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The Political Economy of Environmental Policy with Overlapping Generations

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The Political Economy of Environmental Policy with Overlapping Generations[*]

Larry Karp[†]   Armon Rezai[‡]

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Abstract

A two-sector OLG model illuminates previously unexamined intergenerational effects of a tax that protects an environmental stock. A traded asset capitalizes the economic returns to future tax-induced environmental improvements, benefiting the current asset owners, the old generation. Absent a transfer, the tax harms the young generation by decreasing their real wage. Future generations benefit from the tax-induced improvement in environmental stock. The principal intergenerational conflict arising from public policy is between generations alive at the time society imposes the policy, not between generations alive at different times. A Pareto-improving policy can be implemented under various political economy settings.

Keywords: Open-access resource, two-sector overlapping generations, resource tax, generational conflict, environmental policy, dynamic bargaining, Markov perfection

JEL, classification numbers: E24, H23, Q20, Q52, Q54

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1 Introduction

Most analysis of stock-related environmental problems uses assumptions that imply that people alive today must make sacrifices in order to preserve consumption opportunities for those alive in the future. This analysis, in focusing on the conflict between agents who live at different points in time, tends to ignore the conflict between different types of agents alive when the policy is first implemented. An overlapping generations (OLG) model with endogenous asset prices turns this conventional view on its head: all generations may benefit from environmental policy, provided that the winners alive today compensate those who would, absent compensation, be harmed by the policy.

Ramsey models that contain an environmental stock and endogenous capital, e.g. climate models such as DICE (Nordhaus 2008), typically use a one-commodity setting in which output can be either consumed or invested. When investment is positive – as is always the case in equilibrium – the normalization that sets the commodity price to 1 implies that the asset price is fixed, also at 1. In these models, environmental policy affects the trajectory of an environmental stock, (e.g. temperature or carbon stocks), which in turn affects the future productivity of capital and thereby affects current investment. In this setting, the trajectory of capital is endogenous but the price of capital is fixed. The fixed asset price means that these models exclude a potentially important mechanism whereby policy-induced changes in future productivity effect the level and distribution of welfare.

For the purpose of examining the role of asset prices, we study a model that reverses these assumptions: there is a fixed or exogenously changing stock of capital and no depreciation, forcing the price of capital to be endogenous and responsive to policy-induced changes in future productivity. For given environmental stocks, stricter environmental policy reduces current real aggregate income, exactly as in previous models. Stricter policy also lowers the current *real* wage and rental rate of capital; in that respect the welfare effect of policy is symmetric across factors of production. However, by increasing future rental rates via improved environmental stocks (relative to Business as Usual, or BAU), the stricter policy increases the *price* (as distinct from *rental rate*) of the asset. In the particular model that we study, the higher price more than offsets the lower current rental rate, and environmental policy increases the welfare of current owners of capital. Although policy has symmetric effects on the current real wage and rental

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rate, there is a basic asymmetry between capital and labor: the price of the former reflects future productivity, whereas the price of the latter depends only on current productivity. This difference drives the welfare effects that we examine.

In our OLG setting, agents live for two periods and can use an environmental tax. The current old generation owns capital, which it sells to young agents. Because a tax lowers current aggregate real income and increases the welfare of old agents, it necessarily decreases the first period utility of young agents. Young agents might benefit from the policy-induced environmental improvement in the second period of their life. In circumstances that are relevant to most environmental problems, this offsetting improvement does not compensate the young for the loss of first period utility. Therefore, absent transfers, environmental policy increases lifetime welfare of the old asset-rich and lowers the lifetime welfare of the young asset-poor in the first period. However, the first-period old generation can retain all of the benefits of the higher asset values and compensate the young merely by giving them a larger share of the revenue from the environmental tax, compared to the share that future young generations will obtain.

In this way, the old rich pay the young poor to accept stricter environmental policy. They make this transfer not because of a moral imperative, but because it is in their interest to do so: absent the transfer, the young have no reason to agree to implement the policy.

We consider two types of policy settings. In the first, we obtain analytic results, summarized above, for arbitrary perturbations from BAU. We then use numerical methods to study the Markov perfect equilibrium (MPE) in a dynamic political economy setting. These numerical results support and illustrate the analytic results. In each period of this game, the current old and young generations pick a current tax to maximize their aggregate lifetime welfare. They understand that this tax affects the evolution of the environmental stock, which influences the equilibrium tax chosen in the next period. Taxes affect future environmental stocks, and thus affect the future rental rate; that in turn affects the current asset price, and thereby affects the welfare of current generations. Recent papers use similar dynamic settings to study MPE in political economy games that involve redistribution and/or the provision of a public good (Hassler et al. 2003, 2005 and 2007, Conde-Ruiz and Galaso 2005, Klein et al. 2008, Bassetto 2008). Ours is the first paper to use this kind of political economy setting to study equilibrium policy in an environmental setting.

The literature that examines environmental policy in OLG models has neglected the particular role of asset prices that we emphasize. Kemp and van Long (1979) and Mourmouras (1991) are among the first to use the Samuelson (1958) and Diamond (1965) OLG framework to assess the economics of renewable resources. Mourmouras (1993) demonstrates that a social planner can implement welfare-improving conservation measures in a model with environmental externalities and capital accumulation. The emphasis of these policies is to implement non-decreasing ("sustainable") consumption paths. Howarth (1991, 1996), Howarth and Norgaard (1990, 1992), Howarth (1998), Krautkraemer and Batina (1999), and Rasmussen (2003) analyze welfare aspects of sustainable consumption paths in OLG models. John et al. (1995) discuss the steady state inefficiencies due to intergenerational disconnectedness in the presence of private goods with negative externalities; John and Pecchenino (1994) consider the transitional dynamics in this setting. Marini and Scaramozzino (1995) analyze the intertemporal effects of environmental externalities and optimal, time-consistent fiscal policy. Laurent-Lucchetti and Leach (2011) note that current owners of capital capture the benefits of policy-induced innovation.

Bovenberg and Heijdra (1998, 2002) and Heijdra et al. (2006) show that the issuance of public debt can be used to achieve intergenerational transfers, leading to Pareto improvements; they examine the difference in distributional impacts of profit, wage, and lump-sum taxes. Our contribution emphasizes the role of asset price effects and shows that Pareto-improving tax policy can be implemented and sustained through an endogenous political process. In particular, an environmental tax can improve current generations' welfare even in the absence of a government that uses bonds to distribute income across generations.

Many papers use the fact that asset prices depend on adjustment costs, without, however, developing the idea that asset prices can provide an incentive for the current generation to improve the welfare of future generations (Huberman 1984, Huffman 1985 and 1986, and Labadie 1986).

Apart from the OLG structure, our dynamic general equilibrium model is similar to that of Copeland and Taylor (2009). It is close to that of Koskela et al. (2002) in its OLG structure, but differs by separating conventional capital and the renewable resource into different sectors and by allowing for open-access in the latter. Galor (1992) discusses existence and stability properties of a two-sector OLG model and Farmer and Wendner (2003) provide an extension with heterogeneous capital. Our assumption of a fixed capital

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stock reduces much of the complexity of that model.

Guesnerie (2004), Hoel and Sterner (2007) and Traeger (2012) examine the importance, to dynamic environmental policy, of imperfect substitutability between goods that rely (primarily) on either environmental or on man-made capital. In a two-good (general equilibrium) model, this imperfect substitutability manifests as changes in the relative price.

The general equilibrium structure is important for another reason as well. One-good (composite commodity) models obscure the manner in which environmental services contribute to factor productivity. For example, Integrated Assessment Models (IAMs) typically assume that an environmental stock affects all factor returns proportionally. Our model shows that environmental services can affect *real* returns even to a factor that is used exclusively in a sector that does not use environmental services. This indirect effect occurs because environmental services affect an endogenous commodity price index. One good in our model uses the environmental stock, the other good uses capital, and both sectors use labor. Environmental policy affects the allocation of labor, thereby affecting the nominal rental rate of capital. Policy also affects the price index, thereby affecting the real rental rate. Through these general equilibrium effects, environmental policy affects the price and rental rate of capital, even though the sector that uses capital does not employ environmental services.

There already exist two challenges to the conventional view that environmental policy requires sacrifices by those alive today. First, by correcting multiple market failures jointly, it is possible to protect the environment without reducing current consumption (a "win-win" opportunity). Second, there may be opportunities to rebalance society's investment portfolio, reducing saving of man-made capital and increasing saving of environmental capital in a way that leaves all generations better off than under BAU (Foley 2009, Rezai et al. 2012). Our model has neither of those features.

After presenting the model and deriving the main analytic results, Section 4 discusses the robustness of the results. For emphasis, we mention two issues here. First, it is obvious that environmental policy will never increase the value of *all* capital assets. Our use of a single capital stock means that we cannot identify the effect of environmental policy on different types of capital. However, almost all environmental (theory and policy) papers that consider capital use a single capital stock; computable general equilibrium models are an exception. The assumption that increased environmental services increase the real return to this capital stock is also standard, although as noted above,

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the mechanism by which this increase occurs in our model is novel.

The second issue is more important. In a two-period OLG model, a reasonable length of a period is 35 years. Over that span of time, much of the current stock of capital – perhaps as much as 90% – will be replaced. If the replacement rate were 100%, i.e. if the current stock fully depreciates in a single period, then there is no asset to be transferred, so future productivity increases could not benefit current asset owners. The 100% depreciation rate is a limiting case, as is the 0% depreciation rate that we use. How empirically important are our results, when the actual depreciation rate is between the two, but closer to the upper than the lower limit? The answer to this question requires a much more complicated model than the one studied here, one involving both depreciation and endogenous investment. Our work in progress establishes that the qualitative results presented here remain important even for depreciation rates of 90% per 35-year period. In particular: a tax policy affects the current asset price; the asset market provides a means of transferring benefits that are realized in the future, to agents currently alive; and a tax can increase the lifetime welfare of agents alive in the first period, even if all of the productivity increases arising from the tax occur after their death. Space limitations make it impractical to discuss in any detail the model with depreciation and endogenous capital.

2   Model

There is a single distortion, missing property rights, and a single endogenously changing stock, the environment. Agents live for two periods and they care only about their own lifetime welfare; their only means of influencing the future is to change their current use of the environmental stock. These assumptions bias the model against Pareto-improving environmental policy. However, an environmental tax with appropriate allocation of tax revenues creates a Pareto improvement and can be implemented in a political economy equilibrium.

One sector, "manufacturing", produces a good M using mobile labor and a sector-specific input, capital. The stock of capital is fixed, $K \equiv 1$; later we relax this assumption. The other sector produces a good F using labor and an endogenously changing resource stock, x. (We suppress time indices where no confusion results.) There are perfect property rights for the stock of manufacturing capital, and no property rights for the resource stock. In

the absence of an environmental policy, mobile labor competes away all rent in the resource-intensive sector.

