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By

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Abstract

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Formal developments of normative theories typically claim that the guidance they give is universal: for any agent, and any way the world could be, there is a way she is permitted (according to that theory) to act. Yet when we consider an agent facing an open, indeterminate future, cases are possible in which what is permitpted depends on what she actually does. These situations follow the letter of the law while seeming to violate its spirit. A famous example, discussed by Gibbard and Harper (1978), comes from Somerset Maugham: while in Damascus, you learn that Death is coming to collect your soul. Your one option is to flee to Aleppo. But you are confident that Death never misses her quarry: if you flee to Aleppo, Death will be there. But if you stay in Damascus, Death will be there too.

If Death is going to Damascus, you should go to Aleppo, and if Death is going to Aleppo, you should go to Damascus. So for any way the world could be, there is a way you should act. Yet there is a clear sense in which there is nothing you can do: since Death's destination depends on yours, no act is such that you ought to have done it, *given that you do it*. The norms of rationality in cases of this structure—and cases with the opposite structure, where available acts deontically validate themselves—are the subject of much recent work in ethics, decision theory, and the metaphysics of persons. I show how a model theory for the natural language modals OUGHT and MAY can incorporate these notions of deontic validation and self-defeat. Because MAY tracks the concept of permissibility brought out by act-dependent cases, its inferential properties reflect the language-*independent* intuitions we have about choiceworthiness, in cases like Death in Damascus.

This theorizing makes contact with natural language in the form of my solutions to two

infamous puzzles about deontic modal language, free choice permission (Kamp 1973) and Ross's puzzle (Ross 1941). Free choice permission is the apparent validity of the classically *invalid* inference from MAY(ϕ or ψ) to both MAY ϕ and MAY ψ, and Ross's paradox is the apparent *in*validity of the classically valid inference from OUGHT ϕ to OUGHT(ϕ or ψ). The first step to a unified solution to these puzzles is precisely to leverage the notion of permissibility corresponding to deontic self-validation. The second component is a generalization of classical logic. On my account, the interpretation of a disjunction depends on which of its atomic disjuncts are true at the actual state—where the 'actual state' can be keyed to the *future*-actual state an agent chooses when she acts. While classical consequence is preserved for sentences without modals, this analysis sets up a match between the act one brings about and the contents of statements describing that act's deontic status: for example, in futures where the agent chooses to ϕ, it is ϕ that she has permission to do. This allows the stronger-than-classical conclusions of free choice permission to follow, and blocks the inference in Ross's puzzle. It also predicts the positive entailment properties of disjunction under OUGHT, while preserving the inviolability of the role classical disjunction plays in our reasoning. This intuitively appealing combination has, in previous work, proved difficult to achieve.

To my parents.

Contents

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Chapter 0

Introduction

Deontic modal operators mark permission, obligation, and ideality in natural language:

(1) You ought to call your mother.
OUGHT (you call you mother)

(2) You may sit down.
MAY (you sit down)

The compositional semantics of these operators, which appear in many natural languages, is of intrinsic interest.[1] A philosopher of language might hope, quite generally, that their semantics provides a glimpse into the nature of permission, obligation, and ideality. For the puzzle-solver, deontic modals hold out a series of challenges.

In this dissertation, I propose a unified solution to two named puzzles involving disjunction and deontic modals: Ross's puzzle (Ross 1941) and free choice permission (Kamp 1973). Ross's puzzle is the apparent failure of OUGHT ϕ to entail OUGHT (ϕ OR ψ): from

(3) Jim ought to warn them.
OUGHT W

it seems you cannot conclude

(4) Jim ought to warn them or stay at home.

[1] Following tradition in modal logic and much philosophy of language, I analyze these constructions as operators on sentences, in contrast to the theories in, for example, Schroeder (2011) and Harman (1973). Syntactically, I take it that these operators occupy the T head (for Tense) in English. See Adger (2003, Ch. 5).

$\text{OUGHT}\ (W\ \text{OR}\ S)$

Free choice permission is the apparent fact that $\text{MAY}(\phi\ \text{OR}\ \psi)$ entails both $\text{MAY}\ \phi$ and $\text{MAY}\ \psi$: from

(5) Jim may go or stay.
$\text{MAY}\ (G\ \text{OR}\ S)$

it seems you *can* conclude

(6) Jim may go and Jim may stay.
$\text{MAY}\ G \wedge \text{MAY}\ S$

The puzzle, then, is the unaccounted-for strength of OR in the scope of these modals. I work towards my explanation of the facts from two directions: one which focuses on disjunction, and one which focuses on deontic modality. Along the way, I unify this familiar data with two additional inference patterns, which I invite the reader to test his or her intuitions on. The first I call (Conditionals-M): from $\text{MAY}\ (\phi\ \text{OR}\ \psi)$, conclude *if not* ϕ, $\text{MAY}\ \psi$:

(7) You may have coffee or tea, therefore: if you do not have coffee, then you may have tea.
$\text{MAY}(C\ \text{OR}\ T) \Rightarrow$ if $\neg C$, then $\text{MAY}\ T$.

The second I call (Conditionals-O): from $\text{OUGHT}\ (\phi\ \text{OR}\ \psi)$, conclude *if not* ϕ, $\text{OUGHT}\ \psi$:

(8) You ought to have coffee or tea, therefore: if you do not have coffee, then you ought to have tea.
$\text{OUGHT}(C\ \text{OR}\ T) \Rightarrow$ if $\neg C$, then $\text{OUGHT}\ T$.

(Conditionals-M) and (Conditionals-O) suggest that the deontic status of the act described by a disjunct under a deontic modal can depend on what else the agent does.[2] If that is right, then they are instances of *act dependence*: the normative status of each disjunct is future-contingent. For example, whether I may have coffee *depends* on whether I have tea. Act dependence seems to be part of the data of free choice permission and Ross's

[2] John MacFarlane notes that this claim depends on how the conditional is interpreted. An *information-sensitive* interpretation of the conditional might only establish that (e.g.) whether you may have tea depends on whether it is *known* that you are not having the coffee. See MacFarlane & Kolodny (2010) and Yalcin (2007, pg. 998 ff.) for a discussion of such an information-sensitive conditional, the former particularly in the context of deontically modalized consequents. My own semantics for a conditional supporting (Conditionals-M) and (Conditionals-O) is presented in Chapter 5.

puzzle, and it is key to the semantic solution to all four puzzles that I offer in Chapter 5.

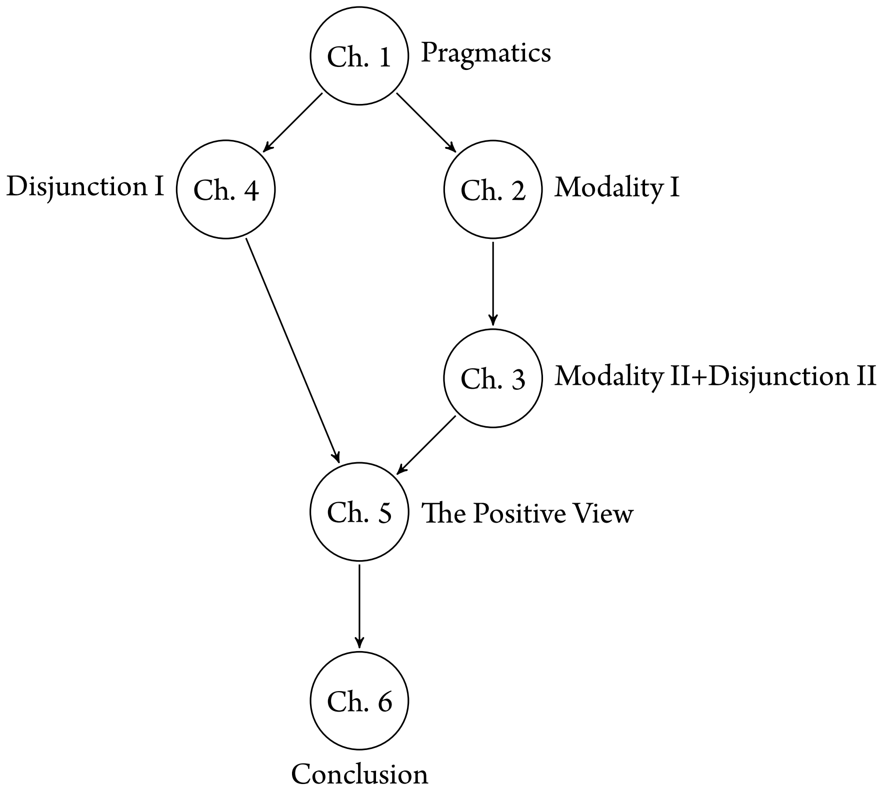
Chapter 1 begins to create space for my semantics by raising a problem for a popular alternative approach free choice permission: that of Kratzer and Shimoyama (2002). This approach has received a good deal of discussion, endorsement and elaboration in recent work (Aloni & van Rooij, 2004; Alonso-Ovalle, 2006; Chierchia, 2006; Fox, 2007; Geurts, 2009; von Fintel, 2012). I suggest that the general form of the Kratzer and Shimoyama explanation is not extensionally adequate, and discuss some replies and some ramifications.

Chapter 2 opens with a presentation my own view of how the linguistic data for OUGHT and MAY are connected. I then turn to Cariani (2011)'s approach to Ross's puzzle. While Cariani's view is very different from my own, it allows me to discuss how the two named puzzles are related, and how a satisfactory joint solution might look. (In particular, I argue that an approach which solves one without the other would be quite undesirable.) Cariani's view gives disjunction a role to play beyond its traditional Boolean contribution, but as I discuss, he does so by way of postulating a pragmatic effect *of* traditional Boolean disjunction that is subsequently "recycled" into the semantics of OUGHT.

In Chapter 3, I lay the foundations for my own non-Boolean view of disjunction. My focus here is merely on the *negative* entailment properties of OUGHT and MAY: the failure of felt entailment from OUGHT ϕ to OUGHT (ϕ OR ψ) and from MAY ϕ to MAY (ϕ OR ψ). The former is sufficient to explain Ross's puzzle—in fact, it *is* Ross's puzzle—and though the latter is not sufficient to explain free choice permission, it is an important step along the way. I advance a minimal semantic entry for disjunction that is sufficient to block these entailments, while maintaining nonnegotiable features of the deontic modal operators and of classical logic. As the reader will see, this is not yet a full theory of OR, but it does advertise the advantages of a two-dimensional approach, on which disjunction is Boolean on the diagonal but not otherwise. Disjunction, in this chapter, is Boolean on the *diagonal* in the sense that two worlds relative to which intensions are calculated are identical.

Chapter 4 approaches non-Boolean disjunction from a completely different direction, one motivated by a phenomenon known as "the simplification of disjunctive antecedents" (Nute 1974). Several explanations for this phenomenon have been proposed in the literature, and I do not make an attempt to canvas them. Rather, I use the phenomenon as a springboard for connecting classical disjunction to the dynamic analysis of indefinites proposed by Heim (1992). In both cases, a disjunction is capable of expressing just one of its disjuncts, rather than the Boolean union of the disjuncts, and in both cases, classical logic is nonetheless preserved throughout the nonmodal fragment of the language. This is unsurprising, but will help make intelligible a claim I refine in Chapter 5: that in a world where p is true and q is false, the disjunction 'p OR q' has the same semantic value as p—even when these world-relative semantic values are intensional (that is, richer than mere truth-values). What this shows is that a fragment of the analysis in Chapter 5 can be motivated by data that has nothing to with deontic modality.

Chapter 5 lays out an argument for the positive view of disjunction and the modals from the beginning. I focus on a pair of cases from the decision theory literature illustrating the phenomenon of act dependence, where what an agent ought to do depends on what she does. Building on contemporary work in decision theory, I argue that in these cases the practical upshot of a set of norms is strictly more permissive than in non-decision dependent cases: what matters is the relationship between what you are required to do *after* you act, and what you have done. This notion of permissibility, which owes a significant debt to Richard Jeffrey's norm of ratifiability (Jeffrey, 1983), forms the basis for my novel analysis of MAY. The framework is then combined with the foregoing generalization of the classical semantics for disjunction, which explains the inferences in terms of semantic consequence. I end in Chapter 6 by by considering objections to the view, and sketching some further avenues of research, and highlighting some features of the formal system.



Flow of Chapters

Chapter 1

Free Choice Permission and the Counterfactuals of Pragmatics

Suppose I say to you[1]

(1) You may have the gin or the whiskey.
MAY(G OR W)

As you help yourself to the latter, I cry, "Stop! You can't have the whiskey!"

It seems that I have contradicted myself. For (1) appears to entail

(2) You may have the gin and you may have the whiskey.
MAY $G \wedge$ MAY W

But why? This is not generated by our straightforward semantics for MAY and a Boolean semantics for OR—which, in this chapter, I will mark with the modal diamond '$\Diamond$' and the classical logical symbol '$\vee$'—is the Puzzle of Free Choice Permission (Kamp, 1973).

A simple version of the classical semantics for modals and disjunction is as follows: given a model M which is a triple $\langle W, R, V\rangle$ where W a nonempty set of possible worlds, R is a serial accessibility relation on W, and V is a function that maps atomic wffs to truth-values relative to elements of W:[2]

[1] Material from this chapter was previously published in *Linguistics and Philosophy*, vol. 37, no. 4 (2014). I follow the convention of that article, and the convention of Kratzer and Shimoyama, in marking the use-mention distinction with single quotes and marking speech acts with double quotes, including for the terms MAY OUGHT, and OR. These are more easily treated without quotes in the formal development of the positive theory.

[2] These clauses are reprised in I will call the "standard modal theory" in Chapter 3, §1.

$w \vDash p$ iff $V(p,w) = 1$.
$w \vDash \neg\phi$ iff $w \nvDash \phi$.
$w \vDash \phi \wedge \psi$ iff $w \vDash \phi$ and $w \vDash \psi$.
$w \vDash \phi \vee \psi$ iff $w \vDash \phi$ or $w \vDash \psi$.
$w \vDash \Diamond\, \phi$ iff $\exists w'$ s.t. wRw' and $w' \vDash \phi$.
$\Box\, \phi := \neg\Diamond\neg\phi$.

Hence when consequence ($\vDash$) is defined as the preservation of truth at w,[3] $\Diamond(\phi$ OR $\psi) \nvDash \Diamond\phi \wedge \Diamond\psi$.

What, then, is the source of the compellingness of the inference from (1) to (2)? My topic in this chapter is an influential pragmatic explanation for the felt entailment, due to Kratizer and Shimoyama (Kratzer & Shimoyama, 2002), which has received a good deal of discussion, endorsement, and elaboration in recent work (Aloni & van Rooij, 2004; Alonso-Ovalle, 2006; Chierchia, 2006, 2013; Fox, 2007; Geurts, 2009; von Fintel, 2012).

I begin by presenting the Kratzer and Shimoyama solution, and contextualizing it within a classical Gricean view of pragmatics. I then argue that the general form of the explanation does not cover the full range of the phenomenon; it relies on counterfactuals about alternative utterances that do not obtain in some contexts where the Free Choice effect is observed.

1.1 The Explanation

Suppose two books are under discussion, an algebra book and a biology book. I say: "you may borrow the algebra book or the biology book": "$\Diamond(A \vee B)$". Kratzer & Shimoyama (2002) offer a pragmatic explanation for the generation of the felt entailment to $\Diamond A \wedge \Diamond B$ for this case, framing the reasoning from a hearer's point of view. The argument has been influential enough to bear direct quotation:

[3] More precisely: $\Gamma \vDash \psi$ iff, for any model $\mathscr{M}$ and any $w \in W_{\mathscr{M}}$, if $\mathscr{M}, w \vDash \phi$ for all $\phi \in \Gamma$, then $\mathscr{M}, w \vDash \psi$. Again, this definition is reprised in Chapter 3.

2 books are under discussion: an algebra book and a biology book. I say

> "You can borrow one of those two books."[a]
> Alternative set chosen: May$\{A, B\}$
> Truth-conditional content: $\Diamond(A \vee B)$

[You reason as follows:] she picked the widest set of alternatives, $\{A, B\}$. Why didn't she pick $\{A\}$, which would have led to a stronger claim? Suppose $\Diamond A$ is false. Then she should have made the stronger claim $\Diamond B$. Why didn't she? It couldn't be because the exhaustivity inference $\neg\Diamond A$ is false. Assume, then, that $\Diamond A$ is true. The reason why she nevertheless made the weaker claim $\Diamond(A \vee B)$ would now have to be that the exhaustivity inference $\neg\Diamond B$ is false. We infer $\Diamond A \rightarrow \Diamond B$. Parallel reasoning for why she didn't pick $\{B\}$ leads to $\Diamond B \rightarrow \Diamond A$. [Finally, $\Diamond(A \vee B), \Diamond A \rightarrow \Diamond B, \Diamond B \rightarrow \Diamond A \vDash \Diamond A \wedge \Diamond B$].
(Kratzer & Shimoyama, 2002, pg.18-19).

[a] K&S discuss the German 'kann' in this passage, which they translate as both 'can' and '$\Diamond$'. I assume here, as they do, that the same explanation is applied to MAY.

The important idea from Kratzer and Shimoyama (hereafter "K&S") is that, when a well-defined set of possibilities is under discussion (for example, the two books) *and* speakers are presumed to be both fully cooperative and well-informed (as the owner of the books would be), *exhaustivity inferences* are triggered. An exhaustivity inference embodies the generalization that, if the speaker didn't assert "p," where p is in the set of salient alternatives, then p is false. That is what explains why the speaker didn't say (e.g.) "you can borrow book A" in the Free Choice case: it would have implicated, contrary to fact, that *you can borrow book B* was false.

An exhaustivity inference is the strongest possible form of inference licensed by application of Grice's Maxim of Quantity (Grice, 1975): "make your contribution as informative as required." Such inferences mine significance from a speaker's act of omission: from her *declining* to assert some salient, more informative alternative p. Suppose the speaker said "q", and "p" is a salient alternative to "q." There are two immediately obvious reasons for omitting to make the assertion "p": (i) the speaker fails to know that the proposition "p" expresses in context is true, or (ii) positively knows that it is false. The meaning of what the speaker did say is then strengthened, either with $\neg K_S p$ (speaker's lack of knowledge that p), in the first case, or $K_S \neg p$ (speakers knowledge that not-p), in the second case. These two moves in succession are familiar enough to have been dubbed the "Standard Recipe" for generating implicatures from the Maxim of Quantity (Sauerland, 2004; Geurts, 2009). An exhaustivity implicature results in the case where both steps go through, and the conclusion that $\neg p$ is reached.

What of the more *general* form of Gricean inference licensed by the Maxim of Quantity? We can distill the maxim into a kind of rational constraint on utterance-interpretation:

> **Quantity Constraint**. If a speaker asserts "q," then for all p logically stronger than q such that p is a relevant alternative to q, there must be some reason the speaker refrained from asserting "p." (Gamut, 1991, pg. 205)

K&S apply the Quantity Constraint to the Free Choice premise $\Diamond(A \vee B)$, with a twist that generalizes on the Standard Recipe. They suggest that the reason the speaker refrained from asserting the stronger alternative "$\Diamond A$" is not that the proposition "$\Diamond A$" expresses is false, or unknown to the speaker (since it is both true *and* known, in the Free Choice case); rather, the problem is that the assertion "$\Diamond A$" would *itself* trigger a misleading exhaustivity implicature: the implicature that $\neg\Diamond B$. Since, in a Free Choice case, the speaker wishes to communicate $\Diamond A$ *and* $\Diamond B$, this would be an undesirable implicature for the hearer to draw.

With that in mind, we can recast Kratzer and Shimoyama's reasoning in full-blown form:

> Why did the speaker say "$\Diamond(A \vee B)$" rather than the stronger "$\Diamond A$"? We consider two cases: the speaker knows $\Diamond A$ is false (Case 1), or the speaker knows that $\Diamond A$ is true (Case 2). Case 1: then $\Diamond B$ must be true, and the speaker must know this. But then why didn't she say "$\Diamond B$," which would have been stronger? There is no explanation; the speaker would be in violation of the Quantity Constraint. So it must be that the speaker does not know that $\Diamond A$ is false. So, treating this as a *reductio*, move on to (Case 2): the speaker knows that $\Diamond A$ is true. Then why didn't she say "$\Diamond A$," which would have been stronger? Here there *is* a possible explanation: if she had said "$\Diamond A$," I would have concluded, via an exhaustivity inference, that $\neg\Diamond B$. Maybe she wanted to avoid that inference. Likewise, if she had said "$\Diamond B$," I would have concluded, via an exhaustivity inference, that $\neg\Diamond A$. Maybe she wanted to avoid that too. Further, perhaps she wanted to avoid both of these inferences because she thinks their conclusions are false. Hence if the speaker is rational then she thinks both $\Diamond A$ and $\Diamond B$ are true.

If K&S are right, the Free Choice effect is really a special kind of quantity implicature: it can be paraphrased, without loss, by considering alternative possible permission-giving *assertions* ("$\Diamond A$", "$\Diamond B$") and their counterfactual effects. Hence it appears to be assimilable to classical Gricean explanations of implicatures along the lines of:

Alice: Did you enjoy your blind date last night?
Otto: The movie was nice.
(implicature: Otto did not enjoy the date.)

The reasoning, of course, is that if Otto had enjoyed the date, he would have said so. This Gricean explanation avoids the wild semantic hypothesis that the proposition expressed by "the movie was nice" is truth-conditionally incompatible with the proposition expressed by "Otto enjoyed the date": a satisfyingly semantically conservative result.

Another interesting consequence follows, if the K&S reasoning is correct: we have an example of a case where Gricean reasoning takes into account the Gricean tendencies of other speakers. On such a picture, in conjuring extra meaning from the non-assertion of a stronger alternative, rational speakers take into account not only *the propositions expressed* by alternative utterances, but also what others would have *inferred* through Gricean mechanisms in situations where those utterances had taken place. For this reason, Chemla & Bott (2014), for example, call the K&S explanation a "second-order implicature." The explanation suggests the following, revised picture of the Gricean maxims: rather than

Maxim of Quantity: Say what is informative.
Maxim of Quality: Say what is true.

we have:

(**Reflective**) **Maxim of Quantity**: Be informative, either by saying *or by implicating* what is informative.
(**Reflective**) **Maxim of Quality:** Be truthful, either by saying *or by implicating* what is true.

These reflective versions of the maxims do justice to Grice's suggestion that

> though [in some cases] some maxim is violated *at the level of what is said,* the hearer is entitled to assume that that maxim, or at least the overall Cooperative Principle, is observed *at the level of what is implicated.* (Grice, 1975, pg. 162-163, emphasis added).

Because Gricean reasoning is supposed to be an exercise of general intelligence, and intelligent agents are generally aware of the rational tendencies of others, this is a satisfyingly rational result: it gives a satisfyingly full-blooded picture of Gricean rationality.[4]

[4]The awareness of the rational tendencies of others involves Gricean explanations in game theoretic considerations. For work in the game-theoretic aspects of implicature derivation, see, *inter alia*, Parikh

1.2 The K&S Explanation has Insufficient Scope

However, the K&S explanation is not sufficiently general to account for the full range of cases in which the Free Choice effect appears.

Notice that Kratzer and Shimoyama stipulate that two books are under consideration, book A and book B, *prior* to the utterance of the disjunctive permission "$\Diamond(A \vee B)$". Because the permission $\Diamond B$ is salient at the moment of utterance, other permission statements that do not mention B (like "$\Diamond A$") implicate via exhaustivity that the corresponding permission statement $\Diamond B$ is false—just as, in the Alice-Otto dialogue, the extant salience of Alice's question makes Otto's silence on the matter of the date especially meaningful. In K&S's original case, where both book A and book B are being actively considered, it is true that:

(C1) If the speaker had uttered "$\Diamond A$", she would have implicated that $\neg\Diamond B$.

But the Free Choice inference has broader scope than this—it is not restricted to contexts in which the disjuncts of the embedded disjunction are already salient. Suppose I say:

(3) You may borrow the algebra book or date my sister.

The Free Choice effect obtains. But we may stipulate that my sister was no way salient before my utterance—in fact, you didn't know I had one. Here, it is implausible to claim—if the claim is a Gricean one—that to have said instead "You may borrow the algebra book" would have been to implicate that you may not date my sister. But this is just what the counterfactual (C1) says.

This limits the scope of the K&S explanation: in the general case, sentences-in-context don't have the salient alternatives they need to have for the explanation to work—the counterfactual (C1) is not true—unless $\Diamond A$ and $\Diamond B$ were already (mutually) salient at the time of utterance.

1.3 A Response: a double-effect?

I think there is a tempting, but ultimately unsuccessful, response to make to this objection. It is to argue that it is the utterance of the disjunctive permission *itself* that creates the salience relations that are needed to make (C1) true. On such a view, it is the speaker's utterance of the embedded disjunction "borrow the algebra book or date my sister" that

(1991, 1992, 2001); Benz et al. (2006); Jäger (2008); Rothschild (2011) and Franke (2013). For a specific application to Free Choice Permission, see Franke (2011).

elevates to salience the two options that the exhaustivity reasoning exploits. The utterance, on such a theory, has two sequential pragmatic effects: it raises a set of alternative possibilities $\{A, B\}$ to salience, and then exploits exhaustivity inferences relative to that alternative-set to generate the implicature from $\Diamond(A \vee B)$ to $\Diamond A \wedge \Diamond B$.

Could this be right? There definitely *are* examples of cases that could correctly be described as double-effects. These occur, for example, with a certain kind of context-sensitive expression. Consider an assertion of

(4) Otto is speaking.

When Otto asserts (4), he makes it the case that he is speaking. The utterance of (4) has two sequential effects: first, it initiates some change in the context—it makes it the case that Otto is the speaker of the context—and then makes some assertion which depends for its truth on the very change that has been made.[5]

Recently, Ephraim Glick (Glick, 2010) has offered such "double-effect" explanations for sentences like

(5) Sarah Vaughan and Ella Fitzgerald are both great singers, but I prefer *the former* to *the latter*.

What fact makes it the case, in context, that the expression "the former" in (5) picks out Sarah Vaughan, and the expression "the latter" picks out Ella Fitzgerald? Nothing other than the utterance of (5) itself. Glick comments, "In general, the contextual facts that determine the values of context-sensitive expressions need not be facts that are available to the audience, or that even obtain, before the utterance begins" (pg. 11). We can add that it is easy to mistake this phenomenon for a semantic effect, rather than a pragmatic one, because the sentence in (5) carries the needed context-modifying effects along with it. Whenever (5) is uttered, it *creates* just the contextual features it needs for its expressions to have the right context-sensitive referents—so that the results appear to be context-*in*sensitive.

The suggested response to the objection, on Kratzer and Shimoyama's behalf, then, is this. The alternative permissions $\Diamond$(borrow the Algebra book) and $\Diamond$(date my sister) do *not* need to be salient in context prior to the utterance of the Free Choice premise. Rather, the utterance of the disjunctive permission "$\Diamond$((borrow the Algebra book) $\vee$ (date my sister))" raises them to salience, and the K&S reasoning proceeds as above.

[5]The first effect corresponds to what Stalnaker calls "the modification of the prior context": "The prior context that is relevant to the interpretation of a speech act is *the context set as it is changed by the fact that the speech act was made, but prior to the acceptance or rejection of the speech act*" (Stalnaker, 1999c, pg. 101, emphasis added).

Double-Effect Response Fails for Counterfactuals

I do not think this attempt to extend the K&S sketch of the Free Choice Inference will work. In reasoning counterfactually about what the alternative assertion "$\Diamond A$" would have implicated if it had been uttered instead of "$\Diamond(A \vee B)$", the reasoner continues to illicitly make use of facts about what *actually* occurred. If it is the utterance of the disjunctive permission $\Diamond(A \vee B)$ *itself* which raises the alternative-set $\{A, B\}$ to salience at the time of utterance, then in the counterfactual situation in which "$\Diamond A$" was asserted instead of "$\Diamond(A \vee B)$"—the counterfactual situation relevant to the antecedent of (C1)—$\Diamond B$ is not a salient alternative. So if "$\Diamond A$" had been uttered instead, it would still not have implicated $\neg\Diamond B$. The counterfactual (C1) is still false.

What is the difference between the good cases of double-effects and the bad? Glick's point about sentence (5) is that contextual features F (denotations of "the former" and "the latter") don't need to obtain *prior* to an utterance that exploits them. And this is true. But the K&S gambit is that contextual features F (mutual salience relations) don't need to obtain in the same *possible world* as an utterance that exploits them. This is far less convincing. The difference between an unproblematic double-effect case like Glick's and the one under consideration is the difference between prior vs. posterior contexts (in the first example) and *actual* vs. *counterfactual* contexts (in the second).

In Glick's case, the assertion of (5) exploits the very contextual effects it creates. But in Kratzer and Shimoyama's case, we are trying to establish that an assertion of

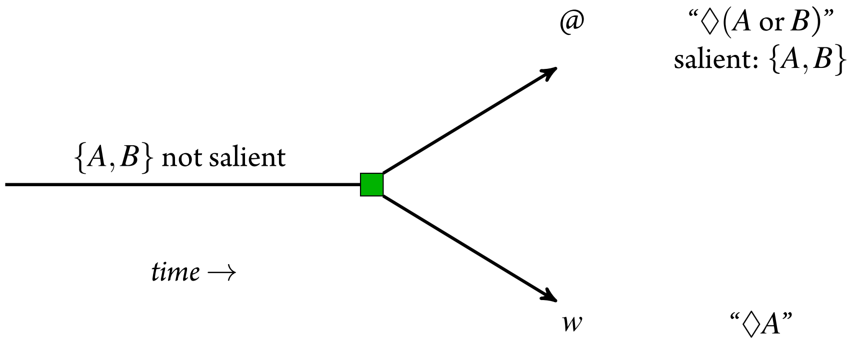
(6) "$\Diamond A$"
"You may borrow the algebra book"

can exploit the alternatives created by a different utterance:

(3) "$\Diamond(A \vee B)$"
"You may borrow the algebra book or date my sister"

...this, recall, is what is needed to establish (C1): that if the speaker had said (6) instead of (3), she would have implicated, via exhaustivity, that $\neg\Diamond B$.

We can use a picture to illustrate the point. Suppose that the actual world and w are exactly alike until the moment of the speaker's decision of what to utter. This branching picture, then, models the speaker's *choice* of what to say. By stipulation, we are in the case where the alternatives $\{A, B\}$ are not salient before the moment of utterance. Granted, (3) can raise the needed alternatives to salience. But that doesn't mean that (C1) is true. For (C1) to be true, it would have to be true that, on the w branch, the alternatives $\{A, B\}$ are salient—this is what we need to establish that *if* the speaker had uttered "$\Diamond A$" *instead*,



she would have implicated (via exhaustivity) that $\neg\Diamond B$. But from the point of view of w, this reasoning makes no sense: B (dating my sister) is *not* salient in w. And what I said in the actual world cannot raise the option of dating my sister to salience in w—how could it, since, by stipulation, w differs from the actual world in virtue of the fact *that I never mentioned her there*? The counterfactual (C1), the antecedent of which directs us to w, is clearly not validated by this model.

1.4 An Evidential Maneuver

I argued above that an utterance of MAY$(A \vee B)$ cannot be relied on to do the causal work of raising the alternatives required for the truth of (C1) to salience—neither from the theorist's, nor from a cooperative hearer's, point of view. Perhaps, though, this claim is more than is strictly needed by the Kratzer & Shimoyama explanation. In this section, I consider a different argument on K&S's behalf, to the effect that (C1) (repeated below) is a reason-giving consideration for deriving Free Choice permission.

(C1) If the speaker had uttered "$\Diamond A$", she would have implicated that $\neg\Diamond B$.

Recall that, in the stipulation of the case, B—dating the sister you didn't know I had—was not salient at the time of the utterance of the Free Choice premise. The suggestion, however, is that the hearer might rationally come to believe that B is salient upon hearing the utterance of the premise, even if it isn't. That means (C1), while it still wouldn't be true in the context under consideration, might be believed by the hearer to be true:

(C2) The hearer believes (C1).

To explain the truth of (C2), the argument appeals to a Lewisian notion of accommodation (Lewis, 1979) instead of a Glick-like double effect. A suggestion in this vein can be seen in Craige Roberts's gloss on accommodation (Roberts, 1996), which she uses in her development of the technical notion of a "question under discussion."[6] Roberts writes:

> if it is clear that an interlocutor presupposes a question or assertion ϕ which is not yet commonly agreed upon but the others have no objection, then they behave as if the common ground contained ϕ *all along*. (Roberts, 1996, pg. 6, emphasis added)

Roberts' gloss suggests that it's insufficient to say that speakers act as if an utterance of q makes its alternatives $Alt(q)$ salient; at least in some cases, speakers act as if the utterance indicates that $Alt(q)$ was salient *before* q was uttered. This is somewhat different from Lewis' own description of accommodation; he writes, "say something that requires a missing presupposition, and straightway that presupposition springs into existence" (Lewis, 1979, pg. 339). Lewis does *not* say that straightaway it is as if that presupposition had been in existence *all along*.[7]

Nonetheless, it is plausible that this can sometimes happen. On the suggested line of argument, the raising-to-salience of B associated with the utterance "$\textsc{May}(A \vee B)$" is *evidential* and not *causal*. This gives the phenomenon a different temporal profile. It's not that the actual utterance, at time t, raises the target alternatives to salience at t^+ (since this, as I have argued, cannot ground their salience in worlds that diverged from the actual world *before* t); the suggestion is instead that the utterance functions as *evidence* for the hearer that $\{A, B\}$ were *already* salient before t, hence still salient in worlds that diverged from w before t. Under a sufficiently "backtracking" notion of relevance along these lines, (C2) might be able to capture *all* of the Free Choice data.

While a judgement that (C2) is false is more subtle than a judgement that (C1) is false, I believe we should be skeptical of (C2) as well; we should be skeptical that relevance facts are really accommodated by rational interlocutors to such a degree. On the view under consideration, the listener must ultimately evaluate the plausibility of counterfactual (C1) in light of her overall evidence. Even if it is true that, in some cases, the hearer *will* revise her previous beliefs regarding relevance, there will be cases where the extant relevance of B is independently *im*-plausible–cases where the listener has strong independent reason

[6] I am indebted to an anonymous referee at *Linguistics and Philosophy* for raising the suggestion I pursue (in somewhat adapted form) in this section, and for the reference to Roberts' work.

[7] This is true throughout Lewis' use of the "straightway" locution (*op. cit.*, pgs. 340, 341, 347, 349, 351, and 356.)

to believe B was not amongst the relevant alternatives at t. Ultimately, my argument that there is a lacuna in the K&S argument rests on cases of this type.

For example, one can use Free Choice permission-type sentences to *flout* the Maxim of Relation—where I use "flout" here in Grice's sense. Suppose you have a chronic habit of borrowing my biology and my algebra books, and expect to get permission to borrow them again; you expect me to say

(7) You may borrow the algebra book or the biology book.

Today, however, I do not want to part with my biology books. In a fit of annoyance at you, I might violate expectations by saying things like:

(8) You may borrow the algebra book or take a hike.

(9) You may borrow the algebra book or go fly a kite.

(10) You may borrow the algebra book or get your own books from now on.

Presumably the second disjuncts of (8)-(10) are not very helpful, because they are not relevant to your purposes.[8] But once again, there is little doubt that the Free Choice effect obtains. For example, (8) generates a felt entailment that you may borrow the algebra book. It *also* generates the felt entailment that you may take a hike—it's just that you're unlikely to want to do so. Returning to our original:

(3) You may borrow the algebra book or date my sister.

...although you might be puzzled as to why dating my sister was mentioned in the first place, you do not *thereby* doubt that (3) communicates that you may date my sister. If the Free Choice effect survives *bold*, deliberate violations of the Maxim of Relation, it is unlikely to depend on this maxim to generate the felt entailment.[9]

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[8] They are also, in the case of (8) and (9), idiomatic. I assume that these idioms, *take a hike* and *go fly a kite*, acquire their characteristic significance in conversation from the fact that they typically flout the Maxim of Relation.

[9] Another worry about a very accommodating notion of relevance is suggested by Grice himself. If we had such a notion, we'd lose the datum Grice made famous with his "lovely weather we're having" example:

> At a genteel party, A says *Mrs. X is an old bag.* There is a moment of appalled silence, and then B says *The weather has been quite delightful this summer, hasn't it?* B has blatantly refused to make what he says relevant to A's preceding remark. He thereby implicates that A's remark should not be discussed and, perhaps more specifically, that A has committed a social gaffe.
> (Grice, 1975, pg. 54)

1.5 Semantics vs. Pragmatics

Free Choice Permission has generated an enormous literature, and the argument here is not directed at approaches rooted in game theory (Franke, 2011), optimality theory (Aloni, 2007a), or epistemic logic (Zimmermann, 2000). It does, however, touch on Kamp's conception of the original puzzle—in its deontic form—as a testing ground for semantic vs. pragmatic models of explanation. I would like to close this chapter by briefly considering some ramifications for this debate.

Closing the Gap

The K&S explanation is Gricean, but many Neo-Gricean and (what we might call) "Post-Gricean"[10] approaches to Free Choice in the current literature (Aloni & van Rooij, 2004; Alonso-Ovalle, 2006; Chierchia, 2006; Fox, 2007; Geurts, 2010) give explanations exhibiting the same problematic structure. Most of the work in this vein endorses Kratzer and Shimoyama's basic explanatory strategy—the strategy that enriches meanings by modus ponens on the counterfactual (C1). An outstanding issue that this ongoing work seeks to address is a perceived gap in the K&S explanation, which I will sketch briefly here.

In §2 I listed two classic reasons why, in keeping with the Quantity Constraint, a speaker who says "*q*" might rationally have refrained from asserting a stronger alternative *p*. They are:

(i) The speaker doesn't know *p*.

(ii) The speaker knows $\neg p$.

To these reasons, as we have seen, Kratzer and Shimoyama add a third:

(iii) An utterance of "*p*," while neither false nor un-knowledgeable, would *implicate* a falsehood.

The gap in the explanation is that reasons (i) and (ii) appear to preclude the Free Choice effect before (iii) has a chance to explain it. For take $q = \Diamond(A \vee B)$ and the stronger alternative $p = \Diamond A$. If we assume (i) and (ii)—the "Standard Implicature Recipe"—then we have the result that the fact that the speaker didn't utter "$\Diamond A$" implicates that the speaker

In a world governed by an extremely deferential notion of relevance, B's implicature would be lost. His audience is forced to conclude that the weather was relevant at the time when A said Mrs. X was an old bag. But this is not what rational interlocutor would infer.

[10] More on Post-Griceanism in the next section.

knows $\neg\Diamond A$. But this is incompatible with the conclusion of the Free Choice inference, $\Diamond A \wedge \Diamond B$.

Why don't considerations (i) and (ii) trigger inferences that rule *out* the Free Choice conclusion before the "second order" consideration (iii) rules it *in*? The family of publications cited above seek to give a satisfactory answer to this question. Any attempt to give a Gricean explanation of Free Choice within a system of implicature calculation must discharge this burden.

If what I have argued in this paper is is correct, there is a larger problem with the explanation. Even if consideration (iii) is a real phenomenon, it isn't the right explanation for Free Choice—at least, not in the general case.

Semantic accounts of the Free Choice effect do not face the task of plugging this gap, because semantic accounts reject the assumption the implicature approach begins with—namely, that the alternative utterances "MAY A" and "MAY B" express propositions which are truth-conditionally *stronger* than the proposition expressed by the Free Choice premise. Whatever its nature, a semantic account would hold that MAY(A OR B) *entails* MAY A $\wedge$ MAY B.[11] The relevant alternatives to what was actually asserted, on such an account, do indeed include such utterances as "MAY A" and "MAY B." But these utterances are no longer *stronger* relevant alternatives to what was asserted, so there is no pragmatic puzzle about why the speaker didn't assert them. From the point of view of a *semantic* approach to Free Choice, there is simply no gap to close.

This observation affects the ultimate balance of considerations for and against a pragmatic approach to Free Choice Permission (see von Fintel (2012, pg. 7 ff.) for broad commentary on the balance of considerations.[12])

In light of the difficulties facing the bid to use (3) to generate an explanation for Free Choice, the gap in the K&S explanation gives us reason to suspect that Free Choice is a semantic phenomenon, rather than a pragmatic one.[13]

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[11] I leave open the nature of such a semantic entailment—whether it is, for example, a relation between sentences, propositions, contents, update functions, etc.

[12] Von Fintel's purpose is to defend the classical semantics for deontic modals (in particular, Kratzer semantics: Kratzer (1977, 1981, 1991b)) from various objections. The line of attack that touches on Free Choice comes from Cariani (2011), who spotlights Ross's Puzzle (Ross, 1941) as a challenge to the monotonicity of deontic modals. More on Ross's Puzzle in the next chapter.

[13] Against this, of course, are other features that give the phenomenon the profile of an implicature, chief amongst which is its apparent suspension in downward-entailing contexts. The fact that this suspension gives Free Choice Permission a signature feature of Quantity Implicatures is emphasized, *inter alia*, by Alonso-Ovalle (2005, 2006), and Kratzer and Shimoyama themselves (Kratzer & Shimoyama, 2002, pg. 14). Aloni confronts the DE data in (Aloni, 2007b, pg. 80-81).

Is K&S An Error Theory?

A different response to my argument is to accept it, but to maintain that Kratzer and Shimoyama are right anyway. The reasoning they provide is, in fact, what underlies the felt entailment from $\textsc{may}(A\ \textsc{or}\ B)$ to $\textsc{may}\,A \wedge \textsc{may}\,B$. I have argued that there are serious obstacles to construing this reasoning as *rational*, in the general case. But sometimes agents engage in reasoning that isn't rational. So the fact that the reasoning is problematic does not mean that it does not bear witness to the psychological processes of speakers and hearers. Perhaps K&S's pragmatic explanation for Free Choice succeeds—descriptively, rather than normatively.[14]

My perspective on implicatures in this paper has been a classic Gricean one, on which calculating the implicatures of utterances is an exercise of generalized rationality. This classical perspective contrasts with recent work from what I shall call a *Post*-Gricean perspective, glossed by Kai von Fintel as "a new perspective on how implicatures work...a way that is not as post-compositional or pragmatic as assumed by (Neo-)Griceans but rather integrated into the recursive grammar" (von Fintel, 2012, pg. 7). On this perspective, the alternatives that factor into quantity-implicature enriched meanings are part of the *semantic value* of lexical items (Chierchia, 2006; Fox, 2007; Klinedinst, 2006).

A proponent of the Post-Gricean view of implicature may respond to my argument from (C1) by holding that I have underestimated the radical nature of the new view—may hold, in fact, that my argument shows the *need* for such a view. If a radical version of the semanticized alternatives view is right, then it is the lexicon, not post-semantic processes, that provide the alternatives for strengthening what was said along the lines of counterfactuals like (C1); knowing these alternatives is not an exercise in common sense reasoning but part of knowing the language itself. So, to put it starkly, it does not *matter* whether the proposed twist on the familiar Gricean moves presents us with plausible instances of the counterfactuals of pragmatics; the alternatives being exploited are part of the "semantic given." Hence theorists like Kratzer and Shimoyama are free to reverse-engineer the alternatives needed for the derivation to go through—not as a matter of rational reconstruction, but as a matter of semantic analysis. It would seem that the argument I have given from (C1) does not gainsay such a position.

While I think this is correct, to leave it at that—that is, at the idea that the K&S explanation is descriptive and not normative—is in one respect too optimistic, and in another

[14]Perhaps some "middle ground" between the descriptive and the normative will carry the day; the K&S pattern may ultimately be recast as an instance of overgeneralization, default processing, or some other pattern studied in theories of cognitive bias (Kahneman & Tversky, 1982). I take it that such routes are partially descriptive and partially normative. Thanks to an anonymous reviewer at *Linguistics and Philosophy* for emphasizing possible autonomy of such an approach.

respect too pessimistic. Let me try to indicate why.

