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PROCEEDING

The branch-cut quantum gravity with a self-coupling inflation scalar field: The wave function of the Universe

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Abstract

This paper focuses on the implications of a commutative formulation that integrates branch-cutting cosmology, the Wheeler–DeWitt equation, and Hořava–Lifshitz quantum gravity. Building on a mini-superspace structure, we explore the impact of an inflaton-type scalar field on the wave function of the Universe. Specifically analyzing the dynamical solutions of branch-cut gravity within a mini-superspace framework, we emphasize the scalar field's influence on the evolution of the evolution of the wave function of the Universe. Our research unveils a helix-like function that characterizes a topologically foliated spacetime structure. The starting point is the Hořava–Lifshitz action, which depends on the scalar curvature of the branched Universe and its derivatives, with running coupling constants denoted as g_i. The corresponding wave equations are derived and are resolved. The commutative quantum gravity approach preserves the diffeomorphism property of General Relativity, maintaining compatibility with the Arnowitt–Deser–Misner formalism. Additionally, we delve into a mini-superspace of variables, incorporating scalar-inflaton fields and exploring inflationary models, particularly chaotic and nonchaotic scenarios. We obtained solutions for the wave equations without recurring to numerical approximations.

KEYWORDS

branch cut cosmology, scalar field, inflation

1 | INTRODUCTION

Alan Guth has suggested that our universe is product of *inflation* (Guth 1981, 2004), a conception based on the existence of states with negative pressure, which effects can be seen in the Friedmann equations (Friedman 1922) in which a positive pressure contributes to the deceleration of the universe, while a negative pressure can cause acceleration.

In order to build physical states of negative pressure, Guth has introduced a scalar field, the inflaton, ϕ. The corresponding energy-momentum tensor, $T^{\mu\nu}$, is given as (Guth 1981, 2004)

$$T^{\mu\nu} = \partial^{\mu}\phi\partial^{\nu}\phi - g^{\mu\nu}\left[\frac{1}{2}\partial_{\lambda}\phi\partial^{\lambda}\phi + V(\phi)\right]. \quad (1)$$

In this expression, $V(\phi)$ represents the potential energy density.

The energy density, $\rho(t)$, and pressure, $p(t)$, in the inflation model are (Guth 2004)

$$\rho(t) = T^{00} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla_i\phi)^2 + V(\phi)\ ,$$
$$p(t) = \frac{1}{3}\sum_{i=1}^{3} T_{ii} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6}(\nabla_i\phi)^2 - V(\phi), \quad (2)$$

As a fundamental result, quantum density perturbations lead to the eternal inflation as a result of a repulsive form of gravity, driving the acceleration of cosmic expansion.

Branch cut gravity (BCG) in turn, a theoretical alternative to the inflation model, based on the mathematical augmentation technique of closure and existential completeness (Manders 1989), represents an analytically continuous extension of general relativity (Einstein 1916, 1917) to the complex plane (Bodmann et al. 2022; Bodmann et al. 2023a; Bodmann et al. 2023b; de Freitas Pacheco et al. 2022; Hess et al. 2023; Zen Vasconcellos et al. 2019; Zen Vasconcellos et al. 2021a; Zen Vasconcellos et al. 2021b; Zen Vasconcellos et al. 2023). Such mathematical procedures have proven extremely useful both in quantum mechanics (Dirac 1937), with direct physical manifestations (Aharonov & Bohm 1959; Wu et al. 2021) and in pseudocomplex general relativity (pc-GR) (Hess 2017; Hess et al. 2016; Hess & Boller 2020; Hess & Greiner 2009), allowing to identify a suppression mechanism of the primordial gravitational singularity and to the prediction of the existence of dark energy outside and inside cosmic mass distributions.

The branch-cut formulation corresponds to the complexification of the Friedman–Lemaître–Robertson–Walker (FLRW) metric (Friedman 1922; Lemaître 1927; Robertson 1935; Walker 1937), resulting in a sum of field equations associated to continuously distributed single-poles with infinitesimal residues, arranged along a line in the complex plane (for details, see ref. Bodmann et al. 2022; Bodmann et al. 2023a; Bodmann et al. 2023b; de Freitas Pacheco et al. 2022; Hess et al. 2023; Zen Vasconcellos et al. 2019, 2023; Zen Vasconcellos et al. 2021a; Zen Vasconcellos et al. 2021b). Through a Riemann integration process, this complexification gives rise to a new scale factor, denoted as $\ln^{-1}(\beta(t))$, which characterizes a topological foliated spacetime structure.

