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ON ENDOGENOUS BUSINESS CYCLES UNDER INCREASING RETURNS TO VARIETY AND SECTOR-SPECIFIC EXTERNALITY

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We examine the theoretical as well as quantitative interrelations between endogenous business cycles, increasing returns to product variety and sector-specific productive externalities within a two-sector real business cycle model. In a calibrated version of our benchmark closed-economy setting, the threshold level of investment externalities needed for equilibrium indeterminacy is found to be monotonically increasing in the degree of market competitiveness. We also study the model's local stability properties (i) when the parameters that govern the level of intermediate-good producers' market power and the strength of variety effects are disentangled; and (ii) in the context of a small open economy. (JEL E30, E32, O41)

I. INTRODUCTION

Since the work of Benhabib and Farmer (1994) and Farmer and Guo (1994), considerable progress has been made over the last two-plus decades in exploring the empirically plausible conditions needed to generate equilibrium indeterminacy and sunspot-driven aggregate fluctuations within real business cycle (RBC) models.¹ In particular, the original Benhabib-Farmer-Guo one-sector model requires an implausibly high level of increasing returns-to-scale in production, *vis-à-vis* estimation results reported by Burnside (1996) and Basu and Fernald (1997), to exhibit a continuum of stationary equilibrium trajectories. Subsequent research,

for example, Benhabib and Farmer (1996), Weder (2000), and Harrison (2001), shows that a representative-agent macroeconomy with two distinct (consumption and investment) goods and sector-specific externalities from productive inputs is able to yield endogenous business cycles under a much less stringent circumstance. In addition, a recent piece by Chang, Hung, and Huang (2011) obtain the qualitatively identical result in the context of a one-sector RBC model with increasing returns to product variety caused by monopolistic competition and free entry/exit of firms.

In this paper, we first build upon Benhabib and Farmer's (1996) analyses and examine the theoretical as well as quantitative interrelations between equilibrium indeterminacy, sector-specific productive externalities and the degree of monopoly power (or the strength of variety effects) within a continuous-time two-sector RBC model. In our baseline closed-economy setting, each production sector has an intermediate-good segment in which monopolistically competitive firms operate under fixed setup costs and fully mobile capital and labor inputs. The equilibrium measure of these intermediate-input producers for consumption or investment goods is endogenously determined through the zero-profit condition. This in turn yields increasing returns to an expansion in product variation *à la* Chang, Hung,

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1. See Benhabib and Farmer (1999) for an early survey on this RBC-based indeterminacy literature.

ABBREVIATIONS

GDP: Gross Domestic Product
RBC: Real Business Cycle

and Huang (2011). A final output is produced within each sector from the set of available differentiated intermediate goods in a perfectly competitive setting.

Using the standard procedure of linearizing equilibrium conditions around the unique interior steady state, we first obtain the analytical expression for the resulting Jacobian matrix of partial derivatives, and then find that the presence of consumption externalities exerts no effect on the model's local dynamics. When agents expect that the rate of return on investment will increase tomorrow, they need incentive to give up consumption today for more capital accumulation. It follows that no productive externality is needed in the consumption sector, from which agents move their capital and labor services, to fulfill their optimistic anticipation. Next, within the empirically realistic configuration that capital's share of gross domestic product (GDP) is lower than that of labor, we analytically derive the *necessary and sufficient* condition for our benchmark close-economy model to possess an indeterminate steady state, around which belief-driven cyclical fluctuations will take place.

To gain further insights of the aforementioned theoretical results, a quantitative investigation of macroeconomic (in)stability is undertaken within a calibrated version of our two-sector RBC model under parameter values that are consistent with post-Korean-war U.S. time series data. Accordingly, a two-dimensional plot is constructed to divide the feasible parameter space into the regions of "Saddle", "Sink" and "Source" as functions of the size of investment externalities versus the magnitude of the price-cost markup ratio. We show that the threshold level for productive externalities in the investment sector that leads to endogenous cyclical fluctuations is monotonically increasing with respect to the degree of market competitiveness. In terms of the underlying economic intuition, consider the two opposing effects on the representative household's intertemporal consumption Euler equation as it becomes optimistic about the economy's future. On one hand, an increase in today's investment expenditures will decrease next period's real interest rate because of diminishing marginal product of capital (*the MPK effect*). This channel in turn invalidates agents' initial optimism. On the other hand, due to market imperfection and productive investment externalities, the social production possibility frontier that traces out the trade-off between consumption and investment

spending is downward-sloping and convex to the origin. As a result, the relative price of investment goods will fall (*the price effect*) upon the household's optimism that shifts capital and labor inputs out of producing consumption goods. This channel in turn justifies agents' spurt to invest more today. Our analysis shows that for a given level of intermediate-good firms' market power, the price effect quantitatively outweighs the *MPK* effect provided the degree of investment externalities is sufficiently high to exceed the lower bound associated with the requisite condition for indeterminacy and sunspots. In this case, the representative agent's rosy expectation will be validated as a self-fulfilling equilibrium.

We also find that *ceteris paribus* when the market competitiveness of intermediate-good producers falls, the curvature for the economy's convex social production possibility frontier becomes more pronounced. As a consequence, for the same magnitude of increases in capital and labor inputs that are moved into the investment sector because of agents' optimism, the resulting decrease in the relative price of investment goods will be larger, which in turn enhances the above-mentioned price effect. It follows that local indeterminacy may arise in our model economy without any investment externalities as long as increasing returns to product variety are sufficiently strong.

With regard to the empirical plausibility on the minimum level of productive externalities required for equilibrium indeterminacy, the RBC macroeconomy that we examine has the versatility to subsume three existing studies: (1) the Benhabib-Farmer two-sector model with perfectly competitive production sectors; (2) the Chang-Hung-Huang one-sector model with monopolistic competition and a social technology that exhibits positive externalities from capital and labor services; and (3) the original Benhabib-Farmer-Guo one-sector model with perfect competition and aggregate increasing returns-to-scale in production. While keeping the calibrated values of other parameters unchanged, we find that the Benhabib-Farmer-Guo one-sector model needs the highest external effect, whereas our two-sector model requires the lowest, and therefore, the most empirically plausible, level of productive externalities to generate belief-driven cyclical fluctuations. In sum, our quantitative analysis highlights two cooperating factors—increasing returns to product variety and sector-specific investment externalities—in

producing multiple, indeterminate equilibria within a RBC model.

Our analysis of the benchmark closed-economy model is related to that in Weder (1998) who also explores equilibrium indeterminacy in a similar two-sector (consumption and investment) RBC setting with monopolistic competition and costless entry/exit of firms. The two studies differ in the following aspects. First, in addition to fixed overhead costs, the sectoral production functions of intermediate goods in Weder's model allow for internal increasing/constant/decreasing returns-to-scale, whereas sector-specific productive externalities are considered in our model. Second, we obtain the necessary and sufficient condition for local indeterminacy after analytically deriving the model's Jacobian matrix, whereas Weder's work does not. Third, Weder conducts numerical business-cycle simulations within his discrete-time macroeconomy driven by sectoral technology shocks and agents' animal spirits, whereas we focus on the occurrence of perfect-foresight competitive equilibria in our continuous-time framework.