It is too *optimistic* because recent experimental results cast doubt on the descriptive adequacy of the solution as well. As Chemla & Bott (2014) report, the Free Choice effect does not pattern experimentally as a Kratzer and Shimoyama-style explanation would seem to predict. Since the K&S explanation is, in a sense that can be made fairly precise, a *second-order* implicature—relying, as it does, on second-order versions of the Gricean maxims—it appears to make a prediction that can be tested in a lab. This would be that processing Free Choice inferences is slower than processing *first*-order quantity implicatures, of which scalar implicatures are the paradigm case. But Chemla and Bott's results indicate that the reverse is true: Free Choice processing is *faster* than scalar implicature processing (Chemla & Bott (2014, pg. 386); see also Chemla (2009)).

While there is reason to be cautious about drawing a direct line between empirical processing times and the puzzle we face here, the result weighs against the second-order aspect of the K&S explanation, according to which the Free Choice effect depends on our ability to calculate the first-order exhaustivity implicature of a different utterance ("$\Diamond A$," relative to the salient set $\{A, B\}$) *first*. In light of Chemla and Bott's results, the second-order hypothesis would seem to suggest that the reasoning is characteristically faster than one of its proper parts.[15]

Finally, the idea that the K&S explanation is descriptive and not normative is also too *pessimistic*, because any descriptively adequate explanation may have a claim to being rational as well, given the goals and nature of communication. It is not, many think, rational to cooperate in a Prisoner's Dilemma, but we can agree that, *if* both players are irrational in the same way (viz., cooperative), then both are better off. Likewise, it is not rational in a case of Lewisian coordination (Lewis, 1969) to be blind to the fact that there are multiple equally good equilibria. But if both players are so blinded *in the same way*, then they are more likely to choose the *same* equilibrium (the one they think is the unique one) and both will be better off. A conservative view on rationality holds the line: one cannot *know* that x is the unique coordination equilibrium, because it isn't; one cannot know that the dominant strategy in the Prisoner's Dilemma is to cooperate, because it's not. But perhaps a less conservative view can hold that what is beneficial for communicators *is* what is rational, in these unusual cases.

[15] An anonymous reviewer from *Linguistics and Philosophy* raises the point that Chemla and Bott's processing prediction does not obviously apply to recursive pragmatic approaches to Free Choice, since on such a view the derivation is implicated in the compositional semantic rules applied to the Free Choice premise. I agree that recursive pragmatics views are not, in general, committed to slower processing times for Gricean inferences than for corresponding "literal" inferences. The issue at hand, however, is the K&S derivation itself, which must exploit the idea that if the speaker had said "$\Diamond A$", she would have implicated $\neg\Diamond B$.

If the K&S explanation is descriptively correct, then in some contexts (like the one relevant to my (3)), the belief that *B* (viz., dating my sister) is salient is similar to the belief that cooperation is rational in a Prisoner's Dilemma, or that *x* is the unique coordination equilibrium (for some particular *x*): not rationally justified, but useful, *if* accepted by both interlocutors, to transmit some information. A conservative line holds that because *B* wasn't salient in the context, and the interlocutors are in a position to know this, then the belief that *B* is salient, however useful, cannot issue in *knowledge*—much less common knowledge. Yet a less conservative line on rationality may disagree.

How much depends on the label "rational"? Gennaro Chierchia (Chierchia, 2004, 2006), writing from the Post-Gricean point of view, frames the idea of lexicalized implicatures as an instance of the "spontaneous logicality of language" (Chierchia, 2006, pg. 548-549). This is the most dramatic version of the Post-Gricean view, on which semantics, and even syntax itself, are influenced by Gricean mechanisms that ultimately descend from Quantity implicatures. Chierchia's use of the term "logicality" is a hat-tip to Grice's use of the term in "Logic and Conversation": it is a reference to the *cooperative rationality* of speakers. In his (Chierchia, 2006) Chierchia presents the Free Choice Inference, and a generalization of the K&S explanation of it, as just such an example of the logicality of language. But if the argument in this chapter is correct, there remains a challenge to construing the reasoning underlying the K&S gloss as logical (in Grice and Chierchia's sense of the term); that is, to construing it as rational.

Chapter 2

Deontic Disjunction and Resolution Sensitivity

2.1 A Wider Range of Data

In the last chapter, we looked at free choice permission:

(FC) $$\text{MAY}(\phi \text{ OR } \psi) \Rightarrow \text{MAY } \phi \wedge \text{MAY } \psi$$

In this chapter, we consider another puzzle involving disjunction under a deontic modal. Suppose I say to you

(1) You ought to take out the garbage.
$\text{OUGHT}\, G$

You reason as follows.

(2) I ought take out the garbage.
$\text{OUGHT}\, G$
Therefore,

(3) I ought to take out the garbage or have some whiskey.
$\text{OUGHT}(G \text{ OR } W)$

It feels as if you have gone wrong; (2) does not seem to entail (3). That this entailment is classically valid, but feels intuitively *invalid*, is Ross's Puzzle (Ross, 1941):

(Ross) $\textsc{ought}\,\phi \not\Rightarrow \textsc{ought}(\phi \text{ OR } \psi)$

It is tempting to see the phenomena in Cases 1 and 2—(FREE CHOICE) and (ROSS)—as related. Ross's puzzle is the purely *negative* datum that OUGHT ϕ appears not to entail OUGHT(ϕ OR ψ). Free choice seems to be the "positive half" of the same datum: there is a clear intuition about *why* (3) doesn't seem to follow from (2): (2) appears to entail, via a free choice inference, that you can fulfill the relevant obligation by taking out the garbage—but that you could *also* fulfill it by having some whiskey instead. This apparent connection between FC and Ross is noted by, amongst others, Portner (2010); von Fintel (2012); Cariani (2011) and von Wright himself (von Wright, 1969).

We can forge a connection between the negative datum in the original case of (ROSS) and the positive datum in (FC) as follows. Use the standard assumption that there is an entailment from OUGHT to MAY:[1]

(Modal Weakening) $\textsc{ought}\ \phi \Rightarrow \textsc{may}\ \phi$

with this in hand, it is simple to sketch why an inference from (2) to (3) can lead to trouble:[2]

(Deontic Tonk)	OUGHT G	(2)
	OUGHT(G OR W)	(3)
	MAY(G OR W)	(Modal Weakening)
	MAY $G \wedge$ MAY W	free choice
	MAY W	conjunction elimination

With (Deontic Tonk), we begin with an assumption OUGHT ϕ and derive MAY ψ for an unrelated atomic ψ. But this is absurd: from the fact that some ϕ is obligatory, we cannot conclude that some unrelated ψ is permissible. What this shows is that, *if* (FC) states a semantic fact, (ROSS) had better state a semantic fact as well; without it, we would have a kind of deontic explosion principle. This reconstruction would allow us to explain what's wrong with the inference spotlighted by Ross in a way that fits the intuition that free choice and Ross are connected, and which suggests that a semantic account of the former entails a semantic account of the latter.

[1] In standard modal logic, this assumption—in the form $\Box\phi \vDash \Diamond\phi$—is a consequence of seriality, the assumption that every world in the model is related to at least one world by the modal accessibility relation.

[2] The name 'Deontic Tonk' is a tribute to the "runabout" connective *tonk* in Prior (1960).

(FC), which involves the word MAY, is a positive datum—a datum about a *felt* entailment. (Ross), which involves the word OUGHT, is a negative datum—a datum about a *lack* of felt entailment (a feeling of non-sequitur). But it is worth emphasizing that this is state of affairs is just an accident of the puzzles that have become widely discussed in deontic logic: we could, if we wanted, isolate a positive and a negative datum for *each* of OUGHT and MAY:

The corresponding negative datum for MAY, we might call (FC-):

(FC-) $\quad\quad \text{MAY}\,A \not\Rightarrow \text{MAY}(A \text{ OR } B)$

To motivate (FC-), consider Case 3: suppose I say to you:

(4) You may take out the garbage.

You reason:

(5) I may take out the garbage.
Therefore,

(6) I may take out the garbage or have some whiskey.

(6) doesn't appear to follow from (5), any more than (3) seems to follow from (2). Moreover, we can make another an argument, similar to the one above, to the effect that (FC) entails (FC-). Suppose for *reductio* the negation of (FC-): suppose, that is, that MAY A entails MAY(A OR B). That would give us

(Deontic Tonk, II)	$\text{MAY}\,G$	Assumption
	$\text{MAY}(G \text{ OR } W)$	Assumption for *reductio*
	$\text{MAY}\,G \wedge \text{MAY}\,W$	free choice
	$\text{MAY}\,W$	conjunction elimination

Here, we begin with an assumption MAY ϕ and derive MAY ψ for an unrelated atomic ψ. This is also absurd. What this shows is that, if (FC) states a semantic fact, (FC-) had better state a semantic fact as well; if it does not, we would have on our hands a second kind of deontic explosion principle.

Finally, (FC) and (Modal Weakening) issue in a positive datum for OUGHT that matches the positive datum for MAY in (FC):

(Ross+) $\textsc{ought}(A \textsc{ or } B) \Rightarrow \textsc{may}\ A \wedge \textsc{may}\ B$

(Ross+) can be derived from the original (FC) and (Modal Weakening) via the reasoning in (Deontic Tonk)—just start at the second step:

$\textsc{ought}(G \textsc{ or } W)$	(3)
$\textsc{may}(G \textsc{ or } W)$	(Modal Weakening)
$\textsc{may}\ G \wedge \textsc{may}\ W$	(FC)

...from the point of view of a theorist who begins from free choice, the virtue of (Ross+) is simply that it rounds out the symmetry of the data.

Thus, taking the position that (FC) is semantic would appear to entail commitment to a semantic account of the rest of the data in Table 1, given (Modal Weakening), our nonexplosion principles, and the transitivity of entailment.

MAY:		
(FC-)	(Negative)	$\textsc{may}\ A \not\Rightarrow \textsc{may}(A \textsc{ or } B)$
(FC)	(Positive)	$\textsc{may}(A \textsc{ or } B) \Rightarrow \textsc{may}\ A \wedge \textsc{may}\ B$
OUGHT:		
(Ross)	(Negative)	$\textsc{ought}\ A \not\Rightarrow \textsc{ought}(A \textsc{ or } B)$
(Ross+)	(Positive)	$\textsc{ought}(A \textsc{ or } B) \Rightarrow \textsc{may}\ A \wedge \textsc{may}\ B$

Table 2.1: Data for OUGHT and MAY.

The foregoing unified perspective is the view of Ross's Puzzle, and of free choice, that I favor. In "OUGHT and Resolution Semantics" (2011), Fabrizio Cariani proposes a semantics for OUGHT which blocks embedded disjunction introduction—hence validating the negative datum in Ross's puzzle. Like (DEONTIC TONK), it also does this by a kind of symmetric entailment from the premise—for $\textsc{ought}(A \textsc{ or } B)$ to be true on Cariani's semantics, it must be the case that something can be said, deontically speaking, for *each* of the embedded disjuncts. However, Cariani stipulates this condition directly into the semantics for OUGHT, rather than allowing it to follow from the semantics for the object-language MAY: when $\textsc{ought}(A \textsc{ or } B)$ is true, we may conclude in the metalanguage that both A and B have some positive deontic status.

Further, on Cariani's account, (OUGHT to disjunct permissibility)—and hence the blockage of disjunction introduction—holds only in virtue of a pragmatic consequence of the prejacent of $\textsc{ought}(A \textsc{ or } B)$'s being a disjunction. In an intensional semantics, the sentence 'I give Otto a glove' can be used to pick out a proposition—say, the set of possible worlds in which Melissa gives Otto a glove.[3] On a classical account of disjunction, the sentence 'I give Otto a left glove or I give Otto a right glove' picks out the same set of possible worlds. So when these sentences are embedded under OUGHT, their respective semantic contributions are identical.

The underlying idea is if a disjunction A OR B expresses a proposition p, it does so in a way that is semantically equivalent to any other classically equivalent way of picking out p, for all the semantics of OUGHT cares—*except* that this way of denoting p raises the propositions denoted by A and B to salience as alternatives. The added stipulation is simply that OUGHT is sensitive to alternatives, so this pragmatic effect is, in effect, "recycled" into the semantics. Disjunction, in particular, is still semantically classical in Cariani's semantics; the difference between a disjunctive prejacent and a classically equivalent non-disjunctive prejacent can only be a pragmatic one.

In this respect, Cariani's explanation for Ross's puzzle is in the same family as pragmatic explanations of free choice found in Kratzer & Shimoyama (2002); Aloni (2002); Aloni & van Rooij (2005); Fox (2007); Chierchia (2006) and Klinedinst (2006). These treatments employ complex operations, in the semantics, on alternatives. But the notion of an *alternative* being used appears to be a pragmatic one—the role alternatives play in these treatments runs in tandem with a resolutely *classical* semantics for disjunction.[4]

I am skeptical of approaches to (FC) and (Ross) in this vein, and in this chapter I will raise some problems for Cariani's view, centering around how to treat the deontic modal MAY and the behavior it displays in (FC). I begin by sketching Cariani's account and attempting to pin down the conceptual basis of his view. I then ask whether Cariani's semantics for OUGHT commits him to any view about MAY. I argue that trouble lurks on either side of this question.

2.2 Cariani's Account of (R)

Cariani's account can be seen as a development of many proposals in the literature on deontic modals. He begins with two widely accepted ideas: the first is that deontic modals exhibit context-sensitivity. This can, inter alia, capture the way deontic modal claims might depend on contingencies—features of the *world* of the context—such as which rules we

[3] I speak here in the jargon of possible worlds, but the same point could be made in any intensional semantics which is not hyperintensional.

[4] I am not sure that all the authors listed would agree with my characterization of their understanding of alternatives as pragmatic. Nonetheless, the basic strengthening mechanism assumes the classical entailment properties of OR and a 'diamond' modal analysis of MAY. See Chapter 1 for discussion.

have voted in favor of. The second is that deontic modals exhibit information sensitivity. Sensitivity to information can capture, for example, whether a subject knows which actions would bring about the outcomes that would be most beneficial. To this, Cariani's semantics adds a distinctive sensitivity to underlying sets of alternatives. These threads are incorporated as follows:

- There is a stable proposition expressed by any prejacent sentence ϕ at context c, called $[\phi]^c$.[5]
- Deontic modals like OUGHT quantify over a domain of epistemic possibilities (MacFarlane & Kolodny, 2010). This set of epistemically possible worlds is called $\mathscr{M}$.
- There is a *partition*—a set of mutually exclusive and jointly exhaustive propositions[6]—$\mathscr{Q}$. Propositions which are cells of $\mathscr{Q}$ are *options* in $\mathscr{Q}$. Propositions which are cells of $\mathscr{Q}$ *or* unions thereof are *visible propositions* in $\mathscr{Q}$.
- The partition $\mathscr{Q}$ combines with the epistemic space $\mathscr{M}$ to give a partitioned information state $\mathscr{Q} \oplus \mathscr{M}$. The cells of this information state are the propositions in $\mathscr{Q}$, each individually intersected with $\mathscr{M}$. We can think of this as the epistemic space $\mathscr{M}$ *at resolution* $\mathscr{Q}$ (Yalcin, 2011), and extend the concept of options and visible propositions to it.[7]

I pause to give an example. Suppose that $\mathscr{Q} = \{$*Obama wins*, *Romney wins*, *Herman Cain wins*$\}$.[8] Seven propositions are visible at this resolution.[9] My information state $\mathscr{M}$ rules out the possibility of Cain's winning. So $\mathscr{Q} \oplus \mathscr{M}$ is a partitioning of my information state into just two live possibilities—$\{$*Obama wins* $\cap \mathscr{M}$, *Romney wins* $\cap \mathscr{M}\}$. There are four visible propositions in this partitioned information state. Note that in addition to *all* the worlds in *Cain wins*, plenty of worlds in the live options of $\mathscr{Q}$ are also knocked out by intersection with what I know—for example, worlds where Obama wins but Paris is in England.

Continuing with Cariani's formalism:

[5]Cariani defines this as a function from worlds to truth-values, and taps Lewis (1980)'s notion of context:"a location—time, place, and possible world—where a sentence is said" in his definition.

[6]The cells of the partition union to logical space, W (13). Presumably it is the latter, otherwise whether $\mathscr{Q}$ is a partition would depend on $\mathscr{M}$.

[7] It's therefore possible that an option may fail to be epistemically possible (that is, for every world in the cell to be in $\mathscr{M}$.

[8]I use italics here to mark propositions.

[9]Including the whole space (the disjunction of all three) and the empty set (the disjunction of none).

- A deontic ranking function takes the pair $(\mathscr{M}, \mathscr{Q})$ as an argument, and orders the individual alternatives in $\mathscr{Q} \oplus \mathscr{M}$ for deontic goodness, using the weak preference relation $\succeq$. $o_1 \succeq o_2$ iff o_1 is at least as good as o_2.

- The $\succeq$-relative ranking of visible propositions in $\mathscr{Q}$ has two aspects. It ranks the options in $\mathscr{Q}$ ordinally—from best to worst, allowing for ties—and also relative to an absolute deontic "benchmark" (Cariani, 2011, pg. 6). The best of these options is $\beta(\mathscr{M}, \mathscr{Q})$.[10]

- All these parameters are "initialized" by the context c, giving us $\mathscr{M}_c$, $\mathscr{Q}_c$, $\succeq_c$ and $\beta_c(\mathscr{Q}_c, \mathscr{M}_c)$.

Finally,

- OUGHT ϕ is true at a context c iff (i) ϕ expresses a proposition, p, which is visible in the partition, (ii) all the alternatives that entail p rank above the benchmark, and (iii) p is entailed by all the best-ranked alternative(s), $\beta_c(\mathscr{Q}_c, \mathscr{M}_c)$.

Hence the official semantic proposal is:

> $c, w \vDash$ OUGHT ϕ iff
>
> **Visibility:** $[\phi]^c$ is visible in $\mathscr{Q}_c \oplus \mathscr{M}_c$.
> **Dominance:** for every o in $\beta_c(\mathscr{Q}_c, \mathscr{M}_c)$, o entails $[\phi]^c$.
> **Strong Permissibility:** for every $o \in (\mathscr{Q}_c \oplus \mathscr{M}_c)$ s.t. o entails $[\phi]^c$, $o \succeq$ benchmark.
> (Cariani, 2011, pg. 14)

Visualizing the Resolution

For convenience, I will be adding some terminology and some pictures which spotlight the moving parts of the proposal. First, the ordering $\succeq$ comes from a ranking function F, which takes the partitioned state $(\mathscr{M} \oplus \mathscr{Q})$ as argument.[11] F is the source of deontic opinion, brought to bear on the partitioned information state $(\mathscr{M} \oplus \mathscr{Q})$. We'll have occasion to refer back to F.

[10] Cariani does not consider, in his Cariani (2011), cases in which options in $\beta(\mathscr{M}, \mathscr{Q})$ are below the benchmark; it is not clear, from his definitions, whether this is possible.

[11] It helps here to specify that $(\mathscr{M} \oplus \mathscr{Q})$ is a set of propositions which union to $\mathscr{M}$.

In addition to the ordering of visible cells in $\mathscr{Q}$ under $\succeq_F$, there is also the benchmark of the relevant ordering, which we will also want to refer back to. I am not sure whether the benchmark is fixed by the deontic worldview F, or by the information state $\mathscr{M}$, or by the options $\mathscr{Q}$; however, we can relativize to all three—using $BENCH_{F(\mathscr{M}\oplus\mathscr{Q})}$— to identify *the benchmark of the ordering F puts on* $(\mathscr{M}\oplus\mathscr{Q})$.

I will use $PERM(F(\mathscr{M}\oplus\mathscr{Q}))$—the set of basic permissibles according to $F(\mathscr{M}\oplus\mathscr{Q})$—to refer to the set of visible options which are ranked above $BENCH_{F(\mathscr{M}\oplus\mathscr{Q})}$. And to keep all relativization above-board, I'll also be renaming β—the function that picks the *best* $\succeq_F$ ranked option(s)—$OPT(F(\mathscr{M}\oplus\mathscr{Q}))$.

So, according to Cariani, at a fixed context c the output of $F_c(\mathscr{M}_c\oplus\mathscr{Q}_c)$ looks like this (where some $\succeq$-related options may be ties):

$$\underbrace{OPT(F(\mathscr{M}\oplus\mathscr{Q}))...\succeq o_i\succeq o_j\succeq o_n\succeq o_m...\succeq BENCH_{F(\mathscr{M}\oplus\mathscr{Q})}}_{\text{options above benchmark}}...o_1\succeq o_2$$

How to evaluate OUGHT ϕ? Since $[\phi]^c$ must be visible in $(\mathscr{M}_c\oplus\mathscr{Q}_c)$, it is equivalent to the union of some set of options in $(\mathscr{M}_c\oplus\mathscr{Q}_c)$; visually, the "$\phi$-options" are strung out along the ordering $\succeq$. Cariani's truth conditions for OUGHT ϕ are simply that (i) each of the ϕ-options is in $PERM(F_c(\mathscr{M}\oplus\mathscr{Q}))$, and (ii) $OPT(F_c(\mathscr{M}\oplus\mathscr{Q}))$ itself is a set of $[\phi]^c$-options.

$$\underbrace{OPT(F(\mathscr{M}\oplus\mathscr{Q}))}_{\phi}\succeq\underbrace{o_i}_{\phi}\succeq o_j\succeq\underbrace{o_k}_{\phi}\succeq o_n...BENCH_{F(\mathscr{M}\oplus\mathscr{Q})}...o_1\succeq o_2$$

It's easy to see how this semantics can explain the negative datum in (Ross), given a Boolean analysis of OR ($\vee$):

(Ross) $\qquad$ OUGHT $A \not\Rightarrow$ OUGHT(A OR B)

...some option in the introduced disjunct B is below benchmark.

The details are these. A counterexample to (Ross), on Cariani's semantics, begins with a true premise that OUGHT A. Hence $[A]^c$ must be visible in context. It proceeds to the non-sequitur conclusion that OUGHT$(A \vee B)$. Hence $[A \vee B]^c$ must also be visible in context.[12] It follows from the definition of a partition $\mathscr{Q}$ that if p is visible in $\mathscr{Q}$ and q is visible in $\mathscr{Q}$ and p entails q, then $p \setminus q$ is visible in $\mathscr{Q}$. Hence $[B]^c$ is visible in $\mathscr{Q}_c$ on the assumption that disjunction is classical. So, if OUGHT A is true and OUGHT$(A \vee B)$ false, both $[A]^c$ and $[B]^c$ are visible in $\mathscr{Q}_c$.

Beyond this, for a counterexample to the inference it is necessary and sufficient to stipulate that, while $[A]^c$ is ranked above the benchmark, $[B]^c$ is ranked below it. Because OUGHT A is true, every $\mathscr{Q}$-visible way of $[A]^c$-ing must be above benchmark (we will return to this point below). Assuming **(Visibility)** is satisfied, the only two conditions left for determining the truth-values of OUGHT A and OUGHT$(A$ OR $B)$ in context are **(Dominance)** and **(Strong Permissibility)**. But if $[A]^c$ satisfies **(Dominance)**, so does $[A \vee B]^c$—if the former is a *necessary* condition on optimality, so is the latter. Hence if the premise is to be true and the conclusion false, $[A]^c$ must satisfy **(Strong Permissibility)** while $[A \vee B]^c$ does not. Cashed out in Cariani's intuitive talk of "ways," the Ross entailment from OUGHT A to OUGHT$(A$ OR $B)$ fails if and only if there is some below-benchmark, visible way of performing the consequent—viz., some below-benchmark, visible option in $[B]^c$.

A Conservative Extension

In glossing Cariani's view, I have restricted myself to talk of whether a certain option $[\phi]^c$ is "ranked above benchmark," without specifying whether it is *permissible*, and without specifying whether MAY ϕ is true. This is because faithfulness to Cariani's account requires staying clear on the fact that his account does not, by itself, make any predictions about the semantics of the object-language MAY.[13] Cariani's semantic account of OUGHT could, in theory, apply to a language that does not have the word MAY or anything like it.

Although the negative datum in Ross's puzzle can be expressed in a language without MAY, it is impossible to capture the rest of the data in Table 1 without it. From the unified perspective with which we began, that is a somewhat unsatisfactory situation. Although Cariani's analysis *could* be applied to a language without MAY, it is *in fact* to be auditioned

[12] Cariani clearly regards (ROSS) as a case where the premise is true and the conclusion *false* (not merely undefined), so this is the case we will explore. See Cariani *op. cit.*, pg. 2: "The anti-boxer suggests that the unacceptability of Ross's inference be taken at face-value...in the relevant context, [OUGHT$(\phi$ or $\psi)$] is false even though [OUGHT(ϕ)] is true."

[13] In particular, he explicitly disavows a connection between OUGHT and MAY via duality (Cariani, 2011, pg. 16)—the standard way, on a classical modal semantics, of extrapolating from an account of OUGHT (modeled as universal quantification) to an account of MAY (modeled as existential quantification).

against an object language which does contain this operator—and moreover, contains the datum (R+), which provides a necessary condition for the truth of OUGHT$(A$ OR $B)$ in terms of MAY.

(Ross+) OUGHT$(A$ OR $B) \Rightarrow$ MAY $A \wedge$ MAY B

Moreover, the way Cariani's view blocks the inference in (Ross) obviously appeals to the *concept* of permissibility, via the (**Strong Permissibility**) clause. It would be desirable to marry the explanatory resources already in Cariani's metalanguage to the extra data in the object language. Can Cariani's apparatus be extended to the positive datum in (ROSS+), once again given an analysis of OR as Boolean $\vee$?

Here we start with the truth of OUGHT$(A \vee B)$ at a context c, and consider the truth-values of MAY A and MAY B at c. How do we know that the sentences in the conclusion MAY $A \wedge$ MAY B even have truth-values at all, since they may be "invisible" in $\mathscr{Q}_c$? Here we cannot play the trick we played before: considered as a set of possible worlds, there is no guarantee that the finer propositions $[A]^c$ and $[B]^c$ are visible in $\mathscr{Q}_c$ whenever their union $[A \vee B]^c$ is. So to even consider the inference, it seems we must make an additional assumption:

> (Disjunct Visibility.)
> If ϕ occurs in a context c and ϕ is a disjunction, then every disjunct of ϕ is visible in ALT_c.

In the guise of a thesis about which alternatives are *relevant* in a context where a disjunction occurs, (Disjunct Visibility) is near universally endorsed by theorists working on Ross's Puzzle and on free choice permission. (For its prominence in the FC literature see Kratzer & Shimoyama (2002); Aloni & van Rooij (2005); Alonso-Ovalle (2006); Chierchia (2006); Fox (2007); Geurts (2009); Klinedinst (2006); Franke (2011); for Ross's Puzzle, see Wedgwood (2006); Follesdal & Hilpinen (1971); for a more general version endorsing the visibility of sub-clauses of ϕ, see Yalcin (2011).) So we will adopt it for now.

If OUGHT$(A \vee B)$ is true and (Disjunct Visibility) holds, then every way of doing $[A]^c$ *and* every way of doing $[B]^c$ is above benchmark in $F(\mathscr{M}_c \oplus \mathscr{Q}_c)$. By the positive datum in (ROSS+), this is also a case where MAY A is true and where MAY B is true; we can relate Cariani's explanation for (Ross) to the semantics of MAY via a fairly plausible constraint on the latter:

(MAY-1) If every $o \in (\mathscr{Q}_c \oplus \mathscr{M}_c)$ s.t. o entails $[\phi]^c$ is above benchmark, then $c, w \vDash$ MAY ϕ.

This is a sufficient condition for the truth of MAY ϕ at c. However, we as yet have no necessary condition.

While (MAY-1) seems unremarkable, it gives rise to a kind of closure principle for deontic permissibility which is lacking in the original account. Cariani's formal apparatus provides a basic notion of permissibility in the metalanguage—that of being above benchmark—which is confined to nonoverlapping cells of a partition. Assuming that the object language MAY also tracks permissibility, (MAY-1) allows us to extend that notion to the whole of $[\phi]^c$, which is a non-atomic, visible consequence of these propositions.[14] We can express this as follows:

> (C): If every visible way of doing $[\phi]^c$ is deontically permissible in c, then $[\phi]^c$ is deontically permissible in c.

We will return to closure principles below; first, though, we turn to an analysis of Cariani's original account of OUGHT.

2.3 Trouble for Strong Permissibility

The requirement of (**Strong Permissibility**) says that an option $[\phi]^c$ cannot be the prejacent of a true OUGHT-claim at c unless *every* epistemically possible $(\mathscr{M}_c \oplus \mathscr{Q}_c)$-visible way of carrying out ϕ is ranked above benchmark. If (**Strong Permissibility**) is indeed a necessary condition for the truth of OUGHTϕ, then, as suggested above, there is a natural extension from (Ross) to (Ross+), via the explanation of *how* the inference in (Ross) fails, when it does.

(**Strong Permissibility**) is, however, a difficult principle to defend. It doesn't seem that we use a principle like this in deciding what to do, and this raises doubts about whether it could really be a hidden feature of what we *ought* to do. In that spirit, consider the case of

> **Professor Punctual.** Professor Punctual is invited to review a book on whose subject matter he is the world's foremost expert. If Punctual accepts the invitation and writes the review, the book will receive a high-quality assessment—this is the best possible outcome. If Punctual accepts and does not write, the delay will constitute an injustice to the author and an embarrassment for the journal. If Punctual declines the invitation, another, less-

[14] It would not be possible in Cariani's framework to say, for example, that if every visible way of doing $[\phi]^c$ is above benchmark in c, then $[\phi]^c$ is above benchmark in c, since only nonoverlapping cells can be in the $\succeq$-ordering.

> qualified person will write a mediocre review. Finally, Professor Punctual is dutiful. He indefatigably fulfills his commitments in a timely manner.

It seems perfectly normal for Professor Punctual to accept, and overwhelmingly natural to say that he *ought* to accept. However, there is a salient way of accepting the invitation to write the review that would bring about the worst possible outcome, in which the review is never written. This is obviously a feature Punctual's case shares with the case of his colleague, Professor Procrastinate (Jackson & Pargetter, 1986). If (**Strong Permissibility**) is really a necessary condition on the truth of OUGHT claims, Punctual should never accept commitments of this kind. So it would *not* seem to be a necessary condition on the truth of an OUGHT ϕ-claim that *every* way of ϕ-ing is above some salient deontic benchmark. Call this *the objection from following-through*: we do not, as a practical matter, limit ourselves to actions such that every way of carrying them out is permissible. Nor does rationality appear to recommend this.

Is it possible that, because of Punctual's punctuality, the option of accepting and failing to write is necessarily *invisible* in context? Consider dialogues with *fronted* alternatives:

(7) a. May I bring some wine to the party?
b. No—the host is allergic. But you ought to bring something.

On a straightforward application of Cariani's semantics, this dialogue is inconsistent. In (9-a), the possibility of bringing wine to the party is explicitly raised and classified as impermissible. But then it seems that it cannot be true that *every* $(\mathscr{Q}_c \oplus \mathscr{M}_c)$-visible way of bringing something to the party is above benchmark. The foregoing dialogue should be just as bad as:

(8) a. May I bring some wine to the party?
b. No—the host is allergic. But you ought to bring something that is wine or something that is not wine.

yet the latter seems clearly worse.

A Less Conservative Extension

What is the proper response to these complaints?[15] It seems there are two routes: first, Cariani can abandon (**Strong Permissibility**). But abandoning it would be to lose both (Ross) and (Ross+)—and hence not to have an explanation for *any* of the data with which we began.

The second option is to double down on (**Strong Permissibility**). The view would be strengthened, though, if we could add yet again to the amount of intuitively *correct* data that a commitment to (**Strong Permissibility**) explains. This is where free choice comes in.[16] In the current state of play, we have a semantics which holds that

(Ross+) $\quad\quad \textsc{ought}(A \text{ or } B) \Rightarrow \textsc{may}\,A \wedge \textsc{may}\,B$

is a semantic entailment. Free choice is the corresponding entailment for MAY:

(FC) $\quad\quad \textsc{may}(A \text{ or } B) \Rightarrow \textsc{may}\,A \wedge \textsc{may}\,B$

Is this a semantic entailment, or merely some sort of pragmatic effect? Note that we are one step away from being able to explain (FC) in its strong (positive) form, thus getting (FC-) into the bargain for free: this is to make (**Strong Permissibility**) both sufficient *and necessary* for the truth of MAY ϕ. I will tentatively add this operator to Cariani's language:

> (STRONG PERMISSION MAY.) $c, w \vDash \textsc{may}\ \phi$ iff
>
> **Visibility:** $[\phi]^c$ is visible in $(\mathscr{Q}_c \oplus \mathscr{M}_c)$.
> **Strong Permissibility:** for every $o \in (\mathscr{Q}_c \oplus \mathscr{M}_c)$ s.t. o entails $[\phi]^c$, $o \succeq$ benchmark.

This entry would close the gap in our semantics for MAY, giving us truth-conditions which are completely determined by the $F(\mathscr{M}_c \cap \mathscr{Q}_c)$ apparatus. And on the assumption of (Disjunct Visibility), it would explain (FC). Although this move doesn't solve the problems I raised in the last section, it may help to shift the balance of considerations, since a

[15] Note that this argument doesn't rely on assuming that bringing wine is an *option* in context—only that it is *visible* in context (the basic options could be finer still). By our sufficient condition on the truth of MAY ϕ, if MAY ϕ is false, then it is not the case that every way of $[\phi]^c$-ing is above benchmark. If it is not the case that every way of bringing wine to the party is above benchmark, then a fortiori it is not the case that every way of bringing *something* to the party is above benchmark.

[16] It is also possible to keep the commitment to (**Strong Permissibility**) by abandoning (DISJUNCT VISIBILITY). But this would also lose us (ROSS+), and reduce the number of cases in which we have an explanation for (ROSS).

Cariani who takes this step (call him Cariani*) can then martial the intuitions in (FC) and (FC-) to bolster his semantics of OUGHT against these complaints.

Extending (**Strong Permission**) to object-language MAY in this way is a venerable approach to the semantics of MAY. It is called "strong permission," and was originally espoused in the deontic logic of von Wright (1951) who, in turn, was originally motivated by the puzzle of free choice permission. von Wright argued that what it means to say "you may ϕ" to someone is to give him or her permission to ϕ "in every way." This paraphrase in terms of *ways* thus matches Cariani's informal gloss on ALT: what it means for agent α to have strong permission to ϕ, in a resolution-sensitive framework, is that every ALT-visible way of ϕ-ing is permissible for α.

2.4 Pressure towards Strong Permission MAY

In this section, I will rachet up the pressure on Cariani's view to take on the above entry for MAY. There are two distinct inducements. The first is that without it, Cariani's explanation for (R+) is amenable to a competing pragmatic explanation. The second is that *with* it, there is a closure principle for deontic permissibility which can *predict* all four of our explananda involving disjunction—(Ross), (Ross+), (FC), and (FC-).

The Pragmatic Angle

First, there there is significant tension involved in rejecting (STRONG PERMISSION MAY). To do this would amount to holding that (ROSS+) is a semantic entailment *while* (FC) *is not*. But it is an awkward fit with the data to claim that

(9) You may post the letter.
So, you may post the letter or burn it.

is a semantically valid (though perhaps somehow pragmatically infelicitous) argument, while

(10) You ought to post the letter.
So, you ought to post the letter or burn it.

is semantically invalid; from the point of view of speaker phenomenology, there is little to tell apart (9) and (10).

The second is that such a position reverses the relationship between (Ross+) and (FC) from the point of view of Gricean pragmatics. (Ross+) can be explained by a standard-issue application of Quantity Implicature reasoning, while (FC) cannot.

Quantity Implicature reasoning invokes Grice (1989)'s Maxim of Quantity: "make your contribution as informative as required." Such inferences mine significance from a speaker's act of omission: from her *declining* to assert some salient, more informative alternative p. Suppose the speaker said "p", and "q" is a salient alternative to "p." There are two immediately obvious reasons for omitting to make the assertion "q":

(i) the speaker fails to know that the proposition "q" expresses in context is true: $\neg K_s q$.

(ii) the speaker positively knows that it is false: $K_s \neg q$.

Postulating that implicatures obtain because of speakers' and hearers' use of these two conditions is a prototypical move in Gricean pragmatics, familiar from Chapter 1; a standard quantity implicature results in the case where both steps go through, and the conclusion that $\neg q$ is reached. What the speaker did say, p, is then "enriched" with $\neg q$, for a total propositional meaning of $p \wedge \neg q$.

This blueprint is sufficient to generate pattern (Ross+) on a classical (i.e. modal box) semantics for OUGHT, if we assume the stronger alternatives to an utterance of "OUGHT(A OR B)" are "OUGHT A" and "OUGHT B." (I use double-quotes to mark speech acts, and put the reasoning in terms of boxes, diamonds, and $\vee$ to mark that we are in the classical semantics):

> The speaker said "$\Box (A \vee B)$." Why didn't she say "$\Box A$," which is stronger, and hence more informative? Perhaps she fails to know that "$\Box B$" is true. (Step i). Furthermore, since she is well-informed about the subject matter, it is reasonable to assume that if she doesn't know it, it's false (Step ii). Hence $\neg \Box A$ is true.

Exactly parallel reasoning for the alternative utterance "$\Box B$" will give us

> $\neg \Box B$ is true.

Finally, we use these conclusions to "enrich" what was said:

$$\Box(A \text{ OR } B), \neg\Box A, \neg\Box B \vDash \Diamond A \wedge \Diamond B.$$

The availability of this standard-issue Gricean explanation for (Ross) and (Ross+)—the same kind that underlies the pragmatic inference from "some" to "not all"—is the reason Ross's Puzzle has been widely classified as a pragmatic, and not semantic, phenomenon (see, for example, Wedgwood (2006); Follesdal & Hilpinen (1971)).

Cariani argues extensively in his paper that the Gricean approach to explaining (Ross) fails—briefly, because of the embedding and retraction behavior of the premise (Cariani, 2011, pg. 17-20). Kai von Fintel responds to these arguments in (von Fintel, 2012). I will not enter the fray here—suffice it to say that the issue is subtle. All I want to do here is to point out that the flatfooted Gricean explanation *definitely* fails for (FC), taking the premise to be a modal diamond, rather than a modal box, consider once again the utterance "MAY(A OR B)" (= "$\Diamond(A \vee B)$") and the stronger alternatives "$\Diamond A$" and "$\Diamond B$."

We reason as follows:

> The speaker said "$\Diamond$ (A or B)." Why didn't she say "$\Diamond$ A," which is stronger, and hence more informative? Perhaps she fails to know that "$\Diamond$ A" is true. (Step 1). Furthermore, since she is well-informed about the subject matter, it is reasonable to assume that if she doesn't know it, it's false (Step 2). Hence $\neg \Diamond A$ is true.

Exactly parallel reasoning for the alternative utterance "$\Diamond B$" will give us

> $\neg\Diamond B$ is true.

But it is a familiar point that an attempt to use these conclusions to "enrich" what was said results in a contradiction:

$$\Diamond(A \vee B), \neg\Diamond A, \neg\Diamond B \models \bot.$$

This is not, of course, a demonstration that Gricean maxims will never explain the free choice effect; as discussed in Chapter 1, attempting to explain the free choice effect through more sophisticated Quantity Implicature maneuvers is a thriving area of research. But the sheer volume and diversity of ongoing work on the subject (Kratzer & Shimoyama, 2002; Aloni & van Rooij, 2005; Alonso-Ovalle, 2006; Chierchia, 2006; Fox, 2007; Geurts, 2009; Klinedinst, 2006; Franke, 2011) suggests a Gricean-style answer will not be easy to find (it is much harder to come up with a recipe that generates the *right* conclusion for the free choice inference, $\Diamond A \wedge \Diamond B$, than it is to find something *wrong* with the rough sketch above.) Judging from a survey of issues that are considered open problems in contemporary pragmatics, it is the connection with free choice that gives Ross's Puzzle real teeth.

2.5 Closure, Multiple Realizability, and Risk

In §1.2, we saw that Cariani's original entry for OUGHT, combined with Disjunct Visibility and (MAY-1):

> (MAY-1): If every visible way of doing $[\phi]^c$ is above benchmark in c, then $c, w \vDash$ MAY ϕ.

could account for (R+). Undergirding (MAY-1) was a relatively uncontroversial closure principle (C):

> (C): If every visible way of doing $[\phi]^c$ is deontically permissible in c, then $[\phi]^c$ is deontically permissible in c.

As we saw above, Disjunct Visibility and (STRONG PERMISSION MAY) can account for (FC), the parallel datum which neutralizes the need for an independent explanation of (R+).[17] But the principle of closure underlying (STRONG PERMISSION MAY) is in fact the *converse* of (C):

> (C-converse): If $[\phi]^c$ is deontically permissible in c, then every visible way of doing $[\phi]^c$ is deontically permissible in c.

This principle, which echoes von Wright's original remarks about strong permission, is quite a strong, and surprising, view about the nature of deontic permissibility.[18]

2.6 Maximin Semantics?

What is the best conceptual underpinning of such a view? It seems that the best answer to this question averts to a Maximin strategy.[19] This sees the multiple realizability of visible propositions as *risk*. Recall that Cariani's way of modeling what context provides gives us a view of visible propositions as either options, which can be directly ranked relative to a benchmark, or as multiply realizable unions of options, whose constituents can be so ranked. In particular, given Disjunct Visibility, a disjunctive prejacent is a multiply realizable proposition, whose realizations will always include its disjuncts; these disjuncts can

[17] Given Modal Weakening (§1).

[18] Moreover, it is not a view about the nature of a particular kind of speech act, as von Wright's view can be construed as a view about a special kind of permission-giving. Rather, strong permissibility is built into the semantic architecture of both OUGHT and MAY.

[19] For a discussion of Maximin strategies in decision theory, see Resnik (1987).

be further broken down into options that are ranked relative to the benchmark. Finally, our use of closure principles reflects the project of extending the notion of being above benchmark to multiply realizable propositions under a unified notion of permissibility, which receives expression in the semantics of MAY.

(C-converse) fits the rationale of Maximin when the multiple realizability of a visible proposition p is viewed as an outcome which could result in any visible option that entails p. A maximin strategy advises an agent to deal with this risk by assuming the worst. Hence, a risky option has a certain value just in case its worst realization has that value. We can use maximin reasoning to argue that $(A \vee B)$ is better than C at a context iff *both* $[A]^c$ and $[B]^c$ are better than $[C]^c$ at that context. It follows that $(A \vee B)$ is better than the benchmark at a context iff *both* $[A]^c$ and $[B]^c$ are better than the benchmark at that context: if MAY$(A \vee B)$ is true at the context, so are MAY A and MAY B. The Strong Permission component of Cariani's OUGHT then falls out of Weakening: the fact that OUGHT entails MAY at a context.
Incorporating this thought into the semantics of MAY ϕ allows us to state its semantic value directly in terms of the language of $\succeq$:

(MAXIMIN SEMANTICS FOR MAY.) $c, w \vDash$ MAY ϕ iff
(i) $[\phi]^c$ is visible in ALT_c;
(ii) $Minimal(\phi) \geq_\beta BENCH$, where $Minimal(\phi) =_{df}$ lowest-$\succeq$-ranked visible member of $[\phi]^{ALT}$.

Defining the clause for MAY this way straightaway gives the result that, where $[A]^c$ and $[B]^c$ are visible options, $c, w \vDash$ MAY$(A \vee B)$ iff both $c, w \vDash$ MAY A and $c, w \vDash$ MAY B. On the assumption that the best option, $OPT(F(\mathscr{M}_c \oplus \mathscr{Q}_c))$, is itself ranked above benchmark, we get (ROSS) and (R+). Maximin holds out the promise a decision-theoretically intelligible rationale which unifies the data.