In this work, based on a recently developed commutative formulation that combines the branch-cut cosmology, the Wheeler–DeWitt equation, and the Hořava–Lifshitz quantum gravity (Bodmann et al. 2023a; Bodmann et al. 2023b), considering a mini-superspace framework, we study the implications of an inflaton-type scalar field and the corresponding potential in the acceleration of the Universe.

2 | HOŘAVA–LIFSHITZ BRANCH-CUT ACTION

The Hořava–Lifshitz approach to commutative quantum gravity incorporates, in the Lagrangian formulation, contributions from the spacetime curvature of high orders, preserving the diffeomorphism property of General Relativity (Kiefer 2012). This property characterizes an isomorphism of smooth varieties, as well as the usual foliation of the Arnowitt–Deser–Misner (ADM) formalism at the limit of the infrared region of the spectrum (García-Compeán & Mata-Pacheco 2022). In combination with the Wheeler–DeWitt (WdW) equation (Witt 1967), the formulation is free of ghosts, making it suitable for describing quantum effects of the gravitational field (García-Compeán & Mata-Pacheco 2022). The solutions of the WdW equation, represented in turn by a geometric functional of compact manifolds and matter fields, describe the evolution of the quantum wave function of the Universe (Hartle & Hawking 1983; Hawking 1982).

The Hořava–Lifshitz action, $\mathcal{S}_{HL}$, is given by (Abreu et al. 2019; Bertolami & Zarro 2011; Bodmann et al. 2023a; Bodmann et al. 2023b; Cordero et al. 2019; García-Compeán & Mata-Pacheco 2022; Hess et al. 2023; Hořava 2009; Vieira et al. 2020):

$$\begin{aligned}\mathcal{S}_{HL} = \frac{M_P^2}{2}\int d^3x\, dt\, N\sqrt{g}\,\big(&K_{ij}K^{ij} - \lambda K^2 - g_0 M_P^2 \\ &- g_1\mathcal{R} - g_2 M_P^{-2}\mathcal{R}^2 - g_3 M_P^{-2}\mathcal{R}_{ij}\mathcal{R}^{ij} - g_4 M_P^{-4}\mathcal{R}^3 \\ &- g_5 M_P^{-4}\mathcal{R}(\mathcal{R}^i_j\mathcal{R}^j_i) - g_6 M_P^{-4}\mathcal{R}^i_j\mathcal{R}^j_k\mathcal{R}^k_i \\ &- g_7 M_P^{-4}\mathcal{R}\nabla^2\mathcal{R} - g_8 M_P^{-4}\nabla_i\mathcal{R}_{jk}\nabla^i\mathcal{R}^{jk}\big)\ . \end{aligned} \quad (3)$$

In the BCG formulation, the action $\mathcal{S}_{HL}$ depends on the scalar curvature of the branched Universe, $\mathcal{R}$, and on its derivatives, in different orders (Abreu et al. 2019; Bertolami & Zarro 2011; Bodmann et al. 2023a; Bodmann et al. 2023b; Cordero et al. 2019; García-Compeán & Mata-Pacheco 2022; Hess et al. 2023; Hořava 2009; Vieira et al. 2020). In expression (3), g_i denote running coupling constants, M_P is the Planck mass, ∇_i represents covariant derivatives, and the branching Ricci components of the three-dimensional metrics may be determined by imposing a maximum symmetric surface foliation (Hess et al. 2023) which gives:

$$\mathcal{R}_{ij} = \frac{2}{\sigma^2 u^2(t)} g_{ij} \ , \quad \text{and} \quad \mathcal{R} = \frac{6}{\sigma^2 u^2(t)}, \tag{4}$$

where the variable change $u(t) \equiv \ln^{-1}[\beta(t)]$, with $du \equiv d\ln^{-1}[\beta(t)]$, was introduced. $K = K^{ij} g_{ij}$ represents in expression (3) the trace of the extrinsic curvature tensor K_{ij} (Bodmann et al. 2023a;Bodmann et al. 2023b; Hess et al. 2023):

$$K = K^{ij} g_{ij} = -\frac{3}{2\sigma N u(t)} \frac{du(t)}{dt}. \tag{5}$$

Applying standard canonical quantization procedures and thus promoting the canonical conjugate momentum into an operator, that is, $p_u \mapsto -i\frac{\partial}{\partial u}$, the Hamiltonian is also elevated to an operator. The canonical quantization Dirac procedure applied to the Einstein–Hilbert action results in a second-order functional differential equation defined in general terms in a configuration superspace, whose solutions depend on a three-dimensional metric and on matter fields (Hartle & Hawking 1983; Hawking 1982; Lukasz 2014; Witt 1967).

