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PARTON FRAGMENTATION MODEL FOR INCLUSIVE PION SPECTRA IN ELECTRON-POSITRON ANNIHILATION

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ABSTRACT

The inclusive pion spectra in e^+e^- annihilation are analyzed in the parton fragmentation model with nonvanishing transverse momenta and with heavy partons. The nonscaling phenomena in the small x region are explained quite satisfactorily in this model.

The recent experimental results at SPEAR in the hadronic inclusive annihilation of electron-positron pair,[1] in particular, the constancy of $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ at the center-of-mass energy above 5 GeV and the jet behavior of the hadrons observed at 7.4 GeV, appear to provide strong evidences in favor of the Bjorken scaling in the spin ½ parton fragmentation model.[2] Within the model, however, the center-of-mass energy of $\sqrt{s} = 7.4$ GeV does not seem to be large enough to warrant the scaling over the entire range of

x $(= 2p_\pi/\sqrt{s})$, as evidenced by the sharp decline in the angular distribution parameter $\alpha(x)$ in the region of $x < 0.5$. At this energy, the pion energy in the region $x < 0.5$ is not sufficiently large to satisfy the condition of exact scaling, i.e., the colinearity between the parton momentum and the momentum of the observed pion. The step-like rise in R between $\sqrt{s} = 3.5$ GeV and 4.0 GeV implies that a new quark or new quarks, heavier than the (p,n,λ) quarks, are produced copiously above 4 GeV. The rise in $s(d\sigma/dx)$ in the small x region may be attributed partly to pair production of heavy partons.

The purpose of this note is to explain inclusive hadron cross sections by introducing a transverse momentum distribution into the fragmentation model, together with a nonvanishing parton mass. The angular distribution is examined as a function of x and s. We find that the nonscaling phenomena in the small x region are explained satisfactorily by the nonvanishing transverse momentum and the mass of partons.

The differential cross section for the inclusive pion annihilation in the e^+e^- center-of-mass frame is given by

$$\frac{d\sigma}{dE_\pi d\Omega} = \frac{\alpha^2}{2s^2}|\vec{p}|\left[2\bar{F}_1(x,s) + \frac{1}{2}\beta^2\sqrt{x^2 + \frac{4m_\pi^2}{s}}\,\sin^2\theta(1 + P^2\cos 2\phi) \times \bar{F}_2(x,s)\right], \quad (1)$$

where β is equal to $|\vec{p}|/E_\pi$ with $\vec{p}$ being the pion momentum, θ the polar angle of $\vec{p}$ from the beam direction, ϕ the azimuthal angle measured from the normal to the plane of the storage ring, and P the beam polarization normal to the plane of the ring. The θ distribution is obtained from (1) as

$$s \frac{d\sigma}{dx\, d\cos\theta} = \frac{1}{2}\pi\alpha^2\beta x\, \bar{F}_1(x,s)\left\{1 + \alpha(x,s)\cos^2\theta\right\}\Big/\left\{1 + \alpha(x,s)\right\}, \tag{2}$$

where

$$\alpha(x,s) = -\beta^2 x\, \bar{F}_2(x,s)\Big/\left\{4\bar{F}_1(x,s) + \beta^2 x\, \bar{F}_2(x,s)\right\} . \tag{3}$$

The integral over θ is carried out to give

$$s\frac{d\sigma}{dx} = \frac{1}{3}\alpha^2\beta\pi x\, \bar{F}_1(x,s)\left[\left\{3 + \alpha(x,s)\right\}\Big/\left\{1 + \alpha(x,s)\right\}\right] . \tag{4}$$

Although measurement has been done in the region of $|\cos\theta| \leq 0.6$ at SPEAR, the large transverse polarization has enabled us to determine $\alpha(x,s)$ at $\sqrt{s} = 7.4$ GeV through the ϕ dependence of $d\sigma/d\Omega$ as given in (1).

We introduce transverse momentum and parton mass into the conventional model of parton fragmentation. Following the standard calculation with p_t and M included, we obtain the structure functions for the inclusive pion production as[3]

$$\bar{F}_1(x,s) = \sum_i \frac{Q_i^2}{8\pi^2}\int dP_T^2 \sqrt{\frac{s}{s - 4P_T^2 - 4M_i^2}}\left(1 - \frac{2P_T^2}{s}\right) f_i(x,s,\vec{p}_t) ; \tag{5a}$$

$$x\bar{F}_2(x,s) = -\sum_i \frac{Q_i^2}{4\pi^2}\int dP_T^2 \sqrt{\frac{s - 4P_T^2 - 4M_i^2}{s}} \times\left(1 - \frac{2P_T^2}{s - 4P_T^2 - 4M_i^2}\right) f_i(x,s,\vec{p}_t) , \tag{5b}$$