To keep the result in check—to keep it from collapsing into an *unrestricted* Strong permission view—we need only say that there may very well not be any resolution available which is *finer* than that needed to make the disjuncts $[A]^c$ and $[B]^c$ visible. This way we avoid the result that e.g.

(11) You may have coffee or tea.
MAY$(C \vee T)$

entails

(12) You may have coffee.
MAY C

which in turn entails

(13) You may have coffee and rob a bank.
MAY$(C \wedge B)$

....simply because $[C]^c$ is visible in context does not mean that $[C \wedge B]^c$ is. Hence this view avoids the problem of *unrestricted conjunction introduction* in the scope of MAY that dogged von Wright's original view (see the discussion in Asher & Bonevac (2005, pg. 306-308)).

2.7 Restricted Downward Closure

The foregoing concludes my arguments in favor of a (STRONG PERMISSION MAY) view that is *generated* by a view of how a notion of deontic permissibility propagates across propositions visible at a contextually determined resolution. What are its drawbacks? The problems for (STRONG PERMISSION MAY) mirror the problems for Cariani's strong permission theory of OUGHT. It seems we can construct Professor cases in which it is true that

(14) Punctual may accept.

but it is false that

(15) Punctual may accept and fail to write.

So it seems, as much as in the OUGHT case, that the requirement *that every way of accepting be permissible* is too strong. Even when we temper this claim with the proviso that it is only the *visible* ways of accepting that must be permissible, we can generate cases with fronted alternatives:

(16) a. May I bring some wine to the party?
b. No—the host is allergic. But you can bring *something*.

Deliberatively, as well, we can raise the problem of following-through. It is implausible that we take a piecemeal approach to action, at each earlier moment minimizing the harm we can do at some later moment: rather, we often undertake actions which will make things go much worse, if we fail to follow through, than they would have been if we had never gotten involved. Since this is a pervasive feature of the kinds of actions we *do* undertake, it is hard to believe that this is not deontically permissible. But these are just the *same* considerations that made the corresponding principle for OUGHT implausible, re-phrased for MAY. So perhaps, from a conceptual point of view, the view is no *worse* off as a result of this extension.

What we have is a nascent semantics for OUGHT and MAY, built upon a ranking of options supplemented by a closure principle. These reflect a uniform stance on multiple realizability: a multiply realizable outcome is evaluated and ranked under the guise of its worst realization (as opposed to, for example, its realization in the nearest possible world(s) where it is true.) This is key to deriving the property by which the view blocks disjunction introduction. As such, it is a species of a genus of deontic logics which reject *upward-closure* inferences for the deontic modals: views which, when $p \vDash q$, accept the inference that if p has a certain deontic status, so does q.[20] It is worth noting that Cariani rejects the symmetry of the data between OUGHT and MAY here, suggesting in a footnote (Cariani, 2011, footnote 27, pg. 26) that the counterexamples to upward closure are more compelling in the OUGHT cases than in the MAY cases. In other words, the entailment the left is more compelling, and has more of a claim to be respected in our semantic theorizing, than the entailment on the right:

Where $p \vDash q$,	MAY p	OUGHT p
	$\therefore$ MAY q	$\therefore$ OUGHT q

I am skeptical this is so. Since I shall go on to deny that OR is Boolean union under the scope of deontic modals, I'll consider nondisjunctive cases of entailment, with 'so':

(17) You may call Karen.
So, you may pick up the phone.

(18) You ought to call the doctor.
So, you ought to pick up the phone.

I think it is undeniable that *both* of these arguments have intuitive force—*equal* intuitive force. There may, of course, be other cases which tell apart the compellingness of upward closure for MAY versus OUGHT; it is difficult to prove that no such case can be found. But in light of the argument given in this chapter, this move seems moot: on the best version of Cariani's view, both inference-patterns fail; the disjunctive case is a counterexample to both. Moreover, in the case of deontic permissibility, once one restricts to visible propositions, the closure pattern is precisely the *reverse* of the upward-closure pattern illustrated above.

[20] See, for example, the views canvassed in Sayre-McCord (1986a).

2.8 Conclusion

This chapter began by introducing Cariani's theory of OUGHT and its underlying architecture. I then subjected the view to an exercise in experimental evolution, by sketching the best extension of its formal apparatus and explanatory approach to the data involving MAY in Table 1.

The result seems to be that in an account that involves data from deontic MAY, it is MAY-statements, rather than OUGHT-statements, that become our guide to the underlying view of value; MAY is our object-language key to the kinds of metalanguage commitments Cariani* needs to make to fully flesh out his view. Finally, I argued that, given the free choice permission data, a Cariani-style view seems pulled inexorably towards a strong permission view of MAY, underwritten by Maximin reasoning. Such a view rests *a lot* of weight on the assumption that disjunction is classical in modal environments. If we are not satisfied with the possible views in this vein—as I am not—the natural place to look next is to non-classical accounts of disjunction.

Chapter 3

Factoring Disjunction Out

In the literature, free choice permission and Ross's puzzle have primarily been interpreted as bearing on the interpretation of deontic modals rather than bearing on the interpretation of OR.[1] My goal in this chapter is to contrast this strategy with a concrete approach to (Ross) and (FC-) from a less-examined angle: a revisionary semantics for disjunction.

First, I contrast two opposed modal views—call them the "box-diamond" theory and a simple Expected Utility (EU) theory—that form the two poles of the debate about these natural language modals. They differ on the question of monotonicity: on whether, when $\phi \vDash \psi$, MAYϕ $\vDash$MAYψ and OUGHTϕ $\vDash$ OUGHTψ. Both theories have confronted (Ross) and (FC-) on the assumption that OR is Boolean union.

Beginning in §4, I sketch a theory of OR in a 2-dimensional semantic framework, which allows us to preserve what is attractive about a monotonic view while giving a semantic account of these two data points. Finally, I respond to an argument from Cariani (2011) that raises a challenge for revisionary theories of disjunction.

3.1 The Standard Modal Theory versus Disjunction

Let us begin with a standard modal approach to deontic modal operators. To simplify our demands on the model—to avoid making stipulations, in particular, about accessibility relations—I consider a language without iterated or embedded modalities.[2]

[1] See, for example, Portner (2010); Cariani (2011); von Fintel (2012), and von Wright himself (von Wright, 1969).

[2] This initial presentation also abstracts away from context-sensitivity in its usual form. See §5 for discussion.

Definition 1 (Syntax)**.** Let $\texttt{At}$ be a set of propositional atoms $p_1, p_2\ldots$ The language of L^0 is as follows: if ϕ is in $\texttt{At}$, ϕ is a wff of L^0; and if ϕ, ψ are wffs of L^0, so too are $(\phi$ OR $\psi)$, $(\phi \wedge \psi)$, and $\neg\phi$. We define language L as follows: all wffs of L^0 are wffs of L. If ϕ is a wff of L^0, then OUGHTϕ, and MAYϕ are wffs of L.

Definition 2 (Models)**.** A **model** M is a triple $\langle W, R, V\rangle$ where W a nonempty set of possible worlds, R is a serial accessibility relation on W, and V is a function that maps elements of $\texttt{At}$ to truth-values relative to elements of W.

The seriality of R for any $w \in W_{\mathscr{M}}$ encodes, following Standard Deontic Logic (SDL), that the set of deontically ideal worlds is never empty: there is always something the agent *may* (deontically speaking) do.

Definition 3 (Semantics for Box-Diamond Modals)**.**
$w \vDash p$ iff $V(p,w) = 1$.
$w \vDash \phi$ OR ψ iff $w \vDash \phi$ or $w \vDash \psi$.
$w \vDash \phi \wedge \psi$ iff $w \vDash \phi$ and $w \vDash \psi$.
$w \vDash \neg\phi$ iff $w \nvDash \phi$.
$w \vDash$ OUGHT ϕ iff $\forall w'$ s.t. $wRw' : w' \vDash \phi$.
$w \vDash$ MAY ϕ iff $\exists w'$ s.t. wRw' and $w' \vDash \phi$.

What counts as deontically ideal relative to a world w—which worlds are R-related to w—may be contingent; it may depend, for example, on what is *known* at w, or what an agent is capable of in w.[3]

It is a result of the semantic entries for these quantificational modals that they are *Upward Monotonic*:

[3] The semantics of MacFarlane & Kolodny (2010) is an example of an information-sensitive theory. The premise semantics of Kratzer (1981) is a generalization of a circumstantial theory, according to which a set of premises determines what counts as good, where these premises may be inconsistent. The result is that worlds may be *ordered* by context, according to how many premises they satisfy. Modulo the Limit Assumption (Lewis, 1973), it will still be the case that OUGHT is a univeral quantifier, and MAY is an existential quantifier, over a modal base, which can be characterized as follows: any world in the modal base satisfies more premises than any world outside the modal base. For Kratzer's discussion of the Limit Assumption, see Kratzer (1981), §3.

(Consequence) $\Gamma \vDash \psi$ iff, for any model $\mathscr{M}$ and any $w \in W_{\mathscr{M}}$, if $\mathscr{M}, w \vDash \phi$ for all $\phi \in \Gamma$, then $\mathscr{M}, w \vDash \psi$.

(Upward Monotonicity (UM)) An operator Mod is upward-monotonic just in case, if $\phi \vDash \psi$, then $Mod\ \phi \vDash Mod\ \psi$.

((UM) for Deontic Modals) If $\phi \vDash \psi$, then OUGHT $\phi \vDash$ OUGHT ψ and MAY $\phi \vDash$ MAY ψ.

This result is discomfited by (FC-) and (Ross), where embedded OR introduction seems to be blocked. For on a Boolean OR, $\phi \vDash (\phi$ OR $\psi)$. On the standard modal theory, to the extent to which (Ross) and (FC-) strike us as problematic inferences, whatever is wrong with them must be explained in the pragmatics, rather than in the semantics.

3.2 EU to the Rescue?

A much different reaction to the data in (FC-) and (Ross) is to use it to overturn the standard modal operator semantics, and to give new entries for OUGHT and MAY that respect these inferences as semantic.

This route models OUGHT and MAY as reflecting the notions of obligatoriness and permissibility that are found in Expected Utility Theory. Expected Utility Theory enjoins an agent perform the act with the highest expected utility, or one of these options, when there are ties.

Given a syntax as in Definition 1, we can provide an Expected Utility model and entries for the modal operators OUGHT and MAY:

Definition 4 (EU Models and Expected Utility)**.** An **EU-model**[4] $\mathscr{M}$ is a tuple $\langle W, Pr, Val, Act, V\rangle$ such that W is a nonempty set of possible worlds and Pr is a probability function on $\mathscr{P}(W)$; for any $w \in W_{\mathscr{M}}$, Val_w is a function $\mathscr{P}(W) \to \mathbb{N}$ which, at a world w, takes a proposition p to a natural number (the utility of p, relative to w);[5] $Act \subseteq \mathscr{P}(W)$ is a set of available acts (closed under union), and V is a function that maps elements of `At`

[4]There are many expected utility models in the literature; the simplified one I present here most closely follows Goble (1996).

[5]It is point familiar from decision theory that an individual's preferences should be modeled by a *family* of such functions, unique only up to positive affine transformation von Neumann & Morgenstern (1944). I abstract from this detail here.

to truth-values relative to elements of W, as above. In addition, we introduce a notation for propositions: given a model $\mathscr{M}$ and a wff ϕ of L^0, $[\![\phi]\!]_{\mathscr{M}} := \{w \in \mathscr{M} : M, w \vDash \phi\}$.

Where $[\![\phi]\!]_{\mathscr{M}} \in Act$, $EU_w(\phi)$ is the **conditional expected utility** of $[\![\phi]\!]_{\mathscr{M}}$: $\sum_w (Pr(w_j | [\![\phi]\!]_{\mathscr{M}})) \cdot Val_w(w_j)$ for all $w_j \in W_{\mathscr{M}}$; the conditional expected utility of $[\![\phi]\!]_{\mathscr{M}}$ is **maximal** relative to an EU-model $\mathscr{M}$ and world $w \in W_{\mathscr{M}}$ iff $[\![\phi]\!]_{\mathscr{M}} \in Act_{\mathscr{M}}$ and $\neg\exists q \in Act_{\mathscr{M}} : EU_w(q) > EU_w([\![\phi]\!]_{\mathscr{M}})$.

Definition 5 (Semantics for EU Modality)**.** Conjunction, negation, and disjunction: the same as in Definition 3.
$\mathscr{M}, w \vDash \textsc{may}_{EU}\ \phi$ iff $EU_w([\![\phi]\!]_{\mathscr{M}})$ is maximal.
$\mathscr{M}, w \vDash \textsc{ought}_{EU}\ \phi$ is true iff $[\![\phi]\!]_{\mathscr{M}} = \bigcup\{q \in Act_{\mathscr{M}} : EU_w(q)$ is maximal$\}$.

EU modals have a swift take on the negative data in (Ross) and (FC-): the problematic inferences are not semantically valid. Whereas the quantificational modals are upward-closed, the EU notion of permissibility is *downward closed*: if $\phi \vDash \psi$, then $\textsc{may}\ \psi \vDash \textsc{may}\ \phi$. Since the expected utility of a multiply realizable option p is the (probability-weighted) average of its realizations, p's EU will be maximal only if the EU of all its realizations is also maximal.[6] Interpreting disjunction as Boolean, we get the result that, for example, if it is EU-permissible to have coffee or tea, then both the coffee option and the tea option must be EU-permissible.

$$\text{if } \phi \vDash (\phi \text{ OR } \psi), \text{ then } \textsc{may}(\phi \text{ OR } \psi) \vDash \textsc{may}(\phi)$$

Because EU permissibility is downward entailing, and EU optimality entails the EU permissibility of any option, Boolean disjunction introduction is also blocked in the scope of $\textsc{ought}_{EU}$. From the EU point of view, given Boolean disjunction, we get (Ross), (FC-) and the positive data (Ross+) and (FC) all in one go. In light of the difficulties for a pragmatic explanation of (FC)—canvassed in Chapter 1—the data of deontic disjunction appears, prima facie, to strongly to support such a view.

3.3 Does Natural Language Semantics Reflect EU Permissibility?

The ease with which the EU modals account for the puzzles of disjunction under modals raises a natural question: has anyone ever embraced these views? To my knowledge, no one has embraced both EU modals as a package, but they have appeared individually in the literature as a response to our puzzles.

[6] I ignore zero-probability options here.

MAY$_{EU}$ imports EU permissibility directly into the object language: if a proposition p is permissible and multiply realizable in context, then every realization, or every *way*, of doing p must be permissible. As discussed in Chapter 2, this notion of permissibility—*strong permissibility*—was proposed by von Wright (1969). EU theory can be seen as an extensive exploration and formal development of von Wright's notion of strong permissibility—the notion of permissibility which, to von Wright's ear, was simply manifest in (some) natural language uses of MAY. Von Wright did not, however, have anything like this to say about OUGHT.

For hints of an inclination towards the presence OUGHT$_{EU}$ in the object language, we can look to Goble (1996), Lassiter (2011) and Cariani (2011). Lassiter notes simply that $EU(\phi) \geq \theta$ does not imply $EU(\phi \vee \psi) \geq \theta$, where θ is some threshold for expected utility (26). Cariani's resolution-sensitive semantics for OUGHT ϕ, discussed in Chapter 3, requires that $[\![\phi]\!]_{\mathscr{M}}$ be an option in context and that every atomic act visible in this proposition be above some "benchmark" of permissibility that is accessible in the metalanguage. This is enough to block disjunction introduction in the scope of OUGHT, on roughly the same grounds as a more straightforward EU semantics would: the failure of the inference is explained by way of holding that the introduced disjunct is not (strongly) permissible. The main difference is whether this type of permissibility *is* (von Wright) or *isn't* (Cariani) identified explicitly with same brand of permissibility that provides the semantics for the object-language MAY.

3.4 Stacking Up Evidence

In Chapter 2, I canvassed two drawbacks for the view that OUGHT and MAY carry a requirement of strong permissibility: the view that, when $\phi \vDash \psi$, the permissibility of ψ entails the permissibility of ϕ.

- **Following Through**: We do not, as a practical matter, limit ourselves to actions such that every way of carrying them out is permissible. Nor does rationality appear to recommend this. (Example: *one borrows money without repayment* entails *one borrows money*, but the permissibility of borrowing does not seem to entail the permissibility of borrowing without repayment.)

- **Fronted Alternatives**: This doesn't reflect the way we talk: even if one *leads* a dialogue by saying that some way of ϕ-ing is impermissible, one may be permitted or obligated to ϕ. (Example: *one drives with one's eyes closed* entails *one drives*, but one may lead a dialogue with the claim that driving with one's eyes closed is impermissible, yet affirm that one may drive.)

This is evidence against a downward-monotonic view of MAY.

There is also evidence *for* upward monotonicity—that is, evidence *for* the classical view or one like it. Two bits of this evidence will be relevant for the dialectic here. First, there is the compellingness of upward-monotone inferences that do not involve disjunction. Two examples, presented in Chapter 3 (repeated below), involved embedded instances of reasoning from means to ends:

(1) You may call Karen.
So, you may pick up the phone.

(2) You ought to call the doctor.
So, you ought to pick up the phone.

Both of these inferences sound good. To this, von Fintel (2012) adds the observation that "flatfooted" conjunctions of the form OUGHT $\phi \wedge \neg$ OUGHT ψ and MAY $\phi \wedge \neg$ MAY ψ, when $\phi \vDash \psi$, sound odd. Once again, we can illustrate with nondisjunctive prejacents:

(3) #Sam may take a ham sandwich, but it's not the case that he may take a sandwich.

(4) #Nicholas ought to take a free trip on the Concorde, but it's not the case that he ought to take a trip on the Concorde. (after von Fintel (2012, pg. 16))

The classical view is equipped to explain what is *wrong* with these sentences on semantic grounds.

3.5 Another Route: Disjunction in 2 Dimensions

The evidence against our upwards-entailing semantic account of the modals rested entirely on disjunctive data and the difficulty of accounting for it with non-semantic means. My interest, in this chapter, is in presenting an argument for blocking embedded disjunction introduction that doesn't rely on MAY being downward entailing—in fact, is compatible with the relevant notion of permissibility being *upward*-entailing at the level of propositions, as it is on the classical modal view. The EU theorist has a shorter way home, of course. But if I can do this, I can offer someone tempted by the EU modals a way to get the data without having to bite the bullets in **Fronted Alternatives** and **Following Through**, as well as a means of preserving the data in (1)-(4). (FC-) and (Ross) can, perhaps, be had for less than the EU theory's price of admission.

Let us (i) keep the idea OUGHT and MAY are upward-entailing, and (ii) reject the Boolean idea that

$$\phi \vDash (\phi \text{ OR } \psi).$$

There are many frameworks which reject unrestricted disjunction introduction (for example, linear logic and some relevance logics). What I propose to explore here, though, is fleshing out (ii) by going to a 2-dimensional semantics, as in Kaplan (1989); Davies & Humberstone (1980). We will do this using the classical framework—our working incarnation of an upward-monotonic theory.

According to the version of 2-dimensional semantics I want to consider, the semantic value of a sentence ϕ in L must be evaluated relative to *two* worlds in $W_{\mathcal{M}}$, an actual world (call this 'y') and an evaluation world (call this 'x'). So instead of

$$w \vDash \phi$$

we have

$$y, x \vDash \phi$$

The relativity of the semantic value of ϕ to a world $y \in W$ allows us to model the idea that a complex sentence ϕ might express different propositions at different possible worlds. For example, if, in w_1, Alice called her mother, but in w_2, Alice forgot to call her mother, we might like to say that "It ought to be that Otto does what Alice actually did" is true in w_1 and false in w_2; we want to say this in virtue of the fact that which proposition "Otto does what Alice actually did" feeds to the deontic modal is determined as a function of the evaluation world (either w_1 or w_2) for the whole sentence. Intuitively, w_1 and w_2 differ, not in respect of what is morally required at each, but in virtue of what is expressed by "what Alice actually did" in each.
[7]

The simplest upgrade of our deontic modals to a two-dimensional system will reflect the sensitivity of propositions expressed to the actual world y, by "carrying over" this value to the point of evaluation at which the prejacent is evaluated.

Definition 6 (Semantics 2D Modals)**.**
$y, x \vDash \text{MAY}(\phi)$ *iff* $\exists x'$ *such that* xRx' *and* $y, x' \vDash \phi$.

[7] I assume here the textbook semantics for 'actually', on which
$y, x \vDash$ Otto does what Alice actually did
iff the action (type) Alice performs in y is identical to the action (type) Otto performs in x. See Yalcin (2015a) for further discussion of 'actually', and Chapters 5 and 6 for further discussion of the proper notion of actuality in a semantics for deontic modals.

$y,x \vDash \text{OUGHT}(\phi)$ *iff* $\forall x'$ *such that* xRx': $y,x' \vDash \phi$.

With these new points of evaluation, we distinguish two relevant notions of consequence, which I will call *diagonal* ($\vDash_D$) and *unrestricted* ($\vDash$), respectively:

Definition 7 (Notions of Consequence). *For any well-formed formulas ϕ and ψ:*
$\Gamma \vDash_D \psi$ *iff for all* $w \in W_{\mathscr{M}}$, *if* $\mathscr{M}, w, w \vDash \phi$ *for each* $\phi \in \Gamma$, *then* $\mathscr{M}, w, w \vDash \psi$.
$\Gamma \vDash \psi$ *iff for all* $x, y \in W_{\mathscr{M}}$, *if* $\mathscr{M}, y, x \vDash \phi$ *for each* $\phi \in \Gamma$, *then* $\mathscr{M}, y, x \vDash \psi$.

Following a common strain in 2D semantics, I will assume that it is *diagonal* consequence that most closely approximates intuitive consequence relations between natural language sentences.[8]

With all this on board, the non-Boolean OR we need, I suggest, is just an OR such that disjunction is Boolean at diagonal points, but not at nondiagonal points.

Proposal 1 (A Non-Boolean OR). $\phi \vDash_D (\phi \text{ OR } \psi)$, *but* $\phi \nvDash (\phi \text{ OR } \psi)$.

To illustrate, we present a toy semantics for L^0 which implements this proposal.

Definition 8 (2D Semantics for L^0).
$y,x \vDash p$ iff $V(p,w) = 1$.
$y,x \vDash \phi \wedge \psi$ iff $y,x \vDash \phi$ and $y,x \vDash \psi$.
$y,x \vDash \neg\phi$ iff $y,x \nvDash \phi$.
$y,x \vDash \phi \text{ OR } \psi$ iff $y = x$ and $y,x \vDash \phi$ or $y,x \vDash \psi$.

For atomics, conjunction and negation, this semantics is, again, the simplest upgrade possible in a 2D framework. For disjunction, we add the simplest version of an "or" which is Boolean on the diagonal and not elsewhere, by simply making it the case that the truth of a disjunction at a point of evaluation $\langle y,x \rangle$ *requires* that $\langle y,x \rangle$ is a diagonal point, and then checks for the original Boolean truth-condition at that point.[9]

Putting It All Together

I claimed above that a non-Boolean semantics for OR could offer a semantic explanation of (FC-) and (Ross) that an upward-monotonic theory could accept. The relevant feature of is that, in 2 dimensions, the modals in Definition 6 require the semantic value of the embedded formula ϕ to be evaluated at nondiagonal points, but only requires the modalized

[8]See, for example, the notion of *real world validity* in Davies & Humberstone (1980).
[9] This toy version is not the best version of the 2D semantics; a more complex version a non-Boolean disjunction will be motivated and defended in the coming chapters.

sentences OUGHT ϕ and MAY ϕ to be evaluated at diagonal points. Our two dimensional truth-conditions for unembedded disjunction and disjunction under MAY, for example, are as follows:

> $\phi \vDash_D (\phi$ OR $\psi)$ iff, for any $\mathscr{M}$ and $w \in W_{\mathscr{M}}$,
> if $w, w \vDash \phi$, then $w, w \vDash \phi$ OR ψ.
>
> MAY$(\phi) \vDash_D$ MAY$(\phi$ OR $\psi)$ iff, for any $\mathscr{M}$ and $y, x \in W_{\mathscr{M}}$,
> if $\exists x'$ such that xRx' and $y, x' \vDash \phi$, then $\exists x'$ such that xRx' and $y, x' \vDash \phi$ OR ψ.

The inference from $\phi \vDash_D (\phi$ OR $\psi)$ to MAY$(\phi) \vDash_D$ MAY$(\phi$ OR $\psi)$ now fails, since in general $y, x' \vDash \phi$ does not guarantee $y, x' \vDash \phi$ OR ψ. In fact, on our toy semantics, when $y \neq x'$, it is *never* the case that $y, x' \vDash \phi$ OR ψ. The inference to OUGHT$(\phi) \vDash_D$ OUGHT$(\phi$ OR $\psi)$ fails for the same reason. The 2D deontic modals are upward monotonic—but only when we consider prejacents ϕ and ψ such that ψ is a *general* consequence, and not merely a *diagonal* consequence, of ϕ. We can preserve the upward-monotonic data that underlies the basic semantics of the quantificational modals, yet still block in-scope disjunction introduction; we just have to endorse a 2-dimensional OR.

A Comparison: "I am here now"

What would it look like to have a logic in which $(\phi$ OR $\psi)$ is a diagonal, but not an unrestricted, consequence of ϕ? Disjunction introduction will pattern with cases in which it is valid to introduce a disjunct *outside* the scope of an upward-entailing deontic modal Mod, but not *inside* its scope. The status of disjunction introduction—the inference from ϕ to $(\phi$ OR $\psi)$—will be something like an *a priori contingent* inference, in the sense of Evans (1977) and Kripke (1980). It is like one's knowledge of the truth of the sentence

(5) I am here now.
IHN

Since (5) is true at all diagonal points, conjoining it with any sentence will preserve truth at a diagonal point. We might call an inference rule that reflects this fact '$\wedge$ *IHN*'-Introduction: from any ϕ, conclude $(\phi \wedge IHN)$.

For example, if 2+2 = 4, then 2+2=4 and I am here now; but from the fact that it is (metaphysically) necessary that 2+2=4, it does not follow that it is (metaphysically) necessary that (2+2=4 and I am here now), since it is not metaphysically necessary that I am here now.

Disjunction introduction—an 'OR ψ' rule—works in a similar way:

'$\wedge$ *IHN*' Introduction	$\dfrac{\phi}{\phi \wedge IHN}$	$\dfrac{Mod\ \phi}{Mod(\phi \wedge IHN)}$
	valid	invalid
'$\text{OR}\ \psi$' Introduction	$\dfrac{\phi}{\phi\ \text{OR}\ \psi}$	$\dfrac{Mod\ \phi}{Mod(\phi\ \text{OR}\ \psi)}$
	valid	invalid

For example, if I am mailing the letter, it follows that I am mailing it or burning it; but from the fact that I *ought* to mail the letter, it does not follow that I ought to mail it or burn it. In the case of Ross's puzzle, that is exactly what our intuitions tell us.

3.6 Coda: Is "Blaming Disjunction" Too General?

In this paper, I've given an overview of the debate over disjunction within the scope of deontic modals, and sketched the ground for a semantic explanation of the data which jettisons Boolean disjunction. I've merely laid a groundwork, of course, for I still haven't even begun to offer an explanation of (FC)—the positive inference for which the failure of in-scope disjunction introduction is merely the negative half. However, what we've done already accomplishes something: it is compatible with upward monotonicity for the modals, and it begins to explain how it is that disjunctions like p OR $\neg p$ might be *unimpeachable*, but also *unembeddable*.

In closing, I'd like to consider an objection, advanced by Cariani (2011), to my approach to (FC-) and (Ross) via disjunction (an approach Cariani calls a "BD" approach, for "blame disjunction.") Cariani's claim bears direct quotation: BD accounts are too general, because they

> do not predict that deontic modals and epistemic modals should give rise to disanalogous predictions. In fact they naturally predict the opposite—that an

> epistemic MUST taking scope over a disjunction should pattern in the relevant respects with a deontic OUGHT in the same position. (21)

It would be bad, I think, if this outcome were predicted by the approach I just sketched. But it isn't predicted, as should by now be clear. Epistemic modals, whatever their precise semantics is, should generate a logic in which sentences true at all diagonal points—the *a priori truths*—are axioms. This is just to say that, for example,

(6) $\Box_e$(I am here now)

should be true at any diagonal point (with '$\Box_e$' marking that the relevant necessity is epistemic) just as its unembedded prejacent should be.[10]

It is a point familiar from Kaplan's remarks that we can capture what is distinctive about a priori truths by looking at what is true at every diagonal point (see, for example, Kaplan (1989, pg. 509).) The most natural way of marking these a priori truths in the object language is with epistemic necessity operators, and indeed a "monstrous" approach to them in the spirit of the simple entry above—where one quantifies over diagonal points, rather than points that are constant in one of the two dimensions—has been proposed as the distinctive feature of epistemic modal operators by Rabinowicz & Segerberg (1994); Perry & Israel (1996); Weatherson (2001); Santorio (2012); Yalcin (2015b), and others. It is epistemically necessary that I am here now, but it is not deontically necessary; it could well be permissible for me to be elsewhere. That is just the pattern we recapitulate on our nascent semantics for OR: OR-introduction is predicted to be valid in the scope of upward-entailing epistemic operators—including, of course, epistemic MUST—but not, again, in the scope of our deontic modals, OUGHT and MAY.

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[10] A simple semantics for this might leverage an modal accessibility relation R, where wRv iff v is compatible with what is known at w. Then a toy semantics for $\Box_e\phi$ might be: $y, x \vDash \Box_e\phi$ iff $\forall w'$ such that xRw': $w', w' \vDash \phi$.

Chapter 4

Donkey Disjunction

4.1 Introduction

In the last chapter, I advanced a proposal about the semantics of disjunction with two appealing features. First, it allows us to explain both (FC-) and (Ross) while maintaining the well-motivated idea that OUGHT and MAY are, as the simple quantificational view of the modals would have it, upward-entailing: when ψ is a (general) consequence of ϕ, then OUGHT ϕ entails OUGHT ψ and MAY ϕ entails MAY ψ. Second, a semantics for disjunction observing this proposal would give rise to a logic which was comfortingly classical throughout the propositional fragment: disjunction introduction would generally be a valid rule.

However, Proposal 1 is a manifestly incomplete characterization of the semantic value of OR, and the toy semantics in Chapter 3 did not have an empirical motivation beyond expedient simplicity. In this chapter, I turn to a new set of data and a new semantic system to provide guidance for filling in the semantics—postponing a return to a two-dimensional system of the type explored above until Chapter 5.

Our inquiry in this chapter starts afresh the question: how alike are disjunction and existential quantification? We will examine the relationship between existential quantification and disjunction from the point of view of natural language. I will briefly motivate, and then sketch, a free-variable approach to disjunction that parallels Heim (1982)'s enormously influential treatment of indefinites as free variables. I call this treatment of disjunction "donkey disjunction," after the sentences which motivated Heim's account of indefinites. On this account, existential quantificational force is not constitutive of the meaning of indefinites and disjunctions; rather, the quantificational force with which they are read may depend on the embedding environment (such as a conditional antecedent) in which they appear.

For reasons I will explain, puzzles involving donkey disjunction are both somewhat harder to find and somewhat harder to solve than corresponding donkey indefinite puzzles. But motivating a nonclassical treatment of disjunction from natural language phenomena—piggybacking on the success of Heim's treatment—will eventually allow us to buttress the theory of disjunction needed for the *deontic* modal puzzles with this brick and mortar.

4.2 Motivating Data

From Indicatives to Counterfactuals

A classic donkey sentence is:

(1) If Arnold owns *a donkey*, he beats *it*.
$\forall x\,[(\text{donkey}(x) \wedge \text{owns}(\text{Arnold}, x)) \rightarrow \text{beats}(\text{Arnold}, x)]$

The puzzle presented by (1) for traditional semantic theories to understand how 'a donkey' in the antecedent of the conditional both controls the anaphor 'it' in the consequent and contributes to the sentence as a whole the strong, universal quantification captured in the FOL gloss. For it would appear that these are mutually exclusive: if we quantifier-raise 'a donkey' high enough to bind 'it'—interpreting the existential as having wide scope over the whole construction—we would get the de re predication:

(1-a) $\exists x\, \text{donkey}(x)\text{: } (\text{owns}(\text{Arnold}, x) \rightarrow \text{beats}(\text{Arnold}, x))$
Some particular donkey is such that, if Arnold owns him, he beats him.

...which is too weak to capture the relevant reading of (1).

The other option is that we keep 'a donkey' low. This has the advantage of capturing the strong, universal quantification in the felt gloss of (1).[1] But this solution is unworkable, because if 'a donkey' has narrow scope with respect to the conditional, the 'it' in the consequent remains unbound:

(1-b) $(\exists x\, \text{donkey}(x) \wedge \text{owns}(\text{Arnold}, x)) \rightarrow (\text{beats}(\text{Arnold}, x))$
If Arnold owns a donkey, Arnold beats x.

The failure of standard theories to escape this classic dilemma is the historical motivation for dynamic theories of anaphora. Heim's solution to the puzzle is to translate 'a donkey'

[1] The relevant FOL fact is that $\exists x Fx \supset \phi$ is equivalent to $\forall x(Fx \supset \phi)$

as 'x donkey'—a free variable. The universal quantification in (1) is imparted by the conditional construction in which this NP appears. The inspiration here is

(2) Always, if Arnold owns *a donkey*, he beats *it*.
$\blacksquare\,[(\text{donkey}(x) \wedge \text{owns}(\text{Arnold}, x)) \rightarrow \text{beats}(\text{Arnold}, x)]$

Where '$\blacksquare$' is Lewis (1975)'s unselective quantification over free variables, and hence the basis, in this case, for the '$\forall x$' we get in (1). The short, programmatic answer to our dilemma, then, is that 'a donkey' and 'it' in (1) are both variables bound by a higher operator; 'a donkey' itself does not need to have (indeed does not have) scope over the pronoun 'it.'

Notice it makes no difference to the signature behavior of donkey anaphora if we change the flavor of the conditional in (1) from indicative to counterfactual. For example:

(3) If Arnold were to buy *a donkey*, he would beat *it*.
$\forall x\ (\text{donkey}\ x)\colon (\text{buys}(\text{Arnold}, x) \mathrel{\Box\!\!\rightarrow} \text{beats}(\text{Arnold}, x))$

I claim this sentence clearly has the truth-conditions listed (using $\Box\!\!\rightarrow$ as a placeholder for the counterfactual conditional), and is thus clearly parallel to (1)-(2).[2] From (3), then, I draw my first conclusion: donkey effects are a feature of both kinds of conditionals.

OR in the antecedent

We now consider conditionals with OR, rather than indefinites, in the antecedent. I take this example from (Elbourne, 2005, pg. 84):

(4) If Mary sees *John* OR *Bill*, she waves to *him*.
(Mary sees John $\rightarrow$ Mary waves to John) $\wedge$ (Mary sees Bill $\rightarrow$ Mary waves to Bill)

In (4), the pronoun 'him' in the consequent is clearly anaphoric. Once again, we could raise 'John OR Bill' high enough to bind the pronoun—giving disjunction wide scope over the whole construction. But in doing that, we'd lose the felt universal quantification—that is, the '$\wedge$' in the gloss of (4)—in favor of the weaker '$\vee$.' On the other hand, if we keep 'John

[2] Seth Yalcin notes that (3) lends itself to a wide-scope, *de re* reading more readily than its indicative counterparts ("If Arnold were to buy a donkey know, he would beat it. So I hope he doesn't buy it!") However, the reading I intend is also clearly available.

OR Bill' low, we achieve strength, at the cost of leaving 'him' in the consequent unbound. It is our classic dilemma once again.

Although I have not yet explained how Heim treats (1) compositionally, let me quickly say what is different between (1) and (4), to preview the difficulties idiosyncratic to the disjunction case. Disjunctions can coordinate a variety of constituents at surface form; in (4), for example, OR coordinates NPs. But FOL can only interpret disjunction as a sentence connective. The standard treatment of the transformation, from Rooth & Partee (1982), is simply to expand a surface coordination of XPs by copying, so that the logical '$\vee$' intervenes between whole clauses at LF. Hence 'Mary sees [John OR Bill]' is expanded to '[Mary sees John $\vee$ Mary sees Bill].'[3] If we do this, though, we have nothing of the appropriate type to serve as a restrictor for the pronoun 'him.' Attributing the point to Stone (1992), Elbourne writes:

> Even if we expand our dynamic theories to allow *or* to introduce a propositional variable when it conjoins sentences, no sense can be made of the notion that the value of such a variable could somehow be taken on by the pronoun ['him'] in the present case. (Elbourne, 2005, pg. 20)

Elbourne's verdict is dire: he concludes that there is "no way" dynamic theories can account for (4).[4]

These examples suggest that donkey disjunction is harder to treat than donkey indefinites: due to the type-mismatch between sentences in the antecedent and a pronoun in the consequent, it is not clear how a single operator, even an unselective Lewisian one, could bind both.

But do such examples show that donkey disjunction is harder to *motivate* than donkey anaphora? No; we can find cases of covariation between disjunctive antecedents and consequents that doesn't suffer from the type-mismatch identified by Stone and Elbourne—that is, cases for which adapting a Heim style approach should be both necessary and sufficient. The relevant data comes from Nute (1974)'s objection to Lewis (1973)'s theory of

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[3]Rooth and Partee do make an exception to this rule to account for donkey-like phenomena when 'or' coordinates indefinite NPs (Rooth & Partee, 1982, pgs. 5-7). In this case they analyze 'a watch or a compass' as introducing a variable over individuals that satisfies the predicate $\lambda x\,.\,watch(x) \vee compass(x)$ (op. cit., example (26)). Although they explicitly cite Heim and Kamp for inspiration, the Rooth and Partee analysis of this OR-coordination differs from the analysis Elbourne suggests, and that I develop, in that the variable introduced remains a variable over *objects* (type e), rather than *propositions* (type t or type $\langle s,t\rangle$).

[4]Stone makes a somewhat similar claim; the killer feature of (4) for Stone is that all the NPs in the antecedent are definite. That is true, but it does not follow that there are *no* indefinites in the antecedent if the disjunction *itself* is an indefinite.

counterfactual conditionals.[5] Lewis's analysis is an ordering semantics: the counterfactual $\phi \mathrel{\Box\!\!\rightarrow} \psi$ is true at w iff, at all the w-nearest ϕ-worlds—a set dubbed $Min_w(\phi)$—ψ is true.[6] Nute's counterfactual is:

(5) If we were to have good weather this summer or the sun were to grow cold by the end of the summer, we would have a bumper crop. (Nute, 776).

Nute argues that, on Lewis's theory, (5) should come out true, while in fact it strikes us as false. Assuming the world at which (5) is evaluated is (or is like) the actual world, *all* the w-close good-weather-or-cold-sun worlds are, in fact, good-weather worlds, and the counterfactual

(6) If we were to have good weather this summer, we would have a bumper crop.

is true. Lewis's theory, Nute continues, can't be right: unlike (6), the felt truth-conditions of (5) seem to require that the counterfactual is true with *each* of the disjoined antecedents.

Here, I suggest, we can happily marry Nute's problem to Heim's approach. As (3) appears to show, the universal force of this counterfactual antecedent ('in *all* nearby ϕ-worlds...') seems to be capable of binding indefinites in the antecedent. If the antecedent of the $\mathrel{\Box\!\!\rightarrow}$ construction universally binds indefinites, and disjunctions are analyzed as indefinites, we should generally expect constructions of the form

$$(p \text{ OR } q) \mathrel{\Box\!\!\rightarrow} r$$

to have truth-conditions equivalent to

$$(p \mathrel{\Box\!\!\rightarrow} r) \wedge (q \mathrel{\Box\!\!\rightarrow} r)$$

...and this is indeed what we find. We have a narrow-scope disjunction read as a wide-scope conjunction: a generalization of the pattern we saw in (1)-(3).

In what follows, then, I will construct a treatment of disjunctions as free variables that parallels Heim's account of indefinites as free variables.

[5]In the literature on conditionals, the question of how to associate these truth-conditions with disjunctive antecedents goes by the name "the simplification of disjunctive antecedents"; see Nute & Cross (2002), §1.7.

[6] For simplicity, in this chapter I stick to a version of Lewis's theory on which the Limit Assumption (Lewis, 1973) holds.

4.3 The Theory

To model donkey disjunction, a new syntax and a new (but conservative) semantics for sentential logic is required. We'll need an assignment function which maps variables to elements of the model, just as we do in the case of the standard semantics for the quantifiers. A disjunction in the language of sentential logic, $p \vee q$, I will write like this:

$$x_i(p, q)$$

where 'x' is a free variable in sentence position (hence of type t), 'i' is a subscript on the free variable 'x,' and p and q are themselves wffs of type t. '(p, q)' is what I will call the variable's *restriction*. Let the language in which this is a well-formed formula be called "LM." The intuitive idea here is that true disjunctions in LM must have witnesses, just like true existentially quantified statements in FOL do. The semantic value of a disjunction will thus be stated in terms of a condition for the disjunction to be satisfied at a point $\langle g, w \rangle$, where w is a standard "possible world" for sentential logic—a complete truth-table for the atomic sentences in the language—and g is an assignment. Furthermore, at a point of evaluation $\langle g, w \rangle$, a propositional variable $x_i(p, q)$ must express the same proposition as some particular member of its restriction. Which member of its restriction this is—which disjunct is its witness—depends on the assignment function g.

Syntax

In order for the restricted free variable notation to serve the same functions as a well-formed LSL disjunction, we must make sure the values in the restrictor sets "bottom out," in the appropriate syntactic (and ultimately semantic) way, with the atomic sentences in the language.

Syntax of LM

The wffs of LM are:

- p, where $p \in \texttt{At}$
- $\phi \wedge \psi$, where $\phi, \psi \in$ LM.
- $\backsim \phi$, where $\phi \in$ LM
- $x_i(\phi, \psi)$ where $x_i \in Var$ and $\phi, \psi \in$ LM.

We define closed sentences by defining a function which maps wffs into the set of variables open in the wff:

- for $p \in \texttt{At}$, $Open(p) = \varnothing$
- for $x_i \in Var$, $\phi, \psi \in$ LM: $Open(x_i(\phi, \psi)) = Open(\phi) \cup Open(\psi) \cup \{i\}$
- $Open(\phi \wedge \psi) = Open(\phi) \cup Open(\psi)$
- $Open(\backsim \phi) = \varnothing$

a sentence ϕ in this fragment is closed iff it has no open variables: $Open(\phi) = \varnothing$.

Heim's Semantics

We've taken the first step towards giving propositional logic a semantics like Heim's. In this section, I review the foundation of Heim's dynamic approach by setting forth her notion of the *context* as a formal object, "filing-cabinet"-like in structure, on which the semantic values of sentences act. Then we will proceed to updating (and simplifying) this notion of context for donkey disjunction.

Files

The leading idea of dynamic semantics is the idea of discourse primacy—the idea that it is primarily whole discourses and not individual sentences that bear semantic properties like truth-conditions (Yalcin, 2012c). Implementing this idea requires some way of formalizing the information carried by the previous discourse. The files in the filing cabinet keep track of this by grouping information ('donkey,' 'man', 'beats x_1') under discourse referents ('x_1,' 'x_2'), gradually building up a set of profiles fit to be mapped to individuals in the domain.

Formally, the work done by the metaphorical filing cabinet is modelled by (i) a set of points $\langle g, w \rangle$, which are assignment function-world pairs compatible with everything that has been asserted about the x_i's so far, and (ii) a domain parameter d_c, which keeps track of the index numbers that have already been introduced. For any index i which is *not* in d_c, anything is possible: for every model-theoretic object o, there is a g' in the context mapping x_i to that o. Call this feature of files *plenum*: this is how the file for a completely discourse-novel variable i—a variable about which nothing has been said—simultaneously records a full range of possibilities and a complete absence of established fact.

