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Journal

Physical Review Letters, 136(13)

ISSN

0031-9007

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Publication Date

2026-04-03

DOI

10.1103/3ml4-gr6h

Peer reviewed

Nanoscale Imaging of Magnetotransport around a Circular p - n Junction in Graphene

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 (Received 8 June 2025; revised 18 December 2025; accepted 29 January 2026; published 2 April 2026)

Magnetoresistance studies of 2D systems are often shaped by the motion of electrons that occupy spatially confined wave functions, such as topological edge modes and disorder-induced bound states. Directly probing how such states form and behave *in situ*, under applied currents, provides a clear way of connecting microscopic physics to the macroscopic transport response. In this Letter, scanning tunneling potentiometry is used to probe the local, current-induced electrochemical potential of carriers in graphene near circular p - n junctions in an out-of-plane magnetic field ranging from 0 to 1.4 T. These measurements provide detailed information about the motion of carriers at the nanometer scale, revealing how it evolves with increasing field. The electrochemical potential displays distinct patterns, such as dipoles, spirals, and concentric disks in weak, moderate, and high fields, respectively. The size and orientation of these patterns can be used to understand how local carrier dynamics change in different transport regimes as well as to directly extract physical parameters such as the electron mean free path and cyclotron diameter.

DOI: [10.1103/3ml4-gr6h](https://doi.org/10.1103/3ml4-gr6h)

Since the discovery of the quantum Hall effect (QHE) [1,2], similar “Hall bar” experiments over the years have uncovered a wealth of other dc magnetotransport phenomena, including nonequilibrium resistance oscillations and superlattice commensurability effects as well as ballistic and hydrodynamic flow [3–10]. These macroscopic transport signatures emerge from properties of the electron wave functions and scattering processes which depend on disorder, Coulomb interactions, and the strength of externally applied fields [11]. At the same time, these properties can also affect the microscopic distribution of current flowing around obstacles and along device edges [12–15]. The connection between the microscopic dynamics of charge carriers and the macroscopic conductivity obtained through bulk measurements is an active research area [16–20].

In the ballistic transport regime, for example, nonlocal correlations related to the cyclotron radius (R_c) are predicted to affect the magnetoresistance and also generate nanometer-scale peaks in the Hall electric field [21,22]. For strong magnetic fields approaching the (fractional) QHE regime, cyclotron trajectories evolve into topological edge modes that are often reconstructed by electron-electron

interactions [23–25]. Meanwhile, periodically etched 2D systems show commensurability resistance peaks that arise from the complex orbits of carriers when R_c is comparable to the pitch [5,6,10]. Barriers formed by strong in-plane electric fields can cause Landau level collapse and modified cyclotron motion in systems with linear dispersion [16,26–28] as well as the formation of snake states that propagate along p - n boundaries [29]. So far, these transport phenomena have not been investigated at the nanometer scale using scanned probes that directly image the potential distribution produced by current flow [13–15].

Previously, atomic force microscopy has been effective in spatially mapping the Hall potential of 2D electron systems—especially in the QHE regime [30–32]—but suffers from limited spatial resolution [33,34]. Besides atomic force microscopy, many other scanned probe techniques were able to visualize submicron features associated with QHE edge and bulk states [35–43] but largely probed systems in their ground state without any transport current. In this Letter, we employ scanning tunneling potentiometry (STP), which is an alternative approach that maps the electrochemical potential (μ_{ec}) induced by an applied source-drain bias [44]. STP can provide subangstrom-resolved images of μ_{ec} , which unveil hidden aspects of the electronic flow [15,45,46].

We use STP to image current flow in graphene and h -BN samples with long transport mean free paths ($l > 2 \mu\text{m}$) at low temperature ($T = 4.5 \text{ K}$). These samples contain a

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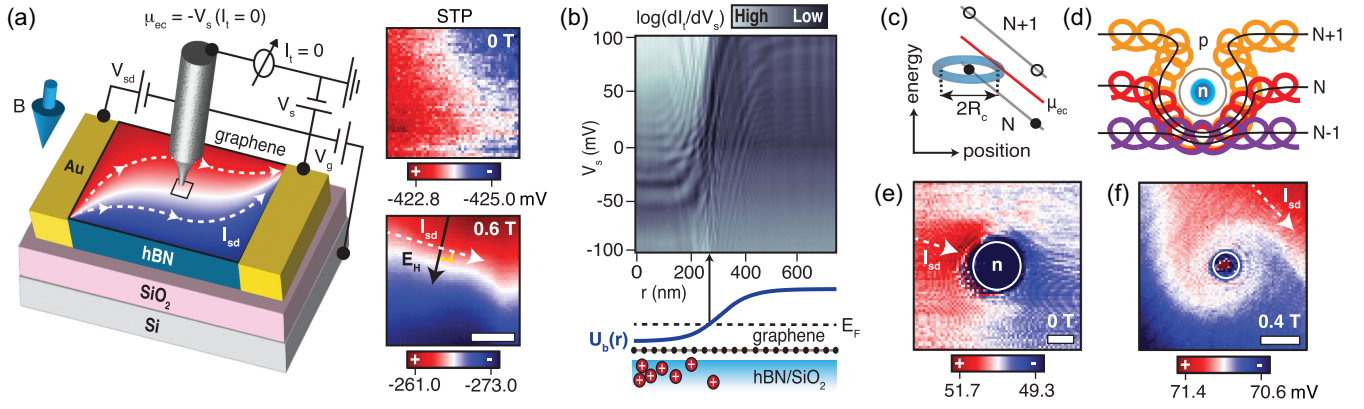


FIG. 1. (a) An STM tip records μ_{ec} (in mV) induced by a current bias I_{sd} . The dashed white arrows indicate the direction of current flow, which is the same for both STP maps (100 μ A). The scale bar is 500 nm. (b) Tunneling dI_t/dV_s spectra along a radial line cut beginning at the center of the p - n junction, with $B = 1.4$ T and $V_g = -30$ V. A cartoon shows the electrostatic potential $U_b(r)$ (blue line) generated by ionized defects in the substrate. The intersection of the Fermi level E_F with $U_b(r)$ sets the p - n junction radius r_{pn} . (c) Energy diagram of LLs in the presence of a uniform Hall field. Both occupied (filled) and unoccupied (hollow) LLs shift along with μ_{ec} . (d) Trochoidlike carrier trajectories in the vicinity of the potential well (not to scale). A Hall field E_H points downward, and carriers drift from left to right via cyclotron motion along equipotential (solid black) lines. (e), (f) STP measurements near an electrostatic barrier at two magnetic fields ($B = 0$ and 0.4 T). $I_{sd} = 200$ and 120 μ A, respectively. The solid circular lines mark the p - n boundary, and the scale bars are 250 nm.

circular p - n junction that acts as a localized barrier for the current. Carrier scattering at the p - n boundary creates a charge pileup that is known as the Landauer residual resistivity dipole (LRRD) [15,45,47,48]. As we apply a magnetic field, the LRRD rotates and twists around the barrier, creating spiral-like textures in the potential contours. At high magnetic fields, we also observe a local enhancement of the electrochemical potential gradient $\vec{\nabla}\mu_{ec}$ approximately one cyclotron diameter $2R_c$ away from the barrier [21,22] ($R_c = \hbar k_F/eB$, where $\hbar k_F$ is the Fermi momentum and e is the elementary charge).

A schematic of the STP measurement is shown in Fig. 1(a). First, we recorded STP data in a large (1.5×1.5 μ m), pristine area of the graphene sample and found that μ_{ec} varies linearly between the electrodes [Fig. 1(a)]. This agrees with semiclassical theory where the gradient of μ_{ec} is related to the current density $\vec{j}$ by Ohm's law: $\vec{j} = -\sigma_{xx}\vec{\nabla}\mu_{ec} + \sigma_{xy}\hat{z} \times \vec{\nabla}\mu_{ec}$. Here, $\hat{z}$ is the unit vector normal to the plane, and σ_{xx} and σ_{xy} are the longitudinal and Hall conductivities, respectively. In a magnetic field, $\vec{\nabla}\mu_{ec}$ is rotated with respect to $\vec{j}$ by the Hall angle θ_H , which can nearly reach 90° as shown in Fig. 1(a). From the measured θ_H and the classical formula $\sigma_{xy}/\sigma_{xx} = l/R_c$, we found an estimate of $l = R_c \tan \theta_H \approx 2$ μ m (Fig. S2 [49]).

Afterward, we introduced a circular p - n junction to the scan area of Fig. 1(a) by applying an STM tip voltage pulse [57]. The resulting potential well $U_b(\vec{r})$ induces a circular p - n junction in the graphene sheet with a gate-tunable radius r_{pn} ranging from 250 ($V_g = -40$ V) to 400 nm ($V_g = -2$ V). In Fig. 1(b), we present dI_t/dV_s spectra of

the local density of states (LDOS) near the barrier. These spectra reveal peaks that match the Landau level (LL) energies $E_N = \Delta\sqrt{N}$ of massless Dirac fermions, where $\Delta = \pm\sqrt{2}\hbar v_F/l_B$, $l_B = \sqrt{\hbar/eB}$ is the magnetic length, and $N = 1, 2, \dots$ is the LL number. We obtained $U_b(r)$ by spatially tracking the LL energies outside and inside the p - n junction (Figs. S3 and S4 [49]).

In equilibrium, the variations of $U_b(r)$ are compensated by those of the local chemical potential $\mu(r)$, such that $-e\mu_{ec}(r) = U_b(r) + \mu(r)$ remains uniform. Under a nonzero source-drain bias, one can expect that μ_{ec} is augmented by the Hall potential $U_H(\vec{r})$ where $-\vec{\nabla}U_H \equiv \vec{E}_H = [\hat{z} \times \vec{j}]/\sigma_{xy}$, and the LL energies acquire the same additional shift [3,8] [Fig. 1(c)]. The corresponding wave functions are localized within the distance $\sqrt{2|N|+1}l_B \approx R_c$ from the equipotential contours of the total potential $U(\vec{r}) = U_b(\vec{r}) - eU_H(\vec{r})$. The structure of these wave functions near the barrier can be illustrated by a classical cartoon [Fig. 1(d)] of trochoidal cyclotron orbits whose guiding centers drift along those equipotential contours. From this cartoon, one can anticipate that μ_{ec} may possess some anomaly near the radius $r = r_{pn} + 2R_c$.

Representative STP images captured after the formation of a circular p - n junction are shown in Figs. 1(e) and 1(f). In zero magnetic field [Fig. 1(e)], μ_{ec} reveals a ballistic LRRD seen in previous work [15]. The LRRD orientation indicates the direction of incident current flow (white dashed arrows) [47]. At $B = 0.4$ T [Fig. 1(f)], the map of μ_{ec} displays a spiral pattern that wraps around the dark blue circle in the center. As in Fig. 1(e), this circle occurs

along the p - n boundary, where sharp variations of the LDOS generate significant thermovoltages between the tip and the sample [15,58]. For simplicity, we focus on the properties of μ_{ec} outside the p - n junction.