where $\vec{p}_t$ is the component of the pion momentum transverse to the parton momentum $\vec{P}$, $\vec{P}_T$ the component of the parton momentum transverse to the pion momentum $\vec{p}$, Q_i the electric charge of the $\underline{i}$th parton, and $f_i(x,s,\vec{p}_t)$ the distribution function of the $\underline{i}$th parton that emits a

pion of momentum $\frac{1}{2}x\sqrt{s}$ and transverse momentum $\vec{p}_t$. Note that P_T and p_t are related to each other through $P_T = p_t\sqrt{(s - 4M_i^2)/sx^2}$. If $f_i(x,s,\vec{p}_t)$ is independent of energy $\sqrt{s}$ and $M_i = 0$ and the $\vec{p}_t$ dependence is like $\delta(p_t^2)$, the structure functions scale precisely.[4] Without detailed knowledge of the distribution function, we make a minimal assumption on it: $f_i(x,s,p_t)$ factorizes as

$$f_i(x,s,\vec{p}_t) = f_i(x,s)\exp(-b_i p_t^2) . \tag{6}$$

The semi-inclusive electroproduction $e^- + p \to e^- + \pi + X$ [5] suggests the Gaussian $\vec{p}_t$ distribution with $b_i \simeq 5\ \text{GeV}^{-2}$.

According to the observed energy dependence of e^+e^- annihilation cross sections, heavy fermion pairs appear to be produced above 3.5 GeV. They may be new heavy quarks, heavy leptons, or both. What we know from experiment at this moment is that the effective mass of particles carrying new degrees of freedom is about 1.8 GeV and that the ratio R rises from $\simeq$2.5 at 3 GeV to $\simeq$5.5 at and above 4.8 GeV. The ground states of "charmed" hadrons[6] decay through weak interactions, and so do heavy leptons, emitting neutrinos.[7] The dynamical mechanism of pion fragmentation for these particles may be different from that for the light quarks. For the purpose of the present phenomenological analysis, however, we treat both heavy quarks and heavy leptons as heavy partons.

Data are available on the inclusive pion spectra at $\sqrt{s} = 3.0$, 3.8, 4.8, 6.2, and 7.4 GeV. At 3.0 GeV, only the light quarks are produced and their masses are presumably small enough for the scaling to hold except for the transverse momentum effect. Setting their masses to zero and choosing $b = 5\ \text{GeV}^{-2}$ for all of p, n, and λ, we have carried out the intregration over P_T^2 in (5a) and (5b).

Owing to the assumption (6), the x distribution due to $f_i(x,s)$ factorizes out of the integral. It is worthwhile to point out that (5a) and (5b) increase as s increases with x fixed since the upper limit of the integral over P_T^2, or equivalently over p_t^2 increases. The rate of increase is most significant at small values of x, while at large values of x the upper limit is practically infinite so that the increase is invisible or even a slight decrease occurs in $s(d\sigma/dx)$ at very high energies owing to other kinematical corrections due to M and p_t. Choosing $f_i(x,s)$ to be common for all of p, n, and λ, we have determined the distribution function $f_\ell(x,s)$ of the light quarks so as to fit the measured inclusive cross section at $\sqrt{s}$ = 3.0 GeV. The result is plotted in Fig. 1a. Statistical errors are rather small, but there may be systematic errors due to acceptance efficiency in detection. In accord with the parton assumption on the light quarks, we assume this $f_\ell(x,s)$ to be the s-independent distribution function of the light partons.

The energy $\sqrt{s}$ = 3.8 GeV is on the way to the upper plateau in R where heavy partons are created with low velocities. Not only the factorization assumption (6), but also the dynamics of parton model itself are in doubt for such slow partons. We therefore leave the data at 3.8 GeV out for the time being. At 4.8, 6.2, and 7.4 GeV, R stays almost constant after the rise by about three units. To avoid complexity, we assume that the masses of all heavy partons are 1.8 GeV and their distribution functions are the same. The rise in $s(d\sigma/dx)$ at small values of x has turned out to be partly kinematical in origin, but largely due to the additional contribution of heavy partons. We have determined the distribution function $f_h(x,s)$ of the heavy

