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HIGH ENERGY ELECTRON-ELECTRON SCATTERING

F. C. Gilbert
(Thesis)
December, 1951

Berkeley, California
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I wish to express my sincere gratitude to Doctor Walter H. Barkas, who has directed this research, for his continued interest and inspirational advice.

I am especially indebted to Mr. Charles E. Violet for processing the emulsions, for many hours of tedious scanning and for invaluable advice and corrections.

I am deeply grateful to Mr. Robert W. Deutsch for his analysis of the various scattering theories, for the time he has spent scanning and for his very helpful counsel.

The emulsion method of studying electron-electron scattering was first found to be practical using fast electrons from a linear accelerator at Stanford University. Doctor Barkas, assisted by Doctor G. E. Becker, first exposed electron sensitive plates to 5 Mev electrons in this experiment. Much of the microscope work of the earlier experiments was done by Professor Lawrence Germain and Mrs. Edith Goodwin, and I am indebted to Mr. A. Oliver for his microphotographs and helpful advice on emulsion technique.

The work was assisted greatly by the continued interest and
encouragement given the film program by Professor R. L. Thornton and I have enjoyed the invaluable cooperation of the synchrotron operating crew under the direction of Mr. George McFarland and Professor A. C. Helmholtz.
I. INTRODUCTION

There have been many electron-electron scattering experiments since the beginning of the 20th century. Almost all of these, which have yielded quantitative results, have been carried out in cloud chambers where it is possible to stereoscopically measure the angles, ranges and curvatures necessary to determine the energies involved.

The first of these experiments was initiated by C. T. R. Wilson\(^1\) in 1923. Using a rapid expansion cloud chamber in a weak magnetic field, he found the electron-electron scattering cross section to be in agreement with classical theory up to his limiting energy (~25 kilovolts).

In 1929 E. J. Williams\(^2\) made a more thorough investigation using electrons from 100 to 1500 kilovolts of energy. He found higher numbers of fast secondary electrons than is predicted from classical consideration. However, in 1930, he compared his results with the newly proposed quantum mechanical treatment as given by N. F. Mott\(^3\) and found agreement within his experimental error.

Two years later (1932) F. C. Champion\(^4\) repeated the above experiment using two cameras to obtain a stereoscopic record of each event. Using electrons from 400 to 1100 kilovolts of energy he found closest agreement with Möller's\(^5\) theory of electron-electron scattering, fair agreement with Mott's theory and poor agreement with the classical or the relativistically corrected classical theory.\(^6\)

The most recent work done in this field is that of G. Groetzingen\(^7\) et. al., at the University of Chicago, in 1950, in which he performed an experiment very similar to Champion's. Combining his data with
Champion's, he found agreement only with Mott's and Möller's theories.

All the above experimenters have been limited to using radioactive chemical sources, and thus their energy range has been limited to a few Mev. The existence of electron accelerators thus suggests the extension of these experiments into the higher energies available from such machines as the betatron, synchrotron and linear accelerator.

The recent development of electron sensitive photographic emulsions (Ilford G-5 emulsion was used for this experiment) and a technique for eradicating accumulated background has made available a substitute for the cloud chamber for detecting high energy electrons. The nuclear emulsion, although too small to detect magnetic field curvature, has a great advantage in simplicity. This paper will attempt to show the application of nuclear emulsions to measuring the electron-electron scattering cross section of 185 Mev electrons.

Historically, the electron is the best known of the fundamental particles. However, an hiatus still exists concerning its actual structure. An electron-electron scattering experiment would seem to be the ideal way to investigate the boundaries of the electron and the possibility of non-Coulomb electron-electron forces. In order to find discrepancies from a Coulomb potential, one would roughly estimate that it is necessary to have an impact parameter of the order of the classical electron radius in the relativistic center of mass system. The de Broglie wavelength, \( \lambda \), for such an impact parameter must therefore be of the order of 19 Bev in the laboratory system. In this experiment, 200 Mev electron primaries were used which have a de Broglie wavelength
of about 10 times the classical electron radius in the relativistic center of mass system. For this wavelength, there still exists a possibility of observing deviation from a Coulomb potential.
II EXPERIMENTAL TECHNIQUES AND PROCEDURES

