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Stochastic Predictive Control with Adaptive Model Maintenance

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Abstract—The closed-loop performance of model-based controllers often degrades over time due to increased model uncertainty. Some form of model maintenance must be performed to regularly adapt the system model using closed-loop data. This paper addresses the problem of control-oriented model adaptation in the context of predictive control of stochastic linear systems. A stochastic predictive control approach is presented that integrates stochastic optimal control with control-oriented input design in order to confer some degree of probing effect to the control inputs. The probing effect will enable generating informative closed-loop data for (online) control-oriented model maintenance. In a simulation study, the performance of the proposed stochastic predictive control approach with integrated input design is demonstrated on an atmospheric-pressure plasma jet with potential biomedical applications.

I. INTRODUCTION

Model predictive control (MPC) has shown exceptional success for high-performance control of complex systems in a wide range of applications [1]. A key challenge in predictive control, however, arises from model uncertainty, which often grows over the lifetime of a MPC application, leading to closed-loop performance degradation (e.g., see [2] and the references therein). In practice, some form of model maintenance in light of its intended control application must often be performed to reduce the model uncertainty and restore the closed-loop performance.

In his seminal work, Feldbaum addressed the problem of (parametric) model uncertainty in model-based control design [3]. Feldbaum recognized the dual effect that control inputs to an uncertain system must have - the *probing effect* to probe the system dynamics for reducing the model uncertainty and the *directing effect* to control the system dynamics. Despite its conceptually appealing features, the *dual control problem* results in a stochastic dynamic programming problem that is computationally intractable even for moderately-sized systems [4].

This paper addresses the problem of parametric model uncertainty in the context of predictive control of stochastic linear systems. To this end, an explicit approach to dual control (see [4]), which involves reformulation of the dual control problem to a tractable optimal control problem with some form of system probing feature, is adopted. Explicit dual control approaches commonly integrate the model-based control design problem with an input design problem to generate sufficiently informative closed-loop data for model adaptation [5], [6], [7], [8], [9], [10]. In a predictive control setting, this typically entails incorporating some measure

of the Fisher information matrix into the optimal control problem. Predictive control with integrated input design allows for seeking systematic trade-offs between the control performance and the extent of probing effect of control inputs for gathering informative data for adequate model uncertainty reduction. A key consideration in input design for control is to explicitly account for the intended control application of the model, instead of improving the general predictive quality of the model [11], [12].

In this paper, a stochastic model predictive control approach with integrated input design (iX-SMPC) is presented for linear systems with stochastic disturbances. The model parameters in the controller are considered to be uncertain, but with known probability distributions. A control-oriented input design cost function is incorporated into a stochastic optimal control problem to allow for tuning the quality of model adaptation toward meeting a prespecified control performance level. The closed-loop data generated by iX-SMPC can be used for identifying the (posterior) probability distribution of model parameters and, consequently, facilitate online model maintenance during predictive control. The probabilistic framework of the iX-SMPC approach enables accounting for not only the system disturbances, but also the probabilistic parametric uncertainties in designing the optimal control inputs. The generalized polynomial chaos framework [13] is used to obtain a deterministic surrogate for the stochastic optimal control problem with integrated input design. In the case of expectation-type state constraints, a quadratic programming (QP) program is derived for the iX-SMPC approach. The performance of the proposed stochastic optimal control approach is demonstrated on an atmospheric-pressure plasma jet for biomedical applications [14].

Notation. $\mathbb{R}$ and $\mathbb{N}$ denote the set of real numbers and natural numbers, respectively; $\mathbb{N}_0 = \mathbb{N} \cup 0$. $(\Omega, \mathcal{F}, P)$ denotes a probability space with Ω , $\mathcal{F}$, and P being the sample space, σ -algebra, and probability distribution function (pdf) on Ω , respectively. $P(\cdot|z)$ denotes the pdf of a stochastic variable conditioned on z . $\mathcal{N}(z; \mu, \Sigma)$ denotes that a stochastic variable z has a Gaussian distribution with mean μ and covariance Σ . $\Pr[\cdot]$ denotes probability ($\Pr_z$ is the probability conditioned on z). $\mathbf{E}[\cdot]$ denotes the expected value ($\mathbf{E}_z$ is the expected value conditioned on z). $\text{tr}[\cdot]$ denotes the trace of a square matrix.

II. PROBLEM SETUP

Consider the linear discrete-time system

$$x_{k+1} = A(\theta_0)x_k + B(\theta_0)u_k + w_k(\omega), \quad (1)$$

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where $k \in \mathbb{N}_0$ denotes the time index; $x \in \mathbb{R}^n$ denotes the system states; $u \in \mathbb{R}^m$ denotes the inputs; $\theta_0 \in \mathbb{R}^p$ denotes the true system parameters; $w(\omega) \sim \mathcal{N}(\omega; 0, Q)$ is zero-mean white process noise with known covariance $Q \in \mathbb{R}^{n \times n}$; and the matrices $A(\theta_0)$ and $B(\theta_0)$ describe the system dynamics. The stochastic states x_k are all measured, but the measurements are corrupted by zero-mean Gaussian white noise with covariance $R_e \in \mathbb{R}^{n \times n}$.

