Authors
Erricolo, Danilo
Capolino, Filippo
Albani, Matteo

Publication Date
2002

DOI
10.1109/aps.2002.1018224

Efficient Computation of Multiple Diffracted Short-Pulses Using Ray Fields

*Danilo Erricolo[1], Filippo Capolino[2] and Matteo Albani[3]

[1]Dept. of ECE, University of Illinois at Chicago, 851 S. Morgan St, Chicago, IL, 60607-7053, USA. derricol@ece.uic.edu

[2] Dept. Information Eng., University of Siena, Siena, Italy. Currently, Dept. of ECE, University of Houston, 4800 Calhoun Rd, Houston, TX 77004-4005, USA. capolino@uh.edu,

[3]Dip. Fisica della Materia e Tecnologie Fisiche Avanzate, Univ. of Messina, salita Sperone 31, 98166 Messina, Italy. malbani@ingegneria.unime.it

I. Introduction

A numerically efficient representation for short-pulse (wideband) field propagation in an environment as in Fig.1a is presented in terms of time-domain (TD) rays. Closed form *uniform wavefront* approximations of the organized multiple diffracted fields are provided, using simple TD transition functions. Focus is given on an arbitrary nth diffracted mechanism, as shown in Fig.1a, with specialization on a second order diffraction mechanism (doubly diffracted (DD) fields) as in Fig.1b. The uniform asymptotic solutions for singly diffracted (SD) and DD fields are valid for early observation times (wavefront approximations). Their time-range of validity may be extended to late observation times when the exciting signal does not contain low frequency components. Higher order diffracted fields are approximated using an efficient numerical convolution between TD-SD fields. For simplicity we will not consider here hybrid mechanisms, such as reflected-diffracted rays, etc. The response of the total field to an impulsive excitation (Dirac delta function) is represented in terms of ray fields as

$$\hat{\psi}^{tot,\delta} = \hat{\psi}^{GO,\delta} + \hat{\psi}^{d,\delta} + \hat{\psi}^{dd,\delta} + \hat{\psi}^{ddd,\delta} + ... + \hat{\psi}^{nd,\delta} + ... \quad (1)$$

in which $\hat{\psi}^{GO,\delta}$ includes all the TD geometrical optics (GO) fields, $\hat{\psi}^{d,\delta}$ includes all the TD-SD fields, $\hat{\psi}^{dd,\delta}$ includes all the TD double diffracted (DD) fields, $\hat{\psi}^{ddd,\delta}$ includes all the TD triply diffracted (DD) fields, and so on. When the source radiates a waveform $\hat{G}(t)$, the impulsive response (1) is used to construct the total radiated field $\hat{\psi}^{tot,G}(t) = \hat{\psi}^{tot,\delta}(t) \otimes \hat{G}(t)$, in which $\otimes$ denotes time convolution. Instead of representing the total diffracted field as a continuous superposition (convolution) of impulsive responses, it might be convenient to approximate it as a discrete superpositions of rectangular-pulse responses. Accordingly, the excitation waveform $\hat{G}(t)$ is expanded as a superposition of rectangular pulses of duration T

$$\hat{G}(t) = \sum_{n=1}^{N} g_n \, \mathrm{rect}(t - nT, T), \; \mathrm{rect}(t, T) = U(t + \frac{T}{2}) - U(t - \frac{T}{2}), \; U(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases} \quad (2)$$

in which the weights $g_n = \hat{G}(nT)$ sample the excitation waveform $\hat{G}(t)$ in the middle of the rectangular pulses. The total field is thus approximated as