I take from Yalcin (2012c)'s gloss on Heim the following definitions:

Definition 1 (Heim Semantic Definitions).

An **assignment-world***, or* **point***, $\langle g, w \rangle$ is a pair of a possible world and a variable assignment.*

A **satisfaction set** *s is a set of assignment-worlds.*

A **domain** *is a set of numerals d corresponding to variable indices.*

*Two assignment functions g and g' are d-***accessible***, $g \backsim_d g'$, iff g and g' disagree at most on the values of $x_i \notin d$.*

A **context set** *(for short:* **context***) c is a pair $\langle s_c, d_c \rangle$ of a satisfaction set and a domain.*[7]

A context set c satisfies **plenum** *iff for any $\langle g, w \rangle \in s_c$ and any $g' \sim_{d_c} g$, $\langle g', w \rangle \in s_c$.*

Update and Truth

Well-formed formulas ϕ are assigned two types of semantic values in Heim's system—one for each component of context. Some sentences, like ones with free indefinites, have the ability to open new files by adding indices to the domain d_c. That effect will ramify into the interpretation of the next sentence(s), because it will contribute to the determination of whether the subscripts on the variables in the next sentence count as novel or familiar with respect to the discourse as a whole. This aspect of the meaning of ϕ is recursively definable in its impact on d_c. The second component of semantic value in Heim's system are context change potentials which shrink the satisfaction set s_c, to smaller satisfaction sets $s_c \llbracket \phi \rrbracket$ compatible with the content of ϕ in c.

Definition 2 (Context Change Potentials). *For any wff ϕ in language LM...*

$s_c \llbracket \phi \rrbracket$ *is a function from satisfaction sets to (smaller) satisfaction sets.*

$d_c[\phi]$ *is a function from domains to (larger) domains.*

These operations apply to context in a strict order: the domain is updated to $d_c[\phi]$ only *after* the satisfaction set has contracted to $s_c \llbracket \phi \rrbracket$.[8] As we have already seen in the case

[7]This notion of context is the descendant of Stalnaker's context *set* (Stalnaker, 1975), a set of worlds compatible with everything that is presupposed by the conversational interlocutors, which speech acts act on.

[8]This is how unembedded indefinites simultaneously manage to have the semantics of existentials (exploiting novel indices) and license anaphora (which must already be members of the domain to be interpretable as bound).

of donkey indefinites, the free variables in indefinites may have narrow scope with respect to other operators, which explains why they can be bound by unselective operators. But how to recover the original, existential truth conditions of *unembedded* indefinites? Heim equates truth for a set of sentences with there being *some* way of matching objects in the domain to discourse referents, such that the object instantiates the full suite of properties in its file. We can translate truth for Heim files into context-index terms (Lewis, 1970) like this:

Definition 3 (Heimian Truth)**.**

Heimian Truth. *$\phi_1 \ldots \phi_n$ are true at* **c,w** *iff there is some $\langle g, w \rangle$ s.t. $w = \mathbf{w}$ and $\langle g, w \rangle \in s_c$ $[\![\phi_1]\!] \ldots [\![\phi_n]\!]$.*

...keeping in mind that context, here, is a context set—hence not a context in the traditional sense of the concrete setting of a speech act, but rather something like a conversational state.

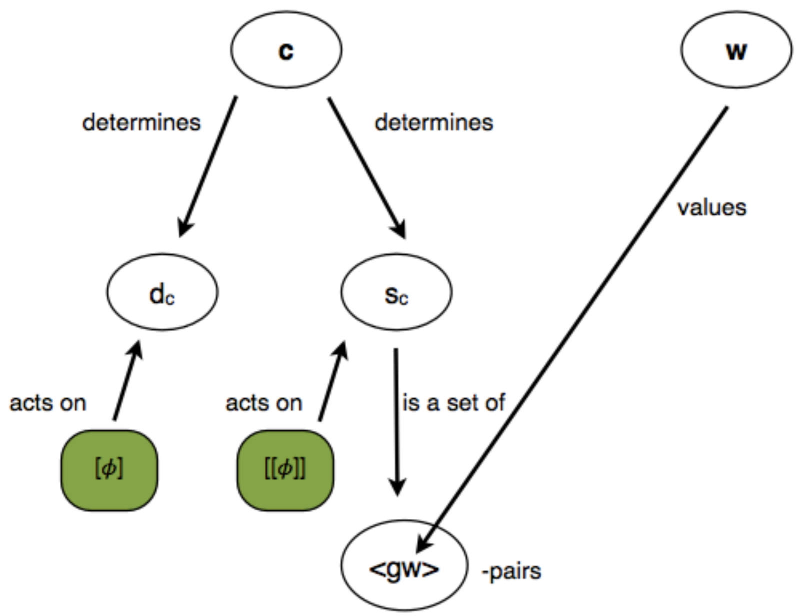


Figure 4.1: Heim's system. Compositional semantic values act on d_c and s_c, while w helps determine truth.

Compositional Semantics

The foregoing concludes our tour of files, Heim's celebrated "intermediate" layer of representation. We have reached the level of the compositional clauses for our object language LM. For a model, we help ourselves, as a regular intensional semantics does, to the following: each n-place predicate F of LM has an intension $I(F)$, which takes n individuals and a possible world w to a truth-value (Yalcin, 2012c, pg. 262).[9]

Definition 4 (Compositional Clauses).

Atomic Predication.

$$s_c [\![F(x_i)]\!] = \{\langle g, w\rangle \in s_c \mid g(x_i) \in I(F)(w)\}$$
$$d_c[F(x_i)] = d_c \cup \{i\}$$

Negation.

$$s_c [\![\neg\phi]\!] = \{\langle g, w\rangle \in s_c \mid \textit{there's no } g' \backsim_{d_c} g \textit{ such that } \langle g', w\rangle \in s_c [\![\phi]\!]\}$$
$$d_c[\neg\phi] = d_c$$

Conjunction.

$$s_c [\![\phi \wedge \psi]\!] = (s_c [\![\phi]\!]) [\![\psi]\!]$$
$$d_c[\phi \wedge \psi] = (d_c[\phi])[\psi]$$

Indicative Conditional.

$$s_c [\![\textit{if } \phi, \psi]\!] = \{\langle g, w\rangle \in s_c \mid \textit{for every } g' \textit{ if } \langle g', w\rangle \in s_c [\![\phi]\!], \textit{ then } \langle g', w\rangle \in s_c [\![\phi]\!] [\![\psi]\!]\}$$
$$d_c[\textit{if } \phi, \psi] = d_c$$

Donkey Disjunction: A Change in View

My purpose in this paper is to extend Heim's analysis to disjunctions. In an extensional semantics for propositional logic, disjunctions do not pick out objects—so the semantics of disjunctive sentences does not naturally lend itself to being described in terms of introducing novel discourse referents or latching on to existing discourse referents.

But there is a somewhat different view of the formalism that lends itself to a stronger parallel, which we can see when we look again at the context change potential for an indefinite predicative sentence $F(x_i)$ ("something is F":)

[9] As with the interpretation function $[\![.]\!]$, arguments of I are implicitly (quasi)-quoted.

$$s_c\,[\![F(x_i)]\!] = \{\langle g,w\rangle \in s_c \mid g(x_i) \in I(F)(w)\}$$

If we take the point of view of an individual point in $\langle g,w\rangle$, there is always some particular individual $o = g(x_i)$ of which F-ness is predicated: the force of the sentence is the force of an existential *de re* predication.[10] From the point of view of each maximally determinate $\langle g,w\rangle$, there is no nonspecific predication; to say that e.g. a donkey is in the yard is to say, of some particular object o, that *it* is a donkey in the yard. On this way of telling the story, each $\langle g,w\rangle$-point in s_c represents an evolving hypothesis about the maximally determinate possible-worlds truth conditions of the sentence $F(x_i)$, constrained by the communicative impact of all the sentences that have been asserted before it.

From this point of view, the existential truth-conditions Heim provides for whole discourses are cast in a light that makes them resemble familiar norms of charitable interpretation: a sentence (more broadly, a discourse) is not to be considered false unless there is *no* way of precisifying it along with the previous discourse—no way of mapping indices to individuals—that makes what was said satisfiable by the model.[11]

Looking forward to an intensional system, our disjunctive variables are variables not over individuals, but over disjuncts, and our new formalism treats the proposition expressed by a disjunction as identical, at any $\langle g,w\rangle$ point, to the proposition expressed by *one* of its disjuncts—it is, if you will, a de re disjunction. From the point of view of each maximally determinate $\langle g,w\rangle$, there is no "non-specific" disjunction. However, since it is not known which of the disjuncts the disjunction will serve to express, each hypothesis is represented in context: if the disjunction is $x_i(p,q)$, there will be some g' s.t. $g'(x_i)$ expresses the proposition that p expresses, and another g'' s.t. $g(x_i)$ expresses the proposition that q expresses. The assumption of plenum—every hypothesis not explicitly ruled out is represented in context—is key, as it is in Heim's original system, to recovering the weak truth-conditions of disjunction.

Going Propositional

How to implement the idea that disjunctions are variables over propositions? Our idea (starting with the atomic case) is that a disjunction $x_i(p,q)$ is like an indefinite—a free variable. Rather than combining with a predicate, this formula must express the same proposition as one of the wffs in its restrictor.[12] We will keep the dynamical impact on d_c the same as in the indefinite. So our first pass is:

[10] In the sense of Quine (1956).

[11] The analogy between file-change semantics and charitable interpretation has also been suggested—though on rather different grounds (an argument from speaker intentions)—by Cumming (2011).

[12] I use p and q here as metavariables over atomic sentences.

Definition 5 (Atomic Disjunction—first pass)**.**
$s_c\ [\![x_i(p,q)]\!] = \{\langle g,w\rangle \in s_c \mid g(x_i(p,q)) = p \textit{ or } g(x_i(p,q)) = q\}$

$d_c[x_i(p,q)] = d_c \cup \{i\}$

The problem with this entry is that the world w does not appear in the description of $s_c\ [\![x_i(p,q)]\!]$. As a result, this satisfaction set change potential does not cut down on the set of worlds w in $s_c\ [\![\phi]\!]$ at all. This cannot be right: disjunctions clearly tell us something about the *world*: they are not mere anaphora-licensing devices, but are in themselves truth-apt.

How should this be fixed? We have the model-theoretic resources of ordinary intensional semantics at our disposal—hence, each n-ary predicate F of LM has an intension $I(F)$, taking n individuals and a possible world w into a truth-value. An atomic formula p is associated with an intensional relation between *zero* objects, a possible world w, and a truth-value. Applying the norm of charity in a way parallel Heim's, the way to restrict the set of $\langle g,w\rangle$ points under the semantic value $[\![x_i(p,q)]\!]$ is to say that x_i expresses the proposition that p *in worlds where* p *is true*, and that it expresses the proposition that q *in worlds where* q *is true*. In worlds w where both p and q are true, the assignment function g simply chooses a witness; it is not constrained by w.[13]

Definition 6 (Atomic Disjunction—second pass)**.**
$s_c\ [\![x_i(p,q)]\!] = \{\langle g,w\rangle \in s_c \mid \textit{either } g(x_i(p,q)) = p \textit{ and } w \in I(p), \textit{ or } g(x_i(p,q)) = q \textit{ and } w \in I(q).\}$

$d_c[x_i(p,q)] = d_c \cup \{i\}$

There are several ways of looking at the relationship between g and w on such a view; we could say that when p or q is true at w, g assigns a particular *truthmaker* to the disjunction (Fine, 2010a,b). Or we could conceive of g's role in terms of converting a world w into a *situation* (Elbourne, 2005): cutting w down to size so that "the witness in $\{p,q\}$" is always a proper definite description. An atomic donkey disjunction $x_i(p,q)$, for example, will rule out of a satisfaction set an assignment-world $\langle g,w\rangle$ where p is false and q is true, yet g assigns $x_i(p,q)$ to p.

[13]This claim raises the natural question of what witness an assignment function chooses (or what witnesses it is constrained to choose between), in worlds where all the disjuncts of a given disjunction are false in w. A norm of charity gives no guidance here, since there is no way of assigning a proposition to the disjunction which is both expressed by one of its disjuncts and true in w. Given Definition 6, no $\langle g,w\rangle$ point where both $w \notin I(p)$ and $w \notin I(q)$ will survive update with $[\![x_i(p,q)]\!]$, regardless of what value the assignment function g assigns to $x_i(p,q)$.

Relationship to Classical Logic

Suppose we incorporate this new entry for disjunction into Heim's system. How much like our familiar, classical propositional logic does the resulting system behave? Here, by "classical propositional logic," I do *not* include the indicative or counterfactual conditionals of natural language, but just a language of atomics closed under negation, conjunction, and disjunction.

To answer this question, we need to look at the LSL—the language of sentential logic—through the lens of $Trans$, a function that translates sentences of LSL into sentences of LM. We begin in the syntax. When we look at translations of well-formed formulas of classical propositional logic, there are no familiar indices—no variables with the same subscripts as other variables. After all, the only free-variable-bearing sentences in the image of LSL under $Trans$ are disjunctions, and disjunctions (like Heimian indefinites) introduce discourse-*novel* variables. This is the first reason the nonclassical behavior predicted by Heimian semantics goes under the radar in LSL: the differential behavior of discourse-*familiar* variables under e.g. negation is never observed.[14]

What of the essentially dynamic structure of Heimian truth? It might seem here that reduction to classical logic is not to be had, for in LSL sentences are true or false at models—assignments of truth-values to the atomic sentences—rather than at Heimian contexts, which are pairs of a world and a context set (the latter itself being a pair $\langle s_c, d_c \rangle$). But this is also an illusion. For propositional logic, a model *is*, in one sense, a possible world—that is, it is simply an assignment of truth-values to the atomic sentences. In our semantics, in the satisfaction set $[\![x_i(p,q)]\!]$, every $\langle g, w \rangle$ pair is such that g must assign a *true*-in-w proposition to the restricted variable. As long as there is some $\langle g, w \rangle$ pair satisfying this constraint, there will be at least one assignment world satisfying the open sentence. On the assumption that we begin with plenum in c, nothing in the discourse could have ruled every such $\langle g, w \rangle$ pair out of s_c. So the sentence will be true at w and c, according to Heim's definition of truth, if it is true in w, according to the classical definition of truth.

Finally, there is the fact that Heim assigns truth-conditions to whole discourses—sets of sentences, rather than to individual sentences. An analogous definition of truth at a

[14] Negation provides a simple illustration of this claim. Suppose our syntax allowed a disjunction $\backsim (x_i(p,r))$ with discourse-familiar variable x_i. If a previous constraint on x_i has limited its interpretation such that $g(x_i) = I(p)$ for any $\langle g, w' \rangle \in s_c$—for example, if the sentence $x_i(p,p)$ appeared earlier in the discourse—then the context-change potential of $\backsim (x_i(p,r))$ is the same as the context-change potential of $\backsim p$. But it is difficult ever to interpret an assertion of the form "Neither p nor r" as having a conversational impact equivalent to an assertion of "not p". The observation from the point of view of the lexicon is that, although indefinites and disjunctions are being treated as parallel in this system, there is no analogue for disjunctions of the definite/indefinite dichotomy—no "definite", or discourse-familiar kind of natural language disjunction.

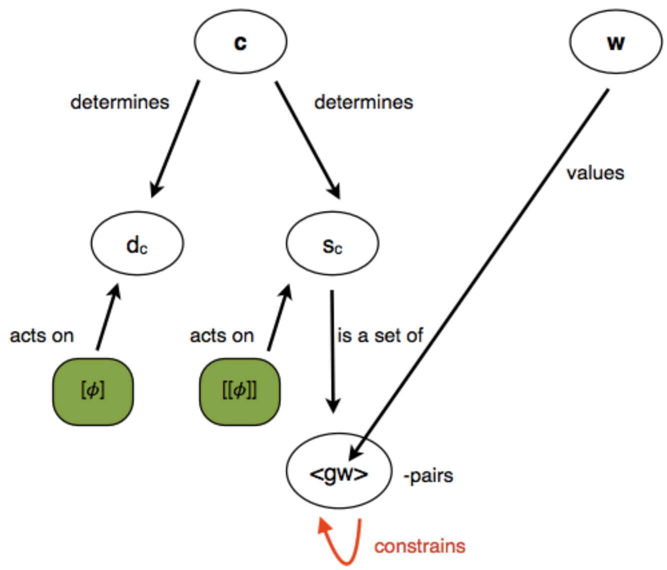


Figure 4.2: Heim's system. Compositional semantic values act on d_c and s_c. Within the satisfaction set, w constrains g.

sentential model should be: a whole set of sentences Γ is true at w iff there is some g s.t. $\langle g, w \rangle$ satisfies *all* sentences in Γ. (By plenum, this assignment function will be present in s_c, if it exists at all). But once again, in the LSL fragment of the language this will be no stronger than a condition on individual sentences in Γ: no subscripts are repeated—every disjunction, like every indefinite, gets a variable with a novel subscript—and because no subscripts are repeated, the members of Γ cannot put conflicting conditions on the same variable.

Under these conditions on the syntax and semantics of *LM*, we can prove that the image of propositional logic in *LM* is semantically conservative. Let the semantic consequence relation $\vDash_H$ over sentences of *LM* to be the preservation of Heimain truth, and let $\vDash_{LSL}$ be the preservation of truth in the language of propositional logic. We can then prove the following

Fact 1. *There is a recursive translation function τ taking sets of wffs $\{\phi_1 \ldots \phi_n, \psi\}$ of LSL into a well-formed discourse $\{\tau(\phi_1) \ldots \tau(\phi_2), \tau(\psi)\}$ of LM, $\phi_1 \ldots \phi_n \vDash_{LSL} \psi$ iff $\tau(\phi_1) \ldots \tau(\phi_2) \vDash_H$*

$\tau(\psi)$.

Setting up this proof is straightforward (see Appendix A).

4.4 Beyond Propositional Logic: The Two Conditionals

Indicative Conditionals

We now have new predictions for indicative conditionals with disjunctions in the antecedent. Recall our problematic conditional from §2 (repeated below):

(7) If Mary sees *John* OR *Bill*, she waves to *him*.
(Mary sees John → Mary waves to John) ∧ (Mary sees Bill → Mary waves to Bill)

The foregoing accomplishes what Elbourne suggested in the first half of his remarks on the case:

> *Even if we expand our dynamic theories to allow* or *to introduce a propositional variable when it conjoins sentences*, no sense can be made of the notion that the value of such a variable could somehow be taken on by the pronoun ["him"] in the present case. (Elbourne, 2005, pg. 20, emphasis added)

We still have our pronoun problem. But we can account for the simpler:

(8) If Mary sees *John* OR *Bill*, Mary waves.
Translation:
if x_1(Mary sees John, Mary sees Bill), Mary waves.
Entailment:
(if Mary sees John, Mary waves) ∧ (if Mary sees Bill, Mary waves)

For (8), the original indicative conditional entry I gave in §3.2.3 (repeated below) now suffices to derive the felt conjunctive truth-conditions.[15]

Indicative Conditional.

[15] In the full-blown Heimian theory, this entry for the indicative conditional is revised to handle phenomena such as indefinites in the consequent (which behave differently than indefinites in the antecedent). Since these complications do not concern us here, for clarity's sake I stick to the simpler entry. Interested readers should see Yalcin (2012c, pg. 265-266) for details.

s_c $[\![\text{if}\ \phi,\ \psi]\!] = \{\langle g, w\rangle \in s_c \mid \text{for every } g' \text{ if } \langle g', w\rangle \in s_c\ [\![\phi]\!], \text{ then } \langle g', w\rangle \in s_c\ [\![\phi]\!]\ [\![\psi]\!]\}$

$d_c[\text{if}\ \phi,\ \psi] = d_c$

I leave it to the reader to verify this.[16]

We are now poised to return to the Nute counterfactual (repeated):

(9) If we were to have good weather this summer or the sun were to grow cold by the end of the summer, we would have a bumper crop.

Translation:
x_1(We have good weather this summer, the sun grows cold this summer) $\Box\!\!\rightarrow$ we have a bumper crop.

Entailment:
(We have good weather this summer $\Box\!\!\rightarrow$ we have a bumper crop) $\wedge$ (The sun grows cold this summer $\Box\!\!\rightarrow$ we have a bumper crop)

The basic thought is simple. It is characteristic of conditionals to quantify universally over free variables in their antecedents. We observed that this appears to be true whether the conditional is indicative or counterfactual. Once we have added assignment-functions g to our points of evaluation—upgrading, as Heim does, the notion of a world w to the notion of an assignment-world $\langle g, w\rangle$—we can ascribe the intuitively correct, conjunctive truth-conditions to (9), without having to rely on hopeful assumptions about nearness of worlds—assumptions that fail in the case of (9). We don't achieve universal quantification over the individual disjuncts indirectly, by way of quantifying over nearby possible worlds w. We quantify over the disjuncts directly, by quantifying universally, as Heim does, over the g parameters that assign values to the disjunctive variables.

[16] Here is a sketch. Suppose the conditional premise is true at c. Then s_c $[\![\text{if}\ x_1(\text{M sees J, M sees Bill}), \text{M waves}]\!]$ is nonempty. By Plenum, there are two types of assignment-functions in s_c: type g_1, such that $g_1(x_1(\text{M sees J, M sees Bill}))$ = M sees J, and type g_2, such that $g_2(x_1(\text{M sees J, M sees Bill}))$ = M sees B. Since s_c $[\![\text{if}\ x_1(\text{M sees J, M sees Bill}), \text{M waves}]\!]$ is nonempty, it follows that for any g_1-type assignment, there is some w such that: if $\langle g_1, w\rangle \in s_c$ $[\![\text{M sees J}]\!]$, then $\langle g_1, w\rangle \in s_c$ $[\![\text{M sees J}]\!]$ $[\![\text{M waves}]\!]$. Likewise for any g_2-type assignment: there is some w such that if $\langle g_2, w\rangle \in s_c$ $[\![\text{M sees B}]\!]$, then $\langle g_2, w\rangle \in s_c$ $[\![\text{M sees B}]\!]$ $[\![\text{M waves}]\!]$. Finally, 'M sees J' is a variable-free atomic sentence, so if $\langle g_1, w\rangle \in s_c$ satisfies this condition, then any $\langle g' w\rangle \in s_c$ does. Likewise for 'M sees B'. Hence any $\langle g', w\rangle \in s_c$ is such that: (i) if $\langle g', w\rangle \in s_c$ $[\![\text{M sees J}]\!]$, then $\langle g', w\rangle \in s_c$ $[\![\text{M sees J}]\!]$ $[\![\text{M waves}]\!]$, and (ii) if $\langle g', w\rangle \in s_c$ $[\![\text{M sees B}]\!]$, then $\langle g', w\rangle \in s_c$ $[\![\text{M sees B}]\!]$ $[\![\text{M waves}]\!]$. Hence the conclusion follows.

Counterfactual Conditionals

I return, at last, to the semantics of Lewis's counterfactual conditional.[17]

Definition 7 (Lewisian Counterfactual). *$w \vDash \phi \mathrel{\Box\!\!\rightarrow} \psi$ iff $\forall w' \in Min_w(\phi)$: $w' \vDash \psi$.*
Where $Min_w(\phi)$ is the set of worlds w' such that:
(i) $w' \vDash \phi$, and
(ii) $\neg\exists w''$ s.t. $w'' <_w w'$ and $w'' \vDash \phi$.

what must be added to this entry to give a counterfactual like (9) the intuitively correct truth-conditions? To make the minimal change, I need to add g to Lewis's entry, making it more like Heim's. Or (looking at it from the other direction) I need to make Heim's entry more like Lewis's, by making it determine a truth-condition on possible worlds, rather than a context change potential.

There are three gaps to be bridged:

1. We borrow from the Lewisian models the similarity ordering $<_w$ on worlds for each world $w \in W$.

2. We pair each Lewisian point of evaluation $w \in W$ with a parameter g, which contributes to the semantic interpretation of disjunction.

3. We make the semantics static, so that there is a stable proposition (a set of points of evaluation) associated with $\phi \mathrel{\Box\!\!\rightarrow} \psi$ at a context c. The conversion rule is $s_c\, [\![\phi]\!] = \{\langle g, w\rangle \in s_c : \langle g, w\rangle \Vdash \phi\}$, where $\Vdash$ is a *local* satisfaction condition for sentences ϕ at assignment-worlds (see Yalcin (2012c, pg. 259) and Stalnaker (1999a)).[18]

For (1), we may take the similarity ordering as primitive. In particular, we do not try to stipulate away the possibility of the similarity ordering in the Nute counterexample, where, for a given antecedent $x_i(p, q)$ and world w, all w-nearest worlds where the Boolean disjunction of p and q is true are worlds where p is true and q is false.

For (2), we tap the semantics of disjunction for the constraint w exerts on g mentioned above: the value $g(x_i)$ assigned to a disjunction $x_i(p, q)$ at $\langle g, w\rangle$ must be true at w. That means any world w in which exactly one of $\{p, q\}$ is true is sufficiently rich in structure

[17] As noted above, our version of the Lewisian counterfactual assumes the Limit Assumption (Lewis, 1973): that is, if there is a set of worlds S which are $\leq_w$-comparable, then some world or worlds in that set is at least as close to w as any other. These world(s) will be members of $Min_w(S)$.

[18] In Appendix A, I show this conversion can also be carried out for the fragment of the language closed under negation, conjunction, and disjunction.

to be converted to an assignment-world $\langle g, w \rangle$: no extra structure is needed to assign a proposition to a disjunction $x_i(p,q)$ at a possible world w.

Given the assumption that every well-formed disjunctive sentence introduces a novel index $i \notin d_c$, and using the rule in (3), a local satisfaction condition of the form

$$\langle g, w \rangle \Vdash \phi \mathrel{\Box\!\!\rightarrow} \psi \text{ iff} \ldots$$

is sufficient to determine context change potential of the form

$$s_c \, [\![\phi \mathrel{\Box\!\!\rightarrow} \psi]\!]$$

Assuming plenum, this latter will be sufficient to determine the former—that is, a local satisfaction condition for *any* local point of evaluation.

Finally, our goal is to ensure that $(x_i(p,q)) \mathrel{\Box\!\!\rightarrow} r$ entails $p \mathrel{\Box\!\!\rightarrow} r$ and $q \mathrel{\Box\!\!\rightarrow} r$. Conceiving of entailment in a static style, as the preservation of truth relative to any context and local point of evaluation, this entry will be sufficient:

Definition 8 (Lewisian Counterfactual+). *$c, \langle g, w \rangle \Vdash \phi \mathrel{\Box\!\!\rightarrow} \psi$ iff $\forall \langle g', w' \rangle \in Min_{c,g,w}(\phi)$: $c, \langle g', w' \rangle \Vdash \psi$.*
Where $Min_{c,g,w}(\phi)$ is the set of $\langle g', w' \rangle$-pairs such that:
(i) $g' \backsim_{d_c} g$,
(ii) $c, \langle g', w' \rangle \Vdash \phi$, and
(iii) $\neg \exists w''$ s.t. $w'' <_w w'$ and $c, \langle g', w'' \rangle \Vdash \phi$.

Proof in Appendix A.1

Definition 8 takes the point of view of a single point of evaluation $\langle g, w \rangle$. That means, by (i), that accessible assignment-functions g' are those which disagree with g only with regards to variables that are not already in the domain d_c. That is right for us, because we are interested in discourse-novel variables in the antecedent (viz., disjunctions and indefinites.)[19] Second, the universal quantification in this entry has the effect of quantifying universally over accessible g's, and only then looking at closeness amongst the worlds w' that can be paired with a given g'. This means that each disjunct will be 'witnessed' by some $\langle g', w' \rangle$ pair in $Min_{c,g,w}$.

[19] But it is worth noting remarking on the results for definites—that is, for indices i which already are in d_c. Here, universal quantification across g' s.t. $g' \backsim_{d_c} g$ does nothing to change the individual assigned to x_i, since this assignment is part of the "fixed" domain. So assuming Heim's analysis of definites is right, definite descriptions (e.g. 'If *the president* were female...') will always wind up having wide-scope on this semantics for the counterfactual. This is obviously not the only reading of "if the president were female...", but this ambiguity is beyond the scope of my purposes here.

...And Beyond

We began with Heim's dynamic treatment of donkey anaphora in indicative conditional antecedents, finding within it motivation for introducing a parameter—in the system of this chapter, an assignment-function parameter g—that assigns a particular witness to formulas with the syntax of indefinites. Locating similar empirical motivation in both indicative and counterfactual conditional antecedents, we then exported that idea to disjunctions, finding ways of accounting for the parallel phenomena in a way that made minimal changes to the received semantics of conditionals. In the next chapter, I'm going to motivate another application of the witnessing approach to disjunction by returning to the deontic modal puzzles.

The theory of disjunction in Chapter 3 gave us part of classical Boolean disjunction that, I argued, we should keep: in a two-dimensional system, we should say that disjunction is Boolean on the diagonal. We gave a merely negative characterization of its behavior off the diagonal: off the diagonal, it is *not* Boolean. The theory argued for in this chapter, while presented in a system which was not two dimensional, nonetheless contributes a clue to the positive characterization of the semantic value of p OR q disjunctions which can be used to flesh out the rest of the picture. For a disjunction to be semantically sensitive to an extra parameter beyond the world of evaluation w is for that second parameter to control which disjunct is passed on to further semantic interpretation. What we have in this is an idea which can be exported into a two dimensional system, to the effect that disjunction's Boolean face is compatible with its expressing a more specific proposition—one which is identical to the proposition expressed by one of its disjuncts—at different points relevant to semantic composition. That is the view we will develop further in the next chapter.

Chapter 5

Deontic Disjunction

5.1 Two Puzzles

Our first named deontic modal puzzle, free choice permission, is usually glossed as a felt entailment from a narrow-scope disjunction under MAY to a wide scope conjunction.

(FC) $\textsc{may}(\phi \text{ OR } \psi) \Rightarrow (\textsc{may}\,\phi) \wedge (\textsc{may}\,\psi)$

But a bit more can be added to this characterization. Most speakers have the strong intuition that while a free choice sentence like

(1) You may have the beer or the wine.
$\textsc{may}(G \text{ OR } W)$

communicates that you may choose the beer and you may choose the wine, it emphatically fails to communicate the permissibility of the corresponding narrow-scope conjunction, which would allow you to have both:

(2) You may have the beer *and* the wine.
$\textsc{may}(G \wedge W)$

This feature goes by the name *exclusivity*:

(Exclusivity) $\textsc{may}(\phi \text{ OR } \psi) \not\Rightarrow \textsc{may}(\phi \wedge \psi)$

This observation is embedded in the scholarship on free choice permission.[1] The tension between sentences like (1) and sentences like (2) is usually glossed as a scalar implicature: while (1) does not preclude the truth of (2), it makes for an uncooperatively uninformative utterance in contexts where (2) is also true.[2]
Our second named puzzle was Ross's Puzzle (Ross, 1941):

(R) $\textsc{ought}\,\phi \not\Rightarrow \textsc{ought}(\phi \text{ or } \psi)$

(R) was strengthened in Chapter 2 with the intuitive observation that a Ross sentence $\textsc{ought}(\phi \text{ or } \psi)$ entails that there must be something to be said, deontically speaking, for each of the embedded disjuncts.

(R+) $\textsc{ought}(\phi \text{ or } \psi) \Rightarrow (\textsc{may}\,\phi) \wedge (\textsc{may}\,\psi)$

This looks like a unification with free choice permission: disjunction under both OUGHT and MAY carry what we can call *an entailment to disjunct permissibility.*

A simple variation on Ross's puzzle, due to Geoffrey Sayre-McCord, suggests that there is yet more to say about Ross sentences. Sayre-McCord observes that, according to the hypothesis that disjunction introduction is blocked in the scope of OUGHT *just in case* the introduced disjunct is impermissible, it would follow that, if ψ is known to be permissible, then

$$\textsc{ought}\,\phi$$

should entail

$$\textsc{ought}(\phi \text{ or } \psi)$$

[1] Barker (2010) writes that a FC sentences like (1) "never" guarantee conjunctions like (2). The failure of (1) to entail (2) is also assumed by Simons (2005), who calls it a "consensus in the literature." Danny Fox, an implicature theorist, makes his psychological explanation of free choice permission *dependent* on a hearer's rejection of (2) (Fox, 2007). Fox's official position thus includes an endorsement of the stronger inference we may call *and-false*:

$$\text{(And-False)} \quad \textsc{may}(\phi \text{ or } \psi) \Rightarrow \neg\,\textsc{may}(\phi \wedge \psi)$$

However, Fox expresses some reservations about this inference—and thus the fact that his explanation of free choice permission relies on it—in the last section of his paper. See (Fox, 2007, pg. 35-36).

[2] Both (FC) and (Exclusivity) have been analyzed as scalar implicatures, the former of an unusual, post-Gricean kind (especially in the wake of Kratzer & Shimoyama (2002).) I discuss arguments against an analysis of (FC) in this vein in Chapter 1.

in context.

But, Sayre-McCord argues, "OUGHT$(\phi$ OR $\psi)$ cannot legitimately be inferred from OUGHT ϕ even when ψ is perfectly permissible" (Sayre-McCord, 1986b, pg. 189). His example is as follows: suppose it is taken for granted that it is perfectly fine for Ralph to go to the movies:

(3) Ralph may go to the movies.
MAY M

is true. It is also true that

(4) Ralph ought to pay back his loan.
OUGHT L

Still,

(5) Ralph ought to pay back his loan or go to the movies.
OUGHT$(M$ OR $L)$

sounds wrong; it does not seem equivalent to (3) and (4). Call this

(SM) MAY $\psi \wedge$ OUGHT $\phi \not\Leftrightarrow$ OUGHT$(\phi$ OR $\psi)$

(SM) captures the observation that sometimes, disjunction introduction in the scope of a deontic modal fails *even when* the introduced disjunct is permissible.

Sayre-McCord's observation seems to show that there is no paraphrase for the OR in a Ross sentence in terms of the *unconditional* deontic status of the disjuncts expressible in the object language—that is, in terms of the deontic status each disjunct has, considered independently of the other one.

OUGHT$(\phi$ OR $\psi)$	$\not\Leftrightarrow$	OUGHT $\phi \wedge$ OUGHT ψ
		both obligatory—too strong
	$\not\Leftrightarrow$	MAY $\phi \wedge$ MAY ψ
		both (merely) permissible—too weak
	$\not\Leftrightarrow$	(OUGHT $\phi \wedge$ MAY $\psi) \vee$ (OUGHT $\psi \wedge$ MAY $\phi)$
		one of each—(SM)

These examples suggest we should switch tracks, and try to directly characterize situations that *are* appropriately described by sentences of the form OUGHT$(\phi$ OR $\psi)$. Suppose that I have come down with a cold this morning and am too sick to host my housewarming party tomorrow night. It seems appropriate to say:

(6) I ought to cancel or postpone the party.

If (6) is true, then canceling the party has a certain deontic status (that of being obligatory), *provided that I do not postpone*, and postponing the party has a certain deontic status (that of being obligatory), *provided that I do not cancel*. That nothing at all is said by (6) about the situation where I both postpone and cancel is shown by the fact that (6) can be true, even though the conjunction, *postpone and cancel*, has no positive normative status at all: it would be rude to my guests to postpone the party only to cancel it later, and it's impossible to cancel a party and *then* to postpone it.

I propose, then, to add to the data associated with Ross sentences like (6): the deontic status of the act described by each disjunct depends on whether the other one is performed. Starting with a Ross sentence as a premise, an agent can reason with future-directed conditionals like this:

(Conditionals-O):
$$\text{OUGHT}\ (\phi \text{ OR } \psi) \Rightarrow \text{If not-}\phi\text{, then OUGHT } \psi;$$
$$\text{OUGHT}\ (\phi \text{ OR } \psi) \Rightarrow \text{If not-}\psi\text{, then OUGHT } \phi.$$

In addition to capturing an intuitively correct entailment of (6), (Conditionals-O) helps us to precisify our discomfort with Sayre-McCord's defective (5), where disjunction introduction in the scope of OUGHT is blocked, even though the introduced disjunct is permissible. Going to the movies is strictly optional—it's just *false* that movie-going achieves the status of an obligation in cases where Ralph fails to pay back his loan. So, according to (Conditionals-O), (5) is in one respect too strong. It is also, in another respect, too weak, which we can see from looking at the second conditional licensed by the schema. In the situation Sayre-McCord describes, paying back the loan is not something Ralph ought to do *on the condition that he doesn't go to the movies*; it's just something he ought to do, full stop.

Another nice feature of (Conditionals-O) is that it holds out the promise of a unification with free choice permission—a better one, since it is more empirically adequate from the OUGHT-side. For consider the property that is formally parallel to (Conditionals-O), with MAY substituted for OUGHT:

(Conditionals-M)
$$\text{MAY}(\phi \text{ OR } \psi) \Rightarrow \text{If not-}\phi\text{, then MAY } \psi;$$
$$\text{MAY}(\phi \text{ OR } \psi) \Rightarrow \text{If not-}\psi\text{, then MAY } \phi.$$

(Conditionals-M) is just a re-framing of free choice permission that incorporates the proviso of (Exclusivity). If I give you permission to have the beer or the wine, then having the beer has a certain deontic status—that of being permissible—*provided that you do not also take the wine.* And vice-versa. As in the OUGHT case, it is certainly *compatible* with what I said that having both beer and wine is permissible. But the truth of a free choice premise, MAY(ϕ OR ψ), does not require this; MAY(ϕ OR ψ) may be true even when doing ϕ *and* ψ is not permitted at all.

I won't argue further for the empirical traction of (Conditionals-M) here. But I think the intuitive appeal of (Conditionals-M)—especially in light of its formal similarity to (Conditionals-O)—is clear enough to make the two inferences a basis for an exploration of the semantic behavior of OR in these modal environments. Table 5.1 gives an interim summary of the intuitions that constitute our data.

May:	
(Failure of OR intro)	MAY $\phi \not\Rightarrow$ MAY(ϕ OR ψ)
(FC)	MAY(ϕ OR ψ) $\Rightarrow$ MAY $\phi \wedge$ MAY ψ
(Conditionals-M)	MAY(ϕ OR ψ) $\Rightarrow$ if $\neg\phi$, then MAY ψ
Ought:	
(Failure of OR intro)	OUGHT $\phi \not\Rightarrow$ OUGHT(ϕ OR ψ)
(R+)	OUGHT(ϕ OR ψ) $\Rightarrow$ MAY $\phi \wedge$ MAY ψ
(Conditionals-O)	OUGHT(ϕ OR ψ) $\Rightarrow$ if $\neg\phi$, then OUGHT ψ

Table 5.1: Data for OUGHT and MAY.

If we are on the right track about (Conditionals-M) and (Conditionals-O), what the data suggest is that the deontic status of the acts described by the disjuncts depends on what else the agent does. If so, they are instances of *act dependence*: the normative status of each disjunct is future-contingent—in particular, contingent on what the agent decides vis-à-vis the other disjunct. Act dependence seems to be part of the data of free choice permission and Ross's puzzle.[3]

In this chapter, I present an account on which act dependence is also one half of a two-part semantic semantic explanation of the data. I begin by choosing a model theory adequate to modeling act dependence, where obligation depends on what is chosen. I

[3] Once again, the type of dependence at issue between act and status—whether it is metaphysical dependence or informational dependence—will depend on how the conditionals are interpreted. I pursue the former alternative here, but for the latter, see e.g. MacFarlane & Kolodny (2010). (See also Chapter 0, footnote 2.)

introduce concepts of obligation and permissibility in this framework inspired by decision theoretic work on the concept of ratifiability (Jeffrey, 1983), and impose on these models a two-dimensional semantic framework familiar from Davies & Humberstone (1980). In §4, I introduce an entry for disjunction, which is equivalent to Boolean disjunction outside the scope of modal operators, but has a different two-dimensional character. I then use these ingredients to validate the patterns in Table 5.1.

5.2 Proto-Semantics

In order to model cases where what an agent ought to do depends on what she chooses to do, we will need models capable of representing

(i) multiple candidates for how the agent will act, and

(ii) multiple candidates for what she ought to do.

Let an *agentive Kripke frame* be a tuple $\mathscr{M} = \langle W, R \rangle$ consisting of a universe of worlds W and a binary accessibility relation R on those worlds. In our frames, two worlds w and w' in W are distinct just in case some choice the agent can make is different in each of them: a world is a maximally decided course of choices beginning at a reference time and continuing through the end of the agent's existence. A *modal base* $s \subseteq W$ (which I will sometimes call a *choice situation*) is a less specific possibility, leaving some future choices undecided. It represents the position from which the agent chooses.

In the service of (i), we assume that the agent represents all actions within the range of her practical abilities as contingent with respect to s. A simple sentence p is circumstantially possible at s if it is true at some world(s) in s,[4] and settled-true at s if it is true at all worlds in s.

Definition 1 (Circumstantial Possibility)**.** *p is* **circumstantially possible at** *s iff there is some world $v \in s$ such that p is true at v.*

Definition 2 (Settled-truth)**.** *p is* **settled-true at** *s iff for all worlds $w' \in s$, p is true at w'.*

The notion of settled-truth in Definition 2 is our approximation of truth at a situation of choice. For example, it is settled-true at some situation s that I will lose the game I am playing if and only if it is true at every world compatible with my practical abilities in that situation—true *no matter what I choose*—that I will lose the game. Conversely, it is

[4] I take the term 'circumstantial' from the influential discussion in (Kratzer, 1981, §5).

circumstantially possible that I will *win* the game if there is some world $w \in s$ where it is true that I win the game.

The R relation is a *deontic accessibility relation*: two worlds w and w' are R-related just in case w' is deontically ideal by the lights of w.[5] Following standard deontic logic ('SDL'[6]), we gloss world-centric obligation and its dual, permissibility, in terms of the R relation:

Definition 3 (Obligation and Permissibility at Worlds)**.**

(3-a) p *is* **obligatory at** w *iff for any world* v *such that* wRv*,* p *is true at* v*.*

(3-b) p *is* **permissible at** w *iff there is some world* v *such that* wRv *and* p *is true at* v*.*

Following Kratzer (1981) and SDL, I assume the R relation is realistic and serial in s:

Definition 4 (Realistic)**.** *R is* **realistic in** *s iff for any w and v, if $w \in s$ and wRv, then $v \in s$.*

Definition 5 (Serial)**.** *R is* **serial in** *s iff for all $w \in s$, there is some v such that wRv.*

Definitions 4 and 5 describe what it is for a world to see another world as ideal, when s is taken to circumscribe the range of practically available options: Definition 4 says a world in s sees another world only if that other world is also circumstantially possible,[7] and Definition 5 says that no world is nihilistic, seeing nothing as ideal.

In service of goal (ii) above, our models allow that two worlds w and v, both in s, may be R-related to different outcomes: there can be a genuine variety of perspectives, amongst future-contingent states, regarding what is deontically ideal. To illustrate, here is a case from the decision-theory literature.

> **Nice Choices at the Spa.** Aromatherapy [$= p$] or body-wrap [$= q$]—which is it to be? You believe that, whichever you choose, you will be very glad you chose it. Mid-aromatherapy, the aromatherapy will seem self-evidently superior [to the body-wrap]. Mid-body-wrap, the body-wrap will seem self-evidently superior [to the aromatherapy]. (Hare & Hedden, 2015, pg. 3)

[5] The thought that deontic relations hold between *choosable* points (rather than anything finer-grained) is suggested by MacFarlane & Kolodny (2010)'s attractively named principle *Ought Implies Can Choose* (pg 132, footnote 28). See also MacFarlane (2013, §11.4), Charlow (2013), Wedgwood (2006), and Williams (1981), who calls the ability to choose the hallmark of the "practical or deliberative OUGHT".