In Figs. 2(a)–2(d), we report additional STP data in magnetic fields varying from $B = 0.2$ to 1.4 T for large hole doping ($n = -2.3 \times 10^{12} \text{ cm}^{-2}$) when the p - n junction radius is relatively small, $r_{pn} \approx 250 \text{ nm}$. At low fields, $B \leq 0.6 \text{ T}$ [Figs. 2(a) and 2(b)], the evolution of μ_{ec} is similar to

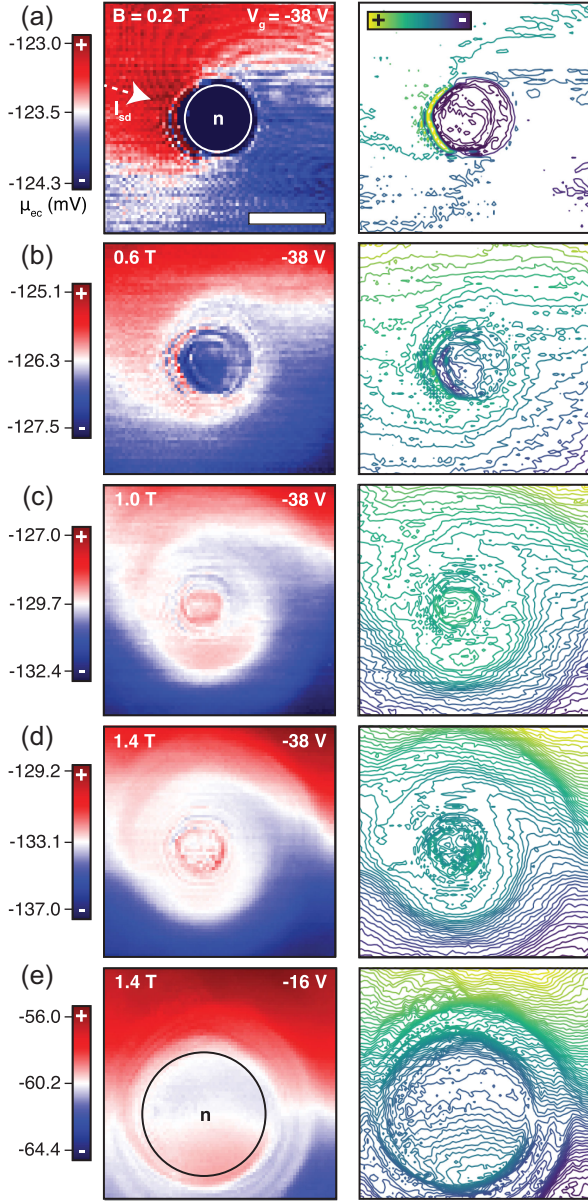


FIG. 2. (a)–(d) STP measurements with $I_{sd} = 100 \mu\text{A}$ around an electrostatic barrier at large hole doping ($V_g = -38 \text{ V}$) and four selected magnetic fields between 0 and 1.4 T. The scale bar is 500 nm. The right side shows equipotential contour plots of the STP maps directly to the left. (d) Additional STP data at low hole doping ($V_g = -16 \text{ V}$), with $B = 1.4 \text{ T}$ and $I_{sd} = 25 \mu\text{A}$.

that shown in Fig. 1. As the field increases, the contours of constant μ_{ec} begin to spiral counterclockwise around the junction, twisting by $\sim 90^\circ$ at $B = 0.2 \text{ T}$ and beyond 180° at $B = 0.6 \text{ T}$. In higher fields of 1.0 and 1.4 T [Figs. 2(c) and 2(d)], the spiral-like patterns in μ_{ec} persist, but other features also emerge. Rather than terminating at the p - n boundary, the spiral arms begin to converge at a larger radius of $r \approx 400 \text{ nm}$. This feature, which we refer to as the “inner disk,” can be seen more clearly in the contour plots of μ_{ec} that show bunching of the equipotentials along the inner disk edge. Also seen in these plots is another disklike feature of a larger radius $r = 700\text{--}800 \text{ nm}$, which we call the “outer disk.” In Fig. 2(e), we show an additional STP map of the same barrier at $B = 1.4 \text{ T}$, except now at lower carrier density ($n = -9.7 \times 10^{11} \text{ cm}^{-2}$) when r_{pn} is larger ($\approx 350 \text{ nm}$) and has merged with the inner disk. Here, the outer disk is especially clear and has a slightly smaller radius than measurements at large hole doping $n = -2.3 \times 10^{12} \text{ cm}^{-2}$ [Fig. 2(d)] (see the complete dataset in Figs. S5–S7 [49]).

For the remainder of this Letter, we present our theoretical modeling of these results. First, we find that Ohmic transport theory reproduces the spiral-like patterns in the low magnetic field regime [Figs. 2(a) and 2(b)] but fails to model the disk features at higher fields [Figs. 2(c)–2(e)]. A more sophisticated analysis using the Boltzmann equation successfully explains the outer disk feature in the high-field limit [Figs. 2(d) and 2(e)]. Lastly, we show that a tight-binding model is able to reproduce the spirals and both disk features simultaneously [Figs. 2(a)–2(e)].

Combining Ohm’s law with charge conservation, $\vec{\nabla} \cdot \vec{j}(\vec{r}) = 0$, we find that μ_{ec} obeys the advection-diffusion equation [59]

$$\sigma_{xx} \nabla^2 \mu_{ec} + \vec{\nabla} \mu_{ec} \cdot (\vec{\nabla} \sigma_{xx} + \hat{z} \times \vec{\nabla} \sigma_{xy}) = 0. \quad (1)$$

We now numerically solve Eq. (1), taking into account the nonuniformity of $\sigma_{xy}(r) \approx -en(r)/B$, where $n(r)$ is the local electron density deduced from the measured $U_b(r)$. For simplicity, we set the Hall angle to a constant, so that $\sigma_{xx} = \alpha |\sigma_{xy}|$ with $\alpha \equiv |\cot \theta_H| = R_c/l \ll 1$. In the numerical solutions, shown in Figs. 3(a) and 3(b), the contours of μ_{ec} exhibit spiraling patterns: They spin counterclockwise approaching the p - n junction, make a U-turn at a turning point radius r_t , and then spin out clockwise moving away from the junction [60]. This behavior is in qualitative agreement with the low field data in Figs. 2(a) and 2(b). However, the Ohmic model fails to reproduce the correct inner and outer disk features observed at high fields; cf. Figs. 2(d) and 2(e).

The Ohmic description of Eq. (1) is strictly valid when $l \ll R_c$. Examination of ballistic physics at short length scales is made possible by solving the Boltzmann kinetic equation. A rigorous solution can be found for a large, hard circular

boundary where the conductivities σ_{xx} and σ_{xy} are constant outside the p - n boundary and $R_c \ll r_{pn}$ [21,22]. This solution is shown in Fig. 3(c), where a circular kink appears in μ_{ec} at an offset of $2R_c$ away from r_{pn} . Before comparing with the data, we note that the hard boundary assumption is best approximated at low carrier densities, where r_{pn} is large and $n(r)$ does not vary significantly outside of the boundary. These conditions are most nearly satisfied in Fig. 2(e), which shows some qualitative agreement with the Boltzmann-corrected solution in Fig. 3(c). However, at high carrier densities as in Fig. 2(d), the data show three circular features in μ_{ec} , while the Boltzmann solution here predicts only two. We attribute this to differences in the turning point radius r_t and p - n junction radius r_{pn} which occur when the hard boundary assumption inadequately models the slowly changing potential $U_b(r)$, requiring a consideration of the full quantum scattering problem.

In order to consider the true barrier shape and carrier wave functions, we performed quantum magnetotransport simulations using a tight-binding model [61]. Figure 3(d) shows the calculated probability distribution of a state $\psi_{N,k}$, which drifts horizontally from left to right in a constant Hall field. The effect of $U(\vec{r}) = U_b(\vec{r}) - e\vec{E}_H \cdot \vec{r}$ on the state $\psi_{N,k}$ is shown in Fig. 3(d), which is the fully quantum version of the semiclassical trochoidal trajectories in Fig. 1(c). LL broadening and electron scattering effects are introduced through random on-site energy shifts uniformly distributed within the interval $(-U_d, U_d)$. We now find μ_{ec} by performing a sum over scattering wave functions weighted with their occupation factors (see Supplemental Material [49]).

Figures 3(e) and 3(f) show the computed profiles of μ_{ec} for two choices of disorder strength U_d . In the ballistic

transport regime ($U_d = 0$), μ_{ec} displays two disk features defined by a bunching of the equipotential lines [Fig. 3(e)]. The first, inner disk is larger than the p - n boundary and corresponds to the radius r_t at which guiding center trajectories divert around the depletion region in the manner of Fig. 1(c). The outer disk corresponds to the inner disk radius plus one cyclotron diameter, $r_t + 2R_c$. In the presence of moderate disorder ($U_d = 20$ meV), the disks are destroyed by scattering [Fig. 3(f)] and replaced by a spiral similar to those predicted by Ohmic simulations in Fig. 3(a). For weak disorder in between these two regimes ($U_d = 5$ meV), the spiral pattern and disk features coexist [Fig. S12(b) [49]].

These simulations provide additional intuition for understanding the field-dependent changes that we observe experimentally. In weak magnetic fields, the effects of LL quantization and the finite size of the cyclotron orbits are obscured by scattering. In stronger fields, when carriers can complete multiple cyclotron orbits before scattering, $l \gg 2R_c$, the behavior of states within individual LLs becomes apparent. We understand the emergence of the inner disk with increasing magnetic field as a consequence of LL quantization, whereby the cyclotron energy gap at the Fermi level grows larger than the disorder energy scale set by U_d , so that states within the disk $r < r_t$ become localized and carrier scattering across the boundary at r_t is suppressed.

