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APPLICATION OF THE METHOD OF INTEGRAL RELATIONS
TO THE CALCULATION OF TWO-DIMENSIONAL INCOMPRESSIBLE
TURBULENT BOUNDARY LAYER

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ABSTRACT

The orthonormal version of the Method of Integral Relations (MIR) was applied to solve for a two-dimensional incompressible turbulent boundary layer. The flow was assumed to be nonseparating. Flows with favorable, unfavorable, and zero pressure gradient were considered, and comparisons made with available experimental data. In general, the method predicted very well the experimental results for flows with favorable or zero pressure gradient; for flows with unfavorable pressure gradient, it predicted the experimental data well only up to a certain distance from the initial station. This result is due to the flow not being in equilibrium beyond that distance. Finally, the scheme was shown to be efficient in obtaining numerical solutions.

Introduction

In the vast body of literature dealing with turbulent boundary layer calculations, two major methods have been employed: the so-called integral method and the differential method. In this report, we investigate a particular solution procedure for the two-dimensional incompressible turbulent boundary layer via the Method of Integral Relations (MIR). In particular, the efficiency and versatility of this method, as applied to turbulent boundary layer calculations, is studied.

Formulation

The method of integral relations has previously been applied to two-dimensional turbulent boundary layer calculations by Abbott and Deiwert [1] and by Murphy and Rose [2] at the 1968 Stanford Conference on turbulent boundary layer calculations. Unfortunately, it was found inferior to other prediction methods presented at the same conference. Here, we shall reformulate the problem by using the modified version of MIR developed by Fletcher and Holt [3]. As a result, most of the shortcomings indicated by Murphy and Rose are overcome.

We start with the two-dimensional incompressible boundary layer equations in the usual notations:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left[(\nu + \frac{\epsilon}{\rho}) \frac{\partial u}{\partial y} \right] \quad , \quad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad , \quad (2)$$

where u and v are the mean velocity components in the x and y directions respectively and p is the mean pressure; ρ is the density of the fluid and ν its kinematic viscosity. The eddy viscosity, ϵ , is defined as

$$-\rho \overline{u'v'} = \varepsilon \frac{\partial u}{\partial y}, \quad (3)$$

where $-\rho \overline{u'v'}$ is the so-called Reynolds stress. The following non-dimensional variables are chosen.

$$U = \frac{u}{u_e}, \quad V = \frac{v \text{Re}^{1/2}}{u_e}, \quad \bar{x} = \frac{x}{L}, \quad \bar{y} = \frac{y \text{Re}^{1/2}}{L}, \quad U_e = \frac{u_e}{u_\infty}, \quad (4)$$

where L and u_∞ are the characteristic length and velocity scales respectively, u_e is the velocity at the outer boundary layer edge, and the Reynolds number Re is defined as

$$\text{Re} = \frac{u_\infty L}{\nu}. \quad (5)$$

The pressure gradient is related to the outer edge velocity by Bernoulli's equation:

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = u_e \frac{du_e}{dx}. \quad (6)$$

Using (4) and (6), the original governing equations (1) and (2) become

$$U \frac{\partial U}{\partial \bar{x}} + V \frac{\partial U}{\partial \bar{y}} = \frac{1}{U_e} \frac{dU_e}{d\bar{x}} (1-U^2) + \frac{1}{U_e} \frac{\partial}{\partial \bar{y}} \left[\left(1 + \frac{\varepsilon}{\mu}\right) \frac{\partial U}{\partial \bar{y}} \right], \quad (7)$$

$$\frac{\partial U}{\partial \bar{x}} + \frac{\partial V}{\partial \bar{y}} = -\frac{U}{U_e} \frac{dU_e}{d\bar{x}}. \quad (8)$$

To apply the Method of Integral Relations, we define a set of linearly independent functions $\left\{f_i(U)\right\}$ such that

$$f_i(1) = 0, \quad i = 1, 2, \dots, N, \quad (9)$$

where N is the order of approximation.