[6] See, for example, the introduction in McNamara (2010).

[7] Suppose, for example, that it is true at every world in s that Bill gets mugged; it is not circumstantially possible to prevent the mugging. Then worlds in s will 'see' worlds where Bill is aided as ideal—even though it would have been better, relative to some enlarged s^+, if Bill had never been mugged at all.

Assume that deontic ideality—the R relation—goes with preference in this case (this is an assumption we will return to later.) Then in (Nice Choices), what you ought to do depends on what you *choose* to do. The Kripke model in Figure 5.1 serves to illustrate the facts.

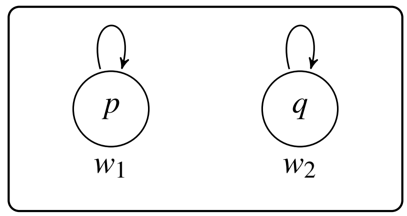


Figure 5.1: A situation s_{nice} representing (Nice Choices).

There is a sense in which you can't go wrong: if you pick aromatherapy, you'll find yourself at w_1, relative to which aromatherapy is the unique ideal option (since w_1Rw_1 and not w_1Rw_2.) So you'll be satisfied with what you did. But if you pick the body-wrap, you'll find yourself at w_2, relative to which the body-wrap is uniquely ideal (since w_2Rw_2 and not w_2Rw_1), so again, you'll be satisfied with what you did. And you know all this before you choose.

Notice that, even though there is something nice in (Nice Choices) about both p and q, p and q are *deontic contraries*—no post-choice perspective that sees p as ideal sees q as ideal, and vice-versa. So there is a sense in which, if you choose (say) to bring about p, the "niceness" of q immediately evaporates: looking back, you *won't* see q as ideal. This is the difference between (Nice Choices) and a model that simply represents p and q as *equally ideal*, from every point of view.[8]

Definition 3 uses the R relation to tell us what is permissible and obligatory at each individual world in s_{nice}. But we want to know what is permissible and obligatory from the *global* perspective the agent occupies *before* she chooses—what is obligatory and permissible relative to the whole of s. It is s, after all, that represents the point of view *from which* she makes her choice.

A simple answer to both questions would be that each normative notion, obligation and permissibility, scales up to s by imposing the conditions in Definition 3 on each individual world in s, just as we said that a fact-describing statement like "I will lose the game"

[8]Another, perhaps more psychologically realistic case of this phenomenon is discussed by Harman (2009), Hare (2011), and Paul (2013, 2015), is the case of conceiving one child rather than another (usually at different times). In each case, you will come to value the child you have (call him *Andrew*), and come to see conceiving *him* as more valuable than conceiving any other child. However, it is also true that if you had had a different child (call her *Bea*), you would be in the same situation, seeing conceiving Bea as more valuable than conceiving any other child. See Chapter 6 for discussion.

is settled-true at s when it is true at each individual world in s. Call these null hypotheses Postulates 1 and 2:

Postulate 1. *p is* **obligatory at** *s iff p is obligatory at every world in s.*

Postulate 2. *p is* **permissible at** *s iff p is permissible at every world in s.*

Postulate 1 looks right to describe the obligation-facts in (Nice Choices). In (Nice Choices), neither p nor q is obligatory at every world, so neither is obligatory relative to the whole modal base. Given that an agent has the freedom to choose between worlds, it does not seem that anything *less* than p's being obligatory at *every* world in s could be sufficient for p's being obligatory at s: if there is even *one* world in s where p is *not* obligatory, the agent could choose that world, and thereby "escape" the obligation.

By contrast, Postulate 2 is less secure. The instinct that you "can't go wrong" in (Nice Choices) remains unexplained on Postulate 2, since it says that (for example) p is permissible at s only if p is permissible by the lights of *every* world in s. Hence by Postulate 2, p is not permissible at s_{nice}, since it is not permissible at w_2. Likewise, q is not permissible at s_{nice}, since it is not permissible at w_1. Whereas instinct holds that *both* of p and q are choiceworthy in (Nice Choices), Postulate 2 tells us that *neither* is. Scaling up the intuitive notion of permissibility from w to s by the null hypothesis thus misses something about the case.

While I do not think Postulate 2 is the correct notion of permissibility at s, it is useful to have a name for the condition it attempts to impose on modal bases, for I think a better notion can be defined in terms of it. Let us call the notion associated with Postulate 2 *admissibility*:

Definition 6. *p is* **admissible at** *s iff p is permissible at every world in s.*

While it is a substantive hypothesis—Postulate 2—that p is permissible at s just in case p is permissible at every individual world in s, it is true by definition that p is *admissible* at s just in case this condition holds.

We can also consider admissibility at subsets s' in s, representing post-choice contexts:

Definition 7. *p is* **admissible at nonempty** *s'* **in** *s iff p is permissible at $\langle s, w \rangle$ for every w in s'.*

The admissibility of p at an s' in s is *persistent* permissibility—permissibility that is inescapable by the lights of one's future choices.[9]

(Nice Choices) highlights cases of preestablished harmony between whether an act is performed and whether that act is *post-choice admissible.* In act-dependent frames, cases of *disharmony* between whether an act is performed and whether it has this status are also possible. Consider:

> **Death**. You live in Damascus and learn that Death is coming to collect your soul. Death always follows his predetermined schedule and Death never misses his quarry. Should you flee to Aleppo $[= p]$? You are confident that, if you flee to Aleppo, Death will be there. But if you stay in Damascus $[= q]$, Death will be there too. (Gibbard & Harper, 1978)
>
> **Nasty Choices at the Spa.** Abdominal-acupuncture $[= p]$ or bee-sting-therapy $[= q]$—which is it to be? Whichever you choose, you will wish that you had chosen the other. (Hare & Hedden, 2015, pg. 13)

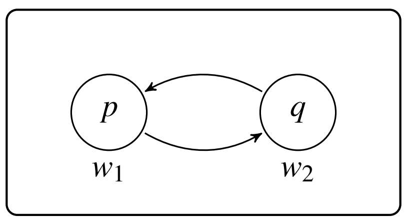


Figure 5.2: A situation s_{nasty} representing (Nasty Choices).

In (Nasty Choices), what the agent ought to do once again depends on what she chooses to do. But now her freedom has become a curse: even though every world in s sees *some* option as ideal, no option sees *itself* as ideal. There is no way the agent can make a choice and be satisfied by the deontic perspective she will occupy *after* she chooses.

Like the predicament in (Nice Choices), the pickle the agent is in in (Nasty Choices) is possible because we are considering what is choiceworthy at a relatively indeterminate state. In typical 'pointed' applications of Kripke frames—where permissibility claims are evaluated at fully determinate worlds—the seriality of R in s (Definition 3) is sufficient

[9] Once p is admissible at s' in s, then any $s'' \subseteq s'$ will be such that p is admissible at s'' in s. The retained relativity to s is for a technical reason: $\langle s', w\rangle$ may fail to be a well-defined point of evaluation if R fails to be serial or realistic (Definitions 4 and 5) in the contracted state.

to guarantee that there is always some intuitively permissible option available, since the seriality of R guarantees that from the point of view of any *world*, something always counts as ideal. It is only from the s-centric perspective that the seriality of R is no longer enough to capture this intuitive thought, since it is from the s-centric perspective that preestablished disharmonies like (Nasty Choices) are possible.

This is why cases like (Nice Choices) and (Nasty Choices) are prominent in the literature on rational choice. The thought is that they bring out a novel feature that choiceworthy acts must have: any choice that is ideal for you must be ideal for you *on the assumption that you perform it*. Richard Jeffrey called the property in question *ratifiability*:

> A ratifiable decision is a decision to perform an act of maximum estimated desirability relative to the probability matrix the agent thinks he would have if he finally decided to perform that act...*Maxim.* Make ratifiable decisions. To put it romantically: 'Choose for the person you expect to be when you have chosen' (Jeffrey, 1983, pg.16).

What is ratifiability in a Kripke frame? We shall say that an option, p, is ratifiable at s if the agent can contract s to some s' such that p is both settled-true and admissible at s' in s: this corresponds to an agent's being able to conditionalize on performing p in such a way that p is deontically ideal from her post-choice standpoint (ideal to her "future self," to use Jeffrey's phrase.)[10]

Definition 8 (Ratifiability). *p is* **ratifiable at** *s iff there is some nonempty $s' \subseteq s$ such that:*

(i) p is settled-true at s' in s;[11]

(ii) p is admissible at s' in s.

Cashing out admissibility in terms of the R-relation, this condition's holding at s is equivalent to its holding at a single world $v \in s$.[12] So Definition 8 simplifies to:

Definition 8 (Ratifiability, simplified). *p is* **ratifiable at** *s iff there is some world $v \in s$ such that:*

[10]Jeffrey's norm is usually glossed in terms of subjective credences, but a more objective gloss on this talk of 'conditionalization" is also available. As the agent acts, she constrains the course of history (whether she knows it or not); we leave open the possibility that the deontic status of an act exhibits objective, *causal* dependence on how history unfolds.

[11]That is, p is true at $\langle s, w \rangle$ for each $w \in s'$.

[12]In the left-to-right direction, just take $s' = \{v\}$. In the right-to-left direction: by nonemptiness of s', there is at least one world $w \in s$ which satisfies the conjunction of 8 and 8; let v be this world w.

(i) p *is true at* v*;*

(ii) $\exists v' \in s$ *such that* vRv' *and* p *is true at* v'*.*

In terms of an agent's ability to navigate between world-relative obligations: p is ratifiable at s just in case it is possible, practically speaking, to choose a p-world that sees a p-world.[13] In place of Postulate 2, I advance

Postulate 3. p *is* **permissible at** s *iff* p *is ratifiable at* s*.*

Postulate 3 differs from Postulate 2 because the ratifiability of p relative to s is distinct from the admissibility of p relative to s. I claim that it is better at accounting for the pretheoretical notion of *permissibility*, and it is the notion I will use, in the coming sections, to model the semantics of MAY. (Nice Choices) and (Nasty Choices) make the case: in (Nice Choices), neither p nor q is s-admissible, but both p and q are ratifiable. To the extent to which we feel that the preestablished harmony between act and status in (Nice Choices) renders both options *permissible* relative to the agent's undecided state, our intuitions are pegging the notion of permissibility to ratifiability, not to admissibility.

Turning our attention to (Nasty Choices), we see that, at every world in s_{nasty}, some nontrivial proposition (either p, or q) is admissible, according to that world. So relative to s_{nasty} as a whole, it is settled-true that

(7) Some option is *admissible*.

But while (7) is settled-true at s_{nasty},

(8) Some option is *ratifiable*.

is settled-false. To the extent to which Nasty cases strike us as hopeless—to the extent to which we feel there is nothing you *may* do in a Nasty case—our intuitions are once again tracking ratifiability, not admissibility.

To endorse the hypothesis that permissibility is ratifiability, relative to an undecided state s, is to implement the hindsight-directed point of view recommended by our cases: to abandon the prospective (and often indeterminate) question, "Is p admissible?" in favor of the retrospective (often more determinate) question, "If I do p, will I *have done* what is admissible?"

Turning to the package of s-obligation and s-ratifiability together—Postulates 1 and 3, united—we see an important difference in perspective. By Postulate 1, world-centric

[13] In the small models considered here, a single choice is sufficient to determine a possible world. But in general, in our Kripke frames, choosing a completely determinate *world* may require a series of choices over time.

obligation is capable of serving as the basic concept of obligation: global conditions on s are derived from it by imposing that concept on each individual world in s. By contrast, Postulate 3 tells us that permissibility can only be fully understood by taking the global perspective as basic: an option a is ratifiable just in case it is *choosably* both true and admissible at s, but whether that conjunction is *choosable* depends irreducibly on the whole of s: it is not a distributive property of individual worlds.[14] Hence Postulate 3 predicts that deontic permissibility is not *persistent*: permissions may fail to endure as s becomes more and more determinate—for example, as the agent executes a series of acts.[15] That is a good fit with the data. For example, it is a good fit with the intuition that you may take the beer and you may take the wine, but these acts are not permissible *come what may*: the latter permission does not persist if you exercise the former one.

Given this, I introduce a toy language. The operator '*O*', for OUGHT, tracks obligation, as defined according to Postulate 1. The operator '*M*', for MAY, tracks permissibility, as defined according to Postulate 3. For expressive completeness, I also introduce an operator '$\Diamond$' for admissibility, as defined according to Definition 6, and '$\blacklozenge$' for circumstantial possibility, as defined according to Definition 1. Summing up what we have so far:

Propositional fragment:

	a is true at $\langle s, w\rangle$	iff	w is an a-world
	$\neg p$ is true at $\langle s, w\rangle$	iff	p is not true at $\langle s, w\rangle$
	$p \wedge q$ is true at $\langle s, w\rangle$	iff	p is true at $\langle s, w\rangle$ and q is true at $\langle s, w\rangle$

Circumstantial Modality:

	$\Diamond p$ is true at $\langle s, w\rangle$	iff	$\exists v \in s$: p is true at $\langle s, v\rangle$

Deontic Modality:

(*Obligation*)	Op is true at $\langle s, w\rangle$	iff	$\forall w' \in s$: if wRw', then p is true at $\langle s, w'\rangle$
(*Admissibility*)	$\blacklozenge p$ is true at $\langle s, w\rangle$	iff	$\exists w' \in s$: wRw' and p is true at $\langle s, w'\rangle$
(*Ratifiability*)	Mp is true at $\langle s, w\rangle$	iff	$\exists v \in s$ such that (i) p is true at $\langle s, v\rangle$, and (ii) $\Diamond p$ is true at $\langle s, v\rangle$

[14] A feature F of a global state s is *distributive* just in case $F(s)$ holds iff $F(\{w\})$ holds for each $w \in s$. See, for example, van Benthem (1986).

[15] Note that this contrasts with the state of affairs for prejacents in the atomic fragment of the language. A prejacent like *agent chooses the aromatherapy* should be read as an eternalist proposition, along the lines of *agent chooses the aromatherapy at t* for a particular time t. Since this prejacent is not a function of an evaluation-time, once it is settled-true at some s, it remains settled-true at any $s' \subseteq s$.

Notice that the truth of Mp at $\langle s, w\rangle$ is independent of w, the world-parameter of the point of evaluation. This encodes the global nature of ratifiability—its dependence only on s.

Since we are considering truth at a context to be akin to settled-truth at s, our attendant notion of semantic consequence is the preservation of settled-truth at s. Hence $\phi \vDash \psi$ iff for any situation s, the truth of ϕ at every world in s guarantees the truth of ψ at every world in s.

5.3 Cases Revisited

Armed with our postulate that MAY tracks ratifiability—the property of being *choosably* both true and admissible—we can sketch two patterns of reasoning in act-dependent models. The first involves OUGHT. The second involves only MAY. Suppose that, for live options p and q:[16]

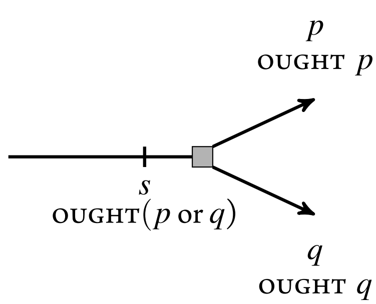
[16]The conditionals in Patterns A and B can be helpfully read as future-directed strict conditionals for (mutually exclusive) p and q: 'if p, then OUGHT p', for example, is true at s in Pattern A because in the largest $s' \subseteq s$ such that p is settled-true at s', OUGHT p is also settled-true at s'.

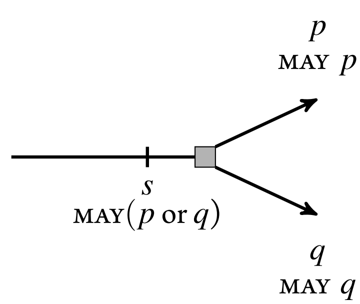
Pattern A:

1. If I make p true, then OUGHT p is true. (Premise)
2. If I make q true, then OUGHT q is true. (Premise)
3. So, if I make p true, p is admissible. (Transitivity of consequence)[17]
4. And, if I make q true, q is admissible. (Transitivity of consequence)
5. So, I can make p both true and admissible. (from 3)
6. And, I can make q both true and admissible. (from 4)
7. So, MAY p is true. (5, Ratifiability)
8. And MAY q is also true. (6, Ratifiability)

Pattern B:

1. If I make p true, then MAY p is true. (Premise)
2. If I make q true, then MAY q is true. (Premise)
3. So, if I make p true, I can make p both true and admissible. (1, Ratifiability)
4. And if I make q true, I can make q both true and admissible. (2, Ratifiability)
5. So I can make p both true and admissible. (from 3)[18]
6. And I can make q both true and admissible. (from 4)
7. So MAY p is true. (from 5)
8. And MAY q is true. (from 6)





I will argue that Pattern A is the underlying form of the inference that gives rise to (R+)—the strengthened version of Ross's puzzle—and that Pattern B is the underlying form of the inference that gives rise to free choice permission. The missing ingredient is

[17]Since Op entails $\Diamond p$ in s whenever R is realistic and serial in s (Definitions 4 and 5).

[18]There is a state where p is done admissibly in s iff there is a state where p is done admissibly *in the region of s throughout which p is true*, so the truth of (3) is sufficient to guarantee the truth of (5); the transition is analogous to the inference from "Amongst the F's, some Fs are G" to "Some Fs are G", when F is nonempty.

the semantics of sentential disjunction, which takes us from OUGHT$(p$ OR $q)$ to Premises 1-2 of Pattern A, and from MAY$(p$ OR $q)$ to Premises 1-2 of Pattern B. If the semantics of disjunction can take us that far, the logic of ratifiability will take care of the rest.

Before I plunge ahead, let me say a little in defense of this strategy. If disjunction can bridge the gap we've framed, we would have a story about how free choice permission sentences and Ross sentences could impose conditions on s that approximate the structure of Nice Cases—the structure, that is, of preestablished harmony between whether an act is performed and whether it has some positive normative status. Since Nice Cases exemplify the kind of impersistent permissibility we seem to get as *outputs*, when free choice permission sentences and Ross sentences are taken as *inputs*, this seems like a promising route to pursue. Moreover, the approach seems not unfeasible, since both patterns characterize the contribution of the premises' common factor—disjunction—in the same way.

To go forward, we need to get from (e.g.)

$$\text{OUGHT}(p \text{ OR } q)$$

being settled-true at s, to the validity of these transitions:

$$\frac{p}{\text{OUGHT}\ p} \qquad\qquad \frac{q}{\text{OUGHT}\ q}$$

That means we need to set up a dependency between which of p or q the agent actually brings about, and the semantic contribution of the sentence embedded under the modal. We must achieve this despite the fact that the modal operator takes a proposition—a *condition* on possible worlds—as an argument. To set up a dependence between what is true in the actual state and the *propositional content* of a sentence like $(p$ OR $q)$, we will require a two-dimensional semantics, in the style of Davies & Humberstone (1980). Our full valuation function will therefore recursively define truth at points of evaluation which are *triples* $\langle s, y, x\rangle$ consisting of a modal base s and a *pair* of worlds y and x in s.

Two-dimensional semantics is motivated by the thought that the pretheoretical notion of sentential truth can be upgraded to a relation in three parts: what it is for a sentence ϕ to be true at w is for the proposition ϕ expresses *at* w to be true *in* w. We can also ask whether that very same proposition might have been true in some other world, v: in asking this question, we hold w fixed in its role as the *actual world*, which contributes to the proposition expressed by ϕ; but we let v play the role of the *world of evaluation*: the world *in which* that proposition's truth is evaluated.

$$\underbrace{s,}_{\text{modal base}} \quad \overbrace{w,}^{\text{actual world}} \quad \underbrace{v}_{\text{world of evaluation}} \quad \vDash \phi$$

In the case of disjunction, we want the proposition expressed by a disjunction to depend on which disjunct is true in w. We can then ask whether *that very disjunct* is true at some other, more ideal world v.

The connection we are trying to make can be put like this. In the previous section, we examined act dependence—the idea that whether an act is obligatory may vary across different worlds, between which the agent can choose. The move to two-dimensional semantics endeavors to mirror this state of affairs by implementing *semantic* act dependence—the idea that a *sentence* may express different *propositions* in different worlds, amongst which the agent can choose. Concretely, when considering-as-actual a world w where only p is true, we want 'p OR q' to be equivalent to p; and when considering-as-actual a world w where only q is true, we want 'p OR q' to be equivalent to q.[19]

I sketch the required treatment of disjunction in the following section. We will then return to the protosemantics of §2 and upgrade it to two dimensions, bringing the rest of the system along for the ride.

5.4 Disjuncts as Witnesses

The semantics for disjunction I lay out in this section develops the idea that any true disjunction is *witnessed* by one of its disjuncts, in the same way that a true existential statement in first-order logic is witnessed by an individual. As promised, we here combine two threads: we take from Chapter 4 the the Heim-inspired idea that the *interpretation* of a disjunction 'p OR q' depends on the way in which the disjunction is witnessed, and from Chapter 3, we take the idea that the disjunction is nontrivially two-dimensional.

We begin by identifying a disjunction's true disjuncts. Let Alt_w (where $w \in W$) be a function that takes two sentences as arguments, returning the sentences from amongst those two that are true in w, whenever there are any. For completeness, I add the condition that Alt_w returns both disjuncts in the case where both are false—the intuition being that, since there are no witnesses for a false disjunction, there is nothing to break the symmetry between the two inputs.

Definition 9 (The Alt_w function)**.** *Alt_w is a function that takes a pair of sentences ϕ and ψ as arguments. It returns the set containing all and only the true-in-w sentences in $\{\phi, \psi\}$, if there are any, and returns $\{\phi, \psi\}$ otherwise.*

[19] The equivalence at stake is equivalence when the y-world, w_3, is held constant: $s, w_3, x \vDash p$ iff $s, w_3, x \vDash (p \text{ OR } q)$ for any $s \subseteq W$ and $x \in W$.

Here is Alt_w for the four world-types that correspond to the standard truth table for p and q:[20]

	p	q	$Alt_w(p,q)$
w_1	T	T	$\{p,q\}$
w_2	T	F	$\{p\}$
w_3	F	T	$\{q\}$
w_4	F	F	$\{p,q\}$

Notice that the truth-at-w of one member of $Alt_w(p,q)$ is both necessary and sufficient for the truth of p OR q at w. So to recover the classical truth-conditions of disjunction, it suffices to quantify existentially over $Alt_w(p,q)$.

Definition 10 (Protosemantics for OR). *$(p$ OR $q)$ is true at $\langle s,w\rangle$ iff there is some $\alpha \in Alt_w(p,q)$ such that α is true at $\langle s,w\rangle$.*

Given our definition of Alt_w, Definition 10 is equivalent to the classical truth-conditions of unembedded disjunction. However, because w appears on both sides of the "is true at" on the right hand side, this restatement opens up the possibility of seeing w as contributing meaningfully to the proposition *expressed* by the disjunction. The Alt_w function allows its parameterizing world, w, to associate a disjunction with a unique witness: p, if p is true and q false in w; and q, if q is true and p false in w. So using Alt_w, we can make p OR q *equivalent* to p in worlds where only p is true, and make p OR q *equivalent* to q in worlds where only q is true.

Definition 10 tells us when p OR q is true at a given world w. This underspecifies a two-dimensional entry for disjunction, because it does not specify *in virtue of what role*—world-as-actual or world of evaluation—w parameterizes the Alt function. Using x, the world of evaluation, will get us Boolean disjunction even in embedded environments; using y, the world-as-actual, will get us the disjunct that is true in the actual world, no matter which world the disjunction is being evaluated in. Let us therefore stipulate that it is y, the world-as-actual, that parameterizes Alt.

Postulate 4 (Two-dimensional OR). *$s,y,x \vDash (p$ OR $q)$ iff there is some $\alpha \in Alt_y(p,q)$ such that $s,y,x \vDash \alpha$.*

[20] Here, I use 'p' and 'q' range over atomics.

When the disjunction in Postulate 4 is embedded under a deontic modal like OUGHT, the modal shifts the world of evaluation, but not the world-as-actual. That means any dependence the embedded sentence displays on the world-as-actual remains anchored to that world. So at any point of evaluation where only p is (actually) true, $\text{OUGHT}(p \text{ OR } q)$ is equivalent to $\text{OUGHT}\, p$, and at any point where only q (actually) is true, $\text{OUGHT}(p \text{ OR } q)$ is equivalent to $\text{OUGHT}\, q$. Likewise, at points where only p is true, $\text{MAY}(p \text{ OR } q)$ is equivalent to $\text{MAY}\, p$, and at points where only q is true, $\text{MAY}(p \text{ OR } q)$ is equivalent to $\text{MAY}\, q$. These are the two sets of transitions in the premises of Patterns A and B in §3.[21]

Postulate 4 is the last postulate we will need to explain free choice permission and Ross's puzzle for mutually contingent p and q. In the Appendix (Appendix B), I will add one additional condition to the definition of Alt_y that is intuitive from the witnessing perspective: in the case where both p_1 and p_2 are true at y, but p_1 *strictly entails* p_2, any $Alt_y(p_1, p_2)$ delivers $\{p_1\}$, the singleton containing 'more specific' of the two disjuncts. (One could appeal here to Armstrong (2004)'s truthmaker Entailment Principle (10): if p entails q, then any truthmaker for p is a truthmaker for q.) This embellishment has no effect on the diagonal profile of disjunction, but it will give us the right results for deontically modalized disjunctions in the case where one disjunct entails the other. Intuitively, the sentence 'I am wearing pants' and the sentence 'I am wearing *red* pants' have the same truthmaker, in a world where both are true: viz., the fact that I am wearing *red* pants; the truth of the more specific fact is enough to guarantee the truth of the less specific one. Anticipating, the permission-statement, 'you may wear pants or red pants,' entails that you may wear *red* pants, in a world where both are true. This more specific permission ('you may wear *red* pants') is sufficient to entail the truth of the less specific one ('you may wear pants.')

5.5 Two Dimensions: Deontic Modality

That was disjunction. In order to really see how the object language works, we need to bring the whole semantics into two dimensions. Deontic modals are classical 'one-dimensional' modals, shifting only the world of evaluation, x, and not the world-as-actual, y: their interpretation holds fixed the proposition expressed by ϕ in evaluating the question, *ought it to be the case that* ϕ*?*

Our old definition of obligation at $\langle s, w \rangle$, Definition 3-a, was

[21] That is, assuming that conditionalizing s on p reduces s to the (or a) *largest* subset $s|p$ throughout which p is true, rather than some smaller subset throughout which e.g. *both* p and q are true. For an explicit connection between this idea and the semantics of an object-language indicative conditional, see, for example, Kratzer (1991a), Yalcin (2007, pg. 998), and MacFarlane & Kolodny (2010, pg. 136).

p is obligatory at $\langle s, w \rangle$ iff $\forall v$ such that wRv, p is true at $\langle s, v \rangle$.

In our full system, sentences ϕ express propositions only relative to some choice of world-as-actual. Using 'y' for this world, Definition 3-a upgrades to

ϕ is obligatory at $\langle s, y, x \rangle$ iff $\forall x'$ such that xRx', the proposition ϕ expresses at y is true at $\langle s, x' \rangle$.

So our new semantic entry is

Definition 11 (OUGHT). *$s, y, x \vDash O\phi$ iff $\forall x' \in s$ such that xRx': $s, y, x' \vDash \phi$.*

This is just our old OUGHT, with an free y parameter added. The OUGHT in Definition 11 is local twice over: its truth at $\langle s, y, x \rangle$ depends on which worlds x is R-related to, and what proposition ϕ expresses, relative to y. So when $O\phi$ occurs unembedded, $O\phi$ is settled-true at s just in case, at every world $w \in s$, the proposition ϕ expresses, *at w*, is obligatory, *in w*.

What of Ratifiability, our companion notion to deontic obligation? In our protosemantics in §2, we endorsed this notion of a proposition's being ratifiable (Definition 8):

p is ratifiable at $\langle s, w \rangle$ iff $\exists v \in s$ such that

(i) p is true in $\langle s, v \rangle$;

(ii) p is admissible in $\langle s, v \rangle$.

once again, in our full system, sentences ϕ express determinate propositions only relative to some choice of world-as-actual. Using 'y' for this world, Definition 8 upgrades to

Definition 12 (MAY). *$s, y, x \vDash M\phi$ iff $\exists v \in s$ such that (i) $s, y, v \vDash \phi$ and (ii) $\exists v'$ such that vRv' and $s, y, v' \vDash \phi$.*

Once again, this is just our old MAY, with a free y-parameter added. Our semantic entry for MAY therefore combines the *global* character of the underlying notion of propositional ratifiability (independence of $M\phi$ from the x-parameter) with the *local* character of propositional content (dependence of $M\phi$ on the y-parameter). The derived settled-truth conditions for unembedded $M\phi$, where the same world plays both roles, can be glossed like this: $M\phi$ is settled-true at s just in case there is some world v in s such that the proposition ϕ expresses, *at v*, is admissible, *in v*.[22]

[22] More technically: $M\phi$ is settled-true at s iff $\exists v \in s$ such that: (i) $s, v, v \vDash \phi$ and (ii) $\exists v' \in s : vRv'$ and $s, v, v' \vDash \phi$.

This is a nice result. The two-dimensional semantics for disjunction, when combined with the underlying inferential properties of ratifiability, will give us proofs of (FC) and (R+)—the inference to the permissibility of each disjunct—for both MAY and OUGHT (see Appendix B, Theorems 2 and 3). When a sentence of the form MAY(p OR q) is settled-true at s, we can describe the agent's situation metaphorically as follows. To be permitted to do p OR q, where p OR q is an disjunction of future contingent actions, is like being issued a *ticket of permission*, bearing p and q, that can be valued in two different ways. If the agent sees to it that exactly one of the disjuncts is true, it is the unique interpretation of the ticket. Looking back from her post-choice perspective, she will see that she has done what is permissible. However, where there is no true disjunct, or too many of them, all that follows is that the ticket has some value or other amongst the possibilities provided by the disjuncts. That is enough to guarantee that, if the agent does neither p nor q, she refrained from *something* she had permission to do, and that, if she does more than one of those things, she did *more* than she had explicit permission to do. From this latter fact, exclusivity follows.

Things are similar in the OUGHT case. Here, the agent is issued a ticket *obligating* her to do p OR q. The one she picks values the propositional content of the disjunction, putting her in a state where she has done what she was obligated to do; so long as she chooses to perform only one of the disjuncts, she will have discharged, at her post-choice context, the content the obligation picks out at that context. But if she does both *p and q*, she does *more* than she was obligated to do. Since the obligation statement has nothing to say about the other disjunct, it is entirely possible that it was *not* obligatory—or, indeed, even permissible. This captures the other set of exclusivity intuitions we had in §1, about the case where I ought to cancel or postpone the party: while each option is permissible, their conjunction may well be *impermissible*.

5.6 Two Dimensions: Consequence

Our target notion of consequence in the protosemantics of §2 was the preservation of settled-truth, or truth at every world in s. The move to a two-dimensional semantics presents us with a further choice: whether we seek the preservation of truth at points where $x = y$, or more generally at any two-dimensional point. Following tradition, I will assume that *diagonal consequence* most closely approximates intuitive consequence relations between natural language sentences.[23] This reflects the motivation we began with: the idea that ϕ's being true at w unpacks (and repacks) into the notion that the proposition expressed by ϕ *at* w is true *in* w. ψ is a diagonal consequence of ϕ just in case, for any s, if

[23]See the notion of *real world validity* in Davies & Humberstone (1980).

ϕ is settled-true at every point $\langle s, y, x \rangle$ in s such that $y = x$, then so is ψ. That gives us a consequence relation that preserves diagonal settled-truth:

Definition 13 (Settled-Diagonal (SD) Consequence). *$\phi \models_{SD} \psi$ iff for any s: if $s, w, w \models_{SD} \phi$ for all $w \in s$, then $s, w, w \models_{SD} \psi$ for all $w \in s$.*

Let us take a closer look at disjunction from the point of view of SD consequence. In the style made familiar by Stalnaker (1999a), we can capture the complete semantic profile of 'p OR q' with a two-dimensional matrix, once again letting w_1-w_4 represent the standard four lines of a truth-table for atomic p and q.[24]

	w_1	w_2	w_3	w_4
w_1	T	T	T	F
w_2	T	T	F	F
w_3	T	F	T	F
w_4	T	T	T	F

Table 5.2: 2D matrix for $\ulcorner p$ OR $q \urcorner$

In w_2, where only p is true, p OR q is equivalent to p. In w_3, where only q is true, p OR q is equivalent to q. The diagonal of the matrix witnesses the T-T-T-F truth-conditions of Boolean disjunction. With diagonal consequence on the table, the status of classical propositional logic is simple to state. Diagonal consequence begins by looking at the truth-values of sentences at diagonal points. Within the propositional fragment of the language, there are no (one-dimensional) modal operators, so nothing ever moves us *off* the diagonal. This is what lies behind

Theorem 1 (Classicality). *For any ϕ in the nonmodal fragment of L: $\models_{SD} \phi$ iff ϕ is a theorem of classical logic.*

The matrix also shows why OR-introduction is not valid off the diagonal—for example, once we start looking at modally embedded environments. For example, it does not preserve truth to infer from the proposition expressed by p to the proposition expressed

[24]The y-axis of the matrix represents world-types in their role as world-as-actual, and the x-axis represents world-types in their role as world of evaluation. Hence, reading across a row will give one the proposition expressed by the sentence, when the y-axis world plays the role of the world-as-actual. See Stalnaker (1999a, pg. 81) for discussion.

by p OR q when y is held fixed at w_3. In w_3, p OR q is equivalent to q. So the inference 'p, therefore (p OR q)' is equivalent at w_3 to the inference 'p, therefore q', which is clearly *not* valid. This is what lies behind

Fact 2 (Failure of OR introduction). *$O\phi \nvDash_{SD} O(\phi$ OR $\psi)$ and $M\phi \nvDash_{SD} M(\phi$ OR $\psi)$.*

This observation targets the intuitive badness of inferences like "you may post the letter; therefore, you may post the letter or burn it." The problem is not (in the first place) that such transitions are misleading, infelicitous, or uncooperative. They are just plain invalid: they can take us from a premise that is settled-diagonal true at s to a premise that is settled-diagonal false at s.

To see how $M(p$ OR $q)$ is stronger than Mp in our system, consider the two permissions in Figure 5.3.

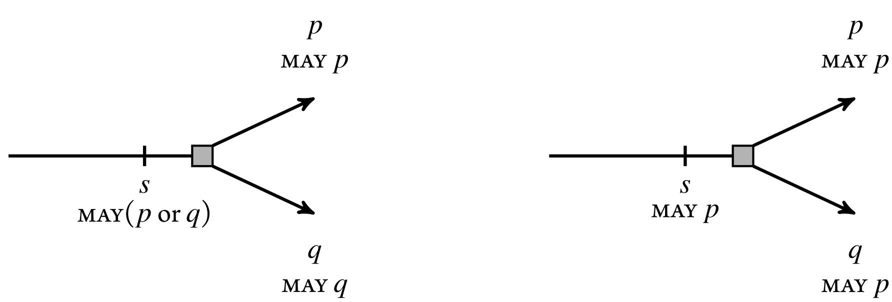


Figure 5.3: Two permissions

On the left, the agent is given permission to do p OR q. Since she is agentially capable of bringing about q, she is capable of occupying the lower branch, where (only) q is true. She is thereby capable of bringing it about that p OR q expresses the proposition that q: and so she is capable of bringing it about that she has done what she had permission to do. On the right, the agent is given permission to do p. Once again, she is agentially capable of bringing about q by taking the lower branch. But p is a propositional constant in our semantics: it expresses the same proposition at every point of evaluation. So she cannot, by *doing* q, bring it about that she has done what she had permission to do.

5.7 Tallying Up

We began with two puzzles: one about deontic permissibility, and one about deontic obligation. Our initial step towards their joint solution was indirect: we began by looking at cases of act dependence, formalizing them in agentive Kripke frames. Our first semantic move was to postulate that object-language MAY tracks the concept of permissibility developed for these cases. Our operator is thus constrained by the language-independent intuitions we have about choiceworthiness, in cases where our future selves disagree. Our second move was a two-dimensional, "witnessing" account of disjunction, which allowed the object language to mirror the dependence of norms on acts by making the *propositional content* of a deontic claim dependent on what becomes actual.

The package of the two moves allows the stronger-than-classical conclusions of free choice permission to follow, and blocks the inference in Ross's puzzle. It also predicts the positive entailment properties of disjunction under OUGHT in terms of MAY (the pattern we called (R+)).

May:	
(Failure of OR intro)	$M\phi \nvDash_{SD} M(\phi \text{ OR } \psi)$
(FC)	$M(\phi \text{ OR } \psi), \blacklozenge\phi, \blacklozenge\psi \vDash_{SD} M\phi \wedge M\psi$
Ought:	
(Failure of OR intro)	$O\phi \nvDash_{SD} O(\phi \text{ OR } \psi)$
(R+)	$O(\phi \text{ OR } \psi), \blacklozenge\phi, \blacklozenge\psi \vDash_{SD} M\phi \wedge M\psi$
Propositional Fragment:	
(Classicality)	for ϕ in the propositional fragment of L, $\vDash_{SD} \phi$ iff ϕ is a theorem of classical logic.

Table 5.3: Semantic entailments in DLC (Deontic Logic with Choice) .

Table 5.3 summarizes the approach taken to the data we began with. Assuming there is an object-language conditional '$\rightsquigarrow$' which is related to $\vDash_{SD}$ by the Deduction Theorem, we can add (Conditionals-M): $M(\phi \text{ OR } \psi) \vDash_{SD} \neg\phi \rightsquigarrow M\psi$, and (Conditionals-O): $O(\phi \text{ OR } \psi) \vDash_{SD} \neg\phi \rightsquigarrow O\psi$. Lacking space to defend such a semantics for the conditional, I leave this inference off the official tally.

Table 5.3 enables a semantic account of some desirable entailments involving deontic modals and disjunction. To this end, I used a notion of consequence for modeling the inferences which was a global notion: diagonal settled-truth throughout the modal base s.

$\Pi \vDash_{SD} \psi$ iff for any M, any $s \subseteq W$ such that R is serial in s:
(if $\forall w \in s : s, w, w \vDash \phi$ for all $\phi \in \Pi$, then $\forall w \in s : s, w, w \vDash \psi$)

An alternative notion of consequence is still diagonal, but *local* (DL consequence):

$\Pi \vDash_{DL} \psi$ iff for any M, any $s \subseteq W$ such that R is serial in s:
$\forall w \in s$: (if $s, w, w \vDash \phi$ for all $\phi \in \Pi$, then $s, w, w \vDash \psi$)

It is worth noting that three of our inferences are not *locally* valid (henceforth, I suppress the "diagonal".)[25]

(Conditionals-M)	$\textsc{may}(\phi \text{ or } \psi) \nvDash_{DL}$ if $\neg\phi$, then $\textsc{may}\,\psi$
(R+)	$\textsc{ought}(\phi \text{ or } \psi), \blacklozenge\phi, \blacklozenge\psi \nvDash_{DL} \textsc{may}\,\phi \wedge \textsc{may}\,\psi$
(Conditionals-O)	$\textsc{ought}(\phi \text{ or } \psi) \nvDash_{DL}$ if $\neg\phi$, then $\textsc{ought}\,\psi$

Table 5.4: Globally, but not locally, valid.

Countermodels can be found in Appendix C. The framework developed here thus encourages an approach according to which global consequence is the favored notion of consequence. This dovetails well with our answer to a postsemantic question (MacFarlane, 2013, pg. 58): how, given a compositional semantics, which defines a technical notion of truth for a sentence ϕ relative to a point of evaluation, do we determine whether a sentence is true at a context c (corresponding to a concerete setting where a speech act might take place)?

Within a typical two-dimensional system system, such as Lewis (1980), context plays the role of "initializing" the values of parameters in the index (which we can mark with a sub-'c'), as well the role of providing semantic values for indexicals. I suggested in §2 that c can initialize the whole of s, without initializing any world within s: it can be a fact about the context that certain choices—represented by certain worlds—are *left open*. Hence I favor answering the postsemantic question in terms of settled diagonal truth:

Definition 14 (Postsemantics). *ϕ is true in c iff $\forall w \in s_c$: $s_c, w, w \vDash \phi$.*

As a result, the global notion of validity in our system is the notion which preserves postsemantic truth.

Fact 3 (Contextual validity). *The inferences in Table 5.4 preserve truth at a context c.*

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[25] Once again, I assume the deduction theorem for (Conditionals-M) and (Conditionals-O).

5.8 Assessing the System: Two Worries

Counterfactual Modalities

How does the nonstandard semantics for OR advanced here interact with the rest of the language—such as other flavors of modality? I suggested in Chapter 3, §6, that epistemic modality be treated as a diagonal modality; in this respect, epistemic modals $\Diamond_e$ and $\Box_e$ will behave like our circumstantial modal operators, ⧫ and ■. Hence epistemic modals will not provide an environment which will display disjunction's nonclassical character, and DLC will not have surprising consequences in this fragment of the language.

Trouble, however, lurks not far behind, in the form of alethic modalities–for example, those created by the antecedents of counterfactuals. Suppose it is true that Otto has a dog and no cats. It seems that on the witnessing account of disjunction, "Otto has a dog or a cat" rigidly expresses the proposition that Otto has a dog. Hence

(9) If Otto had picked out a cat rather than a dog at the animal shelter, Otto would have had a dog or a cat.
$p \mathrel{\Box\!\!\rightarrow} (p \text{ OR } q)$

is predicted to come out false on the view.[26] For the same reason disjunction introduction is blocked in the scope of e.g. OUGHT, it is blocked in the consequent of the counterfactual: from $p \mathrel{\Box\!\!\rightarrow} p$, one may not infer $p \mathrel{\Box\!\!\rightarrow} (p \text{ OR } q)$. Yet (9) seems obviously true.

For ease of discussion, we can consider the counterfactual conditional in (9) from the point of view of Stalnaker's simple selection-functional semantics (Stalnaker, 1975), according to which $\phi \mathrel{\Box\!\!\rightarrow} \psi$ is true at w just in case ψ is true at $f(w, \phi)$, the (unique) nearest state to w where ϕ is true.[27] Stalnaker adds to this recipe a "pragmatic constraint on selection functions," to the effect that any world w in the context set is closer to every other world in w's context set than it is to any world outside the context set (Stalnaker, 1975, pg. 275-276). Worlds in the context set are compatible with everything that is presupposed; assuming, for the moment, that the conversational tone is such that ϕ is presupposed iff ϕ is known to be true, it follows that every world in the context set is a candidate for actuality, and everything outside the context set fails to be a candidate for actuality (it is *counter*-known-factual.) Any possible world where Otto picks out a cat rather than a dog is counterfactual in this sense.

[26] Thanks to John MacFarlane for raising this objection.

[27] Famously, Stalnaker holds this account extends to both indicatives and counterfactuals; we consider only the counterfactual version here. I assume for ease of discussion that the syntax of counterfactuals *is* the syntax of a two-place connective; see Kratzer (1981) for a denial that this is the case for indicatives.

Strictly speaking, DLC semantics as yet makes *no* prediction for what happens in the case of such *counter*-factuals, since we have hitherto examined only sentences which are evaluated at modal bases s which were glossed as (sets of) candidates for actuality. So there is, it seems, some room to maneuver, even in light of (9).

Here is a sketch. We begin with Stalnaker's function on antecedents—the function f that, relative to a world w and sentence ϕ, takes us to the nearest state where ϕ is true. Generalize this to a function on modal bases, which takes us from a modal base s to the nearest modal base s' relative to which ϕ is diagonally settled true. Such a function preserves the idea that there is a unique closest state to s satisfying some constraint, but allows that constraint to be a global, rather than a local one.[28] We require the consequent ψ of a counterfactual to be diagonally settled-true relative to a shifted modal base $f(s,\phi)$.