The presence of this effective “hard wall” at $r_t \geq r_{pn}$ causes charge carriers to pile up as they divert around the barrier, locally enhancing the Hall field at $r_t + 2R_c$. Measuring the distance between the inner and outer disk features to extract R_c at different carrier density and applied fields, we find reasonable agreement with the expected

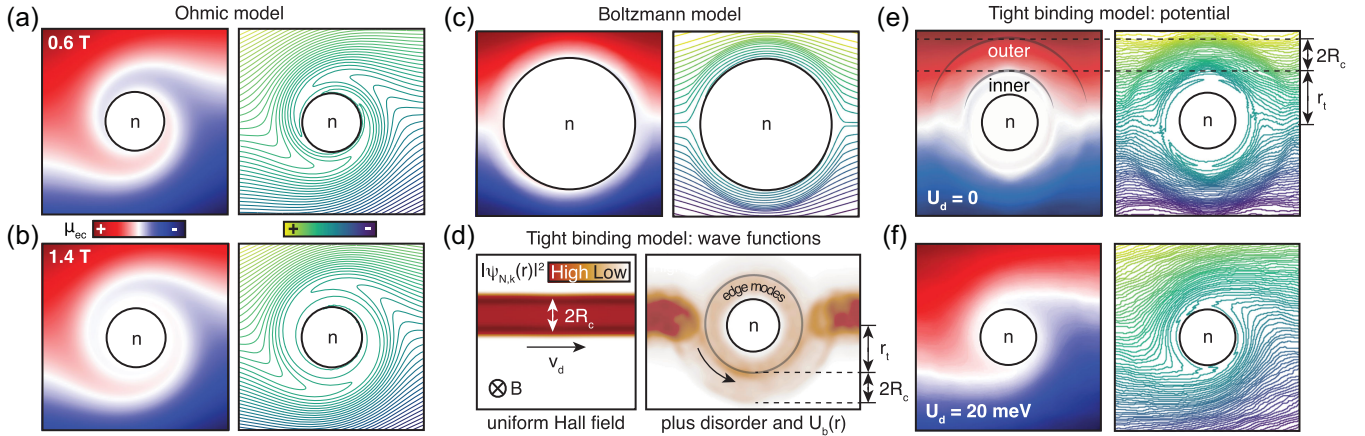


FIG. 3. (a),(b) Calculated μ_{ec} in the Ohmic regime, obtained by solving Eq. (1) at large hole doping ($E_F = -210$ meV) and applied fields (a) $B = 0.6$ T ($\alpha = 0.13$) and (b) $B = 1.4$ T ($\alpha = 0.053$). The black solid line denotes the p - n junction of radius $r_{pn} = 250$ nm. At large distances, the current flows horizontally from left to right. (c) Calculated μ_{ec} using Boltzmann kinetic theory for a larger barrier such that $R_c \ll r_{pn}$. (d) Probability density of a tight-binding wave function with energy $E = -190$ meV at $B = 1$ T in a uniform Hall field, plus $U_b(r)$ and the random disorder potential. (e),(f) Calculated μ_{ec} using the tight-binding model in (e) the disorder-free ($U_d = 0$) and (f) moderate disorder ($U_d = 20$ meV) regimes. The black semicircles mark the inner and outer disks at radii $r = r_t$ and $r_t + 2R_c$, respectively.

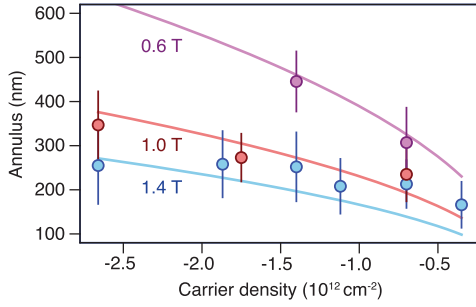


FIG. 4. Measured cyclotron diameter $2R_c$ (outer disk radius minus inner disk radius, or annulus) as a function of carrier density and magnetic field. Error bars include the spatial extent of the local Hall field enhancement, plus the standard deviation of measurements across several line cuts.

value $R_c = \hbar\sqrt{\pi n}/eB$ in Fig. 4 (solid lines). We note, however, that the experimental data overestimate the cyclotron diameter in the high-field, low-carrier-density regime where R_c is smallest. At this point, we cannot conclusively explain this numerical deviation. However, it has been predicted that strong in-plane electric fields $\vec{E}_b = -\vec{\nabla}U_b$ can reduce the effective magnetic field felt by massless carriers in graphene, $B_{\text{eff}} = \sqrt{1 - \beta^2}B$, where $\beta = E/v_F B$ [16,26]. This effect enhances the cyclotron radius and eventually leads to so-called “Landau level collapse” [16,26–28]. It is expected to be most pronounced in the high-field, low-carrier-density regime where R_c is comparable to or smaller than the width of the region where the radial electric field $\vec{E}_b$ is concentrated.

In summary, we have used STP to probe magnetotransport in graphene at $T = 4.5$ K with the help of electrostatic barriers “drawn” with the STM tip. As the magnetic field increases up to 1.4 T, the current flow evolves in the following way: At weak fields and on measurement length scales less than l , transport is ballistic as evidenced by the LRRD in Fig. 1(d). In intermediate fields such that $l \gtrsim 2R_c$, the electron flow is a combination of the lateral diffusion with a slow guiding center drift of the cyclotron orbits. This advection-diffusion transport creates spiral-like current flow around the obstacle. At higher magnetic fields, when $l \gg 2R_c$, the flow crosses over into a “quasiballistic” regime marked by a local enhancement of the Hall field one cyclotron diameter away from the turning point r_t . The latter permits a direct measurement of the cyclotron diameter in a quantum material, which has some advantages over measuring R_c by magnetic focusing [62,63]. Future STP measurements could investigate the role of strong electron-electron interactions in magnetotransport [9,19,20].

Acknowledgments—Work by Z. J. K. and K. J. S. was supported by the U.S. Department of Energy (DOE), Office of Science, Basic Energy Sciences (BES) Program for Materials and Chemistry Research in Quantum Information

Science under Award No. DE-SC0020313. V. W. B. was supported by NSF CAREER Award No. 239478. W. A. B. and M. A. F. were supported by the Office of Naval Research Award No. N00014-20-1-2356. The authors gratefully acknowledge the use of facilities and instrumentation supported by NSF through the University of Wisconsin Materials Research Science and Engineering Center (No. DMR-1720415). K. W. and T. T. acknowledge support from JSPS KAKENHI (Grants No. 21H05233 and No. 23H02052), CREST (JPMJCR24A5), JST, and World Premier International Research Center Initiative (WPI), MEXT, Japan. M. A. F. was also supported through a Google Ph.D. Fellowship (Quantum Computing). This work was supported by a fellowship from the Alfred P. Sloan Foundation.

Data availability—The data that support the findings of this article are openly available [64].

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End Matter

Methods—For the STP measurements, a dc current bias (I_{sd}) is applied across two ends of a graphene or h -BN device, and the resulting variation of μ_{ec} across the surface is measured with a sharp STM tip. The value of $\mu_{ec}(\vec{r})$ at each point $\vec{r}$ is defined by the sample bias (V_s) required to zero the tunneling current (I_t), which we determine by recording an I_t - V_s curve and performing a linear fit. The electrostatic barriers were created by applying a 5-V, one-minute-long STM tip voltage pulse to ionize defects in the h -BN/SiO₂ substrate [57], creating a screened electrostatic potential $U_b(\vec{r})$ in the overlying graphene.

Electronic structure—The difference between the peaks and minima of the experimental LDOS in Fig. 1(b) is relatively small, which means that the LLs overlap significantly. Therefore, the highest magnetic field of 1.4 T in our experiments should be categorized as “classically strong”: It is not high enough to split the LDOS into well-separated LLs, but it is sufficient to achieve a near-90° Hall angle. This characterization is also consistent with our two-probe resistance measurements that reveal Shubnikov–de Haas oscillations but no QHE (Fig. S1 [49]).

At large radii in Fig. 1(b), the spectra reveal peaks that match the holelike LL energies $E_N = -\Delta\sqrt{N}$ of massless Dirac fermions, where $\Delta = \sqrt{2}\hbar v_F/l_B$. As the p - n boundary is approached, the LLs shift in energy $E_N(r) = \Delta\sqrt{N} + U_b(r)$ following $U_b(r)$. Inside the p - n junction, a different set of peaks match the electronlike LL energies which also shift with the potential, as observed in prior work [65]. We obtain an estimate of $U_b(r)$ by tracking $E_1(r)$ as a function of r ; see Figs. S3 and S4 [49]. To get $n(r)$, we use the relations $n = k_F^2/\pi$ and $k_F = (E_F - U_b)/\hbar v_F$.

Ohmic theory—The advection-diffusion (Ohmic) equation [Eq. (1)] applies on length scales larger than R_c if localization effects can be neglected [66]. Since Dirac fermions cannot easily transmit through p - n junctions in a magnetic field [16], we model the junction as an impenetrable hard wall by enforcing the condition $\vec{j} \cdot \vec{\mu}_{ec} \equiv 0$ for $r < r_{pn}$. It is also important to note that, if we try to ignore electron scattering completely by setting $\sigma_{xx} = 0$, in an attempt to model perfectly ballistic transport, then Eq. (1) becomes an identity. This illustrates a singular nature of transport in a strong magnetic field where one *cannot* ignore scattering effects even if the size of the region of interest is smaller than the zero-field mean free path l .

It is also possible to show analytically that the turning point radius r_t of the spirals in the Ohmic results is set roughly by the condition $\delta\sigma_{xy}/\sigma_{xy} \sim U_b(r_t)/E_F \sim \alpha$, where $\delta\sigma_{xy}$ is the

deviation of the local Hall conductivity from its large-distance value [59]. This illustrates a general principle [59] that the current concentrates along the lines $\sigma_{xy}(\vec{r}) = \text{const}$, and so it is repelled by the depletion region surrounding the p - n junction. This is evident in Figs. 3(a) and 3(b), where the turning points of the spiral arms shift outward as the magnetic field increases and α decreases. The Ohmic turning point r_t lends a possible explanation for the inner disk features seen in Fig. 2. However, the experimentally observed inner disks are larger than these simulations predict.

Boltzmann theory—The Boltzmann kinetic equation that we solve has the general form

$$\vec{v} \cdot \vec{\nabla}_{\vec{r}} f_{\vec{p}}(\vec{r}) + \{e\vec{E}(\vec{r}) + e[\vec{v} \times \vec{B}]\} \cdot \nabla_{\vec{p}} f_{\vec{p}}(\vec{r}) = I_{\vec{p}}(\vec{r}), \quad (\text{D1})$$

where $f_{\vec{p}}(\vec{r})$ is the distribution function of carriers depending on both particle position $\vec{r}$ and momentum $\vec{p}$, $\vec{E}$ is the electric field, $\vec{v} = v_F \hat{p}$ is the velocity of charge carriers in graphene, and $I_{\vec{p}}(\vec{r})$ is the collision integral describing the relevant scattering processes. The methods used to solve this equation can be found in Supplemental Material [49]. The physical origin of the disklike correction in Fig. 3(c) is the modified dynamics of electron orbits away from and colliding with the wall: a smooth drift of the former vs skipping motion of the latter.