Young agents receive a wage, income from the resource sector, and possibly a share of tax revenues. They divide their income between consumption of the two goods and purchase of manufacturing capital. The old generation earns the profits of its manufacturing firm, the proceeds from selling the firm, and its share of the tax revenue. Because agents are non-altruistic, the old generation consumes all of its income. The size of the population is fixed and normalized to 1.

The labor and commodity markets are competitive and clear in each period. Employment in the resource sector equals L, and free movement of labor between the sectors ensures that the return to labor there equals the manufacturing wage. Manufacturing is the numeraire good, and the relative price of the resource-intensive good is P. Output in the resource-intensive sector is $F = L\gamma x$, with the constant $\gamma > 0$. Manufacturing output is $M = (1 - L)^\beta$ with $0 < \beta < 1$, so that there are profits (rent) in this sector; β is labor's share of the value of manufacturing output. The old generation owns K, the only asset of the economy, which it sells to the young generation after production occurs.

The open access of the resource sector means that too much labor moves to this sector. An ad-valorem tax, T, on production of the resource-intensive good reduces this misallocation. The revenue accruing to workers in the resource sector, under the tax, equals $P(1 - T)L\gamma x$. Society returns the tax revenue, $R = PTL\gamma x$, in a lump sum, but possibly different shares, to the young and old generations. The next two subsections examine the effect of the tax on nominal and real returns to factors and on the price of capital.

The endogenous variables σ_t and $\tilde{\sigma}_{t+1}$ are the present and the expected next-period value of the firm; χ_t and $\tilde{\chi}_{t+1}$ are the present and expected next-period share of the tax revenue received by the young; and $\tilde{\pi}_{t+1}$ is the expected next-period manufacturing profit. We assume intertemporal additive utility, with the single period utility function $u\left(c_{F,t}, c_{M,t}\right)$, where $c_{i,j}$ is the consumption level of good i at time j. The agent's pure rate of time preference is ρ. The lifetime decision problem of the representative agent who is young in period t, is

$$\max_{c^y_{F,t}, c^y_{M,t}, c^o_{F,t+1}, c^o_{M,t+1}, s_t} u(c^y_{F,t}, c^y_{M,t}) + \frac{1}{1 + \rho} u(c^o_{F,t+1}, c^o_{M,t+1})$$

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subject to the budget constraints in the first and second period of their life:

$$w_t + \chi_t R_t \geq P_t \, c_{F,t}^y + c_{M,t}^y + \sigma_t s_t, \text{ and}$$

$$\tilde{\sigma}_{t+1} s_t + (1 - \tilde{\chi}_{t+1}) \tilde{R}_{t+1} + \tilde{\pi}_{t+1} \geq \tilde{P}_{t+1} c_{F,t+1}^o + c_{M,t+1}^o,$$

where s_t is the fraction of shares that the young agent purchases. In equilibrium, supply of shares equals demand, i.e. $s_t \equiv 1$; hereafter we suppress s_t. The superscripts o and y on consumption variables indicate whether the agent is old or young at a point in time. We suppress those superscripts because the meaning is clear from the context. Agents take as given, or have rational point expectations of:

$$w_t, \, P_t, \, \tilde{P}_{t+1}, \, \sigma_t, \, \tilde{\sigma}_{t+1}, \, \chi_t, \, \tilde{\chi}_{t+1}, \, R_t, \, \tilde{R}_{t+1}, \, \rho, \, \tilde{\pi}_{t+1}.$$

The young agent dedicates all of her time to working and the old agent manages the manufacturing firm.

The utility function is Cobb-Douglas: an agent who consumes $c_{F,t}$ and $c_{M,t}$ obtains utility $u(.) = c_{F,t}^\alpha c_{M,t}^{1-\alpha}$, with α the constant budget share for the resource-intensive good. With $\mu \equiv \alpha^\alpha (1 - \alpha)^{1-\alpha}$ and e equal to expenditures, an agent's single period indirect utility is

$$v(e, P) = \left(\frac{\alpha e}{P}\right)^\alpha \left(\frac{(1 - \alpha) e}{1}\right)^{1-\alpha} = \alpha^\alpha (1 - \alpha)^{1-\alpha} P^{-\alpha} e = \mu P^{-\alpha} e.$$

The assumption of identical homothetic preferences implies that the share of income devoted to each good is independent of both the level and distribution of income; prices do not depend on the distribution of income. The ratio of demand for both goods is a function of this price. The requirements that workers are indifferent between working in either sector, $P(1 - T)\gamma x = w$, and that manufacturing firms maximize profits, determine the wage, the allocation of labor, and supply of both goods. The relative price, P, causes product markets to clear. These equilibrium conditions for the labor and product markets lead to the following expressions for the values of L, w, and P:

$$L = \frac{1 - T}{\frac{1 - \alpha}{\alpha} \beta + 1 - T}, \qquad w = \beta \left(1 + \frac{1 - T}{\frac{1 - \alpha}{\alpha} \beta}\right)^{1-\beta}$$

$$P = \frac{w}{(1 - T)\gamma x} = \frac{\beta \left(1 + \frac{1 - T}{\frac{1 - \alpha}{\alpha} \beta}\right)^{1-\beta}}{(1 - T)\gamma x} \equiv p(x, T).$$

(1)

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Under Cobb-Douglas technology and preferences, the equilibrium allocation of labor and the wage do not depend on the resource stock, x, only on the tax T and the parameters α and β. However, the equilibrium commodity price depends on x via the function γx. Firms' profits, π, the tax revenue, R, and the sectoral values of output, PF and M, also depend only on T and model parameters:

$$\pi = \tfrac{1-\beta}{\beta}w(1 - L), \qquad R = \tfrac{T}{1-T}Lw,$$
$$M = (1 - L)^{\beta}, \qquad PF = \tfrac{\alpha}{1-\alpha}(1 - L)^{\beta}. \tag{2}$$

Systems (1) and (2) determine the static equilibrium of the economy.

2.1 Relation to one-commodity Ramsey models

The real wage, $\mu P^{-\alpha}w$, and the real rental rate, $\mu P^{-\alpha}\pi$, equal the amount of utility that an agent obtains by renting out one unit of labor or one unit of capital, respectively. These real factor returns depend on both the tax and the stock of the resource.

Our model has two features in common with familiar one-commodity Ramsey models: the current tax reduces current aggregate utility and reduces both the current real wage and the real rental rate. We emphasize this similarity, lest the reader mistakenly think that our main results depend on incidental features of the model, e.g. the assumption that only the manufacturing sector uses capital.

Proposition 1 *(i) An increase in the tax at time t reduces aggregate period-t utility. (ii) For a predetermined level of the environmental stock, a higher tax decreases both the real wage and the real rental rate. (iii) A higher environmental stock increases utility and both the real wage and the real rental rate.*

(The appendix contains proofs not found in the text.)

The tax causes labor to leave the resource sector, reducing the nominal wage and increasing nominal profits in manufacturing. The tax also increases the relative price, P, so the tax unambiguously decreases the real wage. The important point is that the tax also reduces the real rental rate, via the effect of the tax on the relative price. The fact that a higher stock increases the real wage is obvious. We emphasize that although the nominal rental
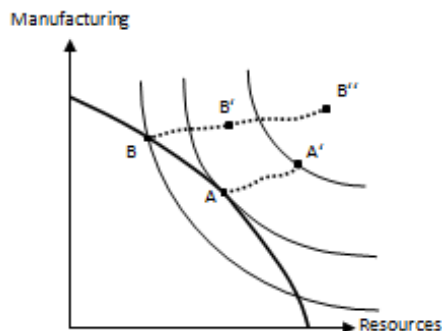
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Figure 1: Consumption expansion paths under BAU $(A - A')$ and under an environmental policy $(B - B' - B'')$

rate is independent of the stock, a higher stock increases the real rental rate. Thus, although a stock or tax change has different effects on the two nominal returns, that change has the same qualitative effect on the two real returns. In this important respect, the model treats the two factors symmetrically. The general equilibrium framework shows how changes in the environmental stock affects the real return to factors that do not directly depend on this stock – the manufacturing capital in our model.

Figure 1 uses a production possibility frontier to illustrate the welfare effect of environmental policy both in standard Ramsey models and in our OLG model. Under BAU, current aggregate consumption is at point A, a level that maximizes current aggregate utility, ignoring the environmental externality. The tax moves consumption to point B, where current aggregate utility is lower. Therefore, some people must have lower *current* utility at B than at A.

Figure 1 illustrates the conventional view that environmental policy creates a conflict between those alive today and those alive in the future. The aggregate (single period) consumption path under BAU moves along the curve from A to A', a trajectory that incorporates changes in both environmental and man-made capital stocks, including technological change (introduced in Section 6). The aggregate consumption trajectory under the environmental policy moves along the curve from point B to B''. Agents alive at the initial time have higher *current* aggregate utility under trajectory AA', and those alive later have higher (single period) aggregate utility under trajectory BB'', so dynastic (rather than OLG) models lead to the conventional view that a

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welfare comparison depends on the social discount rate.

The two previous challenges to the conventional view, the existence of win-win situations or the possibility of reallocating the investment portfolio, imply that environmental policy moves society from trajectory AA' to trajectory $B'B''$. With this move, environmental policy increases aggregate single period utility in every period. Our model rules out both of the previous challenges: there are no win-win opportunities, and the assumption that the environment is the only endogenously changing stock excludes the possibility of reallocating investment across stocks.

Environmental policy in our model lowers aggregate *current* utility in the first period, just as in the standard Ramsey framework. The current old live for a single period, so the tax increases their lifetime welfare if and only if it increases their utility in the current period. The current young are also alive in the next period. Even if the tax lowers their current utility, their lifetime welfare can increase if their utility in the next period increases sufficiently.

Although a higher tax and a higher environmental stock have the same qualitative effects on the real wage and the real rental rate there is a fundamental asymmetry between the two factors: the *price* of capital depends on future rental rates, whereas the price of labor depends on only its current value of marginal productivity. Current owners of capital benefit from the future increases in productivity created by the environmental policy, even though they are not alive to enjoy them directly. Absent transfers, current owners of labor benefit from these future productivity increases only to the extent that they are alive to enjoy them.