The model of system (1) is subject to probabilistic uncertainties in the identified parameters $\hat{\theta}$ and initial conditions x_0 due to imperfect knowledge of the system. According to prediction error identification [15], the identified model parameters have the normal distribution (asymptotic in the sample size $\bar{N}$)

$$\sqrt{\bar{N}}(\hat{\theta} - \theta_0) \sim \mathcal{N}(0, \mathcal{P}_{\theta_0}), \quad (2a)$$

$$\mathcal{I}(\theta_0) \triangleq \mathcal{P}_{\theta_0}^{-1} \triangleq \mathbf{E} \left[\left(\frac{\partial x_k}{\partial \theta} \right)^\top R_e^{-1} \left(\frac{\partial x_k}{\partial \theta} \right) \right], \quad (2b)$$

where $\mathcal{P}_{\theta_0}$ is the covariance matrix of the estimated parameters and $\mathcal{I}(\theta_0)$ denotes the Fisher information (FI) matrix. The covariance matrix $\mathcal{P}_{\theta_0}$ must be evaluated using the true parameter values. The probabilistic model uncertainties $[x_0^\top \hat{\theta}^\top]^\top \in \mathbb{R}^{n_\xi}$ ($n_\xi \leq n + p$) are defined in terms of the standard normal variables $\xi \in \mathbb{R}^{n_\xi}$ that belong to the Hilbert space $\mathcal{L}^2(\Omega, \mathcal{F}, \mathbf{P})$.

The inputs to the system are bounded as

$$u_k \in \mathbb{U} \triangleq \{H_u u_k \leq d_u\},$$

where $H_u \in \mathbb{R}^{s \times m}$; $d_u \in \mathbb{R}^s$; and $s \in \mathbb{N}$ is the number of input constraints. Likewise, each system state $x_{i,k}$ is constrained

$$\mathbb{X}_i \triangleq \{x_{i,k} \in \mathbb{R} \mid c_i x_{i,k} + d_i \leq 0\},$$

where $c_i \in \mathbb{R}$ and $d_i \in \mathbb{R}$ are constants. Due to the unbounded nature of system stochasticity in (1), the above state constraints can only be fulfilled in a probabilistic sense in terms of chance constraints (e.g., see [16] and the references therein).

The goal of this work is to address the model uncertainty problem in the context of predictive control of the stochastic system (1). To this end, the control inputs must be designed not only to realize the desired closed-loop performance, but also to probe the system dynamics such that sufficiently informative closed-loop data are generated for control-oriented model maintenance. Control-oriented model adaptation requires that the model quality is tuned toward the intended control application, instead of improving the intrinsic model quality [11]. At every measurement sampling time, the closed-loop data can then be used for re-identifying the parameters $\hat{\theta}$ in the underlying model of the controller. The model structure is assumed to be known in this work.

Control-oriented input design hinges on characterizing the effect of model quality on the control performance [12]. Define an application cost function $V_{\text{app}}(\theta)$, which quantifies the control performance degradation that results from the mismatch between the true parameters θ_0 and model

parameters θ . The function $V_{\text{app}}(\theta)$ is defined such that its minimum occurs when $\theta = \theta_0$, that is, $V_{\text{app}}(\theta_0) = 0$, $V'_{\text{app}}(\theta_0) = 0$, and $V''_{\text{app}}(\theta_0)$ is a positive semi-definite matrix (e.g., [17]). Control-oriented input design generally involves constraints of the form [12]

$$\mathcal{I}(\theta_0) > V_{\text{app}}(\theta), \quad (3)$$

which indicates that the model parameters must be identified such that the inverse of covariance matrix of the model parameters θ is larger than the application cost function due to plant-model mismatch. When constraint (3) is met, the identified system model will yield admissible control performance. However, (3) can be fulfilled merely in probability since the identified parameters are described by the normal distribution (2a). Notice that $\mathcal{I}(\theta_0)$ and $V_{\text{app}}(\theta)$ are dependent on the unknown true parameters θ_0 .

In this work, the input design constraint (3) is incorporated into a predictive control approach for the stochastic system (1) as a soft constraint. Under the assumption of full-state feedback, the proposed iX-SMPC approach solves the following stochastic optimal control problem (OCP) at every sampling time k

$$\pi^* \triangleq \arg \min_{\pi} \mathbf{E}_{x_k} [J_c(\pi) + \rho J_i(V_{\text{app}}^{-1}(\theta)\mathcal{I}(\theta_0))] \quad (4a)$$

$$\text{s.t.: } \bar{x}_{t+1} = A(\hat{\theta})\bar{x}_t + B(\hat{\theta})u_t + w_t(\omega), \quad (4b)$$

$$\forall t = [0 \ N - 1]$$

$$\Pr_{x_k}[\bar{x}_{i,t} \in \mathbb{X}_i] \geq \beta_i, \quad \forall i = 1, \dots, q, \quad (4c)$$