Supplementary Material for “Nanoscale imaging of
magnetotransport around a circular p-n junction in graphene”

Device geometry

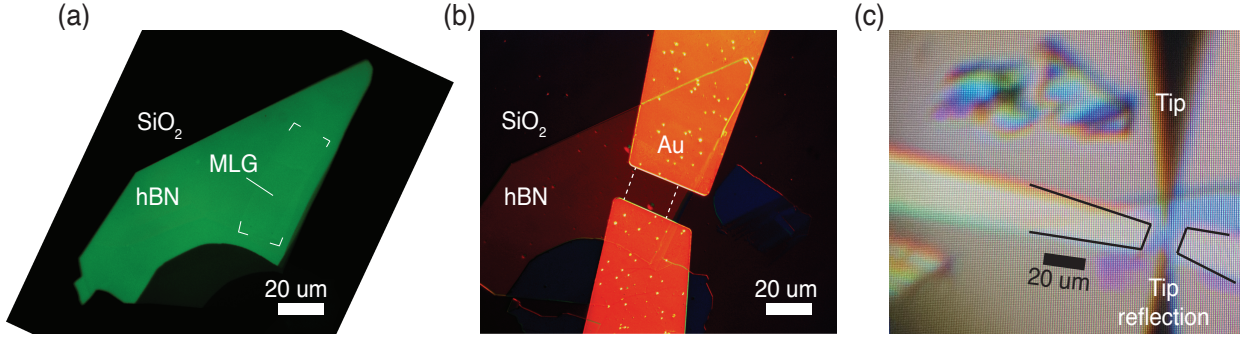


FIG. S1. (a) Optical image of the MLG/hBN/SiO₂ stack. (b) Image of the stack with gold electrodes contacting the graphene. The dashed white lines denote the MLG flake edges. (c) *In situ* image of the STM tip nearing the device. The gold electrodes are outlined in black for visual clarity.

Optical images of the device geometry used for this experiment are shown in Fig. S1. Samples were made by first mechanically exfoliating monolayer graphene (MLG) flakes on Si wafers with 285 nm of SiO₂. These graphene flakes were then stamped on 80 nm-thick exfoliated hBN. The resulting graphene/hBN/SiO₂ heterostructure is shown in Fig. S1(a). We then deposited two Cr/Au electrodes on each end of the flake using electron beam lithography, see Fig. S1(b). Afterwards, the sample was annealed in a furnace with Ar/H₂ (100/10 sccm) at 400°C and a total pressure of 460 mbar for 40 minutes, before loading into the STM prep chamber. The sample was further vacuum annealed at 250°C and 10⁻⁹ mbar for at least 3 hours. The sample was then transferred into the STM cryostat for cooldown and measurement at a base pressure of 4×10^{-11} mbar. Figure S1(c) shows the STM tip approaching the graphene flake right before the measurement.

Two-point magnetoresistance

The devices in this experiment were fabricated with only two gold electrodes to provide a source-drain bias. Therefore, we could not perform a separate characterization of the longitudinal and Hall resistivities (R_{xx} and R_{xy} , respectively). Nevertheless, we have observed Shubnikov-de Haas oscillations in the total resistance R (which includes the contact resis-

tances) as a function of the back gate voltage V_g , shown here. To accentuate the oscillations, we plot the derivative of R with respect to the back gate voltage in Fig. S2(b).

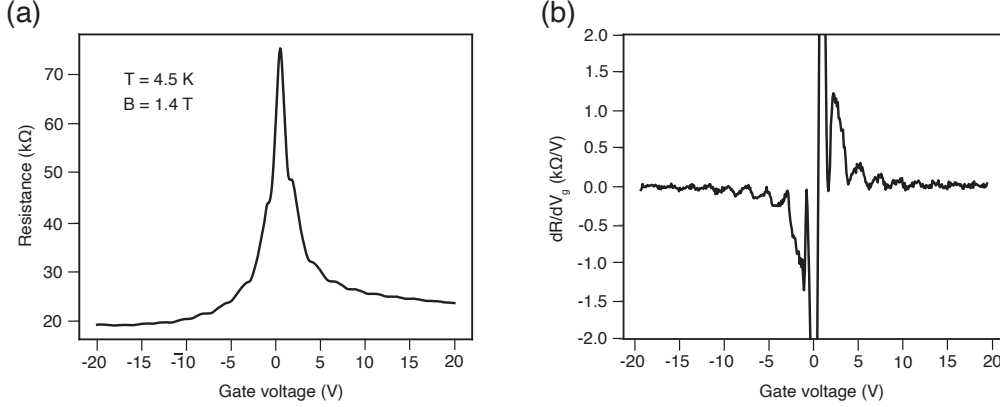


FIG. S2. (a) Gate-dependent two-point resistance measurement at $B = 1.4$ T of a graphene/hBN device used for STP measurements. (b) Derivative of resistance with respect to gate voltage dR/dV_g .

Mean free path estimation

In this section we analyze STP data in the same area as Fig. 2 but recorded before the creation of the electrostatic barrier, see Fig. S3(b)-(f). We also show one STP image recorded after the barrier creation, Fig. S3(a), only to assess the direction of current flow with respect to the scan window. For an isotropic system and a circular barrier, the LRRD in zero magnetic field is oriented in the direction of current flow. Therefore, in Fig. S3(a) the current flows roughly from left to right with a slight downwards tilt. For $B = 0$ T and no barrier in Fig. S3(b), μ_{ec} drops linearly from left to right along the expected direction of the current. By fitting the slope of μ_{ec} in zero magnetic field at different gate voltages, we can apply the Drude conductivity formula $\sigma_{xx} = (e^2/\pi\hbar)k_F l$ to obtain a gate-dependent estimate of the mean free path l . We plot the extracted l in Fig. S3(g), and find that $l > 2$ μm when $V_g < -2$ V. Alternatively, an estimate of l can be obtained by considering the Hall angle as function of magnetic field. To determine the Hall angle, we fit each STP image in Fig. S3(b)-(f) to a linear function of coordinates, and then calculate the angle between the corresponding electric field and the direction of current flow [which we assume to be fixed across all magnetic fields, and aligned with the LRRD in Fig. S3(a)]. We show these results

in Fig. S3(h), plotting the Hall angle as a function of cyclotron energy gap $\hbar\omega_c$ at the Fermi level. According to Ohmic transport theory, the Hall angle is given by

$$\theta_H = \arctan(\omega_c l / v_F). \quad (\text{S1})$$

The best fit to the data points in Fig. S3(h) gives $l = 2.5 \mu\text{m}$ in rough agreement with the previous estimate.

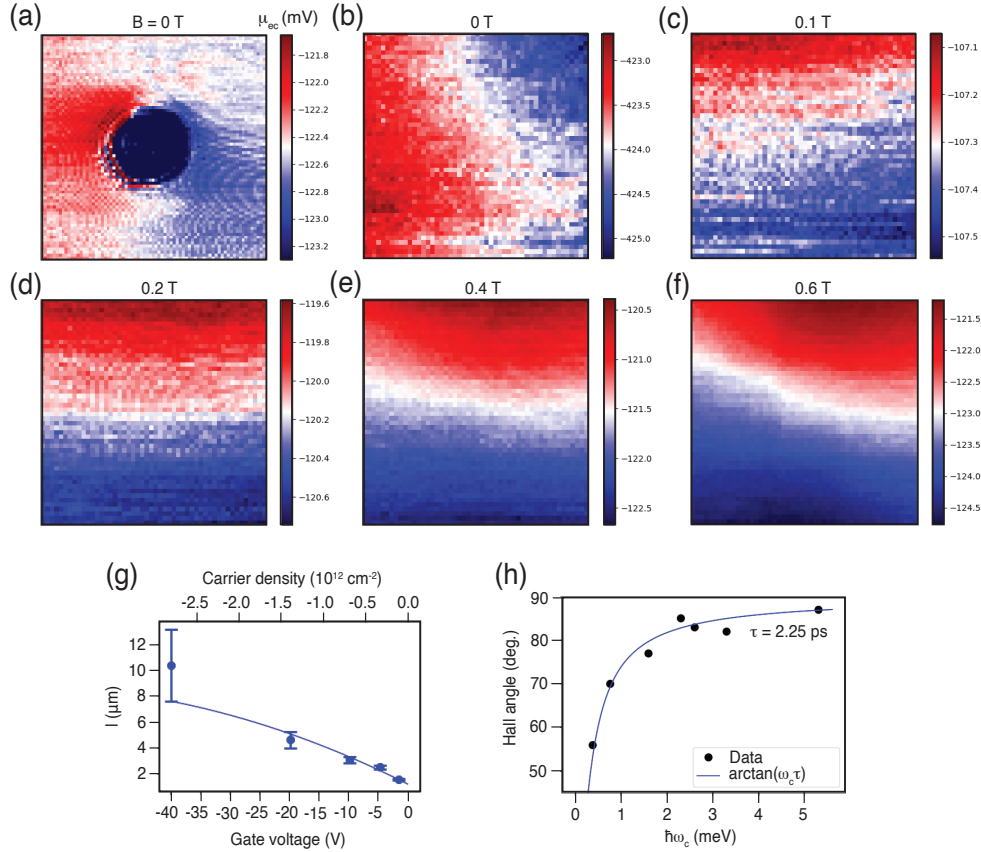


FIG. S3. (a) An STP map in zero magnetic field with an electrostatic barrier [same as Fig. 1(a)]. (b)-(f) STP maps of the same area before the creation of the electrostatic barrier, in magnetic fields increasing up to 0.6 T. (g) Mean free path l extracted from the slope of μ_{ec} in zero magnetic field at different gate voltages by using the Drude conductivity formula. Error bars are derived from the chi-squared uncertainty in the linear fit of μ_{ec} . (h) Hall angle of the STP maps (b)-(f) plotted as a function of cyclotron gap $\hbar\omega_c$ at the Fermi level. These data points are fit to extract another estimate of $\tau = v_F l$.

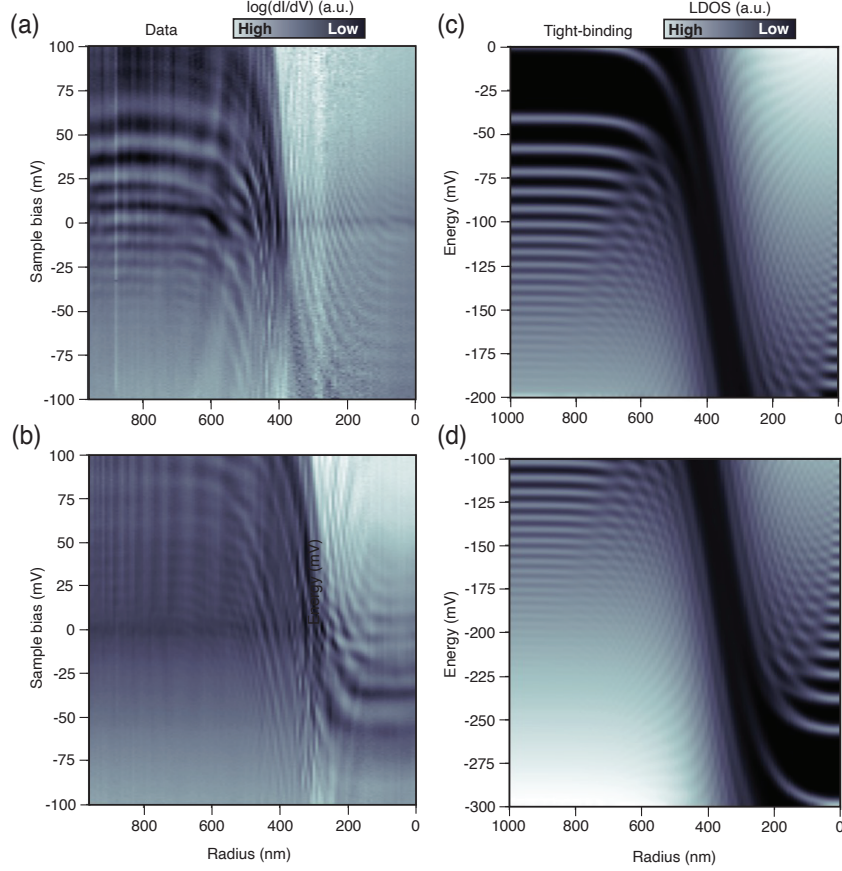


FIG. S4. (a), (b) Log-scale dI_t/dV_b spectra of the electrostatic barrier for $V_g = -10$ and -30 V, respectively. (c), (d) Local density of states calculated from a tight-binding model. Zero on the horizontal axes ($r = 0$) corresponds to the center of the circular barrier.