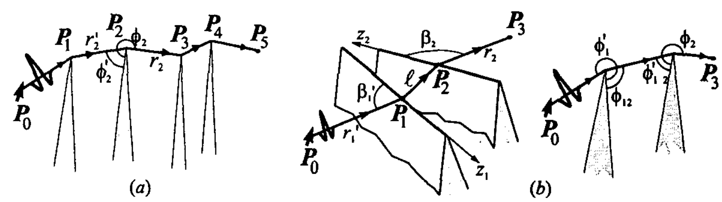

Fig. 1. (a) Multiple diffracted ray. (b) Double diffracted (DD) ray and notation used in [5]

$$\hat{\psi}^{tot,G}(t) \approx \sum_{n=1}^{N} g_n\, \hat{\psi}^{tot,r}(t-nT), \;\; \hat{\psi}^{tot,r} = \hat{\psi}^{GO,r} + \hat{\psi}^{d,r} + \hat{\psi}^{dd,r} + \ldots \tag{3}$$

in which the responses to the rectangular pulse, $\hat{\psi}^{GO,r}$, $\hat{\psi}^{d,r}$ and $\hat{\psi}^{dd,r}$, are evaluated in closed form from the impulsive responses $\hat{\psi}^{GO,\delta}$, $\hat{\psi}^{d,\delta}$ and $\hat{\psi}^{dd,\delta}$ in Sections II and IV. The present formulation has several advantages. First, the representation (3) of the total radiated field $\hat{\psi}^{tot,r}$ is asymptotically uniform, as it was the total impulsive radiated response (1). Second, the various terms are easier to evaluate then (1), since they do not contain singularities due to the Dirac delta excitation. Indeed, the GO field for rectangular excitation $\hat{\psi}^{GO,r}(t) = \mathrm{rect}(t,T)/(4\pi R)$, with R the distance between source and observer, is finite and band limited, imposing the same properties when it is multiply diffracted.

II. Time Domain Single Diffraction (TD-SD)

The impulsive TD-SD field has been presented in [1], [2], [3], while the response to the unit step function $U(t)$ has been presented in [4]. After expressing the rectangular pulse as $\mathrm{rect}(t,T) = U(t+T/2) - U(t-T/2)$, the TD-SD field response to rectangular pulse is obtained as the difference

$$\hat{\psi}^{d,r}(t-T) = \hat{\psi}^{d,U}(t+T/2) - \hat{\psi}^{d,U}(t-T/2), \qquad \hat{\psi}^{d,U}(t) = \frac{1}{\sqrt{rr'(r+r')}} D^{d,U}_{s,h}(t-t^d) \tag{4}$$

in which $\hat{\psi}^{d,U}(t)$ is the response to the unit step function $U(t)$ from [4], or obtained by integrating the results in [1],[2],[3]. $t^d = (r+r')/c$ is the turn-on time (wavefront arrival time) of the diffracted field, and

$$D^{d,U}_{s,h}(t-t^d) = -\frac{1}{2n\sqrt{2\pi}\sin\beta_0} \sum_{m=1}^{4} d^{s,h}_m f^U(x_m, t-t^d) \tag{5}$$

is the TD-SD diffraction coefficient. The TD transition function

$$f^U(x_m,t) = 2\sqrt{\frac{x_m}{\pi}}\arctan\left(\sqrt{\frac{t}{x_m/c}}\right)U(t) \tag{6}$$

is evaluated from the convolution $f^U(x_m,t) = f(x_m,t) \otimes U(t)$ where $f(x_m,t) = x_m/[\sqrt{\pi c t}(t + x_m/c)]$, in which $f(x_m,t)$ is the transition function for impulsive excitation (we have used the notation in [3]). The parameters in (5) are $d^{s,h}_m = \cot(\pi - (-1)^m(\phi-\phi'))/(2n)$, for $m = 1,2$, $d^{s,h}_m = \mp\cot(\pi - (-1)^m(\phi+\phi'))/(2n)$, for $m = 3,4$, $x_1 = 2L\cos^2(2n\pi N^+ - (\phi-\phi'))/2$, $x_2 = 2L\cos^2(2n\pi N^- - (\phi-\phi'))/2$, $x_3 = 2L\cos^2(2n\pi N^+ - (\phi+\phi'))/2$, $x_4 = 2L\cos^2(2n\pi N^- - (\phi+\phi'))/2$, where $L = rr'/(r+r')$ (for spherical wave incidence), $N^\pm$ are the integers that most nearly satisfy the equation $2\pi n N^\pm - (\phi \pm \phi') = \pm\pi$, ϕ' is the incidence angle, r' is the incidence distance, ϕ is the observation angle, r is the observation distance and n is the wedge aperture angle factor.