2.2 The asset price

The young buy manufacturing firms from the old; the asset price affects welfare through expenditure. Systems (1) and (2) enable us to express the young and old generation's expenditure levels, e^y and e^o, as functions of current tax T and the asset price, $\sigma(x, \mathbf{T})$, where $\mathbf{T}$ is the tax trajectory:

$$e^y = w(T) + \chi R(T) - \sigma(x, \mathbf{T}) \quad \text{and} \quad e^o = \pi(T) + (1 - \chi)R(T) + \sigma(x, \mathbf{T}). \quad (3)$$

The first order condition for optimal savings (hereafter, the "no-intertemporal-arbitrage condition") requires that the young's marginal loss in utility from purchasing a unit of the asset in the current period equals their marginal gain in utility from having that asset in the next period. This condition determines the demand for the asset as a function of its current price and the

expectation of next period rental rate and price. This demand function, and the fixed (or exogenously changing) supply of capital, determine the current asset price as a function of expected next period rental rate and price, leading to:

Proposition 2 *The price of a unit of capital is equal to the infinite sum of future discounted utility arising from the firm's future profits, evaluated at current prices.*

A policy change that, for example, increases the asset price, benefits the current asset owners, the old. The changed asset price has no effect on the welfare of asset purchasers, the young. The no-intertemporal-arbitrage condition described above implies that the young pay exactly what the asset is worth to them. Although the change in asset price changes their current expenditures, the offsetting change in future receipts leads to a zero change in their welfare:

Corollary 1 *(i) An unanticipated change in the asset price does not affect the lifetime utility of current and future young generations. (ii) Unanticipated changes in the asset price affect only the current old generation.*

2.3   Resource dynamics

We assume that the resource stock obeys a logistic growth function:

$$x_{t+1} = x_t + rx_t \left(1 - \tfrac{x_t}{C}\right) - L(T_t)\gamma x_t = \left(1 + r\left(1 - \tfrac{x_t}{C}\right) - L(T_t)\gamma\right) x_t$$
$$= (1 + \bar{r}_t(T_t, x_t)) \, x_t; \text{ with } \bar{r}_t \equiv \left(r\left(1 - \tfrac{x_t}{C}\right) - L(T_t)\gamma\right), \tag{4}$$

with r the intrinsic growth rate, C the carrying capacity of the resource, and $\bar{r}$ the endogenous growth rate of the resource. A higher tax conserves the resource because $\frac{dL_t}{dT_t} < 0 \Rightarrow \frac{d\bar{r}_t}{dT_t} > 0 \Rightarrow \frac{dx_{t+1}}{dT_t} > 0$.

We restrict parameter values to ensure that under BAU there exists an interior steady state, x_∞, to which trajectories beginning near that steady state converge monotonically. The necessary and sufficient conditions for this are $1 > \frac{d(1+\bar{r})x}{dx} > 0$, evaluated at $T = 0$, $x = x_\infty$. These inequalities are equivalent to

$$1 < \varsigma < 2 \text{ with } \varsigma \equiv r + \frac{\beta\left(1 - \alpha\right) + \alpha\left(1 - \gamma\right)}{\beta\left(1 - \alpha\right) + \alpha}. \tag{5}$$

The unique non-trivial BAU steady state stock of the resource is

$$x_\infty = C\left(1 - \frac{\gamma L(0)}{r}\right) = C\frac{\varsigma - 1}{r}. \tag{6}$$

The BAU trajectory is monotonic if and only if the initial condition satisfies $x_0 \leq \frac{1}{\varsigma - 1} x_\infty$.

3 Welfare Effects of a Tax

Under BAU, the environmental tax is identically 0. Consider an arbitrary non-negative tax trajectory, the vector $\bar{\mathbf{T}}$, with element $\bar{T}_i \geq 0$. Strict inequality holds for at least one i, including $i = 0$. The index i denotes the number of periods in the future, so $i = 0$ denotes the current period. A non-negative perturbation of the zero tax BAU policy is $\mathbf{T} = \varepsilon\bar{\mathbf{T}}$, with $\varepsilon \geq 0$ the perturbation parameter. A larger ε therefore is equivalent to a higher tax policy. Consideration of delayed policies yields only obvious results, so we assume that $\bar{T}_0 > 0$. In this section we set the fraction of tax revenue given to the young, χ, to be a constant, an assumption we revisit in Section 5. The following proposition provides a sufficient condition for a non-negative perturbation of the BAU policy to improve the welfare of the old generation.

Proposition 3 *For all $\chi \in [0, 1]$, a sufficient condition for the old generation to benefit from a small tax increase is that the initial value of the environmental stock (when the policy begins) satisfies $x_0 \leq \frac{1}{\varsigma - 1} x_\infty$, where ς and x_∞ are defined in equations (5) and (6). The old generation's welfare increases in its tax share, $(1 - \chi)$.*

The sufficient condition, stated in terms of the initial value of the environmental stock, ensures that the BAU stock trajectory approaches the BAU steady state monotonically. Inspection of the proof shows that the old can benefit from a tax even when this restriction does not hold. We have the following immediate result.

Corollary 2 *Under the condition stated in Proposition 3, the tax leads to a fall in first-period welfare of the present young generation.*

Proof. Propositions 1 and 3 state that aggregate current welfare falls while welfare of the old generation rises. Therefore, first-period welfare of the current young must fall. ∎

In general, a price change creates winners and losers. The OLG framework shows that a policy that discourages over-use of a resource benefits asset holders and in the first period harms the young agents. Of course, the policy also changes the consumption of the current-young in the next period, thereby creating the possibility of higher *lifetime* welfare. To avoid uninteresting complications, we assume for the rest of this section that $\bar{T}_1 = \bar{T}_0 > 0$.

Proposition 4 *For a constant $\chi \in [0,1]$, a small increase in tax rates (equivalently, a larger ε) increases lifetime welfare of the present young generation if and only if: (a) it receives less than the entire tax revenue while young ($\chi < 1$), and (b) $\bar{r}(0, x_t) > (1+\rho)^{\frac{1}{\alpha}} - 1$, i.e. the pure rate of time preference is less than the positive welfare effect of lower prices due to the higher resource stock.*

If the renewable resource is being degraded on the BAU trajectory (as is the case for most stock-related environmental problems), then $\bar{r}(0, x_t) < 0$, so condition (b) in the Proposition fails. If in addition $\chi < 1$, the tax policy lowers the young generation's lifetime welfare.

Even if a small tax harms the young, it makes sense to ask whether they would prefer to receive a larger share of tax revenue when young or old:

Proposition 5 *The young generation prefers to receive all tax revenue when it is old ($\chi = 0$) if and only if it benefits from a tax introduction. If the policy lowers their welfare, they prefer to receive all of the tax revenue while young ($\chi = 1$).*

Condition (b) in Proposition 4 implies that the young benefit only if they can increase their welfare by shifting their income into the future. In this case, the young generation wants to shift all its tax receipts into the future, and prefers $\chi = 0$. If condition (b) does not hold and $\chi = 1$, the tax creates a zero first order effect on the young generation's welfare; the first order effects of a tax on the real wage and the tax revenue sum to zero.

In summary, if the environmental problem is that the resource is below its 0-tax steady state and therefore recovering, but just not recovering sufficiently quickly, then the young potentially would support a tax that speeds

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recovery. In that circumstance, both the young and the old generations want all of the tax revenue to go to the old, under the constraint that the share is constant. In the more relevant circumstance where the environmental objective is to keep the resource from degrading excessively, the young would oppose a tax that helps to solve the problem. If such a tax were forced upon them, and the tax share χ were constant, they would prefer to receive all of the tax revenue while young. Thus, in the case that is relevant to most problems involving environmental stocks, this OLG model shows that there is a conflict between generations alive at the time society imposes the tax. The old generation favors the environmental policy because some of the future benefits of that policy are capitalized into the asset value. The current young obtain none of those capitalized benefits, and they do not live long enough to reap significant benefits from the improved environment.

4   Robustness

We are now in a position to discuss the model's robustness. Section 2.1 notes that taxes and environmental stocks have the same qualitative effects on the real rental rate and wage, as in one-commodity Ramsey models. The critical difference between capital and labor (in our model but not in standard Ramsey models) is that the price of capital reflects tax-induced *future* increases in rental rates, whereas the wage reflects none of the future increases in labor productivity. Asset owners therefore capture some of the future productivity gains due to the tax, whereas agents who sell their labor benefit only from the productivity improvements that occur during their lifetime.

As with most theoretical Ramsey models, we have a single type of capital, an assumption motivated by tractability not realism. We do not, of course, think that environmental policy increases the value of all assets; counterexamples are easy to find. These models simply assume that a better environmental stock increases the (real) return to an *aggregate* measure of capital, and that environmental policy improves the future environmental stock. These two assumptions are innocuous.

The crucial assumption is that tax-induced changes in environmental stocks affect the price of currently existing capital. This assumption holds if two more basic conditions hold.

The first condition is that given current stocks of environmental and manmade capital, the production possibility frontier between a composite con-

sumption commodity and a composite investment commodity is strictly concave; equivalently, there are convex investment costs. As noted in the Introduction, most environmental applications of the Ramsey model assume that a composite good can be converted at a constant rate between a consumption and an investment good. This assumption means that the production possibility frontier between the two types of goods is linear, and it implies that the price of capital is fixed at the price of the numeraire consumption good. These models therefore make the opposite assumption to ours: environmental policy cannot possibly affect the asset price. However, it seems at least as reasonable to assume a strictly concave production possibility frontier. In that case, changes in the future environmental stock change the price of new capital.

The second condition is that not all of the capital that exists at the beginning of the period depreciates during that period. If capital fully depreciates during a period, then the current old have no assets to sell to the current young, and cannot benefit from increased future productivity. In that case, capital and labor are exactly symmetric: the price of both depends only on their productivity during the period, not on productivity in subsequent periods.

Our model satisfies both of these conditions: capital does not depreciate, and it is not possible to convert the consumption good to capital: adjustment costs are infinite. Clearly, neither zero depreciation nor infinite adjustment costs are empirically plausible. For example, as annual depreciation of capital ranges from $2 - 6\%$, the amount of an initial stock of capital remaining after 35 years (half of an agent's lifetime) ranges from $49 - 11\%$. The question is not whether our assumptions of zero depreciation and infinite adjustment costs are empirically plausible (they are not), but whether these assumptions are critical to our results.

The answer to this question requires a more general model that includes the possibility of investment, together with depreciation and empirically plausible adjustment costs. A companion paper (in progress) studies such a model, which must be solved numerically. Our central conclusion, that environmental taxes have a meaningful effect on the price of capital and that this relation has a meaningful effect on the welfare of old agents, survives, even at annual depreciation rates of 6%. Thus, the central results of this paper are robust.[1]

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[1] Our functional forms imply that utility is linear in income, so the intertemporal elastic-

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5 Transfers

Propositions 4 and 5 are based on the assumption that the old in each period receive the same share of tax revenue, i.e. that $1 - \chi$ is constant. That assumption is useful for understanding the distributional effect of environmental policy, but it is not reasonable as a policy prescription. The old in the period when the tax is imposed – unlike the old in any other period – capture the future benefits that are capitalized in the asset price (Corollary 1). In addition, the young in future periods benefit from a higher resource stock (relative to BAU) in both periods of their life; the young in the current period benefit from environmental protection in only the second period of their life. Therefore, it is reasonable to treat the old and the young in the period when the policy is introduced differently than their counterparts in future periods. In particular, the current young should receive a larger share of tax revenues, compared to the young in future periods.

