

Hennessy, Mercer, Vrikki, and Wheatley 2019; Ing et al. 2015), one was a mixed methods study (Lim, Lee, Tyson, Kim, and Kim 2020), and two zoomed in on the qualitative research after an intervention study (Smit & van Eerde 2013;Smit et al. 2013); Prediger and Wessel 2013), without quantitative evidence for the effectiveness of the teacher moves.

Based on these 24 articles, we identified the following six categories of teacher moves for enhancing language in the mathematics classroom (see Table 3).

TM1 Plan and prepare collective discussions that focus on mathematical concepts

TM2 Understand and connect students' ideas and mathematics; make them accessible to as many students as possible

TM3 Enhance language practices for learning mathematics

TM4 Encourage student participation in demanding discourse on mathematics

TM5 Pay attention to feedback and evaluation of students' mathematics

TM6 Purposefully use pauses and silence

These categories focus on the purpose or intention of teacher moves and practices. Specific teacher moves can have different purposes and effects depending on the linguistic, social and mathematical context in which they occur.

Table 2 Overview of quantitative studies showing effectiveness in small group settings or whole class settings

Quantitative study with research format	Addressed design principles	Sample and topic	Empirical evidence for the effectiveness
CRCT/RCT = (cluster-) randomized controlled trial, PP = pre-post evaluation, no control group SG = small group settings WC = whole class settings	(P1) Engage in discourse practices (P2) Use various language routines (P3) Connect languages & represent (P4) Include multilingual resources (P5) Use macro-scaffolding (P6) Compare for raising awareness	Topic, Sample size (age group)	Effect size d in intervention group (if reported) and ANOVA-results p and η^2 for differences to control group
Mercer & Sams (2006): CRCT WC	P1, P2, P6	n = 231 (age 9–10) On numeracy	$p < 0.001$, d and η^2 not given
Prediger & Wessel (2013): CRCT SG	P1, P3, P5	n = 72 students (age 12) On fractions	$d = 1.22$, $p < 0.05$ $\eta^2 = 0.13$
Pöhler et al. (2017): CRCT WC	P5, P3, P1	n = 108 (age 12) On percentages	$d = 0.6$, $p < 0.05$ $\eta^2 = 0.141$
Smit & van Eerde (2013;Smit et al. 2013): PP WC	P5, P1, P3	n = 22 (age 10) On diagrams	$d = 1.22$, $p < 0.01$
Schüler-Meyer et al. (2019): RCT SG	P4 (P1, P3, P5 also in control group)	n = 128 (age 12) On fractions	$d = 0.99$, $p < 0.01$ $\eta^2 = 0.41$
Prediger & Neugebauer (2020): CRCT WC	P5, P3, P1	n = 655 (age 12) on percentages	$p < 0.001$ $\eta^2 = 0.011$
Dröse & Prediger (2021): CRCT WC	P6, P5 (strategic scaffolding), P1	n = 167 (age 10) On word problems	$d = 0.87$, $p < 0.01$ $\eta^2 = 0.02$
Shilo & Kramarski (2019): CRCT WC	Use metacognitive self-question prompts enhancing the depth of discourse	n = 824 (age 10/11) On metacognitive discourse, measured transfer on mathematical problem solving skills	$p < 0.01$, $d = 0.78$ $\eta^2 = 0.30$
Short (2017): CRCT WC	SIOP approach sheltered instruction in all subjects	n = 580 (age 10–18), ca. 30 in mathematics (Age 16)	not significant for mathematics, only for social studies
Hagena et al. (2017): RCT	Training of general reading strategies	n = 380 (age 13) On general reading, measured transfer on math modelling	$p > 0.05$, $d = 0.2$ for all groups, no advantage for training group
Ing et al (2015): WC	P1 (Student participation in explaining ideas and building on others ideas positively predicted student achievement. Teacher support has a mediating effect on student participation but no direct impact on achievement.)	n = 71 (age 8–10) On number operations, fractions and decimals	$p < 0.5$