The Hamiltonian, the new complex dynamical variable $u(t)$, representing the helix-like scale factor analytically continued to the complex plane, along with the corresponding conjugate momentum p_u, are then treated as operators, denoted respectively as $\hat{\mathcal{H}}(t)$, $\hat{u}(t)$, and $\hat{p}_\mathrm{u}$. This leads to the formulation of the branching Hamiltonian given by[1] (Bodmann et al. 2023a; Bodmann et al. 2023b; see also Bertolami & Zarro 2011)

$$\mathcal{H} = \frac{1}{2}\frac{N}{u}\left[-p_u^2 + g_r - g_m u - g_k u^2 - g_q u^3 + g_\Lambda u^4 + \frac{g_s}{u^2}\right], \tag{6}$$

with $p_\mathrm{u} = -\frac{u(t)}{N}\frac{du(t)}{dt}$. In this expression, p_u represents the conjugate momentum of the original branching gravitation dynamical variable $\ln^{-1}[\beta(t)]$, g_k, g_Λ, g_r, and g_s represent respectively the curvature, cosmological constant, radiation, and stiff matter running coupling constants (Bertolami & Zarro 2011; Maeda et al. 2010)

$$g_k \equiv \frac{2}{3\lambda - 1}; g_\Lambda \equiv \frac{\Lambda M_{PI}^{-2}}{18\pi^2(3\lambda-1)^2}; g_r \equiv 24\pi^2\,(3g_2 + g_3)\,;$$
$$g_s \equiv 288\pi^4\,(3\lambda - 1)\,(9g_4 + 3g_5 + g_6). \tag{7}$$

The g_r, and g_s running coupling constants can be positive or negative, without affecting the stability of the solutions. Stiff matter contribution in turn is determined by the $p = \omega\rho$ condition in the corresponding equation of state. In Equation (6) we supplemented the Hamiltonian with two additional terms, $g_m u$, that describes the contribution of baryon matter combined with dark matter, and $g_q u^3$, a quintessence contribution. The parametrization of curvature, cosmological constant, radiation, stiff matter, baryon matter combined with dark matter, and quintessence running coupling constants are in tune with the Wilkinson Microwave Anisotropy Probe (WMAP) observations (Hinshaw et al. 2013).

3 | MINI-SUPERSPACE OF VARIABLES

We consider in the following a mini-surperspace of variables $(u(t), \phi(t))$. We adopt for the action of the scalar field the following expression (Kiritsis & Kofinas 2009; Tavakoli et al. 2021)

$$S_\phi = \int_\mathcal{M} d^3x\, dt\, N\sqrt{g}\Big[\frac{1}{N^2}\big(\dot{\phi} - N^i\partial_i\phi\big)^2 \ - \mathcal{V}(\partial_i\phi, \phi)\Big]. \tag{8}$$

Assuming homogeneous and isotropic cosmological settings we have $N_i = 0$ (Kiritsis & Kofinas 2009; Tavakoli et al. 2021), and the action of the scalar field $\phi(t)$, given by expression (8), may be written as

$$S_\phi = \int_\mathcal{M} d^3x\, dt\, N\sqrt{g}\left(\frac{1}{2}\frac{1}{N^2}F(\phi)\dot{\phi}^2 - V(\phi)\right), \tag{9}$$

with $V(\phi)$ denoting the inflation potential and where $F(\phi)$ represents a coupling function. In the following, from the total action determined by adding the Hořava–Lifshitz and the scalar field actions, the Hamiltonian associated with the mini-superspace of variables may be obtained.