Here we consider the role of transfers, when under BAU the resource is degrading, $\bar{r}(0, x_t) < 0$. The proof of Proposition 4 shows that a small tax has only a second order welfare effect on the young if they receive all of the tax revenue while young ($\chi = 1$). We noted above that the old obtain a first order welfare gain even if they receive none of the tax revenue. Given these two results it is not surprising that for a small tax, it is always possible for the old generation to make a transfer to the young, in addition to giving them all of the tax revenue, so that both generations are better off. This means of compensating the young requires that the old give them a portion of the tax-induced increase in the asset value.

An alternative means of compensating the young is to give them a higher share of tax revenue, compared to the future young. One way to do this is to decrease χ (thereby increasing the share that today's young receive in the next period, when they are old) and simultaneously giving today's young the fraction ξ of today's old generation's share of tax revenue. This transfer scheme ($\xi > 0$), which occurs only in the first period, allows the first period old to keep all of the capital gains and the fraction $(1 - \chi)(1 - \xi)$ of tax revenue. In this way, *the future young (rather than the current old) compensate the current young to make the latter willing to accept the tax policy.* We state this formally:

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ity of substitution is infinite. With finite intertemporal elasticity of substitution (as in our companion paper), changes in future productivity can influence current savings decisions, thereby affecting the current level of utility, due to income-smoothing incentives.

Proposition 6 *For constant $\chi < 1$ there exists a tax transfer rate $0 \le \xi^{crit} < 1$ from the present old to the present young such that with $\xi > \xi^{crit}$, a small tax policy with $\bar{T}_0 = \bar{T}_1 > 0$ creates a Pareto improvement.*

Because the young gain under this tax and transfer, an argument parallel to that which establishes Proposition 5 implies that for any $\xi > \xi^{crit}$, both generations prefer $\chi = 0$. In the political economy model studied in Section 7 we therefore emphasize the case $\chi = 0$.

The fact that a tax and transfer combination creates a Pareto improvement for the generations alive at the time society imposes the policy is noteworthy because it arises in a model that appears biased in favor of finding that an environmental policy harms some generation. Agents alive at the time the policy is imposed do not care about the welfare of future generations. In addition, they have only one means of accumulation: protecting the environment. That protection always requires that aggregate first period utility of consumption falls.

Generations sufficiently far in the future are also better off due to a small tax. A small tax has only a second order effect on "static efficiency", the efficiency calculation that holds the trajectory of the resource stock fixed. However, the tax has a first order effect on the steady state resource stock, and that increased stock creates a first order welfare gain in the steady state. Absent transfers, the tax is more likely to benefit future generations compared to the current young generation: the tax-induced higher stock benefits each of the future generations in two periods, whereas it benefits the current young generation in only one period. (See Appendix B.1 for details.)

6   Exogenous Productivity Growth

In the context of most environmental problems, the natural resource is degrading on the 0-tax trajectory. In our model of constant productivity and capital, the world becomes poorer and future generations have lower welfare on that trajectory. This section introduces exogenous productivity growth in both sectors. Let $a \ge 0$ be the growth rate of total factor productivity in manufacturing and $b \ge 0$ the growth rate of efficiency in output per unit flow of the resource. Sectoral output is

$$M_t = e^{at}(1 - L_t)^{\beta} \quad \text{and} \quad F_t = e^{bt} L_t \gamma x_t.$$

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The inequality $a > 0$ can also be interpreted as exogenous growth in the stock of capital. The extraction of the resource is still $L_t \gamma x_t$ (not $e^{bt} L_t \gamma x_t$). This model of resource productivity growth implies that each extracted unit of the resource increases the supply of the resource-intensive commodity. If we think of the resource as being energy, the assumption means that the economy becomes less energy intensive. The assumption of exponential productivity growth simplifies the discussion, but the next proposition also holds if the productivity parameters a and b decrease over time. The exponential productivity growth implies a growth factor of $e^{(a-b)} (1 + \bar{r}(T_t, x_t))$ for the price level and of e^a for all other variables (w_t, R_t, and π_t). For the following proposition we assume that $\chi \in (0,1)$ is constant and that there is no transfer between generations, i.e. $\xi = 0$.

Proposition 7 *A larger value of $a - b$ increases the stringency of the necessary and sufficient condition under which a small constant tax increases the welfare of the young.*

Under proportional growth ($a = b$), the condition for the young to benefit from the tax is the same as when $a = b = 0$. The welfare effect of the tax, for the young, depends on the change in the price level. A *ceteris paribus* increase in $a - b$ increases the next period relative supply of the manufacturing good, thereby increasing the future relative price of the resource-intensive good, P_{t+1}. The higher price lowers the marginal utility of next period income, making it "less likely" that the young are willing to forgo income today in order to have higher income in the next period. For $a > b$, the young would require a higher transfer from the old in order to agree to the tax. If, however, the productivity in the resource sector grows much faster than in the manufacturing sector ($b >> a$), the young might support a tax even when the resource is shrinking on the 0-tax trajectory, and in the absence of a transfer.

7 Political Economy Equilibria

In each period, both generations can gain from a tax, given proper allocation of tax revenues. To find the equilibrium tax, we calibrate the model and solve it numerically under the assumption that χ is a constant – an assumption that we discuss in footnote 3 below. Our baseline sets $\chi = 0$, i.e. the old agents receive all of the tax revenue, a choice motivated by the comment

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below Proposition 6. We assume that in each period agents play a bargaining game with side payments and exogenous threat point. Agents consequently choose the tax to maximize their joint lifetime welfare. We provide sensitivity results and compare the baseline trajectory to that of a social planner who maximizes the discounted stream of single period aggregate real income.

We consider a Markov Perfect equilibrium (MPE). In our stationary model, agents condition the choice of the current tax on the only directly payoff-relevant state variable, the environmental stock. The MPE consists of a policy function mapping the state variable into the tax. If agents in the current period believe that future agents will use that policy function, and if it is optimal for current agents to also set the current tax equal to the value returned by that function, then we have a MPE.

Hassler et al. (2003, 2005, 2007), Conde-Ruiz and Galaso (2005), Battaglini and Coate (2007), Klein et al. (2008), and Bassetto (2008) also study MPE in political economy settings. Hassler et al. 2005, page 1339, note that the probabilistic voting model described in Lindbeck and Weibull (1987) and Perrson and Tabellini (2000) provides an explanation for an equilibrium decision that maximizes current agents' joint welfare. In our model, with two types of agents of equal measure, voting models are not particularly useful; nevertheless, the assumption that these two agents play a bargaining game remains compelling. Current decisionmakers are constrained by the equilibrium decision rules of their successors. This constraint means that the equilibrium tax need not be, and in fact is typically not, Pareto efficient, just as in Battaglini and Coate (2007).

Bargaining between those currently alive does not, of course, resolve the conflict across generations that live during different periods. Generations in the future always prefer that previous generations use a larger tax, to generate a larger environmental stock. Our point is simply that starting with a major unsolved environmental problem, here represented by a zero BAU tax, all generations can be made better off when agents alive at each point are able to use a politically determined tax, even when they do not care about the welfare of those who will live in the future.

7.1 Calibration

We set the parameter α, the share of the resource-intensive commodity in the consumption basket, equal to 0.2. We set $\beta = 0.6$, the approximate wage share in U.S. manufacturing. We set the annual pure rate of time preference

at $2\%/year$ which gives $\rho = 1$ assuming one period lasts 35 years.

We model the renewable resource as easily exhaustible and slowly regenerating, in order to capture the idea that the environmental problem is serious. We choose units of the resource stock, x, such that its carrying capacity is normalized to one, $C = 1$, so that x equals the capacity rate. The productivity parameter γ equals the inverse of the amount of labor that would exhaust the resource in a single period, starting from the carrying capacity $x_0 = 1$. We set $\gamma = 3.33$ and $r = 1.37$ which is equivalent to an uncongested growth rate of $2.5\%/year$. On a 0-tax trajectory the resource continues to degrade to a steady state of $x_\infty = 0.285$. Equation system (7) summarizes the parameter values:

$$\alpha = 0.2;\ \beta = 0.6;\ \rho = 1;\ r = 1.37;\ \gamma = 3.33. \tag{7}$$

For this parameter set, the old generation has a higher expenditure level than the young under BAU for any stock level. Here, the asset-rich and the asset-poor correspond to the rich and the poor. This calibration also ensures that the young agent can always afford to pay the asset price given her wage income along the equilibrium trajectories. The BAU trajectory is monotonic if and only if $x_0 \leq 0.73$. For larger initial conditions, the BAU trajectory drops below the steady state in the first period and then approaches the steady state monotonically from below.

7.2 The Markov perfect equilibrium

The nominal value of national income in period t is $Y(T_t) = P_t F_t + (1 - L)^\beta$ and the aggregate utility in period t (real national income) is $\mu p^{-\alpha}(x_t, T_t) Y(T_t)$; see the proof of Proposition 1 for the explicit form of $Y(T_t)$. Taking as given $\chi = 0$, our goal is to find the equilibrium stationary tax function, denoted $T_t = \Upsilon(x_t)$. The Nash condition requires that given agents' belief that $T_{t+i} = \Upsilon(x_{t+i})$ for $i > 0$, the equilibrium decision for the agents choosing the current tax is $T_t = \Upsilon(x_t)$. Ownership of the asset entitles the owner to profits and revenue from the sale of the asset after production. By purchasing the asset from the old in period t, the agent who is young in period t obtains the utility derived from profits and asset sales when she is old. We denote the level of utility obtained from the sale of assets in the next period

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as $\bar{\sigma}\left(x_{t+1}, \Upsilon\left(x_{t+1}\right)\right) \equiv P_t^{-\alpha}\sigma_t$. This function is defined recursively:

$$\bar{\sigma}\left(x_t, T_t\right) = \frac{1}{1+\rho}\left\{p^{-\alpha}(x_{t+1}, \Upsilon\left(x_{t+1}\right))\pi\left(\Upsilon\left(x_{t+1}\right)\right) + \bar{\sigma}\left(x_{t+1}, \Upsilon\left(x_{t+1}\right)\right)\right\}. \tag{8}$$

Equation (8) states that the utility that the old generation receives in period t, from the sale of assets to the young generation in that period, equals the young generation's present value of the utility from next-period profits, plus the utility from their future sale of the asset. This equation is the utility analog of the no-intertemporal-arbitrage condition used in the proof of Proposition 2.