$$\forall t = [0 \ N]$$

$$u_t \in \mathbb{U}, \quad \forall t = [0 \ N - 1] \quad (4d)$$

$$\hat{\theta} \sim \mathcal{N}(\theta_0, \mathcal{P}_{\theta_0}) \quad (4e)$$

$$\bar{x}_0 = x_k, \quad (4f)$$

where N is the prediction horizon¹; $\pi \triangleq [u_0^\top, \dots, u_{N-1}^\top]^\top$ denotes the control policy, with π^* being the optimal control policy; J_c denotes the control cost function; J_i denotes the input design cost function, which is defined in terms of a scalar experiment design optimality criterion (e.g., A-, D-, or E-optimality); $\rho > 0$ is a scalar weight; q denotes the number of state constraints; and $\beta_i \in (0, 1)$ is the lower bound for the probability level that each state chance constraint (4c) must be satisfied.² In this work, the control cost function is defined by

$$J_c \triangleq \sum_{t=1}^N \|\bar{x}_t - r\|_{\mathcal{Q}}^2 + \sum_{t=1}^{N-1} \|u_t - u_{t-1}\|_{\mathcal{R}}^2, \quad (5)$$

where $\mathcal{Q}$ and $\mathcal{R}$ are symmetric and positive definite weight matrices; and r denotes the reference trajectory for the states. The iX-SMPC approach is implemented in a receding-horizon manner, which implies that at every measurement sampling time k only the set of optimal inputs u_0^* is applied to the stochastic system (1).

¹The prediction and control horizons are assumed to be identical for notational convenience.

²Notice that the state constraints can be satisfied only in probability due to the (unbounded) probabilistic nature of system uncertainties.

The stochastic OCP (4) enables seeking systematic trade-offs between input design for control (J_c) and input design for control-oriented model maintenance (J_i). The information content of closed-loop data can be tuned for re-identification of the model parameters so that the adapted model fulfills a given control performance level (in a probabilistic sense). A key challenge in solving (4) results from the fact that the input design cost function J_i relies on knowledge of the unknown true parameters θ_0 . A common practice in experiment design is to evaluate the experiment design criterion using the best point estimate of parameters [11]. However, this approach cannot account for the effect of parametric uncertainties (i.e., $\hat{\theta} \sim \mathcal{N}(\theta_0, \mathcal{P}_{\theta_0})$) and, therefore, is likely to yield less effective input designs. On the other hand, the stochastic OCP (4) provides a probabilistic framework for predictive control with integrated input design. The iX-SMPC approach not only considers the intrinsic stochasticity of (1) due to disturbances, but also accounts for the model uncertainty associated with the identified parameters $\hat{\theta}$. Next, the methods adopted to obtain a deterministic formulation for (4) are presented.

III. STOCHASTIC PREDICTIVE CONTROL WITH INTEGRATED INPUT DESIGN

A. Input Design Cost Function

The application cost function $V_{\text{app}}(\theta)$ is defined in terms of the difference between the system states when the controller is designed using the true parameters θ_0 and when the controller is designed using a set of perturbed parameters θ [17]

$$V_{\text{app}}(\theta) \triangleq \frac{1}{M} \sum_{i=1}^M \|x_i(\theta_0) - x_i(\theta)\|^2, \quad (6)$$

where M is the number of state measurements. In this work, a second-order approximation of $V_{\text{app}}(\theta)$ is adopted. Since $V_{\text{app}}(\theta_0) = V'_{\text{app}}(\theta_0) = 0$, the approximated application cost function takes the form

$$\begin{aligned} V_{\text{app}}(\theta) &\approx V_{\text{app}}(\theta_0) + V'_{\text{app}}(\theta_0)(\theta_0 - \theta) \\ &\quad + \frac{1}{2}(\theta_0 - \theta)^\top V''_{\text{app}}(\theta_0)(\theta_0 - \theta) \\ &= \frac{1}{2}(\theta_0 - \theta)^\top V''_{\text{app}}(\theta_0)(\theta_0 - \theta), \end{aligned}$$

where the second derivative $V''_{\text{app}}(\theta_0)$ is evaluated in terms of the Hessian of states x_k [18]. $V''_{\text{app}}(\theta_0)$ provides an approximate measure for the control performance degradation due to the plant-model mismatch. Notice that $V''_{\text{app}}(\theta_0)$ implicitly relies on the unknown true parameters.

To evaluate the input design cost function J_i , the true parameters θ_0 are replaced with the normal distribution of the identified parameters, that is, $\hat{\theta} \sim \mathcal{N}(\theta_0, \mathcal{P}_{\theta_0})$. Thus, the FI matrix is approximated by

$$\mathcal{I}_t(\hat{\theta}) = \left(\frac{\partial \bar{x}_t}{\partial \theta} \bigg|_{\hat{\theta}} \right)^\top R_e \left(\frac{\partial \bar{x}_t}{\partial \theta} \bigg|_{\hat{\theta}} \right), \quad (7)$$

where the sensitivities are given by

$$\frac{\partial \bar{x}_{t+1}}{\partial \theta} = A(\theta) \frac{\partial \bar{x}_t}{\partial \theta} + \frac{\partial A(\theta)}{\partial \theta} \bar{x}_t + \frac{\partial B(\theta)}{\partial \theta} u_t.$$

Using the A-optimality experiment design criterion [11], the input design cost function J_i can be defined by

$$J_i \triangleq -\text{tr} \left[\left(V''_{\text{app}}(\hat{\theta}) \right)^{-1} \underbrace{\sum_{t=0}^{N_i-1} \mathcal{I}_t(\hat{\theta})}_{\mathcal{I}_{N_i}(\hat{\theta})} \right], \quad (8)$$

where N_i denotes the horizon over which the input design cost function is defined.

