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A real-time approach for damage identification using hyperchaotic probe and stochastic estimation

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ABSTRACT Among numerous damage identification techniques, those which are used for real-time damage identification have received considerable attention recently. In the current study a real-time damage identification approach is proposed which is applicable both as a vibration-based technique and as a guided-wave technique for structural health monitoring. In fact, the proposed approach makes use of the intrinsic hypersensitivity of hyperchaotic systems to subtle changes in system parameters for identification of damage induced changes in structural systems driven by hyperchaotic dynamics. The proposed approach, as a vibration-based technique, involves applying the augmented state method along with optimal filtering problem to provide simultaneous estimation of the states and parameters of a (possibly) nonlinear structural system driven by a tuned hyperchaotic excitation in order to monitor damage-induced changes. The proposed real-time approach is also expandable to the realm of guided-wave structural health monitoring (SHM) by adapting the method of lines for discretization of the governing partial differential equation (PDE) of hyperchaotic guided waves and using that as a process model in the optimal filtering problem in order to estimate the transmitted hyperchaotic wave and parameters of the model. The extended Kalman-Bucy filter is then used for identification and real-time monitoring of damage-induced changes in parameters of the process model as well as estimating the transmitted waves at measurement points. Numerical simulation using measurements from a finite element model of a cantilever beam shows that the proposed approach with guided hyperchaotic waves is capable of real-time identification of reduction in elastic modulus of an isotropic beam.

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1. INTRODUCTION

Two main strategies are possible for damage identification in structures. The first one which is the active approach needs continuous actuation (or excitation) of monitored structures and real-time measurements and analysis of the resulting response, e.g. acoustic emission. The second strategy which is the passive approach consists of checking the structure periodically and identifying the damage by comparing the initial state to the actual state of the structure. While the passive approach is satisfactory for traditional structures, it is not as desirable for some modern structures, and in particular, “smart structures” which contain active components or layers. For such smart structures, the identification algorithm needs to provide instantaneous updates of the mechanical properties of the structure to the active components to adaptively perform functions of sensing and actuation required. For the online identification of damage, various time-domain approaches have been used in the literature with different degree of success. A few examples include least-squares estimation [1 -3], different filter approaches including the extended Kalman filter [4 - 7], H_∞ filter [8], Monte Carlo filter [9], etc. The Monte Carlo method is capable of dealing with nonlinear systems with even non-Gaussian uncertainties. However, it is computationally expensive due to requiring a large number of sample points. Since the application of least squares estimation (LSE) for nonlinear structural system identification requires displacement and velocity measurements, which may not always be readily available. The extended Kalman filter (EKF) is perhaps the most widely-used vibration-based time-domain techniques for identification of nonlinear systems. While the EKF has good performance when the parameter to be identified is a constant parameter, it is not as successful in identification of changes in time-varying system parameters [10]. A common technique used in the literature for identification of time-varying parameters is an extension of the LSE approach. This technique makes use of a constant [11,12] or time-dependent [13] forgetting factor in LSE. This approach has some drawbacks and shows good performance in some cases; however, it exhibits poor results when the stiffness of the structure has an abrupt change [10]. An adaptive tracking technique based on EKF to identify structural parameters is proposed in [10], which is particularly suitable for tracking the abrupt changes of the system parameters with the purpose of online evaluation of the structural damages. However, the observation equation used in [10] to adopt acceleration measurement assumes the system parameters to be known which is not realistic. Note that to the authors best knowledge the application of filtering problem for real-time damage identification in the literature has been so far limited to vibration-based SHM techniques and discrete structural systems only.

The extended Kalman-Bucy filter is an alternative filtering approach that has been recently reintroduced due to its enhanced capabilities in parameter estimation compared with the extended Kalman filter [14, 15]. On the other hand, an aspect of damage identification which is shown to be crucial from a detectability standpoint is the excitation. When applied as the excitation in some attractor-based damage identification techniques, chaotic and hyperchaotic dynamics due to their intrinsic high sensitivity to subtle changes in the system can often produce better outcome rather than the common stochastic white noise [16 - 22]. In the context of attractor-based damage identification techniques, a hyperchaotic excitation is proven to have an enhanced capability for being an indicator of damage compared with chaotic excitations [21,22].

In the current investigation, a feasible approach for real-time identification of damage is proposed which can be applied both as a guided-wave technique for structural health monitoring (SHM) and as a vibration-based SHM technique. The proposed approach takes advantage of the intrinsic high sensitivity of hyperchaotic dynamical systems to subtle changes in system parameters and combines this advantage with the enhanced estimation capability of extended Kalman-Bucy filter to establish an appropriate tool for real-time damage identification. The proposed approach, as a vibration-based technique, involves applying the augmented state method along with optimal filtering problem to provide simultaneous estimation of the states and parameters of a (possibly) nonlinear structural system driven by a tuned hyperchaotic excitation in order to monitor damage-induced changes. The simulation results show that the proposed approach when used with the extended Kalman-Bucy filter is capable of real-time identification and assessment of damage in nonlinear and hysteretic structures with single or multiple degrees-of-freedom using noise-corrupted measured acceleration response. The current approach is also expandable to the realm of guided-wave SHM for real-time damage identification by applying the method of lines for discretization of the governing partial differential equation (PDE) of guided waves propagation in isotropic media. The governing PDE of propagation of frequency-shifted hyperchaotic guided waves in an isotropic solid after being converted to a set of ODEs constitutes the process model and measurements of the transmitted wave at some equally-spaced points along the structure serves as the observation for the optimal filtering problem.

The extended Kalman-Bucy filter is then used for identification and real-time monitoring of damage-induced changes in parameters of the process model as well as estimating the transmitted waves at measurement points.

This paper is organized as follows: in Section 2, a concise background on optimal filtering problem in general and extended Kalman-Bucy filter in particular, is provided. Then further details is given on tuned hyperchaotic excitations; in Section 3, the application of the current approach to vibration-based SHM is discussed; in Section 4, how the current approach is applicable for guided-wave SHM is studied; in Section 5, in a numerical simulation the proposed approach is applied for online identification and assessment of damage in discrete nonlinear and hysteretic MDOF structures using noise-corrupted measured acceleration response. Also, measurements from a finite element simulation of guided-wave propagation in a cantilever beam are used to numerically verify the capability of the approach for real-time identification of damage-induced reduction in elastic modulus of an isotropic beam.

2. BACKGROUND THEORY

2.1. STOCHASTIC ESTIMATION PROBLEM

In a general parametric identification problem (cf. non-parametric identification) it is assumed that the form of the model is known only approximately due to imperfect knowledge of the dynamical model that describes the motion and/or imperfect knowledge of parameters. The goal is to obtain the best estimate of the state as well as of model parameters based on measured data that has a random component due to observation errors. In addition, a second source of stochastic disturbance typically appears in the state dynamics as so-called process noise. Both the process noise and the measurement noise are assumed to be additive in this paper. The optimal continuous-time filtering problem in the general form considered in this paper can be written as a set of Ito stochastic differential equations as