Table 3 Overview on teacher moves with examples and research base

Teacher move category	Examples of teacher moves	Study with research format
TM1 Plan and prepare collective discussions that focus on mathematical concepts	• Use tasks suitable for enabling discussions • Anticipate student responses during planning • Monitor students' processes during individual/group work • Purposefully select and sequence ideas that are presented • Tailor discussions to their epistemic function	Stein et al. (2008): TF Henning et al. (2012): CD Prediger & Wessel (2013): DR Ferrer et al. (2015): CD Preciado-Babb et al. (2018): DR
TM2 Understand and connect students' ideas and mathematics; make it accessible to as many students as possible	• Explore the details of children's thinking and pose informed probing questions • Use guiding questions providing directions and • promoting confidence • Be responsive and adjust challenges to students' mathematical and linguistic capabilities in the moment • Use gestures, drawings, connect representations and language varieties as supports • Keep student in the role of the responsible speaker • Revoice • Explicate epistemic demands • Chose the language that represents a mathematical idea more transparently	Stein et al. (2008): TF Henning et al. (2012): CD Prediger & Wessel (2013): DR Smit & van Eerde (2013;Smit et al. 2013): DR Prediger et al. (2015): CO, PD Jacobs & Empson (2016): TF, CO da Ponte & Quaresma (2016): TF, CO Teuscher et al. (2016): TF, CD Eckert & Nilsson (2017): CO Erath (2018): CO Martínez (2018): CO Preciado-Babb et al. (2018): DR I & de Araujo (2019): CO
TM3 Enhance language practices for learning mathematics	• Explicate communicative demands • Revoice • Reformulate, paraphrase • Initiate self-repair, ask for clarification / elaboration • Advert to helping materials, make use of connecting language varieties and representations • Provide vocabulary and syntactical structures, Complement words • Act as language model • Elicit students' languages	Prediger & Wessel (2013): DR Smit & van Eerde (2013;Smit et al. 2013): DR Pöhler & Prediger (2015): DR; CD Prediger et al. (2015): CO, PD Erath (2018): CO Martínez (2018): CO Howe et al. (2019): TF, CO, quantitative
TM4 Encourage participation in demanding discourse on mathematics	• Use inquiry questions • Invite students to explain, argue, give reasons, etc. • Focus on students' mathematical ideas rather than encouraging every contribution	da Ponte & Quaresma (2016): TF, CO Hofmann & Ruthven (2018): CD Henning et al. (2012): CD Erath (2018): CO Ing et al. (2015): quant., see Table 2 Kazak et al. (2015): DR Preciado-Babb et al. (2018): DR
TM5 Pay attention to your feedback and evaluation of students' mathematics	• Emphasize joint examination of the quality of students' ideas; establish shared criteria • Handle mistakes and nescience positively and in a future-oriented way • Repeat correct student utterances • Initiate self-repair	Henning et al. (2012): CD Smit & van Eerde (2013;Smit et al. 2013): DR Ingram et al. (2015): CO Erath (2018): CO Hofmann & Ruthven (2018): CD
TM6 Purposefully use pauses and silence	• Pause to allow students to articulate their thinking • Listen to students' contributions	Cohrssen et al. (2014): CO Ingram & Elliott (2016): CO Lim et al. (2020): CO, mixed methods Xu & Clarke (2019): CO

DR = design research study; CD = classroom observation study with given instructional design; CO = classroom observation study without influence on design; TF = theoretical framework; PD = study on professional development

Consequently, particular teacher moves could be included in multiple categories depending on the empirical findings surrounding the move or practice. As Sect. 4.2 highlights, all moves from the 6 categories together contribute to establishing teaching practices and norms that enhance language for learning mathematics, which means that the categories can overlap. Nevertheless, for analytical reasons and for supporting teachers in establishing practices, norms, and moves, it makes sense to treat them separately here.