For the sensitivity analyses, we first disentangle the effects of intermediate-good producers' monopoly power from the strength of product variety, as in Pavlov and Weder (2012). In this extended setting, it can be analytically shown that it is the inverse interrelations between two investment-specific parameters—the degree of increasing returns to specialization and the magnitude of productive externalities—that will govern the model's local dynamics. Our numerical experiments also find that the occurrence of equilibrium indeterminacy requires only one of them to be strictly positive. In addition, the price-to-cost markup ratios for consumption as well as for investment do not play any role in affecting the economy's macroeconomic stability properties. Next, we follow Weder (2001, Section II) and examine a small-open-economy environment in which agents have access to international borrowing and lending at a perfect world capital market. Under the same calibrated parameterizations as those in the closed-economy counterpart, a necessary condition for our model to possess an indeterminate steady state is that the nonzero sector-specific productive externalities are of opposite signs. In addition, the threshold size of negative external effects in the consumption sector that leads to endogenous business cycles will fall as the investment externality rises and becomes more positive. Finally, our analysis confirms the general point that *ceteris paribus* a

small open economy is more susceptible to indeterminacy and sunspots than a closed economy. Since the representative household is able to completely smooth its consumption over time at a constant world real interest rate, it will take a relatively smaller level of market imperfections in technology for a small-open-economy model to exhibit a continuum of equilibrium paths.

The remainder of this paper is organized as follows. Section II describes the economy and analyzes its equilibrium conditions. Section III analytically and quantitatively examines our baseline model's local stability properties. Section IV considers two extensions to re-examine the economy's equilibrium dynamics (1) when the parameters that govern the degree of intermediate-good firms' monopoly power and the level of product variety are differentiated, and (2) in the context of a small open economy. Section V concludes.

II. THE ECONOMY

Our analysis builds upon the continuous-time two-sector RBC model *à la* Benhabib and Farmer (1996) for a closed economy that allows for market imperfection together with free entry and exit of firms. Households live forever, and derive utility from consumption and leisure. The production side of the economy consists of two distinct sectors for consumption and investment goods, respectively. Each sector has an intermediate-good segment in which monopolistically competitive firms operate under fixed setup costs and sector-specific externalities from fully mobile capital and labor inputs. The equilibrium measure of these intermediate-input producers for consumption or investment goods is endogenously determined through the zero-profit condition. This in turn yields increasing returns to specialization or product variety as in Chang, Hung, and Huang (2011). A final output is produced within each sector from the set of available differentiated intermediate goods in a perfectly competitive environment. We postulate that there are no fundamental uncertainties present in the economy.

A. Firms

Our model economy is comprised of two production sectors indexed by $m = c, I$, where c stands for consumption and I stands for investment. Since firms are solving a static profit maximization problem during each period, time

subscripts will be suppressed for notational convenience throughout this subsection. The final good in each sector Y_m is produced from a continuum of intermediate inputs x_{mj} , where $j \in [0, N_m]$ and N_m represents the measure of (or the degree of variety for) intermediate goods utilized within sector m , through the following technology that exhibits constant returns-to-scale:

$$(1) \quad Y_m = \left(\int_0^{N_m} x_{mj}^\lambda dj \right)^{\frac{1}{\lambda}}, \quad m = c, I \text{ and } 0 < \lambda \leq 1.$$

Using p_{mj} to denote the dollar price of the j th intermediate input in sector m , and P_m to denote the dollar price of sectoral output Y_m , the first-order condition for final-good producers' profit maximization problem is given by

$$(2) \quad x_{mj} = \left(\frac{p_{mj}}{P_m} \right)^{\frac{1}{1-\lambda}} Y_m, \quad m = c, I,$$

where the price elasticity of demand for x_{mj} is equal to $\frac{1}{1-\lambda}$. When $\lambda = 1$, the model collapses to one with perfectly competitive markets as in Benhabib and Farmer (1996) and Harrison (2001).

Each intermediate good is produced by a monopolist with the production function that allows for increasing returns-to-scale:

$$(3) \quad x_{mj} = \Psi_m K_m^\alpha L_m^{1-\alpha} - Z_m, \quad m = c, I, \\ 0 < \alpha < 1 \text{ and } Z_m > 0,$$

$$(7) \quad MC_{mj} = \frac{1}{(1 + \theta_m)(x_{mj} + Z_m)} \underbrace{\left[\left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} + \left(\frac{1-\alpha}{\alpha} \right)^\alpha \right] \times R_m^\alpha W_m^{1-\alpha} (x_{mj} + Z_m)^{\frac{1}{1+\theta_m}}}_{=TC_{mj}}, \quad m = c, I,$$

where K_{mj} and L_{mj} are capital and labor inputs employed by the j 'th intermediate-good producer in sector m ; and Z_m represents a constant amount of intermediate goods that must be expended in sector m as fixed setup costs of production before any sale is made. The presence of such overhead costs implies that the intermediate-good technology exhibits increasing returns-to-scale. Moreover, Ψ_m denotes the sectoral production externalities that each intermediate-good firm takes as given, and is postulated to take the following specification as in Chang, Hung, and

Huang (2011):

$$(4) \quad \Psi_m = (K_m^\alpha L_m^{1-\alpha})^{\theta_m}, \quad m = c, I, \text{ and } \theta_c \leq 0 \leq \theta_I,$$

where K_m and L_m represent the within-sector aggregate levels of capital and labor services devoted to producing intermediate inputs, that is, $K_m = \int_0^{N_m} K_{mj} dj$ and $L_m = \int_0^{N_m} L_{mj} dj$; and θ_m denotes the degree of sector-specific productive externalities. Based on Harrison's (2003) empirical findings of zero or negative external effects in the consumption sector, whereas investment goods are produced under positive externalities, the parametric restriction of $\theta_c \leq 0 < \theta_I$ is imposed. We also allow the possibility of $\theta_I = 0$ for the sake of analytical completeness.

Using Equations (2) and (3), together with the assumption that factor markets are perfectly competitive within each sector, it is straightforward to show that the first-order conditions for intermediate-good producers' profit maximization problem are given by

$$(5) \quad R_m = \frac{\lambda \alpha (x_{mj} + Z_m) p_{mj}}{K_{mj}}, \quad m = c, I,$$

$$(6) \quad W_m = \frac{\lambda (1 - \alpha) (x_{mj} + Z_m) p_{mj}}{L_{mj}}, \quad m = c, I,$$

where R_m is the nominal rental rate of capital and W_m is the nominal wage rate in sector m . It can then be derived that the marginal cost of intermediate good j in sector m is

where TC_{mj} denotes the total production cost ($= R_m K_{mj} + W_m L_{mj}$). Therefore, marginal costs of sectoral intermediate inputs x_{mj} are decreasing (increasing) functions when $\theta_m > (<) 0$; and are constants when $\theta_m = 0$. This in turn implies that other than fixed setup costs, additional increasing returns will exist in (3) for x_{Ij} under $\theta_I > 0$ because of diminishing marginal costs associated with investment-specific capital and labor inputs.

Since capital and labor inputs are postulated to be fully mobile across the two production sectors, intermediate-good firms in each sector will face

the same factor prices, that is, $R_c = R_I = R$ and $W_c = W_I = W$. Under the maintained assumption of free entry and exit for intermediate-input producers in both sectors, their profit will be equal to zero at each instant of time. This zero-profit condition in conjunction with (5) and (6) yield the (constant) equilibrium quantity of intermediate input x_{mj} :

$$(8) \quad x_{mj} = \frac{\lambda Z_m}{1 - \lambda}, \quad m = c, I,$$

which also represents the size of an intermediate-good firm that turns out to be independent of any endogenous variable. In what follows, our analysis is restricted to the model's symmetric equilibria within each sector in which

$$(9) \quad p_{mj} = p_m, x_{mj} = x_m, K_{mj} = \frac{K_m}{N_m}, L_{mj} = \frac{L_m}{N_m},$$

$m = c, I$ and for all $j \in [0, N_m]$.

$$(14) \quad C_t + P_t I_t = r_t K_t + w_t L_t + \underbrace{\int_0^{N_{ct}} \pi_{cjt} dj + \int_0^{N_{It}} \pi_{ijt} dj}_{=Y_t},$$