$$d\mathbf{x}(t) = \boldsymbol{f}(\mathbf{x}(t), \mathbf{a}(t), t)dt + \mathbf{G}(\mathbf{x}(t), t)d\boldsymbol{\beta}(t) \quad (1)$$
$$d\mathbf{z}(t) = \boldsymbol{h}(\mathbf{x}(t), \mathbf{a}(t), t)dt + \mathbf{J}(t)d\boldsymbol{\eta}(t),$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state process, $\mathbf{z}(t) \in \mathbb{R}^q$ is the measurement process, $\mathbf{a}(t) \in \mathbb{R}^r$ is a vector of unknown parameters, $\boldsymbol{f}$ is the drift coefficient (process model), $\boldsymbol{G}$ is the diffusion coefficient, $\boldsymbol{h}$ is the measurement model function, $\mathbf{J}(t)$ is an arbitrary time-varying functions independent of $\mathbf{x}$, and $\boldsymbol{\beta}(t)$ and $\boldsymbol{\eta}(t)$ are independent Brownian motion additive stochastic processes with $E[d\boldsymbol{\beta}(t)] = \mathrm{E}[d\boldsymbol{\eta}(t)] = 0$, $E[d\boldsymbol{\beta}(t)d\boldsymbol{\beta}^T(t)] = \mathbf{Q}dt$ and $E[d\boldsymbol{\eta}(t)d\boldsymbol{\eta}^T(t)] = \mathbf{R}dt$ where $E[\,]$ represents the expectation operator. Note that in this paper the system is considered to be disturbed by additive noise only. Therefore, hereafter we will treat stochastic differential equations with the diffusion coefficient only depending on t and not the process $\mathbf{x}$. Under this condition the filtering problem can also be formulated in terms of the stationary zero-mean Gaussian white noise processes formally defined as $\mathbf{v}(t) = d\boldsymbol{\beta}(\mathrm{t})/dt$, $\mathbf{w}(t) = d\boldsymbol{\eta}(\mathrm{t})/dt$ and differential measurement $\mathbf{y}(t) = d\mathbf{z}(t)/dt$ as [23]

$$\dot{\mathbf{x}}(t) = \boldsymbol{f}(\mathbf{x}(t), \mathbf{a}(t), t) + \mathbf{G}(t)\mathbf{v}(t) \quad (2)$$
$$\mathbf{y}(t) = \boldsymbol{h}(\mathbf{x}(t), \mathbf{a}(t), t) + \mathbf{J}(t)\mathbf{w}(t),$$

where $\mathbf{v}(t)$ and $\mathbf{w}(t)$ are assumed to be both mutually independent and independent from the state and observation with constant covariance matrices of $\boldsymbol{Q}$ and $\boldsymbol{R}$, respectively, i.e. $\mathbf{v} \sim N(0, \boldsymbol{Q})$ and $\mathbf{w} \sim N(0, \boldsymbol{R})$. Here in this paper, the stochastic term $\mathbf{v}(t)$ (the "process noise") functions as an approximation for the influence of the unknown dynamics of the process model. The time evolution of the states of the system and the unknown parameters of the stochastic model are to be identified using measurements of the output corrupted by the measurement noise term $\mathbf{w}(t)$.

The Ito and standard forms in Eqs. (1)-(2) are only equivalent, however, because the process and measurement noise are restricted to be additive and not multiplicative. Also, note that $\mathbf{a}(t)$ can be a constant or time-varying vector and it is assumed to be Heaviside function later in this study. Eq. (2) defines a continuous-time state-space optimal filtering model. The purpose of the optimal continuous-time filtering problem is to recursively obtain estimates of the states and parameters from the mean, median, or mode of the time-varying conditional probability density

$$p(\mathbf{x}(\mathrm{t}) \mid \{\mathbf{y}(\tau) \ : \ 0 \leq \tau \leq t\}). \tag{3}$$

To see how the filtering problem can be represented in the context of system identification, once again consider Eq.(2). As mentioned before, the assumed model of the system consists of the nonlinear function $\boldsymbol{f}$ which is a function of the state vector $\mathbf{x}(t)$ and parameters $\mathbf{a}(t)$. Suppose that $\mathbf{a}(t)$ is unknown but is assumed to be piecewise constant. The Kalman-Bucy filter can be used to simultaneously estimate the states $\mathbf{x}(t)$ and the parameters $\mathbf{a}(t)$. The standard method employs the so-called state augmentation method, in which the parameter vector $\mathbf{a}(t)$ is included in an augmented state vector $\mathbf{X}(t) = [\mathbf{x}(t), \mathbf{a}(t)]^T$ while being constrained to have a predefined rate of change (zero here), i.e.

$$\begin{aligned}\dot{\mathbf{X}}(\mathrm{t}) &= \boldsymbol{\mathcal{F}}(\mathbf{X}, t) + \boldsymbol{\mathcal{G}}(t)\mathbf{v}(t) \\ &= \begin{Bmatrix} \boldsymbol{f}(\mathbf{x}(t), \mathbf{a}(t), t) \\ \mathbf{0} \end{Bmatrix} + \begin{Bmatrix} \mathbf{G}(t) \\ \mathbf{0} \end{Bmatrix} \mathbf{v}(t) \\ \mathbf{y}(t) &= \boldsymbol{\mathcal{H}}(\mathbf{X}, t) + \mathbf{J}(t)\mathbf{w}(t).\end{aligned} \tag{4}$$

The parameter vector $\mathbf{a}(t)$ is assumed to initially have a Gaussian distribution with mean $\mathbf{a_0}$ and covariance $\boldsymbol{P}_0$. Note that there is no noise term in the equation for the unknown parameter dynamics. The reason is that the parameters are already assumed to be stationary. Therefore, the augmented state method along with optimal filtering problem provides a pertinent approach for simultaneous estimation of the state and parameters of a (possibly) nonlinear system from noise-corrupted observations. The next section discusses how to acquire the solution to this optimal filtering problem using extended Kalman-Bucy filter.

2.2. THE EXTENDED KALMAN-BUCY FILTER

The nonlinear optimal filtering problem described via the Ito differential form of Eq. (1) is considered, where the nonlinear process and measurement function are now functions of the augmented state $\mathbf{X}$ (as in Eq. (4)), and $\mathbf{J}(t)$ is the identity matrix. In order for the Kalman-Bucy filter to be applicable to the nonlinear system, the dynamics need to be locally linearized. Rather than linearizing about a reference trajectory, the extended Kalman-Bucy filter employs a linearization about the state estimate itself. It can be derived by taking the expectation of the dynamic model and adding a feedback term consisting of the measurement residual times an (as yet) unknown gain matrix, i.e.

$$d\widehat{\mathbf{X}}(t) = E[\boldsymbol{\mathcal{F}}(\mathbf{X}, t)]dt + \boldsymbol{K}(t)[d\mathbf{z}(t) - E[\boldsymbol{\mathcal{H}}(\mathbf{X}, t)]dt]. \tag{5}$$

Defining the observer error as $\boldsymbol{e}(t) = \mathbf{X}(t) - \widehat{\mathbf{X}}(t)$, the differential observation error is obtained as

$$\begin{aligned} d\boldsymbol{e}(t) &= d\boldsymbol{X}(t) - d\widehat{\boldsymbol{X}}(t) = \boldsymbol{\mathcal{F}}(\boldsymbol{X}, t)dt - \\ &E[\boldsymbol{\mathcal{F}}(\boldsymbol{X}, t)]dt - \boldsymbol{K}(t)[\boldsymbol{\mathcal{H}}(\boldsymbol{X}, t)dt - \\ &E[\boldsymbol{\mathcal{H}}(\boldsymbol{X}, t)]dt] + d\boldsymbol{B}(t) ,\end{aligned} \tag{6}$$

where $d\boldsymbol{B}(t) = \boldsymbol{\mathcal{G}}(t)d\boldsymbol{\beta}(t) - \boldsymbol{K}(\mathrm{t})d\boldsymbol{\eta}(t)$ is a Brownian motion process with

$$E[d\boldsymbol{B}(t)\, d\boldsymbol{B}(t)^{\boldsymbol{T}}] = [\boldsymbol{\mathcal{G}}(t)\boldsymbol{Q}(t)\boldsymbol{\mathcal{G}}(t)^{\boldsymbol{T}} + \boldsymbol{K}(t)\boldsymbol{R}(t)\boldsymbol{K}(t)^{\boldsymbol{T}}]dt. \tag{7}$$