A. Exposure

Because of the difficulty of obtaining electrons directly from a synchrotron, the beam of electrons is normally allowed to strike a high Z target, located inside the orbital doughnut, producing bremsstrahlung which then pass out of the machine. In this experiment the resulting x-ray beam was collimated to 1/8 inch diameter before striking the 0.009 inch tantalum target (Figures 1 and 2). The high-energy x-rays produced positron-electron pairs in the tantalum which were sorted in energy by the magnetic field. The field used in the pair spectrometer was 13,000 gauss, which gave a radius of curvature of 20.15 inches for 200 Mev electrons. The field was initially calibrated by means of a proton momentum measurement. (9)

The plates were supported in the spectrometer by small brass holders which could be adjusted such that the electrons from the target entered the emulsion at a slight angle to the surface and perpendicular to the leading edge of the plate. (Figures 1 and 2.) The magnitude of the exposure was measured by means of an ionization chamber placed in the x-ray beam. (For an approximate calculation of the proper exposure see Appendix A.)

B. Processing and Care of Emulsions (10)

1. Eradication

Because of the high background of low energy electrons found on all but freshly prepared electron sensitive emulsions, it was necessary
Fig. 1
Arrangement of the photographic emulsions in the magnetic field of the synchrotron pair spectrometer.
Fig. 2
Photograph showing pair spectrometer in place behind synchrotron.
to eradicate\(^{(8)}\) the emulsions immediately before use. The eradication was accomplished by storing the plates in a warm, high humidity atmosphere for several days before exposure. The temperature was controlled at about 97\(^{\circ}\) F. by immersing a watertight box containing the plates in a thermostatically controlled water bath (Figure 3). The relative humidity was maintained near 100 percent by placing a wet sponge in the box with the plates. Over 90 percent eradication was obtained by this method.

2. Exposure

After eradication, the plates were dried, wrapped, exposed and developed as rapidly as possible to reduce extraneous background. The wrapping consisted of a single layer of black paper held by Scotch Tape.

3. Development

The development procedure was a cold cycle method similar to that used by Occialini\(^{(11)}\) for thick emulsions.

**DEVELOPMENT PROCEDURE**

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Soak (distilled water, room temperature)</td>
<td>1 hour</td>
</tr>
<tr>
<td>*Cold Developer (5(^{\circ}) C.)</td>
<td>1 hour</td>
</tr>
<tr>
<td>*Hot Developer (26(^{\circ}) C.)</td>
<td>30 minutes</td>
</tr>
<tr>
<td>Cold Soak (5(^{\circ}) C.)</td>
<td>1 hour</td>
</tr>
<tr>
<td>Fixing (room temperature, with agitation)</td>
<td>6 hours</td>
</tr>
<tr>
<td>Washing (room temperature)</td>
<td>24 hours</td>
</tr>
<tr>
<td>Drying</td>
<td>24 hours</td>
</tr>
</tbody>
</table>
DEVELOPMENT PROCEDURE (cont.)

*Amidol Developer

<table>
<thead>
<tr>
<th>Substance</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distilled Water</td>
<td>1000 cc</td>
</tr>
<tr>
<td>Boric Acid</td>
<td>35 gm.</td>
</tr>
<tr>
<td>Sodium Sulfite (anhydrous)</td>
<td>18 gm.</td>
</tr>
<tr>
<td>Potassium Bromide (10 percent solution)</td>
<td>8 cc.</td>
</tr>
<tr>
<td>Amidol (Acrol)</td>
<td>4.5 gm.</td>
</tr>
</tbody>
</table>

This procedure gave a grain density of $41.9 \pm 1.0$ grains/100 μ for 200 Mev electrons.

Some difficulty was encountered during periods of low relative humidity when the emulsions cracked and peeled from their glass backing. Two methods were tried to prevent this cracking. The first was to dip the plates in a mixture of Duco cement and lacquer thinner in an attempt to retard evaporation through the surface of the emulsion. When this proved only partially successful, the emulsions were dipped in a 1 percent glycerine solution immediately after processing. The latter procedure proved to be effective, although introducing a slight uncertainty into the emulsion shrinkage factor.

4. Shrinkage Factor

In order to properly determine the ranges and angles of the knock-on electrons, it was necessary to measure the ratio of the thickness of the emulsions before development to the thickness of the emulsions after development. This was accomplished by passing 380 Mev alpha particles through the undeveloped emulsion at an angle of 45° to the emulsion surface, and then measuring the ratio of the horizontal projection to the vertical projection of the alpha track after development.
Fig. 3a
Eradication consisting of: (1) porcelain crock for water bath; (2) plastic cover containing heating elements, thermostat and thermometer; and (3) light-tight and water-tight box.
Fig. 3b
Contents of eradication box: (1) platform holding plate rack and sponge, (2) cover, and (3) cover clamp.
Shrinkage factor, \( F = \tan \phi \)

This ratio gave the shrinkage factor directly as \( 2.5 \pm 0.1 \).