3.1 | Chaotic inflation

The momenta conjugate to the dynamical variables $(u(t), \phi(t))$ can be obtained by definition as $p_q = \partial L/\partial \dot{q}$,

[1]To simplify notation, we do not use in the following the hat symbol and explicit time dependence in most cases.

where L defines the total Lagrangian of the system, resulting in

$$p_u = -\frac{1}{N}u\dot{u}\ ; \quad \text{and} \quad p_\phi = \frac{1}{N}F(\phi)u^3\dot{\phi}. \tag{10}$$

The total Hamiltonian then reads

$$\mathcal{H} = \frac{1}{2}\frac{N}{u}\left[-p_u^2 + g_r - g_m u - g_k u^2 - g_q u^3 + g_\Lambda u^4 + \frac{g_s}{u^2}\right] + \frac{1}{2}\frac{N}{u}\left[\frac{1}{u^{3\omega-1}F(\phi)}p_\phi^2 + 2V(\phi)\right]. \tag{11}$$

The condition $H\Psi(u,\phi) = 0$, where $\Psi(u,\phi) = \Psi(u)\Psi(\phi)$ represents the wave function of the Universe, implies, for $\omega = 1/3$, the following two separable equations

$$\left[-p_u^2 + g_r - g_m u - g_k u^2 - g_q u^3 + g_\Lambda u^4 + \frac{g_s}{u^2} - C\right]\Psi(u) = 0, \tag{12}$$

and

$$\left[\frac{1}{F(\phi)}p_\phi^2 + 2V(\phi) - C\right]\Psi(\phi) = 0. \tag{13}$$

Promoting the canonical conjugate momenta p_u and p_ϕ into operators

$$p_u \to -i\frac{\partial}{\partial u}\ , \quad \text{and} \quad p_\phi \to -i\frac{\partial}{\partial \phi}, \tag{14}$$

we get

$$p_u^2 \to -\frac{\partial^2}{\partial u^2}\ , \quad \text{and} \quad p_\phi^2 \to -\frac{\partial^2}{\partial \phi^2}. \tag{15}$$

Combining Equations (12), (13), and (15), we obtain

$$\left[\frac{\partial^2}{\partial u^2} + g_r - g_m u - g_k u^2 - g_q u^3 + g_\Lambda u^4 + \frac{g_s}{u^2} - C\right]\Psi(u) = 0, \tag{16}$$

and

$$\left[-\frac{\partial^2}{\partial \phi^2} + F(\phi)\left(2V(\phi) - C\right)\right]\Psi(\phi) = 0. \tag{17}$$

In the following we consider chaotic inflation modeled by using the potential $V(\phi) = \frac{1}{2}g_\phi^2\phi^2$ and we assume a polynomial coupling function for the scalar field, $F(\phi) = \lambda\phi^m$, so Equation (17) then reads

$$\left[-\frac{\partial^2}{\partial \phi^2} + \lambda\phi^m\left(g_\phi^2\phi^2 - C\right)\right]\Psi(\phi) = 0. \tag{18}$$

Through consistent computational approaches, we obtained solutions for the Universe's wave function without resorting to numerical approximations, despite the formal difficulties inherent in solving the differential equations associated with the Hořava–Lifshitz formalism, as is common in works found in the literature. The boundary conditions adopted in this work follow conventional canons of convergence, as well as stability and continuity of solutions of differential equations, and are in line with the Bekenstein criterion (de Freitas Pacheco et al. 2022). The sample set of solutions for the wave function, $\Psi(\phi)$, illustrates the impact of different parameters on chaotic inflation. Figure 1 shows typical individual solutions for the $u(t)$-component of the wave function of the Universe, $\Psi(u,\phi)$, given in Equation (16) for different parametrizations for the different terms of the super-Hamiltonian and boundary conditions. Figure 2 shows generic forms of the potential for the chaotic and nonchaotic inflation. On the left, a generic form illustrates the potential for the chaotic inflationary scenario. This scenario typically involves a potential function associated with chaotic inflation dynamics. On the right, the figure presents a typical form of the potential for the original inflationary model, which is based on the Fubini potential.

3.2 | Modeling inflation with a Fubini-type potential

We adopt the following the Fubini potential to simulate inflation

$$V(\phi) = \frac{\beta}{4}(\phi - \phi_c)^4 - \frac{1}{2}g_\phi^2(\phi - \phi_c)^2. \tag{19}$$

Combining this equation with expression (17), we obtain

$$\left[-\frac{\partial^2}{\partial \phi^2} + \lambda\phi^m\left(\frac{\beta}{2}(\phi - \phi_c)^4 - g_\phi^2(\phi - \phi_c)^2 - C\right)\right]\Psi(\phi) = 0. \tag{20}$$

The Fubini potential introduces specific features into the inflationary model, and this visual representation provides insight into the shape and characteristics of the potential associated with the original inflationary model.