III. TD Multiple Diffractions

Multiple diffractions may be computed by cascading TD-SD fields derived in the previous section. Let P_0 be the source, P_{N+1} the observation, and P_1,...,P_N the locations of N diffraction points (see Fig.1a), a pulsed field emitted at P_0 produces a transient field at P_{N+1} computed repeating N times the following steps:

1. Evaluate the field $\hat{\psi}^d_{i-1}(P_i,t)$ incident on P_i, due to the source P_{i-1}.
2. Approximate the incident field $\hat{\psi}^d_{i-1}(P_i,t)$ as

$$\hat{\psi}^d_{i-1}(P_i,t) \approx \sum_{n_i=1}^{N_i} \hat{\psi}^d_{i-1}(P_i, n_i T)\mathrm{rect}(t - n_i T, T) \tag{7}$$

and note that the field $\hat{\psi}^d_{i-1}(P_i, t)$ is sampled at the center of the rectangles $(\hat{\psi}^d_{i-1}(P_i, n_i T))$.

3. Using the previous incident field, evaluate at P_{i+1} the field diffracted at P_i

$$\hat{\psi}^d_i(P_{i+1}, t) \approx \sum_{n_i=1}^{N_i} \hat{\psi}^d_{i-1}(P_{i+1}, n_i T)\, \hat{\psi}^{d,r}_i(P_{i+1}, t) \tag{8}$$

in which the diffracted field $\hat{\psi}^{d,r}_i(P_{i+1}, t)$ is the response to the rectangular pulse excitation (4).

In evaluating the field multiply diffracted by wedges, two peculiar situations may occur: a) two consecutive edges have a common face and the electric field is tangent to the common face (soft polarization); b) two consecutive edges are experiencing a double transition, i.e., two consecutive diffraction points P_i and P_{i+1} are almost aligned with source P_{i-1} and observer P_{i+2}. In such cases, more accurate results are accomplished by introducing a TD version of the double diffraction mechanism that is discussed in the next section.

IV. Time Domain Double Diffraction (TD-DD)

The TD response of a double wedge to a rectangular pulse is derived from the uniform wavefront approximation for the impulsive TD-DD field presented in [5], [6]. Similarly to the SD case (4), the TD-DD field response is obtained as the difference

$$\hat{\psi}^{dd,r}(t-T) = \hat{\psi}^{dd,U}(t+T/2) - \hat{\psi}^{dd,U}(t-T/2) \tag{9}$$

where $\hat{\psi}^{dd,U}$ is the response to a unit step function. Referring to Fig. 1b, the TD-DD field is given by $\hat{\psi}^{dd,U}_{12} \sim A^i(r_1')\, A(r_1', \ell, r_2)\, \widehat{D}^U_{12}(t - t^{dd})$, where A^i is the incident spreading factor [for a spherical source, $A^i(r_1') = 1/(4\pi r_1')$] at P_1, $A(r_1', \ell, r_2) = \sqrt{r_1'}/\sqrt{\ell r_2(r_1' + \ell + r_2)}$ is the DD field spreading factor. $\widehat{D}^U_{12}(t - t^{dd})$ is the *TD-DD coefficient*, evaluated at the retarded time $t - t^{dd}$, where $t^{dd} = (r_1' + l + r_2)/c$ is the turn-on time of the DD field. Similarly to the frequency domain case [5],[7], the double diffraction coefficient is $\widehat{D}^U_{12} = \widehat{D}^{I,U}_{12} + \widehat{D}^{II,U}_{12}$, with

$$\hat{D}^{I,U}_{12}(t) = \sum_{p,q,r,s=1}^{2} \frac{c(\mp 1)^{p+q+r+s}}{8\pi n_1 n_2 \sin\beta_1' \sin\beta_2} \cot\left(\frac{\Phi_1^{pq}}{2n_1}\right) \cot\left(\frac{\Phi_2^{rs}}{2n_2}\right) \hat{T}^{I,U}(\bar{a}_{pq}, \bar{b}_{rs}, w, t) \tag{10}$$