C. Scanning and Measurement Technique

The plates were scanned under \( \sim 500x \) magnification and all events of interest were measured under \( \sim 2500x \) magnification. Spencer binocular microscopes were used which had been fitted with special stages to give accurate measurements for long ranges. The ranges of short knock-ons were measured by means of an eyepiece reticule (1 reticule unit \( \approx \) 1 micron) in combination with the vertical, fine focus, micrometer. The length of primary track scanned was determined by means of the microscope stage coordinates.

To ensure that only electrons which came directly from the convertor were accepted, only tracks whose initial direction lay within \( 2-1/2^\circ \) of the perpendicular were scanned. This criterion included over 90 percent of all the high energy electrons entering the plate. Control plates exposed with no convertor in the beam had less than 1 percent of the
acceptable tracks found on a plate exposed with the convertor.

In order to reduce the percentage of knock-on electrons missed, each track used was scanned independently by two observers and all questionable events were examined by a third observer before a decision was reached. No track was scanned for more than 0.8 cm or beyond a detectable single scatter or a high-energy electron-electron scatter. Tracks were not scanned and no event was recorded within 10 microns of either surface of the emulsion. The average track length in emulsion was 0.4 cm.

In order to insure that certain types of events were not being missed (especially those in which the knock-on electron was nearly vertical in the emulsion), a plot was made of the distribution of the azimuth angles of the knock-ons about the direction of the incident electron. This distribution was found not to be significantly different from a symmetric distribution.

To determine the energy of the knock-on electron, both its range and the angle between its direction and the direction of the incident electron were measured wherever possible. For very low energy electrons the angle became difficult to measure because of nuclear scattering. Therefore, the range was the principal means of determining the energy up to about 0.6 Mev. Above this energy, few knock-ons stayed in the emulsion, but the angle became a practical means of determining the energy. In the region where the angle and range methods overlap a plot was made of the range of the knock-on versus its angle (Figure 4). Assuming 20 percent range straggle, and a 2° error in measuring the
Fig. 4
Range of knock-on electron plotted against angle between knock-on and incident electron. Assuming a standard deviation in angle of $2^\circ$, 68 percent of the events should be found within the dotted lines. Actually 64 percent of the events are found within these limits. The theoretical curve is based upon the range-energy relation as determined by Zajac and Ross.\(^{(14)}\)
angle one would expect 68 percent of the events to fall within one
standard deviation as shown in Figure 4. Actually 64 percent are found
within these limits.

Knock-on electrons of energy less than 30 Kev were not included be-
cause of the small range (<7 microns), and because of the effect of
electron binding energies.
III THEORETICAL CONSIDERATIONS

A. Kinetics of the Electron-Electron Scatter

In this section we shall deal only with the momentum and energy conservation laws as applied to the electron-electron collision. In this manner we shall derive the following relationships which prove valuable in analyzing the experimental results.

\[ \theta_1 \text{ versus } T_1 \]
\[ \theta_1 \text{ versus } \theta_2 \]

The following symbols will be used throughout this paper:

- \( \theta \) - the angle between the incident electron and a secondary electron
- \( T \) - the kinetic energy of an electron
- \( v \) - the velocity of an electron
- \( \beta = \frac{\sqrt{\gamma}}{\gamma} \)
- \( \gamma = \frac{1}{\sqrt{1 - \beta^2}} \)
- \( c \) - the velocity of light
- \( m \) - the rest mass of an electron

Subscripts

- \( o \) - refers to incident electron
- \( 1 \) - refers to lower energy secondary electron
- \( 2 \) - refers to higher energy secondary electron
1. Energy Conservation
   (a) \( T_0 = T_1 + T_2 \)
   \[ m(\gamma_0-1)c^2 = m(\gamma_1-1)c^2 + m(\gamma_2-1)c^2 \]
   \( \gamma_0 + 1 = \gamma_1 + \gamma_2 \)