Because $\chi = 0$, the t-period young also obtain all of the tax revenue in the next period; using the second equation in system (2), we write this revenue as $R(\Upsilon\left(x_{t+1}\right))$; the present value of the utility of this revenue is $\frac{1}{1+\rho}\mu p^{-\alpha}(x_{t+1}, \Upsilon\left(x_{t+1}\right))R(\Upsilon\left(x_{t+1}\right))$

The political economy equilibrium in period t is the solution to the optimization problem

$$\max_{T_t} U^o + U^y = \max_{T_t}\{\mu p^{-\alpha}\left(x_t, T_t\right)Y\left(T_t\right)$$

$$+\mu\bar{\sigma}\left(x_t, T_t\right) + \tfrac{1}{1+\rho}\mu p^{-\alpha}(x_{t+1}, \Upsilon\left(x_{t+1}\right))R(\Upsilon\left(x_{t+1}\right))\} \tag{9}$$

subject to

$$x_{t+1} = \left(1 + \bar{r}_t(x_t, T_t)\right)x_t. \tag{10}$$

Equation (9) states that the objective is to maximize the sum of the lifetime utility of the current old and the current young generation. This maximand equals the utility value derived by both generations from current national income, and from owning the asset and receiving the tax revenue in the next period.

The primitives of the model lead to explicit expressions for the functions $p\left(x, T\right)$ and $Y\left(T\right)$. Equation (8) recursively determines the function $\bar{\sigma}\left(x_t, T_t\right)$. Agents at time t take the functions $\Upsilon\left(x_{t+1}\right)$ and $\bar{\sigma}\left(x_{t+1}, \Upsilon\left(x_{t+1}\right)\right)$ as given, but they are endogenous to the problem. We obtain a numerical solution using the collocation method and Chebyshev polynomials (Judd,
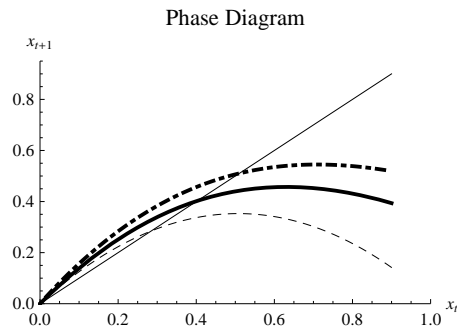
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Figure 2: The phase diagram for the resource stock under the equilibrium tax policy (solid), the 0-tax scenario (dashed), and the social planner (dot-dash).

1998; Miranda and Fackler, 2002); see Appendix B.2.[2, 3]

Figures 2 and 3 summarize the numerical solution to the problem in equations (9) and (10) for the parameter values of equation system (7). The figures also contain information about a social planner's problem, discussed below. Inspection of the discrete time phase diagram in Figure 2 shows that for any current stock, the next period stock is higher under the equilibrium tax compared to under BAU: the environmental policy protects the resource stock. The steady state stock level in the political economy equilibrium is 0.38, compared to the BAU level of 0.28. Under the equilibrium tax, the

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[2]Models of this genus typically have multiple equilibria, as a consequence of the infinite horizon. Experiments suggest that our numerical approach always returns a unique equilibrium. An algorithm that iterates over the value function can be interpreted as the limit as the horizon goes to infinity of a finite horizon model. In view of the generic uniqueness of finite horizon models, the uniqueness of the numerical results is not surprising.

[3] Our assumption that χ is an exogenous constant rather than an endogenous function reduces the dimension of the problem, making it easier to find and describe the equilibrium. There is also an important conceptual reason for preferring the simpler model. If agents currently alive choose the current value of χ as a function of the current state, the term $R(\Upsilon(x_{t+1})$ in the second line of equation would be replaced by $R(\Upsilon(x_{t+1}))(1 - \chi(x_{t+1}))$. The resulting maximization problem does not contain information that makes it possible to find the function $\chi(x)$. To obtain this information, we would have to add structure to the bargaining problem, e.g. by choosing a disagreement point and also excluding side payments, so that all transfers are made by means of χ. An *ad hoc* choice of that kind of structure does not seem more attractive than our decision to use a constant χ. Our analytic results provide a rationale for our choice $\chi = 0$, and Section 7.4 discusses sensitivity analysis on χ.
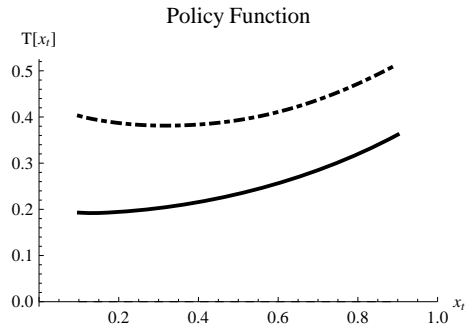
Figure 3: The policy function in the political economy equilibrium (solid) and under the social planner (dot-dash).

stock trajectory is a monotonic function of time. In contrast, under BAU, for large initial values of x, the subsequent level of x is below the steady state. In this situation, the BAU trajectory first overshoots the steady state and then approaches the steady state from below.

The possibility of overshooting helps to explain why the equilibrium tax policy is (slightly) non-monotonic in the stock (Figure 3), and also why the asset value, in units of utility, is monotonic in the stock under the tax policy, but non-monotonic under BAU. (To conserve space, this figure is not presented.) At high values of the resource stock, a high tax prevents the stock from overshooting the steady state, as would occur under BAU. At low values of the resource stock, a high tax helps the resource to regenerate. The equilibrium tax therefore reaches a minimum for an intermediate value of the stock. Under BAU, the possibility of overshooting causes the asset value to be low at high stock values; the asset value is also low when the low resource stock leads to low equilibrium utility. Under efficient bargaining, the equilibrium adjustment of the tax ensures that a higher resource stock leads to higher utility value of the asset.

Figure 4 shows agents' welfare gain under the equilibrium tax, relative to BAU levels. For future generations ($i \geq 1$) the figure shows the welfare gain of the young agent, and for the current generation ($i = 0$) it shows the aggregate lifetime welfare gain for the current young and old generations. The dashed curve corresponds to the initial condition $x_0 = 0.45$ and the solid curve corresponds to $x_0 = 0.9$. For intermediate initial conditions, the welfare gain lies between these two curves. If the economy starts out slightly
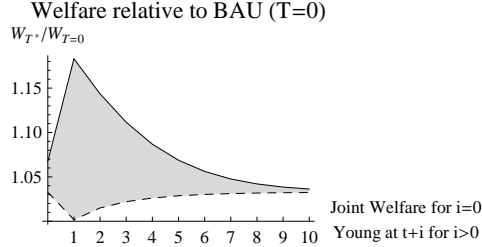
Figure 4: Welfare effects of efficient-bargaining tax policy relative to no-policy scenario for initial resource stock $x_0 = 0.45$ (dashed) and $x_0 = 0.9$ (solid).

higher than the with-policy steady state, agents gain because under BAU welfare would fall to a low level as the resource degenerates. If the initial resource stock is far above the steady state, future generations additionally benefit because the tax prevents overshooting. The fact that overshooting is a problem for high but not for low initial stocks explains why the welfare gain falls when the initial stock is large. The aggregate gain to the first generations is 3 – 7% and the steady state welfare gain is about 3%.

7.3 A social planner

For the purpose of comparing the political economy equilibrium with a familiar optimization problem, we consider the social planner's problem typically used in Ramsey models. This planner maximizes the present value of the stream of single period aggregate utility, $\mu p(x_t, T_t)^{-\alpha} Y(T_t)$. Schneider, Traeger and Winkler (2010) explain the problems with using parameters that describe individual preferences in an OLG setting to calibrate a social discount rate. Nevertheless, we take the social discount rate to be the individual agent's pure rate of time preference, so that the social planner's problem is time consistent. The social planner's problem is

$$\max_{\{T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (1+\rho)^{-t} \mu p(x_t, T_t)^{-\alpha} Y(T_t)$$

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$$\text{subject to } x_{t+1} = (1 + \bar{r}_t(x_t, T_t)) \, x_t \quad \text{with } x_0 \text{ given.}$$

We obtain a numerical solution to the dynamic programming problem associated with this optimization problem, using the parameters above, the collocation method, and Chebyshev polynomials. The dot-dash graphs in Figures 2 and 3 show the phase portrait and the policy function for this social planner. The equilibrium stock and tax trajectories are higher under the social planner, compared to under efficient bargaining. This result is not surprising, given our assumption that agents have no bequest motive. The possibly surprising result is that the selfish agents' equilibrium tax is rather close to the social planner's tax. The social planner's steady state tax is $T = 0.40$, a level slightly higher than the tax that maximizes the steady state lifetime welfare of the young, $T = 0.38$.

As we noted in discussing Figure 1, a policy intervention always decreases *first period* aggregate utility. Thus, interpreting the results of the Ramsey model in the standard manner, implies that policy requires those alive in the present to sacrifice for those alive in the future. Our (standard) choice of the social planner's maximand means that she ignores the asset market. Nevertheless, her policy sequence changes the asset price sequence; in particular, her intervention creates a capital gain for the first-period asset owner. The resulting transfer from the future to the present means that intervention by the social planner increases the *lifetime* aggregate welfare of those alive in the first period. Ramsey models that either ignore the asset market, or adopt assumptions that make that market trivial, ignore the possibility that all generations can gain from policy intervention.

7.4   Sensitivity

Footnote 3 explains our reason for taking χ to be a constant. We find that replacing $\chi = 0$ (used above) with $\chi = 1$ leads to slightly lower taxes for any value of the stock, but does not change qualitative results.

We also computed a variation in which young agents select the current tax and receive all of the surplus, but have to compensate the old generation to ensure that the latter's welfare does not fall below a default level. This default level equals their welfare under the tax chosen in the previous period. The rationale for this model is that inertia favors the existing tax, and that young agents have to compensate the now-old agents to persuade them to

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change the tax that the latter chose when they young. For this experiment we set $\chi = 1$. We find that this variation results in a tax policy very close to, but slightly lower than the policy under the previous formulation with $\chi = 1$. We conclude that our results are not sensitive to changes in χ or to moderate changes in the structure of the political economy model.

We also considered a more extreme variation, in which $\chi = 0$. The old in the first period to propose a transfer rate ξ. Conditional on this choice, the old and the young each propose a constant tax. Due to inertia, society chooses the smaller of these two taxes. We then confirmed numerically that this tax is time consistent. Future young generations would like to lower the tax and future old generations would like to increase it, but the welfare gain that either achieves is insufficient to compensate the other. Therefore, no proposed change achieves consensus. The belief in the initial period that the tax will be constant is therefore confirmed by the equilibrium. The steady state stock is about 2% higher than in the political economy framework (with $\chi = 0$) and 10% lower than under the social planner.