Defining $\widetilde{\boldsymbol{F}}(\mathrm{t})$ and $\widetilde{\boldsymbol{H}}(\mathrm{t})$ to be the Jacobian matrices

$$\widetilde{\boldsymbol{F}}(t) := \left.\frac{\partial \boldsymbol{\mathcal{F}}(\boldsymbol{X}, \boldsymbol{t})}{\partial \boldsymbol{X}}\right|_{\boldsymbol{X}=\widehat{\boldsymbol{X}}} , \qquad \widetilde{\boldsymbol{H}}(t) := \left.\frac{\partial \boldsymbol{\mathcal{H}}(\boldsymbol{X}, \boldsymbol{t})}{\partial \boldsymbol{X}}\right|_{\boldsymbol{X}=\widehat{\boldsymbol{X}}}$$

and linearizing about the current estimate yields

$$\begin{aligned}\boldsymbol{\mathcal{F}}(\boldsymbol{X}, t) &= \boldsymbol{\mathcal{F}}(\widehat{\boldsymbol{X}}, t) + \widetilde{\boldsymbol{F}}(t)(\boldsymbol{X} - \widehat{\boldsymbol{X}}) \\ &\quad + \boldsymbol{r_f}(\boldsymbol{X}, \widehat{\boldsymbol{X}}, t) \\ \boldsymbol{\mathcal{H}}(\boldsymbol{X}, t) &= \boldsymbol{h}(\widehat{\boldsymbol{X}}, t) + \widetilde{\boldsymbol{H}}(t)(\boldsymbol{X} - \widehat{\boldsymbol{X}}) \\ &\quad + \boldsymbol{r_h}(\boldsymbol{X}, \widehat{\boldsymbol{X}}, t)\end{aligned} \tag{9}$$

from which $E[\boldsymbol{\mathcal{F}}(\mathbf{X}, t)] = \boldsymbol{\mathcal{F}}(\widehat{\mathbf{X}}, t) + E[\boldsymbol{r}_f(\mathbf{X}, \widehat{\mathbf{X}}, t)]$ and $E[\boldsymbol{\mathcal{H}}(\mathbf{X}, t)] = \boldsymbol{\mathcal{H}}(\widehat{\mathbf{X}}, t) + E[\boldsymbol{r}_h(\mathbf{X}, \widehat{\mathbf{X}}, t)]$. $\boldsymbol{r}_f(\mathbf{X}, \widehat{\mathbf{X}}, t)$ and $\boldsymbol{r}_h(\mathbf{X}, \widehat{\mathbf{X}}, t)$ represent the remaining higher order terms. Truncating the Taylor series after the first order terms yields the differential observation error as

$$d\boldsymbol{e}(t) = \left[\widetilde{\boldsymbol{F}}(t) - \boldsymbol{K}(t)\widetilde{\boldsymbol{H}}(t)\right]\boldsymbol{e}(t)dt + d\boldsymbol{B}(t). \tag{10}$$

The error covariance matrix can be obtained by differentiating $\mathrm{E}[\boldsymbol{e}(t)\, \boldsymbol{e}(t)^T]$ using the Ito differential rule to obtain

$$d\boldsymbol{P}(t) = \left[\widetilde{\boldsymbol{F}}(t) - \boldsymbol{K}(t)\widetilde{\boldsymbol{H}}(t)\right] \boldsymbol{P}(t)dt + \boldsymbol{P}(t)\left[\widetilde{\boldsymbol{F}}(t) - \boldsymbol{K}(t)\widetilde{\boldsymbol{H}}(t)\right]^{\boldsymbol{T}} dt + \boldsymbol{\mathcal{G}}(t)\,\boldsymbol{Q}(t)\,\boldsymbol{\mathcal{G}}(t)^T dt + \boldsymbol{K}(t)\,\boldsymbol{R}(t)\,\boldsymbol{K}(t)^T dt. \quad (11)$$

The optimal gain matrix $\boldsymbol{K}(\mathrm{t})$ which leads to a minimum variance estimator can be obtained by minimizing the cost function $J = \mathrm{Trace}(d\boldsymbol{P}(t))$ with respect to $\boldsymbol{K}(t)$ as

$$\frac{\partial}{\partial \boldsymbol{K}(\mathrm{t})}\left[\mathrm{Trace}\big(d\boldsymbol{P}(t)\big)\right] = -2\boldsymbol{P}(t)\widetilde{\boldsymbol{H}}(t)^{\mathrm{T}} + 2\boldsymbol{K}(t)\boldsymbol{R}(t)^{\mathrm{T}} = 0, \quad (12)$$

which yields the $\boldsymbol{K}(t)$ matrix as

$$\boldsymbol{K}(t) = \boldsymbol{P}(t)\widetilde{\boldsymbol{H}}^{\mathbf{T}}(t)\boldsymbol{R}(t)^{-1}. \quad (13)$$

Therefore the propagation of the estimate is obtained as

$$d\widehat{\mathbf{X}}(t) = \boldsymbol{\mathcal{F}}\big(\widehat{\mathbf{X}}, t\big)dt + \boldsymbol{K}(\mathrm{t})\left[d\boldsymbol{z}(t) - \boldsymbol{\mathcal{H}}\big(\widehat{\mathbf{X}}, t\big)dt\right], \quad (14)$$

while the following Riccati differential equation is obtained which propagates the error covariance $\boldsymbol{P}(t)$.

$$d\boldsymbol{P}(t) = \widetilde{\boldsymbol{F}}(t)\,\boldsymbol{P}(t)dt + \boldsymbol{P}(t)\widetilde{\boldsymbol{F}}(t)^{\mathrm{T}}dt + \boldsymbol{\mathcal{G}}(t)\,\boldsymbol{Q}(t)\,\boldsymbol{\mathcal{G}}(t)^{\mathrm{T}}dt - \boldsymbol{K}(t)\,\widetilde{\boldsymbol{H}}(t)\,\boldsymbol{P}(t)dt. \quad (15)$$

The estimator so obtained is the extended Kalman-Bucy filter. Unlike the discrete-time extended Kalman filter, the prediction and measurement update steps are combined in the continuous-time extended Kalman filter.

2.3. TUNED HYPERCHAOTIC PROBE

Traditionally, broadband random signals have been widely used for exciting structures. The reason is that the broadband nature of noise ensures a full modal response, ideal for frequency domain approaches to system identification or feature extraction. The motivation for the use of a chaotic signal as the excitation mechanism in damage detection is due to various unique features intrinsic to a chaotic signal. Chaotic signals also tend to possess broadband frequency spectra. However, unlike noise, chaos is deterministic and intrinsically low-dimensional (a stochastic process is infinite-dimensional). In fact many chaotic systems can be as low as three-dimensional when described as a continuous time process. In addition, a chaotic system is defined by a positive Lyapunov exponent (LE) implying extreme sensitivity to small changes in system parameters. Hyperchaos, on the other hand, is defined as chaotic behavior where at least two LEs are positive and thus provides a higher level of sensitivity to changes in parameters of the system. Having all the advantages that make a chaotic signal suitable for being used as an excitation, it is shown in [21,22] that a hyperchaotic signal is even more sensitive to subtle changes in damage severity as a result of the trajectory being permitted to more fully explore the entire phase space. The subtlety of damage-induced changes to a structure further motivates this choice as the mechanism of excitation. Thus, hyperchaotic oscillators can be an alternative excitation mechanism in damage detection when extra sensitivity to damage is required.

In order to better understand the effect of the hyperchaotic dynamic on the damage identification capability of a vibration-based SHM technique, consider a structure driven by a hyperchaotic excitation. The structure can be regarded as a filter affecting the hyperchaotic input signal while the damage manifests itself by changing the design parameters of the filter. Thus the approach makes use of the intrinsic high sensitivity of hyperchaotic systems to subtle changes of the parameters.