Figure 3 show typical individual solutions for the $\phi(t)$-component of the wave function of the Universe, $\Psi(u,\phi)$, given respectively in Equations (18) and (20) for different parametrizations and specific boundary conditions.

The figures represent distinct scenarios corresponding to different particular parameter conditions. These variations in parameter values allow for the exploration of different conditions that influence the behavior of the wave function during chaotic inflation and nonchaotic inflation. Evidently, there is a range of additional parameterizations to be explored in the future. The exploration of diverse parameter combinations contributes to a nuanced

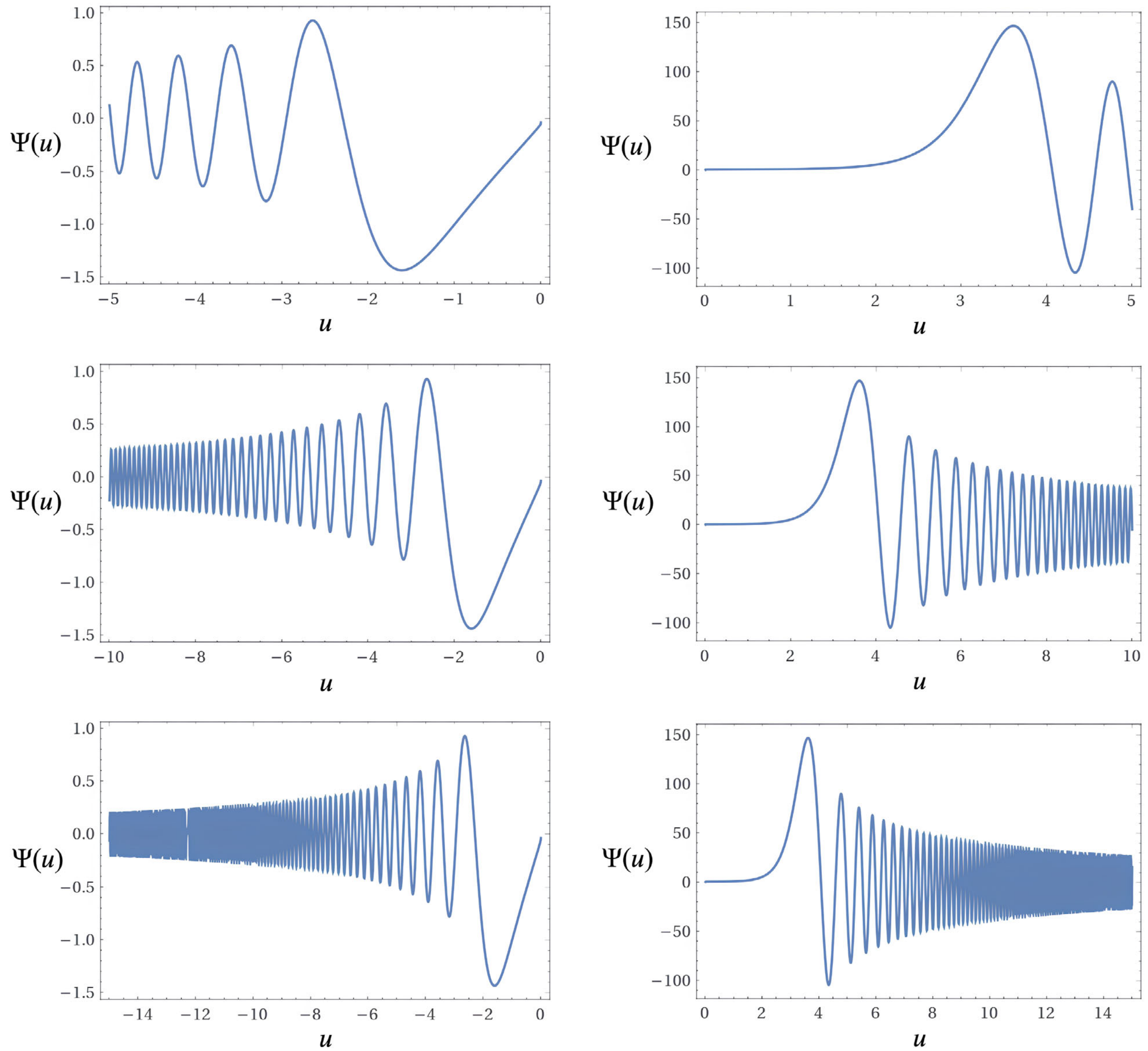

FIGURE 1 Typical individual solutions of the $u(t)$-component ($\Psi(u)$) of the wave function of the Universe, $\Psi(u, \phi)$, using the commutative approach given by Equation (16) for different initial value, boundary conditions and ranges. The boundary condition on the left figures is $\Psi(-1) = -1$ while on the right figures the boundary condition is $\Psi(1) = 1$.