Multiply equation (8) by f_i and equation (7) by f_i' , defined as the first derivative of f_i with respect to U , add and integrate from $\bar{y} = 0$ to $\bar{y} \rightarrow \infty$:

$$\begin{aligned} \frac{\partial}{\partial \bar{x}} \int_0^\infty f_i U d\bar{y} &= \frac{1}{U_e} \frac{dU_e}{d\bar{x}} \int_0^\infty f_i' (1-U^2) d\bar{y} - \frac{1}{U_e} f_i'(0) \left(\frac{\partial U}{\partial \bar{y}} \right)_0 \\ &\quad - \frac{1}{U_e} \int_0^\infty \left(1 + \frac{\varepsilon}{\mu} \right) \left(\frac{\partial U}{\partial \bar{y}} \right)^2 f_i'' d\bar{y} - \frac{1}{U_e} \frac{dU_e}{d\bar{x}} \int_0^\infty U f_i d\bar{y} . \end{aligned} \quad (10)$$

Change the variable of integration from $\bar{y}$ to U and define Z as:

$$Z = \left(\frac{\partial U}{\partial \bar{y}} \right)^{-1} , \quad (11)$$

and equation (10) can be written as:

$$\begin{aligned} \frac{\partial}{\partial \bar{x}} \int_0^1 f_i U Z dU &= \frac{1}{U_e} \frac{dU_e}{d\bar{x}} \int_0^1 \left[(1-U^2) f_i' - U f_i \right] Z dU \\ &\quad - \frac{1}{U_e} f_i'(0) \frac{1}{Z_0} - \frac{1}{U_e} \int_0^1 \left(1 + \frac{\varepsilon}{\mu} \right) \frac{f_i''}{Z} dU . \end{aligned} \quad (12)$$

Equation (12) is the basic integral relation in the present analysis.

We now further define $\{f_i\}$ as a set of orthonormal functions constructed from the Dorodnitsyn functions $(1-U)^k$, $k=1,2,3,\dots$ by the Gram-Schmidt procedure [4]. Hence:

$$f_i(U) = \sum_{k=1}^i c_{ik} (1-U)^k , \quad (13)$$

and

$$\int_0^1 f_k f_j \frac{U}{1-U} dU = \delta_{kj} , \quad (14)$$

with δ_{kj} being the Kronecker delta. Z is then represented by:

$$Z = \frac{b_0 + \sum_{j=1}^{N-1} b_j f_j(U)}{1-U} \quad (15)$$

where $b_0, b_1, b_2, \dots$, are unknown coefficients to be found. Substitute (15) into (12) and invoking (14), we obtain an explicit set of ordinary differential equations in the coefficients $b_0, b_1, \dots, b_{N-1}$:

$$\begin{aligned} \frac{db_0}{d\bar{x}} \int_0^1 \frac{f_i U}{1-U} dU + \frac{db_i}{d\bar{x}} = \frac{1}{U_e} \frac{dU_e}{d\bar{x}} \int_0^1 \left[(1-U^2) f_i' - U f_i \right] Z dU \\ - \frac{1}{U_e} \frac{f_i'(0)}{Z_0} - \frac{1}{U_e} \int_0^1 \left(1 + \frac{\varepsilon}{\mu} \right) \frac{f_i''}{Z} dU, \quad i=1, 2, \dots, N-1 \end{aligned} \quad (16a)$$

and

$$\begin{aligned} \frac{db_0}{d\bar{x}} \int_0^1 \frac{f_N U}{1-U} dU = \frac{1}{U_e} \frac{dU_e}{d\bar{x}} \int_0^1 \left[(1-U^2) f_N' - U f_N \right] Z dU \\ - \frac{1}{U_e} f_N'(0) \frac{1}{Z_0} - \frac{1}{U_e} \int_0^1 \left(1 + \frac{\varepsilon}{\mu} \right) \frac{1}{Z} f_N'' dU \end{aligned} \quad (16b)$$

which can be integrated subject to appropriate initial conditions at some station $\bar{x} = \bar{x}_1$. We shall discuss the initial conditions later.

The above formulation is similar to that in a laminar boundary layer (Dorodnitsyn, [5]). Apart from the difficulty of evaluating the integral

$$\int_0^1 \left(1 + \frac{\varepsilon}{\mu}\right) \frac{1}{Z} f_i''(U) dU, \quad ,$$