2. Momentum Conservation
   (a) \( \sum T_i \)

   (b) \( \sum P_x = 0 \)
   \[ m\beta_0\gamma_0 c = m\beta_1\gamma_1 c \cos \theta_1 + m\beta_2\gamma_2 c \cos \theta_2 \]
   since
   \[ \beta\gamma = \sqrt{\gamma^2 - 1}, \sqrt{\gamma_0^2 - 1} = \sqrt{\gamma_1^2 - 1} \cos \theta_1 + \sqrt{\gamma_2^2 - 1} \cos \theta_2 \]

   (c) \( \sum P_y = 0 \)
   \[ m\beta_1\gamma_1 c \sin \theta_1 = m\beta_2\gamma_2 c \sin \theta_2 \]
   or
   \[ \sqrt{\gamma_1^2 - 1} \sin \theta_1 = \sqrt{\gamma_2^2 - 1} \sin \theta_2 \]
   eliminating \( \theta_2 \) between (b) and (c)

   (d) \( (\gamma_2^2 - 1) = (\gamma_0^2 - 1) + (\gamma_1^2 - 1) - 2\sqrt{(\gamma_0^2 - 1)(\gamma_1^2 - 1)} \cos \theta_1 \)
   eliminating \( \theta_1 \) between (a) and (d)

   \( \gamma_0\gamma_1 - \gamma_0 + \gamma_1 - 1 = \sqrt{(\gamma_1^2 - 1)(\gamma_0^2 - 1)} \cos \theta_1 \)

   (e) \( \cos^2 \theta_1 = \frac{(\gamma_0+1)(\gamma_1-1)}{(\gamma_0-1)(\gamma_1+1)} \)
or
\[
T_1 = \frac{T_0 \cos^2 \theta_1}{1 + \frac{T_0}{2mc^2} \sin^2 \theta_1}
\]

For \(\sin^2 \theta_1 >> \frac{2mc^2}{T_0}\), the equation in (f) reduces to \(T_1 = 2mc^2 \cot^2 \theta_1\).

Since the smallest angle, \(\theta_1\), found in this experiment was 90°, this condition is met by all the events and affords a simple means of calculating the knock-on energy which is independent of \(T_0\). By further algebraic manipulation it may also be shown that

\[
\tan \theta_1 \tan \theta_2 = \frac{2}{\gamma_0 + 1}
\]

and that \(\tan(\theta_1 + \theta_2) = (1 + \frac{2mc^2}{T_0})(\tan \theta_1 + \tan \theta_2)\).

Thus it can be seen, that for \(\theta_1 < 90^\circ\) and \(\theta_2 < 90^\circ\), that \((\theta_1 + \theta_2) < 90^\circ\)
for \(T_0 > 0\). In the classical case for \(T_0 \ll 2mc^2\), \((\theta_1 + \theta_2)\) approaches 90° for all \(\theta_1\) or \(\theta_2 \leq 90^\circ\).

B. Theoretical Cross Sections

The generally accepted formula giving the scattering cross section of electrons by electrons has been derived by Möller.\(^{(5)}\) This formula in terms of the scattering angle \(\Theta^*\) in the relativistic center of mass system is the following:

\[
\sigma(\Theta) \, d\Theta = \frac{(\gamma_0 + 1) \pi \rho_0^2 \sin \Theta^* \, d\Theta^*}{\gamma_0^2 \beta_0^4}
\]

\[
\left\{ \csc^4 \frac{\Theta^*}{2} + \sec^4 \frac{\Theta^*}{2} - \csc^2 \frac{\Theta^*}{2} \sec^2 \frac{\Theta^*}{2} + \frac{(\gamma_0 - 1)^2}{\gamma_0^2} \left[ 1 + \frac{4}{\sin^2 \Theta^*} \right] \right\} 
\]

where \(\gamma_0\) is the classical electron radius \(\frac{e^2}{mc^2}\), \(\beta_0 = \frac{v_0}{c}\), \(\gamma_0 = \sqrt{1 - \beta_0^2}\)
and \(v_0\) is the velocity of the primary electron in the laboratory system.
The first two terms in the bracket correspond to the classical, relativistic Rutherford\(^{(6)}\) scattering formula. The third term is the quantum mechanical exchange term. The inclusion of this term with Rutherford formula gives the relativistic Mott\(^{(3)}\) formula. The fourth term represents retardation and spin interaction effects.