However, in order for hyperchaotic excitation to have the best performance the excitation should be tuned for the structure. There are two tuning criteria based on attractor dimensionality. First, the Lyapunov spectrum of the oscillator must overlap that of the structure. This ensures that changes to the LEs of the structure, i.e. by damage, will alter the dimension of the filtered signal. Second, the dominant exponent associated with the oscillator must be minimized for a given degree of overlap in order to maintain the lowest possible dimensionality. By employing the Kaplan-York conjecture in attractor dimensionality, these criteria become

$$\begin{aligned}|\lambda_M^C| > |\lambda_1^L| \\ |\lambda_1^L| > \textstyle\sum_{r=1}^{p} \lambda_r^C,\end{aligned} \qquad (16)$$

where λ_i^C are the LE exponents associated with the M-dimensional hyperchaotic system, λ_j^L are the exponents of the N-dimensional structure, and p is the number of positive Lyapunov exponents of the M-dimensional oscillator.

3. APPLICATION TO VIBRATION-BASED SHM

The parameter estimation technique based on filtering problem described in Section 2.1 in combination with the idea of using a hyperchaotic probe as the excitation (discussed in Section 2.3) makes a real-time damage assessment approach which is applied as a vibration-based SHM technique in this section.

Conventional extended Kalman filter which is basically a discrete sequential filtering technique has already been used for identification of constant unknown parameters. However, in case of time-varying parameters the EKF does not show a good performance [10]. A commonly used approach to overcome this shortcoming is to apply the EKF with a forgetting factor [24] which is a constant factor used to modify the error covariance matrix of the EKF. As a result of applying this forgetting factor, the Kalman gain matrix is amplified by the inverse of the forgetting factor. This approach exhibits only a limited success in real-time tracking of time-varying system parameters. A major drawback of this approach is that if the forgetting factor is small, it has a better tracking capability, but is very sensitive to noise. Conversely, if the forgetting factor is large, the approach is more robust against noise, but shows less tracking capability. Different modified versions of this approach based on using constant or variable (fading) forgetting factors are proposed [10,25] in the literature with different degrees of success.

The approach used in this study makes use of continuous-time filtering technique and hyperchaotic probe for real-time damage identification. The extended Kalman-Bucy filter introduced in Section 2.2 has been shown [14] to provide an enhanced estimation capability over the EKF technique. Also the hyperchaotic probe is expected to show a better performance when used with filtering techniques compared to what random excitations used in [10] do. In order to adopt the filtering problem discussed in Section 2.1 to damage assessment, the process equation in the optimal filtering problem of Eq. (4) is considered as

$$\boldsymbol{f}(\mathbf{x}(t), \mathbf{a}(t), t) = \boldsymbol{\varphi}\big(\mathbf{x}(t), \mathbf{a}(t)\big) + \mathbf{b}\,\boldsymbol{u}(t), \qquad (17)$$

where $\boldsymbol{\varphi}()$ describes the nonlinear structure of interest, $\boldsymbol{u}()$ is the hyperchaotic excitation force, and the constant coupling matrix $\mathbf{b}$ determines which component of $\boldsymbol{u}()$ to be used as the excitation and which degree-of-freedom of the structure is to be excited. Any hyperchaotic nonlinear systems may be used as an excitation. We use a hyperchaotic version of the well-known Lorenz oscillator shown below, i.e.

$$\begin{aligned}\dot{u}_1(t) &= (\sigma(u_2(t) - u_1(t)) + u_4(t))\delta \\ \dot{u}_2(t) &= (ru_1(t) - u_2(t) - u_1(t)u_3(t) - u_5(t))\delta \\ \dot{u}_3(t) &= (u_1(t)u_2(t) - bu_3(t))\delta \\ \dot{u}_4(t) &= (du_4(t) - u_1(t)u_3(t))\delta \\ \dot{u}_5(t) &= (ku_2(t))\delta\end{aligned} \qquad (18)$$

which exhibits hyperchaotic behavior with 3 positive LEs for $\sigma = 10, \rho = 28, \beta = \frac{8}{3}, d = 2, k = 10$ [26]. Note that if we eliminate states u_4 and u_5 from the first 3 states of the oscillator above, the resulting 3-dimensional oscillator is the well-known Lorenz oscillator which exhibits chaotic behavior (one positive LE) for $\sigma = 10, \rho = 28, \beta = \frac{8}{3}$. The bandwidth control parameter δ is used to tune the LEs of the excitation based on the tuning criteria of Eq. (16). Values of δ that are less than unity decrease the bandwidth of the input, while values greater than unity increase the bandwidth. In order to eliminate transient dynamics when using the hyperchaotic oscillator as an excitation, the oscillator is initiated at a point on the attractor. The matrix $\mathbf{b}$ throughout this paper is chosen in a way that the first component u_1 of the chaotic/hyperchaotic Lorenz oscillators is used for the excitation.

Refer to the examples in Section 5.1 and Section 5.2 to see applications of the current vibration-based real-time technique in numerical simulations.

4. APPLICATION TO GUIDED-WAVE SHM

The idea of combining optimal filtering problem with a hyperchaotic probe is extended to guided-wave SHM in this section.

The classical problem of propagation of stress waves in solid media gives rise to the problem of guided and bulk waves. Although guided waves and bulk waves share the same set of partial differential governing equations, these two classes of wave are fundamentally different. Bulk waves travel in the bulk of material and away from boundaries. Guided waves however propagate along an elongated structure while guided by its boundaries. Mathematically, the introduction of boundary conditions to the problem of bulk waves leads to the guided waves problem. Thus, the solution to the governing equations in guided waves must satisfy some physical boundary conditions in addition to the governing equations. In contrast, there are no boundary conditions that need to be satisfied by the proposed solution in the case of bulk waves. Additionally, waves can be categorized according to the relative configuration of the direction of propagation and the direction of particle oscillations. In that sense, waves can be either longitudinal or shear. Longitudinal waves occur when the direction of particle oscillation is in the direction of wave propagation while shear waves are formed when the direction of particle oscillation is perpendicular to the direction of wave propagation. Obviously, the oscillation particles in shear waves can be either perpendicular to the plane of wave propagation which is called vertical (or transverse) shear wave or in the plane of wave propagation which is called horizontal (or normal) shear waves.

So far, the proposed technique for real-time damage identification was only applicable to discrete structural systems and used low-frequency vibration excitations. Next, we will extend this technique to continuous structural systems and use high-frequency guided-waves. In a thin isotropic and homogeneous linear elastic plate-like structure, the elasto-dynamics equation of waves, regardless of their modes, can be described (using Cartesian tensor index notation) as

$$\mu\,\nabla^2\mathbf{u} + (\lambda+\mu)\ \nabla(\nabla.\mathbf{u}) + \rho\mathbf{f} = \rho\ddot{\mathbf{u}} \tag{19}$$

where $\mathbf{u}$ and $\mathbf{f}$ are the displacement and body force, and ρ, μ and λ are the density, shear modulus, and Lamé constant, respectively. By using Helmholtz decomposition and writing the displacement vector of particles of an isotropic media (away from boundaries) in terms of an irrotational displacement and a pure rotational displacement, the elasto-dynamic equation in the absence of body force can be decomposed into an equation for propagation of dilatational waves (or longitudinal waves) as

$$(\lambda\ + 2\mu)\frac{\partial^2 u}{\partial x^2} = \rho\frac{\partial^2 u}{\partial t^2} \tag{20}$$

and an equation for propagation of distortional waves (or shear waves) as

$$\mu\frac{\partial^2 u}{\partial x^2} = \rho\frac{\partial^2 u}{\partial t^2} \tag{21}$$

In order to extend the proposed real-time damage identification approach to (longitudinal or shear) wave propagation, the optimal filtering problem needs to be applied to a system of particles in a continuous medium instead of a discrete mass-spring-damper system as used before. Therefore, the elasto-dynamic equation of wave propagation needs to replace the equation of motion of MDOF systems in the optimal filtering problem. The main problem that arises here is that the governing equation of guided waves is a PDE that cannot be used with filtering problem. To solve this problem we take advantage of a method called method of lines [27,28] to convert the governing PDE of guided waves to a set of ODEs that can be used with the filtering problem.