B. Uncertainty Propagation using Polynomial Chaos

The generalized polynomial chaos (gPC) framework [13] is used for probabilistic uncertainty propagation. In the gPC framework, the stochastic states are approximated in terms of a series expansion of orthogonal polynomial basis functions of uncertainties. The orthogonality of the basis functions enables efficient computation of the moments of stochastic states based on the expansion coefficients. In this work, a two-step procedure is adopted for propagation of the time-invariant modeling uncertainties and the time-varying system disturbances (see also [19]). First, for a given realization of the additive stochastic disturbances, the gPC framework is used to propagate the normal distribution of the model parameters $\hat{\theta}$ through the system dynamics. This yields the probability distribution of states conditioned on disturbance realizations. Then, the pdf of states is obtained by integrating the latter conditional pdf over the normal distribution of the disturbances.

1) *Propagation of parametric uncertainties:* In the system model (4b), each stochastic state $\bar{x}_{i,t}(\xi)$ is approximated by a finite series expansion of orthogonal polynomial basis functions

$$\bar{x}_{i,t}(\xi) \approx \sum_{j=0}^{l_\xi} \tilde{x}_{i,t,j} \varphi_j(\xi) = \tilde{X}_{i,t}^\top \Lambda(\xi), \quad (9)$$

where $\tilde{X}_{i,t} \triangleq [\tilde{x}_{i,t,0}, \dots, \tilde{x}_{i,t,l_\xi}]^\top$ is the vector of the PC expansion coefficients; $\Lambda(\xi) \triangleq [\varphi_0(\xi), \dots, \varphi_{l_\xi}(\xi)]^\top$ is the vector of orthogonal polynomial basis functions that have a maximum degree m_ξ with respect to the standard normal variables $\xi \in \mathbb{R}^{n_\xi}$; and $l_\xi + 1 = \frac{(n_\xi + m_\xi)!}{n_\xi! m_\xi!}$. The orthogonal polynomial basis functions are defined on the support space of ξ , and belong to the Wiener-Askey scheme of polynomials. This implies that the inner product $\langle \varphi_i(\xi), \varphi_j(\xi) \rangle = \int_{\Omega} \varphi_i(\xi) \varphi_j(\xi) P_\xi d\xi = \langle \varphi_i^2(\xi) \rangle \delta_{ij}$, where δ_{ij} is the Kronecker delta function.

For a particular realization of disturbances $w_t(\omega) \triangleq \bar{w}$, the PC expansions of states are used to approximate the system model (4b) as

$$\tilde{X}_{t+1}^\top \Lambda(\xi) = A(\hat{\theta}) \tilde{X}_t^\top \Lambda(\xi) + B(\hat{\theta}) u_t + \bar{w}, \quad (10)$$

where $\tilde{X}_t \triangleq [\tilde{X}_{1,t}^\top, \dots, \tilde{X}_{n,t}^\top]^\top$ and $\Lambda(\xi)$ is the vector of n stacked $\Lambda(\xi)$. The Galerkin projection [20] is used to project

the truncation error of PC expansions in (10) onto the space of the orthogonal basis functions $\{\varphi_j(\xi)\}_{j=0}^{l_\xi}$. This leads to a set of deterministic ordinary differential equations for describing the coefficients of PC expansions

$$\tilde{X}_{t+1} = A_g \tilde{X}_t + B_g u_t + \tilde{w}, \quad (11)$$

where

$$A_g \triangleq \sum_{j=0}^{l_\xi} A_j \otimes \Psi_j, \quad B_g \triangleq \sum_{j=0}^{l_\xi} B_j \otimes \Psi_j, \\ \Psi_j \triangleq \begin{bmatrix} \psi_{0j0} & \cdots & \psi_{0jl_\xi} \\ \vdots & \ddots & \vdots \\ \psi_{l_\xi j0} & \cdots & \psi_{l_\xi jl_\xi} \end{bmatrix},$$

with A_j and B_j being the projections of $A(\hat{\theta})$ and $B(\hat{\theta})$ onto the j^{th} polynomial basis function, and $\psi_{ijk} = \langle \varphi_i, \varphi_j, \varphi_k \rangle / \langle \varphi_i^2 \rangle$.

2) *Propagation of system disturbances:* The PC expansion coefficients $\tilde{X}_t$ in (11) have a normal distribution with respect to different realizations of disturbances $w_t(\omega)$. Thus, the stochastic coefficients $\tilde{X}_t$ can be described by their mean $\mu_{x,t}$ and covariance $\Sigma_{x,t}$

$$\mu_{x,t+1} = A_g \mu_{x,t} + B_g u_t \quad (12a)$$

$$\Sigma_{x,t+1} = A_g \Sigma_{x,t} A_g^\top + Q_g, \quad (12b)$$

where $Q_g \in \mathbb{R}^{n(l_\xi+1) \times n(l_\xi+1)}$ is a diagonal matrix whose $\{(i-1)n_\xi + 1\}_{i=1}^n$ diagonal entries are the entries of the disturbance covariance matrix Q .