After substituting condition (9) into (1) and (2), we derive that the sectoral production functions are given by

$$(10) \quad Y_m = N_m^{\frac{1}{\lambda}} x_m, \quad m = c, I,$$

which will display increasing returns to an expansion in product variety since $0 < \lambda < 1$; and that the corresponding sectoral prices of intermediate goods are

$$(11) \quad p_m = P_m N_m^{\frac{1-\lambda}{\lambda}}, \quad m = c, I.$$

Finally, using Equations (3), (4), (8), and (9) leads to the equilibrium measure of intermediate-good firms in sector m :

$$(12) \quad N_m = \left(\frac{1 - \lambda}{Z_m} \right) K_m^{\alpha(1+\theta_m)} L_m^{(1-\alpha)(1+\theta_m)}, \quad m = c, I.$$

B. Households

The economy is populated by a unit measure of identical infinitely lived households. Each household maximizes its present discounted

lifetime utility

$$(13) \quad \int_{t=0}^{\infty} \left(\log C_t - \frac{L_t^{1+\gamma}}{1+\gamma} \right) e^{-\rho t}, \quad \rho > 0, \quad \text{and} \quad \gamma \geq 0,$$

where C_t and L_t are the household's consumption and hours worked, respectively; ρ is the subjective rate of time preference, and γ denotes the inverse of the wage elasticity for labor supply. Notice that the instantaneous utility function in (13) is consistent with long-run balanced growth, a feature that is commonly adopted in the RBC literature. Using the consumption good as the economy's numeraire, the real-valued budget constraint faced by the representative agent is given by

where I_t is gross investment, $P_t \left(= \frac{P_{It}}{P_{ct}} \right)$ denotes the relative price of investment to consumption goods, $r_t \left(= \frac{R_t}{P_{ct}} \right)$ is the real rental rate, $w_t \left(= \frac{W_t}{P_{ct}} \right)$ is the real wage rate, π_{mjt} represents the real profit from intermediate-good producer j of sector $m = c, I$ that is returned to the household as lump-sum dividends, Y_t is real national income or GDP, and K_t is the household's capital stock that evolves according to the law of motion

$$(15) \quad \dot{K}_t = I_t - \delta K_t, \quad K_0 > 0 \text{ given},$$

where $\delta \in (0, 1)$ is the capital depreciation rate.

The first-order conditions for the representative household's dynamic optimization problem are

$$(16) \quad \frac{1}{C_t} = \frac{\phi_t}{P_t},$$

$$(17) \quad C_t L_t^\gamma = w_t,$$

$$(18) \quad \frac{\dot{\phi}_t}{\phi_t} = \rho + \delta - \frac{r_t}{P_t},$$

where ϕ_t is the costate variable (measured in terms of utility) that can be interpreted as the shadow value of K_t ; (17) equates the slope of household's indifference curve to the real wage rate, and (18) is the Keynes-Ramsey rule that governs the household's intertemporal choices of consumption.

C. Symmetric Equilibria and Steady State

Since firms use identical production technologies and face the same factor prices across the two sectors, the fractions of capital and labor inputs utilized in the consumption sector are equal,

$$(19) \quad \frac{K_{ct}}{K_t} = \frac{L_{ct}}{L_t} \equiv \mu_t \in (0, 1).$$

We will focus on the model's symmetric equilibria in which the household's and firms' first-order conditions are all satisfied. Without loss of generality, the equilibrium price of the numeraire (consumption) good is normalized to unity, $P_{ct} = 1$. The equalities of demand by households and supply by firms in the consumption and investment sectors are given by $C_t = Y_{ct}$ and $I_t = Y_{It}$. Moreover, both the capital and labor markets will clear whereby $K_{ct} + K_{It} = K_t$ and $L_{ct} + L_{It} = L_t$. Using Equations (5)–(12), (14), and (19), the equilibrium price of investment relative to consumption goods can be expressed as

$$(20) \quad P_t = \frac{\mu_t^{\frac{1+\theta_c-\lambda}{\lambda}} \left(\frac{Z_t}{Z_c}\right)^{\frac{1-\lambda}{\lambda}}}{(1-\mu_t)^{\frac{1-\lambda+\theta_t}{\lambda}}} [K_t^\alpha L_t^{1-\alpha}]^{\frac{\theta_c-\theta_t}{\lambda}},$$

and the economy's aggregate/reduced-form production function or total output is given by

$$(21) \quad Y_t = \lambda(1-\alpha)^{\frac{1+\theta_c-\lambda}{\lambda}} \left(\frac{1-\lambda}{Z_c}\right)^{\frac{1-\lambda}{\lambda}} \times K_t^{\frac{\alpha(1+\theta_c)}{\lambda}} L_t^{1+\gamma-\frac{(\alpha+\gamma)(1+\theta_c)}{\lambda}},$$

where $\frac{\alpha(1+\theta_c)}{\lambda} < 1$ to rule out the possibility of sustained endogenous growth.

It is straightforward to show that our model economy possesses a unique interior steady state. Specifically, the steady-state proportion of factor inputs allocated to the consumption sector, and the household's (aggregate) hour worked and capital stock are

$$(22) \quad \mu_{ss} = 1 - \frac{\alpha\delta}{\rho + \delta}, L_{ss} = \left(\frac{1-\alpha}{\mu_{ss}}\right)^{\frac{1}{1+\gamma}} \text{ and}$$

$$K_{ss} = \left[\frac{\lambda(1-\mu_{ss})^{\frac{1+\theta_t}{\lambda}} \left(\frac{1-\lambda}{Z_t}\right)^{\frac{1-\lambda}{\lambda}} L_{ss}^{\frac{(1-\alpha)(1+\theta_t)}{\lambda}}}{\delta} \right]^{\frac{\lambda}{\lambda-\alpha(1+\theta_t)}}.$$

Given (22), the steady-state expressions of all the remaining endogenous variables can then be easily derived.

III. MACROECONOMIC (IN)STABILITY

In terms of the local stability properties of our model economy, we take linear approximations to its equilibrium conditions in a neighborhood of the steady state to obtain the following dynamical system:

$$(23) \quad \begin{bmatrix} \dot{K}_t \\ \dot{\phi}_t \end{bmatrix} = \mathbf{J} \begin{bmatrix} K_t - K_{ss} \\ \phi_t - \phi_{ss} \end{bmatrix}, \quad K_0 > 0 \text{ given},$$

where $\mathbf{J}$ is the Jacobian matrix of partial derivatives for the transformed dynamical system. It can be shown that the Jacobian's determinant and trace are

$$(24) \quad Det = \frac{\alpha\mu_{ss}(1-\mu_{ss})(1+\gamma)(\lambda\Pi L_{ss})^2[\lambda-\alpha(1+\theta_t)]}{(1+\theta_t)[1-\alpha+(\alpha+\gamma)\mu_{ss}]-\lambda(1+\gamma)},$$

$$(25) \quad Tr = \left(\frac{\Pi}{1-\mu_{ss}} \right) \left\{ (1-\mu_{ss})[\alpha(1+\theta_t)-\lambda] + \frac{\alpha\Omega}{(1+\theta_t)[1-\alpha+(\alpha+\gamma)\mu_{ss}]-\lambda(1+\gamma)} \right\},$$

where

$$(26) \quad \Pi = (1-\mu_{ss})^{\frac{1-\lambda+\theta_t}{\lambda}} \left(\frac{1-\lambda}{Z_t}\right)^{\frac{1-\lambda}{\lambda}} K_{ss}^{\frac{\alpha(1+\theta_t)-\lambda}{\lambda}} \times L_{ss}^{\frac{(1-\alpha)(1+\theta_t)-\lambda}{\lambda}} > 0,$$

$$(27) \quad \Omega = (1-\alpha)(1+\theta_t)(1-\mu_{ss}) \times [\lambda - (1+\theta_t)(1-\mu_{ss})] + \mu_{ss}(1+\gamma) \times [\lambda(1+\theta_t-\lambda) - (1-\mu_{ss})(1+\theta_t)^2] \gtrless 0,$$

and μ_{ss} , L_{ss} , and K_{ss} are given by (22).