Categories TM1 and TM2 can be understood as setting the basis for productive whole-class discussions that enhance language. TM1 refers, on the one hand, to lesson planning beforehand (see the six design principles in Sect. 3 for details on material and instruction design at both micro and macro levels and timelines), in particular choosing appropriate tasks and anticipating student responses (Stein et al. 2008) as well as actively planning whole-class discussions including tailoring them to their epistemic functions (Henning, McKeny, Foley, and Balong 2012). In these studies, the outcome of interest was student participation in classroom discussion. On the other hand, TM1 also refers to teacher activities while students are working individually or in groups that prepare subsequent discussions (Stein et al. 2008). Building on this planning and preparing, Category TM2 points to the importance of taking students' mathematical ideas seriously, which particularly includes putting effort into understanding and connecting these ideas and making them accessible to the whole class (e.g., Van Zoest, Stockero, Leatham, Peterson, Atanga, and Ochieng 2017). For example, connecting language varieties and representations is not only an important design principle (P3) but also an important teacher move for understanding and connecting students' ideas to mathematics concepts and making them accessible to all students. Table 3 gives an impression of the various teacher moves that studies identified as productive in Categories TM1 and TM2. All have in common a strong focus on 'adaptivity'/responsiveness/in-the-moment-adjustments (e.g., Teuscher, Moore, and Carlson 2016; da Ponte and Quaresma 2016; Jacobs and Empson 2016) and/or the idea of assigning competence and responsibility to students (O'Connor and Michaels 1993; I and Araujo 2019).

Some of the moves in category TM2 are so-called 'talk moves' that bridge Categories TM3 and TM4, with their specific focus on encouraging student participation and enhancing language in interaction. A talk move is a specific teacher move that is reflexive in that it "responds to what has gone before; … adds to the ongoing discussion and… anticipates or 'sets up' what will come next" (Sohmer, Michaels, O'Connor, and Resnick 2009, p. 107). These moves can include which questions teachers ask (e.g., Boaler and Brodie 2004) as well as how they follow-up and build on students' responses (e.g., Franke et al. 2007; Jacobs and Empson 2016; Lim et al. 2020; Land, Tyminski, and Drake 2019) and these are collected in category TM3. Much of the recent research has focused more on what teachers do in response to students' answers to their questions, rather than on the questions that initiated these answers. For example, the micro-analysis by Tabach, Hershkowitz, Azmon, and Dreyfus' (2020) of two teachers' probability lessons showed that whilst both teachers often asked students to 'say more', the students of the teacher who prompted for reasoning provided more justifications for their claims, both in their contributions to whole class discussions and in their written assessments at the end of the topic. Lim et al. (2020) showed that students felt their teachers listened to them, and were interested in what they said when they asked follow-up questions often and when they spent more time considering students' answers, but their study focused solely on students' perceptions of these talk moves rather than any impact on students' learning. Many studies make use of O'Connor and Michaels' (1993) notion of revoicing, though there is some variation in the distinctions made between revoicing, repeating and reformulating. These distinctions become important where the difference between who is responsible for what is being said matters, as when revoicing the responsibility remains with the student, where a repetition or reformulation can shift the responsibility to the teacher, with repetition and reformulation acting as evaluations of what students have said. Furthermore, TM3 also includes teacher moves regarding explicating demands, providing support, and acting as a language model (Smit, van Eerde, and Bakker 2013; Smit et al. 2013; Prediger, Quasthoff, Vogler, and Heller 2015; Erath 2018). The cited studies also show that students' language production can be situatively enriched by these supports.

Recent work on encouraging student explanations has focused on teacher moves that support students to explain *what* they did and *why*, rather than just *how* or just by recounting a set of procedural steps, and informs category TM4. This TM is closely linked to design principle P1, since rich discourse practices include explaining and justifying. P4 is also enacted using these moves, where it acknowledges and builds on students' own repertoires and resources. This category therefore highlights the importance of encouraging students to contribute to mathematically and linguistically demanding discussions (Erath 2018; da Ponte and Quaresma 2016; Hofmann and Ruthven 2018), which is seen as providing more learning opportunities for deep and meaning related mathematics learning (e.g., Erath 2017). Ing et al. (2015) focused their quantitative study on teachers' eliciting of student thinking and support for their students' engagement with others' ideas. Although these moves did not directly impact student achievement, they did positively predict the number of times students offered explanations or engaged with other students' ideas, and these student participation moves did positively predict student achievement.

Teacher move categories TM3 and TM4 are closely linked to the first four design principles from Sect. 3, since these moves interactionally enact the four principles of engaging students in rich discourse practices (P1), establishing various language routines (P2), connecting language varieties and representations (P3), and including students' multilingual resources (P4). In some ways, those four design principles define what it means to enhance language practices (TM3) and what constitutes demanding discourse (TM4).