partons so as to fit the measured $s(d\sigma/dx)$ at 4.8, 6.2, and 7.4 GeV. In integrating over P_T^2 in (5a) and (5b) for the heavy partons, we have set $b = 5\ \mathrm{GeV}^{-2}$ in common with the light partons. The function $f_h(x,s)$ has been plotted in Fig. 1b for each energy.[8] It is more localized near $x = 0$ and steeper in slope at small values of x than that of the light partons. The same procedure has been followed in calculating $f_h(x,s)$ by setting heavy parton mass and transverse momenta of both light and heavy partons to be zero. The function $f_h(x,s)$ thus determined has been plotted in Fig. 1c for the sake of comparison. We see clearly that $f_h(x,s)$ with nonzero p_t and M is less energy independent than that with $p_t = M = 0$. The trend is more significant in the small x region where the apparent nonscaling effect in $s(d\sigma/dx)$ is largest. At large values of x, errors accumulate in the difference between two small numbers, $s(d\sigma/dx)_{\text{measured}}$ and $s(d\sigma/dx)_{\text{light partons}}$. It can be said that the distribution function of the heavy partons is also energy independent within some errors above $\sqrt{s}$ = 4.8 GeV.[9] We regard this approximate scaling of $f_h(x,s)$ as a strong support for the parton fragmentation model. The function $f_h(x,s)$ determined from the data at 3.8 GeV lies with large errors close to those of 4.8, 6.2, and 7.4 GeV, provided that it is rescaled properly according to the increment in the ratio R.

With the distribution functions of the light and heavy partons given, we have estimated the inclusive cross section at $\sqrt{s}$ = 30 GeV, the highest energy to be available at PEP, on the assumption that no more new partons be created above 7.4 GeV (see Fig. 2a). As is anticipated, a sharp rise appears towards $x = 0$ as the center-of-mass energy increases.

We are able to calculate the angular distribution parameter $\alpha(x,s)$ to compare with the data at 7.4 GeV (see Fig. 2b). The behavior of $\alpha(x,s)$ is more sensitive to the transverse momentum distribution than to the heavy parton mass M. For the sake of comparison, $\alpha(x,s)$ has been plotted for $b = 1$ and 10 GeV^{-2}. Although data are not available, $\alpha(x,s)$ has been calculated and plotted at $\sqrt{s} = 3.0$ GeV and 30 GeV for $b = 5$ GeV (see Fig. 2c).[10] It is reassuring to see that the angular distribution parameter $\alpha(x,s)$ with $b = 5\ GeV^{-2}$ and with our parton distribution function falls very close to the data points at 7.4 GeV.

We conclude from the present analysis that the parton fragmentation model works quite well below and above the heavy parton production threshold if nonvanishing transverse momentum and heavy parton masses are properly taken into account. The nonscaling phenomena at small values of x in the inclusive pion cross section may be explained entirely by the transverse momentum and the heavy parton mass.

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FOOTNOTES AND REFERENCES

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† Supported in part by the U. S. Energy Research and Development Administration.

1. R. Schwitters, in Proceedings of International Symposium on Lepton-Photon Interactions at High Energies, Stanford, 1975 (to be published).

2. S. D. Drell, D. J. Levy, and T. M. Yan, Phys. Rev. D1, 1617 (1970); J. D. Bjorken, Proceedings of SLAC Summer Institute on Particle Physics, 1973, Vol. 1, p. 1; J. D. Bjorken and B. L. Ioffe, SLAC-PUB-1467 (1974).

3. In carrying out the calculation, we have set only the fraction x of the parton energy be equal to the pion momentum, rather than both energy and momentum of the parton be related individually to those of the pion by x as is done in the conventional model.

4. In the scaling limit, we obtain $2\bar{F}_1(x) = -x\bar{F}_2(x)$, which is the Callan-Gross relation for the inclusive annihilation.

5. J. T. Dakin et al., Phys. Rev. D10, 1401 (1974).

6. We call all of hadronic states with a new quantum number as "charmed" hadrons.

7. M. L. Perl, Lecture given at Institute of Particle Physics Summer School, McGill University, June 1975; SLAC-PUB-1592.

8. Since the smallest value of x measured at $\sqrt{s} = 3.0$ GeV is 0.07, $f_h(x,s)$ can also be determined only at and above $x = 0.07$.

9. Systematic errors in detection efficiency depend on the pion momentum. At a given value of x, therefore, data points at different center-of-mass energies could be subject to corrections of different magnitudes. The systematic errors have not yet been finally determined, but the dispersion of three data points at each x in Fig. 1b may turn out to be less than the systematic errors.

10. The calculated values for the angular distribution parameter $\alpha(x,s)$ at 3.0 GeV are fairly large at large values of x. Since most of the pions have small values of x, a severe cut has to be made to determine $\alpha(x,s)$ experimentally above $x = 0.6$. See R. Hollebeek, Ph.D. Thesis, LBL-3874, May, 1975 (unpublished).

FIGURE CAPTIONS

Fig. 1. The distribution functions of partons. The unit of the ordinates is μb-GeV^2.