and

$$\hat{D}^{II,U}_{12}(t) = \sum_{p,q,r,s=1}^{2} \frac{-\varepsilon_{12} c^2 (\mp 1)^{p+s} (\pm 1)^{q+r}}{32\pi\ell(n_1 n_2 \sin\beta_1' \sin\beta_2)^2} \csc^2\left(\frac{\Phi_1^{pq}}{2n_1}\right) \csc^2\left(\frac{\Phi_2^{rs}}{2n_2}\right) \hat{T}^{II,U}(\bar{a}_{pq}, \bar{b}_{rs}, w, t) \tag{11}$$

where upper/lower sign applies to the soft/hard case, respectively. The TD-DD transition functions

$$\widehat{T}^{\left\{ {I \atop II} \right\},U}(\bar{a}, \bar{b}, w, t) = \left\{ {1 \atop 2\bar{a}\bar{b}/w} \right\} \frac{\bar{a}\bar{b}}{\sqrt{1-w^2}} \left\{ \left[G\left(\bar{a}, \frac{\bar{b} + w\bar{a}}{\sqrt{1-w^2}}, t\right) + G\left(\bar{b}, \frac{\bar{a} + w\bar{b}}{\sqrt{1-w^2}}, t\right) \right] \right.$$
$$\left. \left\{ {+ \atop -} \right\} \left[G\left(\bar{a}, \frac{\bar{b} - w\bar{a}}{\sqrt{1-w^2}}, t\right) + G\left(\bar{b}, \frac{\bar{a} - w\bar{b}}{\sqrt{1-w^2}}, t\right) \right] \right\} \tag{12}$$

are obtained by the convolution $\widehat{T}^{\left\{ {I \atop II} \right\},U}(t) = \widehat{T}^{\left\{ {I \atop II} \right\}}(t) \otimes U(t)$, $\widehat{T}^{\left\{ {I \atop II} \right\}}(t)$ being the transition functions for TD-DD field with impulsive excitation [5],[6], that is calculated in closed form as of $G(x, y, t) = \left\{ \arctan\left[(x/y)\sqrt{1 + t/x^2}\right] - \arctan(x/y) \right\} U(t)$. The various parameters are defined as in [5],[6], $\bar{a}_{pq} = \sqrt{2/c}\sqrt{r_1'\ell/(r_1' + \ell)} \sin\beta_1' \sin[(\Phi_1^{pq} - 2\pi n_1 N_1^{pq})/2]$, $\bar{b}_{rs} = \sqrt{2/c}\sqrt{r_2\ell/(r_2 + \ell)} \sin\beta_2 \sin[(\Phi_2^{rs} - 2\pi n_1 N_1^{rs})/2]$, in which N_1^{pq} and N_2^{rs} are the integers that most nearly satisfy $\Phi_1^{pq} = 2\pi n_1 N_1^{pq}$ and $\Phi_2^{rs} = 2\pi n_2 N_2^{rs}$, respectively; being $\Phi_1^{pq} = \pi + (-1)^p \phi_1' + (-1)^q \phi_{12}$ and $\Phi_2^{rs} = \pi + (-1)^r \phi_2 + (-1)^s \phi_{12}$. Furthermore $w = \sqrt{r_1' r_2/[(r_1' + \ell)(r_2 + \ell)]}$ and $\varepsilon_{12} = \mathrm{sign}(\hat{\beta}_{12}' \cdot \hat{\beta}_{12}) = \mathrm{sign}(\hat{\phi}_{12}' \cdot \hat{\phi}_{12})$. The second order contribution $\widehat{D}^{II,U}_{12}$ becomes particularly important in two possible cases: when two edges have a common face, and when source and observation are almost aligned with two consecutive edges.

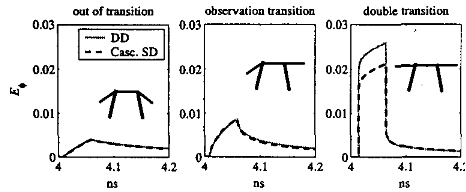

Fig. 2. TD-DD field diffracted at the two edges shown in Fig. 1b. Three different cases: a) out of transitions; b) single transition and c) double transition aspects

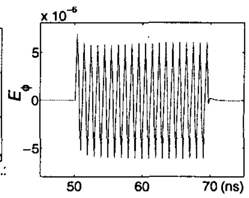

Fig. 3. Diffraction of a modulated rectangular pulse by the four consecutive edges shown in Fig. 1a.