Definition 15 (Selection Function). *Given a strict similarity ordering $<_s$ on modal bases s' for each $s, s' \subseteq \mathscr{P}(W)$, $f(s,\phi)$ is the s' such that (i) $\forall w' \in s'$: $s, w', w' \vDash \phi$, and (ii) $\neg\exists s'' <_s s'$ such that $\forall w' \in s''$: $s'', w', w' \vDash \phi$.*

Definition 16 (Counterfactuals). $s, y, x \vDash \phi \mathrel{\Box\!\!\rightarrow} \psi$ *iff* $\forall w' \in f(s,\phi) : f(s,\phi), w', w' \vDash \psi$.

It follows from Definition 16 that in the consequent of a counterfactual, OR is Boolean. So (9) is true, for the same reason that Boolean disjunction introduction is valid.

This means that the embedding environment created by the antecedent of a counterfactual pries apart the notion of a diagonal point relevant for disjunction from the c-centered notion of a diagonal point relevant to the analysis of indexicals. It is a familiar point that under counterfactual antecedents, indexical-containing diagonal truths can go false:

(10) I am here now.
IHN

(11) If the cafe were closed, I would not be here now.
$C \mathrel{\Box\!\!\rightarrow} \neg IHN$

What is the difference? In the terminology of context and index, our indices consist of three parameters, s, y, and x. Disjunction is sensitive to points in a modal base s that are off-diagonal in the following sense: they are context-index tuples

[28] See, for example, MacFarlane and Kolodny's notion of a maximal ϕ-subset of an information state i (MacFarlane & Kolodny, 2010, pg. 135-136), and Yalcin's notion of an i nearest information state i' that accepts ϕ (Yalcin, 2012b, pg. 1018).

$\langle c, s, y, x \rangle$ such that $x \neq y$.[29]

Counterfactual consequents are not sensitive to points that are off-diagonal in *this* sense, though they may be sensitive to parameters that are not initialized by context:

$\langle c, s, y, x \rangle$ such that $x = y$, but $s \neq s_c$, $x \neq w_c$ and $y \neq w_c$.

These are independent notions. A context-index tuple $\langle c, s, w, w \rangle$, where $s \neq s_c$, is diagonal in the sense that matters for the inferences in Table 5.4. But more would need to be said in order to show that sentences with indexicals like "I am here now" are valid at such a point. Going in the other direction, we cannot understand a context-index pair $\langle c, s, w, w \rangle$, which is diagonal in the second sense, in terms of worlds y and x being initialized by a common context, since context does not, given our indeterminist leanings, initialize local parameters *within* s at all.

The sketch in Chapter 3 comparing "I am here now" (the rule of IHN introduction) and OR introduction was therefore incomplete, since it involved points of evaluation that did not draw a distinction between s_c (what is left open by what is actual) and y (the world-as-actual). The analogy is strict only when considering points that are diagonal in both senses:

$\langle c, s_c, w, w \rangle$.

For these "doubly diagonal" points, I claim, IHN introduction and disjunction introduction will be, as argued in that chapter, relevantly alike.

Are Nice Cases Irrational, or Impossible?

I have argued that we can explain free choice permission and Ross's puzzle by providing a semantics on which free choice permission sentences and Ross sentences impose conditions on a modal base which have the structure of "nice choices": choices where what the agent ought to do depends on what she does. But my leading example, Hare & Hedden (2015)'s Nice Choices at the Spa case, was first and foremost a case where what the agent *will prefer* depends on what she does. What is the relationship between deontic ratifiability and the preference ratifiability in this example—and can a gap between the two raise problems for the account?

One problem, beginning on the side of preference, is that one may have the intuition that preference ratifiability is irrational, in much the same way that intransitive preferences

[29] See, for example, Segerberg (1973), Davies & Humberstone (1980).

seem to be irrational. If all the cases that are used to motivate ratifiability as distinct from admissibility are 'fishy'—rules only slightly crazy agents could be in a position to use—it seems we should be cautious about drawing conclusions from them.

Here, I think, it is worth keeping in mind that to give someone permission is not to describe their preferences as being some way. Nor is it clear that deontic modal statements attempt to promote or cause preferences to be that way.[30] Finally, it is not clear that permission-giving is subject to rational criticism in the way that inconsistent desires might be; a friend who gives you permission to one-box in the Newcomb puzzle (Nozick, 1969)[31] may be giving you permission do to something irrational without himself being subject to rational criticism. So even if one does have the intuition that there is something that borders on irrationality about a single agent's having the preferences in Nice Cases, it is not clear that this is an objection to a view of the communicative function of permission on which disjunctive permissions have this structure—and therefore, license the entailments of interest to us.

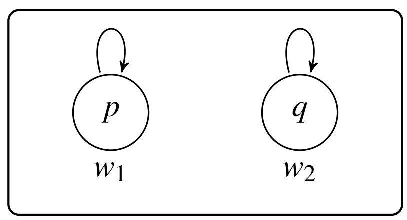

Figure 5.4: (Nice Choices) revisited.

[30]Contra, as I read them, the suggestions of Starr (2014), Silk (2014).

[31] Newcomb's puzzle is a classic thought experiment that distinguishes the case of an available act's raising the evidential probability of a good outcome from the same act's raising the causal probability—serving to *bring about*—that outcome. Here is a typical statement:

> There are two boxes before you, a large opaque box and a small clear box containing $1,000. You may take both boxes, or just the opaque box, and keep whatever is inside the box(es) you take. But: a nearly infallible predictor has put either $1 million or $0 in the opaque box, and she has put the million in the opaque box *just in case* she predicted you would *not* take the extra $1,000.

Should you take one box, or two? The probability that the predictor put $1 million in the opaque box, given that you take only one box, is very high. But choosing one box feels intuitively wrong to many people. The fact that conditionalizing on taking one box raises the probability of getting a million dollars cannot, in Newcomb's Puzzle, reflect any *positive causal influence* the act has on bringing about having a million dollars, since the predictor makes her decision before you do. So, it seems, the decision to one-box rather than two-box is not rational: you pass up a sure $1,000 by one-boxing, and your sacrifice in no way *brings about* better chances at the $1 million.

A second, related worry targets the same act dependent structure at the level of deontic ideality. This objection grants the independence of the R relation from preference, but still holds that it is deeply puzzling whether such structures could really exist: whether there are true *deontic* analogues of the preference structure in cases like Nice Choices at the Spa. Can our explanation of free choice permission and Ross's puzzle really depend on such cases, if they entail a kind of perspectivalism about normativity that cannot be ascribed to a shift in preference?[32]

A substantive answer to this question requires steps towards a positive characterization—though not necessarily a reductive one—of the R relation. In order to show that what one *ought*, deontically, to do can depend on one's acts, one might argue that *reasons*, for example, can have this act-dependent structure. And that is a taller order than it is in the case of preference, for anyone who thinks that reasons are less fickle than preferences.

While I find this metasemantic project—looking to the structure of reasons to ground claims about the nature of the R relation—an attractive one, a full-blooded defense of it is properly outside the scope of the present project. A compositional semantic theory will tell us that complex sentences receive certain (settled-)truth conditions; it will not tell us what makes, or ever could make, those (settled-)truth conditions obtain. It is the former question which is directly relevant to approaching puzzles in language, such as Ross's Puzzle and free choice permission.

From the semantic point of view, the kind of "perspectivalism" at issue is not at all unusual. It is simply the fact that a settled-true sentence may express different propositions at different points of evaluation (for example, at $\langle s_{\text{nice}}, w_1, w_1 \rangle$ vs. $\langle s_{\text{nice}}, w_2, w_2 \rangle$); all I have added here is that the agent can choose between such points. On the theory here defended, if it is a matter of worldly indeterminacy whether the addressee will take the beer or the wine, "you ought to take the beer or the wine" will express the proposition *you ought to take the beer* in worlds where you take the beer, and the proposition *you ought to take the wine* in worlds where you take the wine. The world provides the structure needed for semantic act-dependence, by failing to be determinate in respect of what you will choose.

As to the further question of what could give the R relation the structure in (Nice Choices), it is worth canvassing two extremes. Doubling down on the autonomy of semantics from questions of ground, we might observe that entailment-facts about free choice permission sentences and Ross sentences could hold even if, at the level of metaphysics, *nothing* could ever ground their truth, just as

[32]Some philosophers, including Hare & Hedden (2015), take for granted that there is a 'subjective OUGHT' which does simply mark decision-theoretic expected utility. This would restore the desire structure in (Nice Choices) to a case of the right structure for *some* OUGHT. I address my remarks here to those skeptical of the idea that the subjective OUGHT is an OUGHT of natural language, of the kind that is relevant to e.g. Ross's Puzzle.

(12) That is a round square.

entails

(13) That is a square.

in virtue of its logical form, even though nothing could ever make (12) true. To pair our semantics with a metaphysical view that is skeptical of normative act-dependence might force us to see free choice inferences and Ross inferences along the lines of (12)-(13); this is admittedly queer as a package.

Stepping back from this extreme will require exploring views of normativity that can support the possibility of act-dependence; one such example might be Chang (2013). Chang's "hybrid voluntarist" view of practical normativity holds that the most important distinction between types of practical reason is not a distinction between internalist and externalist reasons, but between those reasons which we can and cannot actively create. "Sometimes," she writes, "the fact that a consideration has the normativity of a reason is given to us, while other times it is a fact of our own making" (14). If one ought to do what one has most reason to do, then one can create truths about what one ought to do by acting. What act-dependence further requires is that one can create a reason for doing *p* by doing *that very thing*.

And so it is appropriate, I think, to close by granting the objector the following. However OUGHT-talk is ultimately to be cashed out, if we are not error theorists about the kind of self-reinforcing cases our account of deontic disjunction appeals to, then our theory may ultimately lead us to ask how relevant grounds could be structured in a self-reinforcing way. It is a natural next step—though one which I maintain is beyond the semantic question—to ask what a *mechanism* for normative ratifiability, as distinct from admissibility, could be.

Chapter 6

Conclusion

I conclude with remarks on the view of disjunction and deontic modality defended in this dissertation, taking the DLC semantics of Chapter 5 and Appendix B as the official view. These include features of and variations on the formal view, features of the wider natural language environment into which DLC semantics must ultimately be incorporated, and outstanding conceptual questions.

6.1 The Empirical Adequacy of DLC

Some Limitations

The focus in this dissertation has been, in one sense, wide—a range of puzzles involving disjunction and deontic modality. This axis of unification is unusual.

Beyond the disjunctive case, free choice effects are widely studied and well known to occur with indefinites. Several languages have lexical alternates of indefinites, equivalent to the English free choice 'any', in sentences which give rise to free choice interpretations:

(1) You may sit in **any** chair.

(2) Puoi prendere **qualunque** dolce.
You may take **any** sweet.
(Chierchia, 2006, pg. 541)

(3) Du kannst dir **irgendeins** von diesen beiden Büchern leihen.
You can you(dat.) **some**-one of those two books borrow.
(Kratzer & Shimoyama, 2002, pg. 25)

(4) **Opjosdhipote** fititis bori na lisi afto to provlima.
Any student can solve this problem.
(Giannakidou, 2001, pg. 25)

There is controversy about whether these "free choice items" (FCI)s are existentials; for example, in the case of the English 'any', there is controversy about whether there are two versions of 'any'—an existential negative polarity item (NPI) and a universal FCI—or one.[1] The unificationist camp subdivides further, according to whether the single 'any' of English is held to be an existential (Partee, 1986; Haspelmath, 1993, 1997; Horn, 2005) or a universal (Reichenbach, 1947; Quine, 1960; Horn, 1972, Ch. 3; Lasnik, 1972). In all these cases, analyses of free choice indefinites can appeal to some special semantic feature of the marked version—for example, of 'any' vs. 'a', in the case of existentialists, and universal FCI 'any' versus NPI 'any' in the case of the universalists—to explain the accompanying felt entailment.

The disjunctive case of free choice permission studied here is, from a distributional point of view, *simpler* than the indefinite case, since there is no lexical alternation to explain: there is little plausibility to the idea that the OR in disjunctive free choice permission statements is lexically distinct from disjunctions appearing appearing elsewhere in the language. From the semantic point of view, it is *harder* than the indefinite case, precisely *because* one cannot assign different semantic values to the words involved in each case; the *super*-unificationist disjunction of DLC must cover the data in downward-entailing and free choice-supporting environments, and also behave plausibly—that is, classically—when unembedded. Indirectly, then, I suggest that DLC semantics supports a strong unificationist view on which (i) free-choice supporting indefinites are existential rather than universal in character, and (ii) the 'a'-'any' alternation in the free choice case does not mark a difference in semantics of the two words, but rather, as in the case of NPIs generally, functions to signal scope.[2] This is the territory occupied by, for example, Partee (1986) and Horn (2005). Just how much support is lent by a successful super-unificationist treatment of free choice disjunction to this camp in the debate on free choice indefinites is one that must be left to further research.

Beyond the case of indefinites, free choice is often studied as a general phenomenon involving disjunction under any flavor of modal, including epistemic modals and generics:

(5) Elephants live in Africa or Asia.

[1] The lexical ambiguity camp for 'any' includes Carlson (1980, 1981); Ladusaw (1979); Morgan (1861, 1862); Dayal (1998), and Horn (1972, Ch. 2).

[2] That is, narrow scope of 'any' with respect to MAY in sentences like (1).

(Nickel, 2010, pg. 480)

(6) It might be raining or snowing.
(Santorio & Romoli, 2015, pg. 9)

This is, to my mind, a strong reason for looking for an implicature-based account of free choice: the phenomenon pertains to disjunction under modality, conceived generally. This would suggest searching for an explanation rooted in something more general than the semantics of deontic modals.[3] Yet my argument made specific appeal to ratifiability, a concept from rational choice theory, combining it with the existential quantification associated with permissibility in deontic modal logic.[4] Does the appeal to ratifiability make the account of free choice defended in this dissertation embarassingly narrow, given examples like (5) and (6)—or can it be extended?

First, if the account cannot be extended to epistemic possibility, that does not, by itself, impugn the explanation in the deontic case. There may be special features of epistemic possibility which lend themselves to a distinct explanation of free choice—whether semantic or pragmatic—in the epistemic case, and likewise for generics.[5]

As to whether an extension *is* possible, I can only sketch a partial answer, as follows. The conceptual defense of my proof of deontic (FC) relied on splitting the notion of deontic possibility at context of decision into two notions: deontic admissibility (possibility come what may) and ratifiability (possibility conditional on the performance of the act). I then argued that deontic ratifiability is sufficient for object language MAY.

To approach the question of whether the same formal proof could be applied while reading the relevant modalities as epistemic, rather than deontic, then, the same two steps should be considered: first, can the admissibility-ratifiability distinction be applied in the epistemic case? And if it can be, is it plausible that epistemic MIGHT tracks the latter?

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[3]On the unity of the phenomena across different flavors of modality, I will also note, for the record, my impression that Ross sentences have less cancelable effects than others.

(i) ?You ought to cancel or postpone the party—in fact, you ought to cancel.

(ii) It must be in the living room or in the kitchen—in fact, it must be in the kitchen.

[4]Mp, in DLC, says that p is permissible just in case it is possibly ratifiable: it is equivalent, at a modal base s, to $\blacklozenge(p \wedge \Diamond p)$ (for one-dimensional p). Argument: by the semantics of M, Mp is settled-true at s iff $\forall w \in s$: $s, w, w \vDash Mp$ iff $\exists v \in s$ such that (i) $s, w, v \vDash p$ and (ii) $\exists v'$ such that vRv' and $s, w, v' \vDash p$. Since atomic semantic values are independent of the y-parameter, (i) simplifies to: $s, v, v \vDash p$, and (ii) simplifies to $\exists v' \in s$ such that wRv' and $s, v'v' \vDash p$.

[5] Following the influential work of Leslie (2008), such an explanation might rely on special features of (talk about) agents' default mode of psychological generalization, rather than on special features of (talk about) e.g. knowledge.

Considering the first question on its own merits, intuitions seem to be torn, in a way that is reflected in the debate on knowledge of future contingents (sometimes called the debate on *middle knowledge*; see DeRose (2010); Craig (1988)). For a proposition p to be epistemically ratifiable, it must be possible for the agent to bring it about; could p nonetheless be ruled out by what the agent knows? If it is possible to have knowledge of future contingents, then the answer is 'yes'; but this is controversial. If the answer is 'no', then epistemic ratifiability seems to be vacuous—a property had by any non-contradictory proposition—and therefore not sufficient for epistemic possibility. I take it that any attempt to interpret our proofs (FC) as applying to the deontic case would need to address this substantive issue.

Returning to the larger question of whether any approach to epistemic possibility is recommended by our system, it is worth noting that to get to an explanation of epistemic free choice effects as in (6), one must not only consider the question of whether epistemic ratifiability is sufficient for epistemic possiblity, but also tell a story about disjunction under epistemic modals. I suggested in Chapter 3 that epistemic modals look at the diagonal semantic values of their arguments, along which, *inter alia*, disjunction is classical; this straightway rules out (FC) in the epistemic case. So there is a positive reason in each direction to treat the two flavors of modality differently: there is reason to treat deontic modality differently from epistemic modality because ratifiability seems more intuitively significant in the deontic case, and there is reason to treat epistemic modality differently from deontic modality, because doing so is a means to preserving a privileged status for the validities of classical logic.

Finally, as emphasized by Schroeder (2011), there is a class of uses of deontic OUGHT—the 'evaluative' OUGHT—that do not seem involve agency, either because there is no agent in the prejacent or because the agent is not capable of guaranteeing that the prejacent comes about.

(7) There ought to be world peace.
OUGHT P.

(8) Larry ought to win the lottery.
OUGHT W.

(In Schroeder's example, (8) is true because the world would be better if Larry won, but he cannot bring about winning.) In Chapter 5, I emphasized that the $\blacklozenge$ modality—the one which contributes the extra premises needed for (R+) and (FC)—should be read as circumstantial modality, which is precisely the modality of what the agent is capable of bringing about. Nonetheless, some such evaluative OUGHT statements seem to give rise to

(R+) readings:

(9) There ought to be a cure for cancer or a way to prevent it.
ought(C or P)[6]

to my ear, (9) gives rise to an (R+) reading in the sense that it carries a felt entailment to the permissibility of each disjunct. But this permissibility cannot be paraphrased in terms of an agentive may:

(10) ?There may be a cure for cancer.
may C

Moreover, given that there is no agent in (9), we cannot analyze the extra premises $\blacklozenge C$ and $\blacklozenge P$ in the (Ross) schema in terms of circumstantial possibility. So it is an open question both whether there is an analogous effect here to be analyzed that can be put in the object language, and how that analysis can be carried out.

Disjunctive Extensions

Another line of ongoing research is to look at the behavior of disjunction in yet more environments—that is, beyond deontic modals and beyond the "donkey conditional" environments examined in Chapter 4.

In addition to solving problems like our Nute counterfactual, the empirical hypothesis that disjunction introduces bind-able variables can be defended on the basis of *wh*-extraction on disjunctions. The particular application is to so-called *sluicing* constructions on disjunctions, such as 'p or q, but I don't know which' (Ross, 1969; Merchant, 2001; Chung et al., 1995). These deleted tense phrase constructions which are unusual in that, rather having an indefinite NP as a correlate, as in

(11) Mary saw a co-worker at the store; I don't know which.

they have a disjunction:

(12) Mary saw John or Bill; I don't know which.

(13) Carlton killed Mr. Boddy or Mr. Boddy committed suicide; I don't know which.

[6] John MacFarlane notes that "it would be acceptable for there to be a cure for cancer" might be a better paraphrase of the LF in (9).

'which' is typically analyzed as a λ-abstractor which binds an open variable position. In Fusco (2015) I pursue the syntactic isomorphy assumption for sluices, meaning that (11) has the following full syntax:

(14) Mary saw a co-worker at the store; I don't know which$_1$ [~~coworker Mary saw t_1~~].

and that the full syntax of (13), at the right level of abstraction, is:

(15) p or q; I don't know which$_1$ [~~p or q~~].

'which [p or q]' therefore contributes a property of propositions to interpretation, just as 'who I saw t' in

(16) That is the man [who$_1$ I saw t_1]

contributes the property of persons to the interpretation of (16).

I argue, adapting a proposal from Adger & Quer (2001) and Groenendijk & Stokhof (1982), that this property is the property of being identical to the proposition that p, if p is true, and being identical to the proposition that q, if q is true. This is witnessing disjunction in yet another form, which can ultimately be traced back to Karttunen (1977)'s analysis of disjunctive questions.

This work targets what may have seemed to many readers like an obvious problem with analyzing the free choice effect as semantic—which is that it appears to be cancellable:

(17) You may have coffee or tea—I don't know which.

This observation is more common in the philosophy literature than in the linguistics literature[7]—I suspect because the linguistics literature is more involved with analyzing lexical FCI and NPI alternation, and the effect is less obviously cancellable with these.

(18) ?You may sit in any chair—I don't know which.

The analysis I propose turns the cancellability data for disjunctive free choice permission cases on its head. For the availability of the sluice simply appears to show that the first sentence in (17) is a wide-scope disjunction, of the form of (13).

(19) You may have coffee or (you may have) tea—I don't know which.

[7] See, for example, the discussion in Simons (2005) the wide-scope analysis of Zimmermann (2000). Rooth & Partee (1982) provide a general recipe for deriving wide-scope disjunction from any coordination of surface constituents of the same type; see Chapter 4, §2 for discussion.

This blocks the argument from cancellability, since free choice effects are only claimed to occur when disjunction takes narrow scope.

Secondly, these constructions cry out for analysis; *what is it that you don't know*, when you assert a sentence like (13)? Clearly, it is not that you don't know the Boolean disjunction $p \vee q$; otherwise any statement like (13) would be a Moorean contradiction of the form

(20) ϕ, but I don't know ϕ.

Answering this question for sluices, I argue, *supports* the nonclassical disjunction I claim is at work in free choice and Ross's puzzle.

6.2 Variations on DLC

In Situ De Re

I have provided only one lexical entry for OR, relying on the difference between diagonal and nondiagonal modal environments to control whether OR displays Boolean (world-shifty) or non-Boolean (world-rigid) behavior. But relying on modal environments to make a similar distinction, the de dicto-de re distinction, has independently recognized shortcomings. Further development of DLC might find support in unification with such systems. The relevant motivating data are 'scope paradoxical' or 'in situ *de re*' readings of natural language sentences, such as

(21) Everyone rich could have been poor.

(a) $\Diamond(\forall x R(x) \rightarrow P(x))$

(b) $\forall x \Diamond(R(x) \rightarrow P(x))$

In addition to the standard two readings (a) and (b), sentences like (21) have a reading on which there is a single world w in which everyone in the actual world y is poor in w. These readings have been influential in the development of both two-dimensional modal logic and in the move to introduce world variables into the syntax of natural language sentences (Fodor, 1970; Percus, 2000). Pursuing the latter route, the LF of the relevant reading of 'everyone rich' in requires an actual world variable w_0 in the syntax. As Percus puts it, w_0 in the object language marks "the world in which the semantics is being practiced" (Percus, 2000, pg. 10).[8]

[8] After this section was written, the suggestion that free choice and Ross sentences contain world-variable in the syntax was also put to me by Adrian Brasoveanu.

(22) Everyone rich$_{w0}$ could have been poor.
$\exists w \forall x(R(x,w_0) \rightarrow P(x,w))$

A variation on the syntax of Ross and free choice sentences allows me to frame the deontic paradoxes as a special case of the in situ de re readings, while also making the system easier to generalize to other types of constructions. Our deontic disjunctions fit the pattern, since OR has narrow scope in them, yet remains anchored to the world-as-actual. To my knowledge, this would be the first application of the in situ de re phenomenon to disjunction; it is a qualified type of lexical alternation view.[9] Such a view might conceive of the syntax of a free choice sentence as follows:

(23) You may have coffee or tea.
$\textsc{may}(C \ \textsc{or}_{w_0}\ T)$

Here, MAY shifts the world of evaluation, but the dependence of $C \ \textsc{or}_{w_0}\ T$ on how matters stand with the world-as-actual is marked by the subscript $_{w_0}$ on OR. Such an analysis may be able to unify two threads of work on free choice phenomena—the lexical alternation view associated with free choice indefinites, and the DLC view. Such an analysis preserves the basic mechanism of the explanation proposed in Chapter 5. An OR which is not subscripted with a variable which tracks the actual world is instead read de dicto, and can play its usual role under operators where OR is resolutely Boolean, such as

(24) Probably p or q.
$\textsc{probably}(p \ \textsc{or}_w\ q)$

$c,P,y,x \vDash \textsc{probably}(p \ \textsc{or}_w\ q)$ iff
$Pr_P(\{w : \exists \alpha \in Alt_w(p,q) \text{ s.t. } c,P,w,w \vDash \alpha\}) > .5$
where P is a probability space determining a probability function Pr_P.
(adapting Yalcin (2007, pg. 1015))

And perhaps for epistemic modals as well:

(25) It must be that p or q.
$\textsc{must}(p \ \textsc{or}_w\ q)$

$i,y,w \vDash \textsc{Must}(p \ \textsc{or}_w\ q)$ iff
$\forall w \in i : \exists \alpha \in Alt_w(p,q)$ such that $i,w,w \vDash \alpha$

[9] For another a more straightforward lexical ambiguity view of OR, see Klinedinst & Rothschild (2012).

where i is a set of epistemically possible worlds.
(adapting MacFarlane & Kolodny (2010, pg. 131))

6.3 Unifying Two Notions of Diagonality

The view built up in Chapter 5 emphasizes global diagonal consequence. In §5.7, I noted two understandings of diagonal points.

- Diagonal-1: $x = y$.
  the world of evaluation is the same as the world-as-actual
- Diagonal-2: $i = i_c$.
  every point in the index is initialized by context

The first notion is internal to the compositional semantics of DLC, representing a point at which the actual world (the y parameter) is initially determined by the same world that determines what is considered-as-ideal (x parameter, giving us access to ideal worlds w such that xRw.)[10] This notion gives us diagonal truth and diagonal consequence, which was a notion of consequence necessary for the preservation of classical logic.

The second notion of diagonality relies on a picture of context as a fully determinate, concrete setting where speech acts take place, and allows one to define a notion of truth at a context which can serve as the aim of assertion. I denied that there is always a unique world of the context, but proposed a weaker notion of context-initialization on which context could initialize s without initializing any world-variables within s.

Given that the two notions of diagonality are distinct, it is an open question how much they can be related, and thus whether the privilege claimed for diagonal-1 consequence in my sense can be maintained. What is the significance of a diagonal-1 point *within* s_c that justifies the role these points play in our postsemantics?

One way to defend this significance is to argue that a diagonal-1 point within s_c represents a *type* of context. By stipulation, it will not be the context of use of the typical free choice or Ross sentence, since it settles agential choices which are undecided at contexts of use. However, a diagonal-1 point $\langle s, w, w \rangle$ could be a context of assessment accessible from a context of use, where the notion of accessibility between contexts can be glossed

[10]In a fuller system, this kind of diagonality might be given a development in terms of the supervenience of the normative on the non-normative: although our formalism allows that two worlds w and w' might agree on an assignment of truth-values to atomic sentences, while disagreeing on which world they are R-related to, this could be restricted. A nondiagonal point, then, would represent a case where supervenience fails to obtain in a suitably naturalistic way.

in terms of the passage of time, as in the branching time framework of MacFarlane (2013, 2008, pg. 226). Changing the terminology slightly, we might say that a pre-choice context (c^-) represents an earlier time, while a post-choice context (c^+) represents a later time (with a smaller range of open possibilities). That is suggested by my description, in Chapter 5, of a given diagonal-1 point as a possible "standpoint of a future self": a context the agent *could* occupy, if she makes a certain choice (or series of choices) from her current context. That will constrain what actuality leaves open for her to a range of certain further choices. Diagonal-1 points $\langle s_{c^+}, w, w\rangle$ represent points which can be initialized by an accessible "endpoint" context c^+, such that s_{c+} is a singleton $\{w\}$. Any diagonal-1 point of evaluation initialized by such an endpoint context must also be diagonal in the diagonal-2 sense. Endpoint contexts represent accessible concrete settings of speech-act consumption that are *fully determinate*, both factually and deontically.

This framing allows a natural connection between the postsemantic view in Chapter 5:

> ϕ is true at a pre-choice context c^- iff, for all $w \in s_{c^-}$: $s, w, w \vDash \phi$

and the inspiration taken from Jeffrey in his discussion of ratifiability (Jeffrey, 1983, pg.16).

> **Jeffrey's Thought**: Facts about choiceworthiness at c^- are determined by facts about choiceworthiness at post-choice contexts c^+ accessible from c^-.

This way of relating the postsemantics to Jeffrey's thought thus countenances post-choice contexts accessible from a pre-choice context, and quantifies universally over them. Agents have reason to care about diagonal points in s_c because, first, each such point represents a context occupy-able by a completely factually and deontically opinionated future self, and secondly, the points of view of these future selves determine how one might follow Jeffrey's maxim to choose for one's (or rather, for *a*) future self.

Assessment Sensitivity?

If diagonal points within s_c are re-construed as contexts of assessment, it is possible to use them in the postsemantics directly, as a relativist like MacFarlane (2013, 2008) would. The resulting semantics could occupy a choice point in the space of possible relativist views which MacFarlane declines to endorse: *content relativism*, the idea that the proposition expressed by a sentence, rather than merely its truth-value, can vary with contexts of assessment (MacFarlane, 2013, pg. 73 ff.).

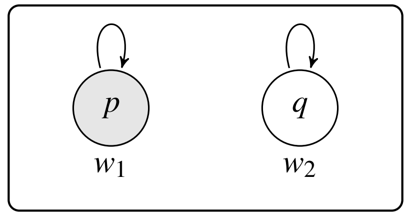


Figure 6.1: $s_{c1} = \{w_1, w_2\}$ and $s_{c2} = \{w_1\}$.

I claimed in Chapter 4 that one may, while preserving classical consequence, associate a disjunction p OR q with a more-determinate-than-classical propositional content—either the proposition expressed by p or the proposition expressed by q—just as a Heimian assignment-world $\langle g, w\rangle$ associates an open sentence $F(x_i)$ with an object-involving truth-condition. But the notion of a proposition did not have a clear role to play in that system. While there are many ways of precisifying the notions of content and proposition, a natural one abstracts over the world of evaluation while keeping the world-as-actual fixed.

Definition 17 (Propositional Content)**.** $|\phi|_{s,y} = \{w' : s, y, w' \vDash \phi\}$

One can rewrite the clause for e.g. OUGHT in a way that highlights the role of propositions:

$$s, y, x \vDash O\phi \text{ iff } \forall w' \text{ such that } xRw', w' \in |\phi|_{s,y}$$

In the example in Figure 6.1, a Ross sentence OUGHT $(p$ OR $q)$, uttered at $c_1 = \{w_1, w_2\}$ is SD-equivalent to OUGHT p in the context of assessment $c_2 = \{w_1\}$, because relative to the point of evaluation $\{w_1\}, w_1, w_1$, p OR q contributes to the interpretation of the modal the same condition on possible worlds that is contributed by p. One might add that the propositional content of ϕ is *determinate* at c iff $|\phi|_{s_c,y} = |\phi|_{s_c,y'}$ for all $y, y' \in s_c$. The content of a disjunction p OR q is determinate at any context of assessment c' where all worlds in s'_c are alike in the truth-values assigned to atomic p and q. One might capture assessment sensitivity as follows:

Definition 18 (Assessment-Sensitive Postsemantics)**.** *ϕ is true as used at c_1 and assessed from c_2 iff, for all worlds w contained in s^*, $w \in |\phi|_{s^*,w}$, where $s^* = s_{c_1} \cap s_{c_2}$.*

DLC is equivalent to a system that supervalues over contexts of assessment, in the following sense: ϕ is true as used at c_1 iff for all c_2 accessible from c_1, $\forall w' \in s_{c2}$, $w' \in |\phi|_{s_{c2},w'}$.

Intuitively: each accessible context of assessment is such that at every world in that context, the propositional content of ϕ relative to that world-context pair is true at that world.

It is worth noting that MacFarlane rejects content relativism by imagining a dispute about the content (rather than the truth) of assessment-sensitive vocabulary. He observes that, when the dispute is between a speaker and a hearer, the speaker is generally taken to be authoritative about content, if not about truth; Supposing a licorice-lover (named *Yum*) says

(26) Licorice is tasty

and then paraphrases by adding

(27) I asserted that licorice is pleasing to my tastes.

MacFarlane asks:

> What proposition shall we say [(27)] expresses, as assessed from [licorice-hater] Yuk's context? Surely it would not be plausible to say that "my tastes," as used by Yum and assessed by Yuk, refers to Yuk's tastes. Yum would simply deny this, and ordinarily this denial would be taken to be authoritative. (74)

Our framework suggests the following reply, in the case of deontic disjunction: free choice and Ross sentences occur in what are essentially many-party communicative situations, consisting not simply of a speaker (occupying context of use) and a hearer (occupying a context of assessment), but a speaker and hearer (at a context of use), and a hearer's future counterparts (at different contexts of assessment).[11] There is no active dispute regarding contents between a hearer and his future counterparts, or between counterparts who occupy different possible worlds.

A further dimension of research, then, is to continue laying out the connections between dynamic semantics and content relativism, considering content relativism as a design feature of certain types of expressions. The proposal finds a natural home in normative language because of the connection to theories which liken the gap between past and future selves to a gap between a rational agent and someone from whom she might receive expert testimony (Jeffrey, 1983; Moss, 2012, 2015; Hedden, 2015) or in whose welfare she might be invested (Briggs, 2010).

[11] I do not claim that this reply has traction in the case of 'tasty' and sentences (26)-(27), since the paraphrasing in that example, and the assessment, is done by agents at a single context.

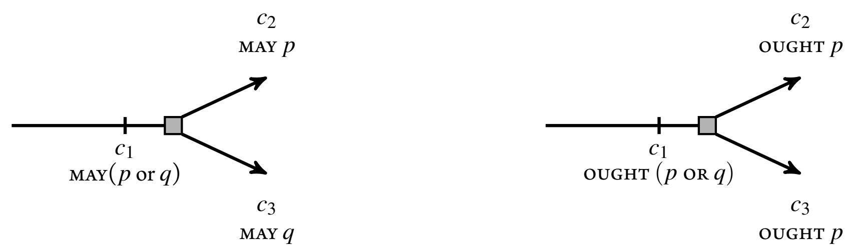


Figure 6.2: Content refinement in time.

6.4 Counterexamples to Ratifiability from EU Theory and Past-directed Attitudes

One objection to DLC semantics, discussed in Chapter 5, raised questions (at the level of preference and deontic ideality, respectively) about the *coherence* of nice cases. A second class of objection instead raises the worry that, while such structures, or analogues thereof, are coherent, the property of ratifiability definable on them is not sufficient for intuitive choiceworthiness—just as I suggested in §1 that the property of epistemic ratifiability is not sufficient for epistemic possibility. Such examples target the attempt to give the DLC semantics of MAY language-independent appeal.

One particular structure at stake, from the decision theory literature, is discussed by Richter (1984), Egan (2007), and Hare & Hedden (2015), amongst others:

> (**Asymmetric Nice Demon**).[12] You can take box A or box B. An infallibly accurate predictor has decided to reward your choice, but to reward your choice *more* if you take box A than if you take box B. You are certain that, if you take box A, there is \$100 in A and \$0 in B; if you take box B, there is \$90 in B and \$0 in A.

This case has the structure of a nice choice: each available act is choiceworthy (in the utility-maximizing sense), given that it is performed. Yet it seems irrational to take box B, thereby winding up with \$90 rather than \$100. Thus in decision-theoretic cases, it seems that ratifiability may not be sufficient for choiceworthiness after all. The natural thought is

[12] After Hare & Hedden (2015, pg. 16)'s Asymmetrically Nasty demon.

	the demon predicted A ($\equiv$ take A)	the demon predicted B ($\equiv$ take B)
take A	\$100	\$0
take B	\$0	\$90

Table 6.1: Payoffs

that once we look at the ratifiable outcomes at a modal base, we may need to re-rank them *relative to each other*.[13]

While a case like (Asymmetric Nice Demon) may show that ratifiability is not generally sufficient for choiceworthiness in EU theory, whether it is grounds for an objection to the deontic MAY of DLC is at best unclear; as noted in Chapters 2 and 3, EU permissibility has features not shared by other flavors of permissibility. The counterexample to decision-theoretic ratifiability gets its structure from the dollar amounts in the payoffs—specifically, the fact that \$100 is more than \$90. Any attempt to construct an asymmetrically ratifiable case for free choice permission or Ross's Puzzle in our model theory would likewise need a quantitative notion of value where such ranking is possible.[14] Yet when I say that you may have the wine, it is not clear that I am locating the wine's value—given that you take it—in some dimension along which it may be outranked. At least, one would like some argument for the existence of such a scale—along the lines, for example, that the gradability of the word 'tall' (to 'taller', 'tallest') encourages a scale-based semantics for the simple adjective 'tall', where otherwise a nongradable, monadic property would apparently do. Hence I think there is reason to be hesitant in using a model with a scale structure like (Asymmetric Nice Demon) as a counterexample to a semantics for MAY.[15]

John MacFarlane (p.c.) suggests the following case, which is closer to the original Nice Choices at the Spa case: suppose that one is offered either aromatherapy, which is pleasant, or a mildly unpleasant pill which causes one to hate the idea of aromatherapy. Con-

[13] If one does this, the resulting view of decision-theoretic choiceworthiness is essentially the "Lexical Ratificationist" view discussed by Egan (2007, pg. 111).

[14] The particular condition suggested by the asymmetric demon case is one where the value of A, given A (viz., \$90), exceeds the value of any other option C, given A, but that this value is still lower than the value of some B, given B (viz., \$100).

[15] While the term *permitted* itself has been argued to be a gradable notion (Portner, 2010; Kratzer, 1981, pg. 12), MAY and other deontic modals cannot be graded in the object language.

ditional on taking the pill, one regards the prospect of aromatherapy as terrible, and so is satisfied with what one did. Yet given the unpleasantness of the pill and the pleasantness of aromatherapy, it is hard to see a sense in which taking the pill is deontically permissible—even if one tries to use a notion of deontic permissibility that is looser than one entailed by a maximization view.

I do not have a strong reply to make in this case, except to emphasize the following. One who holds that deontic ratifiability is sufficient for deontic permissibility need not maintain that one *may* take the pill in this case, or that one may perform the action presented in any other counterexample case. The other possibility left open by the view is that the relevant act (here, taking the pill) is deontically inadmissible: that is, deontically impermissible, come what may. In the case above, my own intuition is that there is sufficient reason to rule out taking the pill even before one acts.

Finally, the ethics literature is also a source for potential counterexamples to sufficiency of ratifiability for permissibility. Harman (2009) discusses the following case, originally from Parfit (1984):

> **Teenage Mother.** A 14-year-old is trying to decide whether to conceive. She knows that if she has a child now, she will love that child, and be glad she conceived at 14. But she also knows that if she waits and has a child in her twenties, when she is better prepared, she will love that (other) child too, and be glad she waited.

Both of the 14-year-old's options are ratifiable from the perspective of her preferences—she will prefer to have done whatever she in fact does, whether that is waiting or not waiting. Nonetheless, Harman argues that a "preference to have a child [now] is not reasonable...since things will be so much worse if she conceives now rather than waiting" (pg. 187). This also fits the profile of a nonquantitative analogue of the Asymmetric Demon case: an act might fit the profile of being (believed) best, on the condition of having been chosen, yet might be "out-competed" by another ratifiable option.

Harman identifies the attractive, but objectionable, principle that is refuted by (Teenage Mother) as one governing reasonable current beliefs about reasonable future preferences:[16]

[16]The thought that one's future self is an *epistemic* expert underlies the discussion of the principle of Belief Reflection (van Fraassen, 1984), the obvious ancestor of the Desire Reflection principle. Within the decision theory literature, a desire reflection-like principle is endorsed by Arntzenius (2008) and rejected by Joyce (2007).

Reflection for Desires—Weaker Version
If a person reasonably believes that in the future she will reasonably prefer that p be true, and she reasonably believes she won't be in a worse epistemic or evaluative position at that time, then *it is reasonable* for her to prefer that p be true. (182)

Wallace (2013), discussing (Teenage Mother) and similar cases, postulates a complementary pair of distinctive backwards-looking attitudes, *regret* and *affirmation*. He argues that they are more principled than desire, in being subject to norms of consistency and resolve relevantly like the norms governing the formation of consistent future-directed intentions (51). Though regret and affirmation have a principled structure—Wallace explicitly compares this structure with the structure of the planning attitudes Gibbard (2003) uses in grounding his discussion of what one *ought* to do—he also claims that they are not sufficient to guide prospective choices. The upshot is that there is no principle linking an agent's attitudes of retrospective affirmation and what Wallace calls "questions of justification" (98-99).

Relevant to a discussion of the relation between deontic modality and these states of mind is expressivism about deontic (and other sorts of) modality, as set forth, for example, in Yalcin (2012a). In that article, the recipe for expressivism about some type of modal expression begins from a certain independently understood and articulated state of mind, and moves to a semantics for modals under 'thinks' or 'believes'-ascriptions; in the case of deontic modality, Yalcin considers the attitudes of wanting (pg. 146) and planning (pg. 147, after Gibbard (2003)). For example, a plan-expressivist about OUGHT begins with an attitude, like Alice's attitude of planning to have coffee or tea, and understands a belief ascription embedding OUGHT, like

(28) Alice believes she ought to have coffee or tea.
$\textsc{Bel}_\alpha\ (\textsc{ought}(C \textsc{ or } T))$

by identifying its truth-conditions with Alice's being in such a planning state. This connects the compound 'believes ought' construction to the structure of a planning state of mind.

From (28), the expressivist moves to the pragmatics and semantics of the unembedded deontic modal in

(29) Alice ought to have coffee or tea.
$\textsc{ought}(C \textsc{ or } T)$

by, in effect, factoring apart contribution of 'believes' and OUGHT in the compound construction. The contribution of OUGHT in unembedded cases ascribes this structure to a suitably enriched notion of the common ground (Stalnaker, 2002) rather than to an agent's doxastic state.

Whether an analogous story for some independently understood attitude can be told for the DLC deontic modals is a question for future research. Plan expressivism is, I venture, not compatible with the semantics of DLC, since *planning* of the straightforward kind is difficult to see in the modal structure of Nice Cases that motivated the semantics of OUGHT and MAY. Once again, it is the structure of ratifiability that is at stake: while one may approve of or prefer doing something just in case one does it, it makes little sense to *plan* to do something just in case one does it. But a story could be told instead that incorporates a retrospective attitude—for example, adapting Wallace, the complex attitude of *knowing one would affirm* p, or, Harman-style, the attitude of *reasonably believing one would be glad if one did* p (from the point of view of a not-inferior epistemic position).[17] These are the candidates for the attitudes ascribed to Alice in e.g.

(30) Alice believes she may choose the aromatherapy or the body-wrap.

The structure of retrospective attitudes, and therefore ultimately the communicative function of sentences like (29) and

(31) Alice may choose the aromatherapy or the body-wrap.
MAY(A OR B)

would therefore be informed by the structure of an independently understood 'backwards-looking' attitude.

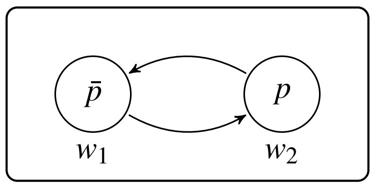
Exploring this option would help to answer the question, raised at the end of Chapter 5, of the grounding of the R-relation appealed to in our modal models. However, such a view would be subject to the apparent *disadvantage* that it is at odds with the intuitions Harman and Wallace (and many others) have in cases like (Teenage Mother).