Tunneling spectra and the LDOS

In Fig. S4(a),(b) we show measured dI_t/dV_s spectra along a radial linecut through the electrostatic barrier of Fig. 2 at two different gate voltages in order to reveal full extent of the potential profile $U_b(r)$. Figure S4(a) is already included in the main manuscript in Fig. 1(b), but we include it again here for completeness. Figure S4(c),(d) shows tight-binding calculations of the LDOS using the Pybinding package [1].

Model for the barrier potential

In order to obtain the potential $U_b(r)$, we track the energy of the first Landau level in Fig. S4(a),(b) as a function of radius from the center of the electrostatic barrier. The ex-

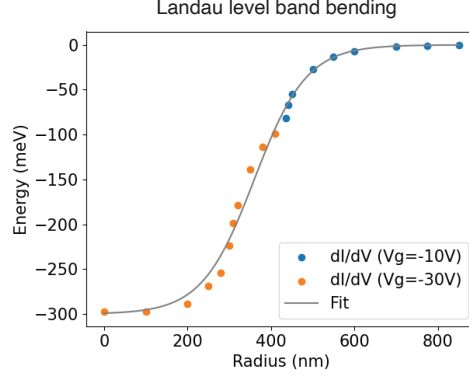


FIG. S5. Barrier potential $U_b(r)$ determined by tracking the LLs in Fig. S4.

tracted $U_b(r)$ is shown in Fig. S5. The orange data points are computed from Fig. S4(b) (bottom of the potential well), while blue data points come from Fig. S4(a) (top of the potential well). A convenient functional fit that captures the qualitative shape of the electrostatic barrier is

$$U_b(r) = 150 \{ \tanh[0.008(r - 360)] - 1 \} \quad (\text{S2})$$

where $U_b(r)$ is in meV and r is in nm. The potential $U_b(r)$ varies smoothly through the p-n junction boundary, and does not exhibit any plateaus associated with compressible regions [2] expected to form in junctions of width much larger than l_B .

Additional STP maps

Additional STP maps are presented in Figs. S6, S7, and S8 for magnetic fields $B = 1.4$, 1.0, and 0.6 T, respectively. The estimated inner and outer disk radii in this dataset are used to produce Fig. 4 in the main text.

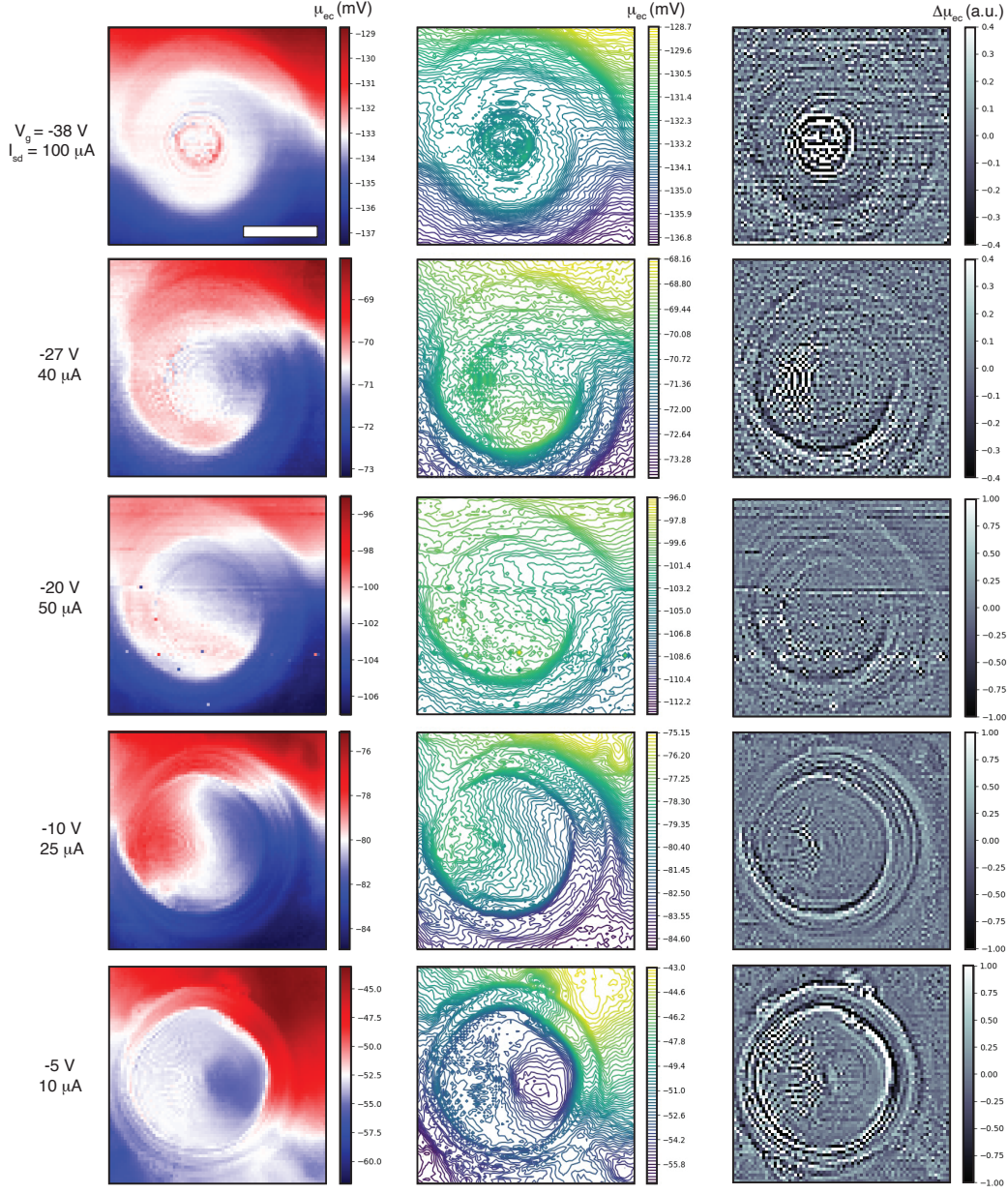


FIG. S6. STP maps at $B = 1.4$ T in order of decreasing carrier density from top to bottom, plotted alongside the corresponding equipotential contours and Laplacians in order to highlight the inner and outer disk features. The scale bar is 500 nm.

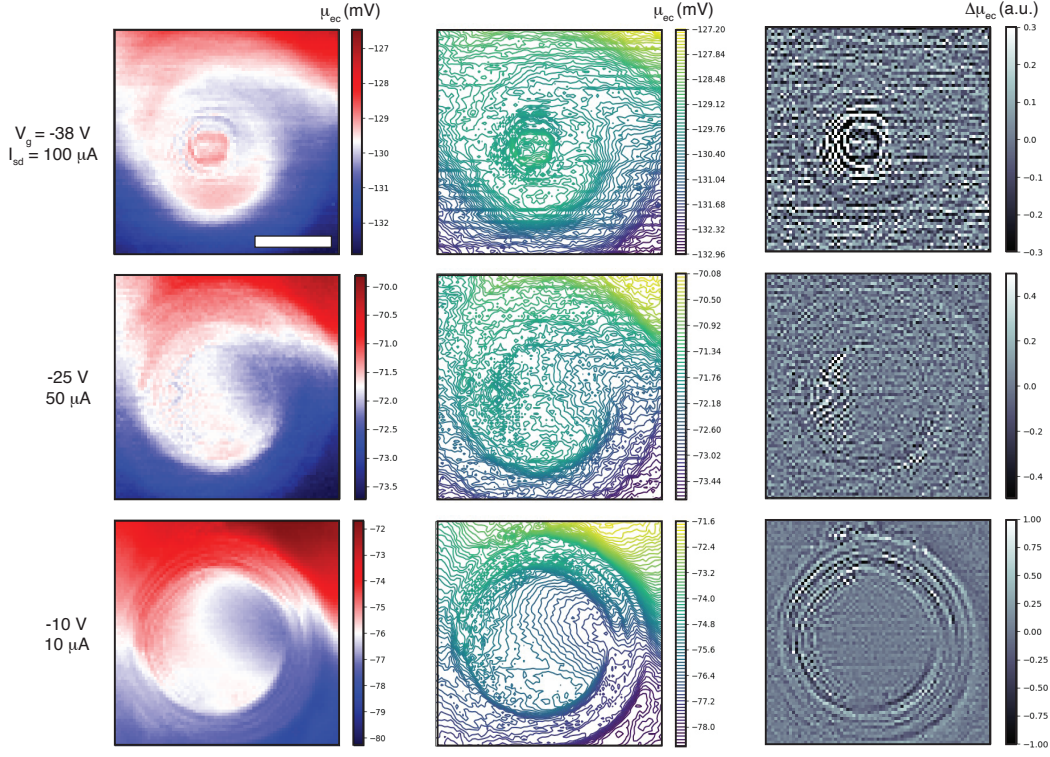


FIG. S7. STP maps at $B = 1$ T in order of decreasing carrier density from top to bottom, plotted alongside the corresponding equipotential contours and Laplacians in order to highlight the inner and outer disk features. The scale bar is 500 nm.