The method of lines is a semi-analytical technique originally developed by mathematicians that basically involves discretizing a given PDE in one or two dimensions while using analytical solution in the remaining dimensions. For instance, consider the governing PDE of longitudinal (pressure wave) wave as described in Eq.(20). The displacement u has a temporal and a spatial dimension, i.e. $u = u(x,t)$. By discretizing the spatial domain of u into N intervals and using finite-difference discretization technique we will have

$$(\lambda + 2\mu)\frac{\partial^2 u(x,t)}{\partial x^2} = \rho \frac{\partial^2 u(x,t)}{\partial t^2} \Rightarrow \tag{22}$$

$$\begin{cases} (\lambda + 2\mu)(\frac{u_0(t) - 4u_1(t) + u_2(t)}{h^2}) = \rho \frac{d^2 u_1(t)}{dt^2} \\ (\lambda + 2\mu)(\frac{u_1(t) - 4u_2(t) + u_3(t)}{h^2}) = \rho \frac{d^2 u_2(t)}{dt^2} \\ \vdots \\ (\lambda + 2\mu)(\frac{u_{N-2}(t) - 4u_{N-1}(t) + u_N(t)}{h^2}) = \rho \frac{d^2 u_{N-1}(t)}{dt^2} \end{cases}$$

where u_i's are displacements at discretization points and h is the discretization length. Therefore, the governing PDE of wave propagation can be written as a set of $N-1$ ODEs for internal discretization points. Defining the state-space vector $\mathbf{U}(t) = [u_1(t), \ \ldots, \ u_{N-1}(t), \dot{u}_1(t), \cdots, \dot{u}_{N-1}(t)]^T$, the optimal filtering problem for the governing equation of wave propagation can be written as

$$\begin{aligned} \dot{\mathbf{U}}(t) &= \boldsymbol{A}(\mathbf{a}(t))\mathbf{U}(t) + \boldsymbol{B}(\mathbf{a}(t))\,\boldsymbol{\mathcal{U}}(t) \\ &\quad + \mathbf{G}(t)\mathbf{v}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{h}(\mathbf{U}(t)) + \boldsymbol{J}(t)\boldsymbol{w}(t) \end{aligned} \tag{23}$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are coefficient matrices obtained based on Eq. (20) and are functions of the unknown parameter $\mathbf{a}(t)$, and $\boldsymbol{\mathcal{U}}(t)$ is the vector of displacement boundary conditions in longitudinal direction. Assuming that the hyperchaotic excitation is exerted in terms of displacement at the boundary point $u_0(t)$ and further assuming that the first component $\mathcal{u}_1$ of the hyperchaotic Lorenz oscillators of Eq. (18) is used for the excitation, $\boldsymbol{\mathcal{U}}(t)$ can be written as

$$\boldsymbol{\mathcal{U}}(t) = [\mathbf{0}_{1\times(N-1)}, \mathcal{u}_1(t), \mathbf{0}_{1\times(N-3)}, u_N(t)]^T \tag{24}$$

This real-time guided-wave SHM approach is further illustrated in a numerical simulation in Section 5.3.

5. SIMULATION RESULTS

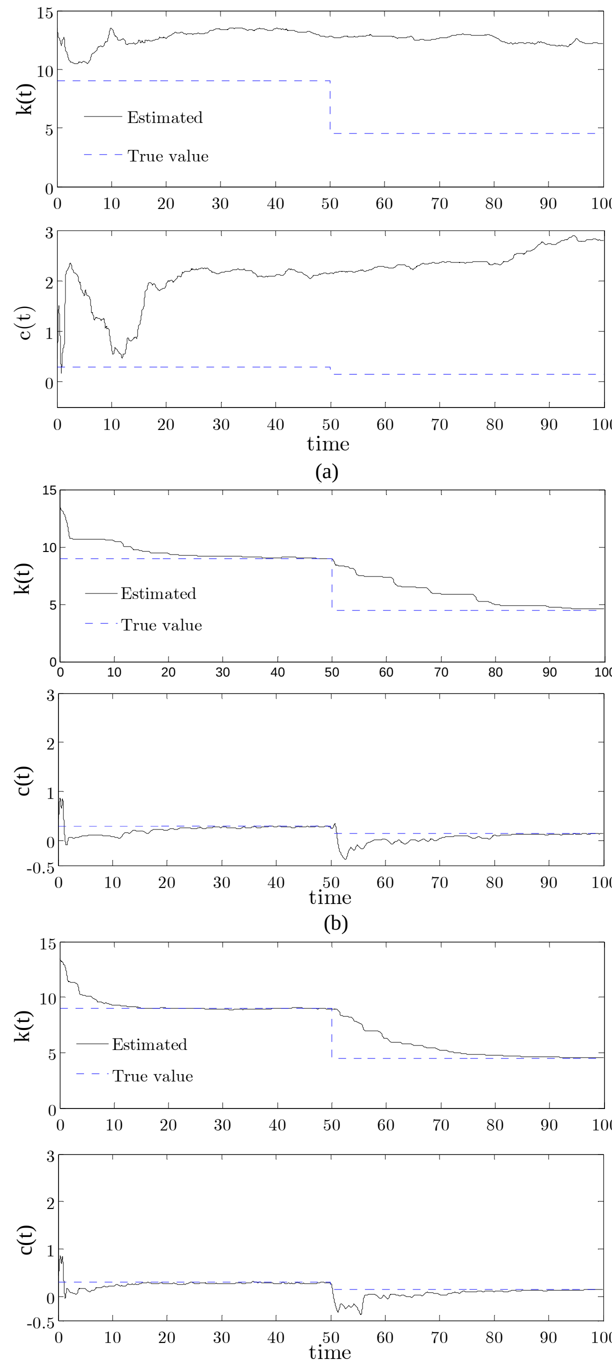
5.1. S-DOF HYSTERETIC NONLINEAR STRUCTURE

Consider a single degree of (SDOF) nonlinear hysteretic Bouc–Wen system subject to the excitation $f_{exc}(t)$

$$\mathrm{m}\,\ddot{x}(t) + \mathrm{c}\,\dot{x}(t) + \mathrm{k}\,r(t) = f_{exc}(t) \tag{25}$$

where $r(t)$ is the Bouc–Wen hysteretic component with

$$\dot{r} = \dot{x} - \beta|\dot{x}\,||r|^{\alpha-1}r - \gamma\dot{x}|r|^{\alpha} \tag{26}$$



(c)

Figure 1. Comparing damage identification with a) random, b) chaotic and c) hyperchaotic excitations in nonlinear system of Eq. (25) using extended Kalman-Bucy filter (damage defined as a 50% stiffness and damping reduction at $t = 50s$). $\boldsymbol{Q} = 1 \times 10^{-8}\boldsymbol{I}, \boldsymbol{R} = 1 \times 10^{-8}\boldsymbol{I}$

The system parameters $\mathrm{m} = 1$, $\mathrm{c} = 0.3$, $\mathrm{k} = 9$, $\beta = 2$, $\gamma = 1$, $\alpha = 2$ are chosen for the simulation. Considering $\mathbf{z} = [x, \dot{x}, r]^T$, the $\boldsymbol{\varphi}$ function in Eq. (17) that forms the process function $\boldsymbol{f}$ of the filtering problem for this system is

$$\boldsymbol{\varphi}(\mathbf{z}(t)) = \left\{ \begin{matrix} z_2(t) \\ (1/\mathrm{m})(-\mathrm{k}\, z_3(t) - \mathrm{c}\, z_2(t)) \\ z_2(t) - \beta |z_2(t)||z_3(t)|^{\alpha-1} z_3(t) - \gamma z_2(t)|z_3(t)|^{\alpha} \end{matrix} \right\}, \tag{27}$$