The pdf of the states $\tilde{x}_t$ can now be approximated by

$$P_{\tilde{x}_t} \approx \int_{\Omega} P\left(\left\{\sum_{j=0}^{l_\xi} \tilde{x}_{i,t,j} \varphi_j(\xi)\right\}_{i=1}^n\right) \mathcal{N}(\tilde{X}_t; \mu_{x,t}, \Sigma_{x,t}) d\tilde{X}. \quad (13)$$

Note that the approximation in (13) is due to the truncation of PC expansions (9). Using (13) and the orthogonality property of the basis functions, the mean and variance of each state are given by

$$\mathbf{E}[\tilde{x}_{i,t}] \approx \int_{\Omega} \mathbf{E}\left[\sum_{j=0}^{l_\xi} \tilde{x}_{i,t,j} \varphi_j(\xi) | w\right] \mathcal{N}(\tilde{X}_t; \mu_{x,t}, \Sigma_{x,t}) d\tilde{X}_i \\ = \int_{\Omega} \tilde{x}_{i,t,0} \mathcal{N}(\tilde{X}_t; \mu_{x,t}, \Sigma_{x,t}) d\tilde{X}_i = \mu_{x_i,t,0},$$

$$\mathbf{E}[\tilde{x}_{i,t}^2] \approx \int_{\Omega} \mathbf{E}\left[\left(\sum_{j=0}^{l_\xi} \tilde{x}_{i,t,j} \varphi_j(\xi)\right)^2 | w\right] \mathcal{N}(\tilde{X}_t; \mu_{x,t}, \Sigma_{x,t}) d\tilde{X}_i \\ = \sum_{j=0}^{l_\xi} (\mu_{x_i,t,j}^2 + \Sigma_{x_i,t,j,j}^2) \langle \varphi_j^2(\xi) \rangle,$$

where $\mu_{x_i,t}$ and $\Sigma_{x_i,t}$ denote, respectively, the vector of mean values and covariance matrix of coefficients of the PC expansion of the i th state.

C. Deterministic Formulation for iX-SMPC

Using the results of the preceding subsections, a deterministic formulation can now be derived for the proposed iX-SMPC approach for the stochastic system (1). The deterministic formulation relies on the input design cost function (8) and the gPC-based prediction model (12). The state chance constraints (4c) are approximated by convex second-order cone constraints [21] (e.g., see [22] for the details).

Proposition 1: When $N_i = N$, the stochastic OCP (4) involves solving the following program at sampling time k

$$\pi^* \triangleq \arg \min_{\pi} \pi^\top H_\pi \pi + f_\pi \pi - \rho \text{tr} \left[(V''_{\text{app}}(\hat{\theta}))^{-1} (P_1 (\pi^\top H_{\pi\theta} \pi + f_{\pi\theta} \pi) P_2) \right] \quad (14)$$

$$\text{s.t.: } \mu_{x,t+1} = A_g \mu_{x,t} + B_g u_t, \quad t = [0 \ N-1]$$

$$\Sigma_{x,t+1} = A_g \Sigma_{x,t} A_g^\top + Q_g, \quad t = [0 \ N-1]$$

$$c_i \left(\sqrt{\frac{\beta_i}{1-\beta_i}} \Sigma_{x_i,t} + \mu_{x_i,t} \right) + d_i \leq 0, \quad i = 1, \dots, q,$$

$$t = [0 \ N]$$

$$u_t \in \mathbb{U}, \quad t = [0 \ N-1],$$

where

$$H_\pi = S_\pi^\top I_\mu^\top I_\mu S_\pi + S_{\Delta\pi}^\top W_\pi S_{\Delta\pi}, \\ f_\pi = 2 \left[\mu_{x,t}^\top S_\mu^\top I_\mu^\top I_\mu S_\pi - R^\top I_\mu S_\pi \right], \\ S_\mu \triangleq \begin{bmatrix} A_g \\ A_g^2 \\ \vdots \\ A_g^N \end{bmatrix}, \quad S_\pi \triangleq \begin{bmatrix} B_g & 0 & \cdots & 0 \\ A_g B_g & B_g & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_g^{N-1} B_g & A_g^{N-2} B_g & \cdots & B_g \end{bmatrix}, \quad (16)$$

$$S_{\Delta\pi} \triangleq \begin{bmatrix} -I_\pi & I_\pi & 0 & \cdots & 0 \\ 0 & -I_\pi & I_\pi & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & -I_\pi & I_\pi \end{bmatrix},$$

$$W_\pi \triangleq \begin{bmatrix} \mathcal{R} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathcal{R} \end{bmatrix},$$

I_μ is a diagonal matrix whose $\{((i-1)n_\xi + 1)\}_{i=1}^n$ entries are 1; R is an $nN \times 1$ vector of the reference trajectory r ; I_π is an identity matrix of dimension $m \times m$; and $S_{\Delta\pi}$ has dimension $(N-1)m \times Nm$.