(a) $f_\ell(c,s)$ of the light partons, p, n, and λ. The data are available starting at $x = 0.07$ and ending at $x = 0.84$. The curve drawn here is smoothed out between these two points.

(b) $f_h(x,s)$ of the heavy partons with mass = 1.8 GeV and $b = 5$ GeV^{-2}.

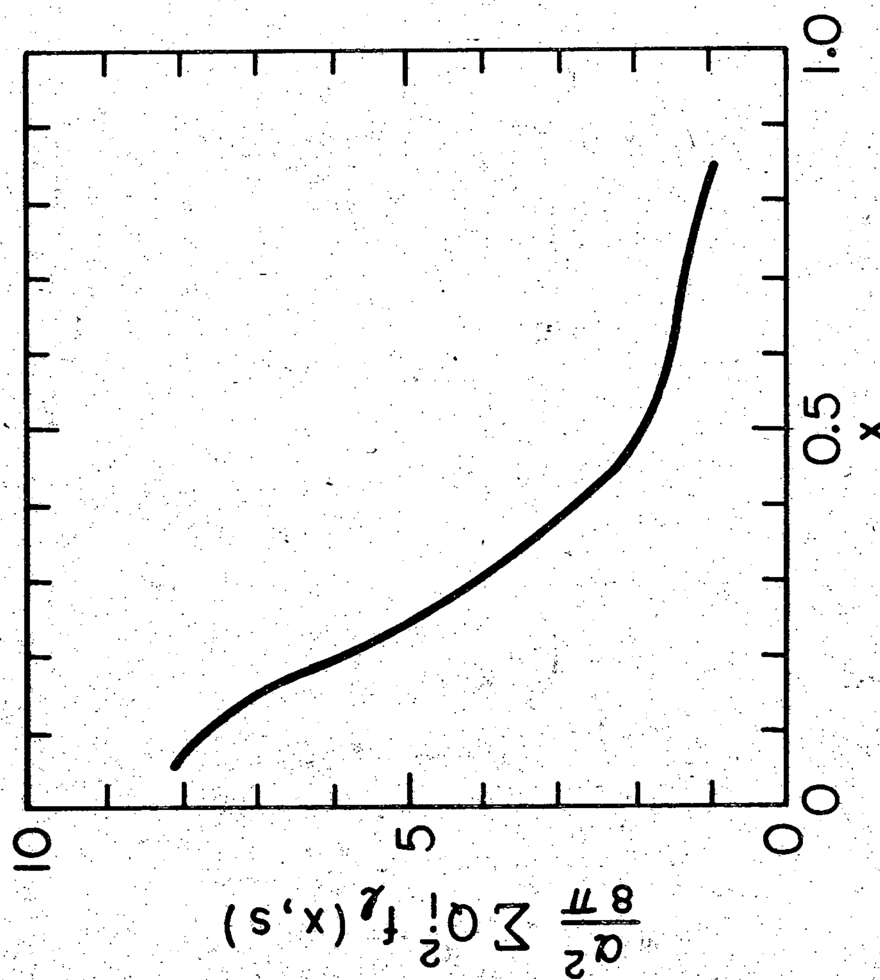
(c) $f_h(x,s)$ of the heavy partons with $M = p_t = 0$, normalized as $f_h(x,s,p_t) = f_h(x,s)\delta(p_t^2)$. Three data points overlap above $x = 0.20$.

Fig. 2. The inclusive cross section $s(d\sigma/dx)$ and the angular distribution parameter $\alpha(x,s)$.

(a) $s(d\sigma/dx)$ at $\sqrt{s} = 30$ GeV calculated with $f_\ell(x,s)$ and $f_h(x,s)$ given in Fig. 1a and 1b, respectively. The curves at 3.0, 3.8, 4.8, 6.2, and 7.4 GeV are those smoothed out data points.