Equation (1) is more conveniently expressed in terms of the parameter, \(A\), defined as the ratio of the kinetic energy given to the secondary or knock-on electron to the kinetic energy of the primary electron. It is not possible to distinguish between the primary and secondary electrons after collision. The knock-on electron is then by definition the lower energy electron after collision. The maximum value of \(A\) is obviously 0.5. By a simple transformation as shown by Møller, (1) becomes:

\[
\sigma (A) \, dA = \frac{2\pi \, r_o^2 \, dA}{\beta_o^2 (\gamma_o-1)} \left[ \frac{1}{A^2(1-A)^2} - \frac{3}{A(1-A)} + \frac{(\gamma_o-1)^2}{\gamma_o^2} + \left[ 1 + \frac{1}{A(1-A)} \right] \right]
\]

The corresponding relativistic Rutherford cross section is:

\[
\sigma (A) \, dA = \frac{2\pi \, r_o^2 \, dA}{\beta_o^2 (\gamma_o-1)} \left[ \frac{1}{A^2(1-A)^2} - \frac{2}{A(1-A)} \right]
\]

The relativistic Mott cross section is:

\[
\sigma (A) \, dA = \frac{2\pi \, r_o^2 \, dA}{\beta_o^2 (\gamma_o-1)} \left[ \frac{1}{A^2(1-A)^2} - \frac{3}{A(1-A)} \right]
\]

For a 200 Mev primary electron, Equation (2) can be approximated by the following:

\[
\sigma (A) \, dA = \frac{2\pi \, r_o^2 \, dA}{\gamma_o} \left[ \frac{1}{A^2(1-A)^2} - \frac{2}{A(1-A)} + 1 \right]
\]
Comparison of Equations (2), (3), and (4) shows that in the region of $A$ less than 0.01, the three equations are indistinguishable. For $A$, in the region between 0.1 to 0.5, the percentage deviation of Rutherford's cross section from Møller's cross section varies from 1 percent to 11 percent, while that of Mott's to Møller's varies from 12 percent to 56 percent. The expected number of knock-ons in photographic emulsions at this primary energy in the entire region from $A = 0.1$ to $A = 0.5$ is about one per 100 cm of track. Therefore, one cannot hope to resolve these three equations without scanning enormous quantities of track. Thus the experiment was reduced to verifying Møller's formula for low energy knock-ons, realizing that the Rutherford and Mott formulae are equivalent for this region.

C. Range Energy Relations for Electrons

The energies of most of the knock-on electrons were determined by measuring their range in emulsion and comparing these with the range-energy relation for electrons in emulsion. The range-energy relation used was that as determined by Zajac and Ross (1949) for NT2 emulsions (Figure 5) up to 0.25 Mev. Above this energy an extrapolation based upon a range energy curve supplied by Eastman Kodak was made up to 1 Mev.
Fig. 5
Range-energy relation used to determine energy on knock-on electron.
IV EXPERIMENTAL RESULTS

A. Data

The histogram in Figure 6 has been obtained in the following manner. The energy range from 30 Kev to 51 Mev was divided into 11 intervals such that the ratio of the magnitude of any interval is twice that of the preceding one (excluding the first, 30-50 Kev). The absolute cross section for an interval was then determined from the number of events found in that interval, the length of track scanned in finding those events, the number of electrons per cubic centimeter of emulsion and the energy width of the specified interval. The number of electrons per cubic centimeter was obtained from the emulsion composition as given by Ilford (See Appendix B).

If Equation (5) is integrated with respect to A one obtains,

$$\int_{A_1}^{A_2} \sigma(A) \, dA = \frac{2\pi r_0^2}{\gamma_0} \left[ \frac{2A-1}{A(1-A)} + A \right]^{A_2}_{A_1}$$

If this is evaluated for $A_1 = (30\times10^{-3}/185)$ and $A_2 = 0.5$, one obtains 8.5 barns for the total integrated cross section. A numerical integration of the results shown in Figure 6 gives an integrated cross section of 7.3 barns, which is 14 percent low.

B. Sources of Error

(1) Error in Cross Section Due to Statistics

The relative statistical error was computed from the number of events, $N$, in an interval by $\frac{0.67}{\sqrt{N}}$. 
ELECTRON-ELECTRON SCATTERING FOR 185 MEV PRIMARY ELECTRONS

ABSOLUTE CROSS SECTION

Fig. 6
Histogram of the experimental results shown with statistical probable errors.
(2) Error in Knock-on Energy Due to Inaccuracies in $\theta_1$.