Proof: The input design cost function J_i (see (8)) can be explicitly defined in terms of the decision variables π .³ Let

$$\varsigma_{t+1} \triangleq \begin{bmatrix} \mu_{x,t+1} \\ \frac{\partial \mu_{x,t+1}}{\partial \theta} \end{bmatrix} = \mathcal{A} \varsigma_t + \mathcal{B} u_t, \\ \mathcal{A} = \begin{bmatrix} A_g & 0 \\ \frac{\partial A_g}{\partial \theta} & A_g \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} B_g \\ \frac{\partial B_g}{\partial \theta} \end{bmatrix}.$$

³Due to space limitation, the procedure of defining the control cost function J_c in terms of the decision variables π is omitted (e.g., see [6]).

The evolution of ς_k over the horizon N is given by

$$\Upsilon_N \triangleq [\varsigma_0^\top \quad \cdots \quad \varsigma_N^\top]^\top = S_{\mu\theta} S_t + S_{\pi\theta} \pi,$$

where

$$S_{\mu\theta} \triangleq \begin{bmatrix} \mathcal{A} \\ \mathcal{A}^2 \\ \vdots \\ \mathcal{A}^N \end{bmatrix}, \quad S_{\pi\theta} \triangleq \begin{bmatrix} \mathcal{B} & 0 & \cdots & 0 \\ \mathcal{A}\mathcal{B} & \mathcal{B} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{A}^{N-1}\mathcal{B} & \mathcal{A}^{N-2}\mathcal{B} & \cdots & \mathcal{B} \end{bmatrix}. \quad (17)$$

The FI matrix $\mathcal{I}_N(\hat{\theta})$ is in fact the sum of the $p \times p$ block diagonal matrices in $\Upsilon_N^\top \Upsilon_N$. Let P_1 and P_2 denote two (constant) permutation matrices such that

$$\begin{aligned} \mathcal{I}_N(\hat{\theta}) &= P_1 \Upsilon_N^\top \Upsilon_N P_2 \\ &= P_1 (\pi^\top S_{\pi\theta}^\top S_{\pi\theta} \pi + \mu_{x,t}^\top S_{\mu\theta}^\top S_{\mu\theta} \mu_{x,t} + 2\mu_{x,t}^\top S_{\mu\theta}^\top S_{\pi\theta} \pi) P_2. \end{aligned}$$

By ignoring the terms that are not dependent on π , $\text{tr}[(V''_{\text{app}}(\hat{\theta}))^{-1} \mathcal{I}_N(\hat{\theta})]$ can be rewritten as

$$\text{tr}[(V''_{\text{app}}(\hat{\theta}))^{-1} (P_1 (\pi^\top S_{\pi\theta}^\top S_{\pi\theta} \pi + 2\mu_{x,t}^\top S_{\mu\theta}^\top S_{\pi\theta} \pi) P_2)],$$

where $H_{\pi\theta} = S_{\pi\theta}^\top S_{\pi\theta}$ and $f_{\pi\theta} = 2\mu_{x,t}^\top S_{\mu\theta}^\top S_{\pi\theta} \pi$. ■

Notice that the approximations involved in obtaining the program (14) consist of truncating the polynomial chaos expansions of states and replacing the state chance constraints with deterministic surrogates.

Proposition 2: When the state chance constraints (4c) are replaced with expectation-type constraints of the form

$$\mathbf{E}[c_i x_{i,t}] \leq d_i, \quad \forall i = 1, \dots, q,$$

the program (14) will become a QP problem.

Proof: The proof follows that of Proposition 1 since the cost function in (14) is quadratic in the decision variables π . The deterministic surrogate for the expectation-type constraints takes the form

$$c_i \mu_{x_i,t} \leq d_i, \quad \forall i = 1, \dots, q,$$

which is linear in π . ■

Algorithm 1 summarizes the receding-horizon implementation of the proposed iX-SMPC approach at every sampling time k under full-state feedback. A gPC-based histogram filter (HF) [23] is used to identify the normal distribution of the model parameters $\hat{\theta}$ using the closed-loop data. The proposed iX-SMPC approach enables control-oriented model maintenance during stochastic predictive control of (1).

IV. APPLICATION OF iX-SMPC TO A PLASMA JET

Atmospheric-pressure plasma jets (APPJs) are a class of nonequilibrium plasma devices that are used in materials processing and biomedical applications [24]. In APPJs, plasma is generated in a flowing carrier gas (usually helium or argon) via the application of pulsed or modulated electric field at a high frequency. Nonequilibrium plasmas contain highly energetic electrons that can induce a complex network of chemical reactions, while the background gas is at a relatively low temperature. Plasma characteristics are often

Algorithm 1 The iX-SMPC Approach (implementation at each sampling time k)

- 1: At k , use the measured states to identify the model parameters $\hat{\theta}$ using a gPC-based HF
- 2: Adapt the matrices A_g and B_g in the gPC-based prediction model (12) using the identified parameters
- 3: Initialize the gPC-based prediction model (12) using the measured states
- 4: Evaluate S_μ , S_π , $S_{\mu\theta}$, and $S_{\pi\theta}$ using (16) and (17)
- 5: Compute the optimal control policy π^* by solving the program (14)
- 6: Apply u_0^* to system (1)

very sensitive to the operating conditions as well as the target surface properties. Abrupt changes in the plasma characteristics (e.g., arching) can drastically increase the heat and current delivery to the target surface, inflicting damage to the surface [25]. Thus, effective regulation of the thermal effects of the plasma on the target surface is paramount, particularly when treating highly sensitive targets such a biological tissue [14].