[17] There is obviously no simple verb like 'plan' to describe such attitudes, but this is not an objection to the application of expressivist methodology to connect such states to deontic modals. For example, Yalcin analyzes the pragmatics of unembedded PROBABLY ϕ-claims by connecting them to the Bayesian state of *b*elieving to a degree greater than 0.5 that ϕ, which state also is not described, in English at least, by a simple verb.

6.5 Some features of the logic of DLC: OUGHT-to-MAY, von Wright-Kanger, and Duality

Finally, I review some deontic features of the logic of DLC. I will use the notation $s \vDash_{SD} \phi$ as shorthand for $\forall w \in s : s, w, w \vDash_{SD} \phi$.

First, is it a conceptual truth that if I ought to ϕ, then I may ϕ? It is standardly taken to be, and in SDL, this is guaranteed by the seriality of the R relation. Despite having serial frames, in our semantics $O\phi \nvDash_{SD} M\phi$ when we consider nontrivially two-dimensional ϕ. For example, the inference fails in the following nasty case:



$s \vDash_{SD} O\neg A$, but $s \nvDash_{SD} M\neg A$
where $s, y, x \vDash A$ iff $x = y$

Figure 6.3: Nasty again.

Our counterexample prejacent, A, is a maximally fragile diagonal truth; it could generally be glossed as *things are exactly the way they actually are*.[18] Recalling that worlds in a circumstantial modal base differ only in respect of some choice the agent makes, we might read A more restrictedly as *the agent does what she actually does*; hence, a rough paraphrase of $O\neg A$ would be that the agent ought to do something she does not actually do. However, that does not mean that doing something she does not actually do is ratifiable, since it is not anywhere both true that she does this, and that doing so is admissible. This does seem to describe a case like (Nasty Choices) in Chapter 5: on the one hand, there is nothing the agent may do; on the other hand, there is something she ought to do—precisely whatever she does not do. For a counterexample prejacent that is available without two dimensional atomic sentences, consider $\phi = \neg(p \text{ OR } \neg p)$. Up above, $s \vDash_{SD} O(\neg(p \text{ OR } \neg p))$ but $s \nvDash_{SD} M(\neg(p \text{ OR } \neg p))$.[19] (It is helpful to read $\neg(p \text{ OR } \neg p)$ as *whichever of p or $\neg p$ the agent does not do.*)

[18] Atomic sentences are defined in the technical appendix so that sentence-letters like A are not possible (since I is a function from the elements of `At` to $\mathscr{P}(W)$, rather than a function from `At` to $W \times W$.)

[19] More generally, any nasty modal base—any case where the only available R relations on a series of worlds in s forms a cycle—will be a case where $s \vDash_{SD} O\neg A$.

But OUGHT entails MAY if we assume that R is both serial and *shift-reflexive* (proof in Appendix A).[20] Conceptually, the shift-reflexivity of the deontic ideality relation encodes the idea that if v is ideal from the point of view of w, it remains so according to the lights of v; worlds that are deontically good from the point of view of some world must approve of themselves.[21] This rules out Nasty Cases. Shift-reflexivity is a constraint on R that many deontic logicians have found independently plausible. In fact, it is the only other frame condition that is added to standard deontic logic: it upgrades SDL to SDL+ (see the discussion in McNamara (2010)). If DLC is right, then, the two independent conditions on standard deontic logic frames are connected in a surprising way: what shift-reflexivity guarantees, given the threat of act dependence, is what seriality can guarantee in its absence. Shift-reflexivity supresses the possibilities for deontic act dependence; assuming it corresponds to making the assumption that if p is permissible, then it remains permissible if you do it. That is one-half of the concept of ratifiability at the local level.

A second candidate for a conceptual truth about morality is that there is always something one may do, morally speaking. The von Wright-Kanger axiom formulates the thought that it is always possible to meet moral demands:

(von Wright-Kanger) $\vDash \Diamond_d \top$

Where $\Diamond_d$ is the standard diamond of deontic logic and $\top$ is an arbitrary tautology in the propositional fragment of the relevant language. In SDL, this too follows from seriality. Giving expression to this thought in DLC, we get:

(von Wright-Kanger 2) $\vDash_{SD} M\top$

While the seriality of R will guarantee that (von Wright-Kanger 2) holds at any modal base, it will not guarantee the following second-order corollary of the original von Wright-Kanger axiom:

(Options) for any p, either $\vDash_{SD} Mp$, or $\vDash_{SD} M\neg p$.

(Options) is false in our nasty case; you may neither do p nor do not-p; although you may do something—$M\top$ is settled-diagonal true—there is nothing more specific you may do.[22] Hence, I suggest, our serial frames do not reflect the intuitive thought that there is always some deontically acceptable *choice* to be made. The addition of shift-reflexivity

[20] R is shift-reflexive in a classic Kripke frame $\langle W, R, I\rangle$ iff $\forall w, v \in W, (wRv \rightarrow vRv)$.

[21] The frame condition in SDL is $O(O\phi \rightarrow \phi)$, or, equivalently, $\Diamond\phi \rightarrow \Diamond\Diamond\phi$.

[22] DLC is therefore a non-normal modal logic Kripke (1963).

validates (Options): by shift-reflexivity, an arbitrary world in s is related to some world v such that vRv. This world is either a p world or a $\neg p$-world.

These two deontic principles—the OUGHT-to-MAY inference and von Wright-Kanger /Options—point to the pathology of nasty-type cases and, perhaps, to the attractiveness of principles that would rule them out. It should be emphasized in this regard that it is *nice* cases which provide the relevant model theory for free choice permission and Ross's puzzle. The use of nasty cases is chiefly dialectical: it is to motivate the claim that the seriality of R—the fact that, from every point of view, something is permissible—is insufficient for there to be a choiceworthy option. In the presence of shift-reflexivity, seriality *is* sufficient for some option ϕ to be such that $M\phi$ is true at a given modal base. In a frame enriched with shift-reflexivity, because $O\phi$ entails $M\phi$, the proof of (FC) follows from the proof of (R+) (see Appendix A), but the explanation for our core entailments is otherwise unchanged.

The last conceptual truth one might consider in the vicinity of deontic logic is the duality of the modals: OUGHT $\neg\phi \equiv \neg$ MAY ϕ. For a counterexample to duality, consider once again a simple Nasty case for the options p and $\neg p$.

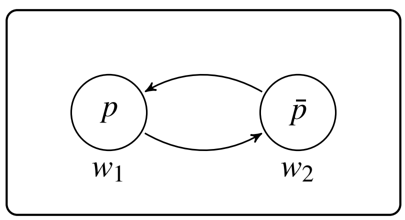


Figure 6.4: Nasty.

$s_{\text{nasty}} \vDash_{SD} \neg M \neg p$, since, if you choose w_2, you will see yourself as having made an unacceptable choice. But it doesn't follow that $s_{\text{nasty}} \vDash_{SD} Op$, since doing p (by going to w_1) would have been just as bad. Unlike OUGHT-to-MAY and (Options), however, duality is not restored by shift reflexivity.

The failure of duality is not unexpected, as it is *admissibility* ($\Diamond$), rather than *permissibility* (M) that was defined to be the dual of OUGHT (O) in Chapter 5, §2. As Appendix B shows, the addition of MAY to the language of DLC strictly increases the expressivity of the basic language (see the "derived modalities" section of Appendix B, which has a richer language than Appendix A.)

Giving up the duality of the natural language modals may seem like a sacrifice (though see Kratzer (1981, §4) and Cariani (2011), who also give up duality.) My best defense is to argue that, from the perspective of ratifiability as our enshrined notion of permissibility,

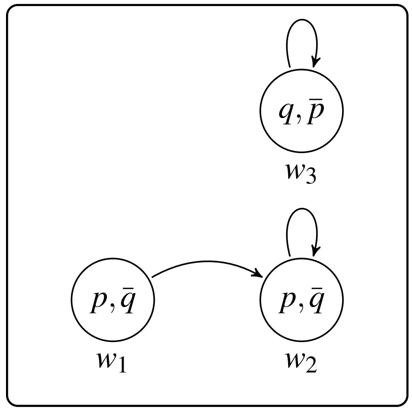


Figure 6.5: Shift Reflexive Failure of Duality: $Mp \nvDash_{SD} \neg O \neg p$

our entry for OUGHT still makes sense. To put it loosely: if we *begin* with the claim that MAY is not a diamond, it still makes sense to say that OUGHT is a box. From the perspective of ratifiability, OUGHT p (for atomic p) is true at s when p's ratifiability is immune to act-dependent revision: if, *no matter which* future standpoint $s' \subseteq s$ you come to occupy, p is ratifiable and $\neg p$ is not, then p is surely obligatory from your present standpoint s. In some decision contexts there may not be any such (nontrivial) p; all propositions may be deontically unstable. But if there is such a p, then you ought to do it.[23]

Since obligation and admissibility are duals, we can go on to consider the dual of MAY in DLC. For one-dimensional ϕ, this is

$$\blacksquare(\phi \vee O\phi)$$

...i.e., necessarily either ϕ is the case or it is obligatory. Insofar as there is a natural concept mapped to this, it is something like: ϕ is *inescapable for the deontically motivated*: if the agent chooses a world where ϕ is not the case, she finds that she was under an obligation to bring it about. Thus the agent is not able to engage in the kind of moral reverse-engineering with

[23]For example, in a language without our special sentence-letter A, there may be no prejacent ϕ for which $O\phi$ is true in Nice and Nasty cases, except the trivial $\top$. So the only statements about obligation true at s_{nasty} and s_{nasty} is $O\top$, which simply means *there is something you ought to do*—without saying what that something is.

respect to ϕ that she can bring about with nice cases: she cannot, with her choices, bring it about that ϕ is both false and not an obligation of hers.[24]

6.6 Reflecting on The Positive View

This dissertation began by examining free choice permission, and ends with a positive view on which choice itself is very much involved in the semantics of modality. The type of choice at issue is not, as a Gricean explanation would have it, a choice about what to say, but a choice about what to do. The methodology of the positive view was to argue for the best concept of agential permissibility available in a framework suitable for modeling cases of act dependence, *before* turning back to the object language. This was to show that our analysis of MAY has independent appeal—albeit not one which is immune from objections and further questions. Having done what is admissible, in the context you occupy once you have done it, has claim to being the primary way deontic permissibility constrains action; it is comparable to the claim that the primary goal of assertion is to express a proposition that is true, in the context you occupy once you begin to speak. This leaves open a number of different things that one might say which are *not* true before one begins to speak. Moreover, it is not the choosing that matters—when *choosing* is taken to mean coming to have a decided view on how to act. Rather, it is the raw act itself, which changes what is actually the case.

In this respect, my account of the two puzzles is in some respects similar to other accounts of Free Choice Permission in the literature (for example, Portner (2010)) which emphasize the performativity of permission-statements. However, there is an important difference, in that these accounts tend to emphasize the agency, qua *authority*, of the speaker, rather than the agency, qua *actor*, of the subject of the modal claim. My account emphasizes both, but the key move is made by focusing on the freedom of the *actor* to make certain future contingent statements true. If witnessing disjunction is the key to understanding Ross's Puzzle and free choice permission, it is because of the practical abilities of an agent facing an open future. Not all performance is speech-act performance; there are more ways to change the world than by talking about it.

[24] Thanks to Alex Kocurek for raising this question.

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Appendix A

LSL and LM (Ch. 3)

Models, Syntax and Semantics for LM

This appendix has two parts. In the first part, which references Chapter 4, §3, we prove that there is a recursive translation function τ taking sets of wffs $\{\phi_1 \ldots \phi_n, \psi\}$ of LSL into a well-formed discourse $\{\tau(\phi_1)\ldots\tau(\phi_2), \tau(\psi)\}$ of *LM*, such that $\phi_1 \ldots \phi_n \vDash_{LSL} \psi$ iff $\tau(\phi_1)\ldots\tau(\phi_2) \vDash_H \tau(\psi)$.

In the second part (Appendix A.1), which references Chapter 4, §4, concerns counterfactuals. We prove that $(x_i(p,q)) \mathrel{\Box\!\!\rightarrow} r$ entails $p \mathrel{\Box\!\!\rightarrow} r$ and $q \mathrel{\Box\!\!\rightarrow} r$.

Syntax of LM

Let `At` be a set of propositional letters $a_1, a_2 \ldots$ and *Var* be a set of variables $x_1, x_2 \ldots$[1] The wffs of LM are: $a \mid {\sim}\phi \mid (\phi \wedge \psi) \mid x_i(\phi, \psi)$. A **discourse** is a set of wffs.

To this, we add the following simplification of Heim's novelty-familiarity condition (**Non-repetition of Subscripts**):

- A discourse $\phi_1 \ldots \phi_n$ satisfies **nonrepetition** iff no variables x_i are repeated in $\phi_1 \ldots \phi_n$.

Models for LM

Models are $\langle W, I \rangle$ pairs, where W is a set of possible worlds. I: `At` $\rightarrow \mathscr{P}(W)$ is an interpretation function. An assignment function is a function g: $Var \times \text{wff} \times \text{wff} \rightarrow \text{wff}$, such that $g(x_i(\phi, \psi)) = \phi$ or $g(x_i(\phi, \psi)) = \psi$.

[1] Thanks to Wes Holliday, Alex Kocurek, and Sridhar Ramesh for help with this appendix.

- Def. An **assignment-world**, or **point**, $\langle gw \rangle$ is a pair of a variable assignment and a possible world.
- Def. A **satisfaction set** s is a set of assignment-worlds.
- Def. A **domain** is a set of numerals d corresponding to variable indices.
- Def. Two assignment functions g and g' are d-**accessible**, $g \backsim_d g'$, iff g and g' disagree at most on values x_i for $i \notin d$.
- Def. A **context set** (for short: **context**) c is a pair $\langle s_c, d_c \rangle$ of a satisfaction set and a domain.
- Def. A context set c satisfies **plenum** iff for any $\langle gw \rangle \in s_c$ and any $g' \sim_{d_c} g$, $\langle gw \rangle \in s_c$.

Semantics for LM

Here are Heim's original entries for atomics, conjunction and negation, to which I add a new entry for disjunction.

- **Atomics.**
  - s_c $[\![a]\!] = \{\langle gw \rangle \in s_c \mid w \in I(a)\}$
  - $d_c[a] = d_c$
- **Negation.**
  - s_c $[\![\sim\phi]\!] = \{\langle gw \rangle \in s_c \mid$ there's no $g' \sim_{d_c} g$ such that $\langle g'w \rangle \in s_c$ $[\![\phi]\!]\}$
  - $d_c[\sim\phi] = d_c$
- **Conjunction.**
  - s_c $[\![\phi \wedge \psi]\!] = (s_c$ $[\![\phi]\!])$ $[\![\psi]\!]$
  - $d_c[\phi \wedge \psi] = (d_c[\phi])[\psi]$
- **Disjunction.**
  - s_c $[\![x_i(\phi, \psi)]\!] = \{\langle gw \rangle \in s_c \mid g(x_i(\phi, \psi)) = \phi$ and $\langle gw \rangle \in s_c$ $[\![\phi]\!]$ or $g(x_i(\phi, \psi))$ $= \psi$ and $\langle gw \rangle \in s_c$ $[\![\psi]\!]$ $\}$
  - $d_c[x_i(\phi, \psi)] = (d_c[\phi])[\psi] \cup \{i\}$

Heimian Truth

Heimian Truth. A discourse $\phi_1 \ldots \phi_n$ is Heim-true at cw iff there is some g such that $\langle gw \rangle \in s_c [\![\phi_1]\!] ... [\![\phi_n]\!]$.

Classical Logic: Models, Syntax and Semantics of LSL

A model is a set of worlds W and a valuation function $I : \mathtt{At} \rightarrow \mathscr{P}(W)$.

Syntax

The wffs of LM are: $a \mid \neg\phi \mid (\phi \wedge \psi) \mid (\phi \vee \psi)$

Semantics

$w \vDash a$ iff $w \in I(a)$
$w \vDash \neg\phi$ iff $w \nvDash \phi$
$w \vDash \phi \wedge \psi$ iff $w \vDash \phi$ and $w \vDash \psi$
$w \vDash \phi \vee \psi$ iff $w \vDash \phi$ or $w \vDash \psi$

The translation function τ, from a set $\phi_1 \ldots \phi_n$ of LSL sentences to a corresponding set of LM sentences

- for a in $Atom$, $\tau(a) = a$[2]
- $\tau(\phi \wedge_{LSL} \psi) = \tau(\phi) \wedge_{LM} \tau(\psi)$
- $\tau(\neg\phi) = \sim(\tau(\phi))$[3]
- $\tau(\phi \vee \psi) = x_i(\tau(\phi), \tau(\psi))$, where all indices appearing in $x_i(\tau(\phi), \tau(\psi))$ are distinct.

After applying this translation function to individual sentences in $\{\phi_1 \ldots \phi_n\}$, we must ensure that the corresponding set of LM sentences is also well-formed: that it observes the

[2] `At` in LSL = `At` in LM
[3] $\neg$ will be an LSL symbol; $\sim$ will be an LM symbol.

nonrepetition of subscripts.

The purpose of this appendix is to prove two theorems:

Theorem 2. For any model $\langle W, I\rangle$, set of wffs $\{\phi_1 \ldots \phi_n\}$ of LSL, context set c satisfying plenum, and world w: $w \vDash \phi_1 \ldots \phi_n$ iff $\tau(\phi_1)\ldots\tau(\phi_n)$ are Heim-true in cw.

Agenda: first we will simplify Heim's compositional semantic entries for the LSL fragment. Then we will simplify Heim's notion of truth.

After proving the theorem above, we can define two notions of consequence, classical consequence ($\vDash_C$) and H-consequence ($\vDash_H$, the preservation of Heimian truth throughout the restricted LM fragment for contexts c satisfying plenum), and show that they are equivalent. So the second theorem is:

Theorem 3. For any set of wffs $\{\phi_1 \ldots \phi_n, \psi\}$ of LSL: $\phi_1 \ldots \phi_n \vDash_C \psi$ iff $\tau(\phi_1)\ldots\tau(\phi_n) \vDash_H \tau(\psi)$.

Simplifying the Compositional Semantic Entries for the LSL Fragment

On a discourse $\phi_1 \ldots \phi_n$ satisfying the nonrepetition of subscripts, we can introduce these simpler entries with $\Vdash$ (local satisfaction):

- **Atomics.**
  - $\langle gw\rangle \Vdash a$ iff $w \in I(a)$.
  - $d_c[a] = d_c$
- **Negation.**
  - $\langle gw\rangle \Vdash {\sim}\phi$ iff there is no g' such that $\langle g'w\rangle \Vdash \phi$
  - $d_c[{\sim}\phi] = d_c$
- **Conjunction.**
  - $\langle gw\rangle \Vdash \phi \wedge \psi$ iff $\langle gw\rangle \Vdash \phi$ and $\langle gw\rangle \Vdash \psi$
  - $d_c[\phi \wedge \psi] = (d_c[\phi])[\psi]$
- **Disjunction.**

- $\langle gw \rangle \Vdash x_i(\phi, \psi)$ iff $g(x_i(\phi, \psi)) = \phi$ and $\langle gw \rangle \Vdash \phi$ or $g(x_i(\phi, \psi)) = \psi$ and $\langle gw \rangle \Vdash \psi$
- $d_c[x_i(\phi, \psi)] = (d_c[\phi])[\psi]) \cup \{i\}$

Lemma 1. $s_c \llbracket \phi \rrbracket = \{\langle gw \rangle \in s_c \mid \langle gw \rangle \Vdash \phi\}$

Proof. By induction. Atomic:

$$s_c \llbracket a \rrbracket = \{\langle gw \rangle \in s_c \mid w \in I(a)\} = \{\langle gw \rangle \in s_c \mid \langle gw \rangle \Vdash a\}$$

Negation:

$$s_c \llbracket \sim\phi \rrbracket = \{\langle gw \rangle \in s_c \mid \neg\exists g' \sim_{d_c} g : \langle g'w \rangle \in s_c \llbracket \phi \rrbracket\}$$

By Nonrepetition of Subscripts in ϕ, this is equivalent to the stronger condition:

$= \{\langle gw \rangle \in s_c \mid \neg\exists g' : \langle g'w \rangle \in s_c \llbracket \phi \rrbracket\}$

$=$ By IH:

$= \{\langle gw \rangle \in s_c \mid \neg\exists g' : \langle g'w \rangle \in s_c$ and $\langle g'w \rangle \Vdash \phi\}$

Conjunction:

$$\begin{aligned} s_c \llbracket \phi \wedge \psi \rrbracket &= (s_c \llbracket \phi \rrbracket) \llbracket \psi \rrbracket \\ &= \text{By IH:} \\ &= \{\langle gw \rangle \in s_c \llbracket \phi \rrbracket \mid gw \Vdash \psi\} \\ &= \text{By IH:} \\ &= \{\langle gw \rangle \in s_c \mid \langle gw \rangle \Vdash \phi \text{ and } gw \Vdash \psi\} \\ &= \{\langle gw \rangle \in s_c \mid \langle gw \rangle \Vdash \phi \wedge \psi\} \end{aligned}$$

Disjunction:

$$\begin{aligned} s_c \llbracket x_i(\phi, \psi) \rrbracket = \{\langle gw \rangle \in s_c \mid\ & g(x_i(\phi, \psi)) = \phi \text{ and } \langle gw \rangle \in s_c \llbracket \phi \rrbracket \\ & \text{or } g(x_i(\phi, \psi)) = \psi \text{ and } \langle gw \rangle \in s_c \llbracket \psi \rrbracket\ \} \\ & = \text{By IH:} \\ & = \{\langle gw \rangle \in s_c \mid g(x_i(\phi, \psi)) = \phi \\ & \text{and } \langle gw \rangle \Vdash \phi \text{ or } g(x_i(\phi, \psi)) = \psi \text{ and } \langle gw \rangle \Vdash \psi\} \end{aligned}$$

□

Simplifying Heimian Truth

Given Plenum in c and Nonrepetition of Subscripts in a discourse, two conditions are equivalent:

- **Heimian Truth.** A discourse $\phi_1 \ldots \phi_n$ is Heim-true at cw iff there is some g such that $\langle gw \rangle \in s_c [\![\phi_1]\!] ... [\![\phi_n]\!]$.

- **Heimian Truth, Simplified.** A discourse $\phi_1 \ldots \phi_n$ is Heim-true at cw iff for each $\phi_i \in \{\phi_1 \ldots \phi_n\}$, $\exists g$ such that $\langle gw \rangle \Vdash \phi_i$.

Lemma 2. *There is some g such that $\langle gw \rangle \in s_c [\![\phi_1]\!] ... [\![\phi_n]\!]$ iff for each $\phi_i \in \{\phi_1 \ldots \phi_n\}$, there is some g such that $\langle gw \rangle \in s_c [\![\phi_i]\!]$*

Proof. ($\Rightarrow$) immediate. ($\Leftarrow$) There is a g_i for each ϕ_i in the discourse such that $\langle g_i w \rangle \in s_c [\![\phi_i]\!]$. By nonrepetition of subscripts in LSL, no distinct ϕ_i, ϕ_j in a discourse will contain the same variables. Hence there is some g which assigns to the variables appearing in each sentence ϕ_i in the discourse the same value as this g_i assigns. Take a g such that for each variable x in each ϕ_i, $g(x) = g_i(x)$, and for each $j \in d_c$, $g(x_j) = g_1(x_j)$. Then by plenum, $\langle g, w \rangle \in s_c [\![\phi_1]\!] ... [\![\phi_n]\!]$. ☐

Lemma 3. *There is some g such that $\langle gw \rangle \in s_c [\![\phi_i]\!]$ for each $\phi_i \in \{\phi_1 \ldots \phi_n\}$ iff for each ϕ_i, $\exists g$ such that $\langle gw \rangle \in s_c$ and $\langle gw \rangle \Vdash \phi_i$.*

Proof. Follows from Lemma 1. ☐

Lemma 4. *$\exists g$ such that $\langle g, w \rangle \in s_c$ and $\langle gw \rangle \Vdash \phi_i$ iff $\exists g$ such that $\langle gw \rangle \Vdash \phi_i$*

Proof. ($\Rightarrow$) immediate. ($\Leftarrow$) Suppose there's a g such that $\langle gw \rangle \Vdash \phi_i$. By nonrepetition, the variables in ϕ_i can't be in d_c. So let $\langle hw \rangle \in s_c$. By plenum, there is a h' such that h' agrees with g on all the variables in ϕ_i while also agreeing on the variables in d_c with h such that $\langle h'w \rangle \in s_c$. But since h' agrees on the variables in ϕ_i with g, $\langle h'w \rangle \Vdash \phi_i$. ☐

Theorem 1. *If a context c satisfies plenum, a well-formed discourse $\phi_1 \ldots \phi_n$ is Heim-true at cw iff for each $\phi_i \in \{\phi_1 \ldots \phi_n\}$, $\exists g$ such that $\langle gw \rangle \Vdash \phi_i$.*

Proof. From Lemmas 2-4. ☐

Equivalence

Lemma 5. *For any model $\langle W, I \rangle$, $w \in W$, and each LM sentence $\tau(\phi_i)$ in a well-formed translation of a set of LSL sentences $\{\phi_1 \ldots \phi_n\}$, the following are equivalent:*

1. $\exists g$ *such that* $\langle g, w \rangle \Vdash \tau(\phi_i)$
2. $w \vDash \phi_i$.

Proof. By induction.

Atomics. $\exists g$ such that $\langle gw \rangle \Vdash \tau(a)$ iff $w \vDash a$. $\tau(a) = a$, so $\exists g$ such that $\langle gw \rangle \Vdash \tau(a)$ iff $w \in I(a)$ iff $w \vDash a$.

Conjunction. $w \vDash \phi \wedge \psi$ iff $\exists g$ such that $\langle gw \rangle \Vdash \tau(\phi \wedge \psi)$.
($\Rightarrow$) $w \vDash \phi \wedge \psi$ iff $w \vDash \phi$ and $w \vDash \psi$. By IH $\exists g$ such that $\langle gw \rangle \Vdash \tau(\phi)$ and $\exists g$ such that $\langle gw \rangle \Vdash \tau(\psi)$. By nonrepetition of subscripts, $\exists g$ such that $\langle gw \rangle \Vdash \tau(\phi)$ and $\langle gw \rangle \vDash \tau(\psi)$. Hence $\exists g$ such that $\langle gw \rangle \Vdash \tau(\phi \wedge \psi)$.
($\Leftarrow$) If $\exists g$ such that $\langle gw \rangle \Vdash \tau(\phi \wedge \psi)$ then $\exists g$ such that $\langle gw \rangle \Vdash \tau(\phi)$ and $\exists g$ such that $\langle gw \rangle \Vdash \tau(\psi)$. By IH, $w \vDash \phi$ and $w \vDash \psi$. Hence $w \vDash \phi \wedge \psi$.

Negation. $w \vDash \neg\phi$ iff $\exists g$ such that $\langle gw \rangle \Vdash \tau(\neg\phi)$.
Equivalent to: $w \vDash \neg\phi$ iff $\exists g$ such that $\langle gw \rangle \Vdash {\sim}\tau(\phi)$.
Equivalent to: $w \vDash \neg\phi$ iff $\exists g$ such that there's no g' such that $g'w \Vdash \tau(\phi)$.
By IH, $w \vDash \phi$ iff $\exists g$ such that $\langle gw \rangle \Vdash \tau(\phi)$.
($\Rightarrow$). If $w \vDash \neg\phi$ then $w \nvDash \phi$. Hence $\neg\exists g$ such that $\langle gw \rangle \Vdash \tau(\phi)$.
($\Leftarrow$). If $\neg\exists g$ such that $\langle gw \rangle \vDash \tau(\phi)$, then $w \nvDash \phi$. Hence $w \vDash \neg\phi$.

Disjunction. $w \vDash \phi \vee \psi$ iff $\exists g$ such that $\langle gw \rangle \vDash x_i(\tau(\phi), \tau(\psi))$.
IH. $w \vDash \phi$ iff $\exists g$: $\langle gw \rangle \Vdash \tau(\phi)$ and $w \vDash \psi$ iff $\exists g$: $\langle gw \rangle \Vdash \tau(\psi)$.
($\Rightarrow$). $w \vDash \phi \vee \psi$ iff $w \vDash \phi$ or $w \vDash \psi$. Suppose that $w \vDash \phi$. Then $\exists g$ such that $g(x_i) = \tau(\phi)$, and $\langle gw \rangle \Vdash \phi$ by IH. The case for ψ is symmetric.
($\Leftarrow$) If $\exists g$ such that $\langle gw \rangle \Vdash x_i(\tau(\phi), \tau(\psi))$, then either $\exists g$ such that $g(x_i) = \tau(\phi)$ and $\langle gw \rangle \Vdash \tau(\phi)$, or $\exists g$ such that $g(x_i) = \tau(\psi)$ and $\langle gw \rangle \Vdash \tau(\psi)$. In the first case, $\exists g$ such that $\langle gw \rangle \Vdash \tau(\phi)$ and so $w \vDash \phi$ by IH, otherwise $\exists g$ such that $\langle gw \rangle \Vdash \tau(\psi)$ and so $w \vDash \psi$ by IH.

From Lemma 5 and Theorem 1, it follows that

□

Theorem 2. *For any model $\langle W, I\rangle$, set of wffs $\{\phi_1 \ldots \phi_n\}$ of LSL, context set c satisfying plenum, and world w: $w \vDash \phi_1 \ldots \phi_n$ iff $\tau(\phi_1) \ldots \tau(\phi_n)$ are Heim-true in cw.*

Consequence Relations $\vDash_H$ and $\vDash_C$

- **Def.** For any set of wffs $\{\phi_1 \ldots \phi_n, \psi\}$ in LM, $\phi_1 \ldots \phi_n \vDash_H \psi$ iff for any model $\langle W, I\rangle$, world $w \in W$, and context set c satisfying plenum, if $\phi_1 \ldots \phi_n$ are Heim-true at cw, then ψ is Heim-true at cw.
- **Def.** For any set of wffs $\{\phi_1 \ldots \phi_n, \psi\}$ in LSL, $\phi_1 \ldots \phi_n \vDash_C \psi$ iff for any model $\langle W, I\rangle$ and world $w \in W$, if $w \vDash \phi_i$ for all $\phi_i \in \{\phi_1 \ldots \phi_n\}$, then $w \vDash \psi$.

It follows from Theorem 2 that

Theorem 3. *For any set of wffs $\{\phi_1 \ldots \phi_n, \psi\}$ of LSL: $\phi_1 \ldots \phi_n \vDash_C \psi$ iff $\tau(\phi_1) \ldots \tau(\phi_n) \vDash_H \tau(\psi)$.*

A.1 The Lewis Counterfactual Conditional + and the Simplification of Disjunctive Antecedents in LM

Syntax

Our syntax is the syntax of LM, with the additional rule that: if ϕ, ψ are wffs, so is $\phi \mathrel{\Box\!\!\rightarrow} \psi$.

Semantics

A **model** is a triple $\langle W, \{\leq_w\}_{w \in W}, I\rangle$, where W and I are as above, and $\leq_w$ is a preorder on W. We assume the Limit Assumption holds.

A **point of evaluation** is a pair $c, \langle g, w\rangle$, where c specifies a list of numerical indices d_c, and $\langle g, w\rangle$ is an assignment-world, as above.

Truth at a Point of Evaluation. We will need only these entries:

Atomics. $c, \langle g, w\rangle \Vdash a$ iff $w \in I(a)$.

Atomic disjunction. $c, \langle g, w\rangle \Vdash x_i(p,q)$ iff either $g(x_i(p,q)) = p$ and $w \in I(p)$, or $g(x_i(p,q)) = q$ and $w \in I(q)$.

Lewisian Counterfactual+ (Definition 7). $c, \langle g, w\rangle \Vdash \phi \mathrel{\Box\!\!\rightarrow} \psi$ iff $\forall \langle g', w'\rangle \in Min_{c,g,w}(\phi)$: $c, \langle g', w'\rangle \Vdash \psi$.
Where $Min_{c,g,w}(\phi)$ is the set of $\langle g', w'\rangle$-pairs such that:
(i) $g' \backsim_{d_c} g$,
(ii) $c, \langle g', w'\rangle \Vdash \phi$, and
(iii) $\neg\exists w''$ s.t. $w'' <_w w'$ and $c, \langle g', w''\rangle \Vdash \phi$.

Conjunction. $c, \langle g, w\rangle \Vdash \phi \wedge \psi$ iff $c, \langle g, w\rangle \Vdash \phi$ and $c, \langle g, w\rangle \Vdash \psi$.

Theorem 4 (Counterfactual SDA). *For any point of evaluation $c, \langle g, w\rangle$: if $i \notin d_c$, then: if $c, \langle g, w\rangle \Vdash x_i(p,q) \mathrel{\Box\!\!\rightarrow} r$, then $c, \langle g, w\rangle \Vdash (p \mathrel{\Box\!\!\rightarrow} r) \wedge (p \mathrel{\Box\!\!\rightarrow} q)$.*

Proof. We assume that $c, \langle g, w\rangle \Vdash x_i(p,q)$ and $i \notin d_c$.
$Min_{c,g,w}(x_i(p,q))$ is the set of $\langle g', w'\rangle$-pairs such that:
(i) $g' \backsim_{d_c} g$,
(ii) $c, \langle g', w'\rangle \Vdash x_i(p,q)$, and
(iii) $\neg\exists w''$ s.t. $w'' <_w w'$ and $c, \langle g', w''\rangle \Vdash r$.

By the semantics for atomic disjunction, there are two kinds of assignment-worlds $\langle g', w'\rangle$ satisfying condition (ii):

$$P\colon \{\langle g', w'\rangle\colon g'(x_i(p,q)) = p \text{ and } w \in I(p)\}$$
$$Q\colon \{\langle g', w'\rangle\colon g'(x_i(p,q)) = q \text{ and } w \in I(p)\}$$

Since $i \notin d_c$, condition (i) does not rule out either type of assignment-world. By (iii), then, $Min_{c,g,w}(x_i(p,q))$ is the union $P' \cup Q'$ of the sets

$$P'\colon \{\langle g', w'\rangle \in P \text{ and } \neg\exists w'' \text{ s.t. } w'' <_w w'.\ \}$$
$$Q'\colon \{\langle g', w'\rangle \in Q \text{ and } \neg\exists w'' \text{ s.t. } w'' <_w w'.\ \}$$

It therefore follows from the counterfactual premise that

$$\forall \langle g', w'\rangle \in P' : c, \langle g', w'\rangle \Vdash r \tag{i}$$

$$\forall \langle g', w'\rangle \in Q' : c, \langle g', w'\rangle \Vdash r \tag{ii}$$

And hence that

$$\forall\langle g', w'\rangle \in P' : w' \in I(r) \tag{i$'$}$$

$$\forall\langle g', w'\rangle \in Q' : w' \in I(r) \tag{i$''$}$$

We now consider truth-at-a-point conditions for counterfactuals with atomic antecedents and consequents. $c, \langle g, w\rangle \Vdash p \mathrel{\Box\!\!\rightarrow} r$ iff $\forall\langle g', w'\rangle \in Min_{c,g,w}(p) : c, \langle g'w'\rangle \Vdash r$. Since p, r are atomic: this condition simplifies to: $\forall\langle g', w'\rangle$ such that $w' \in I(p)$ and $\neg\exists w'' <_w w'$: $c, \langle g', w'\rangle \Vdash r$.

We argue as follows:

1. All assignment-worlds in P' satisfy r at c.

   This is immediate from (i$'$) above.

2. If $\langle g', w'\rangle$ satisfies r at c, then any g-variant of $\langle g'', w'\rangle$ satisfies r at c.

   $c, \langle g', w'\rangle \Vdash r$ iff $w' \in I(r)$ iff $c, \langle g'', w'\rangle \Vdash r$.

3. Every member of $Min_{c,g,w}(p)$ is a g-variant of some member of P'.

   $Min_{c,g,w}(p) = \{\langle g', w'\rangle : w' \in I(p)$ and $\neg\exists w'' <_w w'$ such that $w' \in I(p)\}$. P' is the same set with an extra condition imposed on the g-parameter.

4. Hence, since all P' assignment-worlds satisfy r at c, all members of $Min_{c,g,w}(p)$ satisfy r at c.

   From (1)-(3).

Hence (i) above is sufficient to guarantee that $c, \langle g, w\rangle \Vdash p \mathrel{\Box\!\!\rightarrow} r$. A symmetric argument from (ii) above is sufficient to guarantee that $c, \langle g, w\rangle \Vdash q \mathrel{\Box\!\!\rightarrow} r$. Hence, by the semantics of conjunction, $c, \langle g, w\rangle \Vdash (p \mathrel{\Box\!\!\rightarrow} r) \wedge (q \mathrel{\Box\!\!\rightarrow} r)$.

$\square$

Appendix B

DLC: Deontic Logic with Choice (Ch. 5)

In this appendix we prove three results listed in Chapter 5, §6 (Table 5.3): Free Choice (Theorem 1), Ross (Theorem 2), and Classicality (Lemma 2).

Syntax.

We define two languages, L and L_{nonm} (the nonmodal fragment of L). Let `At` be a set of propositional letters $a_1, a_2...$ The members of `At` are well-formed sentences of L and L_{nonm}. If ϕ, ψ are well-formed sentences of L_{nonm}, so too are $\neg\phi$, $(\phi \wedge \psi)$, and $(\phi \text{ OR } \psi)$. If ϕ, ψ are well-formed sentences of L, so too are $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \text{ OR } \psi)$, $\blacklozenge\phi$, $O\phi$, and $M\phi$.

Semantics.

A **model** M is a triple $\langle W, R, I\rangle$ where W a nonempty set of possible worlds, R is a binary relation on W, and I is a function from the elements of `At` to $\mathscr{P}(W)$ ("the interpretation function").

Given a model M and a set of worlds $s \subseteq W$, we define **the standard intension of** ϕ, $V(\phi)$, on L_{nonm} as follows (where $a \in$ `At`):

$$\begin{aligned} V(a) &= I(a) \\ V(\neg\phi) &= W \setminus V(\phi) \\ V(\phi \wedge \psi) &= V(\phi) \cap V(\psi) \\ V(\phi \text{ OR } \psi) &= V(\phi) \cup V(\psi) \end{aligned}$$

For arbitrary $\phi \in L_{nonm}$ and s, **the standard intension of ϕ in s**, $V_s(\phi)$, is $(s \cap V(\phi))$.
Given some M, a set $s \subseteq W$, a world $w \in s$, and a pair of sentences ϕ_1 and ϕ_2 in L_{nonm}, the **alternative set** in s of w, ϕ_1, and ϕ_2 is

$$Alt_s(w, \phi_1, \phi_2) = \begin{cases} \{\phi_1\} & \text{if } w \in V_s(\phi_1) \text{ and } V_s(\phi_1) \subsetneq V_s(\phi_2). \\ \{\phi_1\} & \text{if } w \in V_s(\phi_1) \setminus V_s(\phi_2). \\ \{\phi_2\} & \text{if } w \in V_s(\phi_2) \text{ and } V_s(\phi_2) \subsetneq V_s(\phi_1). \\ \{\phi_2\} & \text{if } w \in V_s(\phi_2) \setminus V_s(\phi_1). \\ \{\phi_1, \phi_2\} & \text{otherwise.} \end{cases}$$

Our gloss on the first and third case for Alt_s takes its inspiration from an *entailment principle* for truthmakers: if p entails q, then any truthmaker for p is a truthmaker for q (Armstrong, 2004, pg. 10). Here, we hold that in a world w where both ϕ_i and ϕ_j are true, but ϕ_i strictly entails ϕ_j, ϕ_i is a sufficient lone truthmaker for the disjunction at w, since it is sufficient to entail ϕ_j.

A **point of evaluation in M** is a triple $\langle s, x, y \rangle$ such that s is a *serial subset* of W $(\forall w \in s, \exists w' \in s$ such that $wRw')$, and a pair of worlds $y, x \in s$.

Truth at a Point of Evaluation. For any model M and point of evaluation $\langle s, y, x \rangle$ in M, propositional letter a, wffs ϕ, ψ:

$s, y, x \vDash a$	iff	$x \in V_s(a)$
$s, y, x \vDash \neg\phi$	iff	$s, y, x \nvDash \phi$
$s, y, x \vDash (\phi \wedge \psi)$	iff	$s, y, x \vDash \phi$ and $s, y, x \vDash \psi$
$s, y, x \vDash (\phi \text{ OR } \psi)$	iff	$\exists \alpha : \alpha \in Alt_s(y, \phi, \psi)$ and $s, y, x \vDash \alpha$
$s, y, x \vDash \blacklozenge\phi$	iff	$\exists w \in s$: $s, w, w \vDash \phi$
$s, y, x \vDash O\phi$	iff	$\forall x' \in s$: if xRx', then $s, y, x' \vDash \phi$
$s, y, x \vDash M\phi$	iff	$\exists v \in s$ such that (i) $s, y, v \vDash \phi$ and (ii) $\exists v' : vRv'$ and $s, y, v' \vDash \phi$.

Consequence.

There are four notions of consequence available in our system, corresponding to some choice of *local* or *global*, and *diagonal* or *2-dimensional*.
For all sets of sentences Π,

- $\Pi \vDash_1 \psi$ iff for any M, any $s \subseteq W$ such that R is serial in s:
  (if $\forall w \in s : s, w, w \vDash \phi$ for all $\phi \in \Pi$, then $\forall w \in s : s, w, w \vDash \psi$)

	global	local
diagonal	$\vDash_1$	$\vDash_2$
2-dimensional	$\vDash_3$	$\vDash_4$

- $\Pi \vDash_2 \psi$ iff for any M, any $s \subseteq W$ such that R is serial in s: $\forall w \in s$: (if $s, w, w \vDash \phi$ for all $\phi \in \Pi$, then $s, w, w \vDash \psi$)
- $\Pi \vDash_3 \psi$ iff for any M, any $s \subseteq W$ such that R is serial in s: (if $\forall y, x \in s : s, y, x \vDash \phi$ for all $\phi \in \Pi$, then $\forall w \in s : s, y, x \vDash \psi$)
- $\Pi \vDash_4 \psi$ iff for any M, any $s \subseteq W$ such that R is serial in s: $\forall y, x \in s$: (if $s, y, x \vDash \phi$ for all $\phi \in \Pi$, then $s, y, x \vDash \psi$)

We are interested primarily in the preservation of **diagonal settled-truth**, which corresponds to $\vDash_1$: ϕ is settled-true at s iff $\forall w \in s$: $s, w, w \vDash \phi$.

Lemma 1 (Nondisjunctive Stability)**.** For any disjunction-free $\phi \in L_{nonm}$, any $s \subseteq W$, and $x, y, y' \in s$: $s, y, x \vDash \phi$ iff $s, y', x \vDash \phi$ iff $x \in V_s(\phi)$.