Since the dynamical system (23) possesses one predetermined variable K_t , the economy exhibits saddle-path stability and equilibrium uniqueness if and only if the two eigenvalues of $\mathbf{J}$ are of opposite sign ($Det < 0$). When both

eigenvalues have negative real parts ($Det > 0$ and $Tr < 0$), the steady state is a locally indeterminate sink that can be exploited to generate endogenous cyclical fluctuations driven by agents' self-fulfilling expectations or sunspots. The steady state becomes a source when both eigenvalues have positive real parts ($Det > 0$ and $Tr > 0$).

A. Analytical Characterizations

Based on (24)–(25) and the subsequent discussion, this subsection analytically examines the condition(s) under which our two-sector RBC model exhibits equilibrium indeterminacy and belief-driven aggregate fluctuations ($Det > 0$ and $Tr < 0$). We first note that the degree of productive externalities in the consumption sector θ_c does not enter (24) or (25), hence, it exerts no effect on the economy's local dynamics. This finding turns out to be reminiscent of Harrison (2001) under perfectly competitive markets. Intuitively, when agents expect the rate of return on investment to increase tomorrow, they will choose to give up today's consumption for more capital accumulation. It follows that no productive externality is needed in the consumption sector, from which agents move their capital and labor services, to fulfill their optimistic anticipation

is positive when

$$(28) \quad \underbrace{\frac{\lambda(1+\gamma)(\rho+\delta)}{\alpha\delta(1-\alpha) + (1+\gamma)[\rho + (1-\alpha)\delta]} - 1}_{\equiv \theta_I^{\min}} < \theta_I < \frac{\lambda}{\alpha} - 1,$$

which in turn provides a necessary condition for indeterminacy and sunspots. Since $0 < \alpha$, λ , $\mu_{ss} < 1$, $\gamma \geq 0$, and L_{ss} , $\Pi > 0$, our model's Jacobian matrix possess a positive determinant if and only if

$$(29) \quad \frac{\lambda - \alpha(1 + \theta_I)}{(1 + \theta_I)[1 - \alpha + (\alpha + \gamma)\mu_{ss}] - \lambda(1 + \gamma)} > 0.$$

Condition (28) reports the feasible range of investment externalities $\theta_I \geq 0$ that leads to $Det > 0$ when the numerator and denominator of (29) are both strictly positive.² Moreover, given the restriction of $\alpha < 1 - \alpha$, it is straightforward to show that $\frac{\lambda(1+\gamma)(\rho+\delta)}{\alpha\delta(1-\alpha) + (1+\gamma)[\rho + (1-\alpha)\delta]} < \frac{\lambda}{\alpha}$.

Third, since $0 < \mu_{ss} < 1$ and $\Pi > 0$, the sign of the Jacobian's trace is determined by the terms inside the curly braces in Equation (25). Using the expression of Ω as in (27), it can be shown that $Tr < 0$ if and only if

$$(30) \quad \theta_I < \frac{\lambda(1+\gamma)(\alpha\delta - \rho)}{\delta[\alpha(1+\gamma) + (1-\alpha)^2] - \left(1 + \frac{\gamma\rho - \alpha\delta}{\rho + \delta}\right)[\rho + (1-\alpha)\delta]} - 1.$$

about the economy's future. This result thus allows us to set $\theta_c = 0$ for the remainder of this section.

Second, in accordance with the observed evidence that capital income accounts for a smaller percentage of GDP than labor income, our analyses are restricted to empirically plausible specifications with $\alpha < 1 - \alpha$. We then find that under this assumption, the Jacobian's determinant (24)

Finally, we cannot be certain whether (1) $\theta_I^{\min}$ given by (28) is strictly positive under all possible parameterizations; and (2) the right-hand side of (28) or (30) is more demanding because α , γ , δ , λ , and ρ enter these conditions in a rather complicated way. As a result, the necessary and sufficient condition for both eigenvalues of the Jacobian $\mathbf{J}$ to have negative real parts is given by³

$$(31) \quad \max\{\theta_I^{\min}, 0\} < \theta_I < \min\left\{\frac{\lambda}{\alpha} - 1, \frac{\lambda(1+\gamma)(\alpha\delta - \rho)}{\delta[\alpha(1+\gamma) + (1-\alpha)^2] - \left(1 + \frac{\gamma\rho - \alpha\delta}{\rho + \delta}\right)[\rho + (1-\alpha)\delta]} - 1\right\},$$

2. When the numerator and denominator of (29) are both negative, the resulting range of θ_I for $Det > 0$ is inconsistent with our maintained assumption $\alpha < 1 - \alpha$.

3. If $\frac{\lambda(1+\gamma)(\alpha\delta - \rho)}{\delta[\alpha(1+\gamma) + (1-\alpha)^2] - \left(1 + \frac{\gamma\rho - \alpha\delta}{\rho + \delta}\right)[\rho + (1-\alpha)\delta]} - 1 < \theta_I^{\min}$, then there exists no feasible range of investment externalities over which our model exhibits an indeterminate steady state. Accordingly, we rule out this possibility.

FIGURE 1
Local Stability Properties of the Benchmark Closed Economy

under which the model's steady state will become an indeterminate sink.

B. Quantitative Results

In this subsection, we undertake a quantitative investigation of macroeconomic (in)stability within a calibrated version of our two-sector RBC model for combinations of parameters whose values are selected based on empirically observed features of the post-Korean-war U.S. economy. In particular, the labor share of national income, $1 - \alpha$, is chosen to be 0.7; the subjective rate of time preference, ρ , is set to be 0.05; the labor supply elasticity, γ , is equal to 0 (i.e., indivisible labor, *à la* Hansen 1985 and Rogerson 1988, that is infinitely elastic); and the capital depreciation rate, δ , is fixed at 0.1.

Given the above benchmark parameterization, Figure 1 depicts the resulting local stability properties of our model economy as a function of the degree of productive externalities in investment versus the inverse level of intermediate-good firms' monopoly power. In particular, the $\phi_I - \lambda$ space is divided into regions of "Saddle", "Sink" and "Source." Using durables as a proxy for investment goods, we set the upper bound of investment externalities θ_I to be 0.33, which is Basu and Fernald's (1997) aggregation-corrected point estimate for returns-to-scale in the U.S. durables manufacturing industry, on the horizontal axis of Figure 1. In addition, we note that $\frac{1}{\lambda}$ is equal to the markup ratio of price over marginal cost, and that the range for its empirical

estimates lies between 1 and 1.7; see Hall (1986), Domowitz, Hubbard, and Petersen (1988), Morrison (1990), and Chirinko and Fazzari (1994), among others. Based on the midpoint of empirically plausible values on $\lambda \in [0.59, 1]$, we set the lower bound for the vertical axis of Figure 1 to $\lambda = 0.8$.