In this case study, the thermal model of an argon jet given in [14] is adapted by approximating the spatial variations of gas temperature and composition along the jet as three well-mixed volumes in series. The stochastic plasma dynamics are described by

$$x_{k+1} = f(x_k, u_k, \hat{\theta}) + w_k,$$

where the system states $x = [T_{g1} \ T_{g2} \ T_{g3} \ \omega_{g1} \ \omega_{g2} \ \omega_{g3} \ T_s]^\top$ correspond to the gas temperature (T_g) and composition (ω_g) in each well-mixed volume as well as the target surface temperature (T_s); the system inputs $u = [P_{in} \ v_{in}]^\top$ consist of the inlet power P_{in} and argon inlet velocity v_{in} ; the uncertain model parameter $\hat{\theta}$ represents the power efficiency factor due to losses in the circuit and other plasma processes ($\theta_0 = 0.94$); and f denotes the nonlinear system dynamics. To implement the iX-SMPC approach on the argon APPJ, the above nonlinear model is linearized around a desired operating point. The stochastic OCP is formulated as

$$\min_{\pi} \sum_{t=1}^N \| \mathbf{E}[T_{s,t}] - 317 \|^2 - 0.6 \text{tr}[(V''_{\text{app}}(\hat{\theta}))^{-1} \mathcal{I}_N(\hat{\theta})]$$

$$\begin{aligned} \text{s.t.: } \bar{x}_{t+1} &= A(\hat{\theta}) \bar{x}_t + B(\hat{\theta}) u_t + w_t(\omega), & t &= [0 \ N-1] \\ \mathbf{E}[T_{s,t}] &\leq 317 \text{ K}, & t &= [0 \ N-1] \\ [8 \text{ W } 8 \text{ m/s}]^\top &\leq u_t \leq [25 \text{ W } 35 \text{ m/s}]^\top, & t &= [0 \ N-1], \end{aligned}$$

where N corresponds to a prediction horizon of 300 s, and $w_t \sim \mathcal{N}(0, 0.01)$. The stochastic OCP is solved every 10 s when new state measurements become available; the measured states are corrupted with zero-mean white noise with covariance $R_e = \text{diag}[0.3 \ 0.3 \ 0.3 \ 0.001 \ 0.001 \ 0.001 \ 0.3]$. Algorithm 1 is utilized for the receding-horizon implementation of the iX-SMPC approach.

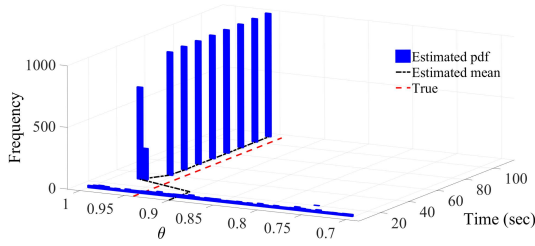


Fig. 1: The evolution of pdf of the identified model parameter $\hat{\theta}$ in a closed-loop simulation run of the iX-SMPC approach.

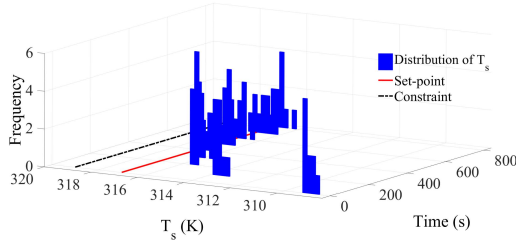


Fig. 2: The evolution of pdf of T_s constructed based on 70 closed-loop simulation runs.

To evaluate the performance of the iX-SMPC approach, 70 closed-loop simulation runs are performed based on different realizations of disturbances w_k .⁴ The evolution of pdf of $\hat{\theta}$ in a closed-loop run is shown in Fig. 1. The figure suggests that there is a reduction in the variance of $\hat{\theta}$ as its distribution converges to a Dirac-delta function (i.e., model uncertainty reduction). However, there is a small bias between the true value of θ and its estimated mean. Fig. 2 shows the evolution of pdf of T_s . The iX-SMPC approach results in adequate reference tracking, while satisfying the state chance constraint in the presence of system stochasticity.

V. CONCLUSIONS

A predictive control approach with integrated control-oriented input design is presented for stochastic linear systems. The objective of the proposed approach is twofold: (i) generating informative closed-loop data for parametric model uncertainty handling, and (ii) regulating the system dynamics. The closed-loop simulations on an atmospheric-pressure plasma jet demonstrate the effectiveness of this approach in terms of online model adaptation.

REFERENCES

- [1] J. H. Lee, "Model predictive control: Review of the three decades of development," *International Journal of Control, Automation and Systems*, vol. 9, pp. 415–424, 2011.
- [2] A. Mesbah, X. Bombois, M. Forgone, H. Hjalmarsson, and P. M. J. Van den Hof, "Least costly closed-loop performance diagnosis and plant re-identification," *International Journal of Control*, pp. 1–13, 2015.
- [3] A. A. Feldbaum, "Dual-control theory. I," *Automatic Remote Control*, vol. 21, pp. 874–880, 1961.