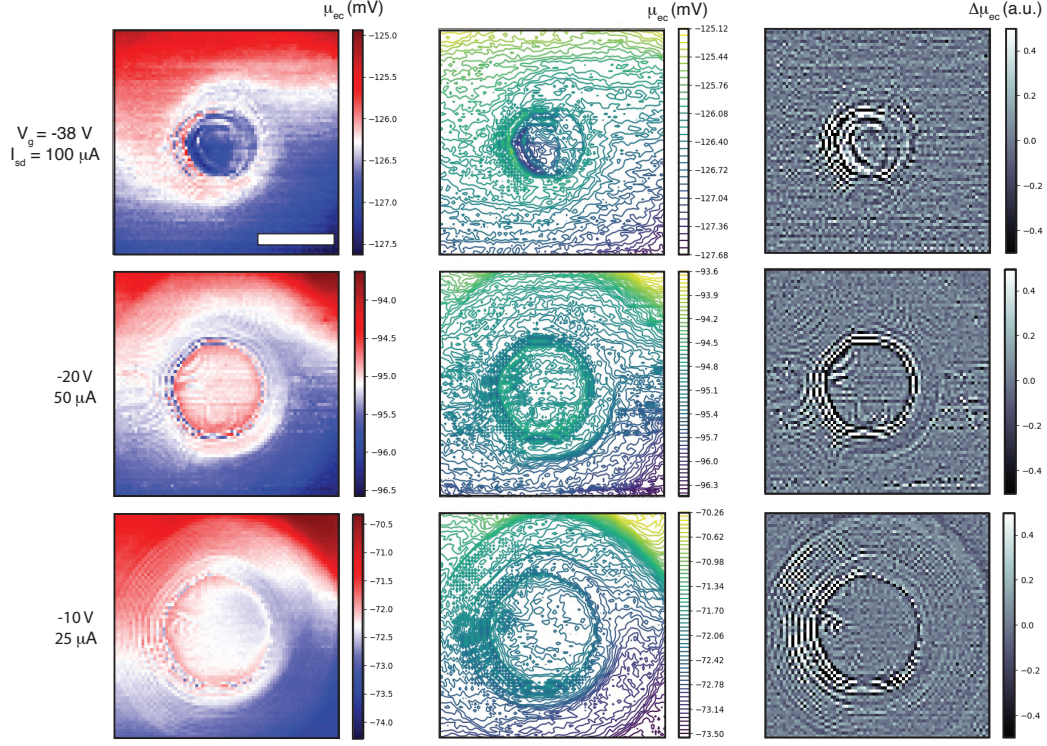


FIG. S8. STP maps at $B = 0.6$ T in order of decreasing carrier density from top to bottom, plotted alongside the corresponding equipotential contours and Laplacians in order to highlight the inner and outer disk features. The scale bar is 500 nm.

Ohmic flow past a realistic barrier potential

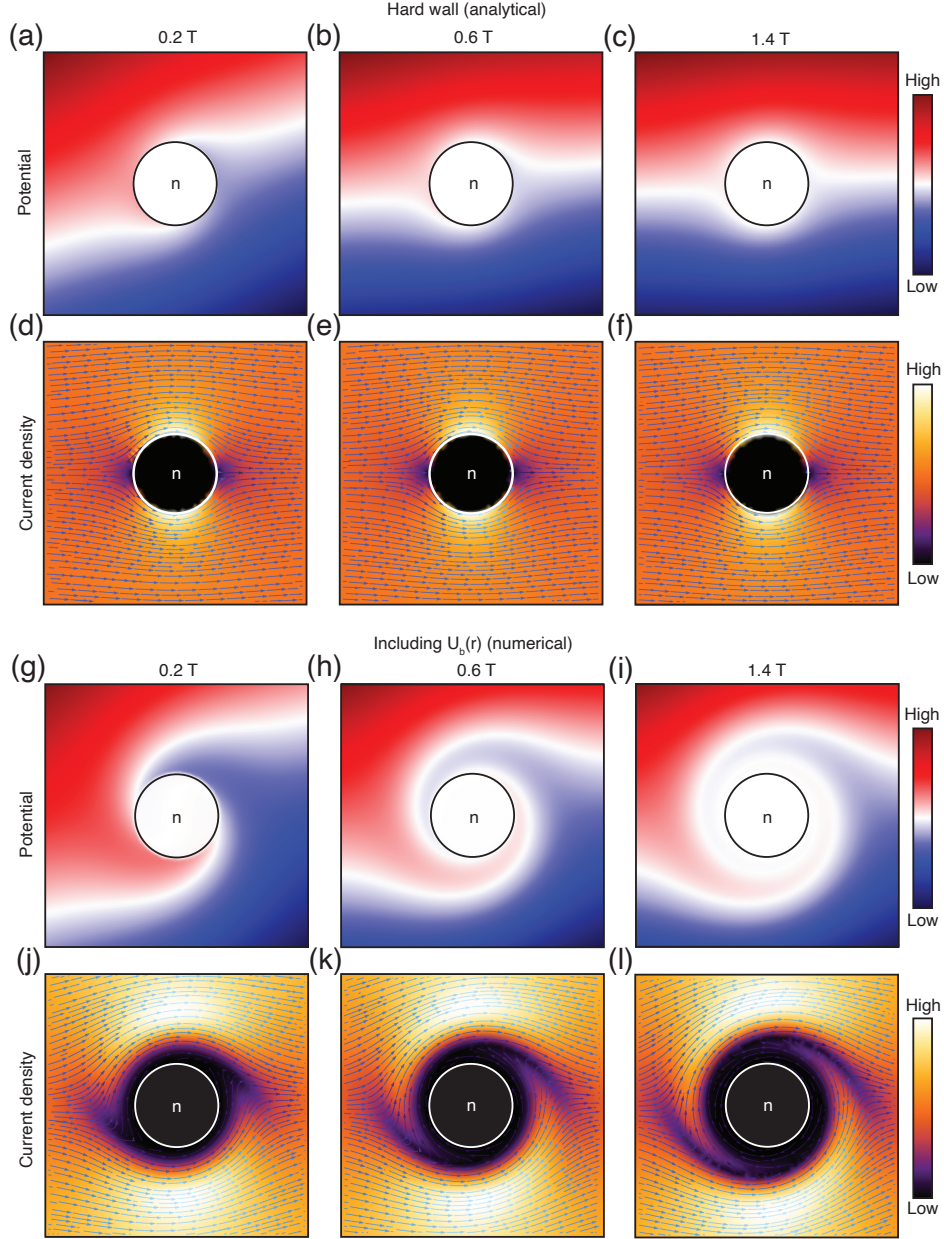


FIG. S9. (a)-(c) Calculated μ_{ec} for Ohmic flow past a circular hard wall in increasing magnetic fields, obtained by using Eq. (2). (d)-(f) Corresponding local current density, which shows no dependence on the magnetic field. (g)-(i) Numerical solution for Ohmic flow past the smooth barrier $U_b(r)$, obtained by solving Eq. (1). (j)-(l) Corresponding current density maps.

For nonzero and constant σ_{xx} , σ_{xy} , Eq. (1) reduces to the Laplace equation, a unique solution of which can be found once suitable boundary conditions are imposed. If the

current is to be uniform and directed along the x -axis far away from the barrier, we get

$$\mu_{\text{ec}}(\vec{r}) \propto -r \cos(\theta + \theta_{\text{H}}) - \frac{r_{\text{pn}}^2}{r} \cos(\theta - \theta_{\text{H}}), \quad (\text{S3})$$

which is the sum of the potential of a constant Hall field (the first term) and a 2D dipole potential, i.e. an LRRD (the second term). Here $\theta = \arcsin(y/r)$ is the polar angle and $r > r_{\text{pn}}$. This solution is plotted in Fig. S9(a)-(c) for fixed l and three magnetic fields relevant to the experiment (such that $\alpha = 0.38, 0.13$ and 0.05 , respectively). However, these solutions do not reproduce spirals with a twist angle larger than 180° or the disk-like features observed at higher B . The corresponding current distribution is shown by the streamlines in Fig. S9(d)-(f).

We contrast these results with Fig. S9(j)-(l), showing the numerical simulations for more realistic profiles of the longitudinal and Hall conductivities using the measured barrier profile of Eq. (S2). The latter reveal a much stronger twisting of the spiral arms (or equipotentials) around the p-n junction, as observed in the experiment. These calculations highlight the importance of taking the correct potential profile $U_{\text{b}}(r)$ into account.

Definition and modeling of the electrochemical potential

The local electrochemical potential $\mu_{\text{ec}}(\vec{r})$ was defined in the main text from the condition of zeroing the STP tunneling current $I_{\text{t}}(\vec{r}) = I_{\text{in}}(\vec{r}) - I_{\text{out}}(\vec{r})$. This empirical definition replaces the usual thermodynamic relation $\mu_{\text{ec}} = \phi - \mu/e$ in terms of course-grained quantities – the electrostatic potential ϕ and chemical potential μ – which are not well-defined on ultrashort length scales probed by STP. Assuming the STM tunneling matrix element is constant and ignoring any tip gating and thermovoltage effects, the current I_{in} due to electrons tunneling into the sample is given by

$$I_{\text{in}}(\vec{r}) \propto \sum_i f_0(E_i + e\mu_{\text{ec}})[1 - f_0(E_i + e\mu_{r,i})]|\psi_i(\vec{r})|^2. \quad (\text{S4})$$

Here $f_0(E)$ is the Fermi function and $\mu_{r,i}$ is the electrochemical potential of the ‘reservoir’ injecting an electron state i with wavefunction ψ_i and energy E_i into the system. The current I_{out} due to electrons tunneling out of the sample is given by a similar expression with μ_{ec} and $\mu_{r,i}$ interchanged. Hence, μ_{ec} satisfies the equation [3, 4]

$$\sum_i [f_0(E_i + e\mu_{\text{ec}}) - f_0(E_i + e\mu_{r,i})]|\psi_i(\vec{r})|^2 = 0. \quad (\text{S5})$$

If the transport across the entire sample is coherent, the role of reservoirs is played by the electric contacts, i.e., the source and the drain, with the electrochemical potentials μ_s and μ_d , respectively. At low temperatures where $f_0(E)$ becomes a step-function, only the states i within the energy window between $-e\mu_s$ and $-e\mu_d$ contribute to the sum. Therefore, $-e\mu_{ec}$ must also be in between these two limiting values. For example, in equilibrium, $\mu_{ec} = \mu_s = \mu_d$. Note that the equilibrium value of $-e\mu_{ec}$ is commonly referred to as the Fermi energy. Out of equilibrium, the Fermi energy does not have a precise meaning. We can choose to define it as, e.g., $E_F = -e(\mu_s + \mu_d)/2$.

One can easily solve Eq. (S5) in the linear-response regime (small bias $\mu_s - \mu_d$) to obtain

$$\mu_{ec} = -\frac{1}{g} \sum_i f'_0(E_i - E_F) |\psi_i(\vec{r})|^2 \mu_{r,i}, \quad (\text{S6})$$

where

$$g(\vec{r}) = -\sum_i f'_0(E_i - E_F) |\psi_i(\vec{r})|^2 = \frac{\partial n(\vec{r})}{\partial E_F} \quad (\text{S7})$$

is the local thermodynamic density of states. If the temperature is low and the potential $U_b(\vec{r})$ acting on electrons in graphene is varying slowly, $g(\vec{r})$ approximately coincides with the LDOS. More precisely, $g(\vec{r}) \simeq g_0(\mu)$, where $g_0(E)$ is the single-particle density of states at energy E in a system with $U \equiv 0$ and

$$\mu(\vec{r}) = E_F - U(\vec{r}) \quad (\text{S8})$$

is the local chemical potential. In quantizing fields, $g_0(E)$ has sharp maxima at the LL energies. In classically strong magnetic fields, function $g_0(E)$ remains smoothly varying, $g_0(E) \simeq C|E|$ with $C = 2/(\pi\hbar^2 v_F^2)$, because LLs overlap.