Time-varying damage is implemented as a 50% abrupt reduction in the stiffness and damping coefficients of the system at time $t = 50s$. The mass of the system is assumed to be known throughout this simulation. The filtering sequence is initiated with values of the state and parameters (k, c) which are 50% deviated from the true values. Three types of excitation $\mathbf{u}(t)$ including white noise, chaotic Lorenz excitation and hyperchaotic Lorenz excitation (Eq. (18)) are applied to the system under identical measurement and process noise covariance $(\boldsymbol{R}, \boldsymbol{Q})$. The standard deviation of the random excitation is chosen to be equal to the RMS of the hyperchaotic excitation. The measurement function $\boldsymbol{h}$ is considered to be the identity function i.e. both displacement x and r and velocity $\dot{x}$ are directly measured. The extended Kalman-Bucy filter is used for real-time identification of the stiffness k and the damping coefficient c of the system. The value of $\delta = 0.542$ can be shown to satisfy the tuning criteria for both the chaotic Lorenz oscillator and hyperchaotic Lorenz oscillator of Eq. (18) and is used for this simulation. The results of the identified parameters with each of the three excitations are shown in Figure 1. As is clear from the figure, in the case of random excitation the change in system parameters is not sensed. However, when chaotic and hyperchaotic excitations are applied the approach successfully identifies the change. Note that in the case of hyperchaotic excitation the filter converges to the true value of the parameter faster than for the case of chaotic excitation.

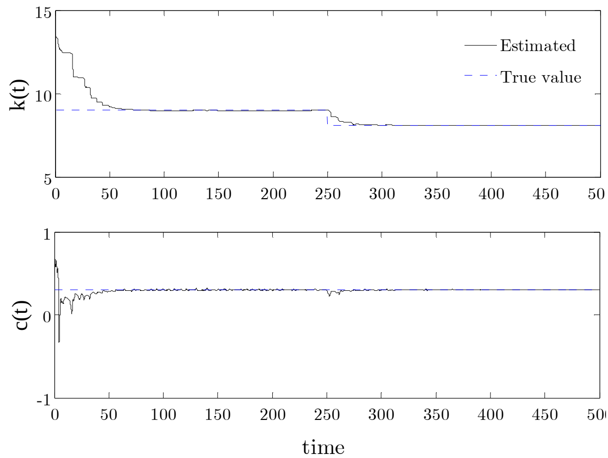


Figure 2. Identification of 10% change in the stiffness of the nonlinear hysteretic system of Eq. (25) from acceleration measurements (Eq. (28)) using hyperchaotic excitation and extended Kalman-Bucy (EKB) filter ($\boldsymbol{Q} = 1 \times 10^{-4}\boldsymbol{I}, \boldsymbol{R} = 1 \times 10^{-4}\boldsymbol{I}$).

In the second simulation the proposed approach is applied for real-time identification of a 10% stiffness reduction in the hysteretic system of Eq. (25). The system parameters are considered as mentioned previously and the hyperchaotic Lorenz oscillator of Eq. (18) is used for the excitation. The value of $\delta = 0.1$ is used for this simulation which can be shown to satisfy the tuning criteria of Eq. (18) for the hyperchaotic Lorenz oscillator and the nonlinear system of Eq. (25). Since measurements of displacement r and x and velocity $\dot{x}$ may not always be readily available, a more common acceleration measurement is considered here. Acceleration measurements in this simulation are provided by using the measurement function $\boldsymbol{h}$ based on the Eq. (25) as

$$\boldsymbol{h}(\mathbf{z}(t), t) = \frac{1}{\mathrm{m}}\left(f_{\mathrm{exc}}(t) - \mathrm{c}\,\dot{x}(\mathrm{t}) - \mathrm{k}\, r(t)\right). \tag{28}$$

Note that the excitation force in Eq. (28) is assumed to be easily measurable via force transducers. The filtering sequence is initiated with values of the state and unknown parameters (k, c) which are 50% deviated from the true values. As is clear from Figure 2, the approach is capable of real-time identification of the 10% stiffness reduction at time $t = 250s$ with good accuracy and fast convergence from measurements of acceleration in the presence of noise.

Although measuring acceleration is more realistic and practical than measuring both velocity $\dot{x}$ and displacement r and x, the $\boldsymbol{h}$ function that is used for measuring acceleration is still not quite realistic for some modern structures. The arguable part is that the values of system parameters m, k and c used in the $\boldsymbol{h}$ function are assumed to be known *a priori*. This is only realistic in the case that the mechanical properties of the structure can be measured or identified before the occurrence of damage. However, if the structure under consideration is a smart structure, then the identification algorithm needs to provide instantaneous updates of the mechanical properties of the structure to some embedded or layered actuators in order for the structure to adaptively perform functions of sensing and actuation. Therefore, the measurement function of Eq.(28) is not applicable to a smart structures. Consequently, in the third simulation the measurement function $\boldsymbol{h}$ is modified to incorporate the estimated values of the system parameters instead of the true values. Assuming the vector of unknown parameters in Eq. (2) to be composed of parameters m, k and c, i.e. $\mathbf{a} = [\mathrm{m}, \mathrm{k}, \mathrm{c}]$, the modified measurement function is

$$\boldsymbol{h}(\mathbf{z}(t), \mathbf{a}(t), t) = \frac{\mathbf{1}}{a_1(\boldsymbol{t})}\left(f_{exc}(t) - a_3(t)\,\dot{x}(t) - a_2(t)\,r(t)\right), \quad (29)$$

where $\mathbf{a}(t)$ is the vector of unknowns which is identical to what used in the augmented state $\mathbf{X}(t)$ in Eq. (4) when forming the process function $\boldsymbol{\mathcal{F}}(\mathbf{X}, t)$ in the process of using the EKB filter. The EKB filter estimates are thus simultaneously used in the measurement function $\boldsymbol{h}$ to relate the measured acceleration to the states of the system ($\dot{x}$, x and r). Note that in this simulation m, k and c are all assumed to be unknown.

Again, the value of $\delta = 0.1$ is used for this simulation and the filtering sequence is initiated with values of the state and parameters (m, k and c) 50% deviated from the true values. Figure 3 shows the simulation results. As is clear from the figure, in this simulation a stiffness reduction of 10% at time $t = 250$ is accurately identified online in the nonlinear hysteretic system without any prior knowledge of the system parameters and by sole measurement of the acceleration response and the excitation force. In fact, the proposed technique first accurately identifies the values of the parameters from noise-corrupted measurements of the acceleration response within the first 250 seconds of excitation, and then successfully monitors the change in the system by identifying the parameter that has changed, the amount of change, and the instant of occurrence of the change. Therefore, the proposed adaptive identification technique is capable of real-time sensing of mechanical properties (m, k and c) in a S-DoF nonlinear smart structure.

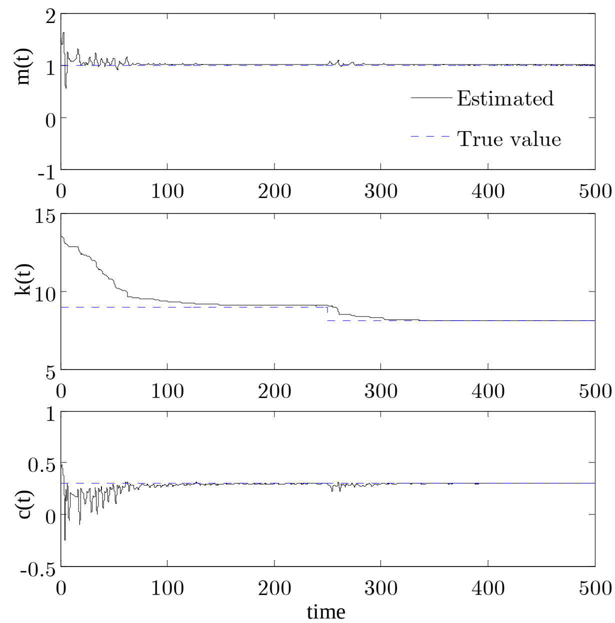


Figure 3. Identification of 10% change in the stiffness of the nonlinear hysteretic system of Eq. (25) from acceleration measurements (Eq. (29)) using hyperchaotic excitation and extended Kalman-Bucy (EKB) filter ($\boldsymbol{Q} = 1 \times 10^{-4}\boldsymbol{I}, \boldsymbol{R} = 1 \times 10^{-4}\boldsymbol{I}$).