Result 1. *For a given level of $\lambda \in [0.95, 1]$, the economy's local dynamics changes from saddle-path stability to equilibrium indeterminacy, and then to complete instability as the degree of investment externalities increases. For a given level of $\lambda \in [0.85, 0.94]$, the model's steady state switches from being a sink to a source as θ_I rises. When $\lambda < 0.85$, the model's steady state is always a source.*

As an illustrative example, when the price-cost markup is set to be 1.03 (or $\lambda = 0.97$) *à la* Basu and Fernald (1997) for the U.S. total private economy, our model exhibits a locally unique equilibrium (a saddle path) for $0 \leq \theta_I \leq 0.0318$. Local indeterminacy occurs for investment externalities in the range of $0.0319 \leq \theta_I \leq 0.1412$.⁴ The steady state turns into a source when θ_I is raised to the interval of $[0.1413, 0.33]$. In this case, any trajectory that

4. When $\gamma = 1$ *à la* Benhabib and Farmer (1996), while keeping the calibrated values of α , ρ , δ , and λ unchanged, indeterminacy results under $0.1149 \leq \theta_I \leq 0.437$. As the labor supply elasticity becomes smaller (or γ rises), it will be more difficult for the household to move labor hours into the investment sector. It follows that endogenous business cycles are less likely to arise.

diverges from this completely unstable steady state may settle down to a limit cycle or to some complicated attracting sets.

Result 2. *When $0.85 \leq \lambda \leq 1$, the threshold level for productive externalities in the investment sector that leads to indeterminacy and sunspots, denoted as $\theta_I^{\min}$ given by (28), is monotonically increasing with respect to the degree of market competitiveness, that is, $\frac{\partial \theta_I^{\min}}{\partial \lambda} > 0$.*

To understand the intuition for this result, we note that the intertemporal consumption Euler equation in the discrete-time version of our model is given by

$$(32) \quad \frac{C_{t+1}}{C_t} = \beta \left[\frac{r_{t+1} + (1 - \delta)P_{t+1}}{P_t} \right],$$

where β is the discount factor. Start from the model's steady state at period t , and suppose that agents become optimistic about the economy's future. Acting upon this change in nonfundamental expectations, the representative household will consume less (C_t falls) and invest more today, which in turn raises next period's capital stock (K_{t+1} rises), hours worked, output, and consumption (C_{t+1} rises). As a result, the left-hand side of (32) becomes higher. For this alternative dynamic path to be justified as a self-fulfilling equilibrium, the (price-weighted) rate of return on K_{t+1} net of depreciation, that is, the right-hand side of (32), needs to increase as well.

As it turns out, the quantitative interdependence between θ_I and λ that governs our model's local stability properties depends crucially on the relative strength of two opposing forces. On one hand, an increase in today's investment that raises K_{t+1} will lead to a lower real interest rate r_{t+1} because of diminishing marginal product of capital ($\frac{\alpha(1+\theta_c)}{\lambda} < 1$; see Equation 21). Therefore, this *MPK effect* causes the right-hand side of (32) to fall. On the other hand, due to the presence of market imperfection as well as non-negative productive externalities in the investment sector, the economy's social production possibility frontier which traces out the trade-off between agents' consumption and investment expenditures is downward sloping and convex to the origin. It follows that its slope (or marginal rate of transformation), which is equal to the relative price of investment goods P_t , will decrease upon the household's optimism that shifts capital and labor inputs out of producing consumption

goods ($\frac{dP_t}{d\mu_t} > 0$; see Equation 20). Consequently, this *price effect* causes right-hand side of (32) to rise.

Results 1 and 2 together demonstrate that for a given level of intermediate-good producers' market power with $\lambda \geq 0.85$, the price effect quantitatively outweighs the *MPK* effect provided the degree of investment externalities is sufficiently high to satisfy the lower bound associated with condition (31). In this case, indeterminacy and sunspot result because the right-hand side of (32) will rise to validate agents' initial anticipated increase in the return on capital. It follows that reducing the investment externalities to be lower than the critical $\theta_I^{\min}$. ($=0.0319$ when $\lambda = 0.97$) is able to stabilize the economy against belief-driven business cycles because of a dominating *MPK* effect. Next, upon further examination of the vertical axis in Figure 1, we observe the following result:

Result 3. *When $0.85 \leq \lambda \leq 0.94$, the model's steady state is an indeterminate sink under no investment externalities ($\theta_I = 0$).*

Since consumption externality is set to zero ($\theta_c = 0$) per our earlier analysis, this result implies that equilibrium indeterminacy will occur without any productive externality in either sector when the degree of increasing returns to specialization (or the price-cost markup ratio) $\frac{1}{\lambda} \in [1.06, 1.18]$ is sufficiently high to generate a quantitatively stronger price effect. As pointed out by Kim (2004), the gains to product variety are a source of additional returns-to-scale in the economy's aggregate production technology. Using Equations (8), (10), (12), and (20), together with $\theta_c = \theta_I = 0$, it can be shown that the economy's period- t production possibility set is

$$(33) \quad Z_c^{1-\lambda} C_t^\lambda + Z_I^{1-\lambda} I_t^\lambda = \lambda^\lambda (1 - \lambda)^{1-\lambda} K_t^\alpha L_t^{1-\alpha}, \\ 0 < \lambda < 1,$$

which depicts a curve that is strictly convex to the origin in the positive quadrant. Figure 2 shows that when the market power of intermediate firms rises (i.e., λ falls), the curvature for the social production possibility frontier becomes more pronounced. As a consequence, for the same magnitude of reductions in capital and labor inputs that are moved out of the consumption sector because of agents' optimism, the resulting decrease in the relative price of investment goods P_t will be larger. This implies that the

FIGURE 2

Social Production Possibility Frontier

aforementioned price effect is strengthened as the intermediate-good markets become less competitive, which in turns may yield indeterminacy and sunspots under no investment externalities.

Finally, it is worth comparing the above numerical results of our model versus those from three previous settings that we have resolved: (1) Benhabib and Farmer's (1996) two-sector model which is a perfectly competitive formulation with $\lambda = 1$ and $Z_c = Z_I = 0$; (2) the one-sector model *à la* Chang, Hung, and Huang (2011) which is an imperfectly-competitive framework with $0 < \lambda < 1$, $P_t = 1$ (for all t) and an aggregate production function given by

$$(34) \quad Y_t = K_t^{\frac{\alpha(1+\theta_K)}{\lambda}} L_t^{\frac{(1-\alpha)(1+\theta_L)}{\lambda}},$$

where θ_K and θ_L represent positive productive externalities from capital and labor services, respectively; and (3) the Benhabib-Farmer-Guo one-sector model which is a perfectly competitive configuration with $\lambda = P_t = 1$, $Z_c = Z_I = 0$ and a social technology as in (34).

As discussed earlier under Result 1, our economy with $\lambda = 0.97$ possesses an indeterminate steady state when $0.0319 \leq \theta_I \leq 0.1412$. To place this quantitative result in perspective, Table 1 also reports the regions for local indeterminacy within the above-mentioned three preceding models, while keeping the calibrated values of α , ρ , γ , and δ unchanged. Notice that the minimum level of productive externalities needed

TABLE 1

Regions of Local Indeterminacy

Chang-Guo-Wang Two-Sector Model ($\lambda = 0.97$ and $P_t \neq 1$)	$0.0319 \leq \theta_I \leq 0.1412$
Benhabib-Farmer Two-Sector Model ($\lambda = 1$ and $P_t \neq 1$)	$0.0638 \leq \theta_I \leq 0.1764$
Chang-Hung-Huang One-Sector Model ($\lambda = 0.97$ and $P_t = 1$)	$0.3858 \leq \theta_L$
Benhabib-Farmer-Guo One-Sector Model ($\lambda = P_t = 1$)	$0.4286 \leq \theta_L$

for indeterminacy and sunspots is the smallest in our two-sector macroeconomy with increasing returns to product variety and investment externalities. Intuitively, Table 1 reflects two complementary factors in producing multiple, indeterminate equilibria within a RBC model. On the one hand, when production takes place in two distinct sectors, agents have the ability to reallocate resources between them to take advantage of increasing returns. Hence, by moving from the one-sector to the two-sector framework, the size of productive externalities required for equilibrium indeterminacy will fall. On the other hand, adding expansions to product variety with $\lambda \in (0, 1)$ raises the elasticity of output with respect to labor, as shown in Kim (2004, Section II.D) or Equations (21) and (34), such that the resulting price effect becomes stronger upon agents' self-fulfilling expectations. As a result, the Benhabib-Farmer-Guo one-sector model requires the highest external effect, whereas our two-sector model needs the lowest, and therefore the most empirically plausible, degree of increasing returns-to-scale in production to generate belief-driven cyclical fluctuations.