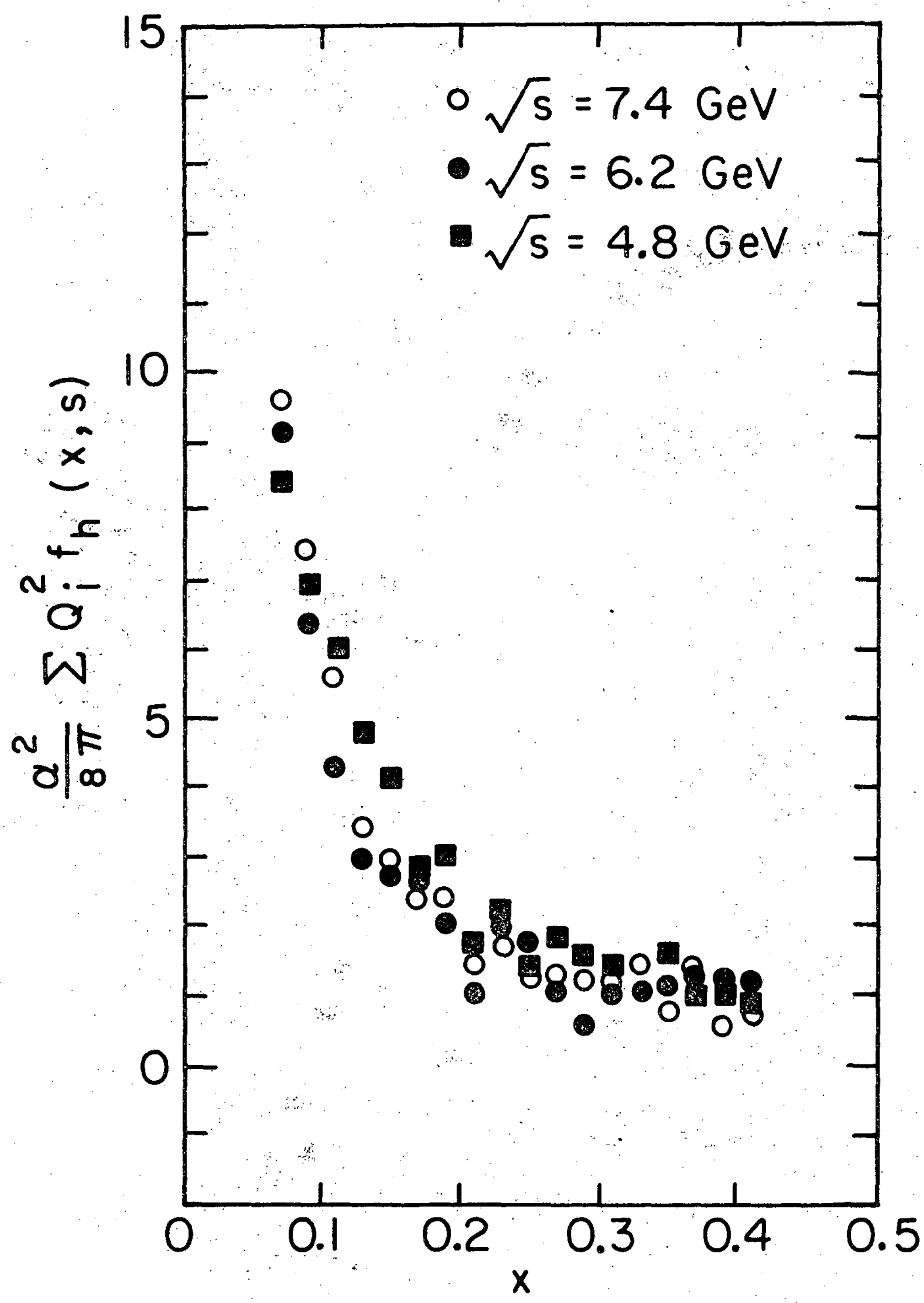
(b) $\alpha(x,s)$ at 7.4 GeV with three different values of transverse momentum distribution, $b = 1$, 5, and 10 GeV^{-2}. The data are taken from Ref. 1.

(c) $\alpha(x,s)$ with $b = 5$ GeV^{-2} at three different values of the center-of-mass energy.