5.2. FOUR-STORY SHEAR-BEAM STRUCTURE

Consider an idealized four-story linear shear-beam type building with floor masses m_i, inter-story stiffnesses k_i, and inter-story viscous damping coefficients c_i where $i = 1, \ldots, 4$. The structure is modeled with a linear discrete spring–mass–damper system where the first spring is connected to the ground. The masses, spring stiffnesses and damping coefficients forming the $\boldsymbol{M}$, $\boldsymbol{K}$ and $\boldsymbol{C}$ matrices are set to $m_i = 5, k_i = 8$ and $c_i = 0.5$. In this example the mass matrix $\boldsymbol{M}$ which denotes the floor masses is assumed to be known and the vector of unknowns is only composed of stiffness k_i and damping coefficients c_i. Therefore, $\boldsymbol{K} = \boldsymbol{K}(\mathbf{a}(t))$, $\boldsymbol{C} = \boldsymbol{C}(\mathbf{a}(t))$. Considering the state-space vector $\mathbf{z} = [\mathbf{x}(t), \dot{\mathbf{x}}(t)]^{\mathrm{T}}$, the $\boldsymbol{f}$ function of Eq. (17) for this system is the linear function

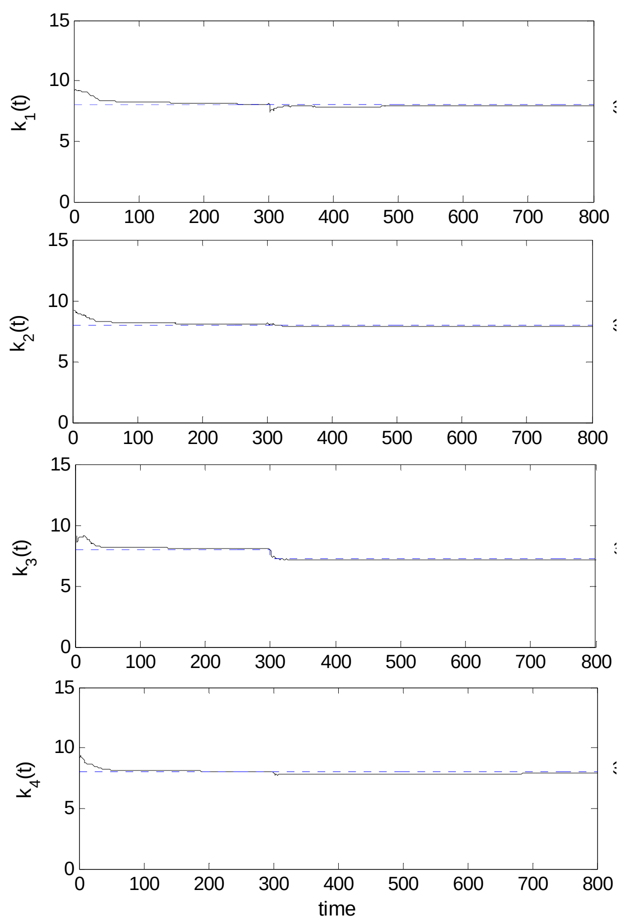
$$\boldsymbol{f}(\mathbf{z}(t), \mathbf{a}(t), t) = \boldsymbol{A}(\mathbf{a}(t))\mathbf{z}(t) + \mathbf{b}\,\mathbf{u}(t) = \begin{bmatrix} \mathbf{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K}(\mathbf{a}(t)) & -\boldsymbol{M}^{-1}\boldsymbol{C}(\mathbf{a}(t)) \end{bmatrix} \mathbf{z}(t) + \mathbf{b}\,\mathbf{u}(t) \quad (30)$$

where $\mathbf{u}$(t) is the response of the hyperchaotic oscillator as shown for example in Eq. (18) and the coupling matrix $\mathbf{b}$ is selected in a way to use u_1 as the excitation which is applied to the fourth mass of the system. The value of parameter δ is selected as $\delta = 0.1$ which can be shown to satisfy the tuning criteria of Eq (16). Damage is introduced as a 10% stiffness reduction in the third spring of the system occurring at time $t = 300s$. It is assumed that only the excitation force and the acceleration of the masses are measured. Thus, since the states of the system consist of velocity $\dot{\mathbf{x}}$ and displacement $\mathbf{x}$, an appropriate measurement function $\boldsymbol{h}$ is required to enable acceleration measurement. The measurement function below enables the technique to be applied when only the excitation force f_{exc} and acceleration are measured.

$$\boldsymbol{h}(\mathbf{z}(t), \mathbf{a}(t), t) = \boldsymbol{M}^{-1}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ f_{\text{exc}}(t) \end{bmatrix} - \boldsymbol{C}(\mathbf{a}(t))\,\dot{\mathbf{x}}(\mathrm{t}) - \boldsymbol{K}(\mathbf{a}(t))\,\mathbf{x}(t) \right). \quad (31)$$

where $\boldsymbol{M}$ is the known mass matrix and $\mathbf{a}(t)$ is the time-varying vector of unknowns which is composed of k_i and c_i. Note that in the above measurement equation the estimates of the system parameters (except for $\boldsymbol{M}$ which is known) are used. This measurement equation thus enables acceleration measurement without *a priori* information about the unknown system parameters.

The filtering sequence is initiated with values of the states and unknown parameters 20% deviated from the true values. The system is excited for 800 seconds by the hyperchaotic Lorenz excitation and the parameters are identified using the current approach. The real-time values of all 8 unknown parameters of the 4-DoF system are depicted in Figure 4. As is clear from the figure, the approach first successfully identifies all 8 unknown parameters of the system within the first 300 seconds. Then the approach successfully identifies 10% stiffness reduction in the third spring of the system. Upon the occurrence of damage, the identified values of some parameters experience a disturbance without losing convergence. However, given enough time they converge to the new true values. The identified stiffness of the third spring clearly monitors the 10% reduction at time $t = 300s$.



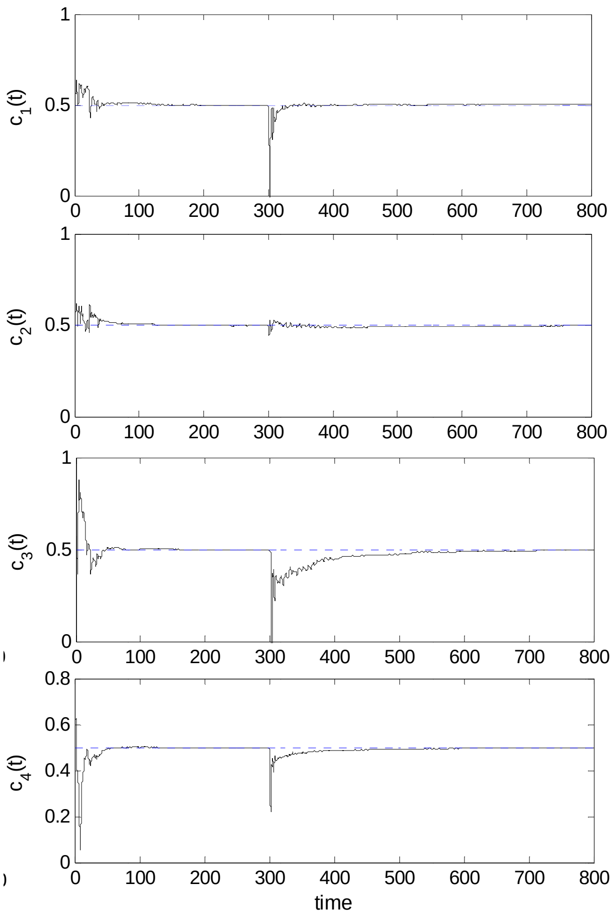


Figure 4. Identification of 10% change in the stiffness 4-DoF linear system of Eq. (30) from acceleration measurements (Eq. (31)) using hyperchaotic excitation and extended Kalman-Bucy (EKB) filter ($\boldsymbol{Q} = \mathbf{1} \times \mathbf{10}^{-4}\boldsymbol{I}, \boldsymbol{R} = \mathbf{1} \times \mathbf{10}^{-4}\boldsymbol{I}$).