in which some kind of eddy viscosity model for ε has to be used, one problem is that the approximation of Z given by (15) is not accurate unless N is rather large. This is mainly due to the highly inflected velocity profile in a turbulent boundary layer as opposed to the smooth profile in the laminar case. Thus, Murphy and Rose [2] pointed out that the usual sequence $(1-U)^j$, $j=1,2,\dots$ is not quite satisfactory, even when a four-parameter profile is used (i.e., $N=4$). Instead, they judiciously assigned much larger exponents to the factor $(1-U)$ and determined in the course of their numerical solution the optimum values of those exponents. This of course destroys the completeness requirement of the original method of integral relations, and it does not guarantee in principle that the approximation will converge to the exact solution if the order of approximation tends to infinity. The sole purpose of their scheme is, to our knowledge, to better represent the Z profile in a turbulent boundary layer while using as few parameters as possible, since the traditional method of integral relations is quite impractical when too many parameters are used. However, the present orthonormal version circumvents this "high order" difficulty and enables one to preserve the completeness requirement while still using as many parameters as possible to represent faithfully the Z profile. Indeed, it has been applied to the two-dimensional laminar boundary layer to as high as $N=15$ with little computer expense [3]. It is therefore believed that the present formulation would be efficient even for large values of N .

Turbulence Modeling

In order to calculate the last integrals in equations (16a) and (16b), a model for the eddy viscosity ε/μ is needed. We use the following expression for the wall region:

$$\frac{\varepsilon}{\mu} = 0.04432 \left[e^{0.4u^+} - 1 - 0.4u^+ - 0.08 u^{+2} \right] , \quad (17)$$

which was first deduced by Spalding [6] and later found independently by Kleinstein [7]. For the wake region, the Clauser model [8] is used.

$$\frac{\varepsilon}{\mu} = 0.0168 \text{Re}_{\delta}^* \quad (18)$$

where $u^+ = u/u_{\tau}$ and $u_{\tau} = \sqrt{\tau_w/\rho}$, the wall frictional velocity. Re_{δ}^* is defined as

$$\text{Re}_{\delta}^* = \frac{u_e \delta^*}{\nu} \quad (19)$$

with δ^* being the usual displacement thickness.

The wall shear stress, τ_w , is given by:

$$\tau_w = (\mu + \varepsilon)_0 \left(\frac{\partial u}{\partial y} \right)_0 \quad (20)$$

where the subscript 0 denotes the wall.

Experimental data shows that the wall region extends to where the non-dimensional velocity U assumes a value of roughly 0.7 (Bradshaw, [9]). In general, we denote this value by U_m . The determination of U_m will be discussed later. Hence the last integral in equation (16a) and (16b) can be evaluated as

$$\int_0^1 (\dots) dU = \int_0^{U_m} (\dots) dU + \int_{U_m}^1 (\dots) dU , \quad (21)$$

where (...) represents symbolically the integrand.

Before discussing the method used to calculate (21), we first derive expressions of u^+ and y^+ in terms of the present variables Z and U . We begin from equation (20):

$$\tau_w = (\mu + \varepsilon)_0 \left(\frac{\partial u}{\partial y} \right)_0 .$$

Since $\varepsilon = 0$ at $\bar{y} = 0$, we have:

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_0 .$$

Using the definition of Z and the nondimensional variables defined in (4), it can be shown that:

$$\begin{aligned} \frac{\tau_w \sqrt{Re}}{\rho u_\infty^2} &= \frac{U_e}{Z_0} \\ \text{or} \quad \left(\frac{u_\tau}{u_\infty} \right)^2 &= \frac{U_e}{Z_0 Re^{1/2}} . \end{aligned} \quad (22)$$

Hence,

$$u^+ = U \sqrt{U_e Z_0 Re^{1/2}} \quad (23)$$

and

$$y^+ = \sqrt{\frac{U_e Re^{1/2}}{Z_0}} \bar{y} . \quad (24)$$

Thus the eddy viscosity can be written in terms of Z and U as follows:

For $0 \leq U < U_m$

$$\frac{\varepsilon}{\mu} = 0.04432 \left[e^{0.4U \sqrt{U_e Z_0 Re^{1/2}}} - 1 - 0.4 U \sqrt{U_e Z_0 Re^{1/2}} - 0.08 U^2 U_e Re^{1/2} Z_0 \right], \quad (25)$$

and for $U_m \leq U \leq 1$

$$\frac{\varepsilon}{\mu} = 0.0168 U_e Re^{1/2} \int_0^1 (1-U) Z dU \quad (26)$$

and the displacement thickness δ^* has been written as

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_e}\right) dy = \frac{L}{Re^{1/2}} \int_0^1 (1-U)ZdU \quad (27)$$

Evaluation of Integrals

The integrals in equations (16a) and (16b) can be computed exactly except for the shear integrals, which appear as the last term in the equations. At present, these shear integrals are evaluated by using a method based on the Gaussian-Legendre rule whose algorithm is available in the Sandia Mathematical Library.