IV. EXTENSIONS

In this section, we will explore two extensions to re-examine the economy's local (in)stability properties (1) when the parameters that govern the degree of intermediate-good firms' monopoly power and the level of product variety are differentiated, as in Pavlov and Weder (2012); and (2) in the context of a small open economy *à la* Weder (2001). These extensions allow us to study the robustness of our theoretical and quantitative results obtained from the benchmark model, as well as further understand the precise mechanisms through which equilibrium indeterminacy may arise within a multi-sector representative-agent macroeconomy.

A. Separation between Monopoly Power and Product Variety

For the sake of analytical simplicity, our baseline model analyzed above adopts a single parameter λ to represent not only the magnitude of returns to specialization, but also the size of market power for monopolistically competitive firms. Similar to Pavlov and Weder (2012),⁵ this subsection examines an extended setting in which these two features are disentangled. In particular, the sectoral production functions for consumption and investment goods are modified to

$$(35) \quad Y_m = N_m^{1+\omega_m} \left(\frac{1}{N_m} \int_0^{N_m} x_{mj}^{\lambda_m} dj \right)^{\frac{1}{\lambda_m}},$$

$$m = c, I, \quad \omega_m \geq 0 \text{ and } 0 < \lambda_m \leq 1,$$

where λ_m determines the elasticity of substitution between intermediate inputs and ω_m measures the degree of increasing returns to specialization in sector m . As in Bénassy (1996) and Pavlov and Weder (2012), the parameters for intermediate-good producers' monopoly power (λ_c and λ_I) are now independent of the strength of the variety effects (ω_c and ω_I).

Next, we follow the same solution procedure as in Sections II and III to find that (1) the steady-state fraction of factor inputs allocated to producing the consumption good μ_{ss} and the household's labor supply L_{ss} are identical to those in (22); (2) the aggregate capital stock at the model's steady state becomes

$$(36) \quad \hat{K}_{ss} = \left[\frac{\lambda_I (1 - \mu_{ss})^{(1+\theta_I)(1+\omega_I)} \left(\frac{1-\lambda_I}{Z_I} \right)^{\omega_I} L_{ss}^{(1-\alpha)(1+\theta_I)(1+\omega_I)}}{\delta} \right]^{\frac{1}{1-\alpha(1+\theta_I)(1+\omega_I)}};$$

(3) the determinant and trace of the resulting Jacobian matrix are

$$(37) \quad Det = \frac{\alpha \mu_{ss} (1 - \mu_{ss}) (1 + \gamma) (\lambda_I \hat{\Pi} L_{ss})^2 [1 - \alpha(1 + \theta_I)(1 + \omega_I)]}{(1 + \theta_I)(1 + \omega_I) [1 - \alpha + (\alpha + \gamma) \mu_{ss}] - (1 + \gamma)},$$

$$(38) \quad Tr = \left(\frac{\hat{\Pi}}{1 - \mu_{ss}} \right) \left\{ (1 - \mu_{ss}) [\alpha(1 + \theta_I)(1 + \omega_I) - 1] + \frac{\alpha \hat{\Omega}}{(1 + \theta_I)(1 + \omega_I) [1 - \alpha + (\alpha + \gamma) \mu_{ss}] - (1 + \gamma)} \right\},$$

5. In Pavlov and Weder's (2012) two-sector RBC model, a unique set of intermediate inputs are used in the production of both consumption and investment goods; acyclical/countercyclical/procyclical markups are examined; and diminishing marginal costs are not considered in the sectoral production functions. By contrast, our model economy is characterized by differentiated sector-specific intermediate goods; constant markups; and the presence of capital/labor productive externalities.

where

$$(39) \quad \hat{\Pi} = (1 - \mu_{ss})^{(1+\theta_I)(1+\omega_I)-1} \left(\frac{1-\lambda_I}{Z_I} \right)^{\omega_I} \\ \times \hat{K}_{ss}^{\alpha(1+\theta_I)(1+\omega_I)-1} \\ \times L_{ss}^{(1-\alpha)(1+\theta_I)(1+\omega_I)-1} > 0,$$

$$(40) \quad \hat{\Omega} = (1 - \alpha)(1 + \theta_I)(1 + \omega_I)(1 - \mu_{ss}) \\ \times [1 - \alpha(1 + \theta_I)(1 + \omega_I)(1 - \mu_{ss})] + \mu_{ss}(1 + \gamma) \\ \times [(1 + \theta_I)(1 + \omega_I) - 1 - (1 - \mu_{ss})(1 + \theta_I)^2(1 + \omega_I)^2] \\ \geq 0;$$

and (4) the necessary and sufficient condition for this extended model to exhibit an indeterminate steady state is given by

(41)

$$\max\{\omega_I^{\min}, 0\} < \omega_I < \min \left\{ \frac{1}{\alpha(1+\theta_I)} - 1, \frac{(1+\gamma)(\alpha\delta - \rho)}{(1+\theta_I) \left\{ \delta[\alpha(1+\gamma) + (1-\alpha)^2] - \left(1 + \frac{\gamma\rho - \alpha\delta}{\rho + \delta}\right) [\rho + (1-\alpha)\delta] \right\}} - 1 \right\},$$

where $\omega_I^{\min} = \frac{(1+\gamma)(\rho+\delta)}{(1+\theta_I)\{\alpha\delta(1-\alpha) + (1+\gamma)[\rho + (1-\alpha)\delta]\}} - 1$ denotes the minimum degree of product variety for investment goods that may lead to equilibrium indeterminacy.⁶

The intuition for the above indeterminacy result can be understood as follows. First, as in Weder (2000), Harrison (2001), Pavlov and Weder (2012), and our benchmark model, no market imperfection is needed in the consumption sector to help fulfill agents' optimistic expectation of a higher rate of return from today's investment spurt. It follows that the consumption-specific parameters $\{\lambda_c, \omega_c, \theta_c\}$ have no bearing on the model's local stability properties. Second, as pointed out by Kim (2004, Section III), the extent of firms' acyclical market power will not affect equilibrium dynamics within a monopolistically competitive RBC model, such as ours or Pavlov and Weder's (2012, Section III.A), under time-invariant setup costs (i.e., Z_c and Z_I) and zero profits at each instant of time. As a result, the markup of price over marginal cost for investment firms λ_I does not matter to macroeconomic (in)stability either. Third, condition (41) illustrates two channels

from the investment sector that are capable of contributing to the occurrence of indeterminacy and sunspots: *ceteris paribus* either an increase

in the strength of variety effects ω_I or an increase in the degree of capital/labor externalities θ_I will endogenously enhance the economy's overall production efficiency.⁷ It is also straightforward to observe that $\frac{\partial \omega_I^{\min}}{\partial \theta_I} < 0$, indicating the presence of a trade-off between these two investment-based parameters in generating nonuniqueness of competitive equilibria. It follows that equilibrium indeterminacy can take place within our extended two-sector macroeconomy when ω_I or θ_I alone is sufficiently high. Using the same benchmark calibrated values of α , ρ , γ and δ as those in Section III, we find that this is indeed the case under θ_I (ω_I) = 0 and $0.0638 \leq \omega_I$ (θ_I) ≤ 0.1764 .⁸

In sum, this subsection analytically shows that once intermediate-good producers' monopoly power is disconnected from the variety effects in our two-sector RBC model, it is the inverse interrelations between two investment-specific parameters—the degree of increasing returns to specialization ω_I and the magnitude of productive externalities θ_I —that will govern the economy's equilibrium dynamics. Interestingly,

6. Alternatively, the indeterminacy condition (41) can be rewritten as

$$\max\{\theta_I^{\min}, 0\} < \theta_I < \min \left\{ \frac{1}{\alpha(1+\omega_I)} - 1, \frac{(1+\gamma)(\alpha\delta - \rho)}{(1+\omega_I) \left\{ \delta[\alpha(1+\gamma) + (1-\alpha)^2] - \left(1 + \frac{\gamma\rho - \alpha\delta}{\rho + \delta}\right) [\rho + (1-\alpha)\delta] \right\}} - 1 \right\},$$

where $\theta_I^{\min} = \frac{(1+\gamma)(\rho+\delta)}{(1+\omega_I)\{\alpha\delta(1-\alpha) + (1+\gamma)[\rho + (1-\alpha)\delta]\}} - 1$.