The relative error in the energy due to inaccuracies in the measurement of the angle, $\theta_1$, can be estimated in the following manner.

$$T_1 = 2mc^2 \cot^2 \theta_1 \quad \text{or} \quad \frac{dT_1}{d\theta_1} = -4 T_1 \csc(2 \theta_1)$$

for small $\theta_1$, $\Delta T_1 = -4 T_1 \csc(2 \theta_1) \Delta \theta_1$

(3) Error Due to Range Straggling.

The standard deviation of the range measurement was assumed to be 20 percent (13) for electrons of the energies used in this experiment.

(4) Error Due to Uncertainty in Energy of Primary Electron.

The principal uncertainty in the energy of the primary electrons is due to the radiation and ionization losses in traversing the emulsion. The average range of the primary electrons before leaving the emulsion was 0.4 cm. This gives a mean total energy loss for the average electron of 30 Mev. Therefore, the average energy of an incident electron causing a knock-on was estimated to be 185 Mev. (See Appendix C.)

If Equation (5) is written in terms of the knock-on energy, $T_1$, for $T_1 \ll T_0$, one obtains:

$$\sigma (T_1) d T_1 = 2\pi \int \frac{mc^2}{T_1^2} d T_1$$

which is therefore independent of the primary energy, $T_0$.

The number of events, the track length scanned, the experimental cross section and the various errors for the energy intervals used in Figure 6 have been tabulated in Table I.

The effect of the error of the measurement of the knock-on energy upon the magnitude of the absolute cross section has been approximately
calculated (Appendix D). However, in all cases it can be neglected in comparison with the statistical error.

### TABLE I

<table>
<thead>
<tr>
<th>$T_1$ (Mev)</th>
<th>Track Length (cm)</th>
<th>$\sigma$ (Barns/Mev)</th>
<th>Error (1) %</th>
<th>Error (2) %</th>
<th>Error (3) Δθ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03 - 0.05</td>
<td>33.4</td>
<td>156</td>
<td>6.3</td>
<td>14</td>
<td>-</td>
</tr>
<tr>
<td>0.05 - 0.10</td>
<td>33.4</td>
<td>39.2</td>
<td>8.0</td>
<td>14</td>
<td>-</td>
</tr>
<tr>
<td>0.10 - 0.20</td>
<td>102.6</td>
<td>12.0</td>
<td>5.8</td>
<td>14</td>
<td>-</td>
</tr>
<tr>
<td>0.20 - 0.40</td>
<td>102.6</td>
<td>2.83</td>
<td>8.5</td>
<td>14</td>
<td>-</td>
</tr>
<tr>
<td>0.4 - 0.8</td>
<td>102.6</td>
<td>0.524</td>
<td>14.0</td>
<td>14</td>
<td>-</td>
</tr>
<tr>
<td>0.8 - 1.6</td>
<td>102.6</td>
<td>9.13x10^{-2}</td>
<td>23.7</td>
<td>14</td>
<td>2° 28</td>
</tr>
<tr>
<td>1.6 - 3.2</td>
<td>102.6</td>
<td>3.99x10^{-2}</td>
<td>25.3</td>
<td>-</td>
<td>2° 29</td>
</tr>
<tr>
<td>3.2 - 6.4</td>
<td>102.6</td>
<td>2.28x10^{-2}</td>
<td>23.7</td>
<td>-</td>
<td>2° 34</td>
</tr>
<tr>
<td>6.4 - 12.8</td>
<td>102.6</td>
<td>2.84x10^{-3}</td>
<td>47.3</td>
<td>-</td>
<td>1-1/2° 33</td>
</tr>
<tr>
<td>12.8 - 25.6</td>
<td>102.6</td>
<td>7.13x10^{-4}</td>
<td>67.0</td>
<td>-</td>
<td>1° 30</td>
</tr>
<tr>
<td>25.6 - 51.2</td>
<td>102.6</td>
<td>3.57x10^{-4}</td>
<td>67.0</td>
<td>-</td>
<td>1° 41</td>
</tr>
</tbody>
</table>

C. Pair Production and Electron Disappearances.

In the course of scanning for electron-electron collisions the following events were also noted. In 102.6 cm of electron track, two events were found in which the primary electron track divided into three tracks (Figure 7), suggesting pair production in the field of the nucleus. By approximate calculation, one would expect 1.1 pairs for this length of track.