Determination of U_m

Up to now, we have only denoted the outer boundary of the wall region by U_m . As mentioned before, experimental results indicate that U_m is roughly 0.7. However, this value of U_m may not produce a continuous curve for ε/μ across the whole boundary layer. To be consistent with the present formulation, then, U_m must be found by determining the intersection of the ε/μ in the inner region with that in the outer region. Hence, equating the two expressions (25) and (26), we have:

$$\begin{aligned} & 0.04432 \left[e^{0.4U} \sqrt{U_e Z_0 Re^{1/2}} - 1 - 0.4U \sqrt{U_e Z_0 Re^{1/2}} - 0.08U^2 U_e Z_0 Re^{1/2} \right] \\ & = 0.0168 U_e Re^{1/2} \int_0^1 (1-U)ZdU \quad (28) \end{aligned}$$

The above equation can be solved iteratively to yield U_m . The advantage of using Spalding's model becomes clear: this model is explicit in U and hence makes it easier to determine U_m . The solution is sought in the neighborhood of 0.7. In the case of flow with adverse pressure gradient, the wake region becomes substantial and U_m is found to be smaller than that of the flow with

favorable pressure gradient or without pressure gradient. Once U_m is determined, the shear integrals can be evaluated as mentioned earlier.

Initial Conditions

The system of ordinary differential equations (ODE) resulting from our formulation of the MIR must be provided with initial conditions on the parameters b_0 and b_j at some initial station, which can be taken as zero by suitable translation of the origin. Unlike laminar boundary layer flows, where the initial conditions are usually given by exact solutions (similarity solution), we have to rely on experimental data to determine the appropriate initial conditions. The easiest way in the present formulation is to calculate these initial values from experimental data. In general, among the experimental data, velocity profile U , boundary layer thickness δ , displacement thickness δ^* , momentum thickness θ , and skin-friction coefficient C_f are available. One then faces the problem of choosing the most "effective" data. Our suggestion on these choices is as follows:

- (a) The skin-friction coefficient, C_f , is the most effective for ensuring the correct behavior of the velocity profile at the wall.
- (b) Second in effectiveness is δ , the boundary layer thickness. Although equation (15) satisfies the boundary conditions $\partial U / \partial \bar{y} \rightarrow 0$ automatically as $\bar{y} \rightarrow \infty$, the value of the boundary layer thickness would not generally match the experimental data. Hence, to match the boundary layer thickness is to secure a more correct boundary condition at the outer edge of the layer.
- (c) Displacement thickness, δ^* , is our third choice of data. In (a) and (b), we specify the boundary conditions at the wall and at the outer edge of the layer. By matching the displacement thickness, we expect to

get a correct global behavior of the velocity profile in the boundary layer.

It was found that using the above three conditions (i.e., C_f , δ , δ^* , for $N = 3$) to compute the flow over a flat plate, a very satisfactory result was obtained. In the case of higher approximations, more data are needed. Generally, a trial-and-error procedure is needed to choose other "effective" data. However, one can pinpoint these data after several attempts by noting that good initial conditions should behave as $|b_0| > |b_1| > |b_2| \dots$ in equation (15). Since $U = 0$ at the wall, equation (15) becomes:

$$Z_0 = b_0 + \sum_{j=1}^{N-1} b_j f_j(0) = b_0 + \sum_{j=1}^{N-1} b_j \left(\sum_{k=1}^j c_{jk} \right) \quad (29)$$

In the course of constructing orthonormal functions from $(1-U)^k$ by the Gram-Schmidt procedure, we found that

$$\left| \sum_{k=1}^{j-1} c_{jk} \right| < \left| \sum_{k=1}^j c_{jk} \right|$$

Intuitively, the contribution to Z from the subsequent term in equation (29) should become smaller as the order of approximation gets higher.

Therefore, as noted, we expect that good initial conditions should yield

$$|b_0| > |b_1| > |b_2| > \dots^*$$

Results and Discussion

Three kinds of flow are tested by the present formulation: (1) flow with zero pressure gradient, (2) flow with adverse pressure gradient, and (3)

* In our calculations, however, this condition was not always met.

flow with favorable pressure gradient. Results for each case are plotted and compared with the experimental data that can be found in Cole and Hirst[10].