Proof. By induction on the complexity of ϕ. We recall that, by the definition of a well-formed point of evaluation, $y, x \in s$. Hence for any s, x: $x \in I(a)$ iff $x \in (s \cap I(a))$ iff $x \in V_s(a)$.
Atomic case. $s, y, x \vDash a$ iff $x \in V_s(a)$ iff $s, y', x \vDash a$.
Conjunction. For the Inductive Hypothesis, assume

$$\begin{array}{lllll} s,y,x \vDash \phi & \text{iff} & s,y',x \vDash \phi & \text{iff} & x \in V_s(\phi) \\ s,y,x \vDash \psi & \text{iff} & s,y',x \vDash \psi & \text{iff} & x \in V_s(\psi) \end{array}$$

Hence

$$\begin{array}{lll} s,y,x \vDash \phi \wedge \psi & \text{iff} & s,y,x \vDash \phi \text{ and } s,y,x \vDash \psi \\ & \text{iff} & s,y',x \vDash \phi \text{ and } s,y',x \vDash \psi \\ & \text{iff} & s,y',x \vDash \phi \wedge \psi \\ & \text{iff} & s \in (V_s(\phi) \cap V_s(\psi)) \\ & \text{iff} & s \in V_s(\phi \wedge \psi) \end{array}$$

Negation. For the Inductive Hypothesis, assume

$$s,y,x \vDash \phi \quad \text{iff} \quad s,y',x \vDash \phi \quad \text{iff} \quad x \in V_s(\phi)$$

Hence

$$\begin{aligned} s,y,x \vDash \neg\phi \quad & \text{iff} \quad s,y,x \nvDash \phi \\ & \text{iff} \quad s,y',x \nvDash \phi \\ & \text{iff} \quad s,y',x \vDash \neg\phi. \\ & \text{iff} \quad x \notin V_s(\phi). \text{ Because } x \in s: \\ & \text{iff} \quad x \in s \setminus V_s(\phi). \\ & \text{iff} \quad x \in V_s(\neg\phi) \end{aligned}$$

□

For the next two Theorems, the following definitions will be useful:

Definition 1. ϕ **diagonally entails** ψ *at* s *iff* $\forall w \in s$, *if* $s,w,w \vDash \phi$, *then* $s,w,w \vDash \psi$.

Definition 2. ϕ, ψ *are* ***diagonally mutually contingent*** *at* s *iff neither diagonally entails the other:* $\exists w,w' \in s$ *such that* $s,w,w \vDash (\phi \wedge \neg\psi)$ *and* $s,w',w' \vDash (\psi \wedge \neg\phi)$.

Theorem 1 (Free Choice)**.** For any disjunction-free $\phi, \psi \in L_{nonm}$: $M(\phi \text{ OR } \psi), \blacklozenge\phi, \blacklozenge\psi \vDash_1 M\phi \wedge M\psi$.

Proof. Suppose $M(\phi \text{ OR } \psi)$, $\blacklozenge\phi$, and $\blacklozenge\psi$ are settled-true at s. Thus, $\exists w \in s$ such that $s,w,w \vDash \phi$ and $\exists w' \in s$ such that $s,w',w' \vDash \psi$. There are two relevant possibilities: either ϕ and ψ are diagonally mutually contingent at s (Case 1), or one diagonally entails the other in s (Case 2).

Case 1. ϕ and ψ are diagonally mutually contingent at s. That is, for some $w_\phi, w_\psi \in s$, we have $s,w_\phi,w_\phi \vDash \phi \wedge \neg\psi$ and $s,w_\psi,w_\psi \vDash \psi \wedge \neg\phi$. In this case, it suffices to show that $M\phi$ is settled-true at s, since the proof that $M\psi$ is settled-true is symmetric.
Since $M(\phi \text{ OR } \psi)$ is settled-true at s, and since $w_\phi \in s$, it follows that $s,w_\phi,w_\phi \vDash M(\phi \text{ OR } \psi)$, i.e., for some $v \in s$:

$$s,w_\phi,v \vDash (\phi \text{ OR } \psi) \tag{i}$$

$$\exists v' \in s : vRv' \text{ and } s,w_\phi,v' \vDash (\phi \text{ OR } \psi) \tag{ii}$$

Because $w_\phi \in V_s(\phi) \setminus V_s(\psi), Alt_s(w_\phi, \phi, \psi) = \{\phi\}$. Hence for any $x \in s$: $s, w_\phi, x \vDash (\phi \text{ OR } \psi)$ just in case $s, w_\phi, x \vDash \phi$. Hence for some $v \in s$:

$$s, w_\phi, v \vDash \phi \tag{i$'$}$$

$$\exists v' \in s : vRv' \text{ and } s, w_\phi, v' \vDash \phi \tag{ii$'$}$$

Thus, by Lemma 1, for arbitrary $w \in s$, $\exists v \in s$ such that

$$s, w, v \vDash \phi \tag{i$''$}$$

$$\exists v' \in s : vRv' \text{ and } s, w, v' \vDash \phi \tag{ii$''$}$$

By the semantic clause for M, it follows that $M\phi$ is settled-true at s. ✓

Case 2. Here, $\blacklozenge\phi$, and $\blacklozenge\psi$ are settled-true at s, and either ϕ diagonally entails ψ at s (that is, $V_s(\phi) \subseteq V_s(\psi)$) or vice-versa; without loss of generality, let it be the case that $V_s(\phi) \subseteq V_s(\psi)$. Because $\blacklozenge\phi$ is settled-true at s, we know $\exists w_\phi \in s : s, w_\phi, w_\phi \vDash \phi$ and hence that $w_\phi \in V_s(\phi)$.

Since $M(\phi \text{ OR } \psi)$ is settled-true at s, and since $w_\phi \in s$, it follows that $s, w_\phi, w_\phi \vDash M(\phi \text{ OR } \psi)$, i.e., for some $v \in s$:

$$s, w_\phi, v \vDash (\phi \text{ OR } \psi) \tag{i}$$

$$\exists v' \in s : vRv' \text{ and } s, w_\phi, v' \vDash (\phi \text{ OR } \psi) \tag{ii}$$

But since, for any w: $s, w_\phi, w \vDash (\phi \text{ OR } \psi)$ iff $\exists \alpha \in Alt_s(w_\phi, \phi, \psi)$ such that $s, w_\phi, w \vDash \alpha$, and since $Alt_s(w_\phi, \phi, \psi) = \{\phi\}$, (i) and (ii) imply:

$$s, w_\phi, v \vDash \phi \tag{i$'$}$$

$$\exists v' \in s : vRv' \text{ and } s, w_\phi, v' \vDash \phi \tag{ii$'$}$$

Since ϕ is nondisjunctive, it follows from Lemma 1 that if $s, w_\phi, v \vDash \phi$ then for arbitrary $w \in s$: $s, w, v \vDash \phi$. Hence for any $w \in s$, $\exists v \in s$ such that

$$s, v, w \vDash \phi \tag{i$''$}$$

$$\exists v' \in s : vRv' \text{ and } s, v', w \vDash \phi \tag{ii$''$}$$

Hence $M\phi$ is settled-true at s. Because ψ is a (local) diagonal consequence of ϕ and ϕ, ψ are non-disjunctive, it follows from Lemma 1 that ψ is a (local) consequence of ϕ even at non-diagonal points (since by Lemma 1, the y-parameter is idle for ϕ, ψ.) Hence from (i$''$), (ii$''$) we may conclude:

$$s, v, w \vDash \psi \tag{i$'''$}$$

$$\exists v' \in s : vRv' \text{ and } s, v', w \vDash \psi \tag{ii$'''$}$$

Hence $M\psi$ is settled-true at s. Hence $M\phi \wedge M\psi$ is settled-true at s. ✓ □

Theorem 2 (Ross)**.** For any disjunction-free $\phi, \psi \in L_{nonm}$: $O(\phi \text{ OR } \psi), \blacklozenge\phi, \blacklozenge\psi \vDash_1 M\phi \wedge M\psi$.

Note: because there are extensions of our system in which $O\phi \nvDash_1 M\phi$, I present the proof of Theorem 2 independently from of the proof of Theorem 1. For discussion, see the Excursus below the proof of Theorem 3.

Proof. Once again, there are two relevant possibilities: either ϕ and ψ are diagonally mutually contingent at s (Case 1), or one diagonally entails the other in s (Case 2).

Case 1. If ϕ and ψ are diagonally mutually contingent at s, then $\exists w_\phi \in s : s, w_\phi, w_\phi \vDash \phi$ and $s, w_\phi, w_\phi \nvDash \psi$. Likewise, $\exists w_\psi \in s : s, w_\psi, w_\psi \vDash \psi$ and $s, w_\psi, w_\psi \nvDash \phi$. In this case, it suffices to show that $M\phi$ is settled-true at s, since the proof that $M\psi$ is settled-true is symmetric.

Since $O(\phi \text{ OR } \psi)$ is settled-true at s, and since $w_\phi \in s$, it follows that $s, w_\phi, w_\phi \vDash O(\phi \text{ OR } \psi)$. Hence $\forall w \in s$, if $w_\phi R w$, then $s, w_\phi, w \vDash (\phi \text{ OR } \psi)$. But since $s, w_\phi, w \vDash (\phi \text{ OR } \psi)$ iff $\exists \alpha : \alpha \in Alt_s(w_\phi, \phi, \psi)$ and $s, w_\phi, w \vDash \alpha$, and since $Alt_s(w_\phi, \phi, \psi) = \{\phi\}$, this implies that $\forall w \in s$, if $w_\phi R w$, then $s, w_\phi, w \vDash \phi$.

$$s, w_\phi, w_\phi \vDash \phi \qquad \text{(i)}$$

Since R is serial in s, $\exists v' \in s : w_\phi R v'$ and

$$s, w_\phi, v' \vDash \phi \qquad \text{(ii)}$$

It follows from Lemma 1 that the y-parameter is idle at v'; hence, for arbitrary $w \in s$, there is some $v \in s$ (viz, w_ϕ) such that

$$s, w, v \vDash \phi \qquad \text{(i}'\text{)}$$

$$\exists v' \in s : vRv' \text{ and } s, w, v' \vDash \phi \qquad \text{(ii}'\text{)}$$

Hence $M\phi$ is settled-true at s. The proof of $M\psi$ is the symmetric, with w_ψ / w_ϕ. Putting both proofs together, $M\phi \wedge M\psi$ is settled-true at s. ✓

Case 2. Here, either ϕ diagonally entails ψ at s (that is, $V_s(\phi) \subseteq V_s(\psi)$) or vice-versa; without loss of generality, let it be the case that $V_s(\phi) \subseteq V_s(\psi)$. Because $\blacklozenge\phi$ is settled-true at s, we know $\exists w_\phi \in s : s, w_\phi, w_\phi \vDash \phi$, and so $w_\phi \in V_s(\phi)$. Note that although $\phi \in V_s(\phi)$

and $\psi \in V_s(\psi), Alt(w, \phi, \psi) = \{\phi\}$.

Our proof of $M\phi$ is the same as above. For the proof of $M\psi$, consider w_ϕ. Because $V_s(\phi) \subseteq V_s(\psi)$ and ϕ, ψ are non-disjunctive, it follows that

$$s, w_\phi, w_\phi \vDash \psi \tag{i}$$

Since $O(\phi \text{ OR } \psi)$ is settled-true at s, it follows that $s, w_\phi, w_\phi \vDash O(\phi \text{ OR } \psi)$. Hence $\forall v'$ such that $w_\phi R v'$: $s, w_\phi, v' \vDash \phi$, and hence (by Lemma 1) that $\forall v'$ such that $w_\phi R v'$: $v' \in V_s(\phi)$. Since $V_s(\phi) \subseteq V_s(\psi)$, it follows that $\forall v'$ such that $w_\phi R v'$, $v' \in V_s(\psi)$. By the seriality of R, $\exists v' \in s$ such that $w_\phi R v'$. Hence $s, w_\phi, v \vDash \psi$. So

$$\exists v' \in s \text{ such that } w_\phi R v' \text{ and } s, w_\phi, v' \vDash \psi. \tag{ii}$$

Hence, $\exists v \in s$ (viz., w_ϕ) such that

$$s, w_\phi, v \vDash \psi \tag{i$'$}$$

$$\exists v' \in s \text{ such that } vRv' \text{ and } s, w_\phi, v' \vDash \psi. \tag{ii$'$}$$

Hence by Lemma 1, for arbitrary $w \in s$, $\exists v \in s$ (viz., w_ϕ) such that

$$s, w, v \vDash \psi \tag{i$''$}$$

$$\exists v' \in s \text{ such that } vRv' \text{ and } s, w, v' \vDash \psi. \tag{ii$''$}$$

Hence $M\psi$ is settled-true at s. Putting both proofs together, $M\phi \wedge M\psi$ is settled-true at s. ✓ □

Theorem 3 (Diagonal Classicality)**.** For any $s \subseteq W, w \in s$, and $\phi \in L_{nonm}$: $s, w, w \vDash \phi$ iff $w \in V_s(\phi)$.

Proof. By induction. The atomic, negation, and conjunction cases are straightforward.

- **Atomic**. $s, w, w \vDash p$ iff $w \in V_s(p)$. ✓
- **Negation**. Assume $s, w, w \vDash \phi$ iff $w \in V_s(\phi)$. Now, $s, w, w \vDash \neg\phi$ iff $s, w, w \nvDash \phi$ iff (since $w \in s$) $w \notin V_s(\phi)$ iff $w \in (s \setminus V_s(\phi))$. ✓
- **Conjunction**. Assume (i) $s, w, w \vDash \phi$ iff $w \in V_s(\phi)$, and (ii) $s, w, w \vDash \psi$ iff $w \in V_s(\psi)$. Now, $s, w, w \vDash (\phi \wedge \psi)$ iff $s, w, w, \vDash \phi$ and $s, w, w \vDash \psi$ iff (Ind Hyp) $w \in V_s(\phi)$ and $w \in V_s(\psi)$ iff $w \in (V_s(\phi) \cap V_s(\psi))$. ✓
- **Disjunction**. We need to show: $s, w, w \vDash (\phi \text{ OR } \psi)$ iff $w \in (V_s(\phi) \cup V_s(\psi))$. Assume for the Inductive Hypothesis that (i) $s, w, w \vDash \phi$ iff $w \in V_s(\phi)$, and (ii) $s, w, w \vDash \psi$ iff $w \in V_s(\psi)$.

  ($\Rightarrow$) If $s, w, w \vDash (\phi \text{ OR } \psi)$, then $w \in (V_s(\phi) \cup V_s(\psi))$.

  If $s, w, w \vDash (\phi \text{ OR } \psi)$, then $\exists\alpha$: $\alpha \in Alt_s(w, \phi, \psi)$ and $s, w, w \vDash \alpha$. For any such s, w, and α: $\alpha \in \{\phi, \psi\}$. Hence if $s, w, w \vDash \alpha$, then $s, w, w \vDash \phi$ or $s, w, w \vDash \psi$. Hence (by Inductive Hypothesis) $w \in V_s(\phi)$ or $w \in V_s(\psi)$. Hence $w \in (V_s(\phi) \cup V_s(\psi))$.

  ($\Leftarrow$) If $w \in (V_s(\phi) \cup V_s(\psi))$, then $s, w, w \vDash (\phi \text{ OR } \psi)$.

  If $w \in (V_s(\phi) \cup V_s(\psi))$, then $w \in V_s(\phi)$ or $w \in V_s(\psi)$. Without loss of generality, assume $w \in V_s(\phi)$. Examining the Alt function, only two cases are relevant: either (Case 1) (i) $V_s(\phi) \supsetneq V_s(\psi)$ holds and (ii) $w \in V_s(\psi)$ holds, or (Case 2) not both (i) and (ii).

  Case 1. If (i) and (ii) both hold, then $Alt_s(w, \phi, \psi) = \{\psi\}$. Hence $s, w, w \vDash (\phi \text{ OR } \psi)$ iff $s, w, w \vDash \psi$. Since $w \in V_s(\psi)$ in this case, the Inductive Hypothesis guarantees that $s, w, w \vDash (\phi \text{ OR } \psi)$.

  Case 2. If (i) and (ii) do not both hold, then $Alt_s(w, \phi, \psi) = \{\phi\}$ or $\{\phi, \psi\}$. In the former case, $s, w, w \vDash (\phi \text{ OR } \psi)$ iff $s, w, w \vDash \phi$, and in the latter case, $s, w, w \vDash (\phi \text{ OR }$

ψ) iff ($s, w, w \vDash \phi$ or $s, w, w \vDash \psi$). Since $w \in V_s(\phi)$, in either case the Inductive Hypothesis guarantees that $s, w, w \vDash (\phi \text{ OR } \psi)$. ✓

□

Lemma 2 (Classicality). For any $\phi \in L_{nonm}$, $\vDash_1 \phi$ iff ϕ is a theorem of classical logic.

Excursus: The Inference from OUGHT to MAY

Definition 3. *Shift (or secondary) reflexivity: R is* **shift-reflexive** *iff* $\forall w, v \in W, (wRv \rightarrow vRv)$.

Theorem 4. *If R is shift-reflexive at the local level,* $O\phi \vDash M\phi$.

Proof. Suppose $s \vDash O\phi$. Then $\forall w \in s$: $s, w, w \vDash O\phi$. Hence (lexicon) $\forall w \in s, \forall w'$ such that wRw': $s, w, w' \vDash \phi$. By seriality, $\forall w \in s, \exists w'$ such that wRw' and $s, w, w' \vDash \phi$. By Shift-Reflexivity, $\forall w \in s, \exists w'$ such that $w'Rw'$ and $s, w, w' \vDash \phi$. For an arbitrary such $w \in s$, consider the corresponding w' such that $w'Rw'$ and $s, w, w' \vDash \phi$. For any such $w \in s, \exists v$ (viz., w') such that:

(i) $s, w, v \vDash \phi$.

(ii) $\exists v'$ (viz., v itself) such that vRv' and $s, w, v' \vDash \phi$.

Hence $\forall w \in s$, there is some such v. Hence $\forall w \in s$: $s, w, w \vDash M\phi$. Hence $O\phi \vDash_1 M\phi$. □

Conceptual gloss: assuming shift reflexivity of R corresponds to making the assumption that if e.g. p is permissible, then it remains permissible if you do it. This is one-half of the concept of ratifiability at the local level. The other half is the converse: if p is permissible *given that you do it* (and doing it is possible), then it is permissible *tout court*.

Appendix C

DLC Without s (Ch. 5)

This appendix presents a slightly different view of the formalism in Chapter 5. There is no s in the index, since nothing in our core language shifts s. (Though see §5.7 for a suggestion in the case of counterfactuals.) Instead, I use a universal accessibility relation S between worlds: wSv just in case v is circumstantially possible from the point of view of w. The more expressive object language has two kinds of modality, one relative to R and one relative to S. Some helpful reductions of our deontic notions are listed along the way. Again, in this appendix we prove three results listed in Chapter 5, §6 (Table 5.3): Free Choice (Theorem 1), Ross (Theorem 2), and Classicality (Lemma 2).

Syntax.

We define two languages, L and L_{nonm} (the nonmodal fragment of L). Let $\texttt{At}$ be a set of propositional letters $a_1, a_2 \ldots$ The members of $\texttt{At}$ are well-formed sentences of L and L_{nonm}. If ϕ, ψ are well-formed sentences of L_{nonm}, so too are $\neg\phi$, $(\phi \wedge \psi)$, and $(\phi \text{ OR } \psi)$. If ϕ, ψ are well-formed sentences of L, so too are $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \text{ OR } \psi)$, $\Box^R_1\phi$, $\Box^S_1$, $\Box^R_2\phi$, $\Box^S_2$ and $@\phi$.

Semantics.

A **model** M is a 4-tuple $\langle W, R, S, \mathscr{V} \rangle$, where W a nonempty set of possible worlds, $R \subseteq W \times W$ is a serial relation on members of W, and $S \subseteq W \times W$ is an equivalence relation on members of W. I assume that for any $x, y \in W$, if xRy, then xSy. (For shorthand, I will use the notation $R[w] := \{v \in W : wRv\}$ and $S[w] := \{v \in W : wSv\}$.) $\mathscr{V} : \texttt{At} \to \mathscr{P}(W)$ is an interpretation function. Given a model M we define **the standard recursive intension** of ϕ, $V(\phi)$, on L_{nonm} as follows:

$$V(a) = \mathscr{V}(a)$$
$$V(\neg\phi) = W \setminus V(\phi)$$
$$V(\phi \wedge \psi) = V(\phi) \cap V(\psi)$$
$$V(\phi \text{ OR } \psi) = V(\phi) \cup V(\psi)$$

For arbitrary ϕ and w, **the standard intension of ϕ in** $S[w]$, $V_{S[w]}(\phi)$, is $(V(\phi) \cap S[w])$. Given some M, a world $w \in s$, and two sentences ϕ_1, ϕ_2 in L_{nonm}, the **alternative set** of w, ϕ_1, and ϕ_2 is

$$Alt(w, \phi_1, \phi_2) = \begin{cases} \{\phi_1\} & \text{if } w \in V_{S[w]}(\phi_1) \text{ and } V_{S[w]}(\phi_1) \subsetneq V_{S[w]}(\phi_2) \\ \{\phi_1\} & \text{if } w \in V_{S[w]}(\phi_1) \setminus V_{S[w]}(\phi_2) \\ \{\phi_2\} & \text{if } w \in V_{S[w]}(\phi_2) \text{ and } V_{S[w]}(\phi_2) \subsetneq V_{S[w]}(\phi_1) \\ \{\phi_2\} & \text{if } w \in V_{S[w]}(\phi_2) \setminus V_{S[w]}(\phi_1) \\ \{\phi_1, \phi_2\} & \text{otherwise} \end{cases}$$

A **point of evaluation in** M is pair of worlds $x, y \in W$ such that xSy. (Note: it follows from this that $S[x] = S[y]$.)

Truth at a Point of Evaluation. For any model M and point of evaluation $\langle x, y\rangle$ in M, propositional letter a, wffs ϕ, ψ:

L_{nonm}:

$x, y \vDash a$	iff	$x \in V_{S[w]}(a)$
$x, y \vDash \neg\phi$	iff	$x, y \nvDash \phi$
$x, y \vDash (\phi \wedge \psi)$	iff	$x, y \vDash \phi$ and $x, y \vDash \psi$
$x, y \vDash (\phi \text{ OR } \psi)$	iff	$\exists \alpha \in Alt(y, \phi, \psi)$: $x, y \vDash \alpha$

Modality 1:

$x, y \vDash \Box_1^S \phi$	iff	$\forall w \in S[x] : w, y \vDash \phi$	$\Diamond_1^S := \neg\Box_1^S\neg\phi$.
$x, y \vDash \Box_1^R \phi$	iff	$\forall w \in R[x] : w, y \vDash \phi$	$\Diamond_1^R := \neg\Box_1^R\neg\phi$.

Modality 2:

$x, y \vDash \Box_2^S \phi$	iff	$\forall w \in S[y] : x, w \vDash \phi$	$\Diamond_2^S := \neg\Box_2^S\neg\phi$.
$x, y \vDash \Box_2^R \phi$	iff	$\forall w \in R[y] : x, w \vDash \phi$	$\Diamond_2^R := \neg\Box_2^R\neg\phi$.

Actuality:

$x, y \vDash @\phi$	iff	$y, y \vDash \phi$

Derived modalities:
$\blacklozenge\phi := \Diamond^S_2 @\phi$
$O\phi := \Box^R_1 \phi$
$M\phi := \Diamond^S_1(\phi \wedge \Diamond^R_1 \phi)$

Derived semantic entries:

$x, y \vDash \blacklozenge\phi$	iff	$\exists v \in S[x] : v, v \vDash \phi$
$x, y \vDash O\phi$	iff	$\forall w \in R[x] : w, y \vDash \phi$
$x, y \vDash M\phi$	iff	$\exists v \in S[x]$: (i) $v, y \vDash \phi$ and (ii) $\exists u \in R[v] : v', y \vDash \phi$.

Consequence.

There are four notions of consequence available in our system, corresponding to some choice of *local* or *global*, and *diagonal* or *2-dimensional*:

	global	local
diagonal	$\vDash_1$	$\vDash_2$
2-dimensional	$\vDash_3$	$\vDash_4$

For all sets of sentences Π,

- $\Pi \vDash_1 \psi$ iff for any M and w:
  (if $\forall v \in S[w] : v, v \vDash \phi$ for all $\phi \in \Pi$, then $\forall v \in S[w] : v, v \vDash \psi$)
- $\Pi \vDash_2 \psi$ iff for any M and w:
  $\forall v \in S[w]$: (if $v, v \vDash \phi$ for all $\phi \in \Pi$, then $v, v \vDash \psi$)
- $\Pi \vDash_3 \psi$ iff for any M and w:
  (if $\forall x, y \in S[w] : x, y \vDash \phi$ for all $\phi \in \Pi$, then $\forall w \in W : x, y \vDash \psi$)
- $\Pi \vDash_4 \psi$ iff for any M and w:
  $\forall x, y \in S[w]$: (if $x, y \vDash \phi$ for all $\phi \in \Pi$, then $x, y \vDash \psi$)

We are interested primarily in the preservation of **acceptance**, which corresponds to $\vDash_1$: ϕ is accepted at M and w iff $\forall v \in S[w]$: $v, v \vDash \phi$.

Lemma 1 (Nondisjunctive Stability). For any disjunction-free $\phi \in L_{nonm}$, and $x, y, y' \in W$ such that ySy': $x, y \vDash \phi$ iff $x, y' \vDash \phi$.

Proof. By induction on the complexity of ϕ.
Atomic case. $x,y \vDash a$ iff $x \in V(a)$ and $y \in S[x]$ iff $x,y' \vDash a$.
Conjunction. For the Inductive Hypothesis, assume

$$\begin{aligned} x,y \vDash \phi &\quad \text{iff} \quad x,y' \vDash \phi \text{ and} \\ x,y \vDash \psi &\quad \text{iff} \quad x,y' \vDash \psi \end{aligned}$$

Hence

$$\begin{aligned} x,y \vDash \phi \wedge \psi &\quad \text{iff} \quad x,y \vDash \phi \text{ and } x,y \vDash \psi \\ &\quad \text{iff} \quad x,y' \vDash \phi \text{ and } x,y' \vDash \psi \\ &\quad \text{iff} \quad x,y' \vDash \phi \wedge \psi \end{aligned}$$

Negation. For the Inductive Hypothesis, assume

$$x,y \vDash \phi \quad \text{iff} \quad x,y' \vDash \phi$$

Hence

$$\begin{aligned} x,y \vDash \neg\phi &\quad \text{iff} \quad x,y \nvDash \phi \\ &\quad \text{iff} \quad x,y' \nvDash \phi \\ &\quad \text{iff} \quad x,y' \vDash \neg\phi. \end{aligned}$$

□

The following definition will be useful:

Definition 1. *ϕ,ψ are* **diagonally mutually contingent at** *w iff $\exists v,v' \in S[w]$ such that $v,v \vDash (\phi \wedge \neg\psi)$ and $v',v' \vDash (\psi \wedge \neg\phi)$.*

Theorem 1 (Free Choice)**.** For any disjunction-free $\phi,\psi \in L_{nonm}$: $M(\phi \text{ OR } \psi), \blacklozenge\phi, \blacklozenge\psi \vDash_1 M\phi \wedge M\psi$.

Proof. $M(\phi \text{ OR } \psi), \blacklozenge\phi, \blacklozenge\psi \vDash_1 M\phi \wedge M\psi$ just in case, for any model M and world w, if $(\forall v \in S[w] : v,v \vDash M(\phi \text{ OR } \psi)$ and $v,v \vDash \blacklozenge\phi$ and $v,v \vDash \blacklozenge\psi)$, then $\forall v \in S[w] : v,v \vDash M\phi$ and $v,v \vDash M\psi$.
There are two relevant possibilities: either ϕ and ψ are diagonally mutually contingent at w (Case 1), or one diagonally entails the other in w (Case 2).

Case 1. ϕ and ψ are diagonally mutually contingent at w. That is, for some $w_\phi, w_\psi \in S[w]$, we have $w_\phi, w_\phi \vDash \phi \wedge \neg\psi$ and $w_\psi, w_\psi \vDash \psi \wedge \neg\phi$. In this case, it suffices to show that $M\phi$ is accepted at $S[w]$: $\forall v \in S[w] : v, v \vDash M\phi$. The proof that $M\psi$ is accepted is symmetric.
By Premise 2, $\forall v \in S[w] : v, v \vDash \blacklozenge\phi$. Hence $\forall v \in S[w], \exists w_\phi \in S[v]$ such that $w_\phi, w_\phi \vDash (\phi \wedge \neg\psi)$. Since $S[w] = S[v]$ and ϕ, ψ are nondisjunctive, it follows from Stability that $\forall v \in S[w], \exists w_\phi \in S[w]$ such that $w_\phi \in V(\phi)$ and $w_\phi \notin V(\psi)$.
By Premise 1, $\forall v \in S[w] : v, v \vDash M(\phi \text{ OR } \psi)$. Hence $w_\phi, w_\phi \vDash M(\phi \text{ OR } \psi)$. Hence $\exists v \in S[w_\phi]$ such that

$$v, w_\phi \vDash (\phi \text{ OR } \psi) \tag{i}$$

$$\exists v' \in R[v] \text{ and } v', w_\phi \vDash (\phi \text{ OR } \psi) \tag{ii}$$

$Alt(w_\phi, \phi, \psi) = \{\phi\}$. Hence $\exists v \in S[w_\phi]$ such that

$$v, w_\phi \vDash \phi \tag{i$'$}$$

$$\exists v' \in R[v] \text{ and } v', w_\phi \vDash \phi \tag{ii$'$}$$

By Nondisjunctive Stability, for arbitrary u: if $u \in V(\phi)$ then for arbitrary z: $u, z \vDash \phi$. Moreover, for arbitrary $z, w_\phi \in S[w], S[z] = S[w_\phi]$. Hence for arbitrary $z \in S[w], \exists v \in S[z]$ such that

$$v, z \vDash \phi \tag{i$''$}$$

$$\exists v' \in R[v] \text{ and } v', z \vDash \phi \tag{ii$''$}$$

Hence ('may'), $M\phi$ is accepted at s. ✓
Case 2. Here, $\blacklozenge\phi$, and $\blacklozenge\psi$ are accepted at $S[w]$, and either ϕ diagonally entails ψ at $S[w]$ (that is, $V_{S[w]}(\phi) \subseteq V_{S[w]}(\psi)$) or vice-versa; without loss of generality, let it be the case that $V_{S[w]}(\phi) \subseteq V_{S[w]}(\psi)$. Because $\blacklozenge\phi$ is accepted at s, we know $\exists w_\phi \in S[w] : w_\phi, w_\phi \vDash \phi$ and hence that $w_\phi \in V_{S[w]}(\phi)$. Since $V_{S[w]}(\phi) \subseteq V_{S[w]}(\psi)$, $Alt(w_\phi, \phi, \psi) = \{\phi\}$.

By Premise 1, $\forall v \in S[w] : v, v \vDash M(\phi \text{ OR } \psi)$. Hence $w_\phi, w_\phi \vDash M(\phi \text{ OR } \psi)$. Hence $\exists v \in S[w_\phi]$ such that

$$v, w_\phi \vDash (\phi \text{ OR } \psi) \tag{i}$$

$$\exists v' \in R[v] \text{ and } v', w_\phi \vDash (\phi \text{ OR } \psi) \tag{ii}$$

Which is equivalent to

$$v, w_\phi \vDash \phi \tag{i$'$}$$

$$\exists v' \in R[v] \text{ and } v', w_\phi \vDash \phi \tag{ii$'$}$$

By Nondisjunctive Stability, for arbitrary u: if $u \in V(\phi)$ then for arbitrary z: $u, z \vDash \phi$. Moreover, for arbitrary $z, w_\phi \in S[w], S[z] = S[w_\phi]$. Hence for arbitrary $z \in S[w], \exists v \in S[z]$ such that

$$v, z \vDash \phi \tag{i$''$}$$

$$\exists v' \in R[v] \text{ and } v', z \vDash \phi \tag{ii$''$}$$

Hence ('may'), $M\phi$ is accepted at s. Furthermore, since ψ is a (local) diagonal consequence of ϕ and ϕ, ψ are non-disjunctive, it follows from Lemma 1 that ψ is a (local) consequence of ϕ even at non-diagonal points. Hence from (i$''$), (ii$''$) we may conclude:

$$v, z \vDash \psi \tag{i$'''$}$$

$$\exists v' \in R[v] : v', z \vDash \psi \tag{ii$'''$}$$

Hence $M\psi$ is accepted at s. Hence $M\phi \wedge M\psi$ is accepted at s. ✓

□

Theorem 2 (Ross)**.** For any disjunction-free $\phi, \psi \in L_{nonm}$: $O(\phi \text{ OR } \psi), \blacklozenge\phi, \blacklozenge\psi \vDash_1 M\phi \wedge M\psi$.

Proof. Once again, there are two relevant possibilities: either ϕ and ψ are diagonally mutually contingent at s (Case 1), or one diagonally entails the other in s (Case 2).

Case 1. If ϕ and ψ are diagonally mutually contingent at w, then $\exists w_\phi \in S[w] : w_\phi, w_\phi \vDash \phi$ and $w_\phi, w_\phi \nvDash \psi$. Likewise, $\exists w_\psi \in S[w] : w_\psi, w_\psi \vDash \psi$ and $w_\psi, w_\psi \nvDash \phi$. In this case, it suffices to show that $\forall v \in S[w] : v, v, \vDash M\phi$, since the proof that $\forall v \in S[w] : v, v, \vDash M\psi$ is symmetric.
By Premise 1, $\forall v \in S[w] : v, v \vDash O(\phi \text{ OR } \psi)$. Hence $\forall v \in S[w] : \forall v' \in R[v] : v', v \vDash (\phi \text{ OR } \psi)$. Hence (instantiating w_ϕ/v), $\forall v' \in R[w_\phi] : v', w_\phi \vDash (\phi \text{ OR } \psi)$. But since $v', w_\phi \vDash (\phi \text{ OR } \psi)$ iff $\exists f : v', w_\phi \vDash f(Alt(w_\phi, \phi, \psi))$, and since $Alt(w_\phi, \phi, \psi) = \{\phi\}$, this implies that $\forall v' \in R[w_\phi] : v', w_\phi \vDash \phi$.
By assumption,

$$w_\phi, w_\phi \vDash \phi. \tag{i}$$

And from the fact that $\forall v' \in R[w_\phi] : v', w_\phi \vDash \phi$ and the seriality of R, we conclude

$$\exists v' \in R[w_\phi] : v', w_\phi \vDash \phi. \tag{ii}$$

It follows from Stability that, for arbitrary $v \in S[w]$, there is some $v' \in S[v]$ (viz., w_ϕ) such that

$$w_\phi, v \vDash \phi. \tag{i$'$}$$

$$\exists v' \in R[w_\phi] : v', v \vDash \phi. \tag{ii$'$}$$

Hence $v, v \vDash M\phi$ for arbitrary $v \in S[w]$. The proof of $M\psi$ is the symmetric, with w_ψ/w_ϕ. Putting both proofs together, $M\phi \wedge M\psi$ is accepted at arbitrary $v \in S[w]$. ✓

Case 2. Here, $\blacklozenge\phi$, and $\blacklozenge\psi$ are accepted at $S[w]$, and either ϕ diagonally entails ψ at $S[w]$ (that is, $V_{S[w]}(\phi) \subseteq V_{S[w]}(\psi)$) or vice-versa; without loss of generality, let it be the case that $V_{S[w]}(\phi) \subseteq V_{S[w]}(\psi)$. By Premise 1, we know $\exists w_\phi \in S[w] : w_\phi, w_\phi \vDash \phi$ and hence that $w_\phi \in V_{S[w]}(\phi)$. Since $V_{S[w]}(\phi) \subseteq V_{S[w]}(\psi)$, $Alt(w_\phi, \phi, \psi) = \{\phi\}$.

By Premise 1, $\forall v \in S[w] : v, v \vDash O(\phi \text{ OR } \psi)$. Hence $\forall v \in S[w] : \forall v' \in R[v] : v', v \vDash (\phi \text{ OR } \psi)$. Hence (instantiating w_ϕ/v), $\forall v' \in R[w_\phi] : v', w_\phi \vDash (\phi \text{ OR } \psi)$. But since

$v', w_\phi \vDash (\phi \text{ OR } \psi)$ iff $\exists f : v', w_\phi \vDash f(Alt(w_\phi, \phi, \psi))$, and since $Alt(w_\phi, \phi, \psi) = \{\phi\}$, this implies that $\forall v' \in R[w_\phi] : v', w_\phi \vDash \phi$.
By assumption,

$$w_\phi, w_\phi \vDash \phi. \tag{i}$$

And from the fact that $\forall v' \in R[w_\phi] : v', w_\phi \vDash \phi$ and the seriality of R, we conclude

$$\exists v' \in R[w_\phi] : v', w_\phi \vDash \phi. \tag{ii}$$

It follows from Stability that, for arbitrary $v \in S[w]$, there is some $w_\phi \in S[v]$ such that

$$w_\phi, v \vDash \phi. \tag{i$'$}$$

$$\exists v' \in R[w_\phi] : v', v \vDash \phi. \tag{ii$'$}$$

Hence $v, v \vDash M\phi$ for arbitrary $v \in S[w]$. Furthermore, since ψ is a (local) diagonal consequence of ϕ and ϕ, ψ are non-disjunctive, it follows from Lemma 1 that ψ is a (local) consequence of ϕ even at non-diagonal points. Hence from (i$'$), (ii$'$) we may conclude that, for arbitrary $v \in S[w]$, there is some $w_\phi \in S[v]$ such that

$$w_\phi, v \vDash \psi. \tag{i$''$}$$

$$\exists v' \in R[w_\phi] : v', v \vDash \psi. \tag{ii$''$}$$

Hence $M\psi$ is accepted at s. Hence $M\phi \wedge M\psi$ is accepted at s. ✓

□

Theorem 3 (Diagonal Classicality)**.** For any model M, world w, and $\phi \in L_{nonm}$: $w, w \vDash \phi$ iff $w \in V(\phi)$.
Since $w \in S[w]$ for any point w, $w \in V(\phi)$ iff $w \in (V(\phi) \cap S[w])$ iff $w \in V_{S[w]}(\phi)$. So we will prove: $w, w \vDash \phi$ iff $w \in V_{S[w]}(\phi)$.

Proof. By induction. The atomic, negation, and conjunction cases are straightforward.

- **Atomic**. $w, w \vDash p$ iff $\in V_{S[w]}(p)$. ✓
- **Negation**. Assume $w, w \vDash \phi$ iff $w \in V_{S[w]}(\phi)$. Now, $w, w \vDash \neg\phi$ iff (since $w \in S[w]$) $w \in S[w]$ and $w \notin V(\phi)$. The first condition always holds, so $w, w \vDash \neg\phi$ iff $w \in (S[w] \setminus V(\phi))$ iff $w \in (S[w] \cap (W \setminus V(\phi))$ iff $w \in V_{S[w]}(\neg\phi)$.
- **Conjunction**. Assume (i) $s, w, w \vDash \phi$ iff $w \in V_{S[w]}(\phi)$, and (ii) $w, w \vDash \psi$ iff $w \in V_{S[w]}(\psi)$. Now, $w, w \vDash (\phi \wedge \psi)$ iff $w, w \vDash \phi$ and $w, w \vDash \psi$ iff (Ind Hyp) $w \in V_{S[w]}(\phi)$ and $w \in V_{S[w]}(\psi)$ iff $w \in (V_{S[w]}(\phi) \cap V_{S[w]}(\psi))$. ✓
- **Disjunction**. We need to show: $s, w, w \vDash (\phi \text{ OR } \psi)$ iff $w \in (V_{S[w]}(\phi) \cup V_{S[w]}(\psi))$. Assume for the Inductive Hypothesis that (i) $s, w, w \vDash \phi$ iff $w \in V_{S[w]}(\phi)$, and (ii) $s, w, w \vDash \psi$ iff $w \in V_{S[w]}(\psi)$.

  $(\Rightarrow)$ If $s, w, w \vDash (\phi \text{ OR } \psi)$, then $w \in (V_{S[w]}(\phi) \cup V_{S[w]}(\psi))$.

  If $w, w \vDash (\phi \text{ OR } \psi)$, then $\exists f$: $w, w \vDash f(Alt(w, \phi, \psi))$. For any w, f: $f(Alt(w, \phi, \psi))$ is nonempty and $f(Alt(w, \phi, \psi)) \subseteq \{\phi, \psi\}$. Hence if $w, w \vDash (\phi \text{ OR } \psi)$, then $w, w \vDash \phi$ or $w, w \vDash \psi$. Hence (by Inductive Hypothesis) $w \in V_{S[w]}(\phi)$ or $w \in V_{S[w]}(\psi)$. Hence $w \in (V_{S[w]}(\phi) \cup V_{S[w]}(\psi))$.

  $(\Leftarrow)$ If $w \in (V_{S[w]}(\phi) \cup V_{S[w]}(\psi))$, then $w, w \vDash (\phi \text{ OR } \psi)$.

  If $w \in (V_{S[w]}(\phi) \cup V_{S[w]}(\psi))$, then $w \in V_{S[w]}(\phi)$ or $w \in V_{S[w]}(\psi)$. Without loss of generality, assume $w \in V_{S[w]}(\phi)$. Hence (Inductive Hypothesis) $w, w \vDash \phi$. Examining the Alt function, only two cases are relevant: either (Case 1) (i) $V_{S[w]}(\phi) \supsetneq V_{S[w]}(\psi)$ and (ii) $w \in V_{S[w]}(\psi)$, or (Case 2) not both (i) and (ii).

  Case 1. If (i) and (ii) both hold, then $Alt(w, \phi, \psi) = \{\psi\}$. Hence $w, w \vDash (\phi \text{ OR } \psi)$ iff $w, w \vDash \psi$. Since $w \in V_{S[w]}(\psi)$ in this case, the Inductive Hypothesis guarantees that $w, w \vDash (\phi \text{ OR } \psi)$.

Case 2. If (i) and (ii) do not both hold, then $Alt(w, \phi, \psi) = \{\phi\}$ or $Alt(w, \phi, \psi) = \{\phi, \psi\}$. Hence either $w, w \vDash (\phi \text{ OR } \psi)$ iff $w, w \vDash \phi$ (in the former case) or $w, w \vDash (\phi \text{ OR } \psi)$ iff $w, w \vDash (\phi \vee \psi)$ (in the latter case). Since $w \in V_{S[w]}(\phi)$, in either case the Inductive Hypothesis guarantees that $w, w \vDash \phi$ and so $w, w \vDash (\phi \text{ OR } \psi)$. ✓

□

Lemma 2 (Classicality)**.** For any $\phi \in L_{nonm}$, $\vDash_1 \phi$ iff ϕ is a theorem of classical logic.

Appendix D

Global and Local (Ch.5)

(R+): $O(p \text{ OR } q), \blacklozenge p, \blacklozenge q \nvDash_2 Mp \wedge Mq$

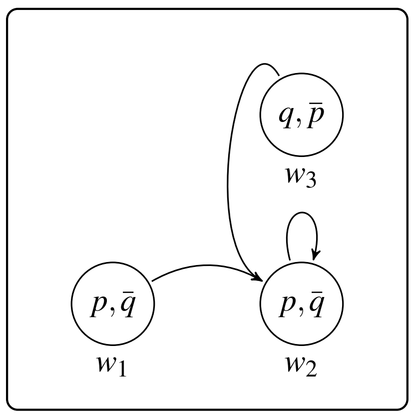

Figure D.1: Local Countermodel to (R+): $\langle s, w_1, w_1 \rangle$

(Conditionals-O): $O(p \text{ OR } q) \not\models_2$ if $\neg p, Oq$
Equivalent by Deduction Theorem to: $O(p \text{ OR } q), \neg p \models Oq$
(Conditionals-M): $M(p \text{ OR } q) \not\models_2$ if $\neg p, Mq$
Equivalent by Deduction Theorem to: $M(p \text{ OR } q), \neg p \models Mq$

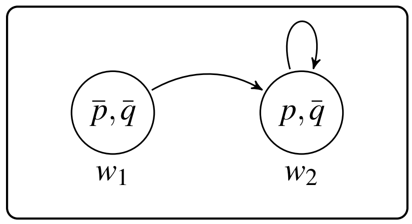

Figure D.2: Local Countermodel: $\langle s, w_1, w_1 \rangle$