7. Pavlov and Weder (2012, Propositions 1 and 2) obtain the qualitatively identical result in their model under constant markups, as in our setting. Since these authors do not consider productive extremities of capital and labor inputs ($\theta_c = \theta_I = 0$), the variety effect in investment ω_I must be strictly positive and sufficiently strong for equilibrium indeterminacy to arise.

8. Since ω_I and θ_I enter condition (41), or its alternative expression given by footnote 6, in a symmetric manner, these parametric regions for indeterminacy exhibit symmetry as well.

numerical experiments find that indeterminacy can prevail without ω_I and θ_I both being strictly positive. In addition, the markup parameters for consumption as well as for investment (λ_c and λ_I) no longer play any role in affecting the model's local stability properties. Although not derived here for space limitation, it can be shown that this market-power irrelevance result will also hold in the one-sector setting of Chang, Hung, and Huang (2011) or in the two-sector formulation of Weder (2000) when elasticity of substitution between intermediate inputs and product variety are separated.

B. Small Open Economy

For the small-open-economy variant of our benchmark model without separating monopoly power versus product variety, we follow Weder (2001, Section II) and postulate that consumption goods are tradeable whereas investment goods are nontradeable. Moreover, agents have access to international borrowing and lending at an exogenously given world real interest rate $r^* > 0$. It follows that the budget constraint faced by the representative household is modified to

$$(42) \quad C_t + P_t I_t + r^* B_t = r_t K_t + w_t L_t + \int_0^{N_{ct}} \pi_{cjt} dj + \int_0^{N_{It}} \pi_{Ijt} dj + \dot{B}_t, \quad B_0 \neq 0 \text{ is given,}$$

where B_t is foreign debt. In addition, the equalities of demand by households and supply by firms for tradeable/nontradeable consumption/investment goods are given by $C_t + r^* B_t = Y_{ct} + \dot{B}_t$ and $I_t = Y_{It}$. Under the standard

assumption that the world interest rate remains stationary, the following knife-edge condition will hold: $r_t - \delta = r^* = \rho$.⁹

Using ϕ_t and η_t to denote the shadow values of physical capital and foreign assets, respectively, it is straightforward to show that the first-order conditions for the household's dynamic optimization problem are

$$(43) \quad \frac{1}{C_t} = \eta_t,$$

$$(44) \quad \phi_t = \eta_t P_t,$$

$$(45) \quad \dot{\eta}_t = 0,$$

together with Equations (17) on labor supply and (18) on intertemporal consumption choices; and two transversality conditions: $\lim_{t \rightarrow \infty} \phi_t K_t e^{-\rho t} = 0$ and $\lim_{t \rightarrow \infty} \eta_t B_t e^{-\rho t} = 0$. Since Equation (45) results in a constant shadow price of foreign bonds, condition (43) implies complete consumption smoothing over time. Hence, there will be no consumption fluctuation along the economy's equilibrium path.

In this small-open-economy environment, the steady-state fraction of factor inputs allocated to the domestically produced consumption good μ_{ss} continues to be the same as that in (22). Moreover, the time-invariance of foreign debt's shadow value leads to $B_t = B_{ss}$, which in turn is equal to its arbitrarily given (nonzero) initial level B_0 , for all t . It can then be shown that at the model's steady state, the household's hours worked $\tilde{L}_{ss}$ and consumption spending $\tilde{C}_{ss}$ are jointly determined by

(46)

$$\tilde{L}_{ss} = \left\{ \frac{\lambda(1-\alpha)}{\tilde{C}_{ss}} \left(\frac{1-\lambda}{Z_c} \right)^{\frac{1-\lambda}{\lambda}} \mu_{ss}^{\frac{1+\theta_c-\lambda}{\lambda}} \left[\frac{\lambda}{\delta} \left(\frac{1-\lambda}{Z_I} \right)^{\frac{1-\lambda}{\lambda}} (1-\mu_{ss})^{\frac{1+\theta_I}{\lambda}} \right]^{\frac{\alpha(1+\theta_c)}{\lambda-\alpha(1+\theta_I)}} \right\}^{\frac{\lambda-\alpha(1+\theta_I)}{(1+\gamma)[\lambda-\alpha(1+\theta_I)]-(1-\alpha)(1+\theta_c)}},$$

9. Weder's (2001, Section II) two-sector small-open-economy RBC model is characterized by perfectly competitive markets, inelastic labor supply, and the presence of economy-wide externalities from aggregate factors of production. By contrast, our model economy exhibits monopolistically competitive firms, variable labor hours, and productive externalities emanated only from sector-specific capital/labor inputs.

and

$$(47) \quad \tilde{C}_{ss} = \frac{\rho B_0(1-\alpha)}{\mu_{ss} \tilde{L}_{ss}^{1+\gamma} - (1-\alpha)};$$

and that the economy's steady-state aggregate capital stock is
(48)

$$\tilde{K}_{ss} = \left[\frac{\lambda(1-\mu_{ss})^{\frac{1+\theta_I}{\lambda}} \left(\frac{1-\lambda}{Z_I} \right)^{\frac{1-\lambda}{\lambda}} \tilde{L}_{ss}^{\frac{(1-\alpha)(1+\theta_I)}{\lambda}}}{\delta} \right]^{\frac{\lambda}{\lambda-\alpha(1+\theta_I)}}.$$

Since (46)–(47) is a highly nonlinear simultaneous equation system that cannot be solved analytically, our subsequent quantitative analyses will be restricted to parametric configurations which yield a unique steady state with positive $\tilde{L}_{ss}$ and $\tilde{C}_{ss}$.

Next, we linearize the model's equilibrium conditions around this steady state to obtain a dynamical system with $\{K_t, \phi_t\}$ as the state vector. As in Weder (2001), this two-dimensional system of differential equations is block-recursive, hence it is independent of the household's consumption decision and foreign debt holding. After some tedious but manageable algebra, we find that the determinant and trace of the resulting Jacobian matrix are given by

$$(49) \quad \underbrace{Det = -\alpha\mu_{ss}\lambda^3 \left(\frac{1-\lambda}{Z_I} \right)^{\frac{2(1-\lambda)}{\lambda}} (1-\mu_{ss})^{\frac{2(1+\theta_I)-\lambda}{\lambda}} \tilde{K}_{ss}^{\frac{2[\alpha(1+\theta_I)-\lambda]}{\lambda}} \tilde{L}_{ss}^{\frac{2(1-\alpha)(1+\theta_I)-\lambda}{\lambda}} \cdot \Phi}_{\text{Negative}},$$