V. Numerical Examples and Conclusions

Numerical results for the diffraction of a rectangular pulse by the double wedge of Fig.1b are shown in Fig. 2. The different ray segments are assumed to be $r_1' = 42cm$, $\ell = 45cm$ and $r_2 = 33cm$ long; while the involved angles are $\beta_1' = \beta_1 = 100°$, $\beta_2' = \beta_2 = 50°$, $\phi_{12} = \phi_{12}' = 110°$. Three cases are considered. In the first case both incidence on the first edge and observation w.r.t. the second edge are out of transition $\phi_1' = \phi_2 = 320°$. In the second case observation is taken at its transition aspect, that is at grazing $\phi_2 = 290°$, while the incidence is still out of transition $\phi_1' = 320°$. The last case considers both incidence and observation at grazing $\phi_1' = \phi_2 = 290°$, thereby DD experiences a double transition. The response to a $T = 50ps$ rectangular pulse was computed in two different ways: 1) by cascading the single diffraction mechanism as explained in Sections II and III (dashed black line); and 2) by using the double diffraction mechanism of Section IV (continuous gray line). These results show that there is strong agreement between the responses obtained with the two methods outside the transition zone; the agreement is still very good when only one aspect is in transition, but greater differences are observed when a double transition occurs. The diffraction of the pulse $\cos(2\pi ft)\mathrm{rect(t)}$, which contains mostly high frequency components, by the four consecutive edges of Fig. 1a is reported in Fig. 3. The duration of the pulse is T=20ns, the carrier frequency f=1GHz, the sampling interval $\Delta t = 0.25$ns and the polarization is soft. The geometrical parameters for this example are $r_1' = 1$m, $r_2' = 2$m, $r_2 = 3$m, $r_4' = 4$m, $r_4 = 5$m, $\phi_1' = 81°$, $\phi_1 = 260°$, $\phi_2' = 100°$, $\phi_2 = 281°$, $\phi_3' = 80°$, $\phi_3 = 260°$, $\phi_4' = 95°$, $\phi_4 = 75°$. The results shown in Fig. 3 were obtained in a fraction of a second for 300 sample points.

In conclusion, numerically efficient results for short pulse propagation in a complex environment have been presented in terms of multiple diffraction. Our TD diffraction propagators are based on discretization of a generic short-pulse in terms of narrow rectangles, and on closed form representations of diffracted fields when the excitation is a narrow rectangular pulse.

References

[1] L. B. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves*, IEEE Press, Piscataway, N. J., 1994,

[2] R. Ianconescu and E. Heyman, "Pulsed beam diffraction by a perfectly conducting wedge: Exact solution," *IEEE Trans. Antennas Propagat.*, vol. 42, no. 10, pp. 1377–1385, Oct. 1994.

[3] P. R. Rousseau and P. H. Pathak, "Time-domain uniform geometrical theory of diffraction for a curved wedge," *IEEE Trans. Antennas Propagat.*, vol. 43, no. 12, pp. 1375–1382, Dec 1995.

[4] T. W. Veruttipong, "Time domain version of the uniform GTD," *IEEE Trans. Antennas Propagat.*, vol. 38, no. 11, pp. 1757-1764, Nov 1990.

[5] F. Capolino and M. Albani, "Time domain double diffraction at a pair of coplanar skew edges," *Antennas Propagat. Soc. Intl. Symp.*, vol. 2, pp. 2062–2065, 2000.

[6] F. Capolino and M. Albani, "Impulsive pyramid-vertex and double wedge diffraction coefficients," *Antennas Propagat. Soc. Intl. Symp.*, vol. 2, pp. 200–203, 2001.

[7] F. Capolino, M. Albani, S. Maci, and R. Tiberio, "Double diffraction at a pair of coplanar skew edges," *IEEE Trans. Antennas Propagat.*, vol. 45, no. 8, pp. 1219–1226, Aug 1997.

D. Erricolo's contribution to this work was supported by the National Science Foundation under Grant ECS-9979413