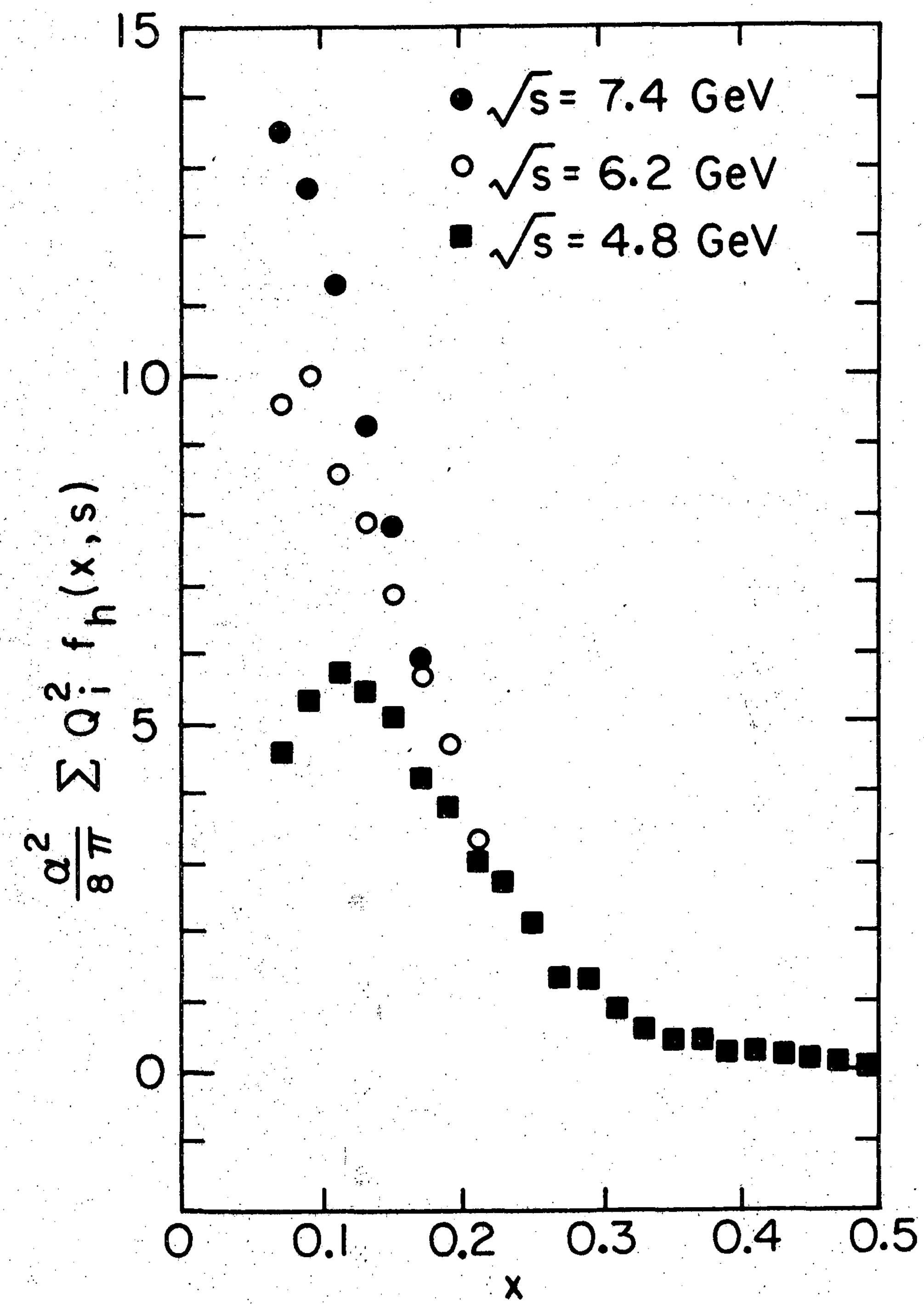
XBL7510-8580

Figure 1 (a)