8   Discussion

Many discussions about environmental policy start from the presumption that this policy requires current sacrifices in order to protect future generations. The two existing challenges to this presumption are that there may be win-win situations, and that it may be possible to reallocate current savings in order to make agents in each period better off. We provide a different perspective, using a model that excludes both of the existing challenges to the conventional view.

The key to our result is that the policy-induced increases in the future environmental stock (relative to BAU) increase the value of the traded asset, capital, and thereby benefit current owners of that asset. This benefit more than offsets the deadweight cost, arising from the tax, born by asset owners. The tax therefore increases the welfare of the old generation, who owns the traded asset. Asset prices are a means of transferring gains in the future to the present period. The young generation bears only the deadweight loss in the first period of their life. If the environmental stock is decreasing on the BAU trajectory, the net effect of the tax is to reduce their welfare. However, if the old give the young a sufficiently larger share of the tax revenue – compared to the share that future young generations will obtain – both generations are

better off. Future generations are also better off because of the improved environmental quality.

We presented these results using a generic renewable resource model, but the main motivation for the research arises from the controversy surrounding climate policy, and in particular the extent to which meaningful policy requires a welfare loss to agents currently alive. The generic renewable resources model has the virtue of simplicity and familiarity, but it is not directly applicable to questions of climate policy. For that purpose, we need a model with investment and depreciation, and the production possibility frontier between a composite consumption good and a composite investment good must be strictly concave; equivalently there must be convex adjustment cost for investment. Ongoing research confirms that under empirically plausible levels of depreciation and adjustment costs, the qualitative insights of the simpler model presented here, survive.

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A   Proofs

A.1   Proof of Proposition 1

Proof. (Sketch) (i) Using systems (1) and (2), the nominal value of national income in period t is

$$Y\left(T_t\right) = P_t F_t + (1-L)^\beta.$$

We multiply nominal national income by $\mu P^{-\alpha}$ to convert dollars to utils; $P = p(x_t, T_t)$ is a function of both the tax and the environmental stock. Using the equilibrium expressions for $Y(T)$ and $p(x,T)$, The single period aggregate utility is

$$U\left(x_t, T_t\right) \equiv \mu p(x_t, T_t)^{-\alpha} Y\left(T_t\right) = x_t^\alpha \phi\left(T_t\right),$$

$$\text{with } \phi\left(T_t\right) \equiv \mu \left(\frac{\beta\left(1+\frac{1-T}{\frac{1-\alpha}{\alpha}\beta}\right)^{1-\beta}}{(1-T)\gamma}\right)^{-\alpha}.$$

Differentiating with respect to T and simplifying gives, for $T \neq 0$,

$$\frac{dU}{dT} = -\mu P^{-\alpha}\frac{(1-\alpha)\beta LT}{(1-T)^2}Y < 0. \tag{11}$$

(ii) The tax decreases the nominal wage, w, and increases the equilibrium relative price, P, and therefore decreases the real wage.

The real rental rate is $\mu P^{-\alpha}\pi$. The tax lowers the equilibrium nominal wage, increasing nominal profits, π, but it also increases the commodity price. Using the fact that preferences are homothetic and that the wage share is constant, we have

$$\mu P^{-\alpha}\pi = \mu P^{-\alpha}(1-\beta)\frac{Y}{1-\alpha}.$$

Differentiating this with respect to T gives, for $T \neq 0$,

$$\frac{d\mu P^{-\alpha}\pi}{dT} = \mu\frac{1-\beta}{1-\alpha}\frac{dP^{-\alpha}Y}{dT} = -\mu P^{-\alpha}\frac{(1-\beta)\beta LT}{(1-T)^2}Y < 0.$$

(iii) A higher stock does not affect the nominal wage but it decreases the equilibrium relative price, so it increases the real wage. A higher stock does not alter nominal profits, but decreases the commodity price, thereby increasing real profits. ∎

A.2   Proof of Proposition 2

Proof. The subscript on $\mathbf{T}_t$ denotes that the first element of the trajectory of taxes is the tax in period t. The price of a firm this period is σ_t and the expectation of the next-period price is $\tilde{\sigma}_{t+1}$. In equilibrium the young generation buys one firm today and sells it in the next period. With intertemporally additive, homothetic lifetime utility, the present value of total utility of the young agent is:

$$U_t^y = \mu P_t^{-\alpha} e_t^y + \tfrac{1}{1+\rho}\mu \tilde{P}_{t+1}^{-\alpha}\tilde{e}_{t+1}^o = \mu \times$$

$$\left(P_t^{-\alpha}\left(w_t + \chi_t R_t - \sigma\left(x_t, \mathbf{T}_t\right)\right) + \tfrac{1}{1+\rho}\tilde{P}_{t+1}^{-\alpha}\left((1-\tilde{\chi}_{t+1})\tilde{R}_{t+1} + \tilde{\pi}_{t+1} + \tilde{\sigma}(x_{t+1}, \mathbf{T}_{t+1})\right) \right).$$

$$(12)$$

If a young person buys a unit of the factory today, costing σ_t, the loss in utility is $\mu P_t^{-\alpha}\sigma_t$ assuming that $\sigma_t < w_t$.[4] Purchase of one factory today increases expenditures next period by $\tilde{\pi}_{t+1} + \tilde{\sigma}_{t+1}$; the increase in the present value of utility next period due to the purchase of the factory is $\tfrac{1}{1+\rho}\mu \tilde{P}_{t+1}^{-\alpha}\left(\tilde{\pi}_{t+1} + \tilde{\sigma}_{t+1}\right)$. The equilibrium price-of-factory sequence requires that excess demand for the asset is 0, which, under rational expectation, requires satisfaction of the no-arbitrage condition

$$P_t^{-\alpha}\sigma_t = \frac{1}{1+\rho}P_{t+1}^{-\alpha}\left(\pi_{t+1} + \sigma_{t+1}\right). \tag{13}$$

Write this no-arbitrage condition, equation (13), as

$$\sigma_t = \frac{1}{1+\rho}\left(\frac{P_t}{P_{t+1}}\right)^{\alpha}\left(\pi_{t+1} + \sigma_{t+1}\right)$$

or

$$\sigma_{t+i} = \frac{1}{1+\rho}\left(\frac{P_{t+i}}{P_{t+1+i}}\right)^{\alpha}\left(\pi_{t+i+1} + \sigma_{t+i+1}\right),$$

so

$$\sigma_t = \frac{1}{1+\rho}\left(\frac{P_t}{P_{t+1}}\right)^{\alpha}\pi_{t+1} + \frac{1}{1+\rho}\left(\frac{P_t}{P_{t+1}}\right)^{\alpha}\left[\frac{1}{1+\rho}\left(\frac{P_{t+1}}{P_{t+2}}\right)^{\alpha}\left(\pi_{t+2} + \sigma_{t+2}\right)\right].$$

---

[4]Throughout our derivations, we assume that such a non-negativity constraint is not binding.

By repeated substitution obtain

$$\sigma_t = \sum_{j=1}^{S} \left(\frac{1}{1+\rho}\right)^j \left[\left\{\Pi_{s=0}^{j-1}\left(\frac{P_{t+s}}{P_{t+s+1}}\right)^\alpha\right\}\pi_{t+j}\right] + \left(\frac{1}{1+\rho}\right)^S \left[\left\{\Pi_{s=0}^{S-1}\left(\frac{P_{t+s}}{P_{t+s+1}}\right)^\alpha\right\}\sigma_{t+S}\right]$$

If the system converges to a steady state, then the second term goes to 0 as $S \to \infty$ and

$$\sigma_t = \sum_{j=1}^{\infty} \left(\frac{1}{1+\rho}\right)^j \left[\left\{\Pi_{s=0}^{j-1}\left(\frac{P_{t+s}}{P_{t+s+1}}\right)^\alpha\right\}\pi_{t+j}\right].$$

Note that

$$\Pi_{s=0}^{j-1}\left(\frac{P_{t+s}}{P_{t+s+1}}\right)^\alpha = \left(\frac{P_t}{P_{t+j}}\right)^\alpha$$

Using this relation we have

$$\sigma_t = P_t^\alpha \sum_{i=1}^{\infty}(1+\rho)^{-i}P_{t+i}^{-\alpha}\pi_{t+i}, \tag{14}$$

π is independent of the stock and, for fixed T, constant. Under this condition the expression for the asset price reduces to

$$\sigma_t = \pi P_t^\alpha \sum_{j=1}^{\infty}\left(\frac{1}{1+\rho}\right)^j P_{t+j}^{-\alpha}. \tag{15}$$

∎

A.3   Proof of Corollary 1

Proof. (i) The imposition of the no-arbitrage condition simplifies the lifetime welfare expression of the young, equation (12), to:

$$U_t^y = \mu\left[p(T_t, x_t)^{-\alpha}(w(T_t) + \chi_t R(T_t)) + \frac{p(T_{t+1}, x_{t+1})^{-\alpha}}{1+\rho}(1 - \chi_{t+1})R(T_{t+1})\right]. \tag{16}$$

The no-arbitrage condition implies that the young generation's utility is independent of the asset price. A loss in utility from the higher asset price in the first period equals the discounted utility gain from increased profits and asset price in the second period. As a consequence, the young generation's

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expenditure equals wage income in the first period and their share of the tax revenue in the first and second period. Their welfare considerations are limited to these expenditure components and the price effects.

(ii) The same holds for all future generations. Asset prices enter only the welfare expression of the current old generation. The current owners of the asset capture all future benefits reflected in a changed asset price. ∎

A.4   Proof of Proposition 3

Proof. Using equation (4), and the definitions of ς and x_∞ in equations (5) and (6), the BAU trajectory is monotonic if and only if the initial condition is less than or equal to the root of $(1 + \bar{r}(0, x)) x = x_\infty$, which is equivalent to $x_0 \leq C \frac{\varsigma - 1}{r}$, or $x_0 \leq \frac{1}{\varsigma - 1} x_\infty$.