The following teacher move categories TM5 and TM6 are not specific to learning mathematics. However, they play a crucial role as they contribute to establishing practices and norms outlined in Sect. 4.2.

Category TM5 combines teacher/talk moves related to the evaluation of and feedback to student utterances. Hofmann and Ruthven (2018) suggested that teachers place emphasis on the joint examination of the quality of students' mathematical ideas and establish shared criteria for doing so, rather than conducting an immediate teacher evaluation. Other studies report on productive teachers' talk moves related to handling mistakes and students' nescience: these moves entail teachers reacting positively and in a future-oriented way, and giving face-saving evaluations aimed also at encouraging students to share their ideas, as well as repeating correct student utterances (Erath 2018; Ingram, Pitt, and Baldry 2015; Smit & van Eerde 2013;Smit et al. 2013; Henning et al. 2012). In their final caveat, Jacobs and Empson (2016) highlighted the importance of teacher move category TM5 and pointed out that students may feel uncomfortable if their thinking is probed and pushed consistently (TM2 and TM4 in line with establishing an enquiry or argument-oriented classroom; see Sect. 4.2) and that these teaching moves therefore work only in a constructive classroom atmosphere.

The last category, TM6, includes teacher moves that involve pauses, silence or wait time. e.g., for eliciting longer and more sophisticated student contributions. These moves offer time within classroom interactions for both teachers and students to think about what they want to say, or about what is being said. In terms of supporting or enhancing students' language within the mathematics classroom, these moves that are focused around silence involve both giving students' opportunities to participate, potentially in an extended or more sophisticated way, and also allowing more time for students and teachers to make sense of the mathematical ideas being expressed and building on them. But, as Ingram and Elliot (2016) pointed out, "mechanistically leaving pauses of at least three seconds is not a productive strategy" (p. 50). Thus, especially these moves (as all others) must be tailored to the established classroom norms and the purpose of the interactions in order to be productive.

From a more psychological perspective, Teuscher et al. (2016) emphasized teachers' focus on student thinking as a basis for high quality interactions with students, by introducing Piaget's construct of decentering. They advocated that teachers need to work constructively with students' individual conceptions and ways of thinking by shifting the focus from what students do to what meaning students have or construct. They criticized that existing research on discourse patterns such as revoicing "foreground teacher actions in ways that do not attend to the depth and content of student responses and thinking" (p. 436), and this research does not investigate reasons why a teacher decides in the interaction to apply a specific move.

4.2 Teaching practices and norms that enhance language for learning mathematics

In Sect. 4.1, we focused on specific discursive moves that teachers, and students, could use to enhance language for mathematics learning in their classroom practices. In order to avoid a too isolated perspective on moves, in this section we focus more broadly on teacher practices and classroom or mathematical norms where there is empirical research detailing how those can enhance learning in mathematics. These practices include establishing norms of interaction (Bauersfeld, Krummheuer, and Voigt 1985; Yackel and Cobb 1996) between students, or between student and teacher (e.g., Wood, Williams and McNeal 2006; Kazak, Wegerif, and Fujita 2015; Makar, Bakker, and Ben-Zvi 2015), which bring together many of the teacher moves discussed in Sect. 4.1 but embed them (as requested by Teuscher et al. 2016) in a larger picture.

Many moves collected in the teacher move categories TM1, TM2, and TM4 contribute to establishing an inquiry-based or argumentation-oriented classroom, as repeatedly advocated in research on enhancing meaning-related, conceptual, deep mathematics (e.g., Yackel and Cobb 1996; Wood et al. 2006; Stein et al. 2008; Hofmann and Ruthven 2018) as well as in studies with a focus on enhancing language (e.g., Smit & van Eerde 2013;Smit et al. 2013; Prediger and Wessel 2013; Moschkovich 2015b; Erath et al. 2018). A main characteristic of inquiry-based or argumentation-oriented classrooms is that children are asked not only to recount the procedures they used to solve a problem but also to provide reasons for their thinking (Wood et al. 2006). Thus, students are encouraged to explain their mathematical ideas and construct arguments (a rich discourse practice as described by P1 and demanding discourse in TM4). Furthermore, whole class discussions aim at making (students') mathematics accessible to all students and to comparing and connecting different ideas (see TM2; this aspect is also evident in P2 for language routines and P5 for macro-scaffolding). Going beyond reporting solutions by explaining meanings or arguing for a strategy is mathematically and linguistically demanding for all students (Erath et al. 2018), but offers important language learning opportunities particularly for language learners (Moschkovich 2015b) that are productive if students are supported in accomplishing their contributions (TM2, TM3 and TM6, as well as design principles P3, P4, and P5). This aspect implies that conducting whole-class discussions in inquiry-based or argumentation-oriented classrooms is also highly challenging for teachers and necessitates planning and preparing (as is evident in TM1 and in all six design principles), as well as establishing norms