The thermodynamic relation (S8) can be generalized by replacing E_F with $-e\mu_{ec}$:

$$-e\mu_{ec}(\vec{r}) = \mu(\vec{r}) + U(\vec{r}). \quad (\text{S9})$$

This equation, mentioned in the main text, may apply when quantum effects are unimportant. At the very least, it is restricted to long length scales. Equation (S6) applies at arbitrary length scales.

The assumption of quantum coherence made in the beginning is essential because otherwise describing a nonequilibrium system using wavefunctions ψ_i is not possible. For example, if the coherence length is short, the semiclassical Eq. (1) should be used instead. In our

experiments we likely deal with an intermediate scenario where electrons do not maintain coherence over the entire sample but do so over the small scanned region. We can model this regime by modifying Eqs. (S5) and (S6) as follows. Instead of restricting $\mu_{r,i}$ to the binary choice between μ_s and μ_d , we assign it an intermediate value determined from the solution of the semiclassical Eq. (1) at the boundary of the simulation region that surrounds the p-n junction.

Boltzmann correction

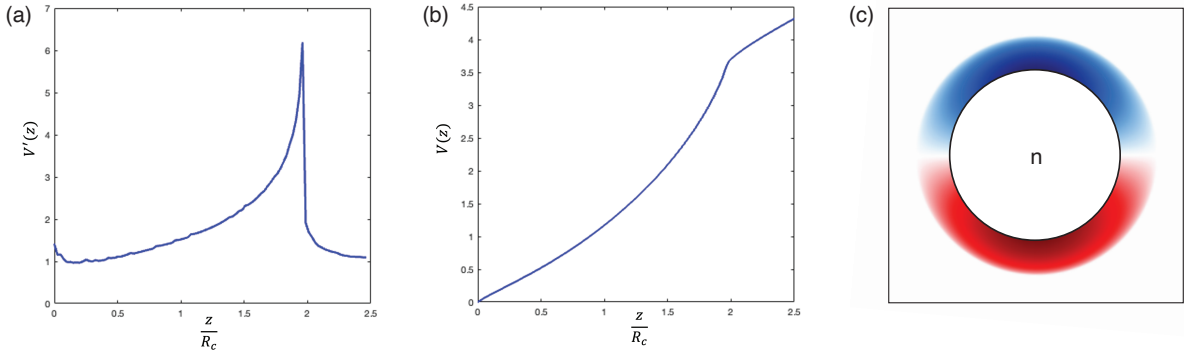


FIG. S10. (a) Electric field $V'(z)$ as a function of distance from the hard wall. A sharp peak appears at $z = 2R_c$. In the bulk $z > 2R_c$ the Hall field E_H is 1 (in units E_x/α for $\alpha = 0.03$). (b) Electrochemical potential as a function of distance from the hard wall. (c) The correction that is added to the Ohmic solution, Eq. (S3), to get the result shown in the main text [Fig. 3(c)].

In Ref. [5] magnetotransport in a long channel of fixed width and uniform equilibrium electron density was considered. In that problem the Ohmic solution to the electrochemical potential is simply $V(y) = -E_H y$, where $y \geq 0$ is the distance from the edge of the channel. Using the Boltzmann kinetic equation the authors of Ref. [5] calculated the profile of the electrochemical potential $V(y)$ which shows a deviation from this Ohmic solution near the edge. We repeated these calculations and the results are shown in Fig. S10. They illustrate that the boundary correction is appreciable only at $y < 2R_c$, as expected. This correction is negative and has a small kink at $y = 2R_c$.

In the problem of the circular barrier the distance from the hard wall is given by $y \equiv$

$r - r_{\text{pn}}$. We construct the boundary correction to the electrochemical potential as follows:

$$\Delta\mu_{\text{ec}}(\vec{r}) = -\frac{\partial}{\partial r} \mu_{\text{ec}}^{\text{LRRD}}(\vec{r}) \Big|_{y=y_0} \times \left[\frac{V(y)}{E_{\text{H}}} + y - \frac{V(y_0)}{E_{\text{H}}} - y_0 \right], \quad (\text{S10})$$

where $\mu_{\text{ec}}^{\text{LRRD}}(\vec{r})$ is given by Eq. (S3). We use this expression in the annulus $r_{\text{pn}} < r < r_{\text{pn}} + y_0$. At $r \geq r_{\text{pn}} + y_0$, we set $\Delta\mu_{\text{ec}} \equiv 0$. We choose a sufficiently large $y_0 = 5R_c$ such that the discontinuity in the radial derivative of $\Delta\mu_{\text{ec}}(\vec{r})$ at $y = y_0$ is negligible. Note that the normalized ratios V/E_{H} in Eq. (S10) are E_{H} -independent.

Quantum transport simulations

In order to account for possible quantum corrections and effects related to the full potential $U_{\text{b}}(r)$ not considered in the Boltzmann model of transport, we numerically calculate the electron wavefunctions within a tight-binding model [6] and use them to construct an approximation of μ_{ec} . A schematic of the tight-binding model is shown in Fig. S11(a). We define a modified honeycomb lattice with the carbon-carbon bond length increased by a factor of 14 ($0.142 \times 14 \text{ nm}$) and hopping energy decreased by a factor of 14 ($2.8/14 \text{ eV}$) [7]. Doing so makes simulating micron-scale systems possible with limited computer memory, while preserving the Fermi velocity and Landau level spectrum relevant to the experiment. The rectangular simulation region is 1000×800 unit cells, each containing two carbon atoms, which corresponds to a physical size of around $3.4 \times 2.7 \mu\text{m}$ — significantly larger than the STP scan window in our measurements ($1.5 \times 1.5 \mu\text{m}$). This region is attached on opposite ends to semi-infinite leads which inject carriers into the system. A magnetic field is introduced by using a lattice Peierls substitution in the Landau gauge, $\vec{A} = By\hat{x}$, which preserves translational symmetry along the x -direction. In the absence of disorder, the eigenfunctions are confined in the y -direction and resemble plane waves in the x -direction. We model carrier scattering with phonons and impurities by assigning a random, onsite disorder potential to each lattice site drawn from a uniform distribution with maximum amplitude U_{d} . The electrostatic barrier is included in the center of the scattering region as an onsite potential using the functional form $U_{\text{b}}(r)$ introduced previously. Lastly, we solve all wavefunctions in the presence of a uniform Hall field in the y -direction for a couple important reasons: without a Hall field, it is difficult to simulate all of the quasibound resonances associated

with $U_b(r)$, which occur at very specific energies and require an impractical energy resolution to handle. With a Hall field, these sharp quasibound resonances broaden significantly and can be included by simulating across a more reasonable energy mesh. Secondly, the Hall field removes LL degeneracy, and the tight-binding wavefunctions can be interpreted in a straightforward, semiclassical way (guiding center motion). The probability density distribution $|\psi_{N,k}(\vec{r})|^2$ of a representative wavefunction we computed is shown in Fig. S11(b). It is consistent with the semiclassical cartoon of Fig. 1(c), where the guiding center of the electron cyclotron orbit traverses the system along an open equipotential contour of $U(\vec{r})$. The nonuniformity of $|\psi_{N,k}(\vec{r})|^2$ seen in Fig. S11(b) is due to elastic intra-LL scattering by the random potential. Because there is no inelastic scattering (e.g. phonon emission) in our tight-binding model, and because $U(\vec{r})$ breaks LL degeneracy, the random potential cannot scatter electrons between modes with different quantum numbers N or k . Therefore, electron backscattering is suppressed and the bulk magnetoresistivity is effectively zero. Additional steps must be taken to include diffusion. This is accomplished through a special approximation: we use an artificially large Hall field to induce elastic inter-LL transitions $N \rightarrow N \pm 1$ [see schematic in Fig. S11(c)] [8, 9]. These transitions involve lateral guiding center shifts by distances up to $2R_c$, and can be activated by setting the Hall field above the threshold $E_H \approx \hbar\omega_c/2R_c$. Here, $\hbar\omega_c = \hbar v_F/R_c$ is the cyclotron gap at the Fermi level. Traditionally, these inter-LL transitions are invoked to explain characteristic magnetoresistance oscillations observed in high-mobility 2D systems [9, 10]. In our case, we use inter-LL scattering as a tool for modeling dissipation in our tight-binding system. With a larger Hall field, the numerical wavefunctions now contain multiple overlapping LLs that fill the scattering region, see Fig. S11(d),(e). For a given energy, Kwant separately outputs the scattering wavefunction associated with modes injected from the left lead (Fig. S11(f), $|\Psi^L(E, r)|^2$) and right lead (Fig. S11(g), $|\Psi^R(E, r)|^2$). In the former, the particle density decreases from the top to bottom of the scattering region, since right moving carriers near the bottom edge tend to be backscattered into the left moving edge modes. In the latter, the density gradient is reversed, since carriers are injected via the left moving edge mode and get scattered into right moving modes (i.e. carriers diffuse from the bottom edge to the top). This nonuniform distribution of carriers can be expressed as

$$|\Psi^{L,R}(E, \vec{r})|^2 = \sum_{\mathcal{E}(N,k)=E} |\Psi_{N,k}(\vec{r})|^2 f_{N,k}^{L,R}(\vec{r}) \quad (\text{S11})$$