The old generation's remaining lifetime welfare consists of the utility it obtains from current consumption,

$$U_t^o(\varepsilon) = \mu \left(p(x_t, T_t)^{-\alpha}(1 - \chi)R_t + \sum_{i=0}^{\infty}(1 + \rho)^{-i} p(x_{t+i}, T_{t+i})^{-\alpha} \pi_{t+i} \right).$$
(17)

We start with the derivative of the second term in U^o, the return to holding the asset. We differentiate each term in the sum by $T_i = \varepsilon \bar{T}_i$, recognizing that T_i has a direct effect on $\pi_{t+i} p_{t+i}^{-\alpha}$ and an indirect effect, via its effect on x_{t+j}, on $\pi_{t+j} p_{t+j}^{-\alpha}$ for $j > i$. We use $T_i = \varepsilon \bar{T}_i$, so $dT_i = \bar{T}_i d\varepsilon$.

$$\frac{d \sum_{i=0}^{\infty}(1 + \rho)^{-i} \pi_i p_{t+i}^{-\alpha}}{d\varepsilon} = \frac{\partial \pi_t p_t^{-\alpha}}{\partial T_t} \bar{T}_t$$

$$+ (1 + \rho)^{-1} \left[\frac{\partial \pi_{t+1} p_{t+1}^{-\alpha}}{\partial T_{t+1}} \bar{T}_{t+1} + \frac{\partial \pi_{t+1} p_{t+1}^{-\alpha}}{\partial x_{t+1}} \frac{\partial x_{t+1}}{\partial T_t} \bar{T}_t \right]$$

$$+ (1 + \rho)^{-2} \left[\frac{\partial \pi_{t+2} p_{t+2}^{-\alpha}}{\partial T_{t+2}} \bar{T}_{t+2} + \frac{\partial \pi_{t+2} p_{t+2}^{-\alpha}}{\partial x_{t+2}} \left(\frac{\partial x_{t+2}}{\partial T_{t+1}} \bar{T}_{t+1} + \frac{\partial x_{t+2}}{\partial x_{t+1}} \frac{\partial x_{t+1}}{\partial T_t} \bar{T}_t \right) \right]$$

$$+ (1 + \rho)^{-3} \left[\frac{\partial \pi_{t+3} p_{t+3}^{-\alpha}}{\partial T_{t+3}} \bar{T}_{t+3} + \frac{\partial \pi_{t+3} p_{t+3}^{-\alpha}}{\partial x_{t+3}} \left(\frac{\partial x_{t+3}}{\partial T_{t+2}} \bar{T}_{t+2} + \frac{\partial x_{t+3}}{\partial x_{t+2}} \frac{\partial x_{t+2}}{\partial T_{t+1}} \bar{T}_{t+1} + \frac{\partial x_{t+3}}{\partial x_{t+2}} \frac{\partial x_{t+2}}{\partial x_{t+1}} \frac{\partial x_{t+1}}{\partial T_t} \bar{T}_t \right) \right]$$

$$+ ....$$

We simplify this expression using the fact that at $\varepsilon = 0$, $T_0 = T_1 = ... = 0$. Evaluating the different expressions along the BAU trajectory, we have

$$\frac{\partial \pi_i p_i^{-\alpha}}{\partial T_i} = 0; \quad \pi_0 = \pi_1 = ... = \pi; \quad \text{and} \quad \frac{\partial \pi_i p_i^{-\alpha}}{\partial x_i} = \eta x_i^{\alpha - 1}, \text{ with } \eta \equiv \alpha \pi \left(\frac{w}{\gamma} \right)^{-\alpha} > 0.$$

Using the convention that $\prod_{j}^{j-1} z_j = 1$, we write the i'th term in the sum above as $(1+\rho)^{-i}\theta_i$, with

$$\theta_i \equiv \eta x_{t+i}^{\alpha-1} \left[\sum_{j=0}^{i-1} \left\{ \frac{\partial x_{t+i-j}}{\partial T_{t+i-j-1}} \bar{T}_{t+i-j-1} \left(\prod_{k=0}^{j-1} \frac{\partial x_{t+i-k}}{\partial x_{t+i-k-1}} \right) \right\} \right].$$

The initial condition is x_t and the BAU steady state is x_∞. The assumption that $x_t < \frac{1}{2}\frac{\varsigma}{\varsigma-1}x_\infty$, with $\frac{1}{2}\frac{\varsigma}{\varsigma-1} > 1$ by inequality (5), implies that $\left(\prod_{k=0}^{j-1} \frac{\partial x_{t+i-k}}{\partial x_{t+i-k-1}} \right) > 0$. By assumption, $\tilde{T}_{t+i-j-1} \geq 0$ with strict inequality for some $i - j - 1 \geq 0$, and we have $\frac{d\bar{r}_t}{dT_t} > 0 \Rightarrow \frac{\partial x_{t+1}}{\partial T_t} > 0$. Consequently $\theta_i \geq 0$ with strict inequality holding for some i.

The old also receive a share of the tax revenue. The effect of a tax increase on current tax revenue is

$$\left. \frac{dP^{-\alpha}(1-\chi)R}{d\varepsilon} \right|_{\varepsilon=0} = \underbrace{(1-\chi)R\frac{dP^{-\alpha}}{d\varepsilon}}_{=0} + (1-\chi)P^{-\alpha}\left. \frac{dR}{d\varepsilon} \right|_{\varepsilon=0}$$

$$= (1-\chi)P^{-\alpha}\frac{\alpha\left(1+\frac{\alpha}{\beta(1-\alpha)}\right)^{-\beta}}{1-\alpha}\bar{T}_t > 0 \qquad (18)$$

Given that the derivatives of both terms in U^o are positive, a small tax trajectory increases the welfare of the old generation. For a positive current tax, $R_t > 0$, and the old generation's utility strictly increases in its share of the tax revenue. ∎

A.5   Proof of Proposition 4

Proof. The lifetime welfare of the young, from equation (16), is

$$U_t^y(\varepsilon) = \mu p(\bar{T}_t\varepsilon, x_t)^{-\alpha}\left(w(\bar{T}_t\varepsilon) + \chi R(\bar{T}_t\varepsilon) + \frac{\left(1+\bar{r}(\bar{T}_t\varepsilon, x_t)\right)^{\alpha}}{1+\rho}(1-\chi)R(\bar{T}_{t+1}\varepsilon) \right).$$

Differentiating this expression with respect to ε gives

$$\frac{dU_t^y}{d\varepsilon} = \frac{d}{d\varepsilon}\mu P_t^{-\alpha}(w(\bar{T}_t\varepsilon)+\chi R(\bar{T}_t\varepsilon)) + \frac{d}{d\varepsilon}\left[\mu P_t^{-\alpha}\frac{\left(1+\bar{r}(\bar{T}_t\varepsilon, x_t)\right)^{\alpha}}{1+\rho}(1-\chi)R(\bar{T}_{t+1}\varepsilon) \right]$$

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We know that the first order tax-effect output measured in utils is zero simply because the pre-tax allocation maximizes current aggregate utility. Given the constancy of shares in the Cobb-Douglas production function in manufacturing, the two remaining components of income, w and R, also have to add up to zero: $\frac{dP^{-\alpha}w}{d\varepsilon}\Big|_{\varepsilon=0} + \frac{dP^{-\alpha}R}{d\varepsilon}\Big|_{\varepsilon=0} = 0$. Using the fact that $R(0) = 0$ such that $\frac{dP^{-\alpha}R}{d\varepsilon}\Big|_{\varepsilon=0} = P^{-\alpha}\frac{dR}{d\varepsilon}\Big|_{\varepsilon=0}$ and the assumption that the first two tax rates are equal, the expression simplifies to

$$\frac{dU_t^y}{d\varepsilon}\Big|_{\varepsilon=0} = \mu\left[P_t^{-\alpha}(-1+\chi)\frac{dR}{d\varepsilon} + P_t^{-\alpha}(1-\chi)\frac{(1+\bar{r}(\bar{T}_t\varepsilon, x_t))^\alpha}{(1+\rho)}\frac{dR}{d\varepsilon}\right]_{\varepsilon=0}$$

$$= \mu P_t^{-\alpha}(1-\chi)\left(\frac{(1+\bar{r}(0,x_t))^\alpha}{1+\rho} - 1\right)\frac{dR}{d\varepsilon}\Big|_{\varepsilon=0}$$

The young generation loses income $-P_t^{-\alpha}\frac{dR}{d\varepsilon}$ in the first period through an increase in the tax, but is able to recuperate $\chi P_t^{-\alpha}\frac{dR}{d\varepsilon}$ in the form of tax revenues. It gains $P_t^{-\alpha}(1-\chi)\frac{(1+\bar{r})}{(1+\rho)}\frac{dR}{d\varepsilon}$ in the next period. The first-order response of tax revenue to a small tax introduction is positive: $\frac{dR(\bar{T}_t,\varepsilon)}{d\varepsilon}\Big|_{\varepsilon=0} = \frac{\alpha}{1-\alpha}\left(1+\frac{\alpha}{\beta(1-\alpha)}\right)^{-\beta}\bar{T}_t > 0$. Under the assumption that $\bar{T}_0 = \bar{T}_1$, we establish the following condition under which a small positive tax increases the initial young agent's *lifetime* welfare (16):

$$\frac{dU_0^y}{d\varepsilon}\Big|_{\varepsilon=0} > 0 \Leftrightarrow (1-\chi)\left(\frac{(1+\bar{r}(0,x_0))^\alpha}{1+\rho} - 1\right)\bar{T}_0 > 0. \qquad (19)$$

With $\chi \in [0,1]$, a small tax increases the lifetime welfare of the young generation if and only if $\chi < 1$ and $(1+\bar{r}(0,x_t))^\alpha > (1+\rho)$. A small tax creates a zero first order welfare effect for the young generation that receives all tax revenue ($\chi = 1$). Condition (b) in the Proposition is equivalent to $\bar{r}(0,x_t) > (1+\rho)^{\frac{1}{\alpha}} - 1$. For $\rho > 0$, the expression on the right side of the previous inequality is positive. Thus, a necessary condition for the young to benefit from a tax is that the resource is below its 0-tax steady state, and is in the process of sufficiently strong recovery. ∎

A.6   Proof of Proposition 5

Proof. The last equation in system (1) implies that $p(T_t, x_{t+1})^{-\alpha} = p(T_t, x_t)^{-\alpha}(1+\bar{r}(T_t, x_t))^\alpha$. This equality and the fact that the young generation's welfare is

linear in χ, from equation (16), implies that

$$\frac{dU_0^y}{d\chi} < 0 \Leftrightarrow \left(\frac{(1 + \bar{r}(T_0, x_0))^\alpha}{1 + \rho} - 1\right)\bar{T}_0 > 0. \qquad (20)$$

We also have $\frac{d\bar{r}_t}{dT_t} > 0$. This inequality and inequalities (19) and (20) imply that if the young benefit from a small tax, then they prefer to receive all of the tax revenue when they are old, i.e. they prefer $\chi = 0$. In contrast, if the young are harmed by a small tax, then provided that the tax is small they prefer to receive all of the tax revenue when young ($\chi = 1$). ∎