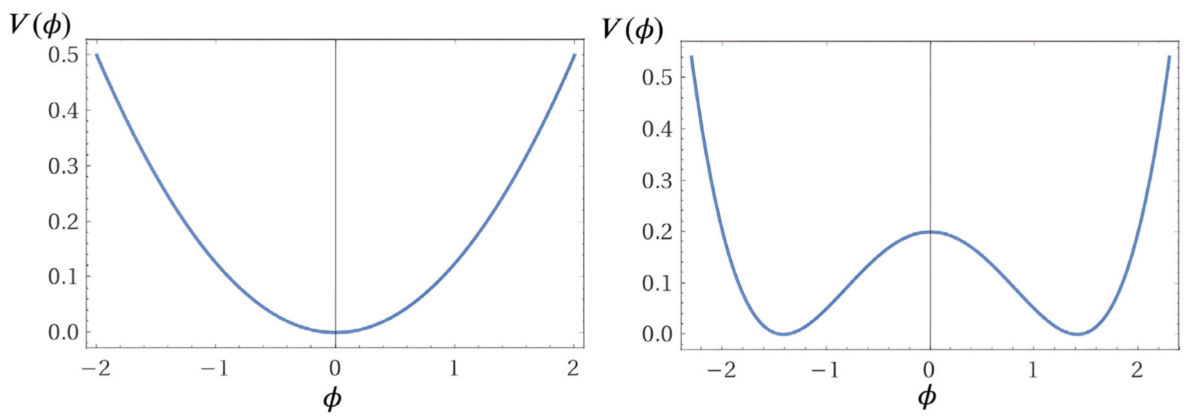

FIGURE 2 On the left, generic form of the potential for the chaotic inflationary scenario. On the right, a typical form of the potential for the original inflationary model, based on the Fubini potential (de Alfaro et al. 1976).

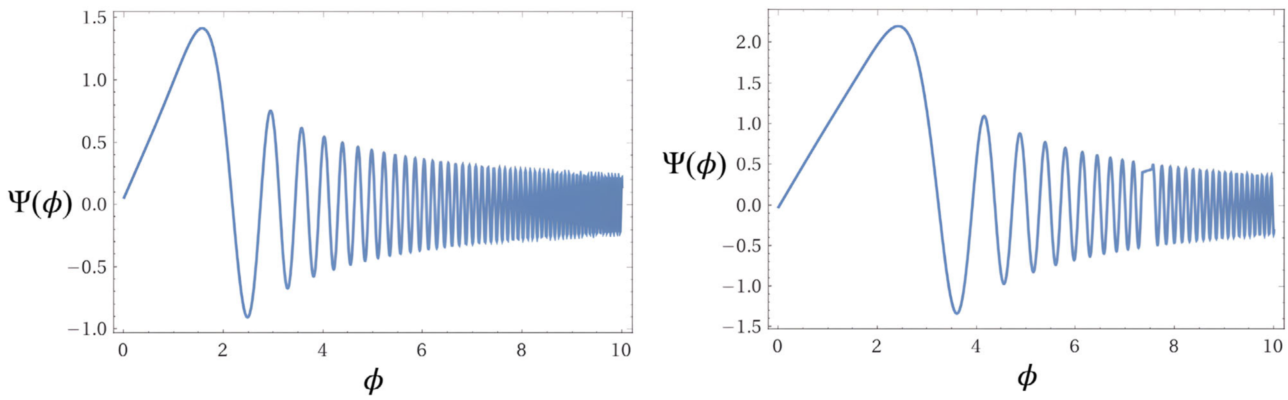

FIGURE 3 Typical individual solutions for $\Psi(\phi)$, the Φ-component of the wave function of the Universe, $\Psi(u, \phi)$. The figure on the left corresponds to chaotic inflation according to Equation (18), with the initial condition $\Psi(1) = 1$ and $\lambda g_\phi^2 < 0$. The figure on the right corresponds to nonchaotic inflation, according to Equation (20), with the initial condition $\Psi(1) = 1$, $\lambda\beta < 0$, $\lambda g_\phi^2 < 0$, and $\lambda C < 0$. Although the solutions exhibit some similarity to Figure 1, it is important to highlight that the presence of the inflaton field anticipates the effects of the acceleration of the Universe towards the boundary region of separation between the two evolutionary cosmic phases: the current phase and its mirror counterpart. This is in comparison with results that do not include the presence of this scalar field.