in which value is placed on others' ideas and on productively making mathematics together (Yackel and Cobb 1996; Hofmann and Ruthven 2018; TM5). Teaching practices in argumentation-oriented classrooms have also been examined in terms of how they contribute to the evolution and development of norms. For example, Makar et al. (2015) looked at how specific scaffolding practices, including responding to students' thinking and handing over responsibility to the students, contributed to the establishment of argumentation-based inquiry norms over time.

Other aspects of teachers' practice worthy of consideration are planning and task design, reflected in TM1 and in the focus of Sect. 3. Ferrer, Doorman, and Fortuny (2015) demonstrated that systematically preparing classroom discussions enabled the teacher to create and exploit students' opportunities to learn mathematics related to similarity. Moreover, teaching practices considered by recent research as enhancing students' language and communication in mathematics classroom include assessing students' understanding, building common experiences, and empowering students (I and de Araujo 2019).

Whereas studies contributing to the teacher moves collected in Table 3 highlight the importance of increasing students' active participation in collective discussions, Xu and Clarke (2019) considered alternatives to student talk as measures of student participation, and pointed out the relationship between cultural contexts and practices or norms that are supportive of students' learning mathematics. These alternatives include choral responses as well as silent participation and listening. By contrasting data from Shanghai and Melbourne, Xu and Clarke (2019) showed that public scaffolding of students' fluency with mathematical language through choral responses and student listening can be as effective as encouraging students to discuss mathematics with each other in terms of students using mathematical terms and phrases themselves.

Other studies on teaching practices and norms for enhancing language focused on taking into account cultural values from minority communities in order to support marginalized students. For example, Hunter and Hunter (2018) emphasized the role of cultural values for shaping norms that open the space for all students (in their case for Maori and Pasifika students) to engage in demanding mathematical practices. They offered various related teacher actions in the "communication and participation framework" that echo teacher move categories TM2, TM3, and TM4. Another focus is set on enacting language-as-a-resource. For example, Martínez (2018) and Schüler-Meyer et al. (2019) identified teaching practices for enacting the language-as-resource orientation (in three immersion classrooms) that relate to TM2 and TM3, with a focus on using students' languages for gaining a deeper understanding of mathematical meaning (for non-immersion contexts see literature contributing to design principle P4, including multilingual resources).

Whereas early research on norms of interaction in mathematics classrooms focused on describing and implementing (sociomathematical) norms that facilitate students' mathematical conceptual development (e.g., Yackel and Cobb 1996), recent studies also focus on *how* to establish them. Hofmann and Ruthven (2018) offered explanations for why it is so difficult to change classroom norms. They concluded that for successful implementation it might be necessary to explicitly address new norms for interaction (see also Erath 2017 on the related notion of explaining practices), and they called for shifting from "focusing on students' mathematical ideas, rather than simply encouraging students to make contributions, and emphasizing joint examination of the quality of those ideas and establishing shared criteria for doing so, rather than making an immediate evaluation" (Hofmann and Ruthven 2018, p. 510). Furthermore, Selling (2016) identified eight types of moves that make mathematical practices explicit without being prescriptive or reducing students' opportunities to engage in them. Aspects such as connecting different students' engagement (TM2), making (communicative or epistemic) expectations explicit (TM2, TM3), and paying attention to the feedback and evaluation move (TM5), are reinforced and complemented by "Explaining the goal or rationale for engaging in a mathematical practice […] Framing students' engagement in mathematical practices expansively […] Referring to a teaching narrative about mathematical practices" (Selling 2016, p. 524).