$$(50) \quad \underbrace{Tr = \lambda^2 \left(\frac{1-\lambda}{Z_I} \right)^{\frac{1-\lambda}{\lambda}} (1-\mu_{ss})^{\frac{1+\theta_I}{\lambda}} \tilde{K}_{ss}^{\frac{\alpha(1+\theta_I)-\lambda}{\lambda}} \tilde{L}_{ss}^{\frac{(1-\alpha)(1+\theta_I)}{\lambda}} \cdot \frac{\Psi}{\Delta}}_{\text{Positive}},$$

where

$$(51) \quad \Phi = \alpha(1-\alpha)(\alpha+\gamma)(1+\theta_c)(\theta_c-\theta_I)(1-\mu_{ss}) + [\lambda(1+\gamma) - (1-\alpha)(1+\theta_c)] \cdot \Gamma \geq 0,$$

$$\Gamma = (\alpha+\gamma)[\alpha(1+\theta_I) - \lambda][1+\theta_I - \lambda + (1-\mu_{ss})(\theta_c-\theta_I)] + (1-\alpha)(1+\theta_I - \lambda)[\alpha(1+\theta_I) + 1 + \theta_c - \lambda],$$

$$(52) \quad \Psi = \alpha\mu_{ss}(1+\theta_I)[\lambda(1+\gamma) - (1-\alpha)(1+\theta_c)] \\ + (1-\mu_{ss})\{\Gamma + \alpha(1+\theta_I)[(\alpha+\gamma)(\theta_c-\theta_I)\mu_{ss} \\ - (1-\alpha)(1+\theta_I - \lambda)]\} \geq 0,$$

$$(53) \text{ and } \Delta = [\lambda(1+\gamma) - (1-\alpha)(1+\theta_c)]$$

$$\times [1 - \lambda + \theta_c(1 - \mu_{ss}) + \theta_I\mu_{ss}] + (1 - \alpha) \\ \times (\theta_c - \theta_I)(1 - \mu_{ss})(1 + \theta_c - \lambda) \geq 0.$$

As a result, the necessary and sufficient conditions for indeterminacy and sunspots are (1) $\Phi < 0$ (hence $Det > 0$), and (2) $\frac{\Psi}{\Delta} < 0$ (hence $Tr < 0$). In sharp contrast to our closed-economy specifications *à la* Sections III and IV.A, Equations (49)–(53) show that the level of consumption-specific productive externalities θ_c now appears in the Jacobian matrix and thus affects the model's equilibrium dynamics. Since model parameters enter the expressions of Φ , Ψ , and Δ in a rather complicated manner, we will conduct numerical experiments to quantitatively explore the economy's local stability properties.

Under identical calibrated values of α , ρ , γ , and δ as those in the closed-economy counterpart, together with the price–cost markup fixed at 1.03 (or $\lambda = 0.97$), Figure 3 divides the $\theta_I - \theta_c$ space into regions of “Saddle”, “Sink” and “Source,” where $\theta_c \leq 0 \leq \theta_I$ per the parametric restriction

FIGURE 3
Local Stability Properties of the Small Open Economy

of Weder (2001) with perfect competition and inelastic labor supply. In particular, combining (44) and (45) leads to $\phi_t = \eta_t \dot{P}_t$, which can then be used to rewrite the consumption Euler Equation (18) as

$$(54) \quad \frac{\dot{P}_t}{P_t} + \frac{r_t}{P_t} - \delta = \rho.$$

Upon the household's optimistic anticipation of a higher rate of return from today's investment, the relative price of investment goods will fall ($\dot{P}_t < 0$) as capital and labor inputs move out of the consumption sector. Unlike within a closed economy, agents do not need to curtail their current consumption because they have access to a perfect international bond market, which in turn leads to time-invariant paths of η_t and C_t . To justify this alternative trajectory as a self-fulfilling equilibrium, the investment-good price must rise along the transition path reverting the economy back to its original steady state. It follows that the overall returns to physical capital, as shown in the left-hand side of (54), will be maintained to equal the household's utility discount rate ρ . Since the existence of positive investment externalities ($\theta_I > 0$) contributes to the initial decrease in P_t , the requisite reversal for price appreciation calls for a dampening effect that can be generated by negative productive externalities in the consumption sector ($\theta_C < 0$). When such an opposite movement does not take place because of zero consumption externalities, the continual decline of P_t induces further increases in investment

activity that will eventually violate the transversality condition on capital accumulation. As a result, along the horizontal axis of Figure 3 with $\theta_C = 0$, the model's steady state becomes a totally unstable source that is surrounded by infeasible explosive trajectories in its neighborhood.

On the other hand, the vertical axis of Figure 3 shows that equilibrium indeterminacy can occur under no investment externalities ($\theta_I = 0$) together with $-0.117 \leq \theta_C \leq -0.106$. Intuitively, within our small-open-economy model subject to a constant world real interest rate, the presence of increasing returns to product variety or intermediate-input producers' market power (*i. e.* $\lambda \neq 1$) alone will cause the relative price of investment goods to decrease in response to agents' rosy expectations. Per the preceding paragraph, negative consumption-specific productive externalities are needed to reverse this price depreciation in order to validate the household's initial optimism. The above discussions also imply that when the degree of investment externalities rises and becomes more positive, the threshold level of $\theta_C < 0$ which leads to an indeterminate steady state will fall and become more negative ($\frac{\partial \theta_C^{\min}}{\partial \theta_I} < 0$), as illustrated by the downward-sloping curve for the lower bound of the "Sink" region in Figure 3. Finally, this subsection confirms the general point that a small open economy is more susceptible to indeterminacy and sunspots than its closed-economy counterpart; see Lahiri (2001), Weder (2001), and Meng and Velasco (2003, 2004), among

others. Since the representative agent is able to completely smooth its consumption over time through international lending and borrowing, it will take a relatively smaller degree of market imperfections in technology, such as monopolistic competition and/or productive externalities, for a small-open-economy model to exhibit a continuum of equilibrium paths with endogenous business cycle fluctuations.

V. CONCLUSION

This paper has examined how the theoretical as well as quantitative interrelations between the level of increasing returns to product variety versus the degree of sector-specific productive externalities affect the equilibrium dynamics of a RBC model with two distinct production sectors: consumption and investment. We analytically derive the necessary and sufficient condition for the baseline closed-economy formulation to possess an indeterminate steady state and thus a continuum of stationary sunspot equilibrium paths. Under a benchmark parameterization that is consistent with post-Korean-war U.S. time series data, the minimum magnitude of investment externalities that leads to endogenous business cycles is shown to be monotonically increasing with respect to the degree of market competitiveness. We also find that when intermediate-good firms' monopoly power is sufficiently strong, local indeterminacy may arise in our model without any externality for producing investment goods. Finally, in comparison with three predecessors that we consider, our two-sector macroeconomy requires the lowest, and therefore, the most empirically plausible, magnitude of productive externalities to generate belief-driven aggregate fluctuations. In terms of sensitivity analyses, we examine the model's local (in)stability properties (1) when the parameters that govern the degree of intermediate-input producers' market power and the strength of variety effects are separated; and (2) in the context of a small open economy. These extensions allow us to study the robustness of our theoretical and quantitative results obtained from the benchmark setting, as well as further understand the precise mechanisms that may yield multiple equilibria within RBC models.

This paper can be extended in several directions. In particular, it would be worthwhile to incorporate additional features that have been shown to influence macroeconomic (in)stability properties in a two-sector RBC model, such

as differentiated capital-labor intensities in the sectoral production functions *à la* Meng and Velasco (2003, 2004), no-income-effect preferences *à la* Guo and Harrison (2010), government spending on goods and services *à la* Chang et al. (2015, 2019), and progressive income taxation *à la* Guo and Harrison (2015), among others. These possible extensions will enhance our understanding of how model parameters and/or fiscal policy rules govern the region of local indeterminacy within a multisector representative-agent macroeconomy. We plan to pursue these research projects in the near future.

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