Two events were also found on two separate plates in which the
Fig. 7
Microphotograph mosaic of an electron-positron pair apparently produced in the field of a nucleus by an $\sim 185$ Mev electron
primary electron track vanished in the center of the emulsion. Figure 8 is a photograph of one of the disappearances. The lengths of track before disappearance were 0.7 and 1.5 mm. The experimental arrangement and selection criteria rule out the possibility that these tracks were positrons. It is improbable that the tracks traversed an insensitive volume of the emulsion since the single grain background remains uniform and other primary tracks have no apparent change in grain density in the region of the disappearance. A short distance back on one of the disappearing electrons there is a knock-on coming off in the forward direction confirming the assumed direction of this primary; this rules out the possibility of a Compton electron in the backward direction for this case. The fact that the endings are near the center of the emulsion reduces the probability of not observing a large angle scatter out of the emulsion. The effective track length in which these two disappearances were found is somewhat less than 102.6 cm, since some of the track used was initially scanned in the backward direction. The mechanism by which a high energy electron could disappear in emulsion has not been satisfactorily explained.
Fig. 8
Microphotograph of the disappearance of an ~135 Mev electron near the center of the emulsion.
V SUMMARY

A new approach to the problem of measuring the electron-electron scattering cross section has been attempted using electron sensitive nuclear emulsions in place of the usual cloud chamber. Two hundred Mev electrons obtained by magnetic analysis of pairs converted in the synchrotron beam were allowed to impinge upon Ilford G-5 emulsions. The relatively high density of electrons (∼10²⁴/cc) in emulsion, reduces the length of track necessary to obtain sufficient statistics. The primary electron tracks were followed under approximately 500x magnification, and 427 electron-electron scattering events were recorded in which the scattered electron of lower energy had an energy greater than 30 Kev. The knock-on energy was determined by measuring either the range or the angle between the knock-on and primary tracks. The average energy of a primary electron causing a knock-on was less than 200 Mev due mainly to radiation losses in the emulsion, and was estimated to be 185 Mev. The observed absolute differential cross section, as a function of knock-on energy, was found to be consistent with Moller's theoretical cross section, although the integrated cross section was 14 percent low. At this primary energy, an insufficient number of events of large energy transfer were observed to detect exchange, spin, and retardation effects, and actually only the classical relativistic theory was verified.

In scanning 102.6 cm of electron track two pairs apparently produced in the field of a nucleus by an high energy electron were found. Also two primary electrons were found to disappear near the center of the emulsion. No heavy particle events were observed.
An estimate of the exposure necessary to give a usable plate is as follows:

Assume \( \frac{dN}{dE} = A \frac{Q_T}{E} \), where \( dN \) is the number of quanta in the energy interval \( dE \), and \( A \) is a constant. \( Q_T \) is the total energy in the beam.

\[
Q_T = \int_{0}^{E_m} E \frac{dN}{dE} dE = \int_{0}^{E_m} A dE = A E_m
\]

Thus \( A = \frac{Q_T}{E_m} \) where \( E_m \) is the maximum x-ray energy. Defined in this manner, \( A \) is a measure of the integrated beam strength frequently designated by "equivalent quanta".

Let \( \phi \) be the pair production cross section, and \( N_o \) be the number of atoms in the target within the beam. Then the number of pairs produced by quanta \( dN \) is \( N_o \phi \frac{dN}{dE} dE = N_o \phi A \frac{dE}{E} \).

Assume that the electrons produced have a flat distribution from \( E' = 0 \) to \( E' = E \), where \( E' \) is the electron energy. Then the number of electrons in an energy interval \( dE' \) formed by x-rays of energy \( E \) in the energy interval \( dE \) is given by \( N_o \phi A \frac{dE}{E^2} dE' \). Therefore the total number of electrons in an energy interval \( dE' \) is given by

\[
\int_{E'}^{E_m} AN_o \frac{\phi}{E^2} dE' dE.
\]

Since \( \phi \) is a function of \( E \) choose an average \( \phi \) for the interval \( E = 200 \text{ Mev} \) to \( E = 330 \text{ Mev} \). (Berkeley Synchrotron) Therefore,

\[
\int_{E'}^{E_m} AN_o \frac{\phi}{E^2} dE' dE = N_o \phi A \left[ \frac{1}{E'} - \frac{1}{E_m} \right] dE'.
\]
Assuming an ideal exposure of one electron per 100 square microns in an emulsion of 200 micron thickness gives a total of 1524 electrons for a three inch plate. The energy spread $\Delta E$ corresponding to a plate thickness of 200 microns for 200 Mev electrons in the pair spectrometer field is 80 kilovolts. The number of atoms $N_0$ of tantalum within the 1/8 inch beam diameter for a 9 mil foil is $1.11 \times 10^{20}$. An approximate value for the pair production cross section, $\phi$, in tantalum is $3.1 \times 10^{-23}$ cm$^2$ per atom between 200 and 330 Mev. This gives a value for $A$ of approximately $2 \times 10^8$ equivalent quanta.
APPENDIX B