In total, multiple studies underline the need for productive teaching practices and establishing norms so that the intentions underlying the designed teaching materials can really be turned into an enacted curriculum. The teacher moves form an important tool for realizing teaching practices and norms, but only if they are attuned to a larger plan (Teuscher et al. 2016).

5 Recommendations for future research

Although a considerable and insightful body of literature has been discussed in Sects. 3 and 4, which can provide different kinds of empirical foundations for designs and teaching practices, there is still substantial further research needed, as briefly sketched in this section.

Our recommendations for future research start with a summary of the limitations of the current research base. As mentioned in earlier sections, there were few empirical studies and even fewer studies relating language learning to students' mathematical learning in a measurable way. In general, we recommend more empirical studies, more studies that use or include quantitative data and analyses, and more studies that consider the impact or effects of particular

topic-specific learning environments. Researchers will need to consider carefully which conceptualizations of language and language development match their focus of learning mathematics, which research questions are best addressed by qualitative analyses; and they will need to include more research questions that are best answered by quantitative studies, and also to include research designs that address impact in the field, not only with single teachers, but scaling up to many teachers. Whether the studies are qualitative or quantitative, we need more studies that provide qualitative or quantitative results on impacts of design principles or teaching practices across a range of classrooms, or that examine how changes in these design principles or teaching practices can impact students' learning of mathematics.

One of the challenges of researching the design and enactment of instruction that enhances language for mathematics learning is gathering empirical evidence at scale of the relevance of different features or practices to students' learning of mathematics. Particularly lacking are studies that examine the impact or effectiveness of particular principles over time. Whilst there is a long tradition of measuring student outcomes such as mathematical reasoning, mathematics attainment or student attitudes towards mathematics, measuring aspects of classroom discourse at scale is relatively recent (Howe et al. 2019; Ing et al. 2015). These measures often focus on the structure of students' participation, such as the number of times they speak and how long their turns are, rather than the content and mathematical features of what students are talking about, or students' understanding of mathematical concepts (as summarized in Erath and Prediger 2021). More complex studies are rare but include the Learner Perspective Study discussed by Xu and Clarke (2019) and Clarke, Emanuelsson, Jablonka, and Mok (2006), in Howe, Mercer, and Hennessy's Classroom dialogue project (Howe et al. 2019), and work in the SimCalc project by Pierson (2008).

The six principles of instructional design we have outlined above focus the design of tasks and curriculum resources, as well as ways of using these tasks and curriculum resources in the classroom. The examination of the implementation of these designs is also not a simple matter as classroom studies need to consider the teacher's moves and practices, teacher-student dialogue, student–student dialogue, mathematical concepts, mathematical practices, and so on, which makes it difficult to isolate the effects or impact of any *one* aspect on student learning. Future research should strengthen the efforts to combine the analysis of dimensions that have formerly been researched separately.

We also see the field as needing to move towards not only in designing research-based professional development experiences for teachers that address both mathematics and language, but also conducting research on whether and how those experiences work, for example examining how and when teachers change or improve their teaching practice. A few examples of current work on teacher professional development that supports teachers in learning to foster language in mathematics classrooms include studies by Amador and Bennett (2015), Michaels and O'Connor (2015), Prediger et al. (2015), Adler and Ronda (2015), Aquino-Sterling, Rodríguez-Valls, and Zahner (2016). Some of these studies (e.g., Michaels and O'Connor 2015), have not only developed a professional development program for enhancing teachers' moves for facilitating productive discussions, but have also shown efficacy. However, more qualitative and quantitative research on teachers' learning pathways towards enhancing language is required.

The contributions in this special issue span a large variety of research approaches, age levels (from Grade 2 to Grade 12, preservice teacher education as well as in-service professional development) and mathematical topics (counting, multiplication, fractions, proportionality, percentages, linear functions, and others). Some papers focus on multilingual learners only, many also on monolingual learners. The papers draw upon different theoretical backgrounds, most of them centering around the role of language in collective meaning-making processes (Zahner et al. 2012) and in the interplay of tasks and classroom discourses. In these various ways they all contribute to the declared aim of the special issue and go beyond describing existing practices and challenges and instead *provide an empirical foundation for innovation in classrooms*.

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