understanding of the dynamic behavior exhibited by the wave function of the Universe. By specifically focusing on the initial condition $\Psi(1) = 1$, the figures discern between distinct scenarios of the wave function's unfolding indicating distinct evolution trajectories under chaotic and nonchaotic inflationary conditions, which exhibit a duality in parameter configurations. This duality sheds some light on the sensitivity of the chaotic and nonchaotic inflationary dynamics to these critical factors. Such insights are pivotal for comprehending the intricate interplay between the chosen parameters and the resulting behavior of the wave function of the Universe.

4 | SUMMARY AND FINAL REMARKS

This paper explored the implications of a commutative formulation integrating branch-cutting cosmology, the Wheeler–DeWitt equation, and Hořava–Lifshitz quantum gravity. Building on a mini-superspace structure, the study investigated the influence of an inflaton-type scalar field on the wave function of the Universe. The analysis focused on the dynamical solutions of branch-cut gravity within a mini-superspace framework, emphasizing the scalar field's impact on the evolution of the Universe's scale factor, parameterized by the dimensionless scale factor $\ln[\beta(t)]$. The formulation is based on the Hořava–Lifshitz action, which depends on the scalar curvature of the branched Universe and its derivatives, with running coupling constants denoted as g_i. We derived the corresponding Wheeler–DeWitt equations and solved them without recurring to numerical approximations. The approach preserves the diffeomorphism property of General Relativity and is compatible with the Arnowitt–Deser–Misner formalism. The study also explored a mini-superspace of variables, incorporating scalar-inflation fields and investigating chaotic and nonchaotic inflationary models. The paper provides insights into the topological foliated spacetime structure and its evolution over time, contributing to our understanding of cosmic acceleration.

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AUTHOR CONTRIBUTIONS

Conceptualization: C.A.Z.V.; *Methodology*: C.A.Z.V., B.A.L.B., P.O.H., J.A.deF.P., D.H., F.W., and M.M.; *Software*: C.A.Z.V., B.A.L.B., M.R., M.M.; *Validation*: C.A.Z.V., B.A.L.B., D.H., P.O.H., J.A.deF.P., and F.W.; *Formal analysis*: C.A.Z.V., B.A.L.B., P.O.H., J.A.deF.P., D.H., and F.W.; *Investigation*: C.A.Z.V., B.A.L.B., P.O.H., J.A.deF.P., M.R., G.A.D., M.M., and F.W.; *Resources*: C.A.Z.V.; *Data curation*: C.A.Z.V. and B.A.L.B.; *Writing—original draft preparation*: C.A.Z.V.; *Writing—review and editing*: C.A.Z.V., B.A.L.B., P.O.H., J.A.deF.P., D.H., G.A.D., M.R., M.M., and F.W.; *Visualization*: C.A.Z.V. and B.A.L.B.; *Supervision*: C.A.Z.V.; *Project administration*: C.A.Z.V. All authors have read and agreed to the published version of the manuscript.

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REFERENCES

Abreu, E., Mendes, A., Oliveira-Neto, G., Ananias Neto, J., Rezende Rodrigues, J., & Silva de Oliveira, M. 2019, *Gen. Relativ. Gravit.*, *51*(95), 27.

Aharonov, Y., & Bohm, D. 1959, *Phys. Rev.*, *115*(3), 485.

Bertolami, O., & Zarro, C. A. 2011, *Phys. Rev. D*, *84*, 044042.

Bodmann, B., Hadjimichef, D., Hess, P., et al. 2023a, *Universe*, *9*(10), 428.

Bodmann, B., Zen Vasconcellos, C., de Freitas Pacheco, J., Hess, P., & Hadjimichef, D. 2022, *Astron. Nachr.*, *344*(1-2), e220086.

Bodmann, B., Zen Vasconcellos, C., Hess, P., de Freitas Pacheco, J., Hadjimichef, D., Razeira, M., & Degrazia, G. 2023b, *Universe*, *9*(6), 278.