Calculation of Electron Density in Ilford G-5 Emulsion

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>DENSITY (gm/cc)</th>
<th>ELECTRON DENSITY (Electrons/cc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>1.85</td>
<td>4.85 x 10^{23}</td>
</tr>
<tr>
<td>Bromine</td>
<td>1.36</td>
<td>3.59 x 10^{23}</td>
</tr>
<tr>
<td>Iodine</td>
<td>0.024</td>
<td>0.06 x 10^{23}</td>
</tr>
<tr>
<td>Carbon</td>
<td>0.27</td>
<td>0.81 x 10^{23}</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>0.056</td>
<td>0.34 x 10^{23}</td>
</tr>
<tr>
<td>Oxygen</td>
<td>0.27</td>
<td>0.81 x 10^{23}</td>
</tr>
<tr>
<td>Sulphur</td>
<td>0.010</td>
<td>0.03 x 10^{23}</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>0.067</td>
<td>0.20 x 10^{23}</td>
</tr>
</tbody>
</table>

10.69 x 10^{23} electrons/cc.
APPENDIX C


(1) Ionization Loss
The ionization loss was estimated from the slope of the range energy curve for minimum ionization electrons as determined by Zajac and Ross. The change in ionization from 1 Mev to 200 Mev is negligible. This loss is approximately 2 Mev in 0.4 centimeter.

(2) Radiation Loss
The radiation length in emulsion is assumed to be about 2.9 cm. Since the average length of electron track followed into the emulsion in this experiment is 0.4 cm, the total energy loss for an average 200 Mev electron is given by \( \frac{0.4}{2.9} \times 200 \approx 28 \) Mev.

(3) The total energy loss for the average electron (range in emulsion of 0.4 cm) is approximately 30 Mev. The average energy for this electron during its passage through the emulsion is then given by 200 - 15 or 185 Mev.
APPENDIX D

Calculation of the Effect of the Energy Resolution upon the Magnitude of the Absolute Cross Section.

The following symbols will be used in the derivation:

- $\delta$ - probable error of energy resolution
- $R$ - curve of resolving power
- $T_1'$ - energy of knock-on electron at which cross section is being measured
- $\rho = \delta / T_1'$
- $f$ - ratio of measured cross section to theoretical cross section

Let the energy spread, $(R)$, be approximated by a triangular resolving power, as illustrated in Figure 9.

![Diagram showing cross section and resolving power curves](image-url)
\[ R = \frac{1}{2\delta^2} \left[ T -(T' - \sqrt{2} \delta) \right] \text{for } T_1' - \sqrt{2} \delta \leq T_1 \leq T_1' \]
\[ R = \frac{1}{2\delta^2} \left[ T -(T' + \sqrt{2} \delta) \right] \text{for } T_1 \leq T_1' \leq T_1' + \sqrt{2} \delta \]

\( R \) defined as above has an area of one and contains half its area within the limits \( T_1' \pm \delta \).

\( f \) is then given by:
\[
\frac{(T_1' + \sqrt{2} \delta)}{\sigma (T_1')}
\int_{(T_1' - \sqrt{2} \delta)}^{(T_1' + \sqrt{2} \delta)} R (T_1) \sigma (T_1) \, dT_1
\]

Assume that \( \sigma (T_1) \) is of the form \( \sigma = A/T_1^2 \).

Then,
\[
f = \left( \frac{T_1'}{A} \right)^2 \left\{ \int_{T_1'}^{T_1} \frac{A}{(T_1)^2} \left[ T_1 - (T_1' - \sqrt{2} \delta) \right] \, dT_1 \right. \\
+ \left. \int_{T_1'}^{T_1} \frac{A}{(T_1)^2} \left[ T_1 - (T_1' + 2 \delta) \right] \, dT_1 \right\}
\]

or
\[
= \frac{1}{2\rho^2} \log \left( \frac{1}{1-2\rho^2} \right)
\]

where \( \rho \) represents the relative error in the determination of the energy of the primary electrons. Some representative values of \( f \) are shown in Table 2.
In this experiment the effect of this error has been neglected in comparison with the statistical errors present.
REFERENCES