5.3. FINITE ELEMENT BEAM MODEL AND HYPERCHAOTIC WAVES

Consider a cantilever beam with a uniform cross-section made of isotropic elastic material with a time varying elastic modulus $E_x(t)$. Damage is introduced as 10% reduction in elastic modulus of the beam at time $t = 20$ millisecond. The proposed approach is applied for damage identification through first identifying the true value of the elastic modulus from a 20% deviated initial guess and then monitoring the elastic modulus in real-time to assess the amount of reduction. To this end, the PDE of wave propagation in the cantilever beam is converted to a set of ODEs using a 10-point equally-spaced discretization and the method of lines as shown in Figure 5. By using $\delta = 1000$, the frequency content of the hyperchaotic oscillator of Eq. (18) is shifted by a few kilohertz (Figure 6) and the first component of the oscillator, $u_1(t)$, is applied as actuation at the free end of the beam. The hyperchaotic wave is applied at the free end of the beam in longitudinal direction and the transmitted wave measured at 9 internal measurement points along the beam are used as observation in the optimal filtering problem.

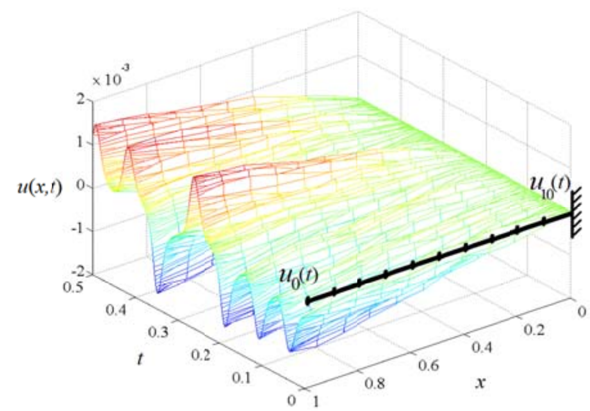


Figure 5. Discretization of partial differential equation of wave propagation in a cantilever beam using the method of lines.

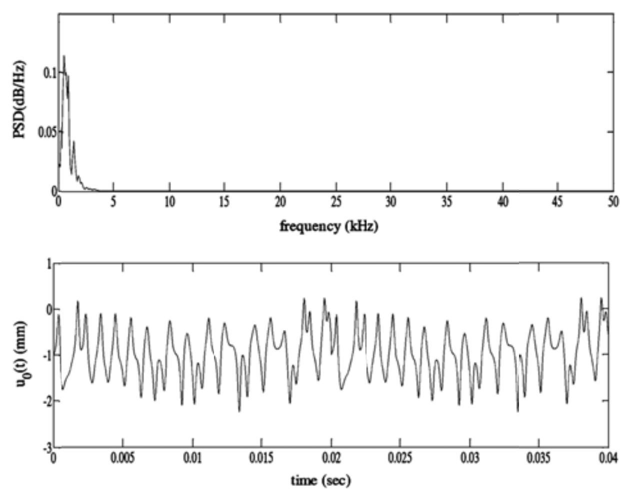


Figure 6. Frequency-shifted hyperchaotic waveform ($\delta = 1000$) applied at the free end of the cantilever beam.

In order to simulate the measured transmitted waves at measurement points along the beam, a finite element (FE) model of the beam with a time-varying elastic modulus is developed in Abaqus CAE finite element software. The FE model of the cantilever beam is excited at the free end in longitudinal direction for 40 ms using the hyperchaotic waveform as shown in Figure 6 and the transmitted waves at nodes corresponding to the measurement points 1 to 9 (Figure 7) are collected. The elastic modulus of the FE model experiences a 10% reduction at time $t = 20$ ms. The collected displacement data after adding a Gaussian noise is used as the simulated measurements for the optimal filtering problem.

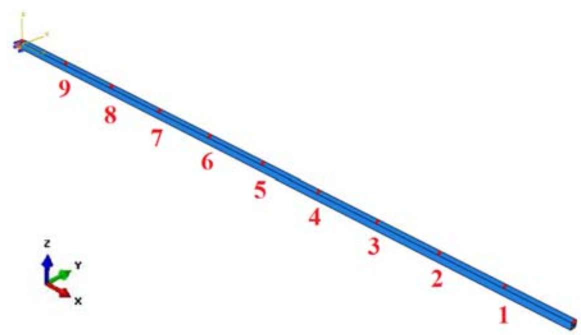


Figure 7. Measurements of the transmitted wave simulated using a FE model in Abaqus/CAE

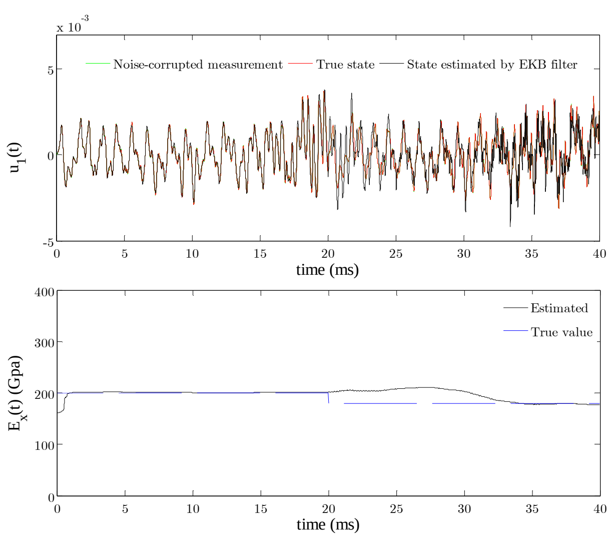


Figure 8. Identification of 10% change in the elastic modulus of a beam continuous system using hyperchaotic wave and extended Kalman-Bucy (EKB) filter ($\boldsymbol{Q} = 2 \times 10^{-6}\boldsymbol{I}, \boldsymbol{R} = 1 \times 10^{-10}\boldsymbol{I}$)

The optimal filtering problem as described in Eq. (23) along with the extended Kalman-Bucy filter are employed to estimate all the 18 states of the problem (displacement and velocities of the transmitted wave at measurement points 1 to 9) as well as the elastic modulus of the cantilever beam. The estimated transmitted wave (displacement) at the first measurement point, $u_1(t)$, along with the estimated parameter, $E_x(t)$ obtained using the proposed approach are depicted in Figure 8. As seen in the figure, the proposed approach first successfully identifies the elastic modulus of the beam from a 20% deviated initial guess within the first 20 milliseconds, and then successfully identifies 10% reduction in the elastic modulus of the beam. Also, a good estimate of the transmitted wave is acquired by the proposed approach.

6. CONCLUSIONS

The current study combines hyperchaotic excitation, which has been previously shown to produce improved outcome when applied as the excitation in some attractor-based damage identification techniques, with the stochastic estimation technique of extended Kalman-Bucy filter, which has been recently reintroduced due to its enhanced estimation capabilities over similar filtering techniques. As a result, a novel real-time approach for identification of damage in structural systems is developed that can be used in smart or self-healing structures for real-time identification of damage. The proposed real-time approach is shown to be applicable both as a vibration-based technique and as a guided-wave technique for structural health monitoring. The current technique is also capable of monitoring changes in the identified parameters by determining the parameter that has changed, the amount of change, and the instant of occurrence of the change.

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