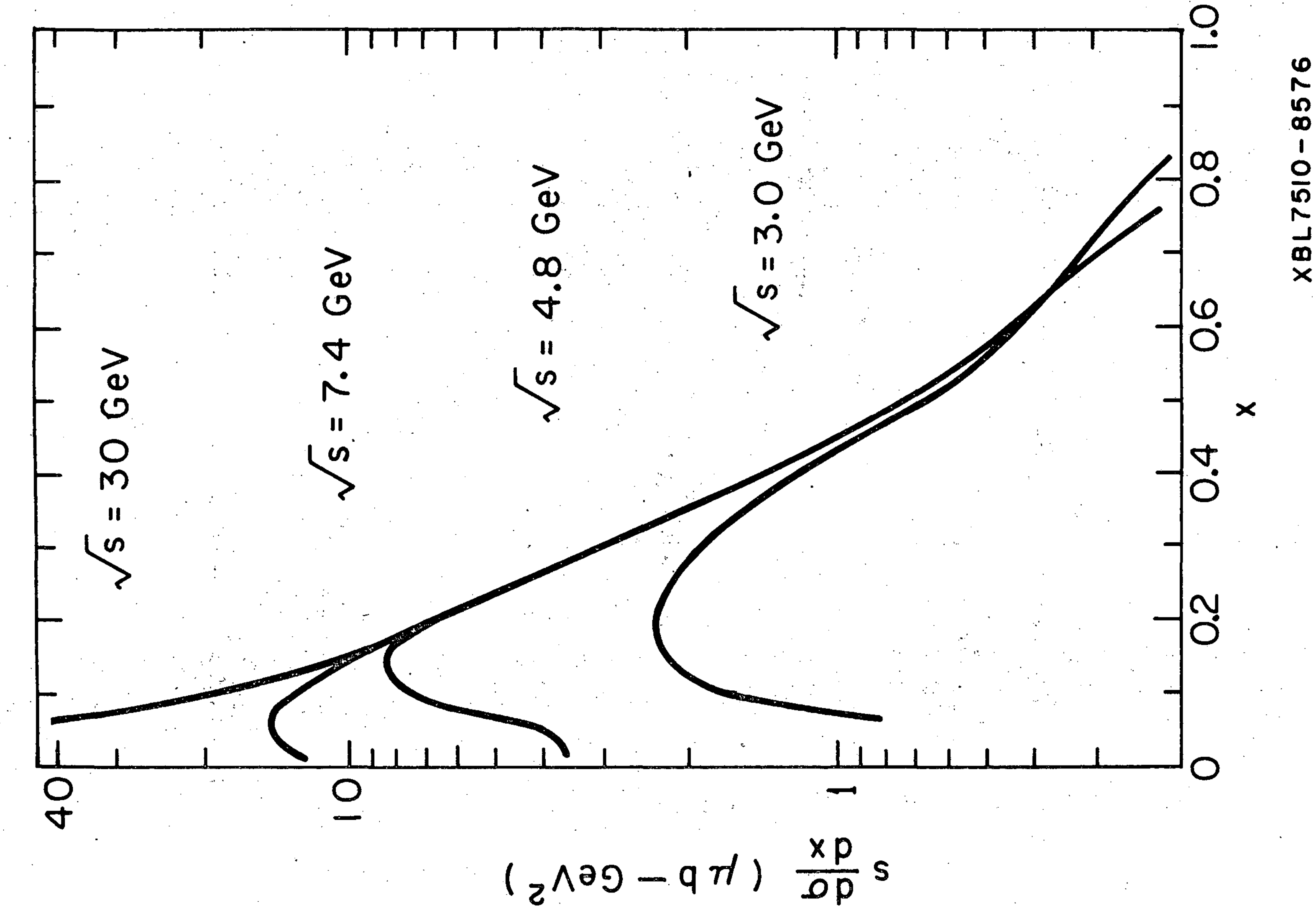
XBL7510-8575

Figure 1 (b)



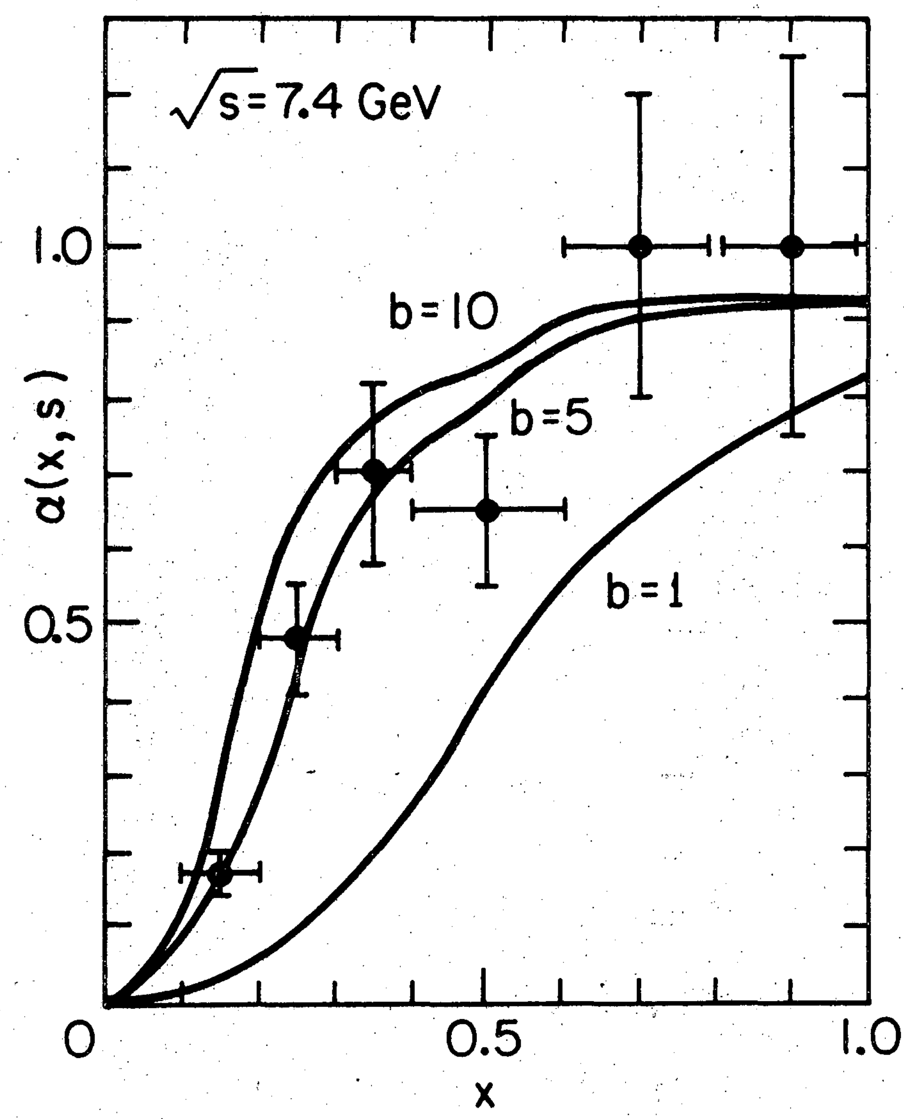
XBL7510-8579

Figure 1 (c)