A.7   Proof of Proposition 6

Proof. With ξ the share of the old generation's tax revenue transferred to the young in the period when the tax is first imposed, the first period's tax receipts are now $(\chi_0 + (1 - \chi_0)\xi)R_0$ for the young and $(1 - \chi_0)(1 - \xi)R_0$ for the old. Under the assumption that current and next period tax rates are changed by the same small amount and that χ is constant, an argument that parallels the derivation in Appendix A.5 leads to the following condition for the young to benefit from the combined transfer and tax:

$$\left.\frac{dU_0^y}{d\varepsilon}\right|_{\varepsilon=0} > 0 \Leftrightarrow (1 - \chi)\left(\frac{(1 + \bar{r}(0, x_0))^\alpha}{1 + \rho} - (1 - \xi)\right)\bar{T}_0 > 0. \qquad (21)$$

Setting $\xi = 0$, equation (21) reproduces equation (19). For

$$\xi > \xi^{crit} \equiv 1 - \frac{(1 + \bar{r}(0, x_0))^\alpha}{1 + \rho}$$

the young strictly prefer the combined tax and transfer compared to the BAU status quo. Even if the resource is degrading on the BAU trajectory, $\xi^{crit} < 1$. Therefore, by transferring less than their entire share of the tax revenue to the young, the old can make the young better off under a small tax. Because Proposition 3 states that the tax improves the old generation's welfare even if they receive none of the tax revenue, the old are obviously better off under the combined tax and transfer, compared to the status quo. ∎

A.8   Proof of Proposition 7

Proof. Using a derivation parallel to that contained in Appendix A.5, we have

$$\left.\frac{dU_0^y}{d\varepsilon}\right|_{\varepsilon=0} > 0 \Leftrightarrow (1-\chi)\left(\frac{e^{-(a-b)\alpha}\left(1+\bar{r}(0,x_0)\right)^\alpha}{1+\rho} - 1\right)\bar{T}_0 > 0.$$

The second inequality is equivalent to

$$\left(\frac{1+\bar{r}(0,x_0)}{e^{(a-b)}}\right)^\alpha > 1+\rho. \tag{22}$$

The left side of inequality (22) is decreasing in $a-b$, so an increase in $a-b$ decreases the set of parameter values and initial conditions under which the inequality is satisfied, i.e. the circumstances under which the young benefit from the tax. ∎

B   Appendix for Referee

This appendix collects information not intended to be published.

B.1   Future generations

Merely in order to avoid uninteresting complications, we assume that for future generations the tax is constant: $\bar{T}_0 = \bar{T}_1 = \bar{T}_2....$ The life-time welfare of the next young generation is

$$U_1^y(\varepsilon) = \mu p(\bar{T}_1\varepsilon, x_1)^{-\alpha} \left(w(\bar{T}_1\varepsilon) + \chi R(\bar{T}_1\varepsilon) + \frac{\left(1 + \bar{r}(\bar{T}_1\varepsilon, x_1)\right)^\alpha}{1 + \rho}(1 - \chi)R(\bar{T}_2\varepsilon) \right).$$

Differentiating this expression with respect to ε gives

$$\frac{dU_1^y}{d\varepsilon} = \frac{d}{d\varepsilon}\mu P_1^{-\alpha}(w(\bar{T}_1\varepsilon) + \chi R(\bar{T}_1\varepsilon)) + \frac{d}{d\varepsilon}\left[\mu P_1^{-\alpha}\frac{\left(1 + \bar{r}(\bar{T}_1\varepsilon, x_1)\right)^\alpha}{1 + \rho}(1 - \chi)R(\bar{T}_2\varepsilon) \right]$$

Using the simplifications of Appendix A.5, especially the fact that $R(0) = 0$, and the fact that $\frac{\partial P_1^{-\alpha}}{\partial x_1} = \alpha P_1^{-\alpha}x_1^{-1}$, the expression simplifies to

$$\frac{dU_1^y}{d\varepsilon}\bigg|_{\varepsilon=0} > 0 \quad \Leftrightarrow \quad \bar{T}_1 P_1^{-\alpha}(1 - \chi)\left(\frac{(1 + \bar{r}(0, x_1))^\alpha}{1 + \rho} - 1\right)\frac{dR}{d\varepsilon}\bigg|_{\varepsilon=0} > -\bar{T}_0 w(0)\frac{\partial P_1^{-\alpha}}{\partial x_1}\frac{\partial x_1}{\partial T_0}$$

$$\Leftrightarrow \quad (1 - \chi)\left(\frac{(1 + \bar{r}(0, x_1))^\alpha}{1 + \rho} - 1\right)\bar{T}_0 > -\left(w(0)\alpha x_1^{-1}\frac{\partial x_1}{\partial T_0}\right)\left(\frac{1}{dR/d\varepsilon|_{\varepsilon=0}}\right)\bar{T}_0$$

Comparing this condition to inequality (19), we see that when the stock is degrading (i.e. $\bar{r}(0, x_0) < 0$), a small tax is more likely to benefit the next period's young generation compared to today's, which always loses in the absence of transfers. The difference arises for two reasons: A lower stock increases the BAU growth rate, $\frac{d\bar{r}(0, x_t)}{dx_t} = -r < 0$, so that the left side is less negative. The right side of the inequality above is negative. Therefore, the condition here is weaker than the condition in inequality (19). In fact, it is satisfied for any initial stock value in the calibration used in Section 7.

B.2   Numerical Method

We approximate $\Upsilon\left(x_{t+1}\right)$ and $\bar{\sigma}\left(x_{t+1}, \Upsilon\left(x_{t+1}\right)\right) \equiv \Phi\left(x_{t+1}\right)$ as polynomials in x_{t+1}, and find coefficients of those polynomials so that the solution to

$$\max_{T_t} \mu P^{-\alpha}\left(x_t, T_t\right) Y\left(T_t\right) +$$
$$\tfrac{1}{1+\rho}\mu\left\{P^{-\alpha}(x_{t+1}, \Upsilon\left(x_{t+1}\right))\pi\left(\Upsilon\left(x_{t+1}\right)\right) + \Phi\left(x_{t+1}\right) + R(\Upsilon\left(x_{t+1}\right))\right\}$$

subject to equation (10) approximately equals $\Upsilon\left(x_t\right)$. We use 13-degree Chebyshev polynomials evaluated at 13 Chebyshev nodes on the $[0.1, 0.9]$ interval. At each node the following conditions have to be approximately satisfied

$$\Phi\left(x_t\right) = \tfrac{1}{1+\rho}\left\{p^{-\alpha}(x_{t+1}, \Upsilon\left(x_{t+1}\right))\pi\left(\Upsilon\left(x_{t+1}\right)\right) + \Phi\left(x_{t+1}\right) + R(\Upsilon\left(x_{t+1}\right))\right\}$$

$$\tfrac{d}{dT_t}\left[\mu P^{-\alpha}\left(x_t, T_t\right) Y\left(T_t\right) + \tfrac{1}{1+\rho}\Omega\right] = 0$$

$$\text{with}\quad \Omega \equiv \mu\left\{P^{-\alpha}(x_{t+1}, \Upsilon\left(x_{t+1}\right))\pi\left(\Upsilon\left(x_{t+1}\right)\right) + \Phi\left(x_{t+1}\right) + R(\Upsilon\left(x_{t+1}\right))\right\}$$
$$(23)$$

subject to $x_{t+1} = \left(1 + \bar{r}_t(x_t, T_t)\right) x_t$ and $T_t = \Upsilon\left(x_t\right)$. Each node gives two non-linear equations in the coefficients of the two polynomials. If the number of nodes equals the degree of approximation (i.e. the number of coefficients of each polynomial), the system of non-linear equations can be solved using a root-finding method. We employ Mathematica's FindRoot command which solves the system in less than a minute on a standard personal computer. We increased the number of nodes and degree of approximation to 16 in the social planner's problem to arrive at satisfactory levels of accuracy.

Evaluating the equation system (23) using the solution approximations gives a statistic for the goodness of fit. The figures 5 and 6 illustrate that the residual errors are 5 orders of magnitudes below the solution values.

In the text we presented the solution for $\chi = 0$ for reasons explained in Section 5. Here we discuss the $\chi = 1$ case, where in each period the young receives all of the tax revenue. This change reduces the incentive for generations to preserve the resource and consequently lower tax rates are chosen at all levels of the stock, x. Figure 7 plots the policy function for the efficient bargaining problem for $\chi = 0$ (solid) and $\chi = 1$ (dotted). At its maximal difference, the $\chi = 0$ policy function lies 20% below the case reported in the text. This considerable difference in the policy function,
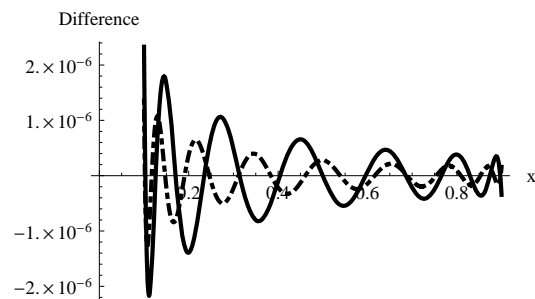
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Figure 5: Deviation of asset price approximation from true value outside of approximation nodes for the efficient bargaining (solid) and the social planner's (dot-dashed) problems
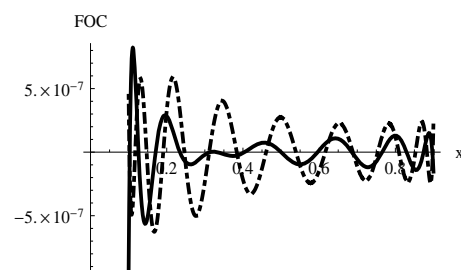

Figure 6: Deviation of policy function approximation from true value outside of approximation nodes for the efficient bargaining (solid) and the social planner's (dot-dashed) problems
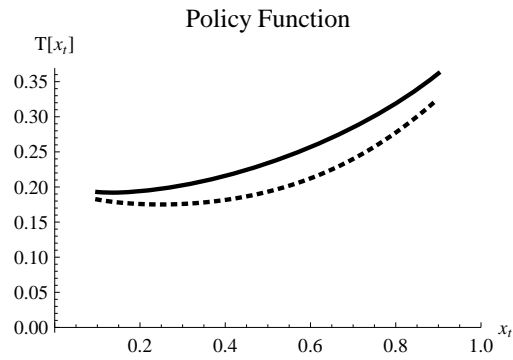
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Policy Function

Figure 7: Policy Function for $\chi = 0$ (solid) and $\chi = 1$ (dotted).

however, has little impact on the value function or the transition equation of the stock variable. The steady state under the less conservative tax policy is at 0.38 which is only 5% under the $\chi = 0$ equilibrium level.