Flow with zero pressure gradient (ID 1400).[†] Figures 1 to 5 show comparisons between the predicted and measured values of the various flow quantities. Surprisingly good results were obtained with the approximation $N = 3$ just by using the initial conditions that matched C_f , δ , and δ^* . This supports the "effectiveness" of the data C_f , δ , and δ^* in the determination of initial conditions. The computer time of this approximation is about 5 sec in a CDC-7600 machine. Results of the approximations for $N = 4$ and $N = 5$ also show excellent agreement with measured values.

It should be pointed out that the CPU time for $N = 5$ (45 sec) is much greater than that for $N = 4$ (8 sec). This is due to the stiffness of the system of ODE's in equations (16a) and (16b), in which the coefficients of C_{ik} in equation (13) get larger as N gets higher. Nevertheless, the CPU times for the present formulation are smaller than those quoted in Murphy and Rose [2], taking into consideration the different computers used.

Flow with adverse pressure gradient (ID 1100). The flow, ID 1100, is not an equilibrium turbulent boundary layer beyond $x = 3m$, which was indicated by Murphy and Rose [2]. This is because the experimental data for θ , H , C_f show that they cannot satisfy the momentum integral equation.

$$\int_0^x \frac{d\theta}{dx} dx + \int_0^x (2+H) \frac{\theta}{U_\infty} \frac{dU_\infty}{dx} dx = \int_0^x \frac{C_f}{2} dx \quad (30)$$

Hence, we shall not discuss the predicted results beyond $x = 3m$ since the flow-satisfying equations (1) and (2) must satisfy equation (30).

[†] The identity number refers to that used in Reference 10.

Figures 6 to 10 show comparisons between the predicted and measured values. The case $N = 3$ could not predict this flow accurately, unlike the first case. It was found that the wake region was substantial in the stream-wise direction. In order to represent the velocity profile more accurately, one should use high-order approximations. Results of $N = 5$ show good agreement with the measured values.

Flow with favorable pressure gradient (ID 1300). This is a simple flow that can be accurately predicted even in the case $N = 3$ with the initial conditions matching C_f , δ , and δ^* . Results are shown in figures 11 to 15. No attempt was made to use high-order approximations in this case.

Conclusion

The application of the orthonormal version of the Method of Integral Relations to turbulent boundary layers has been examined. The present method circumvents the "high order" difficulty and preserves the completeness requirement.

Three sample runs have been given. Results are shown to be in good agreement with experimental data. In addition, solutions for these flows require only 8 sec on a CDC-7600 computer when $N = 3$ or 4 . This is very rapid.

A manner of determining the initial conditions from the experimental data at the initial station has been suggested and has proven to be effective. A shortcoming of this method, however, must be mentioned. It appears difficult to find initial conditions when we use higher-order approximations, say $N=6$, since the velocity profile is quite flat in turbulent boundary layers near the wall region. This high-order interpolating polynomial may be oscillatory. Furthermore, the corresponding system of ODE's becomes quite stiff and difficult to integrate. This shortcoming remains to be overcome in future research.

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FIGURE CAPTIONS

- Fig. 1. Initial velocity profile and its development (ID 1400). (This and subsequent ID numbers are from Ref. 10.)
- Fig. 2. Skin-friction coefficient (ID 1400).
- Fig. 3. Shape factor (ID 1400).
- Fig. 4. Displacement thickness (ID 1400).
- Fig. 5. Momentum thickness (ID 1400).
- Fig. 6. Initial velocity profile and its development (ID 1100).
- Fig. 7. Skin-friction coefficient (ID 1100).
- Fig. 8. Shape factor (ID 1100).
- Fig. 9. Displacement thickness (ID 1100).
- Fig. 10. Momentum thickness (ID 1100).
- Fig. 11. Initial velocity profile and its development (ID 1300).
- Fig. 12. Skin-friction coefficient (ID 1300).
- Fig. 13. Shape factor (ID 1300).
- Fig. 14. Displacement thickness (ID 1300).
- Fig. 15. Momentum thickness (ID 1300).

Fig. 1

XBL 813-452

XBL 813-453

Fig. 2

XBL 813-444

Fig. 3

XBL 813-445

Fig. 4

XBL 813-446

Fig. 5

Fig. 6

XBL-812-345

Fig. 7

XBL-812-346

Fig. 8

XBL-812-347

Fig. 9

XBL-812-348

Fig. 10

Fig. 11

XBL 813-447

XBL 813-448

Fig. 12

XBL 813-449

Fig. 13

XBL 813-450

Fig. 14

XBL 813-451

Fig. 15