⁴In closed-loop simulations, the plant consists of the stochastic linear system simulated using the Matlab Toolbox SDETools [26].

- [4] B. Wittenmark, "Adaptive dual control methods: An overview," in *Proceedings of the 5th IFAC Symposium on Adaptive Systems in Control and Signal Processing*. Budapest, 1995, pp. 67–72.
- [5] G. Marafioti, R. R. Bitmead, and M. Hovd, "Persistently exciting model predictive control," *International Journal of Adaptive Control and Signal Processing*, vol. 28, pp. 536–552, 2013.
- [6] C. A. Larsson, M. Annergren, H. Hjalmarsson, C. R. Rojas, X. Bombois, A. Mesbah, and P. E. Moden, "Model predictive control with integrated experiment design for output error systems," in *Proceedings of European Control Conference*, Zürich, Switzerland, 2013, pp. 3790–3795.
- [7] M. Forgone, X. Bombois, and P. M. J. Van den Hof, "Data-driven model improvement for model-based control," *Automatica*, vol. 52, pp. 118–124, 2015.
- [8] T. A. N. Heirung, B. Foss, and B. E. Ydstie, "MPC-based dual control with online experiment design," *Journal of Process Control*, vol. 32, pp. 64–76, 2015.
- [9] C. A. Larsson, C. R. Rojas, X. Bombois, and H. Hjalmarsson, "Experimental evaluation of model predictive control with excitation (MPC-X) on an industrial depropanizer," *Journal of Process Control*, vol. 31, pp. 1–16, 2015.
- [10] V. Bavdekar and A. Mesbah, "Stochastic model predictive control with integrated experiment design for nonlinear systems," in *Proceedings of the 11th IFAC Symposium on Dynamics and Control of Process Systems (DYCOPS)*. Trondheim, Norway, 2016, pp. 49–54.
- [11] M. Gevers, "Identification for control: From the early achievements to the revival of experiment design," *European Journal of Control*, vol. 11, pp. 335–352, 2005.
- [12] H. Hjalmarsson, "From experiment design to closed-loop control," *Automatica*, vol. 41, pp. 393–438, 2005.
- [13] D. Xiu and G. E. Karniadakis, "The Wiener-Askey polynomial chaos for stochastic differential equations," *SIAM Journal on Computing*, vol. 24, pp. 619–644, 2002.
- [14] D. Gidon, D. B. Graves, and A. Mesbah, "Model predictive control of thermal effects of an atmospheric pressure plasma jet for biomedical applications," in *Proceedings of the American Control Conference*, Boston, 2016, pp. 4889–4894.
- [15] L. Ljung, *System Identification: Theory for the User*. Prentice Hall, New Jersey, 1999.
- [16] A. Mesbah, "Stochastic model predictive control: An overview and perspectives for future research," *IEEE Control Systems Magazine*, Accepted, 2016.
- [17] C. A. Larsson, "Toward applications oriented optimal input design with focus on model predictive control," Ph.D. dissertation, KTH Royal Institute of Technology, Stockholm, Sweden, 2011.
- [18] A. Ebadat, M. Annergren, C. A. Larsson, C. R. Rojas, B. Wahlberg, H. Hjalmarsson, M. Molander, and J. Sjöberg, "Application set approximation in optimal input design for model predictive control," in *Proceedings of European Control Conference*, Strasbourg, France, 2014, pp. 744–749.
- [19] U. Konda, P. Singla, T. Singh, and P. D. Scott, "State uncertainty propagation in the presence of parametric uncertainty and additive white noise," *Journal of Dynamic Systems, Measurement and Control, Transactions of the ASME*, vol. 133, pp. 051009–1–10, 2011.
- [20] R. Ghanem and P. Spanos, *Stochastic Finite Elements - A Spectral Approach*. Springer, New York, 1991.
- [21] G. C. Calafiore and L. E. Ghaoui, "On distributionally robust chance-constrained linear programs," *Journal of Optimization Theory and Applications*, vol. 130, pp. 1–22, 2006.
- [22] A. Mesbah, S. Streif, R. Findeisen, and R. D. Braatz, "Stochastic nonlinear model predictive control with probabilistic constraints," in *Proceedings of the American Control Conference*. Portland, 2014, pp. 2413–2419.
- [23] V. Bavdekar and A. Mesbah, "A polynomial chaos-based nonlinear Bayesian approach for estimating state and parameter probability distribution functions," in *Proceedings of the American Control Conference*, Boston, 2016, pp. 2047–2052.
- [24] C. Tendero, C. Tixier, P. Tristant, J. Desmaison, and P. Leprince, "Atmospheric pressure plasmas: A review," *Spectrochimica Acta Part B: Atomic Spectroscopy*, vol. 61, pp. 2–30, 2006.
- [25] J. J. Shi, X. T. Deng, R. Hall, J. D. Punnett, and M. G. Kong, "Three modes in a radio frequency atmospheric pressure glow discharge," *Journal of Applied Physics*, vol. 94, pp. 6303–6310, 2003.
- [26] A. D. Horchler, *SDETools: A Matlab Toolbox for the Numerical Solution of Stochastic Differential Equations (SDEs)*, 2015.