Cordero, R., Garcia-Compean, H., & Turrubiates, F. J. 2019, *Gen. Relativ. Gravit.*, *51*, 138.

de Alfaro, V., Fubini, S., & Furlan, G. 1976, *Nuovo Cimento A*, *34*, 569.

de Freitas Pacheco, J., Zen Vasconcellos, C., Hess, P., Hadjimichef, D., & Bodmann, B. 2022, *Astron. Nachr.*, *344*(1-2), e220070.

Dirac, P. 1937, *Proc. R. Soc. A*, *160*(900), 48.

Einstein, A. 1916, *Ann. Phys.*, *49*, 769.

Einstein, A. 1917, *Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie*, König.-Preuss. Akad. Wiss (Berlin, Germany).

Friedman, A. 1922, *Z. Phys.*, *10*, 377.

García-Compeán, H., & Mata-Pacheco, D. 2022, *Universe*, *8*(4), 237.

Guth, A. 1981, *Phys. Rev.*, *D23*, 347.

Guth, A. 2004, in: *Measuring and Modeling the Universe*, ed. W. L. Freedman, Vol. *2*, Cambridge Univ. Press (Cambridge, UK).

Hartle, J., & Hawking, S. 1983, *Phys. Rev. D*, *28*, 2960.

Hawking, S. 1982, *Pontif. Acad. Sci. Scr. Varia*, *48*, 563.

Hess, P. 2017, in: *Centennial of General Relativity: A Celebration*, ed. C. A. Z. Vasconcellos, World Scientific Pub. Co. (Singapore).

Hess, P., & Boller, T. 2020, in: *Topics on Strong Gravity: A Modern View on Theories and Experiments*, ed. C. A. Z. Vasconcellos, World Scientific Pub. Co. (Singapore).

Hess, P., & Greiner, W. 2009, *Int. J. Mod. Phys. E*, *18*(1), 51.

Hess, P., Schäfer, M., & Greiner, W. 2016, *Pseudo-Complex General Relativity*, Springer International Publishing (Cham).

Hess, P., Vasconcellos, Z., de Freitas Pacheco, J., Hadjimichef, D., & Bodmann, B. 2023, *Astron. Nachr.*, *344*(6), e220101.

Hinshaw, G., Larson, D., Komatsu, E., Spergel, D., & Bennett, C. 2013, *Astrophys. J. Suppl. Ser.*, *208*(2), 19.

Hořava, P. 2009, *Phys. Rev. D*, *79*, 084008.

Kiefer, C. 2012, *Quantum Gravity*, 3rd. ed., Oxford University Press (Oxford).

Kiritsis, E., & Kofinas, G. 2009, *Nucl. Phys. B*, *821*, 467.

Lemaître, G. 1927, *Anna. Soc. Sci. Bruxelles*, *A47*, 49.

Lukasz, A. 2014, *Appl. Math. Phys.*, *2*(3), 73.

Maeda, K.-i., Misonoh, Y., & Kobayashi, T. 2010, *Phys. Rev. D*, *82*, 064024.

Manders, K. 1989, *J. Philos.*, *86*(10), 553.

Robertson, H. 1935, *Astrophys. J.*, *82*, 248.

Tavakoli, F., Vakili, B., & Ardehali, H. 2021, *Adv. High Energy Phys.*, *2021*, 6617910.

Vieira, H., Bezerra, V., Muniz, C., Cunha, M., & Christiansen, H. 2020, *Phys. Lett. B*, *809*, 135712.

Walker, A. 1937, *Proc. Lond. Math. Soc.*, *42*, 90.

Witt, B. D. 1967, *Phys. Rev.*, *160*, 1113.

Wu, K.-D., Kondra, T., Rana, S., et al. 2021, *Phys. Rev. Lett.*, *126*(9), 090401.

Zen Vasconcellos, C., Hadjimichef, D., Razeira, M., Volkmer, G., & Bodmann, B. 2019, *Astron. Nachr.*, *340*, 857.

Zen Vasconcellos, C., Hess, P., de Freitas Pacheco, J., Hadjimichef, D., & Bodmann, B. 2023, *Astron. Nachr.*, *344*(6), e220079.

Zen Vasconcellos, C., Hess, P., Hadjimichef, D., Bodmann, B., Razeira, M., & Volkmer, G. 2021a, *Astron. Nachr.*, *342*(5), 776.

Zen Vasconcellos, C., Hess, P., Hadjimichef, D., Bodmann, B., Razeira, M., & Volkmer, G. 2021b, *Astron. Nachr.*, *342*(5), 765.

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