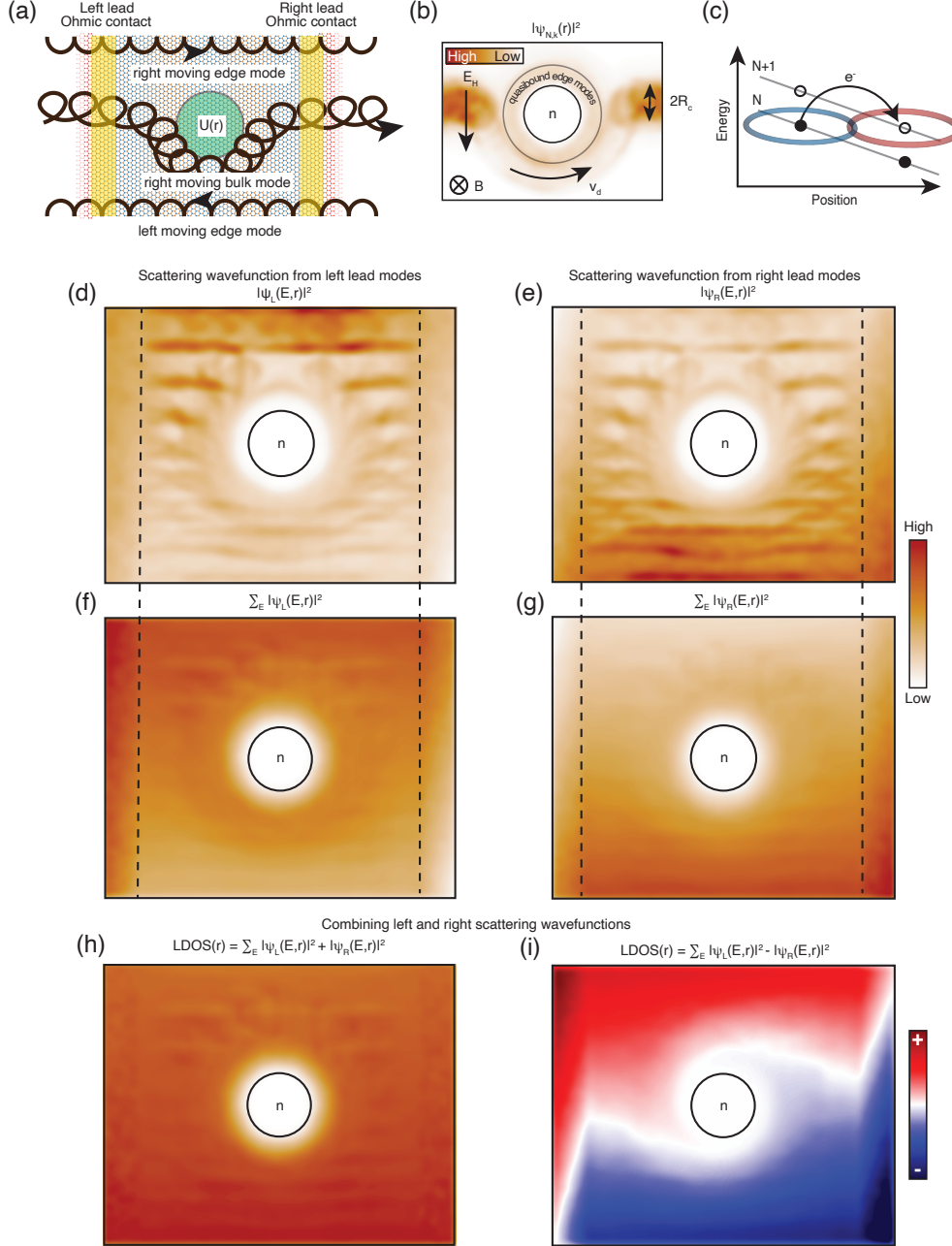


FIG. S11. (a) Schematic of the tight-binding model (not to scale). The magnetic field points into the page, and a uniform Hall field points downwards. (b) Scattering wavefunction in a single LL. (c) Inter-LL scattering occurs when wavefunctions overlap. (d)-(e) Scattering wavefunctions at fixed energy $E = -180$ meV due to modes in the left and right leads, respectively. The dashed black lines mark the edges of the Ohmic contacts. (f)-(g) Summing many scattering wavefunctions, like the ones in (d),(e), in a small energy window around the Fermi level. (h) Adding (f) and (g) together gives the local density of states. (i) Subtracting (f) and (g) gives an approximation of the electrochemical potential μ_{ec} .

where $\mathcal{E}(N, k)$ is the LL dispersion, and $f_{N,k}^{L,R}(\vec{r})$ are the distribution functions for modes injected from the left and right leads. By definition we have $f_{N,k}^L + f_{N,k}^R = 1$, and the local density of states in a narrow energy window around the Fermi level is given by

$$\text{LDOS}(E_F, \vec{r}) = \sum_{E \sim E_F} |\Psi^L(E, \vec{r})|^2 + |\Psi^R(E, \vec{r})|^2 = \sum_{\mathcal{E}(N,k) \sim E_F} |\Psi_{N,k}(\vec{r})|^2 \quad (\text{S12})$$

and shown in Fig. S11(h). The local density of states is uniform away from the electrostatic barrier, and decreases near the p-n junction (as expected for a depletion region). There is no particle density within the n-doped disk, since carriers cannot easily transmit through the p-n junction in a magnetic field [11]. We approximate the electrochemical potential by taking an antisymmetric combination of the left and right scattering states,

$$\mu_{\text{ec}}(\vec{r}) = \sum_{E \sim E_F} |\Psi^L(E, \vec{r})|^2 - |\Psi^R(E, \vec{r})|^2 = \sum_{\mathcal{E}(N,k) \sim E_F} |\Psi_{N,k}(\vec{r})|^2 \delta f_{N,k}(\vec{r}) \quad (\text{S13})$$

and the result is plotted in Fig. S11(i).

Current density distribution

Figure S12 shows the calculated current density from the tight-binding simulations, in addition to the electrochemical plots that were already shown in Fig. 3 of the main text. The local current density is enhanced within the annulus defined by the space between the inner and outer disks. Much like the disk features that we observe in the electrochemical potential, this feature in the current density is a finite size effect caused by the pile-up of cyclotron trajectories that divert around the barrier.

For completeness, we also include additional plots in the low disorder, intermediate regime [Fig. S12(b)], which is not shown in the main text. In this regime the spiral textures and disk features coexist.

Tip gating effects

Local electrostatic gating by the STM tip is an inherent challenge in STM measurements of materials with low carrier density – especially 2D materials – and the authors have previously described the effects of tip gating in previous works [12, 13]. These reports show that tip-induced band bending is strong near the CNP of graphene and becomes less significant

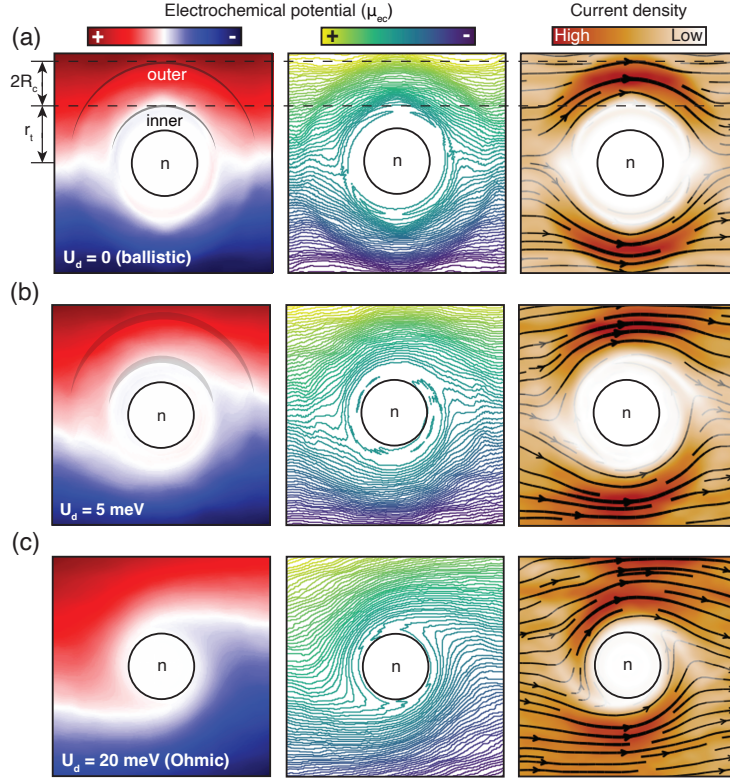


FIG. S12. Calculated electrochemical potential and current density in the (a) ballistic, (b) low disorder, (c) and moderate disorder regimes.

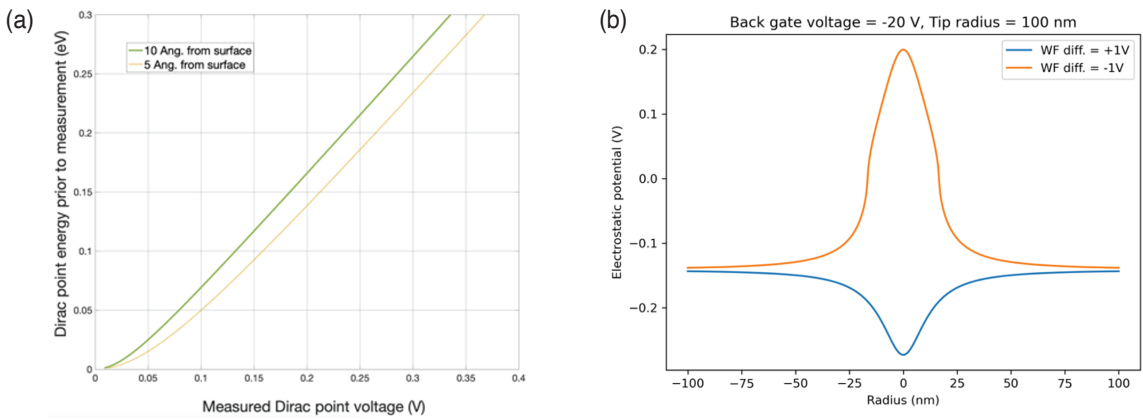


FIG. S13. (a) Shift in the Dirac point energy in the presence of an STM tip (with zero work function offset from the sample). (b) Electrostatic potential on the graphene surface as a function of radius away from a spherically shaped STM tip.

upon finite doping. In order to visualize this effect, we plot in Fig. S13(a) how the Dirac point energy is shifted locally beneath an STM tip, assuming no work function difference. This plot shows that for measured Dirac point voltages of 75 meV and above, the magnitude of tip gating is largely constant. As a result, tip gating within each STP map does not vary appreciably except for very close to the circular p-n barrier - we estimate <50 nm based on the potential profiles of our barriers.

Using similar electrostatic calculations, we can also estimate how the tip forms locally varying potential profiles due to its sharp apex which could, in principle, scatter current locally to distort the observed electrochemical potential profiles. To do this, we modeled the tip as a conducting sphere with a radius of 100 nm, sitting 1 nm above the surface, and with tip-sample work function mismatch of 1 eV. The results of this calculation are shown in Fig. S13(b). Depending on the sign of the work function difference, the tip creates a small p-doped or n-doped region below the STM tip. We note that this potential profile decays to its large-distance value at radii larger than 25 nm, so it will not significantly perturb the flow of current at radii larger than 25 nm. Meanwhile, the features we measure in this work are largely greater than 100 nm. Moreover, the work function mismatch used here is a large overestimation of the expected work function difference; in these experiments the tip is prepared by poking in a Cu(111) surface prior to measurements depositing Cu on the tip apex which is expected to closely match the graphene work function, as described in previous works by others [14]. As a result, the magnitude of the calculated profiles is expected to be 1-2 orders smaller, and thus create a smaller perturbation.

An additional possible effect of tip gating is that when the tip is close to the p-n boundary, the shape of the p-n boundary will be distorted by the tip's potential. As seen in the Fig. S13(b), the tip causes minor potential shifts at lateral distances greater than 25 nm from the tip apex. For electrostatic barriers with a steep potential profile $U_b(r)$, such as the ones studied in this work, tip doping is not expected to change the p-n boundary radius locally by more than 25 nm. This distance is small compared to radius of the unperturbed barrier (> 200 nm), the scan window size (1000 nm), and the cyclotron radii or carriers probed in this work. Thus, such small-scale changes would not significantly distort our

measurements of the electrochemical potential away from the boundary.

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