√s = 30 GeV
√s = 7.4 GeV
√s = 4.8 GeV
√s = 3.0 GeV
s dσ/dx (μb − GeV²)
40
10
1
0
0.2
0.4
0.6
0.8
1.0
x
XBL7510-8576

Figure 2 (a)



XBL7510-8577

Figure 2 (b)

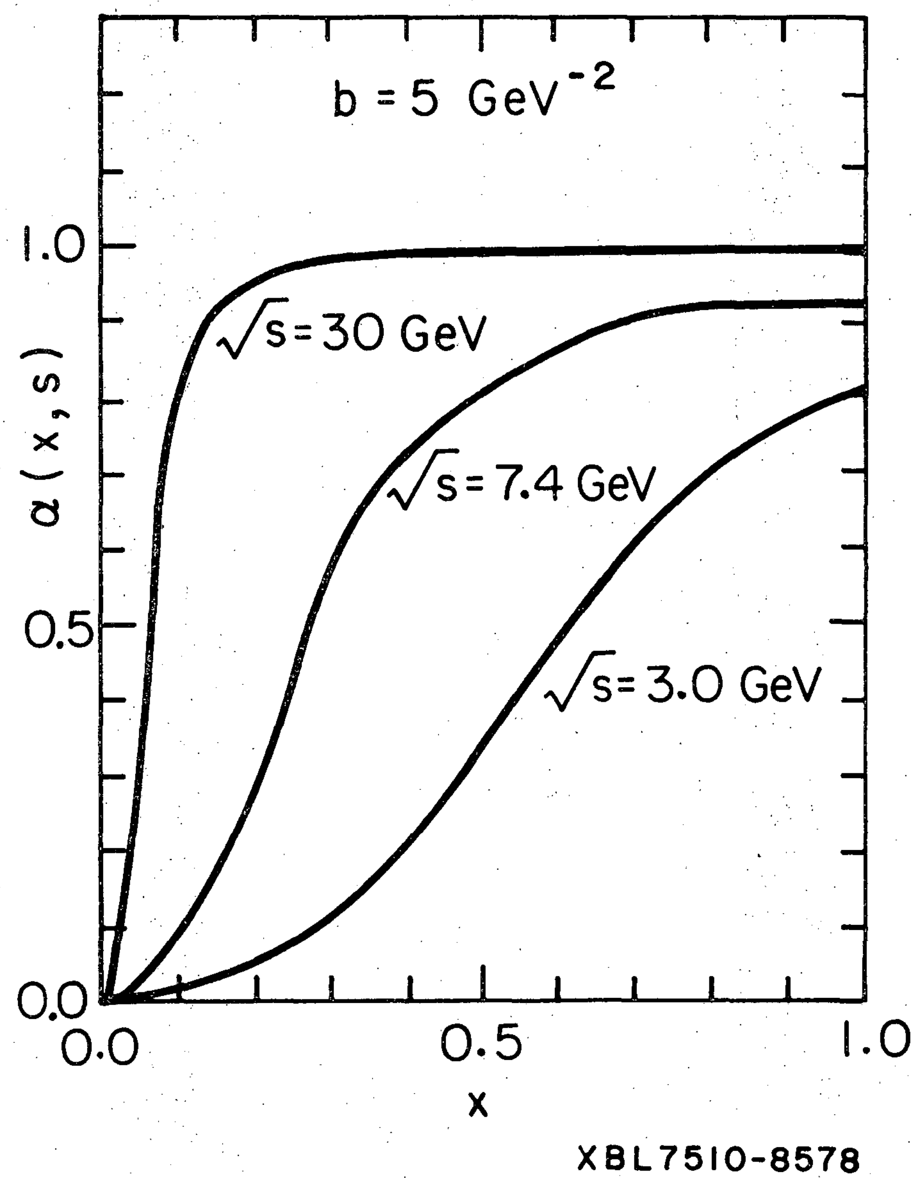

b = 5 GeV^{-2}
1.0
0.5
0.0
$\alpha(x,s)$
$\sqrt{s}$= 30 GeV
$\sqrt{s}$= 7.4 GeV
$\sqrt{s}$= 3.0 GeV
0.0
0.5
1.0
x
XBL7510-8578

Figure 2 (c)

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